DEFAULT PENALTY AS DISCIPLINARY AND SELECTION MECHANISM IN PRESENCE OF MULTIPLE EQUILIBRIA

By

Juergen Huber, Martin Shubik, and Shyam Sunder

October 2009

COWLES FOUNDATION DISCUSSION PAPER NO. 1730

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS
YALE UNIVERSITY
Box 208281
New Haven, Connecticut 06520-8281

http://cowles.econ.yale.edu/

DEFAULT PENALTY AS A DISCIPLINARY AND SELECTION MECHANISM IN PRESENCE OF MULTIPLE EQUILIBRIA[1]

Juergen Huber, University of Innsbruck
Martin Shubik, Yale University
and
Shyam Sunder Yale University

Abstract

Closed exchange and production-and-exchange economies may have multiple equilibria, a fact that is usually ignored in macroeconomic models. Our basic argument is that default and bankruptcy laws are required to prevent strategic default, and these laws can also serve to provide the conditions for uniqueness. In this paper we report experimental evidence on the effectiveness of this approach to resolving multiplicity: society can assign default penalties on fiat money so the economy selects one of the equilibria. Our data show that the choice of default penalty takes the economy to the neighborhood of the chosen equilibrium. The theory and evidence together reinforce the idea that accounting, bankruptcy and possibly other aspects of social mechanisms play an important role in resolving the otherwise mathematically intractable challenges associated with multiplicity of equilibria in closed economies. Additionally we discuss the meaning and experimental implications of default penalties that support an active bankruptcy-modified competitive equilibrium.

JEL: C73, C92, D51, E42, G21, G33

Keywords: Bankruptcy penalty, financial institutions, fiat money, multiple equilibria, experimental gaming

First Draft, October 2, 2009.

---

[1] Financial support by the Austrian Forschungsfoerderungsfond (FWF, grant P-20609) is gratefully acknowledged. Email addresses: Juergen.huber@uibk.ac.at, martin.shubik@yale.edu, and shyam.sunder@yale.edu.

1. INTRODUCTION

Closed exchange and production-and-exchange economies may have multiple equilibria. In spite of its importance for policy decisions, questions about this multiplicity are often set aside in dynamic models of the macro-economy. While learning theories and experimental gaming have attempted to address the problem of equilibrium selection, in mathematical general equilibrium theory how equilibria are selected in presence of multiplicity remains an interesting but unresolved challenge. This paper is an attempt to understand the role of financial institutions, such as bankruptcy laws and accounting rules, in dynamically resolving the multiplicity problems in closed economies. To this end, we conduct a laboratory experiment and report evidence that through choice of a default (bankruptcy) penalty, the outcome of a closed economy can be directed to any targeted element in the set of its equilibria.

In attempting to construct a process model of a general equilibrium system as a playable game, if any form of borrowing whatsoever is present it is necessary to introduce default penalties to prevent strategic bankruptcy; but the bankruptcy conditions may also provide a way to select among multiple equilibria if they are present. In the static models of general equilibrium theory there is no "nice" general condition that selects a unique equilibrium in an economy with only the usual restrictions on smooth concave utility functions.

The following five mathematically describable conditions (all highly restrictive) are known to be sufficient for the existence of a single competitive equilibrium in a closed exchange economy with n agents and m commodities:

1. There is a single agent ($n = 1$).

2. There is a single commodity ($m = 1$).
3. All individuals have the same utility function.
4. There is gross substitutability among all goods.
5. There exists a commodity that is in positive supply that is desired by all and whose worth enters into the utility functions of all as a linear separable term.

Here we explore the fifth condition, and more specifically the role of institutional constraints like default penalties as instruments of equilibrium selection. Qin and Shubik (2008) suggest that penalty conditions are reasonable when one attempts to convert a general equilibrium structure into a playable game. They demonstrate that trading in markets with a fiat money and default penalties is a sufficient way to construct a process model that selects among multiple equilibrium points. In a dynamic model of an economy where individuals are strategically free to borrow, well-defined rules and penalties on default are a logical necessity. They are also an institutional fact in every economy (see Karatzas, Shubik, Sudderth and Geanakoplos, 2006).

In this paper we experimentally examine the possibility to engineer the outcome of a three-equilibrium exchange economy (constructed by Shapley and Shubik, 1977[2]) through the choice of financial institutions in the form of the default penalty regime. The selection of penalties or a value for a government money is equivalent to the fifth condition listed above. We find that the assignment of a proper value to a fiat money (which can be interpreted as a default penalty when net money holding is negative) yields laboratory outcomes in proximity to a predictable unique equilibrium.

[2] The Shapley and Shubik (1977) example has been generalized by Bergstrom, Shimomura and Yamato (2008) so that many other examples of economies with three equilibria can easily be generated.

In contrast with the important goals of macroeconomic applications approached from the “top down,” in this paper we are concerned with a “bottoms up” approach in utilizing a microeconomic approach to studying market economies. Building rigorous foundations of macroeconomics calls for the static general equilibrium models to be integrated with general process models. Strategic market games help us achieve this goal because setting them up forces us to specify complete and consistent process models. By their very nature they are amenable to examination by both mathematical analysis and experimental gaming.

Selection of a single equilibrium from a set of equilibria raises another question: is there any societal reason to favor a specific equilibrium over the others? One reasonable condition is to select the equilibrium which calls for the minimum cash flow. Any process model that is a playable game must specify how trade takes place and thus provides the conditions to be able to calculate the cash flows. By selecting the equilibrium that requires the least amount of money, society would economize on the use of “trust pills” as individual trust is a prerequisite for acceptability of fiat money. As a New England saying puts it: “In God we trust, all others pay cash.”

The paper is organized as follows: Section 2 introduces the Qin and Shubik (2008) economy with multiple equilibria; Section 3 describes three variations of the economy as playable games and presents testable hypotheses; Section 5 presents the results followed by conclusions in Section 6.

2. AN ECONOMY WITH MULTIPLE EQUILIBRIA

In this paper we examine the outcomes of an economy with two commodities and two types of traders modeled as a strategic market game with three competitive equilibrium points, one of which is unstable under Walrasian dynamics. Consider the model graphically illustrated in Figure 1. It displays an exchange economy with three competitive equilibria in an Edgeworth Box. The initial endowment (x, y) of each trader of Type 1 is (40,0) and the initial endowment of each trader of Type 2 is (0,50). The utility functions of the individuals are, respectively,

$$U_1(x,y) = x + 100(1 - e^{-y/10}) \text{ and}$$

$$U_2(x,y) = y + 110(1 - e^{-x/10}).$$

The initial endowment point is the upper left of the box with coordinates (40,0) and (0,50). The dotted lines represent the minimal individually rational indifference curves going through the initial endowment point. The Pareto optimal set of outcomes is given by C_1D_1V C_2D_2. The two curves that intersect three times on the Pareto Set are the response curves for each trader, calculated by varying price and asking each trader how much she would be willing to trade at each price. Supply equals demand only at the points of intersection of the two curves as is indicated by the three equilibria.

(Insert Figure 1 about here)

With only one trader on each side it can be regarded as a model of barter. With *n* traders on each side it provides the simplest model of an economy where a market price can be formed by aggregating many bids and offers. Here the same figure can be regarded as representing type symmetric trade outcomes in a market with *n* players on each side.[3]

Ordinary individuals rarely make conscious economic decisions at a global level. Therefore, for understanding and analyzing an economy populated by agents whose behavior is mostly local, it is possible that the multiple equilibria obtained from global optimization in a formal mathematical model may be misleading or irrelevant. Moreover virtually all

[3] Type symmetry means that all traders of the same type take the same action. Thus, instead of needing a diagram in 2n dimensions the 2-dimensional diagram is sufficient.

experimental gaming has been conducted with open or partial equilibrium systems and we cannot assume that those results necessarily generalize to closed systems. On the other hand, global optima may form domains of attraction even in environments dominated by local behavior (e.g., Gode and Sunder, 1993). Whether this is the case remains an empirical question on which the present exploration can be expected to shed some light.

3. THE GAME AND ITS MODIFICATIONS

We consider a simple game with two subject types with endowments of (40,0) and (0,50) of goods (A,B). All subjects know their initial endowments and the earnings functions, and bid and offer some of their endowments for exchange. We conduct three experimental treatments, the first to establish control about the behavior of the economy in presence of multiple equilibria and about the irrelevance of the choice of numeraire in a pure exchange economy[4]. The second treatment is to verify whether the choice of different levels of the default penalty leads to the respective desired equilibrium, and the third treatment is to explore whether any desired point on the contract curve can be approached. Each treatment has two or more sub-treatments. Unlike the three CEs where trades always balance in value and can be illustrated by points on the Pareto surface in two dimensions, the equilibrium for the third case is a point in three dimensions showing the change in money holdings.

In the first treatment the goods are traded in a pure exchange economy consisting of a single market without money. In Treatment 2 a money is introduced to facilitate trade in two markets (one for each commodity) and we reparametrize this multiple equilibrium economy by changing the default penalty/value of money so that each of the three equilibria, in turn, becomes the unique competitive equilibrium (see Qin and Shubik, 2008). Finally, in

[4] In economics the selection of a numeraire is a far less innocent an assumption than is usually thought (see Smith and Shubik, 2005 and Shubik and Smith, 2007 for a discussion).

Treatment 3 the default penalties are set so that the unique equilibrium coincides with none of the conventional CEs, where the budgets balance for every agent, but with an equilibrium point with active bankruptcy that we call "an active bankruptcy modified general equilibrium."[5]

3.1. Treatment 1 (T1a, T1b): One market to exchange two goods

A commodity money has the property that it can be used to facilitate exchange in every market. In treatment T1 the decision by a player of type 1 is to bid a quantity of good A to buy good B at an exchange ration (price) to be determined by the market, while type 2 player offers an amount of good B for sale in exchange for good A at the same to-be-determined-in-the-market price. When there are only two goods and only one market either can serve as a money, as one can define price in terms of either. Price can be regarded as either the number of units of good A that can be exchanged for one unit of good B, or number of units of good B that can be exchanged for one unit of good A. In the first instance A serves as the numeraire or money, in the second instance B is the numeraire. To explore whether the selection of the numeraire influences market outcomes we conduct two sub-treatments: good A is the numeraire in treatment T1a, and Good B serves that function in treatment T1b. Holdings of goods are carried over from one period to the next. The game ends when total trading volume for either of the two commodities falls below a negligibly small level (arbitrarily set at 0.2 units) in a period, i.e., when no trader wishes to further change his/her position and trading almost stops.

[5] Technically this is a noncooperative equilibrium that only coincides with the conventional general equilibrium when there is a continuum of agents and the penalties are set so that no one chooses strategic bankruptcy.

Subjects are provided with a 51 x 51 (range 0-50 and 0-50) payoff matrix that portrays the final earnings space (see Appendix A for condensed versions). In addition they receive the explicit formulae for the payoffs, which are:

$$U_1(x, y) = x + 100(1 - e^{-y/10}) \text{, and}$$

$$U_2(x, y) = y + 110(1 - e^{-x/10}) \text{.}$$

where x and y are the quantities of goods A and B held by subjects.

In theory, which of the two commodities is regarded as a money should make no difference to outcomes of the economy. In treatment T1 we explore whether this holds in the lab. Furthermore, of the three equilibrium points the middle one is unstable under Walrasian dynamics, but stable under the dynamics in the strategic market game participants play in the lab sessions. Experimental literature frequently reports a tendency towards middle (or average) outcomes (Spear, Gode and Sunder, 2004), which could also play a role here.

There is no generally agreed upon learning theory for such a situation. Without specifying the dynamics, several conjectures are possible, including convergence toward one of the equilibrium points predicted by the static theory. In the single-market models, since we use the buy-sell mechanism,[6] the dynamics is neither price adjustment (Walrasian) nor quantity adjustment (Marshallian) alone. Kumar and Shubik (2004) point out that its dynamics includes elements of both, and has distinct stability properties. They have noted many variants of convergence beyond either Walrasian or Marshallian adjustment.

<u>Conjecture 1:</u> *In Treatment 1 the process fails to converge to any of the three equilibria.*

[6] If A is regarded as the money we may say the traders of Type 1 bid money and Type 2 offer goods, hence "bid-offer". If B is regarded as the money it is vice-versa. For an examination of buy-sell and two other minimal market mechanisms, see Huber et al. (2009).

Although we can mathematically describe the full state space and calculate the equilibria, we are not able to, from the theory, to select a single equilibrium when there are only two commodities.

<u>Conjecture 2</u> *In Treatment 1 the middle CE is favored.*

Walrasian stability considerations do not favor the middle equilibrium point, but it is reached by trading similar amounts of goods 1 and 2 for each other and we thus conjecture that this may be the market outcome.

<u>Conjecture 3</u> *In Treatment 1 the choice of the medium of exchange or numeraire does not influence the outcomes (prices and distribution of goods).*

At this level of abstraction when only two consumable non-durable commodities are present there are no obvious micro-variables such as durability, portability, weight, etc. present that favor one good over the other as the medium of exchange.

We ran T1a and T1b with two cohorts of 10 student subjects each. Each cohort played two runs, i.e., after the first run, a second run was played. The only difference between the runs was that the initial endowments of subjects were reversed, i.e., those endowed with (40,0) in the first run had (0,50) in the second run and vice versa. The notation is run 1-1 for the first run of cohort 1 and run 1-2 for the second run of this cohort (and 2-1 and 2-2 for the second cohort).

3.2. Treatment 2 (T2a, T2b, T2c, T2a-R, T2b-R, T2c-R): Two Markets and Varied Marginal Value of Money

In Treatment 2 a linearly separable money *M* is introduced in addition to goods A and B, and the utility functions are modified by adding a monetary good *z* with constant marginal utility normalized to one:[7]

$$U_1(x, y, z) = \mu_1 z + x + 100(1 - e^{-y/10}) \text{ and}$$

$$U_2(x, y, z) = \mu_2 z + y + 110(1 - e^{-x/10})$$

where the μ's are parameters and the initial endowments now include an amount of money. This amount equals or exceeds the transactions amount needed at any one of the CEs. This change, i.e., the introduction of money with a positive value (default penalty when net money holdings are negative) leads to a new unique equilibrium. The location of this equilibrium depends on the two μ's. For interpretation of the parameters μ_1 and μ_2 note that in an exchange economy with a fiat money there are two quite different forces that support the valuation of the fiat. The first is expectations of the future worth of money in exchange, which is essentially dynamic (see Bak, Norrellyke and Shubik, 1999). The second involves the magnitude of the penalties imposed by a society on individuals who default on their debts. In equilibrium in a society that uses a fiat, money must have the marginal utility of a unit of income equal at least to the marginal disutility of ending with a unit of debt.

Although it appears that we have introduced a linear separable money that could be regarded as some form of commodity money with value, it is a zero-interest loan by the government that must be redeemed at the end of the game. A penalty is assigned against those who fail to return their full original loan, and a reward is given to those who return

[7] In order to make a meaningful comparison between any equilibria we need to normalize the economies so that the value of the total amount of goods in the economy is the same amount, W, under all equilibrium prices.

more. The positive bonus may be regarded as reflecting the expectation that the future valuation of a paper money would maintain its marginal utility.[8]

We may rewrite the utility functions in the form:

$$U_1(x, y, z) = z + \frac{1}{\mu_1}(x + 100(1 - e^{-y/10}))\text{, and}$$

$$U_2(x, y, z) = z + \frac{1}{\mu_2}\,(y + 110(1 - e^{-x/10}))\,.$$

If the marginal disutility of debt is less than the marginal utility of income, it would pay individuals to borrow more and to default. If it is greater than or equal to the marginal utility of income it does not pay to default. In this experiment, we set the terminal value of money (which represents expectations) and the default parameter to be equal, i.e., subjects earn points for positive money holdings and get points deducted for negative money holdings.

In this economy subjects trade in two markets (one each for goods A and B) for money. The trader strategy has two dimensions, with type 1 offering a quantity of good A for sale and bidding a quantity of fiat money to buy good B (and vice-versa for traders of type 2). The introduction of fiat money with the parameters μ_i is enough to guarantee a unique competitive equilibrium point for non-zero amounts of money (see Qin and Shubik 2008).

While μ_1 is always set to 1, μ_2 is varied in three sub-treatments to attain the three equilibrium points.[9] Specifically, in treatments T2a, T2b, and T2c, we set μ_2 = 0.28, 0.75, and 5.07, respectively, since these values correspond to the marginal value of income[10] at the

[8] There are two ways to model the return to the individuals with extra fiat money: give them no terminal payoff or give them a payoff that can be interpreted as the expected future value of money. We choose the latter because it is more consistent with the infinite horizon models; the former is well defined but only consistent with the proposition that the asset fiat money has no value if there are no further transactions.

[9] With the same bankruptcy laws applicable to all, how could the penalties differ across traders? As demonstrated by the bankruptcies of General Motors and Chrysler in Spring 2009, default penalties are tailored by the legal process yielding very different opportunity costs for different agents.

[10] A default penalty needs to be at least this strong to discourage default.

three competitive equilibria of the economy, and examine the effect of this variation in μ_2 on outcomes of the economy.[11]

To assure comparability with the results from T1 we first conduct one run each for T2a, T2b, and T2c. However, learning possibilities are limited in these treatments, as holdings of goods are not reinitialized. Even in a static theory this makes a difference. If we reinitialize the holdings, traders have the opportunity to learn costlessly. Reinitialization clearly makes learning easier, as different strategies can be tried and individual decisions can be improved. In the other instance, i.e., when holdings of goods are carried over, a subject does not have the opportunity to recover from poor decisions made in the past. In particular, as the competitive equilibrium moves with each change in endowment point the no-reinitialization process is stacked against going to the initial CE. To account for this, and to observe whether learning takes place, we conducted two runs for each of three sub-treatments T2a-R, T2b-R, and T2c-R (where "R" standing for reinitialization with 15 independent periods in each run (starting with endowments of 40/0/100 or 0/50/100 of A/B/money); the sum of all period earnings determined the final payout in dollars.

Conjecture 4 *In Treatment 2 the system converges and can be made to converge to any of the three equilibria guided by the selection of parameter μ.*[12]

In Treatment 2 there is an ideal money with linear transferable utility. Where the parameters μ come from has to be explained. In the structure here it is the expected value of money at the end of the game fixed by the experimenter. In the theory it can be interpreted as

---

[11] Multiple equilibria are rare in general as was shown by Debreu. For this example Kumar and Shubik (2003) performed a sensitivity analysis to show precisely the somewhat narrow range of changes in the distribution of endowments of the two player types that would preserve the property of multiple equilibria. In other words a slight redistribution of resources given to a trader (more of one commodity and less of the other) will produce an economy with one equilibrium unless the redistribution is within an appropriately narrow range,

[12] If the μ (μ_1, μ_2) are not selected to coincide with the Lagrangians the books are balanced by a transfer of money as is shown in the third treatment.

expectations, or (when borrowing is permitted and thus negative holdings are possible) it can be interpreted as default parameters set by a society. In either instance, if these parameters are under the control of the society (or experimenter in the game) and there is a setting associated with each of the three equilibria, the parameters may be selected in a way that fixes the value of money associated with one of the equilibria.

As the money is a linear term, equilibrium can be reached with any net money holdings, depending on how prices evolve. We conjecture that subjects with relatively high marginal utility for good holdings (i.e., those endowed with good B in T3a and those endowed with goods A in T3c) will be ready to incur negative net money holdings (i.e., spend more than they earn), while those with low marginal value of goods (and thus relatively high value for money) will hold on to money and thus end with positive net money holdings. However the theoretical possibility of zero net money holdings should occur for penalties set appropriately for any one of the three CEs. Thus for them the null hypothesis is.

<u>Conjecture 5</u> *In Treatment 2 net money holdings will beequal to the equilibrium level of zero.*

We have six sub-treatments to T2. Each of T2a, T2b, and T2c is run with one cohort of 10 students each and trading ends once trading volume drops to a negligible amount. Each of the three sub-treatments T2a-R, T2b-R, and T2c-R is run with two cohorts of 10 students each that played a predetermined number (15) of independent periods. No subject participated in more than one of the sub-treatments.

3.3. Treatment 3 (T3, T3-R): Two Markets with Money

In Section 3.2 it was suggested that by the appropriate selection of parameters μ_1 and μ_2 the economy could be guided to select any one of the three equilibrium points. A question not addressed was what happens if the parameters selected do not coincide with the marginal values of income at any of the three equilibria?

An equilibrium will exist for any parameter selected but it will not be one of the three CEs of the original economy. Any sufficiently large selection of penalty which is different from the CE penalties will lead to a unique equilibrium with a net transfer of money from one class of agents to the other.[13] We examine this possibility in T3 where we purposely set $\mu_1 = \mu_2 = 1$ in order to consider a case where the solution should be a unique equilibrium with allocations different from all three equilibria in the original model (Treatment 1). When both μ's are set to 1 the unique equilibrium coincides with the joint maximum, i.e., the point where the sum of the earnings of the two trader types is maximized.

The formal mathematical theory of general equilibrium has no money and no errors. It does not provide a process structure. Here money plays a critical role in formulating the dynamic structure of an economy. The value of supply does not need to equal the value of demand. The difference is covered by a net transfer of money.

One might ask why bother with penalty levels other than those that support equilibria. One reason is to stress our concern with the role of rules and institutions in the economy. We believe that the government has only general knowledge about the preferences and assets in the economy which is normally insufficient accurately to guess a penalty level that would support one of multiple equilibria. If it guesses incorrectly the number of bankruptcies would signal that it needs to adjust the penalties. This could be tested experimentally by having the

[13] Thus, for a complete representation we would need a three-dimensional diagram.

government as a player trying to select an appropriate penalty but having some uncertainty concerning endowments and preferences.[14]

Conjecture 6 *In Treatment 3 the unique equilibrium defined by the default penalties* μ_1 *and* μ_2 *is approached.*

We conduct one sub-treatment where holdings of goods are carried over (T3) and one where holdings are reinitialized after each period (T3-R). Two runs with one cohort of 10 subjects are conducted for T3 and one run for T3-R. All experiments were carried out using a program written in z-Tree (Fischbacher, 2007), and average payment was 20 dollar for each subject in each of the approximately 60-minute sessions. All subjects were undergraduate business or economics students. About half of the runs were conducted at Yale University, while the other runs were conducted at the University of Innsbruck.

4. RESULTS OF THE EXPERIMENTS

For our three treatments we conducted ten sub-treatments (T1a, T1b, T2a, T2b, T2c, T2a-R, T2b-R, T2c-R, T3, T3-R) with a total of 20 runs. Each run was conducted with 10 subjects for a total of 200 subjects. In all treatments five subjects started with an endowment of 40/0 while the other five started with 0/50 of goods A/B. There was no money in Treatment 1. The money endowment was 100 units per trader in Treatments 2 and 3.

4.1. Treatment 1

In Treatment 1 goods A and B are traded for each other. Holdings of goods are carried over from one period to the next and there is no money. Good A serves as the unit of

[14] When there is no exogenous uncertainty active bankruptcy is caused by inappropriate penalties or human error. In an economy with exogenous uncertainty an optimal bankruptcy law can only be reflected by taking into account society's attitude towards risk. It is a form of public good; and even without human error it will involve active bankruptcy.

accounting in T1a and good B is the numeraire in T1b. Trading stops when total offered volume in either of the two goods is below 0.2 units in a period.

The four panels of Figure 2 present the Edgeworth boxes of the holdings of goods A and B in the four runs of T1a where A is the numeraire. The black squares mark the three equilibrium points and the dashed line is the contract curve. The grey triangles mark the joint maximum earnings, i.e., the point where the sum of the earnings of both trader types combined is maximized. The small circles show individual holdings of goods at the end of a period and the black line with black diamonds show the development of average holdings of goods over periods (each diamond shows the holdings at the end of a period). Two panels on the left show the first run of a subject cohort, while the panels on the right show the respective second run. We observe that three of the four runs end in the vicinity of the contract curve. Of the three equilibria and the joint maximum, the end points of all four runs are closest to the joint maximum. Of the three equilibria, the end points are closest to the middle equilibrium in three runs and about midway between the middle and the lower left equilibrium in one run. On the whole, neither the joint maximum nor any of the three equilibria are a compelling candidate to serve as domains of attraction for these economies.[15] Rather a vague "trend to the middle" seems to drive the results. This confirms Conjecture 1 (no approach to an equilibrium) and rejects Conjecture 2 (approach to middle equilibrium) for T1a.

(Insert Figure 2 about here)

Figure 3 presents the same data for Treatment 1b. Recall that this treatment is identical to Treatment 1a with the only exception that good B is the numeraire. We see in Figure 3 that – as predicted by theory – this does not change results significantly. Two of the

[15] For a detailed discussion of stability see Kumar and Shubik 2004.

four economies come close to the joint maximum, but none of the three equilibria serve as a strong domain of attraction for these economies. Even the middle equilibrium does not do very well in spite of the fact that in the presence of three equilibrium predictions, the middle point has a statistical advantage of the two extremes in being closer to the observed data. Again Conjecture 1 is confirmed while Conjecture 2 is rejected.

(Insert Figure 3 about here)

Does the choice of the numeraire make a difference for the outcome of the experiment? In a first test we use a Mann-Whitney U-test to compare final holdings of good A between the four runs of T1a and the four runs of T1b. The same is repeated for the final holdings of good B. We do not find any significant differences, as the p-values are 0.2 or higher (N = 40). This observation confirms Conjecture 3 that the choice of numeraire does not significantly affect the market outcome.

A key question in considering any market mechanism is whether the outcome is efficient. Here we define efficiency as the sum of all subjects' earnings as a percentage of the maximum earnings achievable, i.e., this maximum is 100 percent in each treatment.[16] Figure 4 presents the time series of trading volume and efficiency in Treatment 1. The four panels on the top and in the middle rows present the development of cumulative trading volume (Good A in the top row, Good B in the middle row; Good A as numeraire on the left, and Good B as numeraire on the right panels). Cumulative trading volume generally approaches the joint maximum prediction (dark continuous line) and the experienced subject economies (shaded graey) are generally closer to the joint maximum than the inexperienced subject

[16] Note that the specific numbers of these maximum earnings are equal in T1a and T1b, but otherwise vary from sub-treatment to sub-treatment, as μ is varied. Actually, in T1 any point on the Pareto surface could be regarded efficient, but for simple two-dimensional graphing as an approximation we utilize the joint maximum, i.e. the point were the sum of the earnings of both trader types is maximized.

economics. Each subject has only the option to sell units of the good she is initially endowed with; cumulative trading volume series are concave because trading volume declines as subjects approach their desired portfolio of goods.

(Insert Figure 4 about here)

The bottom panel of Figure 4 shows the development of efficiency in T1. With lower trading volume in the second runs, efficiency also increases more slowly. On average the same level of efficiency is reached in the first and the second runs (96.1 vs. 96.8 percent; the difference is statistically insignificant). Overall we find that efficiency reaches high levels and trading in all runs stopped either in the period with the highest efficiency, or one period after the maximum was reached.

Summing up, in Treatment 1 we find that it does not matter which of the two goods in this exchange economy is chosen as the numeraire. Efficiency in all markets is high, demonstrating that such simple markets serve well as coordination mechanisms.

4.2. Treatment 2

In Treatment 2, the salvage value of money (default penalty when negative) is varied (μ_2 = 0.28, 0.75, and 5.07) to reach the three separate equilibria in T2a, T2b, and T2c, respectively. We conduct one run each for T2a, T2b and T2c, where holdings of goods and money are carried over from one period. Each run was conducted with a different cohort of ten students each.

To allow learning and observe possible learning effects we then implemented three other sub-treatments, where holdings of goods are re-initialized after each period (μ_2 is varied again with values 0.28, 0.75, and 5.07). Thus, subjects start each period with 40/0/100 or 0/50/100 of goods A/B and money, and they have only one transaction to reach their desired holdings of the goods and money. As holdings are re-initialized we label these sub-treatments "R" and thus refer to them as T2a-R, T2b-R, and T2c-R. We conduct two runs for each of the three sub-treatments for a total of six runs, each with a different cohort of students. 15 independent periods are conducted for each of the runs. No student participated in more than one run presented in this paper.

Figure 5 presents the development of individual and average end-of-period holdings of goods A and B for T2a (left panel), T2b (center), and T2c (right panel). We still show all three competitive equilibria of the economy, however, by defining the salvage value of money there is a unique equilibrium in each sub-treatment. This unique equilibrium is shown as a black square, while we still display the former equilibria in white squares and the former joint maximum (white triangle) for the sake of easier comparison across treatments. The paths in the three sub-treatments are distinct from each other, and each path approaches the vicinity of its respective equilibrium (and away from the other two equilibria). To test whether manipulation of salvage values/default penalties for money (parameter μ_2) can select different equilibria as claimed in Conjecture 4, we use Mann-Whitney U-tests comparing the ending holdings of good A for T2a with those of T2b, T2a vs. T2c, and T2b vs. T2c. This is repeated for good B. All six tests produce p-values smaller than 0.05, and Conjecture 4 on the choice of default penalty causing the economy to converge to the corresponding equilibrium is not rejected.

(Insert Figure 5 about here)

The three sub-treatments differ with respect to the trading volume required to reach the respective equilibrium. In T2a (μ_2 = 0.28), which holdings of goods relatively more valuable for traders endowed with good B, those endowed with A should sell most of their holdings of A to the other traders, while those endowed with B should hold on to most of their goods. To reach equilibrium each A-holder should sell 36.78 of his 40 units of A, while each B-holder should sell only 10.23 of his 50 units of B. To provide more dynamic information the development of cumulative market trading volume per period is displayed in Figure 6 (market volumes should be five times the per capita trades given above). We see in each market the expected pattern of relatively high trading volume in the beginning and a subsequent drop in trading volume until trading stops between period 9 and 12 (as volume in one good drops below 0.2). We also see that the predicted market volumes provide some support for the observed market volumes in all three sub-treatments in the three panels of Figure 6. As predicted by theory in T2a, trading in A is more active than in B, while the opposite holds in T2c (in T2b the volumes are balanced, as predicted by theory).

(Insert Figure 6 about here)

We do not display the development of efficiency in a separate figure, as the results are very similar to those observed in T1: efficiency increases over time and trading stops in the period with the highest overall efficiency or one period later. The efficiency levels reached are 98.2, 98.2, and 95.6 percent, respectively, in T2a, T2b, and T2c.

Treatment 2-R (holdings of goods and money re-initialized)

We now turn to the three sub-treatments where holdings of goods and money are re-initialized after each period. Two runs were conducted for each sub-treatment. Figure 7 displays period-by-period holdings of goods A and B in T2a-R, T2b-R, and T2c-R (with the final period shaded in grey). The lines allow us to follow the outcome of trading, i.e., the average end-of-period holdings over the sequence of periods. The left panel shows first run for each of the three sub-treatments, and the right panels presents the second. All six runs were conducted with different subjects. The paths in the three sub-treatments are quite distinct from one another, and each run approaches its respective equilibrium over time.[17] The T2b-R and T2c-R equilibria are essentially reached in the second period, while in T2a-R it took a few periods longer to approach the equilibrium. This demonstrates that the selection of the default penalty is suitable to select among the multiple equilibria, thus corroborating the result from T2 that Conjecture 4 is not rejected.

(Insert Figure 7 about here)

In all three sub-treatments the largest gain in efficiency occurred during the first period, moving from autarky to the market economy. As can be seen in Figure 8 this is followed by smaller increases in efficiency over subsequent periods (period 0 is efficiency associated with autarky and 100 percent is the efficiency of the respective competitive equilibria). The parameter μ_2 played a major role in the achievement of efficiency. In T2a-R ($\mu_2 = 0.28$) it took 10 (4) periods for efficiency to reach 90% in the first (second) run. T2b-R and T2c-R ($\mu_2 = 0.75$ and 5.07, respectively) reached high efficiency levels of more than 90% already during the very first period.

(Insert Figure 8 about here)

These results of T2 and T2-R broadly confirm the results from T1—the introduction of a money allows convergence to the unique equilibrium that is defined by the value/default penalty associated with the money.

[17] Note that each diamond marker in Figure 7 shows the holding achieved in a period starting every period with the endowment point in the northwest corner. We have joined the diamond markers with a line to indicate the sequence of periods in order to point out that the outcomes got generally closer to the respective equilibrium holders in the later periods of the runs. In contrast, in the treatments without re-initialization, the northwest corner was the endowment point only at the beginning of period 1, and the change in holdings in all subsequence periods was incremental relative to the end of the preceding period.

Net Money Holdings

In the sub-treatments of T2 and T2-R the respective equilibrium can be reached with net money holdings of all traders at zero or at any other desired level, as money holdings are a result of prices which are set endogenously by traders' bids and offers. Net money holdings of zero are achieved when the ratio between the prices of the two goods is equal to the respective μ_2.[18] However, this would lead to a very uneven distribution of final points earned, e.g., in T2c (and T2c-R) with $\mu_2 = 5.07$ subjects starting with good B would have to buy 7.74 units of good A at a price five times higher than the price they get for each of the 39.26 units of B they sell in equilibrium. They would end up with relatively small holdings of the goods and thus earn only 10 percent of the points that A-holders earn.

Figure 9 presents the development of net money holdings over time in all six sub-treatments (the runs of T2a (and T2a-R) are in the left panel, T2b is in the center and T2c on the right). The panels also show the GE holdings (always zero). We see that the GE-proposition does not serve as a good benchmark in most of the runs, as net money holdings are never zero and mostly move away from zero over time. Only in T2b do net money holdings remain relatively close to zero. Net money holdings of subjects in the last period are significantly different from zero in eight of the nine runs of T2 ($p < 0.01$, Mann-Whitney U-Tests, N=5 for each test, the only insignificant result is T2b). Thus Conjecture 5 is rejected.

(Insert Figure 9 about here)

Conjecture 5, while consistent with the theory leave out both the imperfections of learning and error to be expected in even as simple an environment as this. Furthermore, the perceived extreme nonsymmetry of the three CEs is such that we might expect a deviation from the balanced budget condition to be present, especially for the two extreme equilibria.

[18] Recall that $\mu_1 = 1$ in all treatments.

Further experimentation is called for to resolve the attribution of the sources of lack of a balanced budget.

4.3. Treatment 3

In Section 4.2 we have demonstrated that varying the default penalty can bring the economy close to the selected one of the three equilibria of the original model. A question that is not yet answered is: what would happen if the default penalties were set to a level other than the three that correspond to the three equilibria. In Treatment 3 we set $\mu_1 = \mu_2 = 1$ to explore this question. This game has a unique equilibrium which coincides with the joint maximum earnings. To ensure comparability with T2 we conducted this treatment once with holdings of goods and money carried over from one period to the next (two runs with the same subjects, T3), and once with holdings of goods and money re-initialized (one run, T3-R).

Figure 10 presents the development of holdings of goods over time in the two runs of T3. Both runs are quite similar and end in the vicinity of the unique equilibrium (joint maximum). Final holdings of goods are not significantly different from the holdings in the joint maximum in both runs (Mann-Whitney U-test, $p > 0.1$ in both runs, N=10), and Conjecture 6 is not rejected.

(Insert Figure 10 about here)

This is corroborated by the run we conducted with re-initialization, presented in Figure 11. On the left panel we see that the average final holdings of traders are in the vicinity of the unique equilibrium, though they never hit it perfectly. The right panel shows that efficiency increases over time (it is high in the first period, lower in the next two, and increases steadily from period 3 to the end).

(Insert Figure 11 about here)

Equilibrium prediction is that the final net money holdings will be -2.3 for traders endowed with 40/0 and +2.3 for traders endowed with 0/50 of goods A/B. In the two runs of T3 average final money holdings turn out to be -3.9 and +3.9 for traders endowed with 40/0 and 0/50, respectively (in T3-R final money holdings are -1.4 and +1.4, respectively). Results are consistent with the equilibrium predictions.

As in Treatment 1 trading volume in T3 drops rapidly (see left panel of Figure 12) and efficiency rises over time (right panel of Figure 12). Again we see that efficiency and trading volume paths are slightly lower in the first few periods of the second run, and the second run lasts longer (11 vs. 8 periods in the first run) and ends with (insignificantly) higher efficiency. These facts are all comparable to in the results of T1.

(Insert Figures 12 about here)

4.4. A Summary of Results

Conjecture 1 Confirmed. The process does not converge to an equilibrium.

Conjecture 2 Rejected. While in Treatment 1 most paths end closer to the middle CE than to any other equilibrium even the middle equilibrium does not serve as a strong point of attraction.

Conjecture 3 Confirmed. In Treatment 1 the choice of the money does not (statistically significantly) influence the outcome (efficiency or distribution of goods).

Conjecture 4 Weakly confirmed. In Treatment 2 the system converges and can be made to converge to any of the three equilibria through the selection of the value of μ and initial endowment of money. Convergence to the "outer" equilibria takes longer than to the middle equilibrium.

Conjecture 5 Rejected . Net money holdings differ from the equilibrium level of zero.

Conjecture 6 Confirmed. The unique equilibrium is approached in Treatment 3.

5. CONCLUSION

This paper examines the role of salvage value/default penalty of a fiat money in selection from a multiplicity of equilibria. In the laboratory markets reported here two goods were traded for each other (in Treatment 1) or for a money (in Treatments 2 and 3). Treatment 1 corroborated the conjecture that, in presence of three equilibria, the outcomes of the economy are noisy, lie somewhere in the middle of the range of the three equilibria, and do not converge to the close proximity of any of them. It also confirmed that when there are only two commodities with identical features in an economy, it makes no difference which one is chosen to serve as the numeraire (the commodity that serves as the money is used to bid in the market to buy the other good).

Treatment 1 having established empirical support for the theoretical indeterminacy, Treatments 2 and 3 were designed to test the main hypothesis: salvage value/default penalty of a fiat money can be chosen to achieve any of the given equilibria of the economy, or more generally, any desired point on the contract curve. Thus the central experimental task is to examine the suggestion from theory that the institutional arrangements in a society provide the means to resolve the possibility of multiple equilibria in an economy. Results from Treatment 2, where three different default penalties were chosen, show that the economies approached the respective equilibria. There is empirical support for the attitudes of macroeconomists who do not regard the non-uniqueness of competitive equilibria as a major applied problem. A society implicitly solves the uniqueness problem in the guidance of a competitive economy by the selection of default penalties that link the value of money directly to the preferences of individuals. The need for society to add the institutional details and extra parameters is forced by the need to specify how to handle all outcomes from a dynamic process.

Treatment 3 demonstrated that by proper selection of a default penalty any desired equilibrium can be selected from the multiple equilibria present. However, we stress that although a societal selection of the extra parameters is sufficient to obtain a unique equilibrium, unless the parameters coincide with the values of the Lagrangian variables at an equilibrium of a static exchange economy, the static equilibrium solution to the new game will not coincide with any of these equilibria.[19]

Summarizing, the first treatment indicated that none of the three equilibria provided much predictive power. Treatment 2 showed how a pair of socially engineered parameters could serve to select any of the equilibria, but this requires a "fine tuning" of the equilibrium values and detailed knowledge of the preferences and parameters of the economy. In a society with dispersed knowledge and perennial political and bureaucratic battles, neither such knowledge nor the fine tuning seems feasible. Treatment 3 investigated what happens if the marginal values of money fixed by the penalties do not coincide with the values at the three CEs. A different unique equilibrium point is predicted by the theory and reflected in the results from the lab.

[19] We expect that if other penalties are selected the players will tend towards the equilibrium which will be fully three dimensional as the net trade will not balance to zero, but will involve a transfer of money between the agents. In order to show this we would need a three dimensional diagram. A more detailed discussion of this point is in Qin and Shubik (2008).

References

Bak, P. S. F. Norelykke, and M. Shubik. 1999. "Dynamics of Money", *Physical Review E* 60 (3): 2528-2532.

Bergstrom, Theodore C., Ken-Ichi Shimomura, and Takehiko Yamato. 2008. "Simple Economies with Multiple Equilibria," University of California Santa Barbara Working Paper. 07'09'08

Fischbacher, U. 2007. "z-tree: Zurich toolbox for ready-made economic experiments" *Experimental Economics* 10(2), 171-178.

Gode, D. K., and S. Sunder. 1993. "Allocative Efficiency of Markets with Zero Intelligence Traders: Market as a Partial Substitute for Individual Rationality." *The Journal of Political Economy* 101, no. 1 (February 1993): 119-137.

Gode, D. K., S. Spear, and S. Sunder. 2004. "Convergence of Double Auctions to Pareto Optimal Allocations in Edgeworth Box," Carnegie Mellon University Working Paper.

Grandmont, J. M. 1983. *Money and value*. Cambridge: Cambridge University Press.

Hansen, L. and J.L Heckman. 1996. "The empirical Foundations of Calibration." *Journal of Economic Perspectives* 1: 87-104

Hicks, J.,R. 1946 *Value and Capital*. 2nd ed. Oxford Clarendon. Original 1939

Huber, J., M. Shubik, and S. Sunder. 2009. "Three Minimal Market Institutions with Human and Algorithmic Agents: Theory and Experimental Evidence" Working paper.

Huber, J., M. Shubik, and S. Sunder. 2008. "The value of a fiat money with an outside bank: an experimental game." Working paper.

Karatzas, I., M. Shubik, W. Sudderth, and J. Geanakoplos. 2006. "The Inflationary Bias of Real Uncertainty and the Harmonic Fisher Equation" *Economic Theory* 28 (3), 481-512.

Kumar, A. and M Shubik. 2004. "Variation on the Theme of Scarf's Counter-Example" *Computational Economics* 24 (1). 1-19.

Kumar A. and M. Shubik 2003. "A computational analysis of core convergence in a multiple equilibria economy." *Games and Economic Behavior,* vol. 42, #2: pp. 253-266.

Qin, C. Z., and M. Shubik. 2008. "A credit mechanism for selecting a unique competitive equilibrium." Cowles Foundation discussion paper No. 1539RRR, Yale University.

Scarf, H. E., with T. Hansen. 1973. *The computation of economic equilibria*. New Haven: Yale University Press.

Shapley, L. S., and M. Shubik, 1977. "An Example of a Trading Economy with Three Competitive Equilibria." *Journal of Political Economy* 85 (4): 873-75.

Shubik, M. and D. E. Smith 2007 "Structure, Clearinghouses and Symmetry" , *Economic Theory*, vol. 30, #3, pp. 587-597.

Smith D.E and M Shubik,. 2005 "Strategic Freedom, Constraint, and Symmetry in One-Period Markets with Cash and Credit Payment" *Economic Theory*, vol. 25, #3, pp. 513-551.

Smale, S. 1976. "A convergent process of price adjustment and global Newton methods." *Journal of Mathematical Economics* 3: 1-14.

Figure 1: An Exchange Economy with Two Goods and Three Competitive Equilibria

(Reproduced from Shapley and Shubik 1977, Fig. 1, p. 874)

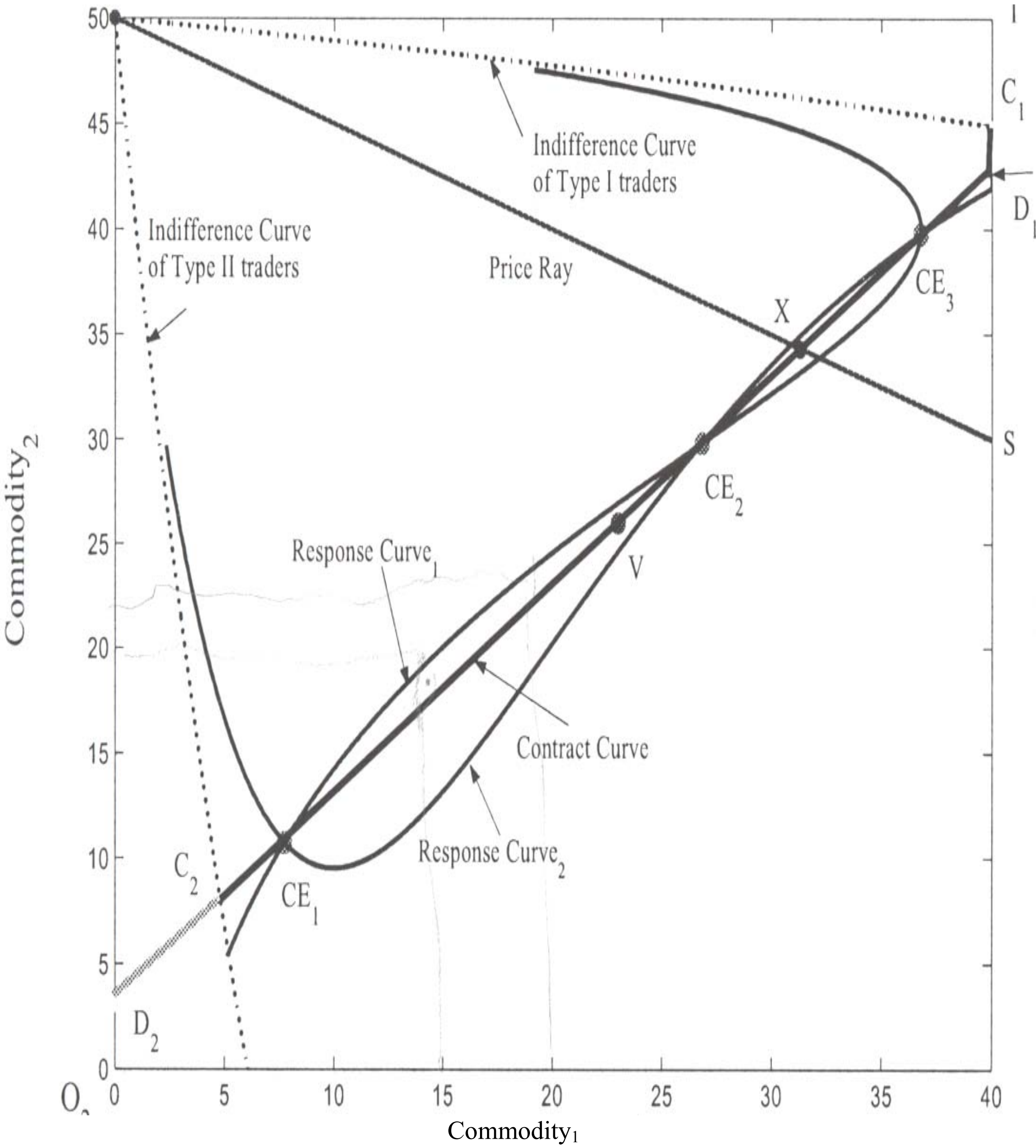


Figure 2: Holdings of Goods A and B in the Four Runs of Treatment 1a (with Good A as the Numeraire)

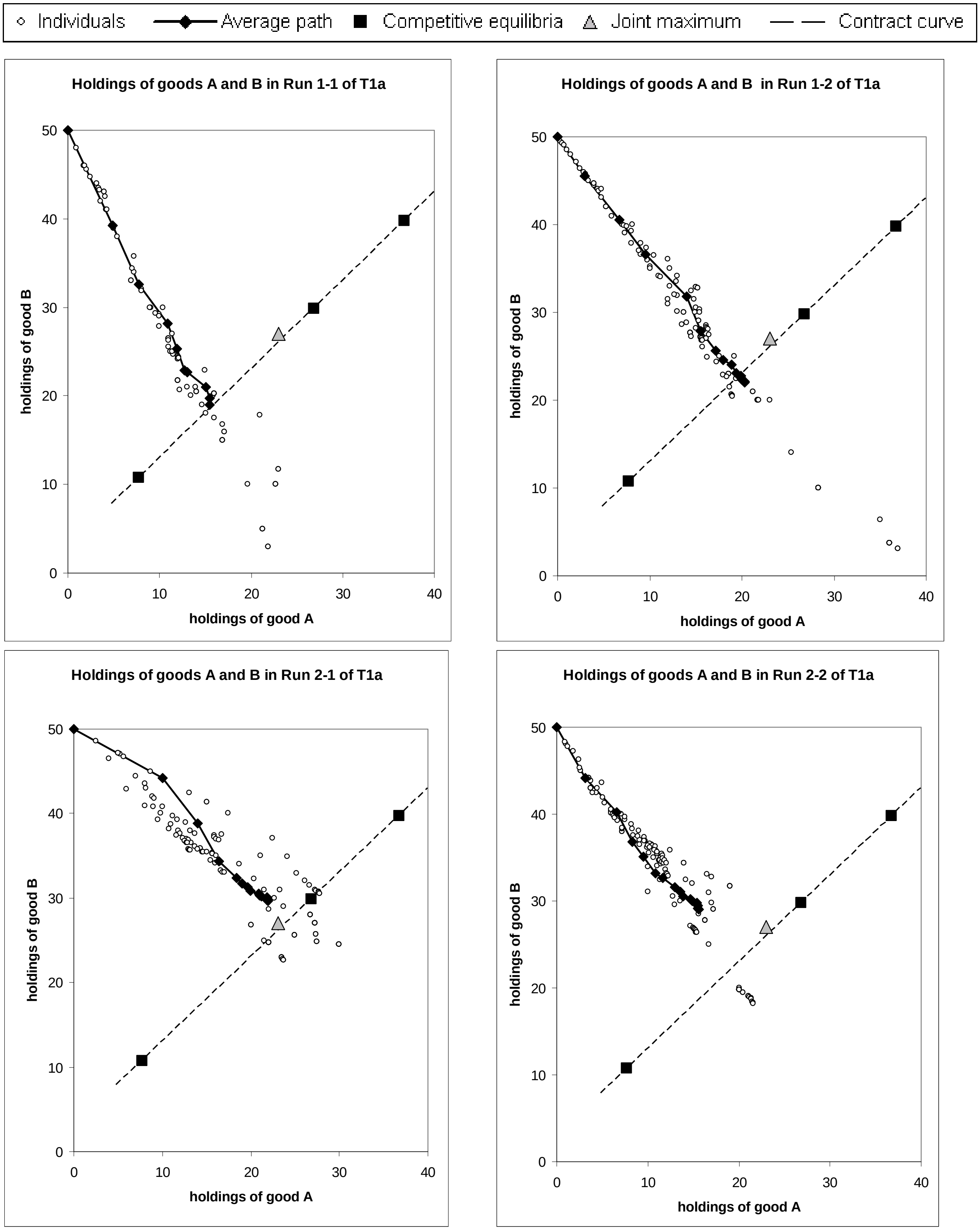


Figure 3: Holdings of Goods A and B in the Four Runs of Treatment 1b (with Good B as the Numeraire)

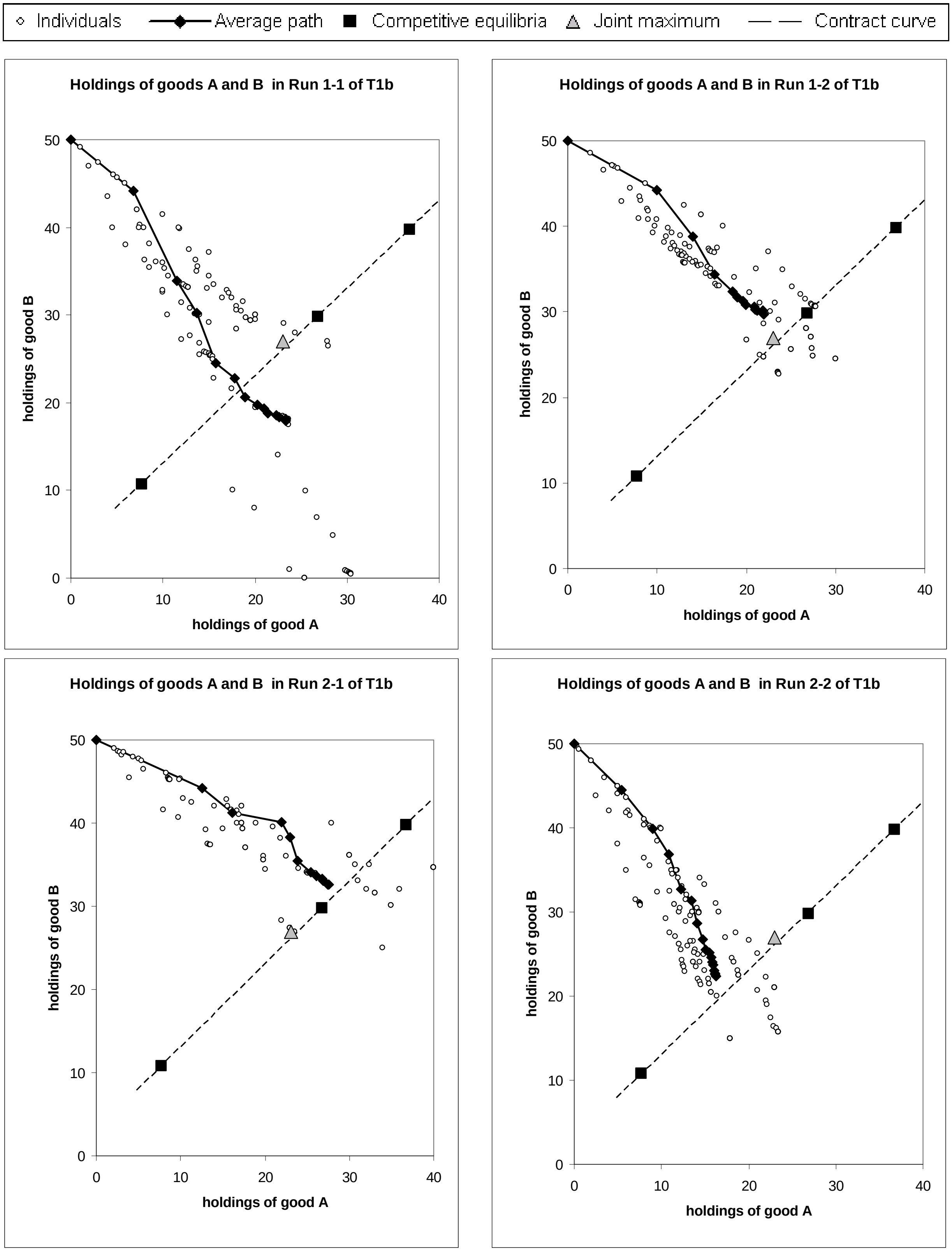


Figure 4: Time Series of Cumulative Trading Volume (Top Panels) and Efficiency (Bottom Panels) per Period in Treatment 1. T1a is presented on the left side, while T1b is on the right side. The Second Run of each Student Cohort is shaded in grey.

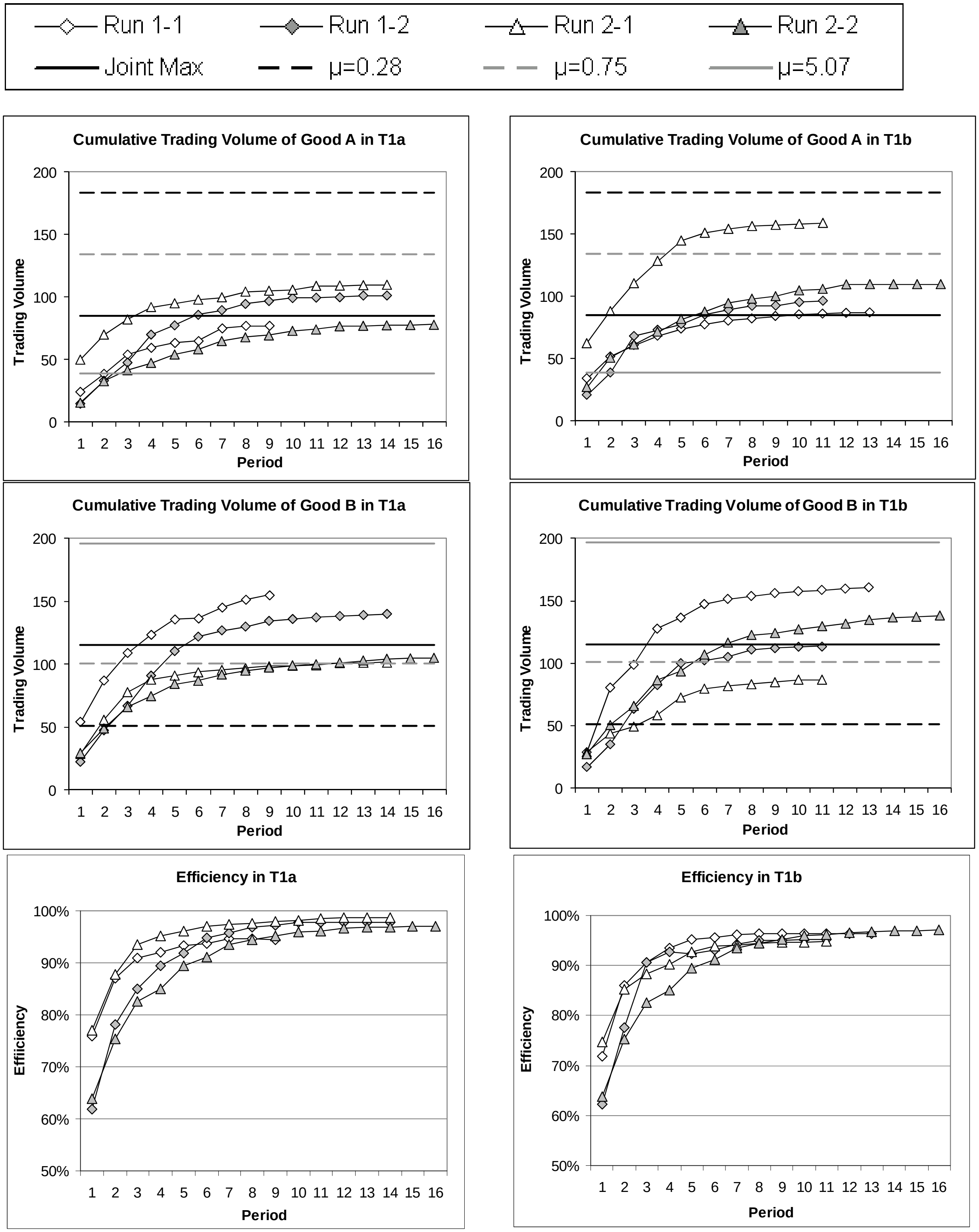


Figure 5: Individual and Average Holdings of Goods A and B in Treatment 2 (holdings of goods and money carried over from one period to the next)

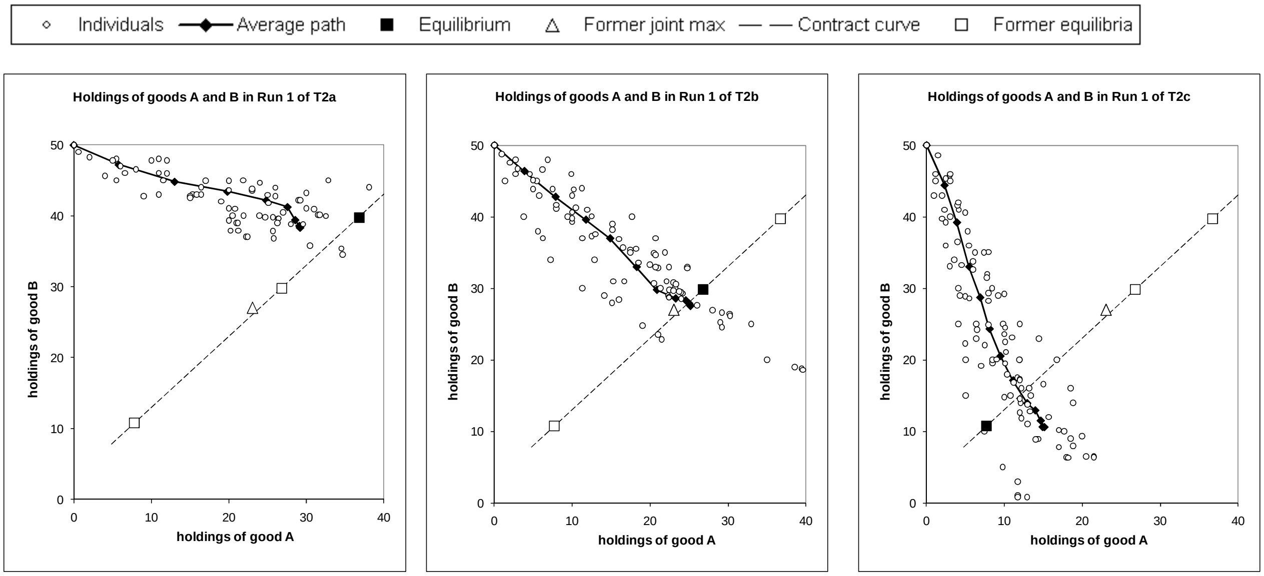


Figure 6: Time Series of Cumulative Trading Volume in Treatment 2. T2a is presented on the left side, T2b in the center, and T2c on the right.

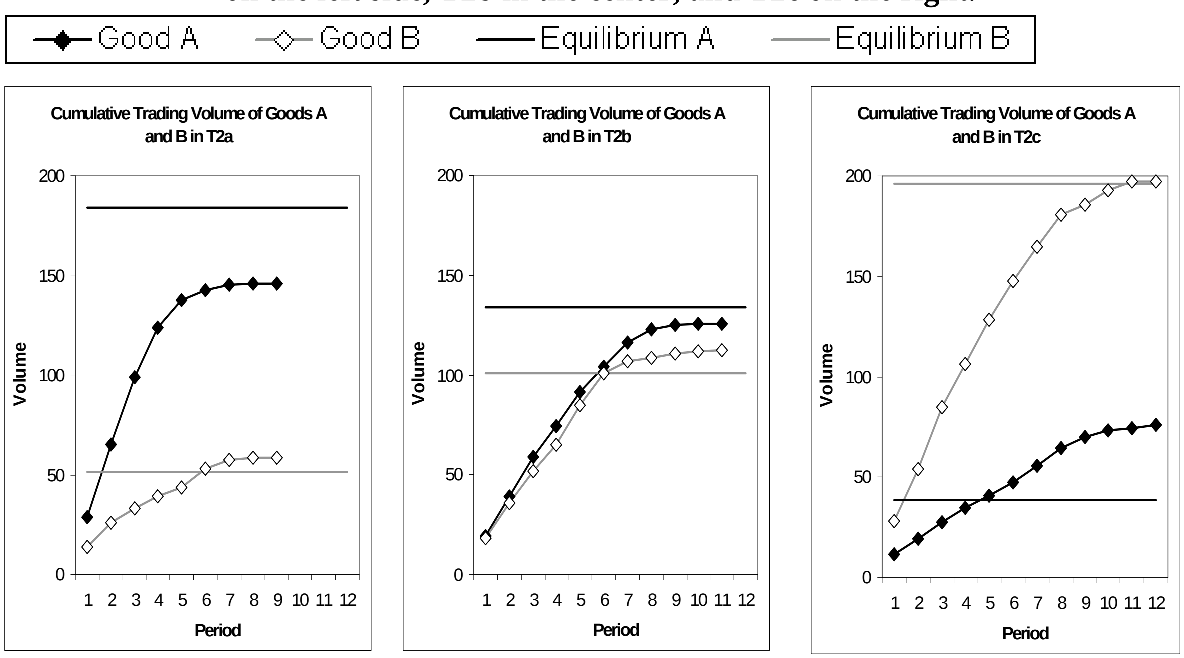


Figure 7: Path of Average Holdings of Goods A and B in Treatment 2-R (with holdings reinitialized)

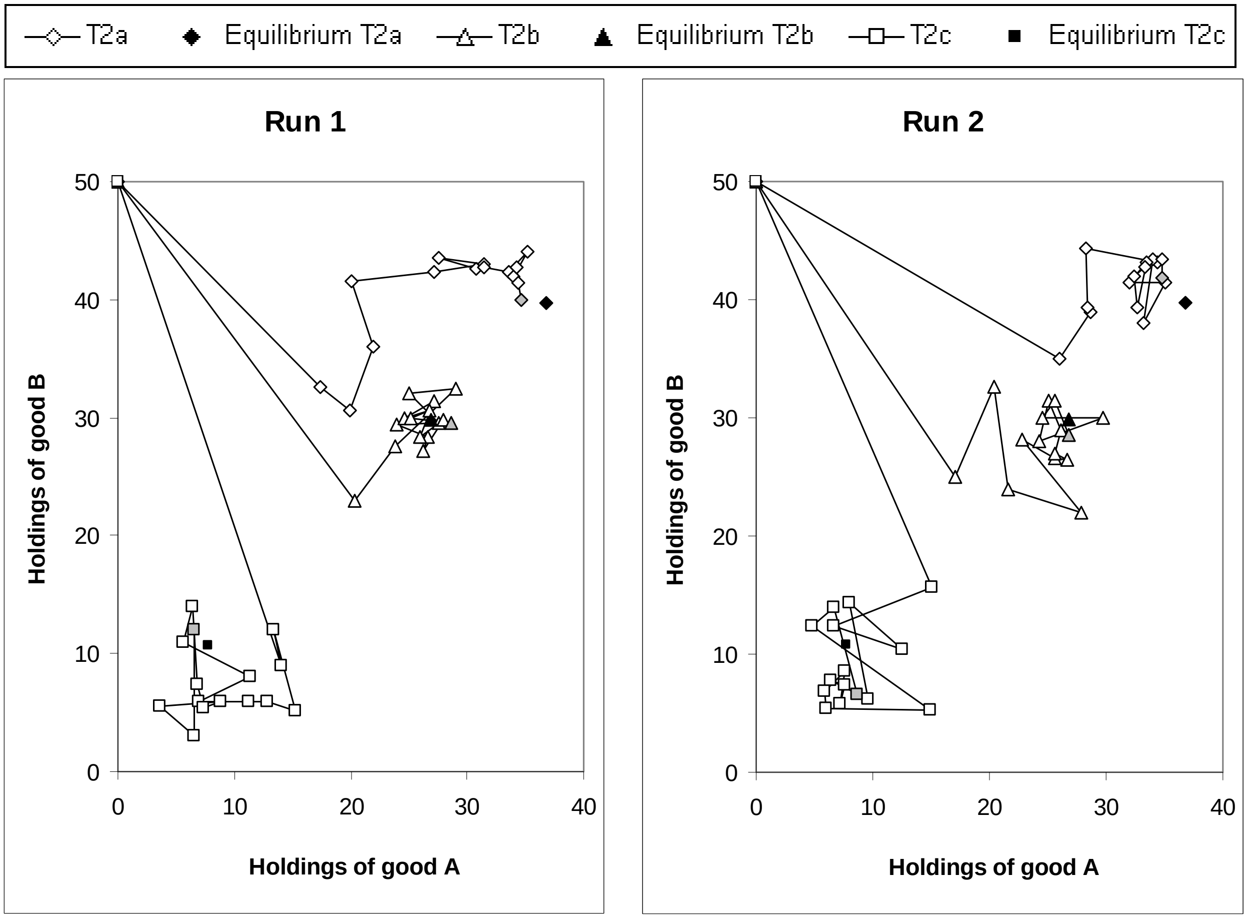


Figure 8: Time Series of Efficiency in Treatment 2-R (with holdings reinitialized) (Period 0 = autarky)

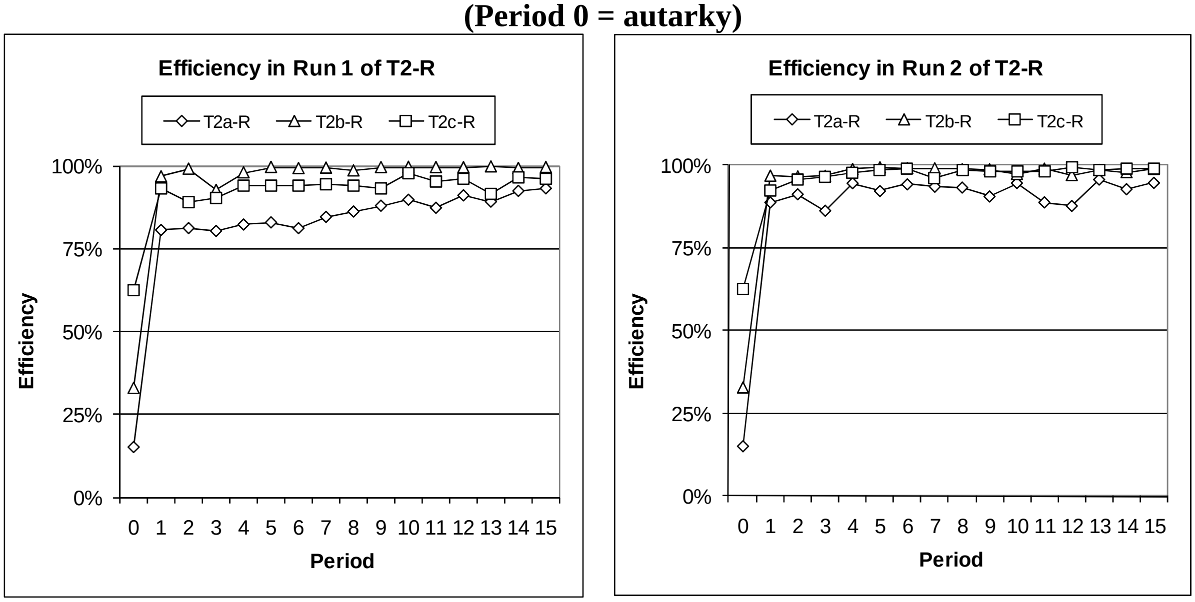


Figure 9: Development of Average Net Money Holdings for Subjects endowed with Good A in Treatment 2 (holdings carried over) and T2-R (holdings re-initialized)

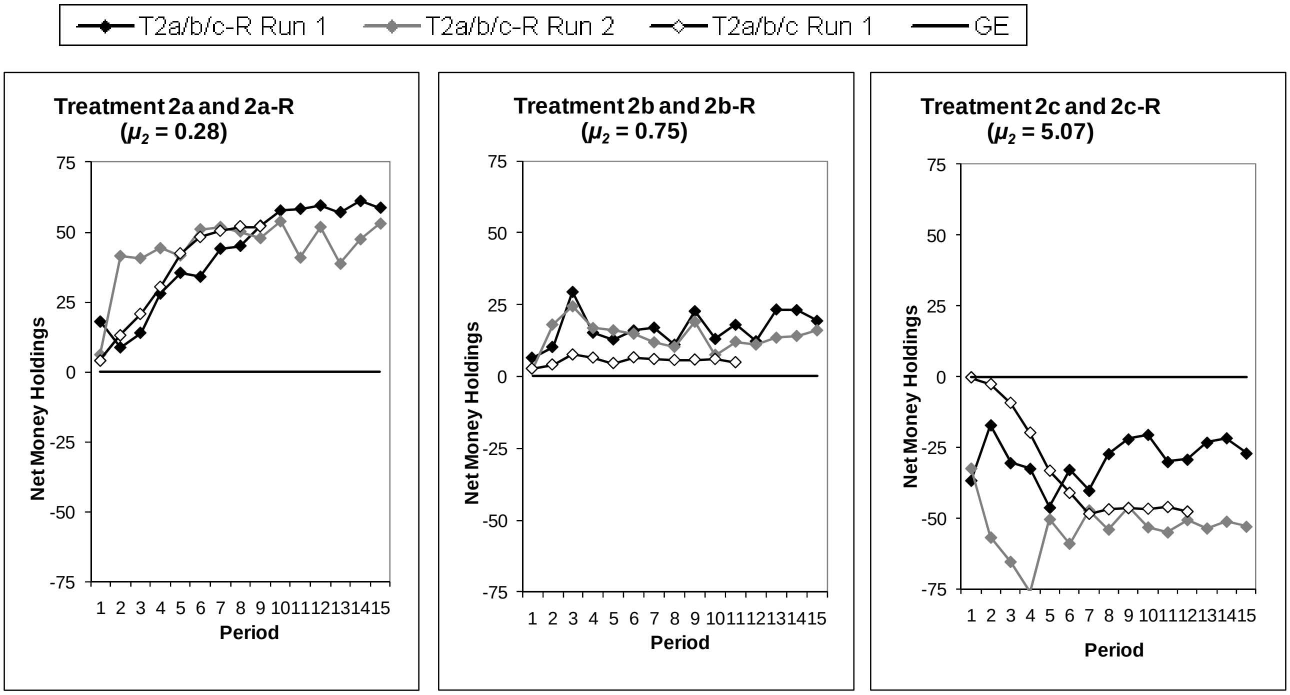


Figure 10: Holdings of Goods A and B in the two Runs of Treatment 3

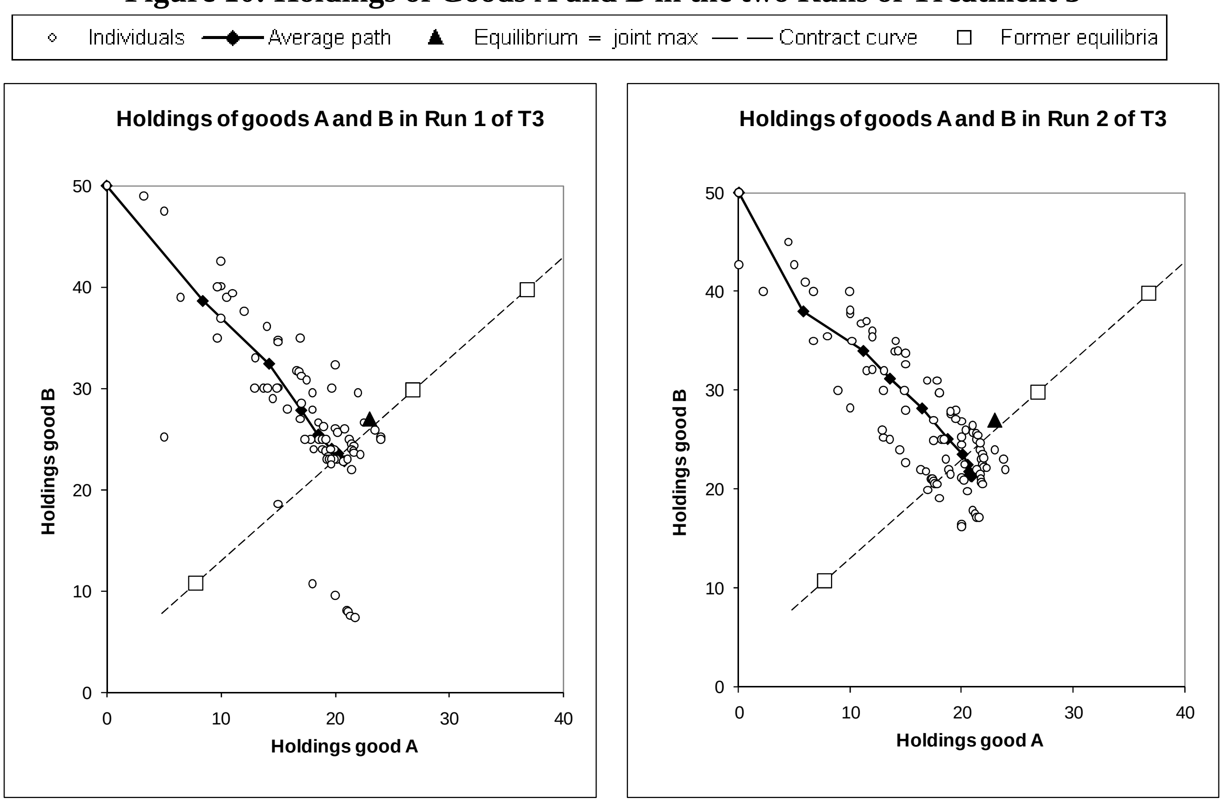
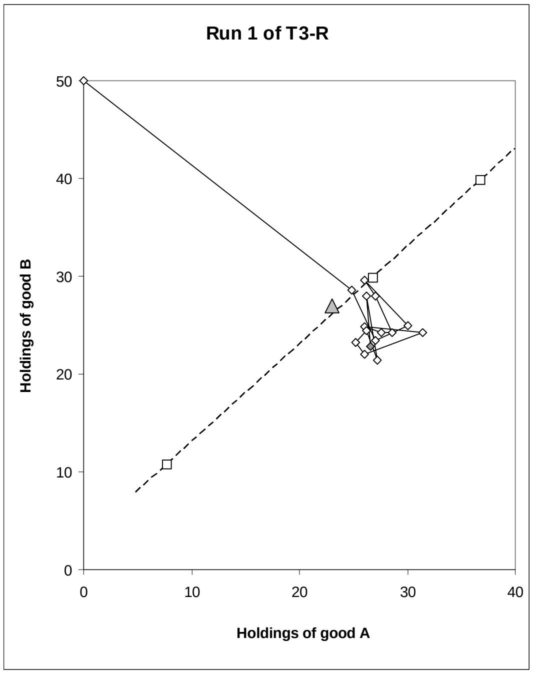


Figure 11: Holdings of goods A and B (Left Panel) and Efficiency (Right Panel) per Period in Treatment 3-R.



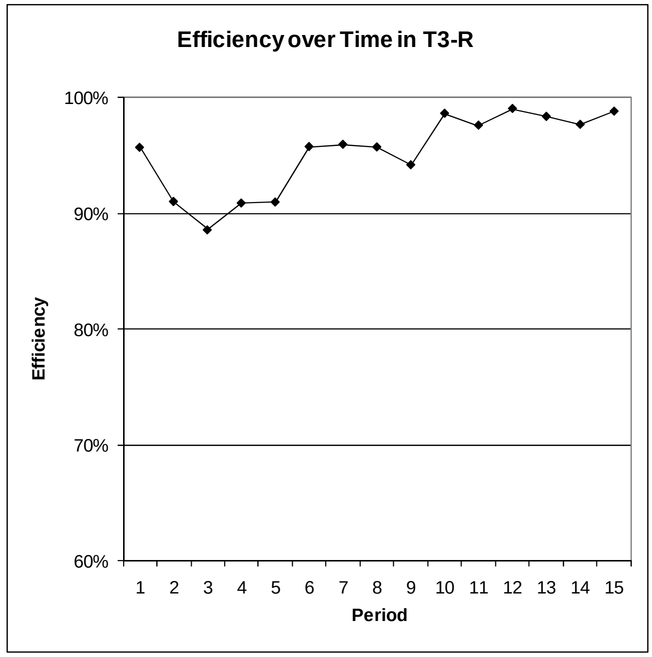
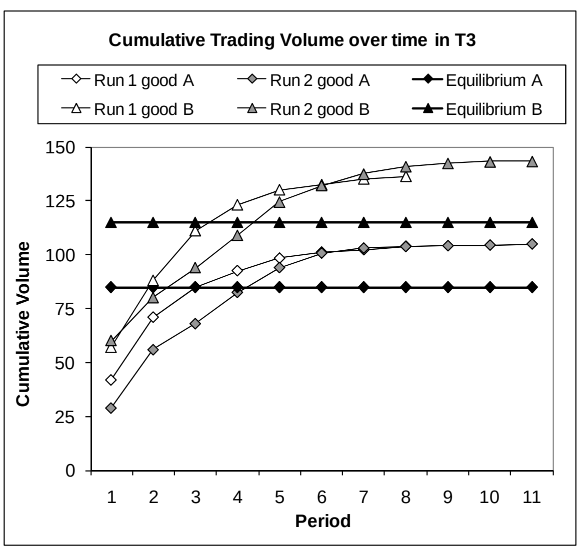
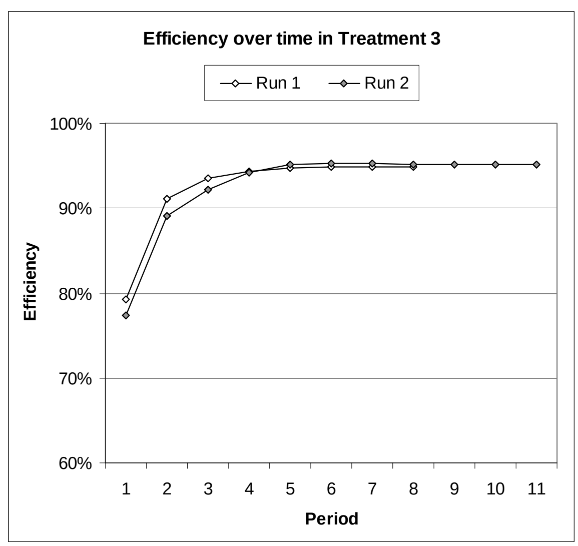


Figure 12: Time Series of Trading Volume (Left Panel) and Efficiency (Right Panel) per Period in Treatment 3. The Second Run is shaded in grey.





Appendix A: Instructions for Treatment 1

General

This is an experiment in market decision making. If you follow these instructions carefully and make good decisions, you will earn more money, which will be paid to you at the end of the experiment.

This experiment consists of one or more sessions of multiple periods and has 10 participants. At the beginning of each session of the experiment, each of the five participants will receive 40 units of good A, and each of the other five will receive 50 units of good B. You will then have the opportunity to trade these goods for up to 20 periods. If the level of trading activity becomes low (less than 0.2 units of either A or B) the session will be terminated earlier. After one session is terminated, it may be followed by another session.

Each participant is free to sell any or all the goods he/she owns for units of the other good. Holdings of A and B are carried over from the end of one period to the beginning of the next period, so you can adjust your holdings in each period. Holdings are not carried over from the end of one session to the beginning of the next. You can only sell the good you were initially endowed with and you can only buy the other good.

During each period we shall conduct a market in which the price per unit of B expressed in units of A will be determined. Good A will be used as “accounting unit” (money, gold) to calculate prices, i.e. prices of B will be expressed in units of A. All units of A and B which have been put up for sale will be exchanged at this price. The following paragraphs describe how the price per unit of A and B will be determined.

In each period, you are asked to enter the units of your endowed good (A or B) that you are willing to exchange for units of the other good (see the center of Screen 1). The number of units you offer cannot exceed your current holdings of that good.

The computer will calculate the sum of the amounts of good A offered by all participants (= Sum_A). It will also calculate the total number of units of B offered for sale (Sum_B), and determine the unit price of B expressed in units of A, $P_B = Sum_A/Sum_B$.

If you offered b_A units of A for sale, you will get b_A/P_B units of good B. If you offered b_B units of B for sale, you will get b_B*P_B units of good A.

Screen 1: trading screen for a trader endowed with good B

At the end of each period (after trading) the points you would earn for the goods you own are calculated. Specifically traders initially endowed with A earn:

Points = A + 100 * $(1-e^{(-B/10)})$

And traders endowed with B earn

Points = B + 110 * $(1-e^{(-A/10)})$

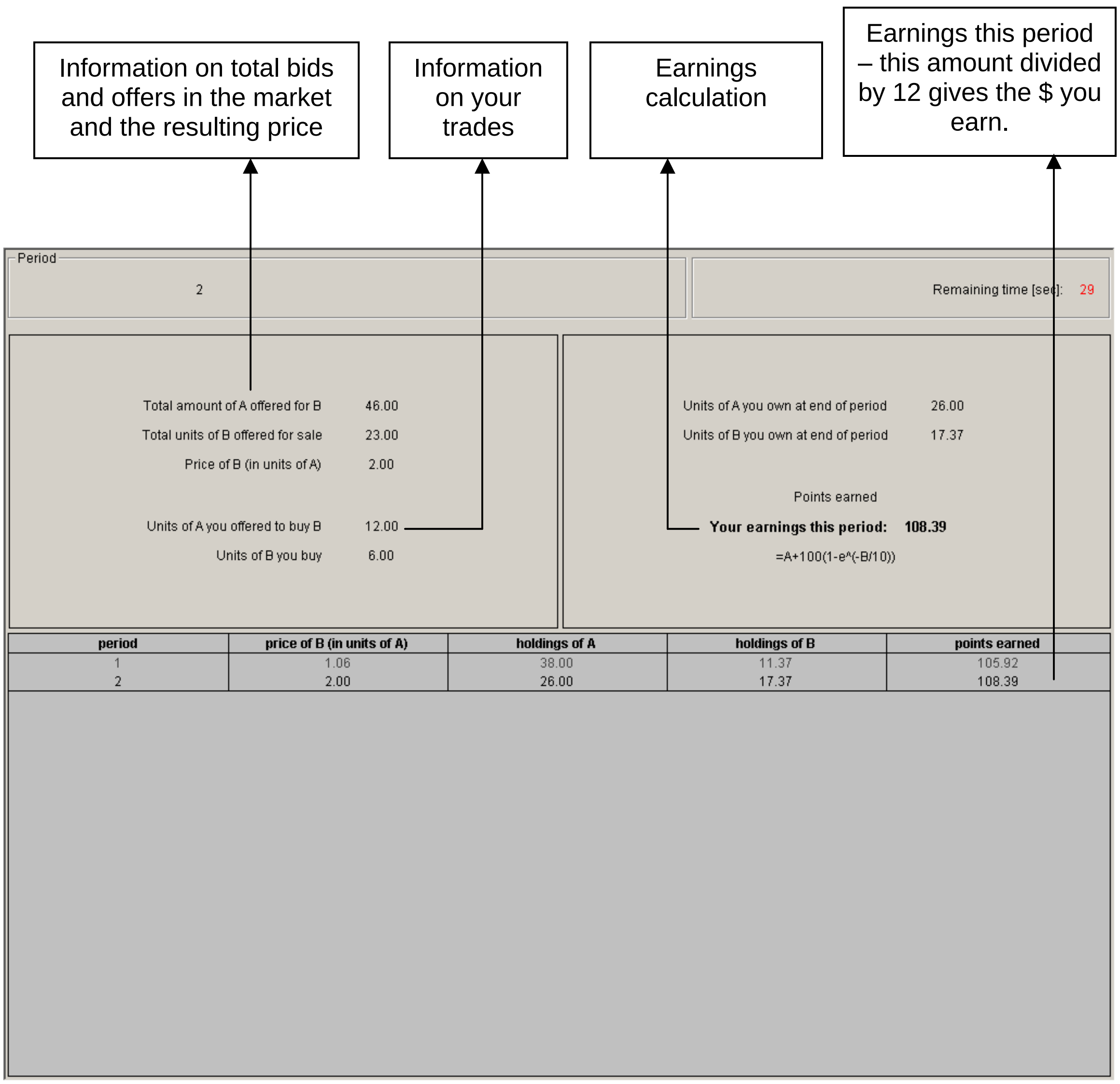
Example 1: If you were endowed with A and have 30 units of A and 15 units of B after trading you earn

30 + 100 $(1-e^{(-15/10)})$ *= 110 points.*

Example 2: If you were endowed with B and have 10 units of A and 25 units of B after trading you earn

25 + 110 $(1-e^{(-10/10)})$ *= 94.5 points.*

Screen 2 shows an example of calculations for Period 2. There are 10 participants in the market, and half of them started with 40 units each of A, the other half with 50 units each of B. Here we see the screen of a subject starting with good B.



The earnings of only the last period of each session (shown in the last row of the last column in the lower part of Screen 2) will determine the dollars you earn. Your points in the last period of each session will be added up and divided by 12 to determine the dollars you earn.

How to calculate the points you earn:

The points earned each period by those initially endowed with good A are calculated as:

Points = $A + 100 * (1-e^{(-B/10)})$

And the points earned each period by those initially endowed with good B are calculated as:

Points = $B + 110 * (1-e^{(-A/10)})$

The following tables may be useful to understand this relationship. They show the points resulting from different combinations of goods A and B.

Table for those initially endowed with A:

		Units of good A you hold at the end of a period										
		0	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**
Units of B you hold	**0**	0.0	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0	50.0
	5	39.3	44.3	49.3	54.3	59.3	64.3	69.3	74.3	79.3	84.3	89.3
	10	63.2	68.2	73.2	78.2	83.2	88.2	93.2	98.2	103.2	108.2	113.2
	15	77.7	82.7	87.7	92.7	97.7	102.7	107.7	112.7	117.7	122.7	127.7
	20	86.5	91.5	96.5	101.5	106.5	111.5	116.5	121.5	126.5	131.5	136.5
	25	91.8	96.8	101.8	106.8	111.8	116.8	121.8	126.8	131.8	136.8	141.8
	30	95.0	100.0	105.0	110.0	115.0	120.0	125.0	130.0	135.0	140.0	145.0
	35	97.0	102.0	107.0	112.0	117.0	122.0	127.0	132.0	137.0	142.0	147.0
	40	98.2	103.2	108.2	113.2	118.2	123.2	128.2	133.2	138.2	143.2	148.2
	45	98.9	103.9	108.9	113.9	118.9	123.9	128.9	133.9	138.9	143.9	148.9
	50	99.3	104.3	109.3	114.3	119.3	124.3	129.3	134.3	139.3	144.3	149.3

Table for those initially endowed with B:

		Units of good A you hold at the end of a period										
		0	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**
Units of B you hold	**0**	0.0	43.3	69.5	85.5	95.1	101.0	104.5	106.7	108.0	108.8	109.3
	5	5.0	48.3	74.5	90.5	100.1	106.0	109.5	111.7	113.0	113.8	114.3
	10	10.0	53.3	79.5	95.5	105.1	111.0	114.5	116.7	118.0	118.8	119.3
	15	15.0	58.3	84.5	100.5	110.1	116.0	119.5	121.7	123.0	123.8	124.3
	20	20.0	63.3	89.5	105.5	115.1	121.0	124.5	126.7	128.0	128.8	129.3
	25	25.0	68.3	94.5	110.5	120.1	126.0	129.5	131.7	133.0	133.8	134.3
	30	30.0	73.3	99.5	115.5	125.1	131.0	134.5	136.7	138.0	138.8	139.3
	35	35.0	78.3	104.5	120.5	130.1	136.0	139.5	141.7	143.0	143.8	144.3
	40	40.0	83.3	109.5	125.5	135.1	141.0	144.5	146.7	148.0	148.8	149.3
	45	45.0	88.3	114.5	130.5	140.1	146.0	149.5	151.7	153.0	153.8	154.3
	50	50.0	93.3	119.5	135.5	145.1	151.0	154.5	156.7	158.0	158.8	159.3

Appendix B: Instructions for Treatments 2-R and 3-R (only μ varied)

General

This is an experiment in market decision making. If you follow these instructions carefully and make good decisions, you will earn more money, which will be paid to you at the end of the session.

This session consists of several periods and has 10 participants. At the beginning of each period, each of the five participants will receive 40 units of good A, and each of the other five will receive 50 units of good B. In addition each participant will receive 100 units of money at the start of each period. In each of some 10 to 20 period you will have the opportunity to offer your goods for sale and to buy the other goods.

Each participant is free to offer for sale any part or all the goods he/she owns each period. You earn points for your holdings of good and money at the end of each period. Holdings of goods and money are not carried over from period to period; you start each period with 100 units of money and either 40 units of A or 50 units of B.

During each period we conduct a market in which the price per unit of A and B will be determined. All units of A and B put up for sale will be sold at their respective price, and you can buy units of A and B at the same price. The following paragraphs describe how the price per unit of A and B will be determined.

In each period, you are asked to enter the cash you are willing to pay to buy the good you do not own (say A), and the number of units of the good you own that you are willing to sell (say B) (see the center of Screen 1). The cash you **bid to buy cannot exceed your money balance (100)**, and the units you **offer to sell cannot exceed your holdings of that good (40 of A or 50 of B)**. You receive the income from the sale of any goods to be paid in money at the end of each period.

The computer will calculate the sum of the amounts of good A offered by all participants (= Sum_A). It will also calculate the total number of units of money offered to buy the goods ($\$Sum_A$) and determine the price of A expressed in terms of money, $p_A = \$Sum_A/Sum_A$. The same is done with good B.

If you offer q_A units of A for sale, you will get an income of $q_A * P_A$. If you bid b_A units of money to purchase A, you will get b_A/P_A units of good A.

Screen 1: trading screen for a trader endowed with good B

Both goods are perishable and must be either sold or consumed in the current period. The number of units of A and B you own at the end of the period, c_A and c_B (unsold units of owned good and purchased units of the other good) will be consumed and determine the number of points you earn for the period. Traders initially endowed with A earn:

$$\text{Points} = (1/\mu) * (A + 100 * (1-e^{(-B/10)})) + \text{NET MONEY}$$

COMMENT: $\mu = 1$ in T2, 0.28 in T3a, 0.75 in T3b, and 5.07 in T3c.

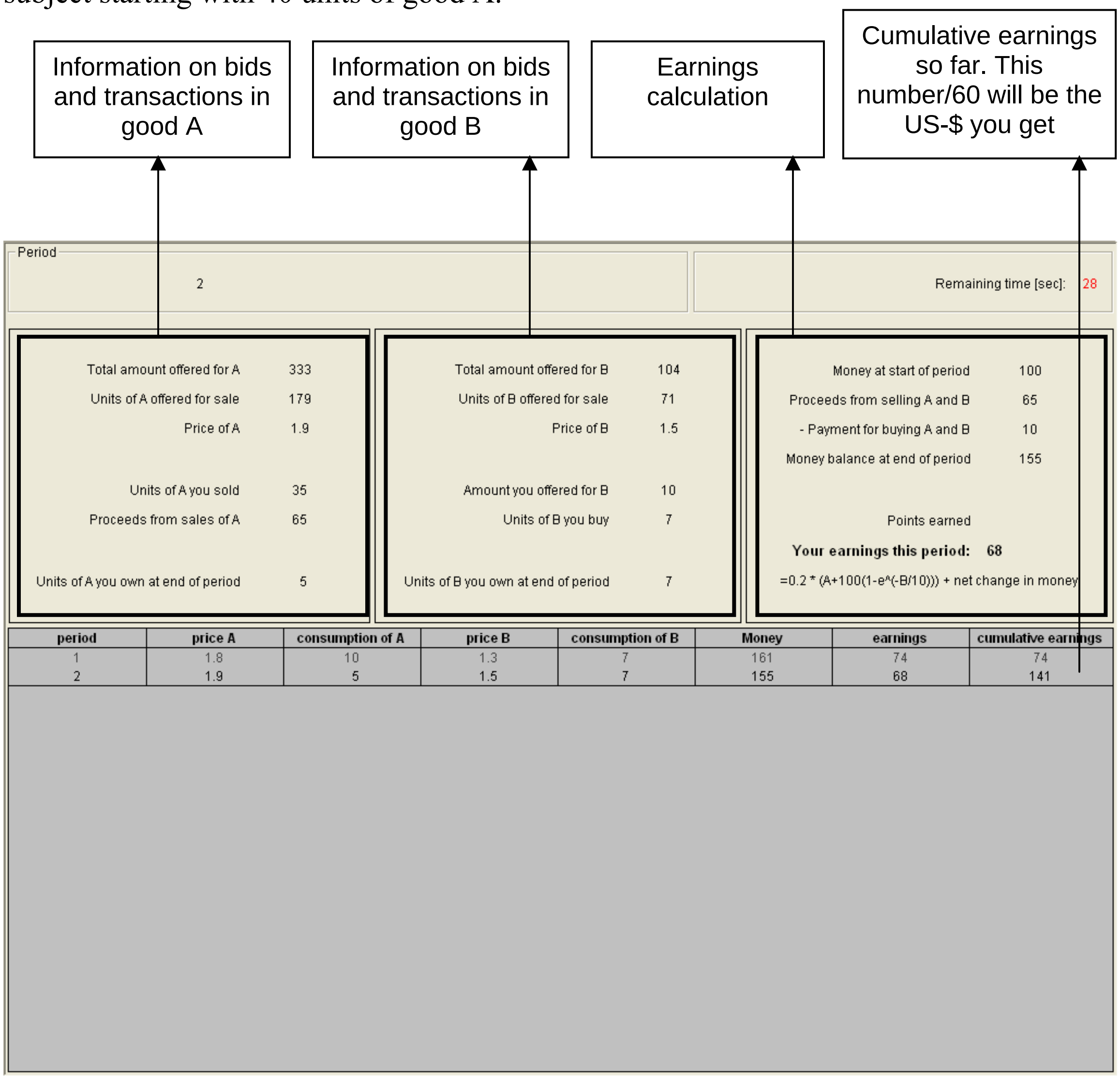
And traders endowed with B earn

$$\text{Points} = B + 110 * (1-e^{(-A/10)}) + \text{NET MONEY}$$

Example: If at the end of any period you are endowed with B and have 30 units of A and 15 units of B you earn 15 + 110 (1-e*$^{\wedge(-30/10)}$*) = 119.5 points.*

Your cash balance holdings will help determine the points you earn. At the end of each period the starting endowment of 100 units of money will be deducted from your final money holdings. The resulting net holdings (which may be negative) will be added to (or subtracted from) your total points earned.

Screen 2 shows an example of calculations for Period 2. There are 10 participants in the market, and half of them have 40 units of A, the other half 50 units of B. Here we see a subject starting with 40 units of good A.



The earnings of each period are added up in the last column. At the end they will be converted into real Dollars at the rate of 60 points = 1 US$ and this amount will be paid out to you.

How to calculate the points you earn (in Treatment T2c):

The points those initially endowed with A earn each period are calculated as:

$$\text{Points} = (A + 100 * (1-e^{\wedge(-B/10)})) + \text{Net Money}$$

And the points those initially endowed with B earn each period are calculated as:

$$\text{Points} = 1/5.07 * (B + 110 * (1-e^{\wedge(-A/10)}) + \text{Net Money}$$

The following tables may be useful to understand this relationship. They show the resulting points from different combinations of goods A and B (assuming net money to be zero).

Table for those initially endowed with A:

Units of B you hold	Units of good A you hold at the end of a period										
	0	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**
0	0.0	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0	50.0
5	39.3	44.3	49.3	54.3	59.3	64.3	69.3	74.3	79.3	84.3	89.3
10	63.2	68.2	73.2	78.2	83.2	88.2	93.2	98.2	103.2	108.2	113.2
15	77.7	82.7	87.7	92.7	97.7	102.7	107.7	112.7	117.7	122.7	127.7
20	86.5	91.5	96.5	101.5	106.5	111.5	116.5	121.5	126.5	131.5	136.5
25	91.8	96.8	101.8	106.8	111.8	116.8	121.8	126.8	131.8	136.8	141.8
30	95.0	100.0	105.0	110.0	115.0	120.0	125.0	130.0	135.0	140.0	145.0
35	97.0	102.0	107.0	112.0	117.0	122.0	127.0	132.0	137.0	142.0	147.0
40	98.2	103.2	108.2	113.2	118.2	123.2	128.2	133.2	138.2	143.2	148.2
45	98.9	103.9	108.9	113.9	118.9	123.9	128.9	133.9	138.9	143.9	148.9
50	99.3	104.3	109.3	114.3	119.3	124.3	129.3	134.3	139.3	144.3	149.3

Table for those initially endowed with B:

Units of B you hold	Units of good A you hold at the end of a period										
	0	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**
0	0.0	8.5	13.7	16.9	18.8	19.9	20.6	21.0	21.3	21.5	21.6
5	1.0	9.5	14.7	17.8	19.7	20.9	21.6	22.0	22.3	22.4	22.5
10	2.0	10.5	15.7	18.8	20.7	21.9	22.6	23.0	23.3	23.4	23.5
15	3.0	11.5	16.7	19.8	21.7	22.9	23.6	24.0	24.3	24.4	24.5
20	3.9	12.5	17.7	20.8	22.7	23.9	24.6	25.0	25.2	25.4	25.5
25	4.9	13.5	18.6	21.8	23.7	24.8	25.5	26.0	26.2	26.4	26.5
30	5.9	14.5	19.6	22.8	24.7	25.8	26.5	27.0	27.2	27.4	27.5
35	6.9	15.4	20.6	23.8	25.7	26.8	27.5	27.9	28.2	28.4	28.5
40	7.9	16.4	21.6	24.7	26.6	27.8	28.5	28.9	29.2	29.3	29.4
45	8.9	17.4	22.6	25.7	27.6	28.8	29.5	29.9	30.2	30.3	30.4
50	9.9	18.4	23.6	26.7	28.6	29.8	30.5	30.9	31.2	31.3	31.4