INFORMATION AGGREGATION, INVESTMENT, AND MANAGERIAL INCENTIVES

By

Elias Albagli, Christian Hellwig, and Aleh Tsyvinski

August 2011

COWLES FOUNDATION DISCUSSION PAPER NO. 1816

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS
YALE UNIVERSITY
Box 208281
New Haven, Connecticut 06520-8281

http://cowles.econ.yale.edu/

Information Aggregation, Investment, and Managerial Incentives*

Elias Albagli — USC Marshall
Christian Hellwig — Toulouse School of Economics
Aleh Tsyvinski — Yale University

August 10, 2011

Abstract

We study the interplay of share prices and firm decisions when share prices aggregate and convey noisy information about fundamentals to investors and managers. First, we show that the informational feedback between the firm's share price and its investment decisions leads to a systematic premium in the firm's share price relative to expected dividends. Noisy information aggregation leads to excess price volatility, over-valuation of shares in response to good news, and undervaluation in response to bad news. By optimally increasing its exposure to fundamental risks when the market price conveys good news, the firm shifts its dividend risk to the upside, which amplifies the overvaluation and explains the premium. Second, we argue that explicitly linking managerial compensation to share prices gives managers an incentive to manipulate the firm's decisions to their own benefit. The managers take advantage of shareholders by taking excessive investment risks when the market is optimistic, and investing too little when the market is pessimistic. The amplified upside exposure is rewarded by the market through a higher share price, but is inefficient from the perspective of dividend value.

* We thank Bruno Biais, John Campbell, V.V. Chari, Amil Dasgupta, Zhiguo He, Richard Kihlstrom, Peter Kondor, Markus Brunnermeier, Stephen Morris, Alp Simsek, Jeremy Stein, Jean Tirole, and Pietro Veronesi. We also thank seminar participants at Harvard, EIEF, Yale, Wharton, UCSD, UCLA, Toulouse, the NY FED, the Society for Economic Dynamics 2010 meeting, the LSE 2010 capital market dysfunctionality meeting, the XII Torcuato di Tella Workshop in International Economics and Finance, and the Cowles Foundation 2010 general equilibrium meeting for very helpful comments and discussions. Hellwig gratefully acknowledges financial support from the European Research Council (starting grant agreement 263790) . Tsyvinski is grateful to NSF for support and EIEF for hospitality.

1 Introduction

A key role played by asset prices is aggregation of information about the value of firms. By pooling together the dispersed knowledge of individual actors, prices provide information that shapes investor expectations and portfolio decisions. To the extent that the information conveyed in prices is not already known within the firm, prices also affect the firm's assessment of investment projects and guide real allocations. If information aggregation is perfect, asset prices fully reflect current expectations of future dividends and provide a parsimonious way of conveying information and guiding real investment. Moreover, linking managerial compensation to share prices alleviates conflicts of interest and aligns the managers' incentives with the best interest of shareholders.

This paper reconsiders the role of asset prices in aggregating information, guiding real investment, and shaping managerial incentives, when prices aggregate dispersed information imperfectly. We consider a setting in which a firm's shares are traded in a financial market, and the share price emerges as a noisy signal pooling the dispersed information investors have about the firm's value. The firm then takes an investment decision based on the information conveyed by the share price. The investors in turn anticipate the firm's decision in their trading strategies. In equilibrium, the resulting feedback determines the firm's share price, investment decision, and its dividend value.

We show that the informational feedbacks between the share price and the firm's investment decision lead to a systematic premium in the firms' share price relative to its expected dividend value. With noisy information aggregation the market-clearing process introduces a systematic bias, or "information aggregation wedge", through which the market interprets the information contained in the price as being more informative than it truly is. This amplifies the response of share prices to the information that is aggregated through the market, generating over-valuation when the news conveyed by the price are positive, and under-valuation when the news are negative. Moreover, the over-valuation resulting from good news dominates the under-valuation from bad news, whenever the firm's underlying dividend risk is tilted towards the upside.

The firm optimally conditions its investment on the share price by increasing the shareholders' exposure when the market is optimistic and reducing it when the market is pessimistic. This increases the shareholders' exposure to upside risk, and thereby induce a premium in the share price: on average, the share price exceeds expected firm value. This feedback results from the informational value of the price signal and arises even when the firm acts in the best interests of its final shareholders.

Moreover, we argue that explicitly linking managerial compensation to share prices gives managers an incentive to manipulate the firm's decisions to their own benefit. The managers take advantage of shareholders by taking excessive investment risks when the market is optimistic, and investing too little when the market is pessimistic. The amplified upside exposure is rewarded by the market through a higher share price, but is inefficient from the perspective of dividend value creation. The inefficiency can become arbitrarily large when the information friction is important and the market price is noisy, because the information aggregation frictions not only amplify the variation in prices, but also reduce the correlation between prices and fundamentals. The combination of high investment volatility and low correlation of investment with fundamentals implies large efficiency losses.

More specifically, we consider a model in which a firm's manager decide whether to undertake an investment project after observing the firm's share price in a financial market. The market is subject to noisy information aggregation and limits to arbitrage, as in Albagli, Hellwig, and Tsyvinski (2011). We solve the model in a closed form and derive a simple expression for the wedge across states of nature. We show that the dividend function in our model of endogenous investment is convex, resulting in a positive expected wedge. We further decompose the expected wedge into two key components. First, the reaction of market prices to expected fundamentals determines the magnitude of the conditional wedge for a fixed level of investment. Second, the variability of firm's posterior expectation about fundamentals captures the value of market information for firm's investment problem. Our expression for the wedge illustrates the role of two central elements through which the informational feedback generates a share price premium: the dispersed nature of information and the value of market information for firm's decision.

The option value inherent in firm's ex post use of market information convexifies its expected dividends (e.g., Dixit and Pindyck 1994). As the firm takes on more exposure to the fundamentals in good states than in bad states, the information aggregation wedge is asymmetric and larger in absolute value when it is positive. The feedback from information aggregation to firm decisions leads to share prices that are higher than expected dividends from an ex ante perspective. This positive information aggregation wedge emerges even when firms' management acts in the best interest of its shareholders, shareholders are perfectly rational, and the investment decisions of the firm are efficient.

Within our model, we then consider the effects of tying managerial compensation to share prices. Specifically, we assume the manager's objective is to maximize a weighted sum of expected dividends and the price. Because of the information aggregation wedge and its sensitivity to investment and upside risk, decisions that maximize firm's share price generally do not maximize its expected dividend. The manager will cater investment policies to those traders who have the largest impact on market prices, responding more aggressively to the information conveyed through the price. The manager overinvests and increases the exposure when prices are high and the wedge is positive, but under-invests and reduces exposure when prices are low and the wedge is negative. In contrast to the case in which the manager acts in the shareholder's best interest, compensation tied to the price results in higher share prices and lower expected dividends. The extent of this manipulation is increasing in the degree to which compensation is linked to share prices, and is again fully consistent with trader rationality.

Finally, we extend the model in several dimensions. We explore the role of the assumptions on the dividend structure and investment costs. We then consider an environment in which a signal about market-specific information is also observed by the firm. This decreases firm's reliance on market prices, reducing the option value component of market information as well as the temptation to manipulate the wedge when managerial incentives are tied to prices.

Related Literature

Our companion paper (Albagli, Hellwig, and Tsyvinski, 2011) develops a general model of information aggregation in which agents have heterogeneous beliefs and face limits to arbitrage. That paper shares similar features with a large literature on noisy information aggregation in rational expectations models but the formulation of the asset market allows for much more flexible specification of dividends, and thereby offers rich asset pricing implications. There, we discuss the information aggregation wedge in great detail in a model with exogenous dividends, and show how unconditional price and dividend levels are linked more generally to the curvature of the dividend function. The key difference with the present paper is that here the dividend function is endogenous because of the investment decision of the firm. This allows us to focus on the informational feedback from share prices to investment decisions and managerial incentives, and on the potential distortions arising from the combination of the wedge with this feedback, when incentives are tied to share prices.

Our analysis is related to the literature on the negative efficiency implications of tying managerial compensation to share prices. Stein (1989) develops a model where managers' utility depends on both current stock prices and future earnings, which is akin to our way of introducing managerial concerns with short-term market valuations. In his model, shareholders observe reported earnings that are subject to manipulation: managers can inflate them by "borrowing" from future earnings, but at a net cost for the firm. The equilibrium features signal-jamming by managers who boost current reports to raise shareholders valuations and stock prices, leading to an inferior equilibrium outcome given the costly nature of manipulation. In a related paper, Stein (1988) motivates managerial concerns with short-term stock prices arising from takeover bids. Even when managers' and shareholders' incentives are aligned, asymmetric information between these parties can lead to low market valuations of firms due to temporarily low profits, threatening takeover at disadvantageous prices. In this context, costly earnings dressing by managers can be justified to reduce temporary price dips that invite corporate raiders.

Benmelech, Kandel, and Veronesi (2010) revisit optimal managerial compensation in a dynamic REE setting where manager's effort postpones the decline in growth opportunities of a firm. When growth rates slow down, share price compensation incentivizes managers to conceal the true state by over-investment in negative NPV projects. Price-based incentives thus imply a tradeoff: while inducing high effort in early stages, it leads to concealment and suboptimal investment in the later part of the firm's life-cycle. The central difference of our model with Benmelech et al. (2010) and Stein (1988, 1989) is that investment distortions in our setup do not relate to misreporting, but rather arise from the excessive weighing of market information in the signal extraction problem of managers. In other words, managers do not fool shareholders (either on or off the equilibrium path), but instead cater real investment decisions to the opinions of those shareholders which determine prices in equilibrium.

Our model also relates to the literature on REE models with the feedback effect in which real decisions depend on the information contained in the price. Leland (1992) addresses efficiency considerations in a model with insider trading, where information aggregation affects the level of available funding to the firm. Dow and Gorton (1997) study a dynamic model of feedback effects in a setup where prices accurately reflect public information and distortions arise from differences in horizons between managers and shareholders.[1] Dow and Rahi (2003) study risk-sharing and welfare in a setting with endogenous investment in a CARA-Normal setup. Tractability requires imposing restrictions on the information structure, namely that the firm's decision can be directly inferred from the share price. More recently, Goldstein and Guembel (2008) study strategic behavior of traders and the feedback effect, showing how manipulative short-selling strategies that distort firm's investment decision can be profitable. Dow, Goldstein, and Guembel (2010) study feedback effects when information production is endogenous. Since speculators' incentives to produce information increase with the ex-ante likelihood of an investment opportunity, small changes in fundamentals can cause large shifts in investment and firm value. There are two important differences with those

[1] Subrahmanyam and Titman (1999) consider a one-way feedback effect in a model in which investment depends on, but does not affect, share prices.

models. First, in our model of endogenous investment we derive an explicit characterization of the environment in which both the firm and market have private information. Second, we focus more on the resulting asset pricing implications of our model, as well as the link between asset prices, expected dividends and managerial incentives.

Finally, recent related work has focused on the interaction of asset prices and real firm decisions in settings where agents' weighting of information departs from bayesian behavior. Goldstein, Ozdendoren, and Yuan (2010) discuss efficiency considerations and trading commonality (frenzies) arising from the socially suboptimal weighting of private and exogenous public signals about fundamentals. Angeletos, Lorenzoni, and Pavan (2010) model the interaction between early investment choices by entrepreneurs and the later transfer of firm property to traders. An informational advantage that originates from the dispersed nature of entrepreneurial information induces a speculative motive that causes excess non-fundamental volatility in real investment and asset prices. An important difference with these two papers is that we allow all agents to simultaneously condition on equilibrium prices when making trading and real investment decisions. As a result, there are no strategic complementarities that deviate the weighting of public and private sources of information from the optimal signal extraction problem, which is at the heart of the mechanism highlighted by these authors.

Section 2 introduces our general model and derives the wedge between price and expected dividends. Section 3 develops our two main results that the information feedback from prices to investment decisions leads to over-valuation of share prices on average, and that tying managerial incentives to share prices leads to inefficient investment. Second 4 considers various extensions and robustness exercises. Section 5 concludes.

2 REE Model with Endogenous Investment

This section consider a model in which the dividend of a firm depends on a real investment decision by the firm's manager. Claims to the dividend are traded in a financial market that opens prior to the investment choice. The manager's investment decision will condition on the equilibrium price since it aggregates valuable information about the forthcoming dividend. Traders, in turn, anticipate the impact that prices have on investment and on the dividend outcome giving rise to an endogenous two-way feedback between asset prices and real investment. The formulation of the financial market builds on Hellwig, Mukherji and Tsyvinski (2006) and our companion paper Albagli, Hellwig and Tsyvinski (2011).

2.1 Payoffs, Information and Market Structure

We formulate the environment as a Bayesian game between a firm (manager), a unit measure of risk-neutral, privately informed traders, and a 'Walrasian auctioneer'. Each informed trader is initially endowed with one share of a firm. The firm's dividend, disbursed at the final stage of the game, takes the form $\pi : \Theta \times Y \times A \to \mathbb{R}$, where $\theta \in \Theta = \mathbb{R}$ denotes firm's unobserved fundamental, $y \in Y \subseteq \mathbb{R}$ denotes firm's manager private information, and $A \subseteq \mathbb{R}$ denotes a compact set from which the firm chooses an action $a \in A$ after observing y and its share price. The function $\pi(\cdot)$ is strictly increasing in θ.

Actions occur throughout four stages: $t = 0, 1$, 2 and 3. At the first stage, $t = 0$, nature draws the stochastic fundamental θ, which is normally distributed according to $\theta \sim \mathcal{N}(\mu, \lambda^{-1})$. Each informed trader i then observes a noisy private signal x_i about firm's fundamental. This signal is normally distributed according to $x_i \sim \mathcal{N}(\theta, \beta^{-1})$, and is i.i.d. across traders (conditional on θ). In addition, nature draws y, which is distributed according to $G : Y \times \Theta \to [0, 1]$, where $G(\cdot|\theta)$ denotes the the cdf of y, conditional on θ, and $g(\cdot|\theta)$ the corresponding pdf.

At stage 1, the firm's manager commits to a decision rule $a : Y \times \mathbb{R} \to A$ which is conditioned on the manager's private information $y \in Y$, and the share price $P \in \mathbb{R}$. The manager's payoffs are given by a linear combination of the realized dividends and the share price, $\alpha P + (1 - \alpha)\pi$, with weight $\alpha \in [0, 1]$ attached to the price.

At stage $t = 2$, traders participate in an asset market and decide whether to hold or sell their share at the market price, P. Specifically, trader i submits a a price-contingent supply schedule $s_i(\cdot) : \mathbb{R} \to [0, 1]$, to maximize her expected wealth $w_i = (1 - s_i) \cdot \pi(\cdot) + s_i \cdot P$. By restricting supply to $[0, 1]$, we assume that traders can sell at most their endowment, and cannot buy a positive amount of shares. Individual trading strategies are then a mapping $s : \mathbb{R}^2 \to [0, 1]$ from signal-price pairs (x_i, P) into the unit interval. Aggregating traders' decisions leads to the aggregate supply function $S : \mathbb{R}^2 \to [0, 1]$,

$$S(\theta, P) = \int s(x, P) d\Phi(\sqrt{\beta}(x - \theta)), \tag{1}$$

where $\Phi(\cdot)$ denotes a cumulative standard normal distribution, and $\Phi(\sqrt{\beta}(x - \theta))$ represents the cross-sectional distribution of private signals x_i conditional on the realization of θ.[2]

Nature then draws a random demand of shares by "noise traders". We assume the demand shock has the form $D(u) = \Phi(u)$, where u is normally distributed with mean zero and variance

[2]We assume that the Law of Large Numbers applies to the continuum of traders, so that conditional on θ the cross-sectional distribution of signal realizations ex post is the same as the ex ante distribution of traders' signals.

δ^{-1}, $u \sim \mathcal{N}(0, \delta^{-1})$, independently of θ. This specification is from Hellwig, Mukherji, and Tsyvinski (2006), and allows us to preserve the normality of posterior beliefs and retains the tractability of Bayesian updating.

Once informed traders have submitted their orders and the exogenous demand for shares is realized, the auctioneer selects a price P that clears the market. Formally, the market-clearing price function $P : \mathbb{R}^2 \to \mathbb{R}$ selects, for all realizations (θ, u) a price P from the correspondence $\hat{P}(\theta, u) = \{P \in \mathbb{R} : S(\theta, P) = D(u, P)\}$.

After observing the price, the manager learns her private signal y and conditions on both P and y to choose her action according to her previously committed investment rule $a : Y \times \mathbb{R} \to A$.

Finally, the firm's dividend $\pi(\cdot)$ is realized at stage $t = 3$, and the proceeds are distributed to the final holders of the firm's shares.

Let $H(\cdot|x, P)$ denote the traders' posterior cdf of θ, conditional on observing a private signal x, and a market-clearing price P. A *Perfect Bayesian Equilibrium* (PBE) consists of a shareholder's supply function $s(x, P)$, a price function $P(\theta, u)$, a decision rule $a(y, P)$ for the manager, and posterior beliefs $H(\cdot|x, P)$, such that (i) the supply function is optimal given the shareholder's beliefs $H(\cdot|x, P)$ and the anticipated investment rule $a(y, P)$; (ii) $P(\theta, u)$ clears the market for all (θ, u); (iii) $a(y, P)$ solves manager's decision problem; and (iv) $H(\cdot|x, P)$ satisfies Bayes' Rule whenever applicable.

Discussion: This general formulation embeds several special cases that highlight the role of specific assumptions in our set-up.

1. The formulation of the asset market is as in our companion paper. Its key feature is that the combination of heterogeneous investor beliefs and limits to arbitrage will induce a systematic departure between the market price, and the conditional expectations of dividends. This wedge will play a role for the informational feedbacks emphasized in this paper, as well as for the conflicts of interests that arise when prices differ from expected dividends.

2. If $\alpha = 0$, then the manager's and final shareholder's objective coincide. The manager maximizes expected dividends. Under this benchmark, manager's decisions will make ex post efficient use of the information conveyed by prices. When instead $\alpha > 0$, the manager places some weight on the price in forming the optimal investment choice. In this case, the frictions in the asset market resulting from heterogeneous investor beliefs, limits to arbitrage and noisy information aggregation will induce a conflict of interest between the manager and the final shareholders.[3] This formulation also allows for compensation contracts explicitly tied to observable market prices.

[3] The current formulation of the manager's objective function $\alpha P + (1 - \alpha)\, \pi(\theta, y, a)$ is chosen for its simplicity

3. The pre-commitment assumption plays a role for our results in the case of price-based incentives ($\alpha > 0$), and when prices differ from expected dividend values. An ex post choice of the investment decision takes the price P as given, whereas the prior commitment to a rule allows the manager to internalize the effect of its investment decision on market prices. We think of this precommitment assumption as follows: the firm's decision making is based on internal reporting, compensation rules and decision making procedures that are updated less frequently than the decisions themselves. In a dynamic environment, the design of such procedures then internalizes the impact of such decisions on future market prices.

4. Our model is flexible and general enough to incorporate rich differences and asymmetries in information between the firm and the market. For example, if $\pi(\theta, y, a) = \pi(\theta, y', a)$, for all y, y', and all (θ, a), then y is a noisy signal of the underlying fundamental θ which has no direct payoff implications for the firm. If on the other hand $G(y|\theta) = G(y)$, for all θ, then the firm has no additional private information about θ. However, the information contained in y is relevant to firm's decision problem. The manager's private information can thus be flexibly interpreted as either directly payoff relevant, or additional inside information on θ, depending on the circumstances.

2.2 Equilibrium Characterization in the Financial Market

We characterize the equilibrium in two parts. This subsection describes trading strategies and characterizes the equilibrium in the financial market for an arbitrary investment rule by the manager. In the next subsection, we pose the manager's decision problem, and discuss features of the investment decision.

Suppose that firm's manager's decision is characterized by an arbitrary decision rule $a(y, P)$. Define $\hat{\pi}(\theta; P) = \int \pi(\theta, y, a(y; P))\, dG(y|\theta)$ as the expected dividends of the share, conditional on the realization of θ and the price P. The only difference between this dividend function and the one used in Albagli, Hellwig and Tsyvinski (2011) is the dependence of $\hat{\pi}(\theta; P)$ on P through the impact of P on the manager's decisions. Otherwise the characterization is the same.

Trader's risk-neutrality implies that share supply decisions are equal to either 0 or 1 almost everywhere – an order to hold ($s_i = 0$) or sell the share ($s_i = 1$) at P. The trader's expected value of holding the share at price P, and conditional on private signal x is $\int \hat{\pi}(\theta; P)\, dH(\theta|x, P)$. The

and convenience. The framework and the conceptual insights generalize to arbitrary specifications of the manager's utility, and can therefore also be used to incorporate other agency frictions between managers and shareholders.

Here, we take these payoff structures as given. The question of how to optimally structure incentives in the presence of market frictions is an important question for future work, but beyond the scope of the current paper.

private signals x are log-concave, and the posterior beliefs $H(\cdot|x, P)$ are first-order stochastically increasing in x, for any P that is observed in equilibrium. Since $\pi(\cdot)$ is increasing in θ, this implies that $\int \hat{\pi}(\theta; P)\, dH(\theta|x, P)$ is monotone in x, and the traders' decisions are characterized by a signal threshold function $\hat{x} : \mathbb{R} \to \mathbb{R} \cup \{\pm\infty\}$,[4] such that

$$s(x_i, P) = \begin{cases} 1 & \text{if } x_i < \hat{x}(P), \\ 0 & \text{if } x_i > \hat{x}(P), \\ \in [0,1] & \text{if } x_i = \hat{x}(P), \end{cases} \tag{2}$$

so a trader sells if $x_i < \hat{x}(P)$ and holds if $x_i > \hat{x}(P)$. We call the informed trader who observes the signal equal to the threshold, $x = \hat{x}(P)$, and who is therefore indifferent, the *marginal trader*. Aggregating the individual supply decisions, the market supply is $S(\theta, P) = \int_{-\infty}^{\hat{x}(P)} 1 \cdot d\Phi(\sqrt{\beta}\,(x - \theta)) = \Phi(\sqrt{\beta}\,(\hat{x}(P) - \theta))$. Since $D(u) \in (0, 1)$, in equilibrium, $\hat{x}(\cdot)$ must be finite for all P on the equilibrium path. Equating demand and supply, we characterize the correspondence of market-clearing prices:

$$\hat{P}(\theta, u) = \left\{ P \in \mathbb{R} : \hat{x}(P) = \theta + \frac{1}{\sqrt{\beta}} u \right\}. \tag{3}$$

From now on, we focus on equilibria in which the price is conditioned on (θ, u) through the state variable $z \equiv \theta + 1/\sqrt{\beta} \cdot u$. The equilibrium beliefs are characterized in the next lemma. All proofs are in the appendix.

Lemma 1 (Information Aggregation) *(i) In any equilibrium with conditioning on z, the equilibrium price function $P(z)$ is invertible. (ii) Equilibrium beliefs for price realizations observed along the equilibrium path are given by*

$$H(\theta|x, P) = \Phi\left(\sqrt{\lambda + \beta + \beta\delta} \left(\theta - \frac{\lambda\mu + \beta x + \beta\delta\hat{x}(P)}{\lambda + \beta + \beta\delta} \right) \right). \tag{4}$$

Part (i) of Lemma 1 shows that in any equilibrium, the price function must be invertible with respect to z, implying that the observation of P is equivalent to observing z. If the price function is not invertible, then some price realization P would be consistent with multiple realizations of z. But (3) implies that P then cannot be consistent with market clearing in all these states simultaneously.

Part (ii) of the lemma exploits the invertibility to arrive at a complete characterization of posterior beliefs $H(\cdot|x, P)$. With invertibility, we can summarize information conveyed by the price through z, and note that conditional on θ, z is normally distributed with mean θ and variance

[4] By extending the real line to $\pm\infty$, we allow for pure strategies of always holding or always selling regardless of the private signal. This however is inconsistent with market-clearing and thus will not arise on the equilibrium path.

$(\beta\delta)^{-1}$. Thus, the price is isomorphic to a normally distributed signal of θ, with a precision that is increasing in the precision of private signals, and decreasing in the variance of demand shocks.

Using this characterization of posterior beliefs, we characterize the optimality condition for the marginal trader. Because $\int \hat{\pi}(\theta; P)\, dH(\theta|x, P)$ is monotone in x, the conjectured threshold strategy is optimal if and only if

$$P = \int \hat{\pi}(\theta; P)\, dH(\theta|\hat{x}(P), P) = \int \hat{\pi}(\theta; P)\, d\Phi\left(\sqrt{\lambda+\beta+\beta\delta}\left(\theta - \frac{\lambda\mu + \beta(1+\delta)\hat{x}(P)}{\lambda+\beta+\beta\delta}\right)\right).$$

Using the market-clearing condition and lemma 1, we rewrite this indifference condition to find the price in terms of the market signal z:

$$P(z) = \mathbb{E}(\hat{\pi}(\theta; P)\,|x = z, P(z)) = \int \hat{\pi}(\theta; P)\, d\Phi\left(\sqrt{\lambda+\beta+\beta\delta}\left(\theta - \frac{\lambda\mu + (\beta+\beta\delta)z}{\lambda+\beta+\beta\delta}\right)\right), \quad (5)$$

where expectations are taken with respect to θ. The first line in equation (5) implicitly defines the price function as the dividend expectation of the marginal trader. On the right hand side of the equation, $P(z)$ appears both through the public signal z that it conveys about θ and through its impact on firm's decision $a(y, P(z))$. The second line in the equation exploits the result from Lemma 1 that observing $P(z)$ and z are equivalent under price invertibility, arriving to an explicit formulation of the price in terms of the endogenous signal z.

For given $a(y, P)$, the market equilibrium exists if and only if there exists a monotone solution $P(z)$ in (5). Although $\hat{\pi}(\theta; P)$ is monotone in θ, this is not sufficient for monotonicity of $P(z)$, due the feedback of P into firm's decisions. In what follows, we disregard the monotonicity requirement at first, and then verify ex post whether it holds at the proposed equilibrium price function.

The main implication from this equilibrium characterization is that the share price generally differs from the share's expected dividend value. The latter is defined as the posterior expectation of $\hat{\pi}(\theta; P)$ that conditions only on the publicly available information z:

$$V(z) = \mathbb{E}(\hat{\pi}(\theta; P)\,|P(z)) = \int \hat{\pi}(\theta; P)\, d\Phi\left(\sqrt{\lambda+\beta\delta}\left(\theta - \frac{\lambda\mu + \beta\delta z}{\lambda+\beta\delta}\right)\right) \quad (6)$$

We label the difference between the price and the expected dividend the *information aggregation wedge*: $W(z) \equiv P(z) - V(z)$. The wedge arises from heterogeneous information and limits to arbitrage in the financial market. The price equals the dividend expectation of the marginal trader who is indifferent between keeping or selling her share. This trader conditions on the market signal z, as well as a private signal, whose realization must equal the threshold $\hat{x}(P)$ in order to be consistent with the trader's indifference condition. The trader treats these two sources of information as mutually independent signals of θ.

The market-clearing condition introduces a shift in the identity of the marginal trader in response to either shocks to fundamentals or noise trading. A positive fundamental shock shifts all signals up but leaves constant the supply of shares that are available to informed traders. To clear the market, the identity of the marginal trader must then shift in parallel with the fundamental. A positive noise trade shock on the other hand leaves the distribution of private signals unchanged but increases the external demand for shares. For the market to clear, the identity of the marginal trader must then shift towards a higher signal. In both cases, the increase in z that is conveyed through the price is reinforced by an identical increase in the marginal trader's private signal $\hat{x}(P) = z$, which is necessary in order to equate demand and supply of shares. The marginal trader's expectation $\mathbb{E}\left(\pi(\cdot)|x = z, z\right)$ thus behaves as if she received one signal z of precision $\beta\left(1+\delta\right)$ instead of $\beta\delta$. In contrast, the expected dividends $\mathbb{E}\left(\pi(\theta)|z\right)$ conditional on P (or equivalently z) weighs z according to its true precision $\beta\delta$.

It is important to note that heterogeneous beliefs and limits to arbitrage are both necessary ingredients for the information aggregation wedge. Free entry by risk-neutral uninformed arbitrageurs would equate the share price with its expected dividend value: $P\left(z\right) = V\left(z\right)$. Similarly, if the informed traders had access to a signal $z \sim \mathcal{N}\left(\theta, \left(\beta\delta\right)^{-1}\right)$ that is common to all, then they hold identical beliefs, and in equilibrium the market can only clear if they are indifferent between holding or selling the security, which again eliminates the wedge.

2.3 Optimal Investment Decisions

At stage $t = 1$, before the market opens, the manager commits to an investment rule $a\left(y, P\right)$, but anticipating the equilibrium price function $P\left(z\right)$. We state this problem formally by allowing the firm to choose its investment rule subject to the constraint that is implied by the above characterization of the equilibrium price:

$$\max_{a(\cdot,\cdot)} \int \left[\alpha P\left(z\right) + \left(1-\alpha\right)\pi\left(\theta, y, a\left(y, P\left(z\right)\right)\right)\right] dG\left(y|\theta\right) d\Phi\left(\sqrt{\beta\delta}\left(z-\theta\right)\right) d\Phi\left(\sqrt{\lambda}\left(\theta-\mu\right)\right) \quad (7)$$
$$\text{s.t} \quad P\left(z\right) = \mathbb{E}\left(\pi\left(\theta, y, a\left(y, P\left(z\right)\right)\right)|x = z, z\right).$$

That is, the manager chooses a price-contingent decision rule subject to the constraint that the price function is an equilibrium price function. Using the fact that $P\left(z\right)$ must be invertible, we reformulate the manager's decision rule as a function of y and z. After changing the order of integration between θ, y and z, the optimal decision is characterized by the solution to the following pointwise optimization problem, for given (y, z):

$$\max_{a(y,z)} \int \left[\alpha P(z) + (1-\alpha)\,\pi\left(\theta, y, a\left(y, P(z)\right)\right)\right] dH_F(\theta|y,z)$$
$$\text{s.t.} \quad P(z) = \mathbb{E}\left(\pi\left(\theta, y, a(y,z)\right)|x = z, z\right),$$

where the manager's posterior conditional on firm-specific information y and market information z, denoted $H_F(\cdot|y,z)$, is

$$H_F(\theta|y,z) = \frac{\int_{-\infty}^{\theta} g(y|\theta)\, d\Phi\left(\sqrt{\lambda+\beta\delta}\left(\theta - \frac{\lambda\mu+\beta\delta z}{\lambda+\beta\delta}\right)\right)}{\int_{-\infty}^{\infty} g(y|\theta)\, d\Phi\left(\sqrt{\lambda+\beta\delta}\left(\theta - \frac{\lambda\mu+\beta\delta z}{\lambda+\beta\delta}\right)\right)}.$$

Notice that objective function further simplifies to a weighted average of $P(z)$ and $V(z)$: $\alpha P(z) + (1-\alpha) V(z)$. The expected dividend value on the other hand is $V(z)$. Therefore, as long as $\alpha = 0$, or $V(z) = P(z)$, there is no conflict of interest between the manager and the shareholders. In the case of $\alpha = 0$, the managers and shareholders' incentives are perfectly aligned from the start. In this case, the decision rule that the manager commits to ex ante is also the one that maximizes expected dividends ex post, i.e. the issue of precommitment also doesn't play a role. If $V(z) = P(z)$, the incentives are aligned because the market perfectly equates prices with expected dividend value. In this case, α has no direct influence on incentives. However a positive value of α may lead to better alignment of manager and shareholder incentives in the presence of (unmodeled) additional agency frictions.

The characterization of the solution to the manager's decision problem leads to a few immediate observations that already clarify the impact of the information in prices on the firm's decision. Specifically, the optimal $a(y,z)$ satisfies

$$a(y,z) \in \arg\max_{a \in A} \alpha \mathbb{E}\left(\pi(\theta, y, a)|x = z, z\right) + (1-\alpha)\,\mathbb{E}\left(\pi(\theta, y, a)|z\right).$$

First, by LeChâtelier's Principle, the maximized objective is always less concave and more convex, the more variables the decision is conditioned on. In the present case, the information feedback arises through the conditioning of $a(y,z)$ on z, which captures the information value of the decision. In this sense, the informational feedback is value-enhancing for the manager. Whether it also enhances expected dividends and/or the price depends on the value of α, and the shape of $\pi(\theta, y, a)$. At $\alpha = 0$, the informational feedback is beneficial for the final shareholders, but also increases the price level, as the sensitivity of investment to z remains below the level the market prefers. The convexification of dividends through the response of investment to the market then implies that the upside risks dominate the price, so that shares on average trade at a premium.

Second, the formulation illustrates how the combination of information aggregation frictions with price-based incentives leads to a conflict of interest: the decision that maximizes the price need not, and generally is not the same as the one which maximizes expected dividends. Intuitively, the marginal trader attaches too much informational content to the price and would like the manager to respond excessively to the price signal. The higher is α, the more the manager will cater to the market's views and generate excess sensitivity in prices. Because this brings the investment closer to the level that would be optimal from the marginal trader's perspective, it will increase prices but reduces dividend value possibly even to a level where any response of investment to share prices becomes harmful. The manager's incentive to manipulate the price to his own benefit and the shareholders' detriment by committing to an investment rule is thus directly linked to an incentive scheme that rewards managers for the share price performance.

In the remainder of this paper, we compare investment incentives under dividend value maximization with those induced by price-based incentives, and make these observations explicit in the context of a simple example.

3 A Binary Action Model

This section develops a particular example of an endogenous investment game in which the manager makes a binary investment choice. This framework can be nested in the general model of the previous section, and serves to illustrate the asset pricing implications of the interplay between heterogeneous beliefs and endogenous investment, as well as the consequences on investment efficiency of tying managerial compensation to share prices.

Assume that the firm's decision is binary, $a \in \{0, 1\}$, where $a = 1$ denotes the decision to invest, and $a = 0$ denotes the decision not to invest. The dividend of the firm is given by:

$$\pi(\theta, F; a) = \rho \cdot \theta + a \cdot (\theta - F), \tag{8}$$

where $\rho > 0$. The dividend has two components. The first is an exogenous effect $\rho \cdot \theta$ of the fundamental, θ, on payoffs.[5] The second component is endogenous. If the firm chooses to invest it incurs a cost F, but its revenue increases by θ. We assume that F is independent of θ, distributed with cdf $G(\cdot)$ and density $g(\cdot)$. Let $\underline{F}$ denote the lower bound of the distribution of F ($\underline{F}$ can be equal to $-\infty$). The cost F is observed privately by the manager before choosing investment (i.e., $y = F$ in terms of our previous notation). The *firm-specific* cost F summarizes characteristics

[5] We further discuss the role of the exogenous component in Section 4.1.

of the project about which the firm holds precise information (for example, proprietary technical specifications). The *market-specific* fundamental θ relates to conditions about which knowledge is dispersed throughout the market (for example, demand for a new product).[6]

Suppose for now that firm's investment decision is characterized by a threshold rule $\tilde{F}(P)$:

$$a(F,P)=\begin{cases} 1 \text{ if } F\leq \tilde{F}\left(P\right); \\ 0 \quad \text{otherwise.}\end{cases} \tag{9}$$

The firm invests if and only if the investment cost is below the threshold $\tilde{F}\left(P\right)$. For now, we leave $\tilde{F}(P)$ deliberately general to characterize the market equilibrium. Below we show that the general specification of managerial payoffs as a linear combination of π and P is consistent with the threshold investment rule in (9). Following the same steps as above, traders' supply decisions are characterized by a threshold rule $\hat{x}(P)$, which satisfies:

$$\begin{aligned} P &= \rho\int\theta dH(\theta|\hat{x}(P),P)+\int[a(F,P)\cdot(\int\theta dH(\theta|\hat{x}(P),P)\ -F)]dG(F) \\ &= \left(\rho+G\left(\tilde{F}\left(P\right)\right)\right)\int\theta dH\left(\theta|\hat{x}(P),P\right)-\int_{\underline{F}}^{\tilde{F}(P)}FdG\left(F\right). \end{aligned} \tag{10}$$

The first integral in the upper line of equation (10), $\rho\int\theta dH(\theta|x,P)$, is the marginal trader's expectation of the dividend if the firm does not invest. The second term in the first line is the expected impact of investment on the dividend. For each pair (F,P), the marginal trader considers the difference between the posterior expectation of the fundamental and the investment cost, $\int\theta dH(\theta|x,P)\ -F$. Since the trader does not observe F, the expectation is an integral over the investment range $F\leq\tilde{F}\left(P\right)$. Equation (10) compares the cost of holding the share, P (the left-hand side) to the expected dividends (the right hand side). The price enters the expected dividend through its impact on marginal trader's expectation of θ, and by its influence on firms' investment threshold $\tilde{F}\left(P\right)$.

With invertibility of the price function, we redefine the investment threshold as a function of z: $\tilde{F}(z)=\tilde{F}(P)$. Using the market-clearing condition $z\equiv\hat{x}(P)$, and the characterization of shareholder beliefs in Lemma 1, we characterize the equilibrium of the endogenous investment model:

Proposition 1 (Equilibrium share price and expected dividend) *For an investment thresh-*

[6]See Miller and Rock (1985) and Rock (1986) for further discussions on market- and firm- specific sources of information.

old $\tilde{F}(z)$, define $P(z)$ by:

$$P(z) = \left(\rho + G\left(\tilde{F}(z)\right)\right) \frac{\lambda\mu + \beta(1+\delta) z}{\lambda + \beta + \beta\delta} - \int_{\underline{F}}^{\tilde{F}(z)} F dG(F). \tag{11}$$

If $P(z)$ is strictly increasing, the asset market equilibrium is characterized by the price function $P(z)$ and traders' threshold function $\hat{x}(p) = z = P^{-1}(p)$. The expected dividend conditional on public information z is given by

$$V(z) = \left(\rho + G\left(\tilde{F}(z)\right)\right) \frac{\lambda\mu + \beta\delta z}{\lambda + \beta\delta} - \int_{\underline{F}}^{\tilde{F}(z)} F dG(F). \tag{12}$$

The equilibrium price and expected dividend in Proposition 1 can be decomposed into three terms. First, $\rho \cdot \frac{\lambda\mu+\beta(1+\delta)z}{\lambda+\beta+\beta\delta}$ and $\rho \cdot \frac{\lambda\mu+\beta\delta z}{\lambda+\beta\delta}$ denote the expected dividend if the firm does not invest, from the marginal trader's and manager's perspective. Second, $G\left(\tilde{F}(z)\right) \cdot \frac{\lambda\mu+\beta(1+\delta)z}{\lambda+\beta+\beta\delta}$ and $G\left(\tilde{F}(z)\right) \cdot \frac{\lambda\mu+\beta\delta z}{\lambda+\beta\delta}$ are the additional expected payoff if the firm invests. The third term $\int_{\underline{F}}^{\tilde{F}(z)} F dG(F)$ is the expected investment cost.

In the next two subsections, we consider two separate cases for manager's objective and compare the resulting threshold functions $\tilde{F}(z)$, equilibrium prices, and expected dividend values.

3.1 The Benchmark Case: Dividend Maximization

We now characterize the equilibrium in which manager's objective is to maximize the expected dividend $\pi(\theta, F; a)$, corresponding to the case where $\alpha = 0$ in the general set-up. The manager invests if (and only if) the realization of the cost F is (weakly) lower than the posterior of the fundamental $\mathbb{E}(\theta|P)$. The investment threshold is given by

$$\tilde{F}(P) = \tilde{F}(z) = \int \theta dH(\theta|z) = \frac{\lambda\mu + \beta\delta z}{\lambda + \beta\delta}. \tag{13}$$

The equilibrium is given by Proposition 1 after replacing $\tilde{F}(z) = \mathbb{E}(\theta|z)$. For the discussion below, it is convenient to redefine the state z in terms of the posterior expectation $Z \equiv \mathbb{E}(\theta|z)$. We then rewrite the price, the expected dividend, and the wedge in terms of this posterior expectation Z:

$$P(Z) = (\rho + G(Z))(\mu + \gamma(Z - \mu)) - \int_{\underline{F}}^{Z} F dG(F), \tag{14}$$

$$V(Z) = (\rho + G(Z)) Z - \int_{\underline{F}}^{Z} F dG(F), \tag{15}$$

$$W(Z) \equiv P(Z) - V(Z) = (\gamma - 1)(Z - \mu)(\rho + G(Z)). \tag{16}$$

The parameter $\gamma > 1$ is given by

$$\gamma \equiv \frac{\beta + \beta\delta}{\lambda + \beta + \beta\delta} / \frac{\beta\delta}{\lambda + \beta\delta}, \tag{17}$$

and corresponds to the ratio of Bayesian weights assigned to the market signal z by the marginal trader, and the manager (or an uninformed outsider). The next lemma establishes a sufficient condition for the invertibility of the price function:

Lemma 2 (Invertibility of the Price Function) *The price function is invertible if* $\rho + G(F) + g(F)(F - \mu) > 0$, *for all* $F \geq \underline{F}$.

Lemma 2 states a sufficient condition for price invertibility in the case of expected dividend maximization. Price non-invertibility, which is caused by price non-monotonicity, can arise if the marginal trader's valuation of investment is locally decreasing in z. If $\rho = 0$, this is inevitably the case whenever $G(F)/g(F)$ is non-decreasing and converges to 0 as $F \to -\infty$. The condition in Lemma 2 imposes a lower bound on the sensitivity of the dividend to the fundamental through the exogenous payoff component $\rho \cdot \theta$. We further expand on this issue in Section 4.1.

The wedge $W(Z)$ can be decomposed as a product of two terms. The first term given by equation (16) corresponds to the difference between marginal trader's and manager's posterior beliefs about θ: $(\gamma - 1)(Z - \mu)$. This term determines the sign of the wedge and follows from our discussion in Section 2. The price, $P(z)$, is the expectation of dividends by the marginal trader who observes z both as private *and* public information of θ. Thus, the price reacts more strongly to the market information relative to the dividend expectation of the manager, $V(Z)$.

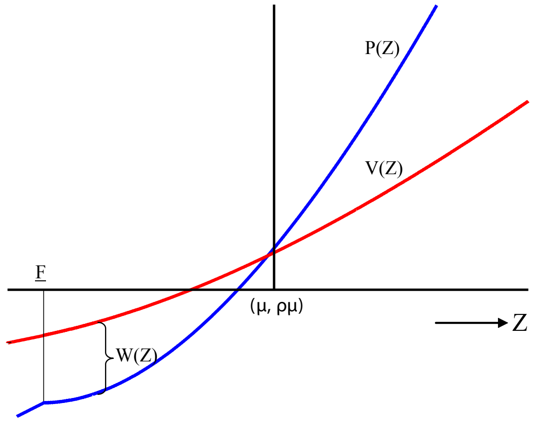
The second term is the marginal effect of the fundamental, θ, on the expected dividend: $(\rho + G(Z))$. This term includes the exogenous effect of the fundamental (ρ) and the endogenous effect from investment ($G(Z)$). It captures the value of market information for the investment decision: the market signal determines manager's posterior belief of θ and the probability of investment $G(Z)$, which determines the net effect of θ on the dividend and, hence, the absolute magnitude of the wedge.

Figure 1 plots the price (solid line), the expected dividend (dashed line), and the wedge as a function of the state variable Z. The expected dividend $V(Z)$ is increasing and convex, and the price function is also increasing given the condition imposed in Lemma 2. The information aggregation wedge, conditional on Z, is negative for $Z < \mu$ (marginal trader overreaction to low states), positive for $Z > \mu$ (overreaction to high states), and zero at $Z = \mu$, where the posterior of θ for the marginal trader and the manager coincides.

To compute the unconditional wedge, we integrate over the prior distribution of Z, which yields

$$\mathbb{E}(W(Z)) = (\gamma - 1)\, Cov(G(Z), Z) > 0, \tag{18}$$

Figure 1: Price, Expected Dividend and Wedge



since $G(\cdot)$ is increasing. Expression (18) can be rearranged as

$$\begin{aligned}\mathbb{E}(W(Z)) &= (\gamma-1)\int_{-\infty}^{\infty} G(Z)\,\frac{Z-\mu}{\sigma_Z}\phi\left(\frac{Z-\mu}{\sigma_Z}\right)dZ \\ &= (\gamma-1)\,\sigma_Z\int_{0}^{\infty}\left(G(\mu+\sigma_Z u)-G(\mu-\sigma_Z u)\right)u\phi(u)\,du, \qquad (19)\end{aligned}$$

where $\sigma_Z^2 = \beta\delta/(\lambda+\beta\delta)\cdot\lambda^{-1}$ is the ex ante variance of the firm's posterior Z. Expression (19) explicitly shows the three factors that are necessary and sufficient for obtaining a positive expected wedge, and how together they determine its magnitude. First, information heterogeneity among traders ($\gamma > 1$) is required to obtain a conditional wedge $W(Z)$. If the shareholders in the financial market had identical beliefs, then the price would would still convey the shareholder's common information to the firm, and the firm would still value this information and act on it, but there would no longer be any wedge.

Second, a positive wedge requires ex ante uncertainty about the firm's posterior Z, i.e. $\sigma_Z^2 > 0$. The variance of the firm's posterior measures how strongly the information conveyed by the market affects the firm's beliefs about θ; σ_Z^2 represents the value of market information for the investment decision and is increasing in the precision of the market signal $\beta\delta$ and the prior variance of the fundamental λ^{-1}. Intuitively, precise private information (high β), or low variance of noise trading (high δ) increase the likelihood that movements in z are due to innovations in θ. This makes z a more reliable signal and increases the sensitivity of manager's posterior Z to changes in z. Also, learning about θ is more important the larger its ex-ante variance (λ^{-1}). The unconditional wedge is increasing in the variance σ_Z^2, which captures the amount of learning from market prices.

Third, the firm's investment choice must be sensitive to the market information - that is, the shape of the distribution $G(\cdot)$ matters for the wedge. This distribution measures how much the firm's investment decision responds to the market information: the higher the variance of the firm-specific cost F (i.e. the flatter $G(\cdot)$ is around the prior mean of F), or the more certain the prior expectation of the firm's investment decision (i.e. if $G(Z)$ is close to 0 or 1 around the prior mean μ), the less the firm's investment probability is going to respond to changes in market information, and hence the less the firm's exposure to θ varies with the price. If the investment decision was completely fixed before the observation of the price, the investment probability would be constant. The wedge would be symmetric around the prior mean $Z = \mu$, and its ex ante expectation equal to zero. The expected information aggregation wedge is therefore large from an ex ante perspective when (i) the prior uncertainty about the firm's investment decision is high, and (ii) the realization of the market signal generates a significant update in the investment probability $G(Z)$. Uncertainty about the firm's investment decision is highest when $G(F) = \mathbb{I}_{F>\mu}$, i.e. when the firm's investment cost is known to be equal to the prior mean μ (meaning that ex ante the firm would be indifferent), and any update from the price can swing the investment decision in either direction.[7]

The next proposition summarizes the comparative statics of the information aggregation wedge, the expected price, and the dividend in terms of γ and σ_Z^2. We will defer a more complete discussion of the role of $G(\cdot)$ until section 4.1.

Proposition 2 (Comparative Statics) *(i) For a given value of γ, $\mathbb{E}(P(Z))$, $\mathbb{E}(V(Z))$, and $\mathbb{E}(W(Z))$ are increasing in σ_Z^2. (ii) For given value of σ_Z^2, $\mathbb{E}(V(Z))$ does not depend on γ; $\mathbb{E}(P(Z))$ and $\mathbb{E}(W(Z))$ are increasing in γ.*

The unconditional price, dividend, and wedge are all increasing in the prior uncertainty about the firm's posterior, σ_Z^2. Recall that the unconditional expected dividend is larger when the manager learns more information from the market (higher σ_Z^2). Moreover, since the marginal trader's posterior is more sensitive to z than manager's, the impact of σ_Z^2 on the average price is stronger than on the expected dividend. This raises the unconditional wedge. The unconditional dividend

[7]The value of market information for the firm highlights the close relation of our setup with real options (e.g., Dixit and Pindyck, 1994). Firm's payoff depends on the realization of a random variable (θ) and on an endogenous choice of investment (a). The price is a public signal of the variable θ for the firm. The firm can better match a good realization of θ by investing, while limiting the negative effects of a low realization by not investing. The value of the investment option depends on the precision of information, and the prior uncertainty about the random variable. An important novel result of our model is that the option value of information also leads to expected prices to be higher than expected dividends when traders hold heterogenous beliefs in equilibrium.

is independent of the difference between manager's and marginal trader's expectations (γ). The expected price and, hence, the wedge, scale up γ.

The primitive parameters β, δ, and λ affect expected dividends $\mathbb{E}(V(Z))$ through σ_Z^2, which is increasing in both the precision of the market signal ($\beta\delta$), and in the prior uncertainty (λ^{-1}). Both better market information and a more variable prior increase the value of the real option to invest, raising the unconditional expected dividend. The same parameters affect the unconditional information aggregation wedge $\mathbb{E}(W(z))$ through both σ_Z^2 and γ. Notice, however, that the effects go in opposite directions: γ is decreasing in λ^{-1}, decreasing in β, and decreasing in δ. The overall comparative statics on the unconditional wedge and price are therefore a priori not clear. Prior uncertainty λ^{-1} must be sufficiently high to generate option value from investment, and private information precision β needs to be large to create belief dispersion. Finally, the market information $\beta\delta$ must be retain some value for the firm, yet it cannot be not so precise that it completely crowds out the prior, eliminating the wedge. The next proposition uses expression (19) to provide tight bounds on the magnitude of the expected wedge, for a given distribution $G(\cdot)$ of the firm's investment cost. Furthermore, holding β and λ constant, we show that the expected wedge can become arbitrarily large.

Proposition 3 (Bounds on $\mathbb{E}(W(Z))$) *(i) Suppose that $G(\cdot)$ has continuous density $g(\cdot)$, and let $\|g\| = \max_{F \geq \underline{F}} g(F)$. Then*

$$\mathbb{E}(W(Z)) \leq (\gamma - 1)\sigma_Z^2 \|g\| = \left(\frac{\beta + \beta\delta}{\lambda + \beta + \beta\delta} - \frac{\beta\delta}{\lambda + \beta\delta}\right)\frac{\|g\|}{\lambda} < \frac{\beta}{\lambda + \beta}\frac{\|g\|}{\lambda}.$$

(ii) Holding β and λ constant,

$$\lim_{\delta \to 0} \mathbb{E}(W(Z)) = \frac{\beta}{\lambda + \beta}\frac{g(\mu)}{\lambda}.$$

(iii) For all $K > 0$, there exist $G(\cdot)$ and $\delta' > 0$, such that for any $\delta \leq \delta'$, $\mathbb{E}(W(Z)) > K$.[8]

Part (i) shows that for a given function $G(\cdot)$, there is an upper bound on the magnitude of the expected information aggregation wedge. Part (ii) derives the limit of the information aggregation wedge when $\delta \to 0$, i.e. when the market is infinitely noisy. It also shows that the information aggregation wedge remains positive in the limit, whenever $g(\mu) > 0$, i.e. whenever the distribution

[8]In part (iii) the order of limits does not matter, as long as $g(\mu) \to \infty$ along the limiting sequence. Along a sequence $\{G_n(\cdot)\}$ of normal distributions for the investment cost, with mean F_n and standard deviation σ_{F_n}, respectively, this condition requires that $\lim_{n\to\infty}(\mu - \mu_n)/\sigma_{F_n} = 0$, i.e. that μ_n converges to μ at a rate faster than σ_{F_n} converges to 0.

of F has positive density at the prior mean μ. If the distribution is such that $g(\mu) = \|g\|$, i.e. the distribution of F reaches its peak density at μ, then as an immediate corollary, $\mathbb{E}(W(Z))$ reaches its maximum in the limit where $\delta \to 0$ (i.e. market information becomes completely uninformative). This result stems from the fact that $(\gamma - 1)\,\sigma_Z^2$ is strictly increasing in $\beta\delta$, and reaches a finite limit as $\delta \to 0$, and in this limit, the marginal effect of the firm's posterior Z on the investment probability is highest when the posterior Z is near the prior μ, which occurs with probability 1 in the limit as $\delta \to 0$.

Part (iii) shows that, although for a given function $G(\cdot)$ the information aggregation wedge is uniformly bounded in δ, this bound becomes arbitrarily large if the distribution of the investment cost is concentrated around the prior mean μ, and $\delta \to 0$. From (19), $\mathbb{E}(W(Z))$ is highest when $G(\cdot) = \mathbb{I}_{F>\mu}$, i.e. the investment cost is equal to the prior mean μ of the fundamental θ. As we discussed above, this cost scenario maximizes the informational impact of the price on the firm's investment decision. In this case, the expected wedge is $\mathbb{E}(W(Z)) = (\gamma - 1)\,\sigma_Z/\sqrt{2\pi}$, and because we know from part (ii) that $(\gamma - 1)\,\sigma_Z^2$ converges to a finite limit as $\delta \to 0$, it follows that $(\gamma - 1)\,\sigma_Z$ grows unboundedly large as $\delta \to 0$ and $\sigma_Z \to 0$. A combination of a high degree of prior uncertainty about the firm's investment decision, coupled with a lot of noise in the market price (yet sufficient information such that even a small update through z can have a large effect on the firm's investment probability) can make the expected information aggregation wedge arbitrarily large.

3.2 Tying Managerial Incentives to Share Prices

We now discuss the effects of managerial incentives tied to stock market performance. As is well known, whenever the market equates the share price to expected dividends, $P(Z) = V(Z)$, maximizing prices is equivalent to maximizing expected dividends, and price-based incentive schemes help to align manager incentives with shareholder value mitigating other agency conflicts. Here this result no longer holds because of the wedge between the price and the expected dividend. Since the anticipated investment decision influences the price and the wedge, the manager has incentives to influence her earnings by manipulating the share price through the investment decision. Necessarily such manipulations of share prices through investment are inefficient from the perspective of final shareholder payoffs.

We consider the general case where manager's objective function is given by $(1-\alpha)\pi(\theta, F; a) + \alpha P, \quad \alpha \in [0, 1]$. We maintain the assumption that the manager chooses an initial decision rule $a(F, P)$ prior to the interim stage, to which she remains committed after the financial market clear. For a given value of Z, the manager's problem can then be stated as follows:

$$\max_{\tilde{F}(Z)} \{\alpha P(Z) + (1-\alpha) V(Z)\},$$

where $P(Z)$ and $V(Z)$ are given by equations (14) and (15), and $\alpha \in [0,1]$ measures how strongly incentives are based on the price relative to expected dividends.

The market equilibrium is characterized by Proposition 1: threshold functions $\hat{x}(P)$ for traders and $\tilde{F}(P)$ for manager, and an invertible price function $P(Z)$. The investment threshold is found by maximizing $\alpha P(Z) + (1-\alpha) V(Z)$ pointwise, for all Z, and then checking that the resulting price function is invertible. Formally, we have

$$\begin{aligned}\tilde{F}(Z) &\in \arg\max_{\tilde{F}} \{\alpha P(Z) + (1-\alpha) V(Z)\} = \arg\max_{\tilde{F}} \{V(Z) + \alpha W(Z)\} \\ &= \arg\max_{\tilde{F}} \left\{ \left(\rho + G\left(\tilde{F}\right)\right)[\mu + k(Z-\mu)] - \int_{\underline{F}}^{\tilde{F}} F dG(F) \right\}. \end{aligned} \tag{20}$$

The parameter $k = 1 + \alpha(\gamma - 1)$ measures excess weighting of market information by the manager and captures the strength of the distortion introduced by price-based incentives. The variable $k \in [1, \gamma]$ depends on the size of the information aggregation wedge through γ and the weight given to the prices in the manager's objective function, α. At one extreme, $\alpha = 0$ and $k = 1$ correspond to our benchmark model of dividend maximization (section 3.2). At the other extreme, $\alpha = 1$ and $k = \gamma$: the manager's incentives are based only on the share price.

Taking first-order conditions to determine the investment threshold $\tilde{F}(Z)$ (and checking that price invertibility holds under the same condition as before in lemma 2), we find the following equilibrium characterization.

Proposition 4 (Equilibrium with price-based incentives) *In the PBE with price-based incentives, the investment threshold $\tilde{F}(Z)$, price $P(Z)$, and expected dividend $V(Z)$ are given by*

$$\tilde{F}(Z) = \mu + k(Z-\mu), \tag{21}$$

$$P(Z) = \left[\rho + G\left(\tilde{F}(Z)\right)\right](\mu + \gamma(Z-\mu)) - \int_{\underline{F}}^{\tilde{F}(Z)} F dG(F) \tag{22}$$

$$V(Z) = \left[\rho + G\left(\tilde{F}(Z)\right)\right] Z - \int_{\underline{F}}^{\tilde{F}(Z)} F dG(F) \tag{23}$$

Equations (22) and (23) decompose the effect that the information aggregation wedge has on the price and the expected dividend. Without price-based incentives ($\alpha = 0$; $k = 1$), the stronger reaction of the price to market signals has no impact on the expected dividend. When $\alpha > 0$

($k > 1$), the price increases with α at an efficiency cost that reduces the expected dividend in (23). We formalize the result in the next proposition.

Proposition 5 (Tying managerial incentives to share-prices) *In the PBE with price-based incentives, the following results hold:*

(i) $\tilde{F}'(Z) = k > 1$*: The volatility of investment is increasing in* α *and* γ *(and hence in* k*), for all* $Z \neq \mu$.

(ii) The expected dividend $V(Z)$ *is decreasing, while the share price* $P(Z)$ *and the wedge* $W(Z)$ *are increasing in* α*, for all* $Z \neq \mu$.

(iii) The expected share price $\mathbb{E}(P(Z))$ *and the unconditional wedge* $\mathbb{E}(W(Z))$ *are increasing in* σ_Z^2 *and* γ.

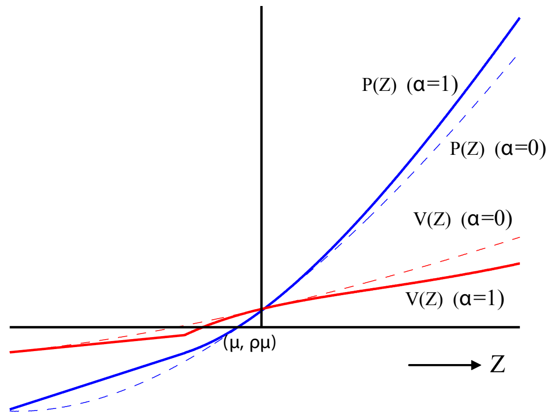
(iv) $V(Z)$ *and* $\mathbb{E}(V(Z))$ *are decreasing in* γ*, while the effect of* σ_Z^2 *on expected dividends,* $\mathbb{E}(V(Z))$ *is ambiguous.*

(v) If $\alpha > 0$*, then* $\lim_{\delta\to 0}\mathbb{E}(W(Z)) = \infty$*, for any continuous, strictly increasing* $G(\cdot)$.

Proposition 5 summarizes the comparative statics for the game with price-based incentives. Part (i) states that investment volatility is increased when incentives are tied to share prices ($\alpha > 0$). The manager reacts more strongly to the information conveyed by the price, as captured by the parameter $k > 1$. Price-based incentives induce the manager to align investment with the beliefs of the marginal shareholder. Therefore, when Z is higher than μ, firm's investment threshold is too high. The firm invests in some states in which the cost F exceeds the expected gains from investment, Z. When Z is lower than μ, the firm's investment threshold is too low. The firm foregoes investment in some states in which Z exceeds the cost F. This results in higher prices but lower expected dividends, for all $Z \neq \mu$ (part ii).

Figure 2 plots the price and expected dividend functions for the extreme cases $\alpha = 1$ (the thick lines) and $\alpha = 0$ (the thin lines). When $\alpha = 1$ ($k = \gamma$) the manager is purely concerned with price maximization. The investment rule in (21) exactly matches the marginal trader's expectation of θ. The firm behaves as if it were run by the marginal trader and achieves the largest share price for each realization of the state Z to the detriment of expected dividends. Indeed, figure 2 shows how the price in this case (the thick, solid line) is always above the one attained under dividend maximization, or $\alpha = 0$ (the thin, solid line). The expected dividend under priced-based incentives (thick, dashed line) is everywhere below its counterpart in the benchmark case (the thin, dashed line). The conditional information aggregation wedge is thus exacerbated and so is the unconditional wedge.

Figure 2: Effect of Price-based Incentives



From an ex ante perspective, this reinforces the effects of γ and σ_Z^2 on the expected share price $\mathbb{E}(P(Z))$ and the unconditional wedge $\mathbb{E}(W(Z))$ (part *iii*) that were positive already in the case with dividend maximization. Things change, however, with regards to the expected dividend value $\mathbb{E}(V(Z))$. First, an increase in γ lowers expected dividend value of the firm by increasing the manager's ability to distort the investment threshold to boost the share price. This is reflected in the fact that k is increasing in γ. Second, the overall effect of information provided through the price is ambiguous, and we can construct both cases in which better market information increases the firm's dividend value, and cases where the opposite is true. In particular, if k is not too high (so that the distortion motive is not too large), and G is highly concentrated around some value close to μ, the loss from the investment distortion is small, relative to the value of the information. On the other hand, if k is high, and $G(\cdot)$ is highly dispersed, then the price-based incentives distort the investment decision sufficiently severely so that a more informative price signal may actually reduce the firm's dividend value. This result follows the standard logic that improved information need not be socially desirable, if economic decisions do not make efficient use of this information.

Finally, part (v) shows that with price based incentives, the information aggregation wedge becomes arbitrarily large, as the market information becomes noisier. As was the case with expected dividend maximization, the comparative statics of different variables w.r.t. the underlying parameters β, δ, and λ are not unambiguous, because of the competing effects of σ_Z^2 and γ (where the latter now also induces inefficient investment decisions). However, in that case, we showed that the expected wedge was bounded, for given $G(\cdot)$. With price-based incentives, this is no longer true. In particular, we can rewrite $\mathbb{E}(W(Z))$ along the same lines as (19):

$$\begin{aligned}\mathbb{E}(W(Z)) &= (\gamma-1)\, cov\left(G\left(\tilde{F}(Z)\right), Z\right) \\ &= (\gamma-1)\,\sigma_Z \int_0^\infty \left(G(\mu+k\sigma_Z u) - G(\mu - k\sigma_Z u)\right) u\phi(u)\, du.\end{aligned}$$

Result (v) then follows immediately from the observation that $\lim_{\delta\to 0} k\sigma_Z = \infty$ for any $\alpha > 0$, and $\lim_{k\sigma_Z\to\infty}\left(G(\mu+k\sigma_Z u) - G(\mu-k\sigma_Z u)\right) = 1$ for all $u > 0$, so that $\mathbb{E}(W(Z))$ is of the same order as $(\gamma-1)\,\sigma_Z$. In contrast to the case with dividend maximization, the impact of Z on the investment probability actually increases without bound, as the market information becomes noisier. The key to this result is that with $\alpha > 0$, the weight the managers put on the market signal is bounded away from 0, even if the market information z is infinitely noisy. But this implies that the variability of the investment threshold $\tilde{F}(Z)$ becomes infinite, even though σ_Z^2 goes to zero. That is, in the limit where market information is pure noise, the manager will no longer update from the price. Yet with price based incentives, the variability of the investment threshold can grow arbitrarily large because the manager is induced to align the investment threshold $\tilde{F}(Z)$ with the marginal shareholder's expectation of θ.

As an immediate corollary, we also have the observation that the introduction of price-based incentive schemes, i.e. a shift from $\alpha = 0$ to $\alpha > 0$, can induce an arbitrarily large increase in the expected share price, a large increase in investment volatility, and a reduction in the dividend value, when market prices are sufficiently noisy.

The analysis above gives an argument against tying executive compensation too closely to market valuations. When dispersed information drives a wedge between prices and expected dividends, and when the wedge responds to firm's endogenous investment decision, price-based incentives lead to inefficient investments that drive up prices but lower firm value.

4 Discussion and Extensions

We now study the robustness of our main results by considering several extensions of our benchmark model. First, we discuss the role of our sufficient condition for the invertibility of the price function (lemma 2). Then, we consider an alternative noise trading assumption which captures limited arbitrage by uninformed traders, and allows us to vary the price impact of the informed traders' orders. Finally, we consider alternative informational environments with additional information about θ - either public or private. The complete analysis for the first and third parts are provided in a separate online appendix.

4.1 The sufficient condition from lemma 2

In Lemma 2, we assumed that $\rho+G(F)+g(F)(F-\mu)>0$, for all $F\geq\underline{F}$, as a sufficient condition to guarantee that the price function was invertible. This is a joint condition on the size of the exogenous dividend component ρ and the distribution of the investment cost $G(\cdot)$, and remained sufficient regardless of the weight $\alpha\in[0,1]$ that the firm placed on maximizing its price vs. its expected dividend. Since the potential for non-monotonicity of $P(\cdot)$ results from the disagreement between the optimal investment decision from the firms' and the marginal shareholder's point of view, the non-invertibility issue is more severe for lower α, and it disappears altogether for $\alpha=1$. Moreover, non-invertibility is an issue only for $F<\mu$, where the firm over-invests, from the marginal shareholder's point of view.

In the online appendix, we characterize equilibria when this condition no longer holds. We consider two separate cases.

First, whenever either $\rho\neq 0$ or $\rho=0$ and $\underline{F}=-\infty$, the realized dividends are a function of θ, for any finite investment threshold, and therefore traders necessarily find it optimal to act on their private information. This implies that $\hat{x}(\cdot)$ is necessarily finite, for all P, P must reveal z in equilibrium, and our characterization of the equilibrium price function remains applicable regardless of ρ or $G(\cdot)$. Since this function is unique, the equilibrium exists only if the candidate price function is invertible.[9]

Non-invertibility then arises whenever the distribution $G(\cdot)$ has sufficiently thin tails: whenever $G(F)/g(F)$ converges to 0 as $F\to-\infty$, $G(F)+g(F)(F-\mu)<0$ for some F and therefore $P(\cdot)$ is locally decreasing, unless ρ is sufficiently large.[10] Non-invertibility also arises when the distribution $G(\cdot)$ has mass points, or when the investment cost F is deterministic. The candidate price function will have a discontinuity at any mass point f of $G(\cdot)$, except when $\alpha=1$ (under pure price maximization, $P(\cdot)$ is continuous and increasing for any distribution $G(\cdot)$). The size of the jump in $P(\cdot)$ at f is $(G_{+}(f)-G_{-}(f))\left(\mu+\gamma\left(\tilde{F}^{-1}(f)-\mu\right)-f\right)$, where $\tilde{F}^{-1}(\cdot)$ denotes the inverse function of the firm's investment threshold. For the investment threshold rules we considered in the previous section, this discontinuity will be negative and $P(\cdot)$ therefore not invertible, whenever $f<\mu$. This implies that the distribution $G(\cdot)$ cannot have any mass points below μ, nor can F be deterministic and less than μ.[11]

[9] Non-existence here refers to non-existence of an equilibrium in which price is conditioned on z only.

[10] Non-invertibility issues do not arise for any value of ρ only if $\underline{F}\geq\mu$.

[11] Checking the sufficient condition of lemma 2 for a sequence of continuous distributions $G(F)$ that approaches a mass point at $f<\mu$ also reveals that in such a case the exogenous component ρ must become infinitely large to avoid price non-monotonicity.

Second, when $\rho = 0$ and $\underline{F} > -\infty$, θ only affects dividends through the investment decision, and the shareholder's private signals only carry information if the firm invests with positive probability. This leads to a feedback between the informativeness of the trader's private signals and the price on the one hand, and the firm's investment decision on the other, with price multiplicity and indeterminacy as a possible consequence. As long as the traders' private signals remain informative, and traders act on them, the price will remain informative and take the same form as characterized throughout. However, for some realizations of Z a second scenario is possible, in which the firm is certain not to invest conditional on seeing a price of 0, the shareholders will not trade on their private signals at such a price and the expected dividends are 0. If $Z < \underline{F}$, the firm would be certain not to invest, so this is the only possible equilibrium outcome. For more optimistic realizations of Z it is possible to sustain trading on private information along with a positive probability of investment in equilibrium. However, trade need not occur at such states: in particular, if $\underline{F} > \mu$ (so that the firm would never invest, based just on its prior information), there exists an equilibrium in which the price is 0, for all values of Z and the firm never invests. There also exists an equilibrium in which traders act on private information, and the firm invests with positive probability for any $Z > \underline{F}$. More generally, when $\underline{F} > \mu$, it is possible to sustain (almost) arbitrary selections from the correspondence $\{0, P(Z)\}$ as equilibrium prices, implying that the information aggregation through the price and the firm's investment decisions are indeterminate.[12]

These multiplicity, indeterminacy and non-existence issues are interesting in their own right, but they are somewhat distracting from the main contribution of our paper. Our baseline model with an exogenous dividend component and a continuous distribution $G(\cdot)$ allows us to focus on the cases in which there is a unique equilibrium.

4.2 Price Impact of Information

So far, the stochastic asset demand was completely inelastic. Here, we generalize our assumption about exogenous asset demand (noise trading) by assuming it comes from uninformed traders: they trade partly for stochastic exogenous motives, and partly in response to gaps between the expected dividend and the price. The model is the same as before, except for the asset demand, which we assume takes the following form:

$$D(u, P) = \Phi\left(u + \eta\left(\mathbb{E}\left(\pi|P\right) - P\right)\right), \tag{24}$$

[12] For $\underline{F} < \mu$, this logic for indeterminacy still applies, but the requirement of invertibility of $P(\cdot)$ places additional restrictions on the selection of the price function between 0 and $P(Z)$. If $\underline{F}$ is sufficiently far from μ, these restrictions can be so severe that an equilibrium no longer exists - as in the case with $\underline{F} = -\infty$ that we discussed above.

with $u \sim \mathcal{N}\left(0, \delta^{-1}\right)$. Uninformed traders' demand is increasing in the expected return conditional on the price, $\mathbb{E}\left(\pi|P\right) - P$, with an elasticity given by η. This specification generalizes our previous formulation to allow for a response of uninformed traders to perceived excess returns on the asset, as well as stochastic trading motives which are unrelated to dividend expectations. The parameter η captures the responsiveness of uninformed traders to the expectation of dividends in excess of prices, or in other words, the extent to which they are willing or able to arbitrage away the difference between expected price and dividend value. In the limit as $\eta \to \infty$, we approach a model with full arbitrage by uninformed traders. Equivalently, η measures the price impact of private information which relates naturally to the concept of market liquidity.

We follow our previous equilibrium characterization and asset prices with minor changes to account for the endogeneity of demand to asset prices. Market-clearing implies $\Phi\left(\sqrt{\beta}\left(\hat{x}\left(P\right) - \theta\right)\right) = \Phi\left(u + \eta\left(\mathbb{E}\left(\pi|P\right) - P\right)\right)$, or $z = \hat{x}\left(P\right) - \eta/\sqrt{\beta} \cdot \left(\mathbb{E}\left(\pi|P\right) - P\right)$. Observing P is thus isomorphic to observing $z \sim \mathcal{N}\left(\theta, \left(\beta\delta\right)^{-1}\right)$, and Lemma 1 continues to hold without any changes. Using the fact that the expected dividend is $\mathbb{E}\left(\pi|P\right) = \mathbb{E}\left(\pi|z\right) = V\left(z\right)$, the equilibrium price function is implicitly defined by the marginal trader's indifference condition

$$P\left(z\right) = \mathbb{E}\left(\pi|\hat{x}\left(P\right), z\right) = \mathbb{E}\left(\pi|z + \eta/\sqrt{\beta} \cdot \left(V\left(z\right) - P\left(z\right)\right), z\right).$$

Thus, for $\eta > 0$, the increased reliance of the market signal that we discussed in sections 2 and 3 as the cause for the information aggregation wedge is partially counter-acted by the uninformed traders' response to the wedge. Using the definition of dividends from section 3 for a given investment threshold function $\tilde{F}\left(Z\right) = \tilde{F}\left(P\left(Z\right)\right)$, and defining the firm's conditional expectation of θ, $Z = \frac{\lambda\mu + \beta\delta z}{\lambda + \beta\delta}$ as the state variable, the equilibrium price, expected dividend value and information aggregation wedge are characterized as

$$\begin{aligned}
V\left(Z\right) &= \left(\rho + G\left(\tilde{F}\left(Z\right)\right)\right) Z - \int_{\underline{F}}^{\tilde{F}(Z)} f dG\left(f\right), \\
P\left(Z\right) &= \left(\rho + G\left(\tilde{F}\left(Z\right)\right)\right)\left(\mu + \frac{\gamma + \frac{\sqrt{\beta}\eta}{\lambda+\beta+\beta\delta}\left(\rho + G\left(\tilde{F}\left(Z\right)\right)\right)}{1 + \frac{\sqrt{\beta}\eta}{\lambda+\beta+\beta\delta}\left(\rho + G\left(\tilde{F}\left(Z\right)\right)\right)}\left(Z - \mu\right)\right) - \int_{\underline{F}}^{\tilde{F}(Z)} f dG\left(f\right), \\
W\left(Z\right) &= \frac{\gamma - 1}{1 + \frac{\sqrt{\beta}\eta}{\lambda+\beta+\beta\delta}\left(\rho + G\left(\tilde{F}\left(Z\right)\right)\right)}\left(\rho + G\left(\tilde{F}\left(Z\right)\right)\right)\left(Z - \mu\right).
\end{aligned}$$

For a given investment threshold $\tilde{F}\left(Z\right)$, the information aggregation wedge is thus inversely related to the uninformed traders' demand elasticity η. Higher η lowers the price impact of private information, and thus the magnitude of the information aggregation wedge. At the extreme with infinite elasticity, price equals expected dividends and the wedge disappears. The other extreme ($\eta = 0$) corresponds to our baseline setup of section 3.2.

If we suppose as before that $\tilde{F}(Z)$ is chosen to maximize $\alpha P(Z) + (1-\alpha) V(Z)$, then the corresponding first-order conditions lead to

$$\tilde{F}(Z) - \mu = \left\{ 1 + \frac{\alpha(\gamma - 1)}{\left[1 + \frac{\sqrt{\beta}\eta}{\lambda+\beta+\beta\delta}\left(\rho + G\left(\tilde{F}(Z)\right)\right)\right]^2} \right\} (Z - \mu).$$

As long as $\alpha = 0$, investment remains undistorted. When $\alpha > 0$, there is over-investment for $Z > \mu$ and underinvestment for $Z < \mu$, but the inefficiency is reduced by the demand elasticity; η. For all η, $\tilde{F}(Z) \in [Z, \mu + k(Z-\mu)]$, with $\lim_{\eta\to\infty} \tilde{F}(Z) = Z$ and $\lim_{\eta\to 0} \tilde{F}(Z) = \mu + k(Z - \mu)$. Efficient investment arises as the uninformed demand becomes infinitely elastic. In this case, it is the uninformed traders who price the shares, arbitraging away the discrepancy between price and expected dividends, conditional on market information.

Therefore, all the previous results regarding over-valuation and price based incentives still apply, but their magnitude is inversely related to the value of η. When η is higher the uninformed traders are better able to arbitrage the difference between expected value and price. This reduces the absolute value of the information aggregation wedge for all realizations of the state Z, and mitigates the distortions induced by price-based incentives on investment decisions.

4.3 Additional information observed by the manager and/or the market

In the online appendix, we also consider two extensions in which the manager observes an additional exogenous signal y about the fundamental: $y \sim \mathcal{N}(\theta, \kappa^{-1})$, and the investment decision is taken after the observation of the price, and conditioned on both y and z. In the first case, we assume that y is observed by both the market and the manager, and therefore also enters the price. In the second case, y is only observed by the firm, and not by the market.

In both cases, we can derive closed-form characterizations for the information aggregation wedge and the investment behavior. The key to the characterization is that the investment threshold remains a linear function of the two signals, and therefore normally distributed from an ex ante perspective. This allows to compute unconditional expectations of prices and dividends, by first changing the variables of integration from y and z to $\tilde{F} = \tilde{F}(y, z)$ and z, and then changing the order of integration between $\tilde{F}$ and z. The solution reveals that the information aggregation wedge remains positive in all cases, even in the presence of additional external information. But there are some subtle and illuminating differences between the two cases.

In the first case where information is symmetric between the market and the firm, the characterization holds for all $\alpha \in [0,1]$. As in our benchmark model without exogenous information, the magnitude of the wedge is larger for higher values of α. Comparative statics w.r.t. the signal precision κ are less clear-cut, but it is the case that as y becomes infinitely precise ($\kappa \to \infty$), the information aggregation wedge disappears, and prices converge to the expected dividend values, for any value of α. In this limit, the external signal completely crowds the price out of the firm's investment decision, i.e. regardless of α, the investment will be entirely determined by y. At the same time, y also crowds out z from the market's conditional expectations of dividends. Therefore, the information aggregation wedge completely disappears, both because prices no longer respond to z, and because conditional on y, the covariance of the investment threshold with the market information z vanishes. In other words, prices in this limit are no longer relevant as signals to guide the firm's decision, nor are they relevant to aggregate market information, since all information is already publicly available through y.[13]

In the second case where the signal is private to the firm, our characterization holds only for the extreme values $\alpha \in \{0,1\}$ of pure price or dividend maximization, as the investment threshold is no longer linear at intermediate values. The main difference with the first case is that now the market no longer observes y directly, and therefore uses z to infer both y (and therefore the likely investment threshold), and θ, the returns to the investment. The market's forecast of the firm's signal y is also distorted relative to the "true" distribution of y. That is, the price is based on expectations of y that are conditional on the market signal z, and conditional on observing a private signal x also equal to z - just as with the forecast of θ, the market assigns additional weight to z in forecasting the firm's signal y. For intermediate values of α, the manager weighs $\mathbb{E}(\theta|y,z)$ and $\mathbb{E}(\theta|y,x=z,z)$ by both α and $1-\alpha$, and by the objective ("true") and market densities of y, conditional on z.

The wedge now consists of two components, both always strictly positive: The first one results as before from the response of investment to market information z, and behaves as in the model with symmetric information, and disappears when the signal y becomes infinitely precise. The second component is new and results from the discrepancy between the market's and an outsider's forecast of y (or equivalently $\tilde{F}$), conditional on z. This component does not vanish when the signal

[13]In fact, in this limit with $\kappa \to \infty$, we are back in a standard external state-space model of asset pricing and firm decisions, in which an exogenous state (signal) y on the one hand fully dictates the firm's efficient investment choice, and on the other hand perfectly dictates the valuation of its cash-flows by the market, with perfect separation of the firm's decision from the cash flow valuation.

y becomes infinitely precise, and more generally, the combination of the two components precludes us from making unambiguous statements about comparative statics. However, it still remains the case that as $\kappa \to \infty$, the investment threshold only depends on y, and no longer varies with z or α, and therefore the price-based incentives no longer influence the investment decision and the informational feedback.

In summary, our main results are robust with some qualifications to richer informational environments. External public or private information, if it is sufficiently precise, will crowd out market signals from the firm's investment decision, which reduces the share over-pricing, and mitigates the impact of price-based incentives on investment and share values. If the additional signal is publicly observed by the market and the firm, then this is the only modification with regards to our benchmark. When instead the signal is private and only observed within the firm, it still has the effect of disciplining investment decisions and reducing the impact of price-based incentives, but the market's forecast of y provides an additional source of the information aggregation wedge, which further raises share prices compared to expected dividends.

5 Concluding Remarks

We develop a model in which a firm's payoffs depends on fundamentals and the choice of investment. Information about the fundamentals is dispersed among traders in a financial market and partially aggregated in firm's share price, upon which the firm conditions its investment choice.

We find that market-generated information enhances firm's value by encouraging investment when high prices communicate good fundamentals, but limiting the losses by discouraging investment when low prices signal poor realizations. However, the interaction between dispersed information and endogenous investment is also the source of a systematic departure between equilibrium share prices and expected dividends –*the information aggregation wedge*: the market price attaches more weight to the market-generated information than is justified by its information content. The higher weight is perfectly consistent with the individual trader's rationality and arises from a compositional shift in the identity of traders that end up holding and pricing the shares. Moreover, because the firm responds to the price by investing more in good states than in bad ones, it exacerbates the price overreaction on the upsides relative to the downside. As a result, the information aggregation wedge is asymmetric, and the share price exceeds on average the expected dividend value of the firm.

Finally, we discuss the role of price-based managerial incentives in the presence of this wedge.

We find that compensation tied to share prices may enhance share overvaluation and induce excess volatility in investment, as managers try to cater investment policies to those traders who have the largest impact on market prices.

Our model has two predictions that align with empirical evidence. First, it suggests that the value enhancing effects of market-specific information in guiding real investment depend importantly on the extent of informed trading activity. This fits the evidence provided by Chen, Goldstein and Jiang (2007) who study the impact of informed trading in the sensitivity of real investment to price changes. They find stronger investment sensitivity in firms whose shares are traded by more informed traders, as measured by PIN (*probability of informed trading* – Easley et al. (1996)).[14] Second, Polk and Sapienza (2009) provide support to our findings regarding the impact of stock-based compensation. They test a "catering" theory using discretionary accruals as a proxy for mispricing,[15] finding a positive relation between share overvaluation and excess investment after controlling for Tobin's Q. This relation is stronger for firms with higher share turnover, which could proxy for traders' short-term horizons. Moreover, they find that firms with high excess investment subsequently have low share returns, the more so the larger is their measure of mispricing. This suggests that such investment behavior is indeed inefficient.

While our model has taken the manager's objective as given, the design of optimal incentive structures in the presence of a wedge between expected dividend and prices remains an important question for future research. The contrast of our results with the standard view by which price-based incentives align shareholder and manager interests is particularly striking, and an immediate question for further interest is how to optimally balance the incentive distortions that arise from agency frictions inside a firm with the market frictions that arise outside, especially if one wishes to use market prices for incentive purposes. Similar questions may be asked regarding regulatory interventions or accounting rules that focus on market value as the relevant measure to evaluate future cash flow risks. More generally, our model of financial markets with noisy information aggregation may provide a useful building block for future work that integrates the formation of prices in a financial market with the frictions that arise from decisions within a firm.

[14] Roll, Schwartz and Subrahmanayam (2009) provide related evidence arguing that developed options markets for a firm's share stimulate the entry of informed traders. They find that firms with deeper options markets have higher sensitivity of corporate investment to share prices, which translates into higher values of Tobin's Q.

[15] Discretionary accruals measure the extent to which a firm has abnormal non-cash earnings. Firms with high discretionary accruals typically have relatively low share returns in the future, suggesting that discretionary accruals artificially drive up prices temporarily.

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6 Appendix

6.1 Proofs for sections 2-3

Proof of Lemma 1. Part (i): By market-clearing, $z = \hat{x}(P(z))$ and $\hat{x}(P(z')) = z'$, and therefore $z = z'$ if and only if $P(z) = P(z')$.

Part (ii): Since $P(z)$ is invertible, observing P is equivalent to observing $z = \hat{x}(P(z))$ in equilibrium. But $z|\theta \sim \mathcal{N}\left(\theta, (\beta\delta)^{-1}\right)$, from which the characterization of $H(\cdot|x, P)$ follows immediately from Bayes' Law. ■

Proof of Proposition 1. Substituting the market-clearing condition $\hat{x}(P) = z$ and the investment threshold $\tilde{F}(z) = \tilde{F}(P(z))$ into (10), a price function $P(z)$ is part of an equilibrium if and only if it satisfies (11) and is invertible. Given the investment threshold $\tilde{F}(z)$, firm's expected dividend value is $V(z) = \left(\rho + G\left(\tilde{F}(z)\right)\right) E(\theta|z) - \int_{\underline{F}}^{\tilde{F}(z)} f dG(f)$. ■

Proof of Lemma 2. Taking the derivative with respect to Z in equation (14) gives

$$P'(\cdot) \quad = \quad \gamma \cdot (\rho + G(Z)) + g(Z)(Z-\mu)(\gamma - 1).$$

Since $\gamma > 1$, we can write the following inequality for values $Z < \mu$,

$$P'(\cdot) \quad > \quad \rho + G(Z) + g(Z)(Z-\mu),$$

which is positive for all Z whenever the distribution $G(F)$ satisfies the condition stated in the Lemma. ■

Proof of Proposition 2. (i) Since $V'(Z) = \rho + G(Z) > 0$ and $V''(Z) = g(Z) > 0$, $V(Z)$ is increasing and convex, so an increase in σ_Z^2 strictly increases $\mathbb{E}(V(z))$. Moreover, notice that $\mathbb{E}(W(z)) = (\gamma-1)\mathbb{E}((G(Z) - G(\mu))(Z-\mu))$. Since $(G(Z) - G(\mu))(Z-\mu)$ is strictly positive for all $Z \neq \mu$, and strictly quasi-convex in Z, an increase in σ_Z^2 strictly increases $\mathbb{E}(W(z))$.The result for $\mathbb{E}(P(z))$ then follows from the statements about $\mathbb{E}(V(z))$ and $\mathbb{E}(W(z))$. Finally, (ii) is immediate given that γ does not affect $V(Z)$, but linearly scales up $W(Z)$. ■

Proof of Proposition 3. (i) From (19), we have

$$\begin{aligned}\mathbb{E}(W(Z)) \quad &= \quad (\gamma-1)\sigma_Z^2 \int_0^\infty \frac{G(\mu+\sigma_Z u) - G(\mu - \sigma_Z u)}{2\sigma_Z u} 2u^2 \phi(u)\,du \\ &\leq \quad (\gamma-1)\sigma_Z^2 \|g\| \int_0^\infty 2u^2\phi(u)\,du.\end{aligned}$$

The result then follows from noting that $\int_0^\infty 2u^2\phi(u)\,du = 1$ and

$$(\gamma-1)\sigma_Z^2 = \left(\frac{\beta+\beta\delta}{\lambda+\beta+\beta\delta} - \frac{\beta\delta}{\lambda+\beta\delta}\right)\frac{1}{\lambda}.$$

(ii) Taking the limit in the expression above as $\delta \to 0$ and $\sigma_Z^2 \to 0$, we have $\lim_{\delta\to 0}(\gamma-1)\sigma_Z^2 = \beta/(\beta+\lambda)\cdot\lambda^{-1}$, and

$$\lim_{\sigma_Z^2 \to 0} \int_0^\infty \frac{G(\mu+\sigma_Z u) - G(\mu-\sigma_Z u)}{2\sigma_Z u} 2u^2\phi(u)\,du = g(\mu)\int_0^\infty 2u^2\phi(u)\,du = g(\mu).$$

(iii) From (19), it is immediate that $\mathbb{E}(W(Z))$ is maximized when $G(\cdot) = \mathbb{I}_{F>\mu}$, an indicator function that assigns 1, whenever $F > \mu$, i.e.

$$\mathbb{E}(W(Z)) \leq (\gamma-1)\sigma_Z \int_0^\infty u\phi(u)\,du = (\gamma-1)\sigma_Z \frac{1}{\sqrt{2\pi}}.$$

This corresponds to a scenario where the investment cost distribution is highly concentrated around the prior mean μ, so that a small change in the firm's posterior can have a large effect on its investment probability. Moreover for all $\varepsilon > 0$, there exists $\varepsilon' > 0$, such that whenever $\max_F \|G(F) - \mathbb{I}_{F>\mu}\| \leq \varepsilon'$, $\mathbb{E}(W(Z)) \geq (\gamma - 1)\sigma_Z/\sqrt{2\pi} - \varepsilon$. But from the above, it follows immediately that $\lim_{\delta\to\infty}(\gamma - 1)\sigma_Z = \infty$, so that $\lim_{\delta\to\infty}\lim_{G(\cdot)\to\mathbb{I}_{F>\mu}}\mathbb{E}(W(Z)) = \infty$. Using the limit characterization from (ii), $\lim_{G(\cdot)\to\mathbb{I}_{F>\mu}}\lim_{\delta\to\infty}\mathbb{E}(W(Z)) = \infty$, as long as $\lim_{G(\cdot)\to\mathbb{I}_{F>\mu}} g(\mu) = \infty$. ■

Proof of Proposition 4. Equation (21) follows immediately from (20). Substituting this into (12) and (11) gives (23) and (22). To check that this is an equilibrium, consider the derivative of the price function: $P'(Z) = \gamma(\rho + G(\mu + k(Z-\mu))) + (\gamma - k)k(Z-\mu)g(\mu + k(Z-\mu))$, which is strictly positive under the assumption from Lemma 2. ■

Proof of Proposition 5. Proof: Part (i) is immediate from the definition of k. For (ii) notice that

$$\begin{aligned}
\frac{\partial V}{\partial \alpha} &= \frac{\partial V}{\partial k}(\gamma-1) = g\left(\tilde{F}(Z)\right)(Z-\mu)(\gamma-1)\left(Z-\tilde{F}(Z)\right)\\
&= -g\left(\tilde{F}(Z)\right)(Z-\mu)^2\alpha(\gamma-1)^2 < 0, \text{ for } Z\neq\mu,\\
\frac{\partial P}{\partial \alpha} &= \frac{\partial P}{\partial k}(\gamma-1) = g\left(\tilde{F}(Z)\right)(Z-\mu)\left(\mu+\gamma(Z-\mu)-\tilde{F}(Z)\right)(\gamma-1)\\
&= g\left(\tilde{F}(Z)\right)(Z-\mu)^2(1-\alpha)(\gamma-1)^2 > 0, \text{ for } Z\neq\mu,\\
\frac{\partial W}{\partial \alpha} &= \frac{\partial P}{\partial \alpha} - \frac{\partial V}{\partial \alpha} = g\left(\tilde{F}(Z)\right)(Z-\mu)^2(\gamma-1)^2 > 0, \text{ for } Z\neq\mu.
\end{aligned}$$

For (iii), we write $\mathbb{E}(W(Z))$ along the same lines as (19) as

$$\mathbb{E}(W(Z)) = (\gamma-1)\sigma_Z\int_0^\infty (G(\mu+k\sigma_Z u) - G(\mu - k\sigma_Z u))u\phi(u)\,du,$$

and the comparative statics for $\mathbb{E}(W(Z))$ then follow immediately. Likewise, we write $\mathbb{E}(P(Z))$ as

$$\mathbb{E}(P(Z)) = \rho\mu + \mathbb{E}\left(\int_{\underline{F}}^{\tilde{F}(Z)} G(f)\,df\right) + \frac{\gamma-k}{k}cov(G\left(\tilde{F}(Z)\right), \tilde{F}(Z)).$$

Both terms in $\mathbb{E}(P(Z))$ are increasing in the variance of $\tilde{F}(Z)$, which is equal to $k^2\sigma_Z^2$, and therefore increasing in both γ and σ_Z^2. In addition, $(\gamma-k)/k$ is increasing in γ, which completes the comparative statics arguments for $\mathbb{E}(P(Z))$ w.r.t. γ.

For (iv), notice that $\frac{\partial V}{\partial \gamma} = \frac{\partial V}{\partial k}\alpha < 0$, and therefore $\mathbb{E}(V(Z))$ is also decreasing in γ. To see that the effect of σ_Z^2 on expected dividends, $\mathbb{E}(V(Z))$ is ambiguous notice that

$$V''(Z) = \frac{2-k}{k}g\left(\tilde{F}(Z)\right) - \frac{k-1}{k}g'\left(\tilde{F}(Z)\right)\left(\tilde{F}(Z)-\mu\right).$$

Now, if $k \in (1,2)$ and $g(\cdot)$ is single-peaked with a maximum at $F = \mu$, then $g'(F)(F-\mu) < 0$ and $V''(Z) > 0$, implying that an increase in the value of market information σ_Z^2 unambiguously increases $\mathbb{E}(V(Z))$. If on the other hand $k > 2$ and $g(\cdot)$ is uniform (with $\underline{F} < \mu$), then $V''(Z) < 0$ for all Z s.t. $g\left(\tilde{F}(Z)\right) > 0$. In this case, if σ_Z^2 is sufficiently low (so that with sufficiently high probability the posterior Z is close to the prior μ), $\mathbb{E}(V(Z))$ is decreasing in the quality of market information σ_Z^2.

(v) The result follows from the expression for $\mathbb{E}(W(Z))$ derived under (iii), and from observing that $\lim_{\delta\to 0} k\sigma_Z = \infty$, so that $\lim_{\delta\to 0}(G(\mu + k\sigma_Z u) - G(\mu - k\sigma_Z u)) = 1$ for all $u > 0$. But then $\lim_{\delta\to 0}\mathbb{E}(W(Z)) = \lim_{\delta\to 0}(\gamma - 1)\sigma_Z/\sqrt{2\pi} = \infty$. ■