DATING THE TIMELINE OF FINANCIAL BUBBLES DURING THE SUBPRIME CRISIS

By

Peter C. B. Phillips and Jun Yu

September 2010

COWLES FOUNDATION DISCUSSION PAPER NO. 1770

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS
YALE UNIVERSITY
Box 208281
New Haven, Connecticut 06520-8281

http://cowles.econ.yale.edu/

Dating the Timeline of Financial Bubbles During the Subprime Crisis[1]

Peter C. B. Phillips
Yale University, University of Auckland,
University of Southampton & Singapore Management University

Jun Yu
Singapore Management University

August 31, 2010

[1]Phillips acknowledges support from the NSF under Grant No. SES 06-47086. Yu acknowledges support from the Singapore Ministry of Education AcRF Tier 2 fund under Grant No. T206B4301-RS. Peter C. B. Phillips, Cowles Foundation for Research in Economics, Yale University, Box 208281, Yale Station, New Haven, Connecticut 06520-8281. Email: peter.phillips@yale.edu. Jun Yu, School of Economics and Sim Kee Boon Institute for Financial Economics, Singapore Management University, 90 Stamford Road, Singapore 178903. Email: yujun@smu.edu.sg.

Abstract

A new recursive regression methodology is introduced to analyze the bubble characteristics of various financial time series during the subprime crisis. The methods modify a technique proposed in Phillips, Wu and Yu (2010) and provide a technology for identifying bubble behavior and consistent dating of their origination and collapse. The tests also serve as an early warning diagnostic of bubble activity. Seven relevant financial series are investigated, including three financial assets (the Nasdaq index, home price index and asset-backed commercial paper), two commodities (the crude oil price and platinum price), one bond rate (Baa), and one exchange rate (Pound/USD). Statistically significant bubble characteristics are found in all of these series. The empirical estimates of the origination and collapse dates suggest an interesting migration mechanism among the financial variables: a bubble first emerged in the equity market during mid-1995 lasting to the end of 2000, followed by a bubble in the real estate market between January 2001 and July 2007 and in the mortgage market between November 2005 and August 2007. After the subprime crisis erupted, the phenomenon migrated selectively into the commodity market and the foreign exchange market, creating bubbles which subsequently burst at the end of 2008, just as the effects on the real economy and economic growth became manifest. Our empirical estimates of the origination and collapse dates match well with the general datetimes of this crisis put forward in a recent study by Caballero, Farhi and Gourinchas (2008).

Keywords: Financial bubbles, Crashes, Date stamping, Explosive behavior, Mildly explosive process, Subprime crisis, Timeline.

JEL classification: C15, G12

There is a very real danger, fellow citizens, that the Icelandic economy in the worst case could be sucked into the whirlpool, and the result could be national bankruptcy (Prime Minister Geir Haarde, televised address to Icelandic Nation, October 8, 2008)

Between 40 and 45 percent of the world's wealth has been destroyed in little less than a year and a half. (Stephen Schwarzman, March 11, 2009)

Federal Reserve policymakers should deepen their understanding about how to combat speculative bubbles to reduce the chances of another financial crisis (Donald Kohn, Federal Reserve Board Vice Chairman, March 24, 2010)

1 Introduction

Financial bubbles have been a longstanding topic of interest for economists, involving both theorists and empirical researchers. Some of the main issues have focused on mechanisms for modeling bubbles, reconciling bubble-like behavior in the context of rational expectations of future earnings, mechanisms for detecting bubbles, and measuring their extent, exploring causes and the psychology of investor behavior, and considering suitable policy responses. While there is general agreement that financial bubbles give rise to misallocation of resources and can have serious effects on real economic activity, as yet there has been little consensus among economists and policy makers on how to address the many issues raised above.

The global financial turmoil over 2008-2009, triggered by the subprime crisis in the US and its subsequent effect on commodity markets, exchange rates and real economic activity, has led to renewed interest among economists in financial bubbles and their potential global consequences. There is now widespread recognition among policy makers as well as economists that changes in the global economy over the last decade, far from decoupling economic activity as was earlier believed, have led to powerful latent financial linkages that have increased risks in the event of a large common shock. The magnitude of the crisis is so large, the mechanism so complex, and the consequences so important to the real economy that understanding the phenomena, exploring its causes and mapping its evolution have presented major challenges to the economics profession. As the headers that lead this article indicate, a substantial percentage of the world's accrued wealth has been destroyed within 18 months of the subprime crisis with manifold effects ranging from the collapse of major financial institutions to the near bankruptcy of national economies. There is also recognition that new empirical methods are needed to improve understanding of speculative phenomena and to provide early warning diagnostics of

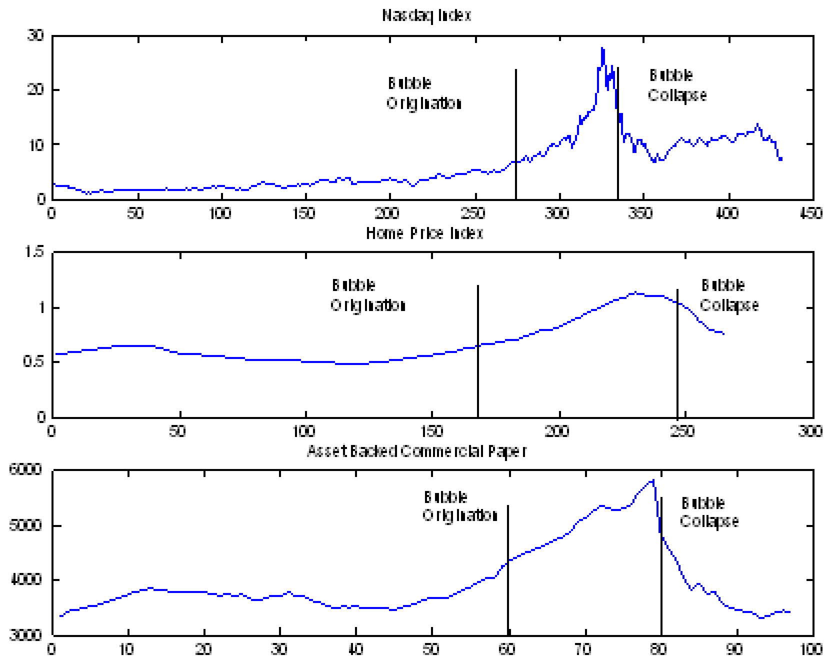

Figure 1: Time series plots of real prices for three financial assets: monthly observations on the Nasdaq index from February 1973 to January 2009; monthly observations on the house price index from January 1987 to January 2009; monthly observations on the outstanding value of asset backed commercial paper from January 2001 to January 2009. All series are normalized by the CPI. The estimated bubble origination and collapse dates are also shown on the figures.

financial bubbles.

The recent background of financial exuberance and collapse with concatenating effects across markets and nations provides a rich new environment for empirical research. The most urgent ongoing questions relate to matters of fiscal, monetary, and regulatory policies for securing financial stability and buttressing real economic activity. In this regard, there have been serious disagreements among economists and policy makers about the effectiveness and consequences of various bailout and recovery plans proposed by North American, European and Asian governments to deal with the financial crisis. Beyond these immediate policy issues are underlying questions relating to the emergence of the phenomena and its evolutionary course through the financial and economic systems. It is these latter issues that form the focus of interest of the present paper.

The subprime crisis is not an isolated empirical event. In a recent article, Caballero, Farhi and Gourinchas (2008a, CFG hereafter) argued that the Internet bubble in the 1990s, the asset bubbles over 2005-2006, the subprime crisis in 2007, and the commodity bubbles of 2008 are all closely related. Similar views of the interconnectedness of the crisis phenomena are held by most economists and media commentators and this interpretation is also generally supported by the timeline in which the various crisis events have unfolded. CFG go further and put forward a sequential hypothesis concerning bubble creation and collapse that accounts for the course of the financial turmoil in the U.S. economy using a simple general equilibrium model without monetary factors but with goods that may be partially securitized. Date stamping the timeline of the origination and collapse of the various bubbles is a critical element in the validity of this sequential hypothesis.

The present paper uses new econometric methodology to test if and when bubbles emerged and collapsed in the stock market, the real estate market, the mortgage market, the commodity market, and the foreign exchange market over the period surrounding the subprime crisis. Many series are studied. In particular, we investigate the bubble characteristics in the Nasdaq index over February 1973 to January 2009, the U.S. house price index over January 1987 to January 2009, outstanding asset backed commercial paper (ABCP) over January 2001 to January 2009, the price of crude oil over January 1999 to January 2009, platinum prices over January 1999 to January 2009, Baa bond rates over January 3, 2006 to January 30, 2009, and Pound/USD exchange rates over March 17, 2006 to March 20, 2009. Figs. 1-3 show the time series plot of the first three, next two, and last two series, respectively. Our methods enable us to determine whether a bubble emerged in each series, date stamp the origination in that event, and correspondingly assess whether the bubble collapsed and the date of that collapse. The empirical date stamps so determined are then matched against the hypothesized sequence of events described in the model of CFG.

The econometric methods used here are closely related to those proposed in Phillips, Wu and Yu (2009, PWY hereafter). In particular, the methods rely on forward recursive regressions coupled with sequential right-sided unit root tests. The sequential tests assess period by period evidence for unit root behavior against mildly explosive alternatives. Mildly explosive behavior may be modeled by an autoregressive process with a root (ρ) that exceeds unity but that is still in the general vicinity of unity. Phillips and Magdalinos (2007a, 2007b, PM hereafter) show that this 'mildly explosive' vicinity of unity can be successfully modeled in terms of deviations of the form $\rho - 1 = c/k_n > 0$, where c is a positive constant and k_n is a sequence that passes to infinity with, but more slowly than, the sample size n, so that $\rho \to 1$. These processes therefore involve only mild departures from strict (rational) martingale behavior in markets. They include submartingale processes of the type that have been used to model rational bubble behavior in finance (Evans, 1991; Campbell, Lo and McKinley, 1998). PM (2007a, 2007b) have investigated this class of process, developed a large sample asymptotic theory, and shown that these models are amenable to econometric inference, unlike purely explosive processes for which no central limit theory is applicable. An important difference between the methods proposed in the current paper and those in PWY lies in the manner in which the initialization is handled. In PWY, the initial condition is fixed to be the first observation in the full sample whereas in the current paper the initial observation is selected based on an information criterion. The use of information criteria in the selection of the the initial observation allows for sharper identification of the bubble origination date.

PWY applied forward recursive regression methods to Nasdaq stock prices during the 1990s, and using sequential tests against mildly explosive alternatives were able to date-stamp the origination of financial exuberance in the Nasdaq market to mid-1995, prior to the famous remark of Alan Greenspan in December 1996 about irrational exuberance in financial markets. This test therefore revealed that there was anticipatory empirical evidence supporting mildly explosive behavior in stock prices over a year prior to Greenspan's remarks. In ongoing work, Phillips and Yu (2009) have developed a limit theory for this date stamping technology and checked the finite sample capability of this procedure to identify and date bubble behavior. The date stamp estimators were shown to be consistent for the origination and collapse of bubble behavior and the dating mechanism was shown to work well in finite samples.

The present paper uses this methodology to explore the sequential pattern of events of the current financial crisis. Dating helps to characterize the phenomena by identifying the individual events and by fixing their extent and sequencing. It may be viewed as a first step in understanding the phenomena and in searching for causes of the behavioral changes involved in bubble origination and collapse. Date stamping also assists in evaluating hypotheses about the concatenation of bubble activity over time and across markets, such as those developed in CFG. The forward recursive regression approach used here enables early identification of the appearance of mildly explosive behavior in asset prices, thereby providing anticipatory evidence of a (local) move away from martingale behavior. This evidence may be used as an early warning diagnostic of (financial) exuberance and thereby assist policy makers in surveillance and regulatory actions, as urged by Fed Vice Chairman Donald Kohn in the header of this article. Similarly, the approach helps to identify a subsequent switch back to martingale behavior as explosive sentiment collapses.

Empirical evidence of emergent mildly explosive behavior is found in many of the time series studied here, and in all of the series (except for the Pound/USD exchange rate) manifesting mildly explosive behavior there is further evidence of subsequent collapse. Figs. 1-3 show

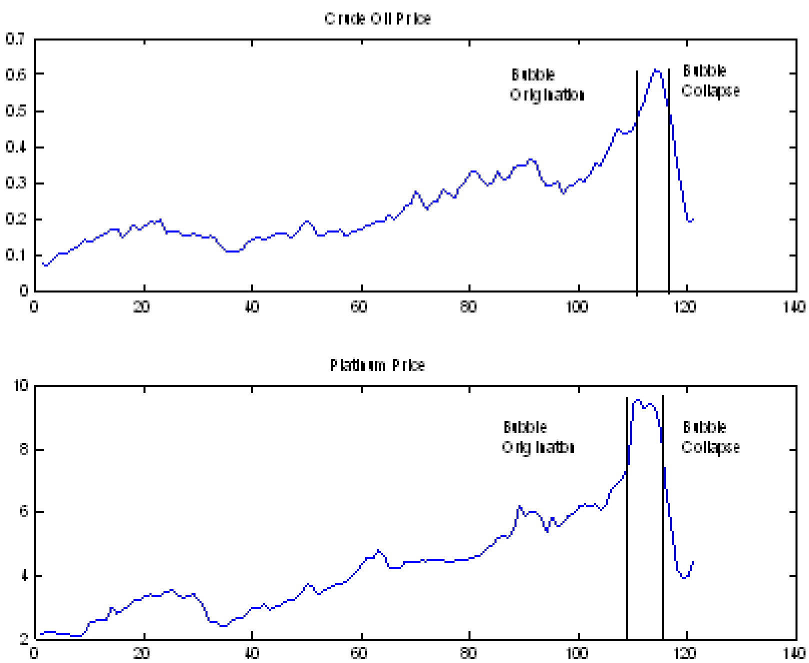

Figure 2: Time series plots of real prices for two commodities: monthly observations of crude oil prices and monthly observations of platinum prices, both from January 1999 to January 2009. Both series are normalized by the CPI. The estimated bubble origination and collapse dates are also shown on the figures.

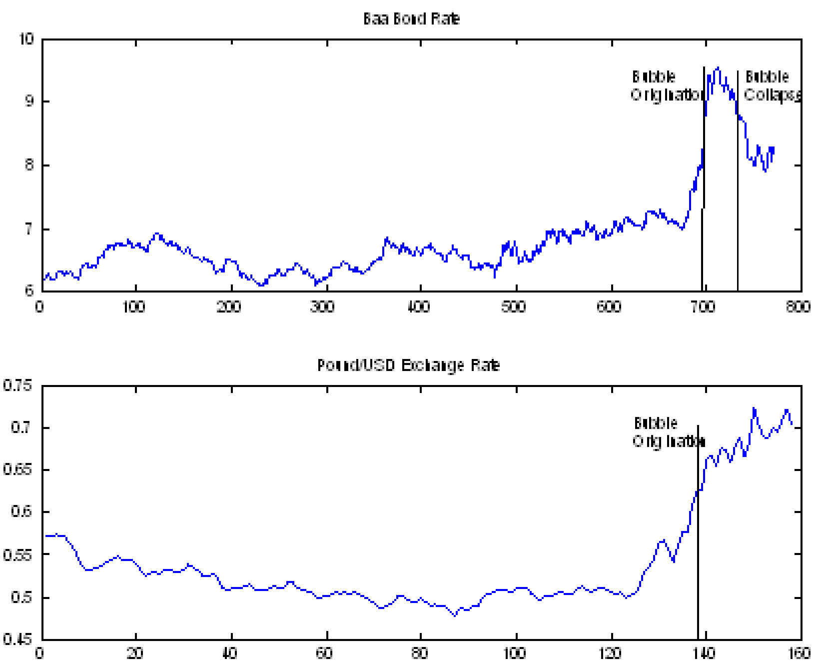

Figure 3: Time series plots of two financial variable: daily observations of Baa bond rates from January 3, 2006 to January 30, 2009 and weekly observations of Pound/USD exchange rates from March 17, 2006 to March 20, 2009. The estimated bubble origination and collapse dates are also shown on the figures.

the origination and collapse dates for the bubbles identified in the seven financial time series mentioned earlier. For the three series depicted in Fig. 1, the bubbles emerged and collapsed prior to the subprime crisis. For the two series depicted in Fig. 2 and the two series depicted in Fig. 3, the bubbles all emerged after the subprime crisis. These findings reveal a sequence of mildly explosive events each followed by a financial collapse that corroborates the sequential hypothesis given in CFG. Consideration of a wider group of related financial series following the eruption of the subprime crisis indicates that bubbles of the type found in the series in Figs. 2-3 are not always evident in other commodities or currencies. Accordingly, the empirical evidence supports a selective migration of the bubble activity through financial markets as the subprime crisis evolved and liquid funds searched for safe havens.

The plan of the paper is as follows. Section 2 reviews the econometric methodology for dating bubble characteristics, discusses rational bubble and variable discount rate sources of financial exuberance, outlines some of the relevant facts concerning the subprime crisis, and relates the timeline implications of the theoretical results obtained in CFG (2008a). Section 3 describes the data that is used in the present empirical study. Section 4 presents the empirical findings and matches the estimates to the theory of CFG (2008a). Section 5 concludes.

2 Bubbles, the Subprime Crisis, and Econometric Dating

2.1 Bubbles and Crashes

In the popular press, the term "financial bubble" refers to a situation where the price of a financial asset rapidly increases and does so in a speculative manner that is distinct from what is considered to be the asset's intrinsic value. The term carries the innuendo that the increase is not justified by economic fundamentals and that there is, accordingly, risk of a subsequent collapse in which the asset price falls precipitously. In such cases, the bubble phenomenon is typically confirmed in retrospect.

A common definition that makes this usage precise is that bubble conditions arise when the price of an asset significantly exceeds the fundamental value that is determined by the discounted expected value of the cash flows that ownership of the asset can generate. However, discount rates may be variable and, as demonstrated below, the time profile of the discount rate can have important effects on the characteristics of the fundamental price and may even propagate explosive price behavior.

An important secondary characteristic of the bubble phenomenon is that during both the run up and run down periods the asset is subject to high volume trading in which the direction of change is widely anticipated (and relied upon), as distinct from normal market conditions in which the asset price follows a near martingale. It is this deviation from martingale behavior that provides a mechanism for identifying both the emergence of the boom phase of a bubble behavior and its subsequent crash.

This distinction is recognized in the rational bubble literature, which characterizes the boom phase of a bubble in terms of explosive dynamics or submartingale behavior. This property contrasts with the efficient market martingale property, which implies unit root time series dynamic behavior. To explain the difference in terms of the commonly used present value model, let P_t be the stock price at time t before the dividend payout, D_t be the dividend payoff from the asset at time t, and r be the discount rate ($r > 0$). The standard no arbitrage condition implies that

$$P_t = \frac{1}{1+r} E_t(P_{t+1} + D_{t+1}), \tag{1}$$

and recursive substitution yields

$$P_t = F_t + B_t, \tag{2}$$

where $F_t = \sum_{i=1}^{\infty} (1+r)^{-i} E_t(D_{t+i})$ and

$$E_t(B_{t+1}) = (1+r)B_t. \tag{3}$$

Hence, the asset price is decomposed into two components, a "fundamental" component, F_t, that is determined by expected future dividends, and a supplementary solution corresponding to the "bubble" component, B_t. In the absence of bubble conditions, $P_t = F_t$. Otherwise, $P_t = F_t + B_t$ and price embodies the explosive component B_t, which satisfies the submartingale property (3). Consequently, under bubble conditions, P_t will manifest the explosive behavior inherent in B_t. This explosive property is very different from the random wandering (or unit root) behavior that is present in F_t when D_t is a martingale and that is commonly found for asset prices in the empirical literature.

Over long periods of time, some asset prices like equities also tend to manifest empirical evidence of a drift component. Unit root time series with a drift can generate periods of run-up if the variance of the martingale component is small and the drift is strong enough. But accumulated gains in such cases are at most of $O(n)$ for sample size n. In practice, of course, the drift component is usually small and is generally negligible over short periods, so the unit root behavior is the dominant characteristic and clear evidence of gains only shows up over long horizons. On the other hand, the run up rate in an explosive process is $O((1+r)^n)$ for some $r > 0$, as in (3), and is therefore much greater. This difference between linear and exponential growth combined with the nonlinear curvature in an explosive process are testable properties distinguishing the two processes. In terms of model (1) and its solution (2), both B_t and P_t increase rapidly during the boom phase of the bubble according to $E_t(B_{t+h}) = (1+r)^h B_t$ and the initialization $B_0 > 0$. But when the bubble conditions collapse and the particular solution disappears, $P_t = F_t$ which corresponds to a sudden collapse in the asset price. If the dividend process D_t follows a martingale, reflecting market conditions generating cash flows, then F_t is similarly a martingale and is cointegrated with D_t. Under such conditions, the presence of an additional "rational bubble" submartingale component B_t in P_t can account for an explosive-type run up in the asset price P_t.

Explosiveness in B_t and hence in P_t suggests that P_t is predictable during an explosive period. While this may be at odds with the efficient market hypothesis, the predictability in stock returns at short horizons is consistent with what has been documented in the recent empirical literature – see, for example, Ang and Bekaert (2006). At longer horizons, explosiveness is subject to collapse, generating a (long run) martingale like feature in the price and making returns more difficult to predict. This latter finding is also empirically documented in Ang and Bekaert (2006).

Importantly, making the discount factor r_t either stationary or integrated of order one does not change qualitatively our analysis because the implications for the statistical properties of F_t, B_t and P_t are the same as with the constant r. For example, if r_t is stationary, (3) becomes

$$E_t(B_{t+1}) = (1 + r_t)B_t. \tag{4}$$

Then, if (3) is fitted, $r = \left(\Pi_{t=1}^{T}(1 + r_t)\right)^{1/T} > 1$, implying an explosive process for B_t and hence P_t, even if F_t itself is not explosive.

2.2 The Effects of a Time Varying Discount Rate

This paper interprets explosiveness in price as sufficient evidence for bubbles and this interpretation holds true under a varieties of assumptions on the discount rate. As indicated above, certain time profiles for the discount rate can have an important effect on the characteristics of the fundamental price. The present section illustrates this possibility by developing a simple propagating mechanism for explosive behavior in the fundamental price under a time varying discount rate.

If dividends grow at a constant rate r_D with $r_D < r$ in (1),[1] the fundamental value of the

[1] This assumption obviously violates the assumption we adopted earlier, namely, constancy, stationarity or integration of order 1.

stock price

$$F_t = \frac{D_t}{r - r_D}. \tag{5}$$

This is the well-known Gordon growth model. It is evident that in this case the fundamental value can be very sensitive to changes in r when r is close to r_D. In fact, the fundamental value diverges as $r \searrow r_D$, so that a price run-up is evidently possible under certain time profiles for the discount rate. This simple Gordon model reveals the potential impact of a time varying discount rate, but it provides no price dynamics. The following argument provides an analytic formulation that shows how an explosive time path in fundamental values can be generated by time variation in the discount rate.

Consider a continuous time version of (5) with time varying discount rate r_t, viz.,

$$F_t = \int_0^\infty \exp(-sr_{t+s}) E_t D_{t+s} ds. \tag{6}$$

Suppose dividends have a constant expected growth rate r_D such that

$$E_t D_{t+s} = \exp(r_D s) D_t, \tag{7}$$

and then D_t is a martingale when $r_D = 0$. Combining (6) and (7)

$$F_t = \int_0^\infty \exp(-s(r_{t+s} - r_D)) D_t ds. \tag{8}$$

Given some fixed time point t_b, constants $c_a > 0$ and $\lambda_1 > \lambda_2 > 0$, let the time profile of the discount rate r_{t+s} for $t \in (0, t_b]$ be as follows:

$$r_{t+s} = \begin{cases} r_D + \frac{t_b - t - s}{s} c_a + \frac{\lambda_1}{s} & \text{for } 0 \leq s < t_b - t \\ r_D + c_a + \frac{\lambda_2}{s} & \text{for } s \geq t_b - t \end{cases}. \tag{9}$$

Then, the discount rate decreases towards some level $r_D + \frac{\lambda_1}{t_b - t}$ as $t + s \nearrow t_b$ and jumps to the level $r_D + c_a + \frac{\lambda_2}{t_b - t}$ immediately thereafter, as shown in Fig. 4. Thus, the time profile of the discount factor has a structural break at t_b in which a higher rate of discounting occurs at t_b. The break itself widens asymptotically as $t \nearrow t_b$.

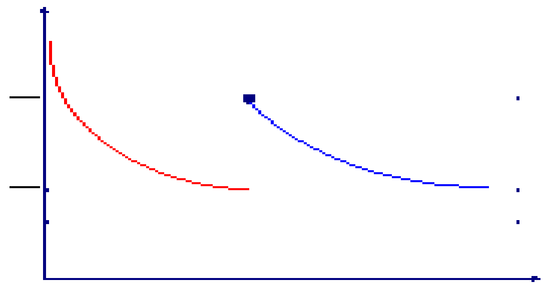

Figure 4: Time Path of the Discount Rate r_{t+s} in (9)

We then have

$$\begin{aligned} F_t/D_t &= \int_0^\infty \exp(-s(r_{t+s}-r_D))ds \\ &= \int_0^{t_b-t} \exp(-c_a(t_b-t-s)-\lambda_1)ds + \int_{t_b-t}^\infty \exp(-c_a s-\lambda_2)ds \\ &= e^{-\lambda_1}\left[\frac{e^{-c_a(t_b-t-s)}}{c_a}\right]_0^{t_b-t} + e^{-\lambda_2}\left[\frac{e^{-c_a s}}{-c_a}\right]_{t_b-t}^\infty \\ &= \frac{e^{-\lambda_1}}{c_a}\left[1-e^{-c_a(t_b-t)}\right] + \frac{e^{-\lambda_2}}{c_a}e^{-c_a(t_b-t)} \\ &= \frac{e^{-\lambda_1}}{c_a} + \frac{\left(e^{-\lambda_2}-e^{-\lambda_1}\right)}{c_a}e^{-c_a(t_b-t)} := \sigma_t, \end{aligned}$$

and the time path of F_t/D_t is explosive over $t \in (0, t_b]$. Over this interval, F_t evolves according to the differential equation

$$dF_t = \left(e^{-\lambda_2}-e^{-\lambda_1}\right)e^{-c_a(t_b-t)}D_t dt + \sigma_t dD_t.$$

Since $c_a F_t/D_t = e^{-\lambda_1} + \left(e^{-\lambda_2}-e^{-\lambda_1}\right)e^{-c_a(t_b-t)}$, we have

$$dF_t = \frac{\left(e^{-\lambda_2}-e^{-\lambda_1}\right)e^{-c_a(t_b-t)}}{e^{-\lambda_1}+\left(e^{-\lambda_2}-e^{-\lambda_1}\right)e^{-c_a(t_b-t)}}c_a F_t dt + \sigma_t dD_t, \quad \text{for } t \in (0, t_b].$$

For t close to t_b the generating mechanism for F_t is approximately

$$\begin{aligned} dF_t &= \frac{\left(e^{-\lambda_2} - e^{-\lambda_1}\right)}{e^{-\lambda_1} + \left(e^{-\lambda_2} - e^{-\lambda_1}\right)} c_a F_t dt + \sigma_t dD_t \\ &= \left\{1 - e^{-(\lambda_1 - \lambda_2)}\right\} c_a F_t dt + \sigma_t dD_t, \end{aligned}$$

which is an explosive diffusion because

$$c_b = \left\{1 - e^{-(\lambda_1 - \lambda_2)}\right\} c_a > 0,$$

since $c_a > 0$ and $e^{-(\lambda_1 - \lambda_2)} < 1$. The discrete time path of F_t in this neighbourhood is therefore propagated by an explosive autoregressive process with coefficient $\rho = e^{c_b} > 1$.

The heuristic explanation of this behavior is as follows. As $t \nearrow t_b$ there is growing anticipation that the discount factor will soon increase. Under such conditions, investors anticipate the present to become more important in valuing assets. This anticipation in turn leads to an inflation of current valuations and price fundamentals F_t become explosive as this process continues.

On the other hand, for $t > t_b$ we have

$$r_{t+s} = \; r_D + c_a + \tfrac{\lambda_2}{s} \quad \text{for } s > 0 \;,$$

and then

$$\begin{aligned} F_t/D_t &= \int_0^\infty \exp(-s(r_{t+s} - r_D))ds \\ &= \int_0^\infty \exp(-c_a s - \lambda_2)ds \\ &= e^{-\lambda_2}\left[\frac{e^{-c_a s}}{-c_a}\right]_0^\infty = \frac{e^{-\lambda_2}}{c_a}. \end{aligned}$$

So, $F_t = \frac{e^{-\lambda_2}}{c_a} D_t$ for $t > t_b$ and price fundamentals are collinear with D_t. When D_t is a Brownian motion or an integrated process in discrete time, F_t and D_t are cointegrated. Thus, after time t_b, price fundamentals comove with D_t.

It follows that the time profile (9) for the discount rate r_t induces a subinterval of explosive behavior in F_t before t_b. In this deterministic setting, it is known as time t_b approaches that there will be an upwards shift in the discount factor that makes present valuations more important. A more realistic model might allow for uncertainty in this time profile and a stochastic trajectory for r_t that accommodated potential upwards shifts of this type.

Econometric dating procedures of the type described below may be used to assess evidence for subperiods of explosive price behavior that are induced by such time variation in the discount factor, just as for other potential sources of financial exuberance.

2.3 Subprime Crisis and Event Timeline

The subprime mortgage crisis is generally regarded as an important triggering element in the ongoing global financial crisis. The subprime event began with a dramatic rise in mortgage delinquencies and foreclosures starting in late 2006 in the US, as easy initial adjustment rate mortgage terms began to expire and refinancing became more difficult at the same time as house prices were falling. The event had wider and, soon, global consequences because of the huge scale of mortgage backed securities (MBS) in the financial system, extending the impact of mortgage failure to the asset positions of investment and commercial banks. The crisis became apparent in the last week of July 2007 when German bank regulators and government officials organized a $5 billion bail out of IKB, a small bank in Germany. We may therefore treat the beginning of August 2007 as the public onset date of the subprime crisis, although the realities in terms of rising mortgage delinquencies commenced earlier.

Much has already been written about the causes of this crisis and a host of factors have been suggested, including poor appreciation of the risks associated with MBS, weak underwriting standards and risk assessment practices in general, increasingly complex financial products, high levels of financial leverage with associated vulnerabilities, shortfalls in understanding the impact of large common shocks on the financial system, and inadequate monitoring by policy makers and regulators of the accumulating risk exposure in the financial markets. We refer readers to Brunnermeier (2008), Greenlaw, Hatzius, Kashyap and Shin (2008) and Hull (2008) for detailed discussions of the subprime crisis and its manifold implications. Our concern in the present paper is with the crisis timeline and, more specifically, the issues of empirically dating the origination and collapse of the various financial bubbles that occurred as the crisis events unfolded.

Prior to the subprime crisis and following the collapse in dot.com stocks in 2000-2001, the housing market in many states of the US sustained rapid increases in valuations fueled by a period of low interest rates, large foreign capital inflows, and high-risk lending practices of financial institutions. In the resulting boom, home ownership in the US increased to 69.2% in 2004 from 64% in 1994 (Callis and Cavanaugh, 2007) and nominal house prices increased by more than 180% over the period 1997-2006 (Panel 2 in Fig. 1). Household debt, as a percentage of disposable income, increased from 77% to 127% over the period 1990-2007 (Economist, November 22, 2008). At the same time, the MBS market, derived from residential mortgages, mushroomed, and major banks and financial institutions around the world invested in securities that were ultimately founded on the U.S. housing market. For example, the nominal outstanding amount of asset backed commercial paper (ABCP) increased by more than 80% over the period July 2004 to July 2007 (See Panel 3 of Fig. 1).

The concatenation of events that occured after the housing market peaked in 2005 and went into decline, followed by the subprime mortgage crisis and subsequent repercussions on financial institutions over 2007-2008 and finally the impact on world trade and real economic activity, is now well known. Securities backed by subprime mortgages lost most of their value, investors lost confidence, and liquidity dried up as money flowed to assets which appeared to have inherently lower risk, such as Treasury bonds, and to other assets like commodities, and currencies such as the U.S. dollar and the Japanese Yen (mainly through the unwinding of the carry trade industry), generating a so-called flight-to-quality. In consequence, commodity prices soared, some currencies like the U.S. dollar appreciated, while others like the British pound rapidly declined. As the crisis deepened, stock markets around the world fell, and commercial banks, mortgage lenders and insurance companies failed. Consumption and investment expenditures dropped, many OECD economies went into serious recession, export driven economies in Asia sustained double digit percentage declines in exports, growth slowed significantly in China, and world trade declined. Concomitant with these real economic effects, global demand for commodities declined and commodity prices fell.

In a recent study, CFG (2008a) proposed a model which seeks to explain the main features of this sequence of complex interlinked financial crises. The CFG model links together global financial asset scarcity, global imbalances, the real estate bubble, the subprime crisis, and the commodity bubble in a general equilibrium macroeconomic environment without monetary factors. The model is based on CFG (2008b) and assumes that the economy has two countries (U and M) and features two goods (X and Z). A key part of the CFG framework is a sequence of hypotheses involving successive bubble creations and collapses, which we briefly review as follows.

Country U is interpreted as the U.S. and country M as the emerging market economies and commodity producers. Good X is a non-storable good, a fraction of which can be capitalized, and is produced by both countries. Good Z is a storable commodity and is produced only by country M. A presumption in the model is that there exists a global imbalance at period t_0. The imbalance can be interpreted as arising from continuing capital flows from emerging markets to the U.S. as the U.S. runs a growing trade deficit with emergent economies, which in turn rely more heavily on export driven growth.

In order to allow country U to have both a large current account deficit and low interest rates, a fundamental assumption that CFG makes is that a bubble developed initially in country

U. In practical terms, this may be viewed as a bubble in the equity, housing and mortgage markets in the U.S., the latter providing financial assets that offer sufficient rewards to be attractive to the rest of the world. Another fundamental assumption is that the bubble bursts at $t = 0$, leaving investors (both locals and foreigners) to look for alternative stores of value. In the first stage, a flight-to-quality reaction migrates the bubble to "good" assets and so the price of commodities (notably, Z) jumps, which results in a significant wealth transfer from U to M. In the second stage, under the assumption that the financial asset crisis and wealth transfer precipitates a severe growth slowdown, the excess demand for the "good" asset is destroyed, leading to a decrease in inventory of the good Z, and the bubble in commodity prices collapses.

Accordingly, this model can describe events in which asset bubbles emerged and subsequently collapsed creating a sequence of bubble effects in one market after another. When the real estate bubble crashed and the value of MBS securities fell substantially, liquidity flowed into other markets creating bubbles in commodities and oil markets as investors transferred financial assets. The deepening financial crisis then sharply slowed down economic growth, which in turn destroyed the commodity bubbles. Obviously, this story makes strong predictions concerning the timing of the origination and the collapse of various bubble phenomena in different markets. To evaluate the evidence in support of such interpretations of the events, consistent date stamping of those events is critical.

2.4 Econometric Dating of the Timeline

> *Bubbles can be definitively identified only in hindsight after a market correction*
> (Economist, June 18, 2005)

The time path of P_t in the rational bubble model (with bubble component B_t) is explosive. Similarly, in the run-up phase of a financial bubble, a pattern of stochastically explosive or mildly explosive behavior is a characteristic feature. The econometric determination of bubble behavior therefore relies on a test procedure having power to discriminate between unit root (or martingale like) local behavior in a process and mildly explosive stochastic alternatives. The same distinction in reverse is required during a bubble collapse. Phillips and Magdalinos (2007a, 2007b, hereafter PM) analyzed the properties of mildly explosive stochastic processes and developed a limit theory for autoregressive coefficient estimation and inference in that context.

PWY (2009) used forward recursive regression techniques and PM asymptotics to test for the presence of mildly explosive behavior in 1990s Nasdaq data and to date stamp the origination and collapse of the Nasdaq bubble. It was shown that a sup unit root test against a mildly explosive alternative obtained from forward recursive regressions has power to detect periodically collapsing bubbles. To improve the power and sharpen date detection, this paper modifies the sup test of PWY by selecting the initial condition based on an information criterion. The new methods are used in combination with the limit theory in Phillips and Yu (2009) which establishes consistency of the dating estimators.

The key idea of PWY is simple to implement and relies on recursively calculated right-sided unit root tests to assess evidence for mildly explosive behavior in the data. In particular, for time series $\{X_t\}_{t=1}^n$, we apply standard unit root tests (such as the coefficient test or the Dickey-Fuller t test) with usual unit root asymptotics under the null against the alternative of an explosive or mildly explosive root. The test is a right-sided test and therefore differs from the usual left-sided tests for stationarity. Contrary to the quotation that heads this section, it is possible by means of these tests to identify the emergence of mildly explosive behavior as it occurs, thereby presaging bubble conditions. It is not necessary to wait for a market correction to identify bubble conditions in hindsight.

More specifically, we estimate the following autoregressive specification by recursive least squares

$$X_t = \mu + \delta X_{t-1} + \varepsilon_t, \;\; \varepsilon_t \sim \text{ iid } (0, \sigma^2), \tag{10}$$

allowing for the fact that the *iid* assumption may be relaxed with the usual (possibly semi-parametric) adjustments to the tests. The null hypothesis is $H_0 : \delta = 1$ and the right-tailed alternative hypothesis is $H_1 : \delta > 1$, which allows for mildly explosive autoregressions with $\delta = 1 + c/k_n$, where $k_n \to \infty$ and $k_n/n \to 0$.

The regression in the first recursion uses $\tau_0 = [nr_0]$ observations, for some fraction r_0 of the total sample where $[\cdot]$ denotes the integer part of its argument. Subsequent regressions employ this originating data set supplemented by successive observations giving a sample of size $\tau = [nr]$ for $r_0 \leq r \leq 1$. Denote the corresponding coefficient test statistic and the Dickey-Fuller t statistic by DF_r^{δ} and DF_r^t, namely,

$$DF_r^{\delta} := \tau \left(\hat{\delta}_\tau (\tau) - 1 \right), \;\; DF_r^t := \left(\frac{\sum_{j=1}^{\tau} \tilde{X}_{j-1}^2}{\hat{\sigma}_\tau^2} \right)^{1/2} \left(\hat{\delta}_\tau (\tau) - 1 \right), \tag{11}$$

where $\hat{\delta}_\tau$ is the least squares estimate of δ based on the first $\tau = [nr]$ observations, $\hat{\sigma}_\tau^2$ is the corresponding estimate of σ^2, and $\tilde{X}_{j-1} = X_{j-1} - \tau^{-1} \sum_{j=1}^{\tau} X_{j-1}$. Obviously, DF_1^{δ} and DF_1^t correspond to the full sample test statistics. Under the null hypothesis of pure unit root dynamics and using standard weak convergence methods (Phillips, 1987), we have the following limit theory as $\tau = [nr] \to \infty$ for all $r \in [r_0, 1]$

$$DF_r^\delta \Rightarrow \frac{\int_0^1 \widetilde{W} dW}{\int_0^1 \widetilde{W}^2}, \quad DF_r^t \Rightarrow \frac{\int_0^1 \widetilde{W} dW}{\left(\int_0^1 \widetilde{W}^2\right)^{1/2}}, \tag{12}$$

where W is standard Brownian motion and $\widetilde{W}(r) = W(r) - \int_0^1 W$ is demeaned Brownian motion.

If Model (10) is the true data generating process for all t, then recursive regressions are unnecessary. In this case, a right-sided unit root test based on the full sample is able to distinguish a unit root null from an explosive alternative. In practice, of course, empirical bubble characteristics are much more complicated than model (10) and involve some regime change(s) between unit root (martingale) behavior with $\delta = 1$ and mildly explosive behavior with $\delta > 1$ and potential re-initialization as market temperature shifts from normal to exuberant sentiment and back again. A distinguishing empirical feature of bubble behavior is that market correction typically occurs as sentiment reverts back and mildly explosive behavior collapses. A model to capture this type of reversion was first constructed by Evans (1991) who argued that conventional unit root tests had little power in detecting periodically collapsing bubbles generated in this manner. As shown in Phillips and Yu (2009), such a model which mixes a unit root process with a collapsed explosive process actually behaves like a unit root process over the full sample (in fact, with some bias toward stationarity as explained below), thereby invalidating the standard unit root test as a discriminating criterion when it is applied to the full sample.

To find evidence for the presence of a bubble in the full sample, PWY (2009) suggest using a sup statistic based on the recursive regression. This involves comparing $\sup_r DF_r^t$ with the right tailed critical values from the limit distribution based on $\sup_{r\in[r_0,1]} \int_0^r \widetilde{W} dW / \left(\int_0^r \widetilde{W}^2\right)^{1/2}$. Similarly, for the coefficient test, one can compare the sup statistic $\sup_r DF_r^\delta$ with the right tailed critical values from the limit distribution based on $\sup_{r\in[r_0,1]} \int_0^r \widetilde{W} dW / \int_0^r \widetilde{W}^2$.

Our approach to finding the timeline of the bubble dynamics also makes use of forward recursive regressions. We date the origination of the bubble by the estimate $\hat{\tau}_e = [n\hat{r}_e]$, where

$$\hat{r}_e = \inf_{s \geq r_0} \left\{ s : DF_s^\delta > cv_{\beta_n}^\delta \right\}, \text{ or } \hat{r}_e = \inf_{s \geq r_0} \left\{ s : DF_s^t > cv_{\beta_n}^{df} \right\}, \tag{13}$$

and $cv_{\beta_n}^\delta$ ($cv_{\beta_n}^{df}$) is the right-side $100\beta_n\%$ critical value of the limit distribution of the DF_r^δ (DF_r^t) statistic based on $\tau_s = [ns]$ observations, and β_n is the size of the one-sided test. Conditional on finding some originating date $\hat{r}_e$ for (mildly) explosive behavior, we date the collapse of the bubble by $\hat{\tau}_f = [n\hat{r}_f]$, where

$$\hat{r}_f = \inf_{s \geq \hat{r}_e + \frac{\log(n)}{n}} \left\{ s : DF_s^{\delta} < cv_{\beta_n}^{\delta} \right\}, \text{ or } \hat{r}_f = \inf_{s \geq \hat{r}_e + \frac{\log(n)}{n}} \left\{ s : DF_s^{t} < cv_{\beta_n}^{df} \right\}. \tag{14}$$

This dating rule for $\hat{\tau}_f$ requires that the duration of the bubble is nonnegligible – at least a small infinity as measured by $\log n$, so that episodes of smaller order than $\log n$ are not considered significant in the dating algorithm for τ_f. This requirement helps to reduce the type I error in the unit root test without affecting the consistency property of the estimator.

The consistent estimation of r_e and r_f requires a slow divergence rate of critical values. For practical implementation, we set the critical value sequences $\left\{ cv_{\beta_n}^{\delta}, cv_{\beta_n}^{df} \right\}$ according to an expansion rule such as $cv_{\beta_n}^{\delta} = (\log\log^2[nr])/2$ and $cv_{\beta_n}^{df} = (\log\log^2[nr])/4$. Both these critical values diverge at a slowly varying rate with $cv_{\beta_n}^{df} < cv_{\beta_n}^{\delta}$. For practically reasonable sample sizes, these critical values are close to the 1% critical values for DF_1^{δ} and DF_1^{t}. For example, when $n = 100$, $cv_{\beta_n}^{\delta} = (\log\log^2 n)/2 = 1.17$ and $cv_{\beta_n}^{df} = (\log\log^2 n)/4 = 0.58$. The 1% critical values for DF_1^{δ} and DF_1^{t} are 1.14 and 0.63, respectively. These critical value expansion rates have been trialed in extensive simulations in Phillips and Yu (2009) and found to give very satisfactory results in terms of small size and high discriminatory power.

Under the mildly explosive bubble model,

$$\begin{aligned} X_t &= X_{t-1} 1\{t < \tau_e\} + \delta_n X_{t-1} 1\{\tau_e \leq t \leq \tau_f\} \\ &\quad + \left(\sum_{k=\tau_f+1}^{t} \varepsilon_k + X_{\tau_f}^* \right) 1\{t > \tau_f\} + \varepsilon_t \, 1\{t \leq \tau_f\} \\ \delta_n &= 1 + \frac{c}{n^{\alpha}}, \quad c > 0, \quad \alpha \in (0,1), \end{aligned} \tag{15}$$

Phillips and Yu (2009) showed that $\hat{r}_e \xrightarrow{p} r_e$ and $\hat{r}_f \xrightarrow{p} r_f$ under some general regularity conditions. Model (15) mixes together two processes, a unit root process and a mildly explosive process with a root above 1 taking the form $\delta_n = 1 + \frac{c}{n^{\alpha}}$. This type of mildly explosive process over $\tau_e \leq t \leq \tau_f$ was originally proposed and analyzed by PM (2007a, 2007b). However, the above system is more complex because it involves regime switches from unit root to mildly explosive behavior at τ_e and from the mildly explosive root back to a unit root at τ_f. At τ_f, the switch also involves a re-initialization of the process and X_t collapses to $X_{\tau_f}^*$, corresponding to a bubble collapse back to fundamental values prevailing prior to the emergence of the bubble. We may, for instance, set $X_{\tau_f}^* = X_{\tau_e} + X^*$ for some $O_p(1)$ random quantity X^*, so that $X_{\tau_f}^*$ is within an $O_p(1)$ realization of the pre-bubble value of X_t.

Under this model specification (15), Phillips and Yu (2009) showed that when $\tau = [nr] \in [\tau_e, \tau_f)$,

$$DF_r^{\delta} = \tau\left(\hat{\delta}_n(\tau) - 1\right) = n^{1-\alpha} rc + o_p(1) \to +\infty,$$

and

$$DF_r^{t} = \left(\frac{\sum_{j=1}^{\tau} \widetilde{X}_{j-1}^2}{\hat{\sigma}_\tau^2}\right)^{1/2} \left(\hat{\delta}_n(\tau) - 1\right) = n^{1-\alpha/2} \frac{c^{3/2} r^{3/2}}{2^{1/2} r_e^{1/2}} \left\{1 + o_p(1)\right\} \to +\infty.$$

Hence, provided $cv_{\beta_n}^{\delta}$ goes to infinity at a slower rate than $n^{1-\alpha}$ and $cv_{\beta_n}^{df}(r)$ goes to infinity at a slower rate than $n^{1-\alpha/2}$, DF_r^{δ} and DF_r^{t} both consistently estimate r_e. Moreover, when $\tau = [nr] > \tau_f$,

$$DF_r^{\delta} = \tau\left(\hat{\delta}_n(\tau) - 1\right) = -n^{1-\alpha} rc \to -\infty, \tag{16}$$

and

$$DF_r^{t} = \left(\frac{\sum_{j=1}^{\tau} \widetilde{X}_{j-1}^2}{\hat{\sigma}_\tau^2}\right)^{1/2} \left(\hat{\delta}_n(\tau) - 1\right) = -n^{(1+\alpha)/2} \frac{c^{1/2} r^{1/2}}{2^{1/2}} \left\{1 + o_p(1)\right\} \to -\infty. \tag{17}$$

Hence, DF_r^{δ} and DF_r^{t} both consistently estimate r_f. Importantly, (16) diverges to negative infinity, so it is apparent that in the post bubble period $\tau > \tau_f$ the autoregressive coefficient $\hat{\delta}_n(\tau)$ is biased downwards, which in this case means biased towards stationarity. This bias is explained by the fact that the collapse of the bubble produces a mean reverting effect in the data, which manifests in the limit theory as a slight bias towards stationarity in the estimated unit root.

We now provide some heuristic discussion about the capacity of these forward recursive regression tests to capture the timeline of bubble activity. The tests have discriminatory power because they are sensitive to the changes that occur when a process undergoes a change from a unit root to a mildly explosive root or vice versa. This sensitivity is much greater than in left-sided unit root tests against stationary alternatives, due to the downward bias and long left tail in the distribution of the autoregressive coefficient in unit root and near stationary cases. By contrast, as is apparent ex post in the data when there has been a bubble, the trajectories implied by unit root and mildly explosive processes differ in important ways. Although a unit root process can generate successive upward movements, these movements still have a random wandering quality unlike those of a stochastically explosive process where there is a distinct nonlinearity in movement and little bias in the estimation of the autoregressive coefficient. Forward recursive regressions are sensitive to the changes implied by this nonlinearity. When data from the explosive (bubble) period are included in estimating the autoregressive coefficient, these observations quickly influence the estimate and its asymptotic behavior due to the dominating effect of the signal from mildly explosive data. This difference in signal between the two periods provides identifying information and explains why the two test procedures consistently estimate the origination date. When the bubble bursts and the system switches back to unit root behavior, the signal from the explosive period continues to dominate that of unit root period. This domination, which at this point is effectively a domination by initial conditions, is analogous to the domination by distant initializations that can occur in unit root limit theory, as shown recently by Phillips and Magdalinos (2009). More than this, the crash and re-initialization give the appearance in the data of a form of mean reversion to an earlier state, so that the estimated autoregressive coefficient is smaller than unity and the classical unit root test statistics diverge to minus infinity, as shown in (16) and (17) above.

2.5 Initialization

To improve the power of the PWY procedure, we modify the methods by selecting the initial condition based on the Schwarz (1978) BIC criterion. In PWY, the initial observation in each recursive regression was fixed to the first observation of the full sample. While this choice is convenient, when time series mix a non-explosive regime with an explosive regime, a more powerful test is obtained if the recursive statistics are calculated using sample data from a single regime for bubble detection. This observation motivates us to use the data to choose the initialization. The method follows an approach to endogenize initialization in time series regression that was suggested in Phillips (1996).

Suppose an origination date $\hat{\tau}_e$ has been identified by the procedure of PWY[2]. Let $n_{\min}$ be the number of observations in a base sample of the observations $\{X_{\tau_e - n_{\min}+1}, \ldots, X_{\tau_e}\}$. The base sample may be constructed by taking some percentage of the sample before $\hat{\tau}_e$. In our applications below we use 10%. For the base sample, we compare the BIC value of two competing models – a unit root model and an autoregressive model. If the BIC value of the unit root model is smaller and the point estimate of δ is larger than 1, we reset the initial condition to $\tau_e - n_{\min} + 1$. Otherwise, we expand the base sample to $\{X_{\tau_e - n_{\min}}, \ldots, X_{\tau_e}\}$, that is another observation is added to the beginning of the sample. Based on the new sample, we again compare the BIC value of the competing models. If the BIC value of the unit root model is smaller and the point estimate of δ is larger than 1, we reset the initial condition to $\tau_e - n_{\min}$. This exercise is repeated until the BIC value of the unit root model is smaller.

[2] In case no bubble is found, no change in the PWY procedure is required. However, a flexible moving window recursive approach is also possible, which allows for variable initializations, and may be more effective in assessing evidence for multiple bubbles.

If the sample eventually becomes $\{X_1, \ldots, X_{\tau_e}\}$ and the BIC value of the unit root model is still larger, we set the initial condition to $t = 1$, which is the same as that used in PWY. If the initialization emerging from this procedure is $\hat{\tau}_0$, then the recursive testing methodology of PYW is applied from $\hat{\tau}_0$. With this initialization, denote the estimate of the origination date $\hat{\tau}_e(\hat{\tau}_0)$ and the estimate of the collapse date $\hat{\tau}_f(\hat{\tau}_0)$. Obviously, $\hat{\tau}_e(1) = \hat{\tau}_e$ and $\hat{\tau}_f(1) = \hat{\tau}_f$. However, if $\hat{\tau}_0 > 1$, it is possible that $\hat{\tau}_e(\hat{\tau}_0) \neq \hat{\tau}_e$ and $\hat{\tau}_f(\hat{\tau}_0) \neq \hat{\tau}_f$. In general it is expected that $\hat{\tau}_e(\hat{\tau}_0) \leq \hat{\tau}_e$ since the backward recursion to locate the initialization $\hat{\tau}_0$ begins from $\hat{\tau}_e$.

Assume the sample is $X_{\tau_e - n_{\min} - n_k + 1}, \ldots, X_{\tau_e}$. The BIC value of the unit root model is

$$\ln\left(\frac{\sum_{t=\tau_e-n_{\min}-n_k}^{\tau_e}\left(\Delta X_t - \overline{X}\right)^2}{n_k + n_{\min}}\right) + \frac{\ln(n_k + n_{\min})}{n_k + n_{\min}},$$

whereas the BIC value of the autogression is

$$\ln\left(\frac{\sum_{t=\tau_e-n_{\min}-n_k}^{\tau_e}\left(X_t - \widehat{\mu} - \widehat{\delta} X_{t-1}\right)^2}{n_k + n_{\min}}\right) + \frac{2\ln(n_k + n_{\min})}{n_k + n_{\min}},$$

where $\overline{X} = \frac{1}{n_{\min}+n_k}\sum_{t=\tau_e-n_{\min}-n_k+1}^{\tau_e} X_t$, $\widehat{\delta}$ and $\widehat{\mu}$ are the OLS estimators of δ and μ from the autoregressive model

$$X_t = \mu + \delta X_{t-1} + \varepsilon_t.$$

It is known that when the criterion is applied in this way BIC can consistently (i.e. almost surely as $n \to \infty$) distinguish a unit root model from a stationary model without specifying transient dynamics (see Phillips, 2008). Using similar methods it can be shown that BIC consistently distinguishes a unit root model from a model with an explosive root. In essence, the use of BIC to select the initialization is equivalent to the use of BIC to choose a break point, although in the present case it is not necessary to specificy transient behavior.

3 Data

Two datasets are studied in the empirical work reported here. The primary data constitute seven financial time series: the monthly Nasdaq composite price index (without dividends) over January 1990 to January 2009; the monthly U.S. house price index over January 1987 to January 2009; the monthly outstanding asset backed commercial paper (ABCP) in the U.S. over January 2001 to January 2009; monthly crude oil prices (in US dollars) over January 1999 to January 2009; monthly platinum prices (in US dollars) over January 1999 to January

2009; the daily Baa bond rates from January 3, 2006 to January 30, 2009; and the weekly Pound/USD exchange rates from March 17, 2006 to March 20, 2009.

A secondary dataset is studied to check whether the empirical bubble characteristics found in the primary series apply to other commodities and exchange rates. The secondary data include some commodity prices such as monthly heating oil, coffee, cotton, cocoa, sugar, feeder cattle prices, all measured in USD and over January 1999 to January 2009, and some exchange rates, such as the weekly Euro/USD exchange rates, the Yen/USD exchange rates and the Cnd/USD exchange rates, all observed over March 17, 2006 to March 20, 2009.

The choice of the sampling periods is judiciously guided by CFG (2008a) because we aim to match the empirical analysis with the predictions made in CFG. The CFG story begins with the internet bubble in the Nasdaq in the 1990s – see page 7 in CFG – and ends with the collapse of all financial bubbles when the economy goes seriously into recession. For the Baa bond rates, it is well known that a relevant event that signaled the effects of the credit crunch is the failure of Lehman Brothers on September 15, 2008. The sampling period is chosen so that we have enough observations before September 15 for the bubble test to have good power. Similar auguments apply to the choice of the sampling period for the exchange rates. However, other sampling intervals, all covering the subprime crisis period, have been used and the empirical findings reported here are reasonably robust to the choice of the sample period.

The Nasdaq composite price index is obtained from finance.yahoo.com. It extends the sample used in PWY by including more recent observations from June 2005 to January 2009. PWY found strong evidence of a bubble in the Nasdaq during the 1990s, associated with the dom.com episode. We extend the sample period in order to check whether there are any subsequent bubbles prior to the subprime crisis. The house price index is the seasonally adjusted S&P Case Shiller composite-10 index obtained from Robert Shiller's website, and represents the maximum time span of this data. The outstanding commercial paper data is for asset backed commercial paper (ABCP) obtained from the Federal Reserve Board. The ABCP time series is a crude indicator of the size of the mortgage and subprime market. The crude oil price series is based on WTI - Cushing, Oklahoma spot prices obtained from the Energy Information Administration website. The platinum price series is obtained from the kitco website. The Baa bond rates are averages of Baa industrial bond rates and are obtained from the Federal Reserve Board. This variable measures the credit risk level and is particularly relevant because, as the crisis unfolded, the sharp drop in the prices and market liquidity of all mortgage-backed securities led a sharp increase in the price of risk and in spreads. Not surprisingly, mutual mistrust amongst counterparties surged and bond rates jumped. Finally, the Pound/USD exchange rates are obtained from the Federal Reserve Board. For the secondary dataset, all the commodity prices are downloaded from EconStats (http://www.econstats.com/index.htm) and all the exchange rates are downloaded from the Federal Reserve Board. All time series, except for the exchange rates and the Baa bond rates, are deflated using the Consumer Price Index (CPI), which is obtained from the Department of Labor. Figs. 1-3 plot all of seven series in the primary dataset. Table 1 reports some summary descriptive statistics for these seven time series, including sample size, sample frequency, sample minimum, date of the minimum, sample maximum, date of the maximum, as well as the coefficient statistic (DF_1^{δ}) and DF-t statistic (DF_1^t) based on the entire sample.

Table 1: Summary statistics

Data	Sample Size	Freq	Min	Date (min)	Max	Date (max)	DF_1^{δ}	DF_1^t
Nasdaq	432	M	1.1002	Sep 1974	27.66	Jan 2000	-4.6476	-1.5614
House	265	M	.4924	Oct 1996	1.1340	Feb 2006	-0.3124	-0.5272
ABCP	97	M	3314.8	Sep 2008	5817.1	July 2007	-2.3631	-1.1098
Oil	121	M	.0730	Feb 1999	.6118	June 2008	-3.6298	-1.5713
Platinum	121	M	2.0879	Aug 1999	9.5841	Mar 2008	-3.4287	-1.5088
Baa	772	D	6.08	Dec/21/06	9.54	Oct/31/08	-1.0436	-0.4544
Pound/USD	158	W	.4775	Nov/9/07	.7240	Jan/23/09	1.9719	1.0594

The real Nasdaq index reached its maximum of 27.66 in January 2000 growing from a minimum of 1.1 in September 1974. The house price index troughed in October 1996 and peaked in February 2006. Interestingly, the minimum value for the ABCP was reached in September 2008, barely a year after its maximum (July, 2007). This timing suggests a strong decline in the index over the period August 2007 to September 2008 when the subprime crisis swept through the mortgage market. The crude oil price and platinum price series follow the same pattern, having their minima in the early part of the sample and reaching the maxima in mid-2008. The rate for Baa is lowest (6.81) on December 21, 2006 and highest (9.54) on October 31, 2008, shortly after the failure of Lehman Brothers on September 15. The Pound/USD exchange rate series is volatile, moving rapidly from 0.4775 on November 9, 2007 to 0.7240 on Jan 23, 2009. At the 5% level, for only one series (namely the Pound/USD exchange rate) is the unit root null rejected in favor of an explosive alternative for the full sample (the 5% asymptotic critical values are, respectively, -0.13 and -0.07 for the two unit root test statistics DF_1^{δ} and DF_1^t).

4 Empirical Results

Three phases have been identified in connection with the subprime crisis. According to CFG (2008a), each phase involves a specific hypothesis that concerns related bubble activity. In the first phase (A), before the subprime crisis publicly erupted, bubbles had emerged and burst in the stock market, the housing market, and mortgage market. These bubbles all played a role in global imbalances. In particular, three hypotheses are in order.

Hypothesis A1: A bubble originated and collapsed in the stock market prior to the emergence of the subprime crisis.

Hypothesis A2: A bubble originated in the housing market following the Nasdaq crash in late 2000 and burst when the subprime crisis emerged in August 2007.

Hypothesis A3: A bubble originated and collapsed in mortgage market securities, the collapse coinciding with the public eruption of the subprime crisis in August 2007.

During the second phase (B), the subprime crisis broke and funds flowed selectively to assets in other markets with lower perceived risk. In consequence, bubbles emerged in certain commodity and foreign exchange markets and credit risk perceptions shot up, leading to the following hypotheses:

Hypothesis B1: Bubbles originated in certain commodity price markets and exchange rates following the eruption of the subprime crisis.

Hypothesis B2: Bubbles originated in the bond market as the subprime crisis unfolded.

In the third phrase (C), as perceptions increased that there would be a potentially serious impact of the financial crisis on real economic activity in the U.S and globally, the financial bubbles in commodity prices and the bond market collapsed. Correspondingly, we have another hypothesis:

Hypothesis C: Bubbles that had arisen in commodity prices and the bond market collapsed.

We now report and discuss the empirical results. First, we check for statistical evidence of the presence of bubble(s) in each of the time series based on the recursively calculated sup statistics $\max DF_r^\delta$ and $\max DF_r^t$. Table 2 reports critical values for these two statistics obtained by simulation for the two sample sizes, 100 and 500. The critical values for $\max DF_r^t$ are nearly identical to those reported in PWY. The critical values for $\max DF_r^\delta$ are about twice as large as those for $\max DF_r^t$. This is not surprising as the critical values for the conventional unit root statistic DF_r^δ are about twice as large as those for DF_r^t (see, for example, Fuller, 2000).

Table 2: Critical values of $\max DF_r^\delta$ and $\max DF_r^t$ obtained in simulations

Sample Size	Test Statistic	10%	5%	1%
500	$\max DF_r^\delta$	2.3525	2.8791	3.9619
500	$\max DF_r^t$	1.1800	1.4603	2.0043
100	$\max DF_r^\delta$	2.3221	2.9470	4.3412
100	$\max DF_r^t$	1.1914	1.5073	2.1899

The first two rows in Table 3 report values for the two statistics based on the seven time series with $\tau_0 = 0.1$. All cases show overwhelming evidence for the presence of bubbles. The p-values are all less than 1% for data other than the crude oil price. For the crude oil price, the p-value is between 1% and 5%. Judging from the magnitude of the two statistics, the bubble characteristics are strongest in the Nasdaq, House prices and Baa.

Table 3: Testing the Presence of Bubbles and Date Stamping[3]

	Nasdaq	Home Price	ABCP	Oil	Platinum	Baa	Pnd/USD
$\max DF_r^\delta$	18.026	5.7668	4.5963	4.2131	5.063	19.156	7.5524
$\max DF_r^t$	8.2106	14.625	4.9612	2.3652	2.565	5.1876	2.6286
$\widehat{\tau}_e(\hat{\tau}_0)$	June/95	Sep/00	Aug/05	Mar/08	Jan/08	Oct/8/08	Oct/24/08
$\widehat{\tau}_f(\hat{\tau}_0)$	Nov/00	June/07	July/07	Aug/08	July/08	Dec/4/08	NA

Next we estimate the origination and collapse dates, τ_e and τ_f and report these in the last two rows in Table 3. Table 4 shows the difference between $\widehat{\tau}_e(\hat{\tau}_0)$ and $\widehat{\tau}_f(\hat{\tau}_0)$, the estimates based on the endogenized initialization, and $\widehat{\tau}_e(1)$ and $\widehat{\tau}_f(1)$, the estimates based on the initialization proposed by PWY. The two sets of estimates turn out to be the same for the Nasdaq, Oil, Plantinum and Baa series. But they differ by 1-4 periods for Home prices, ABCP and Pound/USD (shown in bold face in Table 4). When they are different, $\widehat{\tau}_e(\hat{\tau}_0)$ is always smaller, suggesting that the endogenized initialization indeed gives an earlier warning.

Table 4: Estimates of the Origination and Collapse Dates Based on Endogenized Initializations[4]

	Nasdaq	Home Price	ABCP	Oil	Platinum	Baa	Pnd/USD
$\widehat{\tau}_e(1)$	June/95	Jan/01	Nov/05	Mar/08	Jan/08	Oct/8/08	Oct/31/08
$\widehat{\tau}_e(\hat{\tau}_0)$	June/95	**Sep/00**	**Aug/05**	Mar/08	Jan/08	Oct/8/08	**Oct/24/08**
$\widehat{\tau}_f(1)$	Nov/00	Aug/07	Aug/07	Aug/08	July/08	Dec/4/08	NA
$\widehat{\tau}_f(\hat{\tau}_0)$	Nov/00	**June/07**	**July/07**	Aug/08	July/08	Dec/4/08	NA

[3] The reported estimated τ_e and τ_f are based on DF_t^δ.

[4] The reported estimated τ_e and τ_f are based on DF_t^δ.

Time series plots of the recursively calculated statistics DF_r^{δ} and DF_r^{t} are shown in Figs. 5-11. Superposed on these plots are the critical value paths, $\log\log^2[nr]/2$, $\log\log^2[nr]/4$, the estimated dates $\widehat{\tau}_e(\hat{\tau}_0)$, $\widehat{\tau}_f(\hat{\tau}_0)$, and the onset date for the subprime crisis. Recall that Figs. 1-3 plot $\widehat{\tau}_e(\hat{\tau}_0)$ and $\widehat{\tau}_f(\hat{\tau}_0)$, together with the time series data. In all of these cases, we clearly identify an explosive subperiod in the data.

Some general conclusions can be drawn from the estimates and Figs. 1–3 and 5-11. First, the estimated origination and collapse dates seem to cover a subperiod of significant price run-up in each of the time series. Second, the estimated origination dates are not the same as the apparent beginning of these run-up periods. This may be because a unit root process as well as processes with very mildly explosive roots that are closer to unity than an $O\left(n^{-1}\right)$ neighborhood can also generate mild run-ups but the latter are indistinguishable from a unit root root process. The present tests have substantial discriminatory power for mildly explosive roots beyond those of $O\left(n^{-1}\right)$ neighborhoods and are consistent against such alternatives. Third, both statistics DF_r^{δ} and DF_r^{t} lead to the same or very similar dating estimates, except for house prices, which are discussed below.

Some specific conclusions can be drawn for the individual time series and these are summarized below.

1. For the Nasdaq series, a very significant bubble is found by both DF_r^{δ} and DF_r^{t} statistics over June 1995 to Oct 2000. The estimates $\widehat{\tau}_e$ and $\widehat{\tau}_f$ are identical to those found in PWY, although a different statistic (the augmented Dickey-Fuller test) was employed there, the data sources (both for the index and the deflator) differ, and the sample period is shorter in PWY. Interestingly, no bubble is found over the more recent period even though there was some run up in prices before the subprime crisis. Note that the Nasdaq bubble collapsed several years prior to the subprime crisis. So the results are entirely consistent with hypothesis A1.

2. For the House Price series, again a very significant bubble is found by both DF_r^{δ} and DF_r^{t}, but this time during the 2000s. Compared with DF_r^{t}, the statistic DF_r^{δ} is three months later in identifying the bubble but nine months earlier identifying the collapse of the bubble. In both cases, our estimates of the bubble origination date in the early 2000s strongly support the argument in Baker (2002), who claimed that there was a housing bubble at the time. According to DF_r^{δ}, the bubble collapsed in June 2007 just before the subprime crisis erupted, consistent with hypothesis A2.

3. For the ABCP series, a significant bubble is found by both DF_r^{δ} and DF_r^{t} over August

2005 to July 2007. Note that in this case the origination date $\widehat{\tau}_e$ comes several years later than that of the House Price series, reflecting the lag in packaging mortgages into financial derivatives and related products. The bubble collapsed in July 2007 when the subprime crisis became apparent, consistent with hypothesis A3.

4. For Crude Oil prices, neither DF_r^{δ} nor DF_r^{t} identifies a bubble before the subprime crisis broke out. However, a significant bubble is found by both DF_r^{δ} and DF_r^{t} over March 2008 to July 2008. The bubble emerged in March 2008 after the subprime crisis broke, consistent with hypothesis B1. The bubble collapsed in August 2008, consistent with hypothesis C.

5. For Platinum prices, the recursions of DF_r^{δ} and DF_r^{t} both include two periods (one at the beginning of the sample and the other in the middle of the sample) where the statistics exceed the critical values. However, the durations are so short so that the $\log(n)$ separating rule for minimum bubble duration suggests that these should be interpreted as short-lived run-ups not bubbles. However, a significant bubble is found by both DF_r^{δ} and DF_r^{t} over January 2008 to June 2008. The bubble emerged in January 2008 after the subprime crisis broke, consistent with hypothesis B1. The bubble collapsed in July 2008, consistent with hypothesis C.

6. For the Baa bond rates, while both DF_r^{δ} and DF_r^{t} suggest random wandering behavior for much of the period, both also indicate a short but significant bubble over the period from October 18, 2008 to December 3, 2008. This period corresponds with the period of the rapid acceleration of financial distress, soon after the Lehman Brothers bankruptcy. The bubble emerged in the bond market on October 8, 2008 after the subprime crisis erupted, consistent with hypotheses B2 and C.

7. Finally, for the Pound/USD exchange rate, a significant bubble is found by both DF_r^{δ} and DF_r^{t}. The bubble emerged on October 24, 2008 after the subprime crisis, consistent with hypothesis C. However, the bubble persists until the end of our sample, March 20, 2009 and remains an ongoing characteristic in the data.

In sum, all these tests provide empirical support for hypotheses A - C, showing bubble characteristics in the data that are consistent with the hypotheses. The empirical estimates on the timeline of the crisis also broadly support the predictions made in the CFG (2008a) model. Fig. 12 shows the complete timeline of the bubble process. The timeline shows how bubbles

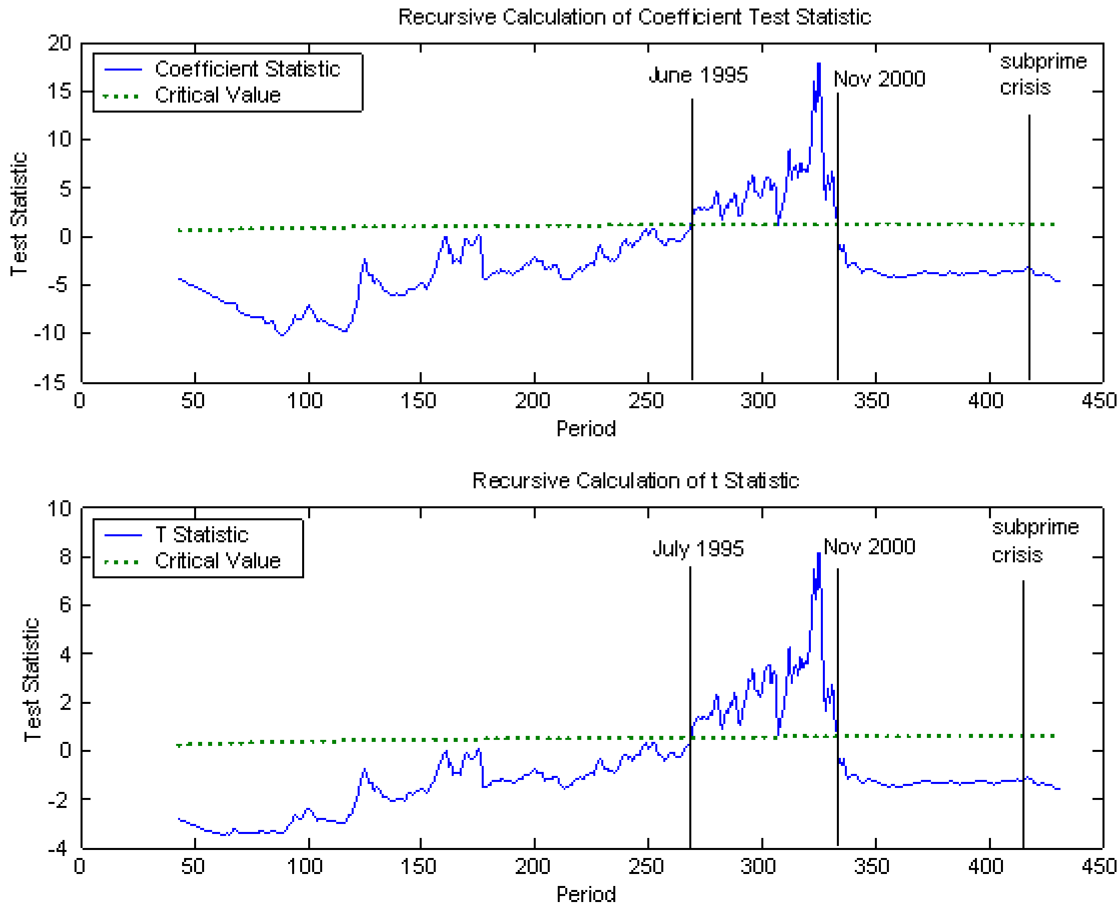

Figure 5: Recursive calculation of the coefficient test and t statistic for the real Nasdaq index from February 1973 to January 2009, obtained from forward recursive regressions.

migrated from the equity market (in particular the Nasdaq index), first to the housing market, and next to the mortgage market before the subprime crisis. After the subprime crisis, the bubbles selectively moved to certain goods in commodity markets and certain currencies in the foreign exchange market.

To assess whether or not bubble characteristics were a generic or specific feature in commodity and foreign exchange markets during the financial crisis, we applied the methods more broadly to many series in a secondary dataset. To preserve space, we present only the summary empirical results in Table 4 without plotting the recursive test statistics.

Although it is clear from the empirical results obtained earlier that funds moved across markets during the crisis period for flight-to-quality and flight-to-liquidity reasons, the results in Table 4 suggest that investors were selective in transferring assets. For example, in the

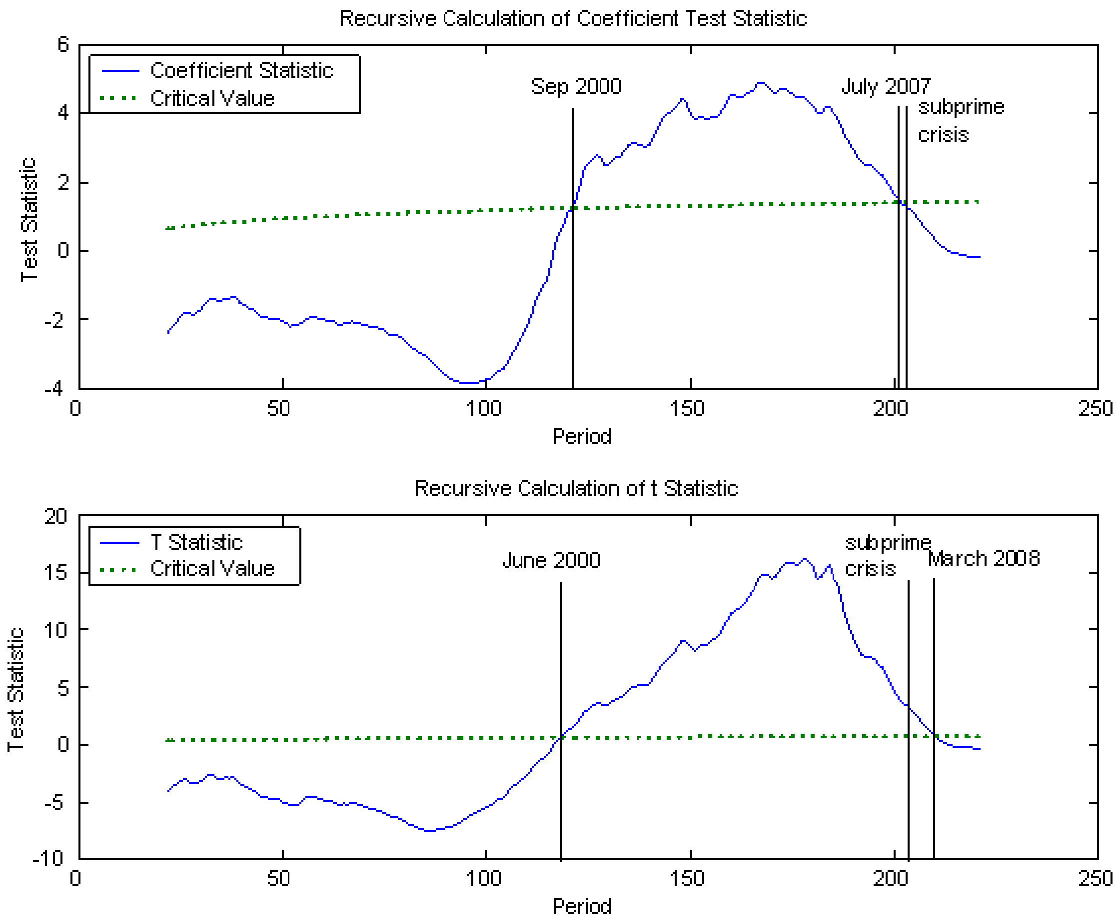

Figure 6: Recursive calculation of the coefficient test and t statistic for the real seasonally adjusted home price composite 10 index from January 1987 to January 2009, obtained from forward recursive regressions.

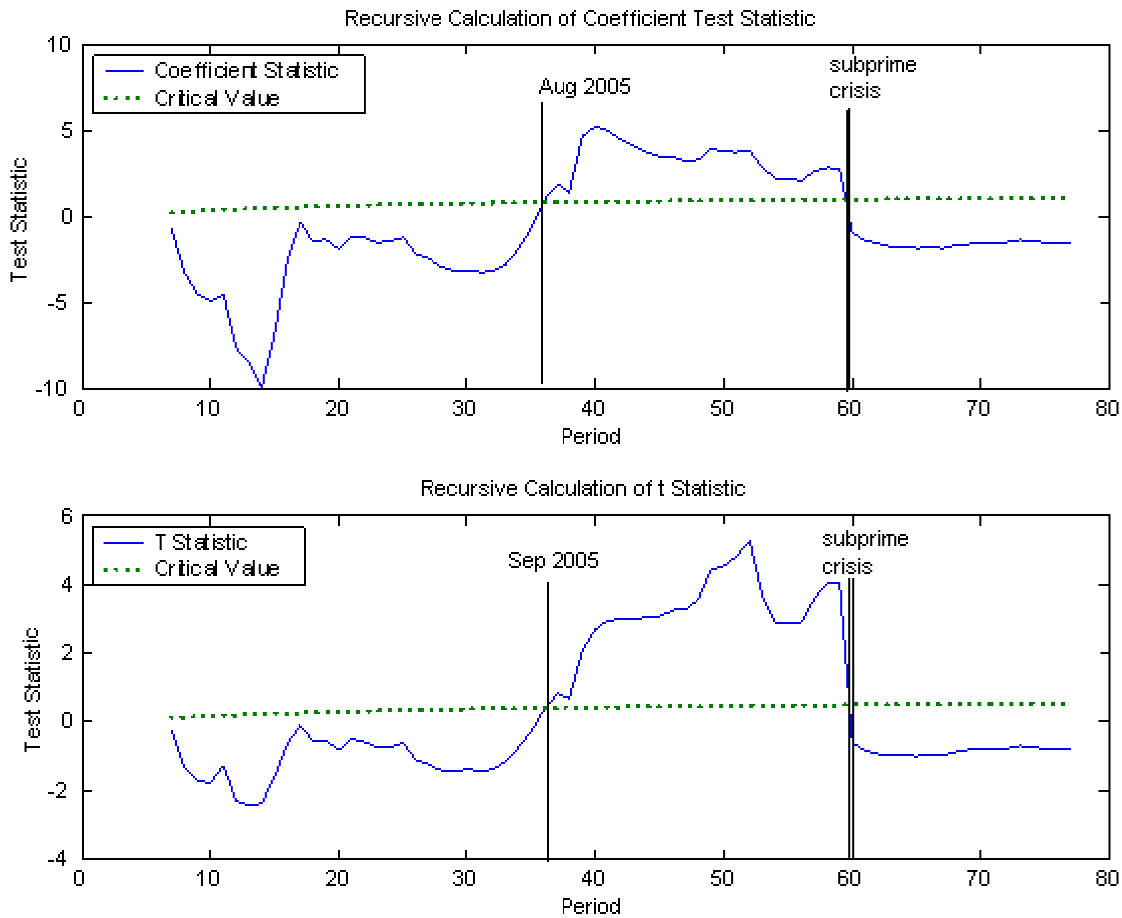

Figure 7: Recursive calculation of the coefficient test and t statistic for the real outstanding values for asset-backed commercial paper from January 2001 to January 2009, obtained from forward recursive regressions.

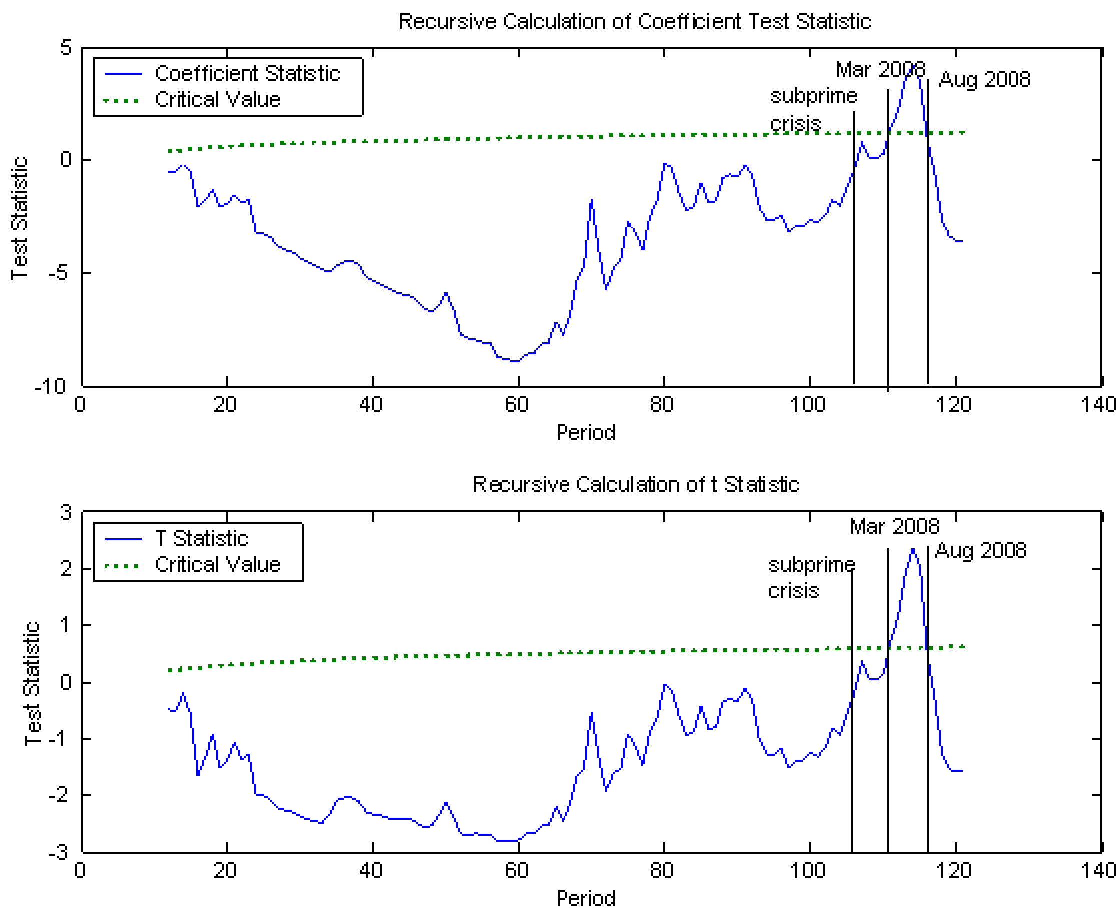

Figure 8: Recursive calculation of the coefficient test and t statistic for the real crude oil price from January 1999 to January 2009, obtained from forward recursive regressions.

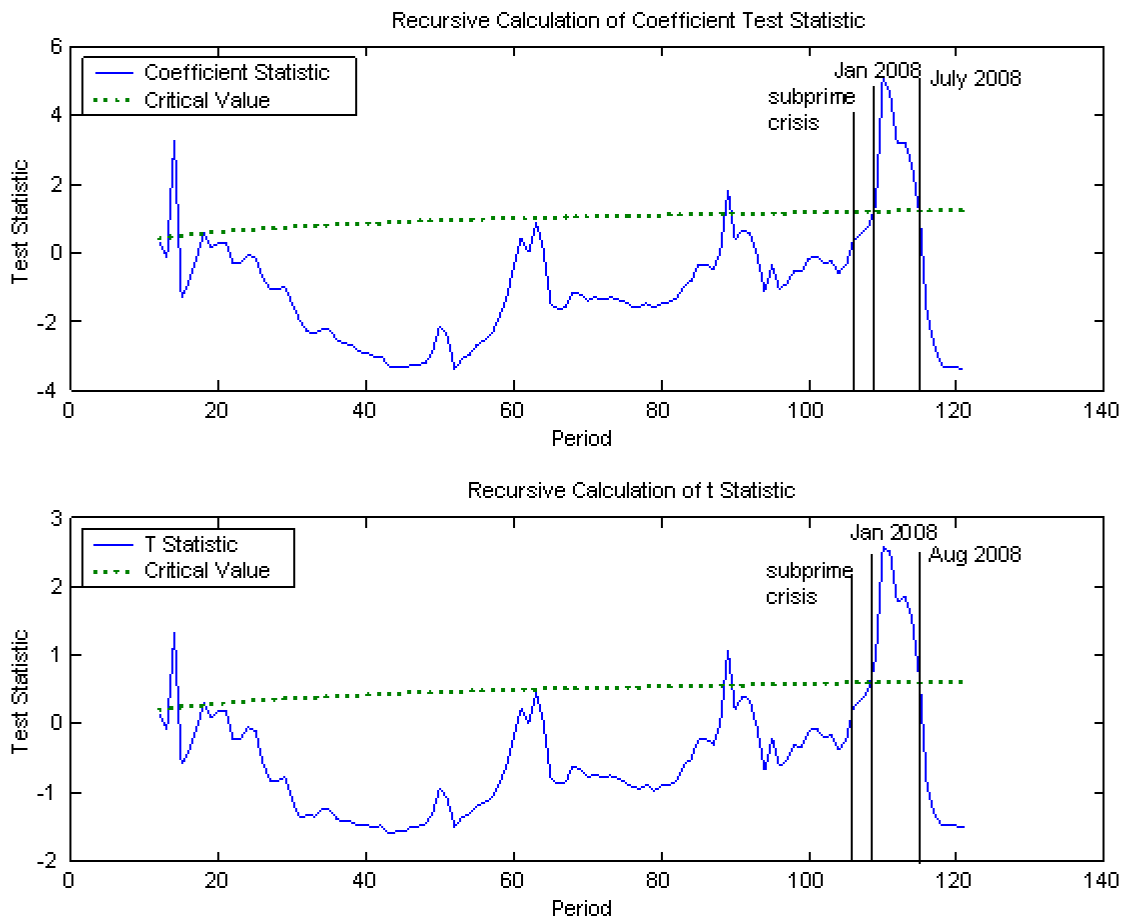

Figure 9: Recursive calculation of the coefficient test and t statistic for the real platinum price from January 1999 to January 2009, obtained from forward recursive regressions.

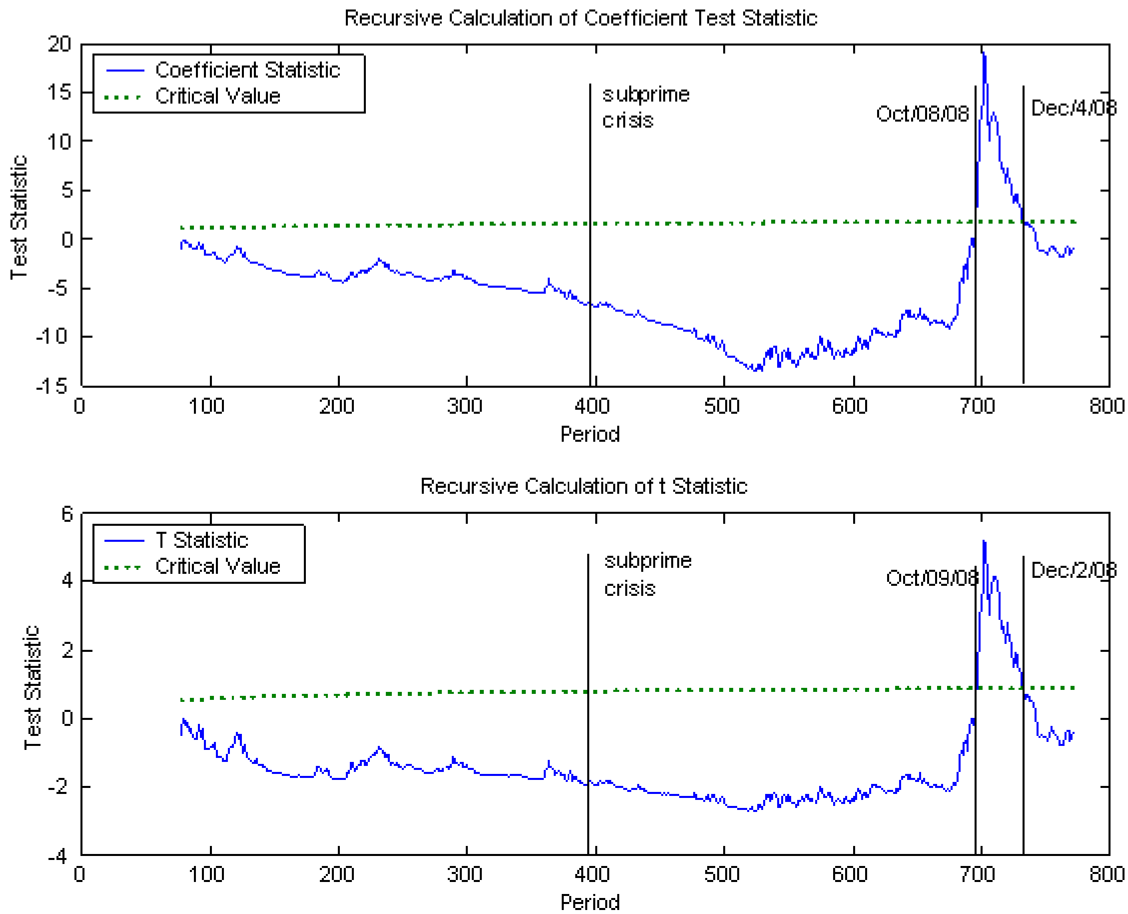

Figure 10: Recursive calculation of the coefficient test and t statistic for the Baa bond rates from January 3, 2006 to January 30, 2009, obtained from forward recursive regressions.

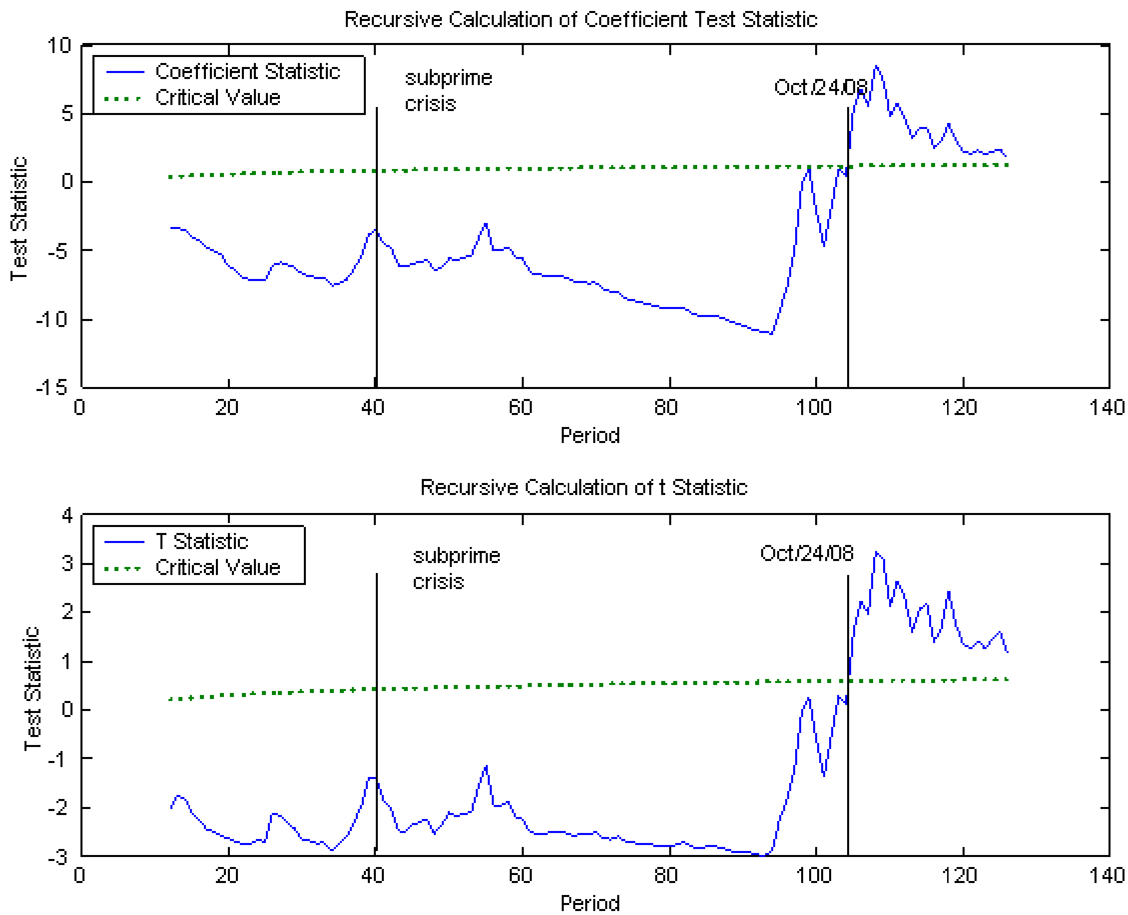

Figure 11: Recursive calculation of the coefficient test and t statistic for the Pound/USD exchange rates March 17, 2006 to March 20, 2009, obtained from forward recursive regressions.

commodity market, we identify a bubble in heating oil prices, with similar origination and collapsing dates as those for crude oil prices. However, we find no evidence of bubbles in coffee, cotton, sugar, and feeder cattle prices. In the foreign exchange market, we locate a bubble in the Cnd/USD exchange rate, which originated on September 21, 2007 and burst on November 23, 2007. Interestingly, the origination date is about one year earlier than that in the Pound/USD exchange rate. However, no bubble is found in the Euro/USD exchange rate. Although the value of the $\max DF_r^{\delta}$ statistic is marginally higher than the corresponding critical value for Cocoa and the Yen/USD rate, detailed analysis of the recursive calculations of the test statistic shows that the run-ups only lasted for a couple of periods and therefore do not survive the $\log(n)$ separating rule for minimum bubble duration.

Table 4: Test Results for the Presence of Bubbles and Date Stamps

	$\max DF_r^{\delta}$	$\max DF_r^{t}$	$\widehat{\tau}_e$	$\widehat{\tau}_f$
Heating oil	6.9092	2.2416	March/08	August/08
Coffee	-1.6035	-0.7002	NA	NA
Cotton	-0.2466	-0.0866	NA	NA
Cocoa	2.4876	0.9872	NA	NA
Sugar	-0.7408	-0.2220	NA	NA
Feeder cattle	1.0336	0.4327	NA	NA
Euro/USD	0.4091	0.3311	NA	NA
Yen/USD	3.8949	1.4247	NA	NA
Cnd/USD	4.0494	2.6956	Sep/21/07	Nov/23/07

5 Conclusions

This paper provides an empirical study of the bubble characteristics in several key financial variables over an historical time period that includes the subprime crisis and its sequel, including global effects. The econometric methods employed are based on recursive regression, right-sided unit root tests and a newly developed dating technology and associated limit theory from Phillips and Yu (2009). These methods enable us to track the timeline of the crisis in terms of the individual series by empirically dating the origination and collapse of each of the bubbles. The dates are matched against the onset date for the subprime crisis as well as a specific sequential hypothesis concerning bubble migrations that are predicted in the theoretical model proposed by CFG (2008a). Our estimates suggest that bubbles migrated from the equity market to the housing market and on to the subprime mortgage derivative market before the crisis broke. After the crisis erupted into the public arena, the pricing bubbles migrated to selected commodity markets and, in some cases, the foreign exchange market, suggesting a flight-to-quality or perceived safe haven phenomena. All these bubbles collapsed as the financial crisis impacted real economic activity. The estimated sequence of the bubble migration phenomenon is broadly consistent with the predictions of CFG (2008a).

The methods used here may also be used to provide early warning diagnostics for market exuberance as they provide consistent tests for mildly explosive behavior. Such diagnostics may assist policy makers in framing early monetary policy responses or other regulatory actions or interventions to combat speculative bubbles in financial markets.

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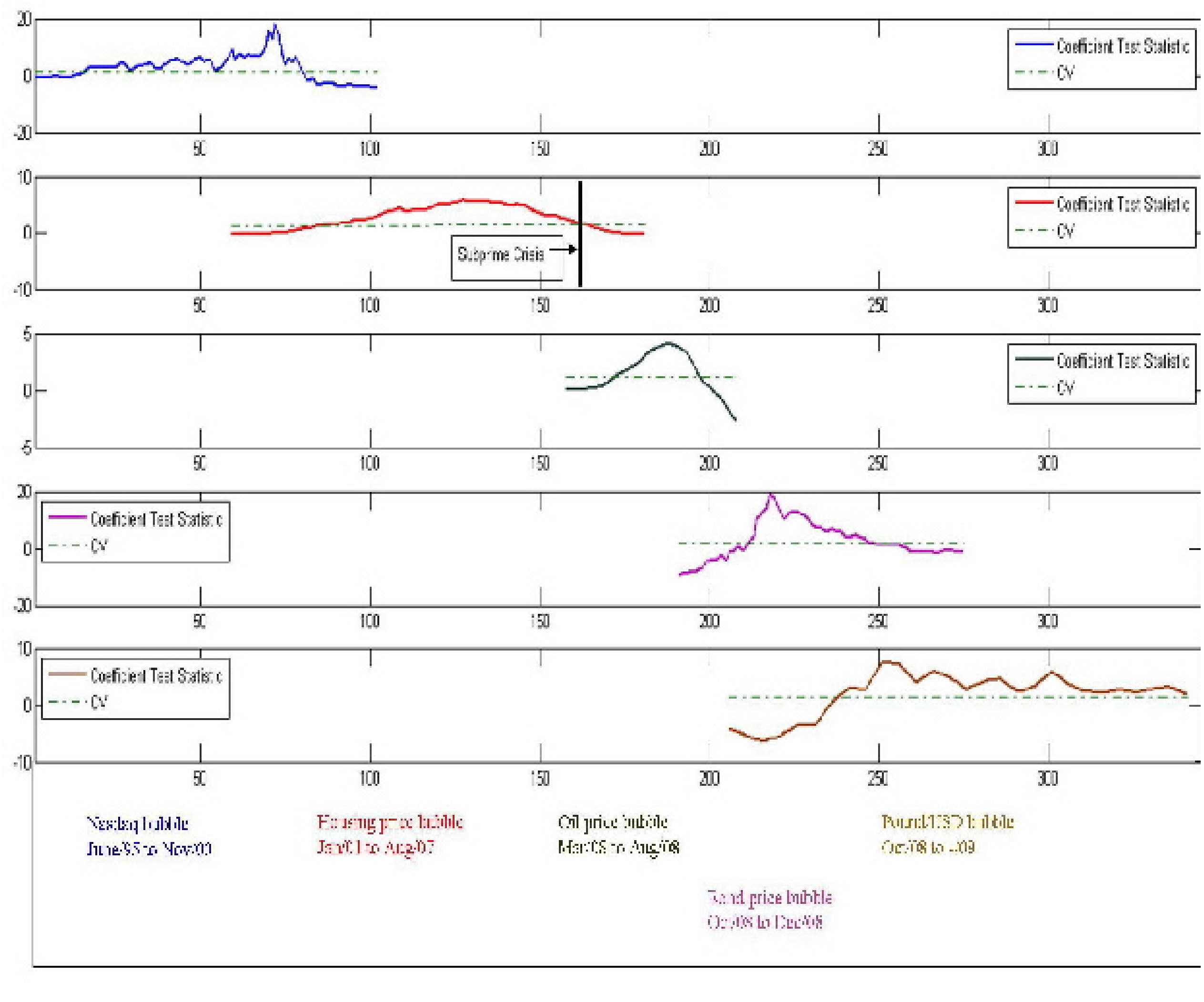

Figure 12: Timeline of financial bubbles in the stock, real estate, mortgate, commodity, bond, and foreign exchange markets. The panels show recursive calculations of the coefficient statistic and critical values highlighting the successive bubble episodes.