VENTURE CAPITAL AND SEQUENTIAL INVESTMENTS

By

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Venture Capital and Sequential Investments*

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Abstract

We present a dynamic model of venture capital financing, described as a sequential investment problem with uncertain outcome. Each venture has a critical, but unknown threshold beyond which it cannot progress. If the threshold is reached before the completion of the project, then the project fails, otherwise it succeeds. The investors decide sequentially about the speed of the investment and the optimal path of staged investments. We derive the dynamically optimal funding policy in response to the arrival of information during the development of the venture. We develop three types of predictions from our theoretical model and test these predictions in a large sample of venture capital investments in the U.S. for the period of 1987-2002.

First, the investment flow starts low if the failure risk is high and accelerates as the projects mature. Second, the investment flow reacts positively to information that arrives while the project is developed. We find that the investment decisions are more sensitive to the information received during the development than to the information held prior to the project launch. Third, investors distribute their investments over more funding rounds if the failure risk is larger.

Keywords: Venture Capital, Sequential Investment, Stage Financing, Intertemporal Returns.

JEL Classification: D83, D92, G11, G24.

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1 Introduction

1.1 Motivation

An innovative project typically has to go through many stages of exploration and development that all require capital outlays before it is completed. Moreover, it carries a substantial failure risk and it is difficult to predict at which point in time evidence might emerge that would lead to its abandonment. While the optimal investment policy depends on the current information available, the progress of research uncovers new information about the project and reduces uncertainty that in turn will influence the optimal continuation strategy. The venture capital industry is a powerful example of the importance of these feedback effects in the financing of innovation. But similar issues also arise for innovative projects within large organizations or in publicly funded research.

The purpose of this paper is to understand the relationship between project-related information and the sequentially optimal investment decisions. We develop a theory that analyzes how investors - venture capitalists or other sponsors that provide the financing and help shepherding a project to success - make optimal dynamic investment decisions as a function of their information about failure risk and potential final value. In our theoretical model, we distinguish between information that investors have before the funding decision and information they receive as the project advances. We analyze the relative impact of *ex ante information* and *interim information*, respectively, on subsequent investment decisions.

We then put the predictions of our model to an empirical test. We use a comprehensive sample of the US venture capital data to examine whether we find support for our predicted relationships. Our empirical findings lend support to the main predictions of our model: First, investors proceed cautiously if the failure risk is high, and they accelerate investment, in spite of the cost of doing so, as projects mature. They also invest faster if they hold favorable information about the project. Second, the investment flow reacts positively to information that arrives during the development of the project, and interim learning seems to be more important for the determination of the investment path and a better predictor of the final outcome than ex ante information. Third, if the failure risk is large then investors tend to adopt a more hands-on approach by adjusting their investment strategy more frequently.

We consider a continuous-time model representing the complete investment cycle of an innovative project under uncertainty, characterized by: *(i)* uncertainty about the likelihood of success; *(ii)* uncertainty about the arrival of the failure event; and *(iii)* interim information about the failure risk and the final value of the project. Our model depicts the progress of the project as a continuous process of development and research. At each point in time, information that the project should be abandoned may arrive. Thus, the model incorporates a simple stopping problem. The signal that the project should be abandoned arises with a given probability, derived from the Pareto distribution with parameter λ. The family of Pareto distributions has the property that the conditional probability of failure is decreasing over time.

In our stylized model, we focus on two essential dimensions of the sequential investment decisions. The first dimension is that investors determine the speed with which the project is undertaken, or the optimal capital flow. The decision about the optimal speed of investment is characterized by the following trade-off: a larger investment flow into the project promises faster success, but is likely to reduce the efficiency of the investment. The investors control the optimal investment flow at every point in time and decide *(i)* on the speed with which they want to develop a project, *(ii)* on the change in the investment pace as the project progresses, and *(iii)* on the adjustment in the financing speed if new information arises that changes the expectation of key parameters, notably failure risk and final value in the event of success.

The second dimension that we take into account is the optimal degree of investor involvement. Venture capitalists typically provide financing in infrequent financing rounds or stages, lasting from a few months to over a year or more. They also define milestones that must be met before a certain fraction of the funds is released. In the venture capital industry, the time of fund managers is often considered as one of the most critical resources (see Michelacci and Suarez (2004) and Inderst and Mueller (2004)). Each financing round necessitates a thorough review and valuation exercise, it typically involves several parties (venture financing is often syndicated among several funds and the managers are involved as well) and a multilateral negotiation process. With these resource constraints and transaction costs in mind, it is then optimal to review the project only at certain intervals, even if this implies a temporarily suboptimal investment path.[1]

We add these considerations to our continuous-time model. Our goal is to specifically understand the intertemporal pattern of stage financing and its interaction with the available information. Critically, the determination of the financing rounds, their expected duration and the associated

[1] Venture capitalists are also involved via continuous monitoring, e.g. through frequent visits and board representation. We view these monitoring activities as complementary to the financing round decisions. But compared with the information on financing rounds, the information on monitoring is soft and typically unavailable or only through questionnaire-based data.

investment flow and the intermediate milestones are endogenous in our model. In this analysis, investors make lumpy investment decisions that are optimized as a function of the expected value and the probability of failure. We model the cost of each investment decision as a loss that is proportional to the current value of the project. With this analysis, we add the following questions to our investigation: *(iv)* How do transaction costs and the need for lumpy investment decisions affect the optimal investment path? *(v)* What is the optimal sequence of stage financing?

The contributions of our theoretical analysis fall into three groups. First, we show that as a project advances and the probability of eventual success increases, investment flows should be optimally increasing. For the same reason, a project with a higher estimated final value or a higher anticipated chance to succeed is also allocated a larger investment flow throughout. With an increase in the probability to succeed, accelerating becomes a more valuable option, even if it makes the investment more costly. In fact, our model shows that the investment flow should be increasing over time as a pure informational effect as the risk of failure recedes. Our model predicts that the observable returns are decreasing even though the increasing investment flows imply an acceleration in the discovery process. Second, we show that the optimal staging sequence depends on the value of the real option to abandon. The higher the estimated final value of the project is, and the larger the estimated success probability, the fewer rounds will be used. Also, echoing our result on the optimal investment path for continuous decisions, the investment flow increases from one round to the next. Third, we show that learning about the expected final value or the failure probability is incorporated in all subsequent investment decisions. If there is a positive news update then the value of the project increases as well as the investment flow. At the same time, the number of subsequent investment rounds decreases, and the capital allocation for each of these rounds increases.

We take these theoretical predictions to a large sample of more than 47,000 venture capital investments in the U.S. for the period 1987-2002, covering an overwhelming majority of all recorded venture investments in the U.S. over that period[2]. The venture capital data are attractive for three reasons: first, they allow us to study these effects in a broad sample of projects across different industries. Second, because of the staged nature of venture financing, interim valuation data are available that contain the estimates regarding the prospects of a particular project. We can use these values and in particular changes in the valuations to extract information about what investors have

[2] We have compared our sample to the data of the National Venture Capital Association (NVCA (2008)) and the PwC/Moneytree/Thomson Reuters survey data available from Thomson Reuters.

learnt since the last capital infusion. Third, the fact that venture-backed projects are *independent* companies makes it possible to track the timing of the stopping decisions or the feedback between information arrival and sequential investment decisions more accurately than, say, for research projects launched *within* organizations in which there is more discretion for window-dressing and the investment information is more opaque.

The results of our empirical investigation lend support to our theoretical predictions as follows. First, we document empirically that as a project advances and the probability of eventual success increases, investment flows are increasing. We show that at the same time, the returns of the projects are decreasing over the investment cycle. Taken together, these two observations imply that learning about the eventual prospect of a project are largely concentrated at the beginning of the investment cycle. Second, our evidence shows that initially, investors seem to have little screening ability about the eventual probability of success, but they seem to hold some information about the final value of the project in the event of a successful completion. We show that as the project advances, in many cases investors get information that leads to a change in the estimated failure risk or exit value of the project, as inferred from the dynamics of the project valuation. Moreover, such information updates lead investors to adjust the investment path optimally: the subsequent investment flow as well as the size of each round and the number of subsequent rounds react in the way predicted by our model. Consistent with our model, we find that investors receive updates over the course of the investment cycle that allow them to better estimate the final value. These updates again give rise to a change in the investment flow and the number and size of subsequent rounds that is consistent with the pattern predicted by our model. Third, we show that the design of financing rounds follows the optimal pattern predicted by our model: the investment size and the investment flow is increasing from one round to the next, and projects with a high initial estimate of the final value or an optimistic appraisal of the probability to succeed will use less rounds than less valuable or more risky projects.

1.2 Related Literature

Our paper is related to three different strands of the literature. First, there is a literature on the role of learning in the financing of innovation. Sorensen (2008) analyzes the decisions of venture capital funds into which industry they invest. Using similar data to ours, he finds evidence that learning and forward-looking expectations drive the investment decisions of the venture capital funds. In Sorensen (2008), each fund makes many investment decisions across many industries.

In the underlying learning model, a statistical multi-armed bandit model, the past investment experience in a given industry allows the venture capital fund to make inferences about future ventures in this industry. The venture capital fund decides on investing into a particular firm as a single decision, and the focus is on the informational externality across ventures. In contrast, we analyze the entire investment cycle of every venture and follow the various funding decisions related to the project over time. The focus of our theoretical and empirical investigation is about how information arrival impacts the sequential funding policy *within* a venture whereas Sorensen (2008) analyze the funding policy *across* ventures. Hochberg, Ljungquist, and Vissing-Jorgensen (2008) discuss the learning impact of a venture capital's past investments on the size and the direction of follow-up funds by the same venture capital firm. They find a positive feedback effect between fund performance and the size of follow-on funds. They also explore the speed of learning of limited partners relative to that of general partners, and argue that the evidence supports asymmetric learning. In contrast to our paper, they do not consider the interaction between learning and investment within a single portfolio firm. Several papers discuss the optimal stopping decision in an investment model with learning. Bergemann and Hege (2005) consider a project with a given failure risk in which the arrival time of the final discovery, and hence the total cost to deliver it, are uncertain. This implies that the value of the project decreases over time until either success arrives or the project is optimally abandoned. They focus on the information rent of the entrepreneur who can divert the continuous investment flow, and show that the project may be financially constrained as these information rents increase in the expected funding horizon. Jovanovic and Szentes (2007) present a paper in which the critical constraint is the expertise of the venture capitalists, similar to Michelacci and Suarez (2004). Because of the opportunity cost linked to their labor input, the venture capitalists abandon projects earlier than would be socially optimal if projects are considered in isolation. In contrast to these papers, we focus in our learning model on the investment flow and the staging sequence.

Second, there is a literature on the optimal dynamic pattern of investments in the presence of a real option to abandon. Berk, Green, and Naik (2004) focus on the evolution of the risk profile that are due to changes from a purely technical risk in the early stages to more diverse sources of risk in later stages. Mostly, the theoretical literature has focused on the use of stage financing as a tool to alleviate agency problems. Fluck, Garrison, and Myers (2007) consider the real option of abandoning a venture capital project and highlight the role of stage financing in this regard. They consider a contract design problem to alleviate moral hazard and show that the entrepreneur's optimal equity share decreases as uncertainty about the project's ultimate success recedes.

There is a substantial empirical literature on stage financing, starting with the seminal analysis of Gompers (1995). Subsequent work analyzes the contingent contract clauses that are either explicit or implied by staging in more detail (see e.g. Kaplan and Stromberg (2003); Bienz and Hirsch (2007)). By and large, these papers confirm many of the theoretical predictions on stage financing. A similar confirmation of the theoretical predictions appears in Tian (2007) who shows that venture capitalists with better access to information about their venture, as proxied by geographical proximity, use staging less frequently.

Finally, there is an extensive literature on the valuation and the returns in venture capital. In contrast to our paper, this literature looks at venture capital as an asset class and studies the returns of venture capital from a performance-based perspective of a diversified investor. Among these studies our paper is most closely related to two studies that calculate company-level returns for venture-related investments, namely Cochrane (2005) and Woodward and Hall (2003). Both studies are interested in understanding the risk-return trade-off, and their focus is on reducing the impact of sample selection bias. Cochrane (2005) calculates returns for each financing round separately and limits the sample to final valuations from IPOs and trade sales, whereas we take an integrated approach that solicits as many observations as possible at each round. Woodward and Hall (2003) include round valuations just as we do, but for a different objective of creating a performance index. Other papers in the risk-return literature, such as Kaplan and Schoar (2005) and Phalippou and Gottschalg (2009) look at returns at the fund level. They focus on cash distributions and thus consider only the final value of exited investments. By contrast, our paper focuses on the interaction between information-driven returns and investments, and looks at interim results and sequential investments at the portfolio-company level.

The remainder of the paper is organized as follows. Section 2 introduces the model of investment under uncertainty and describes the intertemporal payoffs. Section 3 analyzes the optimal investment policies in the basic model. Section 4 augments the analysis in the baseline model to allow for staging decisions and for additional uncertainty about the failure risk of the project. Section 5 develops the hypothesis for the empirical results given the theoretical predictions obtained in the earlier sections. Section 6 describes the dataset and presents some summary statics of the dataset. Section 7 reports the empirical results. Section 8 discusses some open issues and concludes. The Appendix collects the proofs of all propositions in the main body of the text.

2 Model

The development of a new venture is described as a sequential investment model under uncertainty. The true value of the venture or project is assumed to be initially unknown to the entrepreneur and the investors. The true value of venture is either 0 or $Y > 0$. The uncertainty about the true value of the venture is resolved over time.

We model the development of the venture as an investment process in continuous time $t \in [0, \infty)$. The initial state of the project is given by k_0, with $k_0 > 0$, and the state of the project at time t is denoted by k_t. The venture is successful if it reaches a final state K, with $k_0 < K$. If the venture reaches the state K then it generates a value Y. The role of the investment at time t is to increase the state k_t and bring the venture closer to a successful realization. Each venture has a critical, but initially unknown, threshold k^* beyond which it cannot progress. If the critical threshold $k^* \leq K$, then the project stalls as the current state k_t reaches the threshold k^*, or $k_t = k^*$. In this case, the venture fails and the true value of the venture is determined to be 0. If on the other hand, the critical threshold k^* is beyond the final state K, or $k^* > K$, then the venture is developed successfully and generates the positive value Y.

The location of the critical threshold k^*, i.e. the breakdown point of the venture, is uncertain and given by a prior distribution $F(k^*)$. The venture has an ex ante chance of success if the prior probability that the breakdown point k^* is smaller than K has probability less than one, or $F(K) < 1$. Conversely, if $1 - F(K) > 0$, then the probability that the critical threshold is beyond K, and hence occurs after the realization of the value Y, is positive. For the remainder of the analysis we shall assume that the probability of a successful realization is positive, and hence $F(K) < 1$. For the analysis, we shall restrict our attention to prior distributions which are in the class of Pareto distributions.

$$F(k; k_0, \lambda) \triangleq 1 - \left(\frac{k_0}{k}\right)^{\lambda}.$$

The class of Pareto distributions is parameterized by two variables, k_0 and λ. The initial state of the project k_0 is a strictly positive lower bound, and $\lambda > 0$ identifies the skewness of the distribution. For notational ease, we shall suppress the dependence on k_0 and λ and simply write $F(k) \triangleq F(k; k_0, \lambda)$. The prior probability of success, starting at k_0 is now given by:

$$1 - F(K) = \left(\frac{k_0}{K}\right)^{\lambda}.$$

Conversely, the prior probability that the project will fail during the development phase is given by $F(K)$. The conditional probability of failure at k_t, in other words the failure hazard rate $h(k_t)$, is given by:

$$h(k_t) \triangleq \frac{f(k_t)}{1-F(k_t)} = \frac{\lambda}{k_t}.$$

The conditional failure rate is decreasing in the state of the project and a project with a larger λ has a uniformly higher rate of failure and consequently a lower prior (and posterior) probability at every k_t that it reaches the final state K. With slight abuse of notation we frequently refer to λ as the *failure rate* of the venture. We postpone a detailed discussion of the specific role that the Pareto distribution has for the results until immediately after the statement of the results. It suffices for the moment to say that the monotonicity and comparative static properties of the optimal investment policy are independent of the parametrization of the failure risk. The properties of the Pareto distribution are used only for the characterization of the intertemporal profile of the observable returns. With respect to the specific class of distribution for the theoretical model, we chose the Pareto distribution over the exponential distribution as the summary statistics of the venture suggested a decreasing rather than constant failure probability, as would be implied by the class of exponential distributions.

The investment flow i_t at time t controls the rate at which the state of the development k_t is moving forward, through the law of motion:

$$dk_t = \gamma\sqrt{i_t}dt. \tag{1}$$

The current investment flow i_t increases the speed at which the project progresses in a concave manner - or to put it differently, increasing the speed increases the total cost of investment in a convex manner. The concavity of the rate of progress dk_t in the investment flow i_t acts like a convex adjustment cost. It represents the presence of a critical resource, such as research or management. The decision about the optimal speed of investment is therefore characterized by the following trade-off: a larger investment flow into the project promises faster success, but reduces the efficiency of the investment. The parameter $\gamma > 0$ describes the marginal effect of investment on the speed of development and a larger value of γ represents a project that is easier to develop.

The instantaneous failure probability, given the current investment flow i_t, is given by:

$$h(k_t) \cdot dk_t = \frac{f(k_t)}{1-F(k_t)} \cdot \gamma\sqrt{i_t}dt = \frac{\lambda}{k_t} \cdot \gamma\sqrt{i_t}dt.$$

A venture capital project is now described by $(Y, k_0, K, \gamma, \lambda)$. The value Y is the value of the successfully developed project. The initial state k_0 and final state K describe the length of the development process, $K - k_0$, until the venture can go public or be sold. The parameter λ describes the failure rate of the project and γ identifies the marginal productivity of the monetary funds to develop the project.

The value of the venture depends on the investment policy $(i_t)_{t=0}^{T}$. From an ex ante point of view, the project is expected to reveal itself to be either a success or a failure. If the project is a success then the payoff Y will be realized at some future time T which depends on the profile of the investment flow $(i_t)_{t=0}^{T}$. Along the way, the project requires investments which represent the development cost. If, on the other hand, the project is a failure, and hence the critical threshold k^* lies below K, or $k^* < K$, then the investment flow halts as soon as current state reaches $k_t = k^*$. In this case the project incurs development costs until the moment of failure and does not generate any positive returns at all. Conditional on a given investment policy $(i_t)_{t=0}^{T}$, we associate to every time t a position k_t which is reached at time t, provided that the project did not come to halt before k_t. The ex ante expected net present value from an investment policy $(i_t)_{t=0}^{T}$ at time $t = 0$ is given by:

$$(1 - F(K))\, e^{-rT} Y - \int_0^T i_t \,(1 - F(k_t))\, e^{-rt} dt. \tag{2}$$

The first term represents the expected discounted gross return of the project. The ex ante probability of success is $1 - F(K)$ and as the value Y is only realized at time T, the value is discounted at the rate r until time T. The second term accounts for the expected cost of the investment policy $(i_t)_{t=0}^{T}$ during the lifetime of the project. The investment i_t in state k_t at time t occurs if and only if the project has a critical threshold k^* beyond the current state k_t which has an ex ante probability $1 - F(k_t)$. The total expected cost of the project is the integral over the investment flows until time T.

3 Sequential Investment

In this section we characterize the optimal investment policy for the project under uncertainty. The optimal investment policy describes the first best solution to the investment problem under uncertainty. In consequence, at this stage, we are not concerned with possible agency conflicts that might arise between the investors and the entrepreneur. The focus of our analysis is to investigate how the funding policy for the venture should optimally respond to the flow of information which arises during the development of the venture. We analyze the role of the agency conflict on the funding policy in the next section, where the agency conflict introduces a friction into the provision of the funds.

The first-best policy under uncertainty can be analyzed as a dynamic programming problem under uncertainty. The natural state variable of the dynamic program is the state k_t which describes the progress of the project. At every point in time, the investment flow carries a cost equal to the investment, $-i_t$, and generates one of two possible outcomes. The project may either fail at the current position k_t or it will pass successfully through the current position k_t. In the event of a failure, which occurs at the rate $\gamma\sqrt{i_t} \cdot (\lambda/k_t)$, the value of the project drops from the current value, denoted by $V(k_t)$, to 0. In the event of a successful passage the state increases at the rate $dk_t = \gamma\sqrt{i_t}$ and the value of the venture increase by $V'(k_t)$. The dynamic programming equation for the optimal investment policy in continuous time is now given by:

$$rV(k_t) = \max_{i_t \in \mathbb{R}} \left\{ -i_t - \gamma\sqrt{i_t}\frac{\lambda}{k_t}V(k_t) + \gamma\sqrt{i_t}V'(k_t) \right\}. \tag{3}$$

The value of the project depends on the flow of investment i_t in period t through three channels: (i) the direct cost of the investment i_t, (ii) the failure rate $\gamma\sqrt{i_t} \cdot \lambda/k_t$, and ($iii$) the rate of change $\gamma\sqrt{i_t}$ in the position of the project.

The rate of change $\gamma\sqrt{i_t}$ in the position k_t is a concave function of the current investment i_t. The optimal investment policy, therefore, is the result of an optimal trade-off between the speed of investment and the cost of building up the asset. The optimal investment at point k_t is determined by the first order conditions of the dynamic programming equation (3):

$$-1 - \frac{1}{2}\frac{\gamma}{\sqrt{i_t}}\left(\frac{\lambda}{k_t}V(k_t) - V'(k_t)\right) = 0,$$

The optimal investment problem is hence the solution to a linear-quadratic problem and the optimal investment i_t^* is given by:

$$i_t^* = \left(\frac{\gamma}{2}\left(V'(k_t) - \frac{\lambda}{k_t}V(k_t)\right)\right)^2. \tag{4}$$

We can insert the optimal investment flow i_t^* into the value function (3) and obtain an ordinary differential equation for the evolution of the value of the venture:

$$rV(k_t) = \left(\frac{\gamma}{2}\left(V'(k_t) - \frac{\lambda}{k_t}V(k_t)\right)\right)^2. \tag{5}$$

We observe from (4) and (5) that the optimal investment i_t^* is linear in the flow value of the venture at time t :

$$i_t^* = rV(k_t). \tag{6}$$

We can rewrite the differential equation (5) in its canonical form as:

$$V'(k_t) = \frac{\lambda}{k_t} V(k_t) + \frac{2}{\gamma}\sqrt{rV(k_t)}. \tag{7}$$

With a change of variable given by $W(k_t) \triangleq \sqrt{V(k_t)}$, we can transform the above differential equation into a nonlinear first order differential equation which we can solve explicitly by variation of parameters. The explicit solution of the value function is given in the Appendix and we derive the properties of the optimal investment policy $i^* = (i_t^*)_{t=0}^T$ on the basis of this solution.

Proposition 1 (Investment Policy)

1. *The optimal investment policy i^* is increasing and convex in the state k_t.*

2. *The optimal investment policy i^* is decreasing and concave in the failure rate λ.*

3. *The optimal investment policy i^* is increasing and convex in Y.*

The intuition of Proposition 1 is that the value of the project increases with the gradual resolution of uncertainty about its final success. As the project proceeds, the likelihood that the critical threshold k^* is reached before K diminishes and it becomes increasingly likely that the project will reach the final position K. As the current valuation of the venture increases, it is optimal to increase the speed of development to reach the final position K earlier because of the opportunity cost of discounting. This increase in the funding occurs in spite of the associated convex increase in the cost of investment. We noted earlier that the above monotonicity results are independent of specific distributional assumptions about the failure risk. The curvature properties of the optimal policies can be verified to hold for other distributions, such as the class of exponential distributions as well. However, if the failure rate is strongly non-monotone in the state k_t, then the above curvature properties may fail to hold.

We note that our model is built on a central premise: from the perspective of a risk-neutral investor, the expected return of the investor is constant over time and given by

$$R \triangleq 1 + r.$$

In our model, the failure event is characterized by a fall of the value to zero. In the absence of a failure event, we observe a change in the position k_t given by:

$$dk_t = \gamma\sqrt{i_t}dt.$$

The constant return R can therefore be decomposed into a return in the event of a failure, which is given by 0, and the return in the event of a successful continuation, the *surviving return,* denoted by R_t. We therefore have

$$R = \Pr(\text{failure}_t) \ \cdot \ 0 \quad + \quad \Pr(\text{survival}_t) \ \cdot \ R_t.$$

Given that the instantaneous failure probability at time t is given by

$$\gamma \sqrt{i_t} \frac{\lambda}{k_t},$$

the surviving return R_t in period t is defined by the complementary survival probability:

$$R = \left(1 - \gamma \sqrt{i_t} \frac{\lambda}{k_t}\right) \cdot R_t,$$

and can hence be explicitly expressed as:

$$R_t = \frac{R}{1 - \gamma \sqrt{i_t} \frac{\lambda}{k_t}}. \tag{8}$$

Alternatively, we can express the surviving return R_t using the value function given in (3) and describe the surviving return in terms of the net change in the continuation value $\dot{V}_t - i_t$ relative to the value V_t of the project:

$$R_t = \frac{\dot{V}_t - i_t}{V_t}. \tag{9}$$

We can infer from (8) that the surviving return R_t is controlled by the product of the conditional failure probability λ/k_t and the investment intensity $\gamma\sqrt{i_t}$. As the conditional failure probability is declining in k_t and k_t is increasing over time, it follows that the conditional failure probability is declining over time as well. This contributes to a decline in the surviving returns over time. On the other hand, the investment intensity $\gamma\sqrt{i_t}$ is increasing over time. As we saw above, the venture becomes more valuable as the successful completion of the project becomes more likely. The intertemporal profile of the surviving return is then determined by the trade-offs between failure rate and optimal responsiveness of the investment to the arrival of new information.

Proposition 2 (Surviving Returns)

1. *The surviving returns R_t are decreasing over time if the failure rate λ and the final state K are not too large.*

2. *The surviving returns R_t are increasing over time if the failure rate λ and the final state K are too large.*

The above characterization exhaustively describes the possible return profiles of the venture project. The surviving returns are either always decreasing or always increasing. The surviving returns are increasing only if the failure rate and the final state K are too large. In this situation, the investment profile over time is exceedingly convex with very low investment flows until close to the completion of the project. In this case the decreasing conditional failure probability is overwhelmed by the rapid increase in the investment flow as a function of the state k_t.

4 Staging and Learning

The objective of this section is to enrich the analysis of the basic sequential investment problem to account for important aspects in the provision of venture capital funding. Specifically, we analyze (i) the role of staging in the provision of the funds and (ii) the role of uncertainty about the true failure rate of the project.

4.1 Staging

In the basic investment model, the flow of funds into the project was continuously adjusted in response to the arrival of information. This suggests an ongoing and continuous involvement of the investors during the development of the venture. In reality, the funding decisions and the associated negotiations over the valuation of the venture only occur infrequently. Presumably, the transaction costs brought about by multi-party bargaining, contracting and the valuation of the project leads to a discrete number of funding decisions and funding rounds.

We represent the friction associated with the contracting and the agency relationship as a cost proportional to the current project value. We assume that the negotiation for each new funding round is successfully concluded with a probability p strictly less than one. With the complementary probability, $1-p$, an agreement fails to to be completed and the project is abandoned. Alternatively, the probability $1-p$ can be viewed as a transaction cost due to a delay in the continuation of the project. In this case, $1-p$ is the fraction of the value that is lost to discounting due to the delay which comes with the (re-)negotiation of the funding terms.

While our model abstracts from explicit moral hazard considerations, the analysis of the staging decision, or on the optimal degree of investor involvement, could be extended to address agency problems in more detail. For example, we could assume that each time the investor advances money to the manager of the venture, the amount transferred is fully handed over to the manager and cannot be retrieved by current or future investors. Effectively, this adds a rich real options dimension to the staging decision, in that the investor knows that the longer are individual stages, the larger is the risk that a substantial amount of cash will be lost if a failure signal arrives in the midst of a round. This extension would reinforce the downside of keeping individual stages too long, which is currently represented by the impossibility to adjusting the funding speed within a single round. However, this extension would not alter the trade-off surrounding the staging decision qualitatively. We therefore focus for simplicity only on the impossibility to adjust the investment speed which is sufficient to generate the basic trade-off.

With this friction in the negotiation process, a continuous involvement in the investment process becomes too costly for the investors. In fact, we show below that it becomes optimal to reevaluate the investment policy only infrequently. In this world with friction, the optimal funding policy now determines the funding volume over a time interval rather than a funding flow at every instant. A decision about the funding volume therefore determines the *constant flow* i of the investment during the round and the length of the funding round.

Thus, we depict the staging decision as the result of a trade-off between the transaction costs of a new funding round *versus* the flexibility to adjust the speed of investment. The objective of the subsequent analysis is to understand the optimal structure of stage financing based on this trade-off. Importantly, the staging decision is fully endogenous in the sense that both the number of stages as well as their duration are chosen by investors in reaction to their information at the beginning of each round. Consequently, we denote by $i_{l,m}$ the optimal investment flow in stage l if the entire project is financed in m stages, with $l \leq m$. Similarly, we denote by $V_{l,m}(k_t)$ the value function of the project in stage l and state k_t conditionally on funding the entire project in m stages.

If the project is funded in a single stage, i.e. it is funded in the initial state k_0 with the objective of maintaining a given investment level $i_{1,1}$ until the positive or negative termination of the object, then the value function is given by the unique solution of the first order differential equation:

$$rV_{1,1}(k_t) = -i_{1,1} + \gamma\sqrt{i_{1,1}}\left(V'_{1,1}(k_t) - \frac{\lambda}{k_t}V_{1,1}(k_t)\right), \tag{10}$$

subject to the boundary condition $V_{1,1}(K) = Y$. The value function can be explicitly solved:

$$V_{1,1}(k_t) = \left(\frac{k_t}{K}\right)^{\lambda} e^{\frac{k_t - K}{\sqrt{i_{1,1}}}\frac{r}{\gamma}} Y - \frac{\sqrt{i_{1,1}}}{(\lambda - 1)\gamma}\left(k_t - K\left(\frac{k_t}{K}\right)^{\lambda} e^{\frac{k_t - K}{\sqrt{i_{1,1}}}\frac{r}{\gamma}}\right). \tag{11}$$

The optimal investment policy given the initial state k_0 can be obtained implicitly by the first order condition of $V_{1,1}(k_0)$ with respect to $i_{1,1}$. The terms on the rhs of the equation represent the benefit and the cost of pursuing the project at a fixed intensity level $i_{1,1}$. The first term represent the time discounted probability that the project is successfully realized. The second term represents the expected cost of developing the project.

We now consider the optimal determination of stage financing. The value function $V_{1,1}(k_t)$ is determined by the optimal investment funding to complete the venture in a single round starting at k_t. The cost of the stage funding is given by the commitment to a specific investment flow over the round horizon. If the project is developing well, then the investors will react with the infusion of new funds and a new, and presumably higher, investment flow. Given that a renewal of the funding is not certain, but might lead to a failure of the project with probability $1 - p$, the question then becomes, at which level of development k_t does it become optimal to complete the development of the venture in multiple rather than in a single stage of funding.

Next, if the project is to be funded in two stages, then the optimal funding policy starting at the initial position k_0 has to make three distinct choices: (i) it has to determine the initial funding level $i_{1,2}$, (ii) the continued funding level $i_{2,2}$ and (iii) the state k_1 in which the funding is supposed to be renewed. We can solve this problem recursively and for given state k_1 determine the optimal funding speed $i_{2,2}$ to complete the project. The solution to this problem gives us the value function $V_{2,2}(k_1)$ in state k_1. We observe that the value function $V_{2,2}(k_1)$ shares the functional form with $V_{1,1}(k_0)$ in (11). The only difference between these two value functions is that the associated investment level $i_{1,1}$ is determined earlier at k_0 rather than $i_{2,2}$ at k_1. But in either case, the optimal investment choice provides the necessary funds in a single round until the project is completed.

Given the optimal continuation value $V_{2,2}(k_1)$, we can recursively determine the funding level $i_{1,2}$ and the state k_1 in which the funding will be reviewed. The joint decision about the funding intensity $i_{1,2}$ and the length of the funding period k_1 is then given as the solution to the following dynamic programming problem.

$$V_{1,2}(k_0) = \max_{i_{1,2},k_1} \left\{ p \left(\frac{k_0}{k_1} \right)^{\lambda} e^{\frac{k_0-k_1}{\sqrt{i_{1,2}}} \frac{r}{\gamma}} V_{2,2}(k_1) - \frac{\sqrt{i_{1,2}}}{(\lambda-1)\gamma} \left(k_0 - k_1 \left(\frac{k_0}{k_1} \right)^{\lambda} e^{\frac{k_0-k_1}{\sqrt{i_{1,2}}} \frac{r}{\gamma}} \right) \right\}. \tag{12}$$

We observe that the functional form of the dynamic programming equation is again similar to (11). The difference is that the expected gain from the investment flow $i_{1,2}$ is the discounted probability that the next funding round is reached in state k_1, which is represented by $V_{2,2}(k_1)$. Similarly, the investment costs are now accumulated between k_0 and k_1 at the rate of $i_{1,2}$ rather than between k_0 and K at the rate of $i_{1,1}$.

The optimal investment decision in each round is a *joint* optimal control and stopping problem. The control problem is the determination of the investment flow $i_{l,m}$ and the stopping problem is the decision about the state k_l at which a new funding decision should be made. As we are interested in the interaction between the staging decision, the investment decisions, and the information arrival, we seek to determine the optimal staging decision as a function of the current state k_t and the final state K. In particular, we would like to know whether at a given position k_t, it is optimal to undertake the remaining investment for the interval $K - k_t$ in a single round or split it over two rounds? In other words, we seek to determine how the length of the remaining task, identified by $K - k_t$, determines the staging decision. The comparative statics of this decision with respect to K, gives us the required hypotheses for our empirical investigation.

We restrict our analysis here to the optimal determination of funding and renewal with two stages. Yet, due to the recursive structure of the funding problem, the optimality conditions and the qualitative properties of the optimal funding decision extend naturally from two to finitely many funding stages.

Proposition 3 (Optimality of Staging)

1. *For a given final state K and for a given number of stages, $m = 1$ or $m = 2$, the respective investment levels satisfy:*

$$i_{1,2} < i_{1,1} < i_{2,2}.$$

2. *The number of optimal funding rounds is increasing in K.*

The first part of the above results demonstrates that the length of the development determines the number of financing rounds. In particular, as the length of the development process increases the investors ultimately find it in their interest to spread the funding decision over several rounds. The second part of the result considers the structure of the funding conditional on either funding the project in one or in two stages. Clearly, except for a critical value of the final state K, either one of the two staging policies will be optimal and dominate the other one. The result is meant to illustrate the level of the funding policies across different staging policies. The outer inequality, namely $i_{1,2} < i_{2,2}$, reflects the monotonicity in the funding policy which we established earlier for the continuous control problem in Proposition 1. The inner inequalities, $i_{1,2} < i_{1,1} < i_{2,2}$, reflect the flexibility offered by multiple stages. The investors can adjust the investment flow upwards as the prospects of the project improve, whereas in a single stage they loose the flexibility of the upward adjustment and choose an investment flow which is in between the investment flows with multiple stages. The relationship between the staging decision and the investment decision is depicted in Figure 1 and Figure 2.

Insert Figure 1 and Figure 2 Here

If the project is funded in a single stage, then the value function $V_{1,1}$ is continuously increasing until it reaches the terminal value Y. If on the other hand, the project is funded in two stages, then the increase in value is initially smaller as the initial investment is smaller and the project has still to secure the second funding round. If at the stopping point k_1, the funding for a second round can be secured, then the associated value observes an upward jump from $V_{1,2}(k_1)$ to $V_{2,2}(k_1)$, where the value before the jump, $V_{1,2}(k_1)$, and the value after the jump, $V_{2,2}(k_1)$ satisfy the following relationship: $V_{1,2}(k_1) = pV_{2,2}(k_1)$. Given the optimality of staging, we now investigate the temporal structure of the staging. In particular, we are interested in the length of each staging round as we come closer to a successful completion of the venture. Specifically, we show that as the length of the remaining development $K - k_0$, increases then it is eventually optimal to switch from a single stage funding to a multiple stage funding policy. As Figure 2 illustrates, the advantage of the multiple stage funding policy is that it allows the investment flow to be adjusted upwards as the project moves closer to completion.

Proposition 4 (Structure of Staging)

1. *The number of funding stages is decreasing in Y and increasing in λ.*

2. *The length of the first stage is increasing in the funding probability p.*

3. *The flow of funding is increasing in Y over all funding stages.*

An implication of Proposition 3 is that a project with a larger return Y will see fewer rounds of funding, as the delay or impasse resulting from a renewal of the funding leads to a higher opportunity cost for a project with a larger possible return Y.

4.2 Learning

So far, we analyzed the dynamic development of the venture with an essentially binary information structure. Either the project progresses and in consequence its prospect improve, or the venture fails and the funding is terminated. In this final extension we accommodate interim learning while the project is developed. In particular, we consider learning during the project in the sense that the progress of the project uncovers information that may change the expectation about the future failure probabilities (or equivalently about the final value in the event of success). The interim arrival of information is interesting as the current development may give rise to additional information about the expected future risk and value of the project. Consequently, we shall extend the basic model to accommodate the arrival of new information about the likelihood of success. More specifically, we assume that the venture starts with a given failure rate $\lambda > 0$. At a random time, the current failure rate λ is replaced by a new failure rate, which can be either lower or higher than the current failure rate λ, wit $\lambda_l < \lambda < \lambda_h$.[3] We shall assume that the expected true failure rate is equal to the current failure rate, or

$$\lambda = \alpha\lambda_h + (1 - \alpha)\lambda_l.$$

The failure rate λ can therefore be interpreted as the current estimate of the true, but currently unknown failure rate which is given by λ_h with probability α and λ_l with probability $1 - \alpha$. We observe that a jump to lower failure rate λ_l represents a positive shock from the point of view of the investors, and conversely an upwards jump to λ_h represents a negative shock as it lowers the expected value of the venture. The new information about the failure rate is assumed to arrive

[3] The focus on learning about λ will be motivated below in the empirical discussion.

with a constant rate ρ. The dynamic investment problem can is represented by the usual dynamic programming equation:

$$rV(k_t) = \max_{i_t} \left\{ -i_t + \sqrt{i_t}\gamma \left(V'(k_t) - \frac{\lambda}{k_t} V(k_t) + \rho \left((1-\alpha) V_l(k_t) + \alpha V_h(k_t) - V(k_t) \right) \right) \right\}. \quad (13)$$

The investment problem represented by (13) is similar to the earlier model, with the exception of the additional jump terms $V_l(k_t)$ and $V_h(k_t)$. The value functions represent the continuation value of the venture conditional on knowing that the true failure rate is either λ_l or λ_h, respectively. While the continuation values, $V_l(k_t)$ and $V_h(k_t)$, have the same form as the value function in the basic model, the value function before the resolution of uncertainty about the true failure rate, λ_l or λ_h, does not permit an explicit solution as it contains the possibility of a jump to a different failure rate. Nonetheless, the implicit solution allows us to obtain a number of important comparative static results.

Proposition 5 (Survival Probability and Investment)
A positive shock from the expected failure rate λ to the low failure rate λ_l leads to:

1. *an increase in the probability of eventual success; and*

2. *an upward jump in the investment flow.*

A decrease in the failure risk leads to a higher probability of success. As this leads to an upward jump in the value of the venture, the optimal investment policy is adjusted as well. As we saw earlier that the investment policy is a linear function of the value of the venture, the decrease in the failure probability leads to an upward jump in the investment flow.

5 Hypothesis Development

In this section, we summarize the hypotheses of our theoretical model in order to confront them with venture capital evidence.

Initial Valuation, Time of Information Arrival, and Return Dynamics We can use our model to explore typical patterns of learning in venture-backed investment projects. Prior to launching a project, investors hold beliefs about the prospects (e.g. final value at exit) and the risks of the project, but they may also receive information after the project is launched. We distinguish between three hypotheses regarding the arrival of information: *(i)* in the uninformed investor hypothesis, the investors cannot discriminate between the prospects of individual projects and use the expected values for failure risk and the value of success; *(ii)* in the ex ante information hypothesis, the investors can discriminate between the prospects of individual firms and the bulk of the information is available at the project launch; *(iii)* in the interim information hypothesis, the investors obtain valuable information on the project terminal value and failure risk over the course of the investment cycle.

We distinguish between these hypotheses by using the initial valuations and the evolution of the valuations over the venture capital investment cycle. We start from the premise that, at the time of inception, the value of innovative projects consists essentially of the expectation of the future value of the project in the event of success. They typically have little or no assets. Therefore, variations in the present value of the project are mainly explained by differences in the expected final value at exit if the venture is successful, or the estimated probability of success.

Under the ex ante information hypothesis, ex ante information on the failure risk should be impounded in the initial project valuation. We investigate this hypothesis by analyzing whether the initial project value predicts the ultimate success probability. It could also be the case that investors have ex ante information on the final value. In this case we expect variation in the initial values to be correlated with the final values in the event of success. Alternatively, if the initial value of the firm and the ultimate success are uncorrelated, then this lends support to either the uninformed investor or the interim information hypothesis. Under these two hypotheses, we also expect little correlation between initial and final value.

Moreover, we can discriminate between the uninformed investor and the interim information hypothesis by analyzing the relationship between interim valuations and ultimate success: under the interim information hypothesis, we expect that projects with a large value increase during the investment cycle, i.e. high abnormal returns, are more likely to succeed, whereas we expect no correlation under the uninformed investor hypothesis. We also expect, under the interim information hypothesis, that high abnormal returns over the investment cycle are linked to higher exit values, whereas we expect no relationship under the uninformed investor hypothesis. Thus, by comparing the predictive power of initial and interim valuations relative to final outcomes, we can draw inferences on the importance of ex ante information and interim information. The same is true for the relationship between investment behavior (e.g. investment flow following the first round) and final outcome (e.g. IPO, M&A, or going down).

Our model also allows us to analyze the dynamics of the failure risk over the investment cycle. Our model is based on the premise of a risk-neutral investor which implies constant expected returns over the lifetime of the project. That is, in a risk-neutral setting the value increase in each round is just an adequate compensation for the failure risk, and hence should decrease as the project is developed to maturity.[4] For the sake of the argument and in contrast to our assumption, suppose for a moment that the hazard rate of dropping out at any given position $k_t \in (0, K)$ during the investment cycle would be constant. With the increase in investment flows, this would imply an increasing speed in the discovery process over time and hence an increase in the returns of the project, since the return is just an adequate compensation for the dynamics of the failure risk. This thought experiment underlines the strong implications contained in the following two elements of our empirical analysis: *(i)* the investment flows are increasing over time and *(ii)* the returns generally decrease from one round to the next. The only way how these two observations can be reconciled is if the conditional hazard rate is declining at a sufficiently high rate (Proposition 2). This condition is satisfied in our model with a sufficiently large value of the parameter λ of the Pareto distribution.

Investment Flow Our model shows that investments optimally react to the flow of information, and that investment flow will increase if there is less uncertainty about the project outcome. In particular, our model explains that as a venture projects matures, it should exhibit larger investments, and higher outlays over any given period of time.

[4]If the investors were risk-averse, then the expected or unconditional returns should be decreasing as the project matures, given that the failure risk decreases. The magnitude of this effect may not be large for moderate levels of risk aversion. This appears to be roughly consistent with our finding of decreasing means in the expected returns. Our (weak) result of increasing medians contradicts the hypothesis that investors are risk-averse.

The fundamental prediction of our model is that there should be a positive relationship between project valuation and investment flow. This relationship between the project's valuation and the investment flow holds throughout the investment cycle. This effect holds whether the firm's value is high because the final value Y is high or because the failure rate λ is low, or both. The reason is that both a higher final value and a lower failure risk translate into a larger present value of the project, and the model shows that a project's investment flow is closely linked to the current project valuation. Therefore, we expect to find that the investment flow is increasing both in measures of the expected final value and the expected failure risk.

Section 4 extends our model to allow for the interim information hypothesis. Our theoretical analysis explores the possibility that there is interim learning about the failure risk λ of the project. If the firm learns positive news about λ, then this has two consequences. First, a lower λ means an increase in the current value of the project V_t, and hence a positive abnormal return at the time the good news is received. Second, the investment flow should optimally increase. The inverse relationship holds if the firm receives bad news about λ. Thus, the model predicts that subsequent investment flows increase with abnormal returns. The same argument would hold if there were interim learning about the final value of the project Y. Good news about Y translates into an increase in the current value and hence a positive abnormal return, and at the same time leads to an upwards adjustment in the optimal investment flow.

Staging Frequency Section 4 explicitly considered that funding may be provided in lumpy amounts even though investments are made continuously. The renewal of the funding decision enhances the value of the real option to abandon the project. Our analysis shows that shorter financing rounds will occur if the information that investing produces is more valuable for the abandonment decision. In particular, the model explains that the staging frequency should be lower for projects with a higher success probability.

Contracting costs in our model are proportional to the current project value. The analysis shows that the staging frequency should be lower for projects with a high expected exit value. The reason is that the expected loss of adding one round increases in the expected final value, whereas the potential savings if there is early abandonment are constant. Thus, we expect the number of rounds to be a decreasing function of the initial value of the project. This is true whether the variations in the project's value are driven by differences in the expected final value or in the expected failure rate.

Considering interim learning about the project's failure risk, a reduction in the estimated failure probability λ means, first, an increase in the current project value and and hence in the abnormal return. At the same time, in reaction to an increase of the value of the firm, the subsequent financing will be undertaken in fewer rounds. Therefore, the model predicts that the number of subsequent rounds until successful completion decreases with the initial abnormal return. The same negative relationship between the initial abnormal return and the number of subsequent rounds holds if there is interim learning about the final value of the project.

Size and Duration of Financing Rounds A separate set of predictions addresses the duration and capital raised in each financing round. The real option of abandonment is most valuable when the uncertainty about ultimate success is high. As shown in Section 4, as the project advances and investors become more confident about ultimate success, they are willing to travel a longer distance $[k_l, k_{l+1})$ in a single financing round l. Thus, the model leads to the prediction that the size or the volume of the investment rounds is increasing from one round to the next. If the investment flow were constant, then the round duration would also be increasing. However, as Proposition 5 shows, the optimal funding flow/intensity also increases from one round to the next. Therefore, the overall impact on round durations is ambiguous, and they could increase as well as decrease as the project advances.

Moreover, our model predicts that the capital raised in a round is a decreasing function of the failure rate λ, and an increasing function of the expected final value Y and the transaction costs of an additional round, $1-p$. At the same time, the investment flow increases in Y and decreases in λ. Therefore, the model predicts that the investment size increases in Y and decreases in λ, but the impact on round duration is again ambiguous.

Interim learning about the project's failure risk implies that capital raised should increase after a positive shock.

Total Project Duration Our model implies that projects with an above-average initial valuation will have a consistently higher investment flow. Therefore, the model predicts that they will be completed faster.

6 Data Description and Empirical Methodology

Our data of venture capital investments are provided by Sand Hill Econometrics (SHE) and contain the majority of US investments in the period from January 1987 to March 2002. SHE combines and extends two databases, VentureXpert (formerly Venture Economics) and Venture One, which are extensively used in the venture capital literature. According to Gompers and Lerner (1999) and Kaplan, Stromberg, and Sensoy (2002), the VentureXpert data contain the majority of the investments. SHE has spent substantial time and effort to ensure the accuracy of the data. This includes removing investment rounds that did not actually occur, adding investment rounds that were not in the original data, and consolidating rounds, so that each round corresponds to a single actual investment by one or more venture capitalists. Cochrane (2005), Sorensen (2008) and Korteweg and Sorensen (2008) use different versions of this data set. The data in Cochrane (2005) end in June 2000 and the data in Korteweg and Sorensen (2008) and Sorensen (2008) end in 2005.

The data contains firm level information and venture capital investment round level information. At the firm level, we focus on the following variables: a unique firm ID, industry category (health care, IT, retail, or others), and the exit type (IPO, merger & acquisition, out of business, restart or unknown). A firm with unknown exit may be alive at the end of the sample period or exited at a unknown time point before March 2002. The round observations are linked to firms via the unique firm IDs. At the round level, we use the following variables for each round: the date stamp of the round, the business status of the firm during the current round (start up, in development, beta-testing, in clinical trails, shipping, profitable, restart, or unknown), the amount (million dollars) raised in the current round, post money valuation of the firm and an exit dummy that equals one if the current round is an exit round. We also use the round status (seed, first, early, late, mezzanine, restart, IPO, acquisition, busted round, or unknown). While round status classifications are frequently tricky to make, all that matters for our purposes is that they correctly indicate the sequence of investments. Therefore, we verify manually that no round status classifications is erroneous - that is, rounds labeled as “seed” or “first” always precede “early” rounds, “early” rounds precede “late” rounds, etc.

We filter the data by keeping firms that have at least one round before the exiting round, removing firms that exit as a restart or have restart rounds. We further aggregate the business status information by combining in development, beta-testing, and in clinical trails as one status called “in development”, and combining shipping, profitable as “in production”.

An important issue is that only accurate valuation data allow us to estimate error-free returns. It is well-known that valuation data for venture capital suffer from a variety of errors. For example, these data typically do not take into account covenants and contingent contract provisions which can dramatically affect the valuation levels (Kaplan, Stromberg, and Sensoy (2002) and Metrick (2007)). Moreover, intermediate financing rounds are often missing or investments are reported without a valuation, or the exit status is missing or wrongly reported (Kaplan, Stromberg, and Sensoy (2002)). While Sandhill Econometrics has undertaken a considerable amount of effort to remedy these problems and our data presumably fare better in this respect than data bases such as Venture One or VentureXpert, it is likely that our data are still affected by these issues and, therefore, must be considered as noisy. However, the only assumption that we really need for the validity of our findings is that any noise in valuation data is uncorrelated with the variation in the data that is driven by venture characteristics explained in our model. In addition, the size and representativeness of our sample makes us confident that our main findings are not driven by measurement problems or missing data issues.

Finally, it is well known that venture capital valuations are subject to large fluctuations over time and across industries (Gompers and Lerner (2000); Gompers, Kovner, Lerner, and Scharfstein (2008); Ljungqvist, Richardson, and Wolfenzon (2007)). Market fluctuations can also lead to large differences in values between the time a venture capitalist starts exiting from a venture (e.g., in an IPO) and the time when the last part of the investment is sold. For these reasons, we consider only the abnormal returns that are not explained by the typical value gains of comparable ventures at the same time, thus controlling for cyclical and industry-specific valuation effects (see Section 7 for details).

Table 1 reports summary statistics for the firms in the data. Panel A reports the break-down of the firms according to industries and exits. It shows the typical composition of venture capital samples, with more than half of the companies in IT-related activities, 15% in health care and 9% in retail. Around 70% of the companies have unknown exits, and 25% exited via either IPO or a trade sale. Panels B and C report average round frequencies and round durations, respectively, for the same breakdown.

While some of the firms with unknown exits might be alive at the end of the sample period, many others might have already been liquidated by then. Failures are incompletely documented in the data because liquidation is less visible than IPOs or trade sales. If firms with unknown exits are more likely to be already liquidated than alive, excluding all firms with unknown exits from our analysis would lead to a biased sample of venture capital backed firms that overrepresents successful ventures. Further, if the firms with a documented failure systematically differ from firms that have been liquidated but do not have a documented failure, excluding all firms with unknown exits would lead to biased results, particularly concerning the analysis of the determinants of exit types and final values of venture capital backed firms. To mitigate the possible sample selection bias, we distinguish "zombies" - firms that were liquidated before March 2002 but have no documented exit in the data set - from firms with unknown exits. Specifically, for each firm with unknown exit, we estimate the length of the period (in months) for which the capital raised in the last recorded round would keep a firm alive. If the duration between the last recorded round and March 2002 is longer than this "survival time", we assume the firm went down at the end of the "survival time". Otherwise, we assume that the firm is alive at the end of the sample period. The empirical analysis in this paper uses not only firms with documented exits but also firms with estimated exits.

We use the following procedure to estimate the "survival time" after the last recorded round for firms with unknown exits. First, we estimate the amount of capital consumed per month after the last recorded round for each firm. Second, we divide the raised amount in the last recorded round with the estimated monthly capital consumption, and obtain the "survival time" in months. In the first step, we run a round level regression of monthly capital consumption (raised capital divided by the number of months between the current and next rounds, in log) for all rounds except the last recorded rounds, on the amount of capital raised (in log), the post money valuation (in log), industry dummies, business status dummies, and dummies for future exit types. The rationale is that the amount of capital consumed per month by a firm is determined by the amount of capital raised, the size of the firm, the industry and business status of the firm, and the quality of the firm (proxied by the final exit type). The R^2 of the regression is 0.75, which seems to indicate that the monthly capital consumption is explained reasonably well by this regression. The regression results suggest that the amount of capital raised and the post money valuation are positive and significant at the 1% level, which indicates that larger firms consume more capital per month and firms with more capital raised consume more capital per month. In addition, retail firms consume more capital per month than firms in other industries, which is significant also at the 1% level. Firms "in development" and "in production" consume less capital than start up firms, which is significant at the 1% level. Moreover, exit type dummies also have significant coefficients. We run another regression with the post money valuation excluded, and obtain similar results and a R^2 of 0.74. After running these two regressions, we estimate the amount of capital consumed per month after the last recorded rounds for firms with unknown exits, using estimated coefficients from the regressions and the explanatory variables for the last recorded rounds. We use the coefficients from the first regression for the last recorded rounds with post money valuations observed, and use the coefficients from the second regression for rounds with unobserved post money valuations. We set the exit dummies to 0, assuming that the firms' exits would follow the same distribution of exits as the exited firms. In the second step, we divide the raised amount with the estimated capital consumption per month, and obtain the "survival time." If the survival time is long enough to go beyond the end of the sample period, we assume that the firm is alive. If the survival time ends before the end of the sample period, we add an exit round for the firm, assuming that this firm raised $0 in the exit round, and went down with $1 post money valuation.

The results of this reclassification procedure are recorded in the two lines (in italics) at the bottom of each of the Panels A, B and C of Table 1. These lines report the same data as in the two preceding lines ("Down" and "Unknown") of each panel, but use the procedure discussed above to classify firms with exit status "Unknown" as either "Down" or "Alive". As expected, the number of liquidated firms (reported *and* estimated "Downs") increases dramatically - from 938 in Table 1 to 10,857, bringing the total of failed firms to 57.3%. 17.9% of firms are now estimated to be still active or alive, and in 10.6% of firms the venture capitalists exited with an IPO, in 14.5% with a trade sale. The summary statistics for pre-exit rounds and duration before exits also show comparable differences that make them more sensible compared with the raw data (for instance, "Down" firms no longer exhibit a longer duration and more rounds).

Starting with Table 2, all the tables and analysis are based on firms with estimated exits whenever the exits are unknown. Table 2 reports summary statistics for venture capital investment rounds prior to firms' exits. Panel A reports the number of rounds for firms with different exit types in different industries. Panels B, C, and D report the means and standard deviations, which are calculated using rounds with corresponding information available, of pre-financing duration (the number of months between the previous round and the current round), investment amount (million dollars), and the ratio of investment amount to post-money valuation. The table shows that the pre-financing duration, investment amount, and the ratio of investment amount to post-money valuation are similar across industries and different exit types.

7 Empirical Results

Initial Valuation, Time of Information Arrival, and Return Dynamics We start by exploring our alternative hypotheses concerning the typical arrival time of information. Table 3 reports evidence from a comparison of successful projects (firms exiting via IPO or M&A) and unsuccessful ones (firms that are going down). Under the ex ante information hypothesis, valuable information about the prospects of a particular project is mostly known ex ante. If the information were about the success probability, then we would expect projects with higher initial values to succeed more often. As Table 3 shows, this is not the case: successful projects actually have lower initial values compared with unsuccessful ones. Moreover, when we look at investment behavior we also find evidence that is inconsistent with the ex ante information hypothesis. Table 3 shows that there is no significant difference in investment behavior between failed and successful projects. First-round investments and the ratio of first-round investment to initial value are fairly constant across projects, regardless of their ultimate outcome. Further, under the ex ante information hypothesis, the first round investment flow, which reflects investors' ex ante belief of the final exits, should help predict the prospect of the project: a larger investment flow indicates a higher probability of success. Table 3 shows the opposite: IPO/M&A firms have smaller first round investment flows than firms that went down.

Additional evidence is provided by Table 4, which presents results of probit regressions on whether firms exit successfully (IPO or M&A) or exit as failures (including estimated failures). The initial value is not significant or - in one regression - is weakly significant but with the wrong sign according to the ex ante information hypothesis. Thus, the results about initial valuation and initial investment behavior resolutely reject the hypothesis that investors have ex ante discriminatory capabilities about the estimated success probability of a project.

We can also test whether investors hold relevant ex ante information about the expected final value in the case of success. Indeed, initial values and final values are correlated, and Table 5 shows that the initial value has clear predictive power for the exit value (significant at the 1% level) in the event of success. Hence we can conclude that investors have some ex ante information about potential final values, but not about the chances to succeed.

According to the interim information hypothesis, the estimates about the ultimate success probability and/or the exit value of the project evolve with the progress of the project. In many of our tests of the interim information hypothesis, we focus on the information content in the first round for which we have the most observations. From now on, we use the abnormal return from one round to the next to measure the information content of that round. We define the abnormal return as the component of the raw return that is orthogonal to the information already known by investors before the round, including the industry and business status of the project and the status of the round, *and* the common return of the whole venture capital asset class. The common return is assumed to be driven by the financial market instead of the specific prospect of the underlying project. As a result, the abnormal return captures information regarding the specific project that is unknown before the round.

We estimate abnormal returns as:

$$\log(\frac{V_{i,t_{k+1}} - I_{i,t_{k+1}}}{V_{i,t_k}}) = (t_{k+1} - t_k)\rho'\text{Industry}_i + (t_{k+1} - t_k)\lambda'\text{Business}_i + \sum_{s=t_k+1}^{t_{k+1}} (\log(R_{m,s})) + \varepsilon_{i,k+1}.$$

The subscript i, t_{k+1} denotes the month in which round $k+1$ is raised for firm i; $I_{i,t_{k+1}}$ is the amount of capital raised in round $k+1$; $V_{i,t_{k+1}}$ is the post money value for round $k+1$; Industry_i is a 3×1 vector of dummies corresponding to health care, IT, and retail firms; Business_i is a (3×1) vector of dummies corresponding to start up, in development, and in production; $\log(R_{m,s})$ is the log venture capital market returns, which is assumed to be affected by a vector of unknown market factors that vary over time and factor loadings that are constant for all venture capital investments; $\varepsilon_{i,k}$ is the portion of the log return that is not explained by market factors or information already known by investors, and thus is the "abnormal return". Note that in the regression, $\log(R_{m,s})$ is essentially the coefficient of a dummy variable for month s. The regression is similar to the repeat sales regression for the construction of real estate price indexes.[5] We pool all rounds in the data set without missing variables and run the above regression. The monthly abnormal log return for firm i from round $k-1$ to round k, which is denoted by $AR_{i,k}$, is constructed from regression residuals as follows:

$$AR_{i,k} = \frac{\widehat{\varepsilon_{i,k}}}{t_k - t_{k-1}}.$$

As Table 3 shows, one of the most powerful results of our study is that projects that ultimately succeed are likely to receive a positive news update during the initial financing round. The abnormal return following the first round is strongly positive for projects that exit successfully, and significantly negative for all other firms (t-value for the difference: 25.335). The probit analysis in Table 4 confirms this effect when including other explanatory variables with a comparable level

[5] See Bailey, Muth, and Nourseerk (1963) for the original regression and Goetzmann and Peng (2006), among others, for an application to real-estate markets.

of significance. In addition, consistent with the hypothesis that interim information also updates the belief about the final value, Table 5 shows that the abnormal return following the first round is positively related to the exit value. Taken together, these results provide solid support for the interim information hypothesis.

We conclude that investors learn during the first investment round about the failure probability. In fact, information arrival *after* the launch of a project, more precisely during the first round, is a strong predictor of ultimate success, as opposed to ex ante information (initial valuations or first-round investment behavior). In addition, investors also learn about the final value, but parts of their expectation of exit values seems to be ex ante knowledge. The last regression in Table 5 shows that in fact both ex ante information (contained in the initial value) as well as interim information (contained in $AR_{i,2}$) explain the final value. In other words, the final value of a project seems to be partially contained in the ex ante information, and partially to be the result of interim learning as expressed in the abnormal returns over the project's investment cycle.

We turn to the exploration of the dynamics of risk and return. Our basic model assumes that, conditional on survival, the risk of failure is decreasing, as captured by the parameter λ of the Pareto distribution.[6] Panel A of Table 6 appears to lend some support to the assumption of decreasing failure risk since it shows that the survival probability is increasing over time. To test formally whether later rounds have higher survival probabilities, we employ round level probit analyses, using a binary dependent variable that equals 1 for surviving rounds and 0 for failing rounds. The main coefficients are reported in Panel B of Table 6, showing that early and late rounds (second and third rounds) have significantly higher survival probabilities than first/seed rounds (first rounds), and late rounds (third rounds) have significantly higher probabilities than all other rounds, including early rounds (second rounds).[7]

Note that while information content in a round is better measured by abnormal returns, raw returns seem a more appropriate measure for the total failure risk, including the expected and

[6] It is interesting to compare these findings to earlier studies that have looked at the *cross-sectional* distribution of payoffs derived from patent grants. Scherer and Harhoff (2000) found that they are poorly described by a Pareto distribution. Since we are looking exclusively at the distribution of failure risk of ventures *over time,* there is no contradiction between our evidence on the longitudinal distribution and the cross-sectional results by Scherer and Harhoff (2000).

[7] The main explanatory variables are dummies that equal 1 for all early and late rounds (left column) or all second and third rounds (right column), respectively dummies that equal 1 for all late rounds (left column) or all third rounds (right column). We also include industry dummies and dummies for the months when the rounds were raised.

not expected (learned) components. Our learning model implies that round returns for surviving companies should show a decreasing trend as the failure risk subsides. This is indeed the case, as Table 6 shows. In other words, our empirical investigation reveals that venture capital practitioner are right when they use higher discount rates in "first" and "seed" rounds, in order to compensate for a perceived larger risk. The results in Table 6 are in fact even more supportive of our model than these numbers suggest: the probabilities express the average fraction of surviving projects in each round. Our model, however, predicts the survival probability in terms of units of the $[0, K]$-investment cycle, which is the survival probability per invested dollar. Table 7 lends support to our premise of risk-neutral investors, which implies that unconditional expected returns are constant. We find that the means of the unconditional returns are increasing over time whereas the median returns are decreasing. Therefore, we conclude that it is difficult to reject the hypothesis of constant expected returns on which our risk-neutral model is based.[8]

Investment Flows We turn to our predictions on investment flow (investment spending per month). Our model leads to the prediction that investment flows are inversely related to a project's estimated failure risk λ. We do not observe estimated or actual failure risk λ directly, but can approximate them in two ways. First, we observe the ultimate project outcome, which is determined by the actual failure risk. Note that the ex ante information hypothesis indicates that investors' estimated failure risk should correlate with the actual failure risk. If the ex ante information hypothesis is correct, we should expect ultimately successful exits (via IPO or trade sale) to be more likely for projects with an above-average investment flow. As the first three regressions in Table 4 show, there is no evidence for this effect. A possible indirect measure of the estimated failure risk λ is the ratio of final value to initial value. Should the ex ante information hypothesis be true, then the higher the ratio, the larger would be the expected cumulative failure probability and hence λ. Again, our regression results are not significant (and thus not reported in our tables). Therefore, consistent with our earlier results, Table 4 provides strong evidence that investors have no ex ante information regarding the chance of success.

The model also predicts that the investment flow increases with the expected final value of the project. Assuming that the investors hold ex ante information regarding the final values, a test is only possible for successfully completed projects, using their actually realized exit values as a proxy

[8] This is certainly not a conclusive test on the question whether venture investors are indeed risk-neutral (see e.g. Gompers (1996) for a discussion), but it lends support to our simplifying model assumption of risk-neutral investors.

for expected final values. Regression 3 in Table 5 presents the results for this test, which seem to support our hypothesis: the coefficient of the first round investment flow is significant at the 1% level. We conclude that investors seem to have some ex ante information regarding the expected exit values.

In our model, the investment flow reacts to uncertainty, and it will optimally increase if there is less uncertainty about the ultimate project outcome. Table 8 provides strong support for this hypothesis (at the 1% level for both means and medians). Consistent with our model, investment flows are increasing over time. Table 8 also shows that venture projects exhibit larger investment volumes as they advance from one round to the next (at 1% level for both means and medians).

Additional predictions on the investment flow which imply multivariate relationships are tested using OLS regressions; the evidence is provided in Tables 9 and 10. First, the model predicts that the investment flow increases in the valuation of the project, and implies that this positive relationship holds for the first as well as later rounds of the investment process. In Table 9, regressions for all firms (unconditional) and IPO and M&A firms substantiate this positive relationship for the first round. Further, since investors have ex ante information about the final value of the project, which is supported by our earlier results in Table 5, final values should positively correlate with the initial investment flow. This is indeed the case, as the conditional regressions in Table 9 show. Table 10 provides evidence that the positive correlation between project valuation and investment flow holds throughout the investment process. In Table 10, regressions 2 and 5 show strong evidence for this effect (significant at 1% level). It is useful to note that we use the pre-money company value at the beginning of each round. Regressions 3 and 5 in Table 10 show that the optimal investment flows are autocorrelated and thus persistent throughout the project investment cycle (significant at 1% level). This is consistent with our model which predicts the persistence of the autocorrelation as a mirror image of the evolution of project valuation over time. We observe that the regressions 4 and 5 control for the sunk cost (the total investment amount raised before) of investors in firms, which would be significant if investors are not willing to realize losses and tend to continue financing bad projects. While the sunk cost is significant in regression 4, it is not significant in regression 5, which includes the lagged firm value, the lagged investment flow, and the lagged abnormal return. Note that, in Table 10, we use dummies to control for mezzanine rounds because they are likely bridge financing rounds prior to successful exits, and may not reflect the learning phase of the project.

We already presented evidence in favor of the interim information hypothesis. If this hypothesis is true then it has clear implications for investment behavior that can easily be tested: investment flow should be increasing in the most recent abnormal return in each round that we use as a proxy for interim information. If the last observed abnormal return $AR_{i,k}$ is higher, then the project should have received a more positive information update. All regressions in Table 10 support this prediction, and show strong evidence consistent with the interim information hypothesis. The effect seems to have a concave shape as the quadratic term for the abnormal return is negative and highly significant as well.

Staging Frequency With regard to the determinants of the round frequency, our model implies that larger financing rounds will occur if there is less uncertainty resolved for every dollar of investment. It predicts that the staging frequency should be lower for projects with a high success probability.

To test this hypothesis, we need to turn to regressions for completed projects that explain the number of rounds over the entire investment cycle. Since we do not observe the risk variable λ directly, we use the ratio of exit value to initial value for completed projects as a proxy, and assume that investors have ex ante information regarding final values, which is supported by our earlier evidence. The first line in Table 11 shows the results, with the total number of financing rounds as a dependent variable. In all regressions in Table 11, there is a positive and highly significant sign (at 1% level) for our proxy for λ. Moreover, our theoretical results imply a negative sign when we regress the number of rounds on the final value. Regressions 3 and 4 of Table 11 show indeed strong evidence (significant at 1% level) in favor of this hypothesis.

Considering interim learning about the project's failure risk or final value, we predict that a positive information release that makes the project more valuable or less risky leads to less subsequent financing rounds. As regressions 2 and 4 in Table 11 show, this is indeed the case. The relationship is again nonlinear, as witnessed by the quadratic terms of $AR_{i,2}$.

Size and Duration of Financing Rounds Our model leads to very clear predictions on the investment size (capital raised) of each financing round. The model predicts that it is increasing from one round to the next. This is indeed the case for the means and the medians of the investment volume, as Panel B of Table 8 shows. By contrast, the predictions on round duration (in months) of each financing round are ambiguous, since increasing investment volume (the dollar amount provided in a given round) and investment flow have countervailing effects on the round duration. Interestingly, we do not find any clear patterns for round durations (not reported in tables), consistent with our model's ambiguous predictions for round duration.

Our model predicts that a higher project valuation, reflecting either a high expected final value or a low expected failure probability, should translate into a larger investment volume in every round. The result of our regression analysis are presented in Table 12. We find strong evidence in favor of this hypothesis as regressions 2 and 5 show. Also, an above-average investment size in round $k-1$, explained by a high exit value, low failure probability or high contracting cost, should translate into an above-average investment volume in round k. This is indeed the case, as the positive and significant signs for variable $\log(investment_{i,k-1})$ in Table 12 (regressions 3 and 5) shows. Note that we control for the sunk cost and mezzanine rounds in this table as well.

We also explore the implications of the interim information hypothesis for investment volume. The model implies that positive interim information releases should lead to an increase in the capital raised in each subsequent round. This is indeed the case as the highly significant and positive coefficient on the interim learning variable $AR_{i,2}$ shows in Table 13, which again exhibits a nonlinear effect.

Total Project Duration Our model implies that projects with an above-average initial valuation will be completed faster as they benefit from a persistently higher investment flow. We find clear evidence in support of this prediction in Table 13. Table 13 further substantiates that favorable information updates, which are proxied by abnormal returns, and lower failure risk, which is proxied by the reciprocal of $\log(exitvalue_i/value_{i,1})$, increase investment flows and thus help projects to be completed faster.

8 Conclusion

We investigate a stylized model to analyze how investors make optimal dynamic investment decisions in an innovative project as a function of their information about failure risk and potential final value. We consider the complete investment cycle and assume that information leading to the failure of the project may arise at any time, but at a decreasing probability. The investors choose the optimal speed of investment with a convex cost function. They also choose an optimal sequence of financing stages. We model the cost of each investment decision as a loss that is proportional to the current value of the project, so that investment decisions only occur as discrete events.

The results of our theoretical analysis are the following. As the project advances and the probability of eventual success increases, investment flows should be optimally increasing. Therefore, decreasing surviving returns require that the failure risk decreases over time. The optimal staging sequence depends on the value of the real option to abandon: The higher the estimated final value of the project, and the larger the estimated success probability, the fewer rounds will be used. Finally, we show that information updates about the expected final value or the failure probability will be incorporated in all subsequent investment decisions. If the value of the project increases then the subsequent investment flow will increase. At the same time, the number of subsequent investment rounds will decrease, and the capital allocation for each of these rounds will increase.

In our empirical tests of these predictions, we find that investment flows are increasing over time as predicted. Our evidence shows that initially, investors seem to have little ability to predict the eventual probability of success, but have some forecasting ability about the final project value conditional on success. The design of the financing rounds follows the optimal pattern predicted by our model: the investment size and the investment flow is increasing from one round to the next, and projects with a high initial estimate of the final value or an optimistic appraisal are likely to succeed with fewer rounds than less valuable or more risky projects. As the project advances, frequently investors get information that leads them to reappraise the failure risk of the project. We show that such information updates lead them to adjust the investment path optimally: the subsequent investment flow as well as the size of each round and the number of subsequent rounds react in the way predicted by our model.

Several possible extensions could be considered. In our theoretical analysis of interim learning, we focus exclusively on information arrival about the failure rate λ and the subsequent empirical analysis underlines the importance of this type of interim learning. However, it is possible that the investors also learn about the expected final value Y of venture projects that they decide to undertake. Changes in the expectations about future values could further address the phenomenon of fluctuations in the firms' values, a well-known phenomenon in venture financing (for example, acquisition values as well as IPO values fluctuate substantially according to market conditions). Finally, throughout the analysis we have assumed that the venture realizes a terminal value Y in a given terminal state K. But clearly, the timing of the exit and the associated value of the venture at the exit time are important decisions for the investors. Interesting additional predictions could be derived from such extensions that we leave for future research.

9 Appendix

The appendix contains the proofs of all propositions in the main body of the text.

Proof of Proposition 1. We consider the dynamic investment problem with uncertainty. We showed in the main body of the text that the ordinary differential equation resulting from the optimal investment policy can be represented in its canonical form:

$$V'(k_t) = \frac{\lambda}{k_t} V(k_t) + \frac{2}{\gamma}\sqrt{rV(k_t)}. \tag{14}$$

With a change of variable given by $W(k_t) \triangleq \sqrt{V(k_t)}$, we can transform the above nonlinear differential equation into a linear first order differential equation. We observe that

$$W'(k_t) = \frac{1}{2}\frac{V'(k_t)}{\sqrt{V(k_t)}}, \tag{15}$$

and hence $V'(k_t) = 2W(k_t)\,W'(k_t)$. Replacing $V(k_t)$ and $V'(k_t)$ by $W(k_t)$ and $W'(k_t)$ in (14) we get:

$$W'(k_t) = \frac{\sqrt{r}}{\gamma} + \frac{\lambda}{2k_t} W(k_t). \tag{16}$$

The unique solution of the differential equation (16) subject to the boundary condition:

$$W(K) = \sqrt{Y},$$

is given by:

$$W(k_t) = \sqrt{Y}\left(\frac{k_t}{K}\right)^{\frac{1}{2}\lambda} - \frac{2\sqrt{r}}{\gamma(2-\lambda)}\left(K\left(\frac{k_t}{K}\right)^{\frac{1}{2}\lambda} - k_t\right). \tag{17}$$

For the failure rate $\lambda = 2$, the value function is linear in k_t and given by:

$$W(k_t) = \sqrt{Y}\frac{k_t}{K} - \frac{2\sqrt{r}}{\gamma}.$$

Consequently, the value function $V(k_t)$, based on the solution of $W(k_t)$ in (17) is given by $V(k_t) = (W(k_t))^2$. We can immediately establish the properties (1)-(3) of the optimal investment i_t^* by using the linear relationship (6). We can explicitly express the optimal investment in terms of the primitives of the model:

$$i^*(k_t, Y, \lambda) \triangleq r\left(\sqrt{Y}\left(\frac{k_t}{K}\right)^{\frac{1}{2}\lambda} - \frac{2\sqrt{r}}{\gamma(2-\lambda)}\left(K\left(\frac{k_t}{K}\right)^{\frac{1}{2}\lambda} - k_t\right)\right)^2.$$

(*1.*) We obtain by elementary calculus that $\partial i^*/\partial k > 0$ and $\partial i^{*2}/\partial^2 k > 0$.

(*2.*) We obtain by elementary calculus that $\partial i^*/\partial\lambda < 0$ and $\partial i^{*2}/\partial^2\lambda < 0$.

(*3.*) We obtain by elementary calculus that $\partial i^*/\partial Y > 0$ and $\partial i^{*2}/\partial^2 Y > 0$.■

Proof of Proposition 2. The surviving return $R_t \triangleq R(k_t) = 1 + r(k_t)$ is given, using (9) by:

$$1 + r(k_t) = \frac{-i_t + \sqrt{i_t}\gamma V'(k_t)}{V(k_t)}.$$

Using the characterization of the optimal investment given by (6), we get

$$r(k_t) = \frac{-rV(k_t) + \sqrt{rV(k_t)}\gamma V'(k_t)}{V(k_t)} = -r + \frac{\sqrt{r}\gamma V'(k_t)}{\sqrt{V(k_t)}}.$$

Using the same change of variable as in Proposition 1, we find that

$$r(k_t) = -r + 2\sqrt{r}\gamma W'(k_t).$$

Using (16) to replace $W'(k_t)$ we have:

$$r(k_t) = -r + 2\sqrt{r}\gamma\left(\frac{\lambda}{2k}W(k_t) + \frac{\sqrt{r}}{\gamma}\right),$$

and we now ask whether $r'(k_t)$ is positive or negative:

$$r'(k_t) = 2\sqrt{r}\gamma\left(-\frac{\lambda}{2k_t^2}W(k_t) + \frac{\lambda}{2k_t}W'(k_t)\right),$$

and using (16) again to replace $W'(k_t)$ we get:

$$\begin{aligned}
r'(k_t) &= 2\sqrt{r}\gamma\left(-\frac{\lambda}{2k_t^2}W(k_t) + \frac{\lambda}{2k_t}\left(\frac{\lambda}{2k_t}W(k_t) + \frac{\sqrt{r}}{\gamma}\right)\right) \\
&= 2\sqrt{r}\gamma\left(-\frac{\lambda}{2k^2}W(k_t)\left(1 - \frac{\lambda}{2}\right) + \frac{\lambda}{2k_t}\frac{\sqrt{r}}{\gamma}\right) \\
&= \frac{\sqrt{r}\gamma\lambda}{k_t}\left(-\frac{1}{k_t}W(k_t)\left(1 - \frac{\lambda}{2}\right) + \frac{\sqrt{r}}{\gamma}\right). \qquad (18)
\end{aligned}$$

We use the solution for the value function $W(k_t)$ from Proposition 1 to insert it into (18). It suffices to determine the sign of

$$\left(-\frac{1}{k}W(k_t)\left(1 - \frac{\lambda}{2}\right) + \frac{\sqrt{r}}{\gamma}\right),$$

and inserting $W(k_t)$ we get

$$\left(-\frac{1}{k_t}\left(\sqrt{Y}\left(\frac{1}{K}\right)^{\frac{1}{2}\lambda} - \frac{2\sqrt{r}}{2\gamma - \lambda\gamma}\left(\left(\frac{k_t}{K}\right)^{\frac{1}{2}\lambda - 1} - 1\right)\right)\left(1 - \frac{\lambda}{2}\right) + \frac{\sqrt{r}}{\gamma}\right),$$

or

$$\left(-\left(\sqrt{Y}\left(\frac{1}{K}\right)^{\frac{1}{2}\lambda}-\frac{2\sqrt{r}}{\gamma\left(2-\lambda\right)}\left(\left(\frac{k_t}{K}\right)^{\frac{1}{2}\lambda-1}\right)\right)\left(1-\frac{\lambda}{2}\right)\right).$$

The second term is negative for $\lambda > 2$. To sign the first term, we simplify to:

$$-\left(\sqrt{Y}\frac{1}{k_t}\left(\frac{k_t}{K}\right)^{\frac{1}{2}\lambda}-\frac{2\sqrt{r}}{\gamma\left(2-\lambda\right)}\left(\frac{k_t}{K}\right)^{\frac{1}{2}\lambda-1}\right),$$

or

$$-\left(\sqrt{Y}-\frac{2K\sqrt{r}}{\gamma\left(2-\lambda\right)}\right)\left(\frac{k_t}{K}\right)^{\frac{1}{2}\lambda}\frac{1}{k_t}. \tag{19}$$

Thus if K is not too large and λ is sufficiently below 2, then the surviving returns are declining everywhere. Conversely if K is large or λ above or just below 2, then the surviving returns are increasing in k_t, and hence in time, everywhere. ■

Proof of Proposition 3. The optimal investment policy $i_{1,1}$ given the initial state k_0 can be obtained (implicitly) by the first order condition of $V_{1,1}\left(k_0\right)$, given by (11) with respect to $i_{1,1}$, or

$$\frac{dV_{1,1}\left(k_0\right)}{di_{1,1}}=\left(\frac{K}{k_0}\right)^{1-\lambda}-\left(\left(\left(\frac{k_0}{K}\right)^{\lambda}-\left(\frac{K}{k_0}\right)^{1-\lambda}\right)\frac{r}{\gamma}\right)\left(\frac{Y\left(\lambda-1\right)\gamma}{i_{1,1}^2}+\frac{K}{i_{1,1}}\right)-e^{\frac{K-k_0}{i_{1,1}}\frac{r}{\gamma}}=0, \tag{20}$$

Based on (20), the solution to the optimal investment policy $i_{1,1}^*$ for a single stage investment can be shown to be strictly increasing in Y and k_0 and strictly decreasing in λ.

If, in contrast, the project is funded in two stages, then the optimal funding policy starting at the initial position k_0 has to make three distinct choices: it has to determine the initial funding level $i_{1,2}$, the continued funding level $i_{2,2}$ and the funding renewal state k_1. Conditional on the optimal funding level given the renewal stage k_1, the value function in the initial state k_0 is given as the solution to the optimization problem (12):

$$V_{1,2}\left(k_0\right)=\max_{i_{1,2},k_1}\left(\frac{\sqrt{i_{1,2}}}{\left(\lambda-1\right)\gamma}\left(k_0-k_1\left(\frac{k_0}{k_1}\right)^{\lambda}e^{\frac{k_0-k_1}{\sqrt{i_{1,2}}}\frac{r}{\gamma}}\right)+p\left(\frac{k_0}{k_1}\right)^{\lambda}e^{\frac{k_0-k_1}{\sqrt{i_{1,2}}}\frac{r}{\gamma}}V_{2,2}\left(k_1\right)\right). \tag{21}$$

We can insert (11) into (12) to get:

$$V_{1,2}\left(k_0\right)=\max_{\{i_{1,2},i_{2,2},k_1\}}\left\{\begin{array}{c}\frac{\sqrt{i_{1,2}}k_1}{(\lambda-1)\gamma}\left(\frac{k_0}{k_1}-\left(\frac{k_0}{k_1}\right)^{\lambda}e^{\frac{k_0-k_1}{\sqrt{i_{1,2}}}\frac{r}{\gamma}}\right)+p\left(\frac{k_0}{K}\right)^{\lambda}Ye^{\left(\frac{k_0-k_1}{\sqrt{i_{1,2}}}+\frac{k_1-K}{\sqrt{i_{2,2}}}\right)\frac{r}{\gamma}}\\+pe^{\frac{k_0-k_1}{\sqrt{i_{1,2}}}\frac{r}{\gamma}}\frac{\sqrt{i_{2,2}}K}{(\lambda-1)\gamma}\left(\left(\frac{k_0}{K}\right)^{\lambda}e^{\frac{k_1-K}{\sqrt{i_{2,2}}}\frac{r}{\gamma}}-\frac{k_1}{K}\left(\frac{k_0}{k_1}\right)^{\lambda}\right)\end{array}\right\}.$$

We now establish the results of this proposition.

1. We first observe that $i_{2,2} > i_{1,1}$ by the comparative static property of the optimal investment policy $i^*_{1,1}$ obtained above for the single stage funding policy. After all, the funding policy $i_{2,2}$ of the project conditional on renewing the project is like a single stage funding, but at a higher level of the state k_t. Given a final state K, we denote by k^*_0 an initial state at which the value conditional on an optimal single stage funding policy equals the value conditional on an optimal two stage funding policy, and hence $V_{1,1}(k^*_0) = V_{1,2}(k^*_0)$. We show that $i_{1,2} < i_{1,1}$. In the two stage funding policy, the optimal renewal occurs at k_1 and conditional on renewal, we have a value function $V_{2,2}(k_1)$. By construction, we have $V_{2,2}(k_1) > V_{1,1}(k_1)$. We now show that it follows from here that $pV_{2,2}(k_1) \leq V_{1,1}(k_1)$. The proof is by contradiction. If $pV_{2,2}(k_1) > V_{1,1}(k_1)$, then starting at k^*_0, and having the advantage of determining the investment level to optimally arrive at k_1, the two stage funding policy does at least as well as the one stage funding policy which runs through the state k_1 with intensity $i_{1,1}$. As the initial funding policy in the two stage funding seeks to determine the optimal intensity to arrive at a stopping point k_1 with a value $pV_{2,2}(k_1) \leq V_{1,1}(k_1)$, it follows that it will choose a strictly lower investment policy $i_{1,2}$ than $i_{1,1}$. Notice that $i_{1,1}$ was determined to optimally reach the higher value $V_{1,1}(K)$ rather than $V_{1,1}(k_1)$.

2. We first establish the uniqueness of k^*_0 for a given final state K by a single crossing argument. The uniqueness result of k^*_0 given K then translates directly into a uniqueness result about K^* given k_0, where K^* is the unique final state at which the value from an optimal one-stage and two-stage policy, respectively, coincide when starting at k_0. We observe that the value functions $V_{1,1}(k_t)$ and $V_{1,2}(k_t)$ are continuous and differentiable in k_t. We next show that if

$$V_{1,1}(k^*_0) = V_{1,2}(k^*_0), \tag{22}$$

then $V'_{1,1}(k^*_0) > V'_{1,2}(k^*_0)$. We note that the value functions of each program, $V_{1,1}(k^*_0)$ and $V_{1,2}(k^*_0)$, respectively, satisfy:

$$rV_{1,1}(k^*_0) = \left\{-i_{1,1} + \lambda\sqrt{i_{1,1}}\left(V'_{1,1}(k^*_0) - \frac{\lambda}{k^*}V_{1,1}(k^*_0)\right)\right\},$$

and

$$rV_{1,2}(k^*_0) = \left\{-i_{1,2} + \lambda\sqrt{i_{1,2}}\left(V'_{1,2}(k^*_0) - \frac{\lambda}{k^*}V_{1,2}(k^*_0)\right)\right\}.$$

Next we express the value function $V_{l,m}(k^*)$ in terms of the investment level $i_{l,m}$ and the first derivative of the value function $V'_{l,m}(k^*_0)$ and get

$$V_{l,m}(k^*_0) = \frac{\sqrt{i_{l,m}}\lambda V'_{l,m}(k^*_0) - i_{l,m}}{r + \frac{\sqrt{i_{l,m}}}{k}\lambda^2}. \tag{23}$$

We determine the sign of $V_{1,1}'(k_0^*) - V_{1,2}'(k_0^*)$ by analyzing how $V_{1,1}'(k_0^*)$ and $V_{1,2}'(k_0^*)$ differ given the different investment intensities: $i_{1,2} < i_{1,1}$, established in part 1 of this proposition while maintaining the hypothesis of equal values given by (22). We determine how $V'(k_0^*)$ changes as we change the investment level i from $i_{1,2}$ to $i_{1,1}$. For the purpose of this argument, we omit the subscripts $_{l,m}$ and write the value function $V(k_0^*, i)$ to depend on k_0^* and i. Consequently, we rewrite (23) as :

$$V(k_0^*, i) \triangleq \frac{\sqrt{i}\lambda \frac{\partial V(k_0^*,i)}{\partial k_0^*} - i}{r + \frac{\sqrt{i}}{k}\lambda^2}. \tag{24}$$

As we increase i from $i_{1,2}$ to $i_{1,1}$, the value $V(k_0^*)$ is constant by construction. We rewrite (24) to obtain:

$$\lambda \frac{\partial V(k_0^*, i)}{\partial k_0^*} = \sqrt{i} + \frac{V(k_0^*, i)\, r}{\sqrt{i}} + V(k_0^*, i)\frac{\lambda^2}{k_0^*}. \tag{25}$$

We establish the sign of $\partial^2 V(k_0^*, i)/\partial k_0^* \partial i$ by differentiating (25) with respect to i:

$$\lambda \frac{\partial V(k_0^*, i)}{\partial k_0^* \partial i} = 1 - \frac{V(k_0^*, i)\, r}{i}.$$

By construction $\partial V(k_0^*, i)/\partial i = 0$, or alternatively

$$\lambda i \frac{\partial^2 V(k_0^*, i)}{\partial k_0^* \partial i} = i - V(k_0^*, i)\, r.$$

We complete the argument by establishing that:

$$i > V(k_0^*, i)\, r. \tag{26}$$

We observe that if the investors were allowed to determine the investment flow optimally in every instant, then we would have, as established in Proposition 1:

$$i = \frac{\gamma^2}{4}\left(\frac{\partial V(k_0^*, i)}{\partial k_0^*} - \frac{\lambda}{k_0^*} V(k_0^*, i)\right),$$

or

$$i = rV(k_0^*, i).$$

But in fact, as we consider the optimal investment decision subject to staging, the optimal investment $i_{1,m}$ is determined with respect to some average valuation over the course of the investment round, and thus as the value is increasing in the current position k_t, we find that at the beginning of the funding round the investment flow $i_{1,m}$ displays

$$i > \frac{\gamma^2}{4}\left(\frac{\partial V(k_0^*, i)}{\partial k_0^*} - \frac{\lambda}{k_0^*} V(k_0^*, i)\right),$$

which establishes (26). ■

Proof of Proposition 4. *1.* The investment decisions $i_{1,1}$ and $i_{1,2}$ represent solutions to similar problems. The sole difference is that the terminal value of the investment problem of $i_{1,1}$ is given by Y whereas the terminal value of the investment problem of $i_{1,2}$ is given by some fraction of Y, say $q \cdot Y$, with $q \in (0,1)$. The optimal investment $i_{1,2}$ is taking the solution to the optimal stopping problem at k_1 as given. Hence the smaller benefit, $q \cdot Y$ is reached at an earlier stage, namely, $k = k_1$. It follows that we can represent the investment decisions $i_{1,1}$ and $i_{1,2}$ as:

$$i_{1,1} \in \arg\max_{i \in \mathbb{R}_+} \left\{ p_K(i) \cdot Y - c_K(i) \right\},$$

and

$$i_{1,2} \in \arg\max_{i \in \mathbb{R}_+} \left\{ p_{k_1}(i) \cdot q \cdot Y - c_{k_1}(i) \right\},$$

respectively. The term $p_k(i)$ represents the discounted probability that a positive terminal value is realized in position k given an investment flow i and the term $c_k(i)$ represents the associated discounted cost to reach the position k with a constant investment flow i. By hypothesis, the value of these problems is equal at k_0^*, or

$$p_K(i_{1,1}) \cdot Y - c_K(i_{1,1}) = p_{k_1}(i_{1,2}) \cdot q \cdot Y - c_{k_1}(i_{1,2}) \quad . \tag{27}$$

Since the cost of reaching K is strictly larger than reaching k_1, we have

$$c_K(i_{1,1}) > c_{k_1}(i_{1,2}),$$

but this implies by (27) that

$$p_K(i_{1,1}) > p_{k_1}(i_{1,2}) \cdot q. \tag{28}$$

Hence it follows from the envelope theorem that a marginal increase in Y is more beneficial to the single round funding regime by (28), which establishes that k_0^* is decreasing in Y.

The argument for an increase of k_0^* in response to an increase in the failure λ is similar to the above argument regarding Y.

2. The marginal benefit of extending k_1 is increased by an increase in p and hence it leads to an increase in k_1 despite the increase in the marginal cost.

3. This follows immediately from the Proposition 3.1. ■

Proof of Proposition 5. *1.* The probability of success given a constant failure probability λ at k_t is given by

$$P(k_t) \triangleq 1 - F(k_t) = \left(\frac{k_t}{K}\right)^{\lambda}. \tag{29}$$

The probability $P(k_t)$ before the resolution of uncertainty about the failure rate is given by:

$$0 = P'(k_t) - \frac{\lambda}{k_t} P(k_t) + \rho\left((1-\alpha) P_l(k_t) + \alpha P_h(k_t) - P(k_t)\right), \tag{30}$$

and inserting $P_l(k_t)$ and $P_h(k_t)$ from (29) we get:

$$0 = P'(k_t) - \frac{\lambda}{k_t} P(k_t) + \rho\left((1-\alpha)\left(\frac{k_t}{K}\right)^{\lambda_l} + \alpha\left(\frac{k_t}{K}\right)^{\lambda_h} - P(k_t)\right).$$

By (30), the probability $P(k_t)$ is an average of $P_l(k_t)$ and $P_h(k_t)$ and the result follows from $P_l(k_t) > P(k_t) > P_h(k_t)$.

2. The optimal investment policy before the resolution of uncertainty about the failure rate is given as the solution to the dynamic programming equation:

$$rV(k_t) = \max_{i_t}\left\{-i_t + \sqrt{i_t}\gamma\left(V'(k_t) - \frac{\lambda}{k}V(k_t) + \rho\left((1-\alpha)V_l(k_t) + \alpha V_h(k_t) - V(k_t)\right)\right)\right\},$$

with the solution given by:

$$i_t = \frac{\gamma^2}{4}\left(V'(k_t) - \frac{\lambda}{k}V(k_t) + \rho\left((1-\alpha)V_l(k_t) + \alpha V_h(k_t) - V(k_t)\right)\right)^2,$$

and hence the value function is given by

$$rV(k_t) = \frac{\gamma^2}{4}\left(V'(k_t) - \frac{\lambda}{k}V(k_t) + \rho\left((1-\alpha)V_l(k_t) + \alpha V_h(k_t) - V(k_t)\right)\right)^2,$$

and so:

$$i_t = rV(k_t).$$

But as $V_h(k_t) < V(k_t) < V_l(k_t)$, it follows that $i_{t,h} < i_t < i_{t,l}$, which completes the proof.$\blacksquare$

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Table 1 Summary of Firm Characteristics

This table reports the number of firms (Panel A), means and standard deviations of pre-exit rounds per firm (Panel B) and the duration (months) from the first round to the exit round for firms (Panel C). The five top lines in Panel A contain the raw data, in which exits are *documented* as either IPO, M&A, Down (= out of business) and Unknown. In Panel B and C, the four top lines report the raw data. The two bottom lines of each panel A (in italics) report the number of firms with exit Unknown that we reclassify as either Down or Alive, according to the estimation procedure based on round duration laid out in Section 6.

Industry	Health care	IT	Retail	Others	Total
		Panel A: # of firms			
IPO	541	1,039	213	227	2,020
M&A	451	1,654	248	312	2,665
Down	137	556	200	45	938
Unknown	1,757	7,326	1,042	3,187	13,312
Total	2,886	10,575	1,703	3,771	18,935
Down	*1,352*	*5,892*	*1,087*	*2,526*	*10,857*
Alive	*542*	*1,990*	*155*	*706*	*3,393*
		Panel B: Pre-exit rounds per firm: average [standard deviation]			
IPO	3.89 [2.46]	3.43 [2.26]	3.18 [1.92]	2.29 [2.01]	3.40 [2.30]
M&A	3.38 [2.46]	2.93 [2.29]	3.10 [2.65]	2.19 [1.96]	2.94 [2.34]
Down	3.72 [3.02]	3.15 [2.29]	3.02 [1.84]	2.51 [1.94]	3.17 [2.32]
All	3.66 [2.54]	3.13 [2.29]	3.10 [2.20]	2.25 [1.98]	3.14 [2.33]
Down	*2.78 [2.23]*	*2.20 [1.77]*	*2.87 [1.78]*	*1.74 [1.42]*	*2.23 [1.80]*
All	*3.15 [2.38]*	*2.49 [2.00]*	*2.95 [1.96]*	*1.83 [1.55]*	*2.50 [2.02]*
		Panel C: Duration (months) before exit: average [standard deviation]			
IPO	45.83 [26.90]	43.45 [28.40]	34.80 [26.27]	36.57 [28.28]	42.41 [28.00]
M&A	49.19 [32.06]	40.74 [32.16]	37.65 [29.70]	45.15 [35.43]	42.40 [32.50]
Down	74.10 [34.70]	59.46 [41.84]	45.05 [34.58]	49.89 [34.77]	58.07 [40.01]
All	50.61 [31.31]	44.81 [33.60]	38.97 [30.49]	42.19 [33.05]	45.02 [32.90]
Down	*34.11 [29.68]*	*23.55 [24.53]*	*29.97 [25.84]*	*18.70 [19.77]*	*24.38 [29.81]*
All	*39.72 [30.27]*	*29.27 [27.96]*	*31.86 [26.71]*	*22.71 [24.23]*	*29.81 [27.96]*

Table 2 Summary of Pre-exit Round Characteristics

This table reports the number of pre-exit rounds, as well as means and standard deviations of duration prior to each round (months), investment volume per round (million $), and the ratios of investment volume to post-money valuations, for firms in healthcare, IT, retail and other industries. Firms are classified into the exit types IPO, M&A, Down (out of business) or Alive (not yet exited) according to both documented *and* estimated exits.

Industry	Health care	IT	Retail	Others	Total
Panel A: # of Pre-exit Rounds					
Exit: IPO	2,103	3,563	678	519	6,863
Exit: M&A	1,525	4,844	770	684	7,823
Exit: Down	3,752	12,969	3,122	4,401	24.244
Alive	1,629	5,248	650	1,078	8,605
Total	9,009	26,624	5,220	6,682	47,535
Panel B: Pre-financing duration (months): average [standard deviation]					
Exit: IPO	10.77 [9.47]	11.16 [10.09]	10.28 [9.84]	10.80 [10.13]	10.93 [9.87]
Exit: M&A	10.96 [9.70]	10.35 [10.30]	8.80 [7.52]	11.69 [9.85]	10.42 [9.92]
Exit: Down	11.88 [10.39]	10.38 [9.79]	9.61 [9.19]	12.34 [14.25]	10.81 [10.59]
Alive	11.83 [9.93]	10.53 [8.78]	9.93 [8.29]	14.41 [18.10]	11.00 [9.99]
Total	11.43 [9.97]	10.53 [9.75]	9.58 [8.93]	12.36 [13.99]	10.79 [10.24]
Panel C: Investment volume (million $): average [standard deviation]					
Exit: IPO	5.72 [9.28]	7.16 [12.70]	11.35 [20.45]	14.13 [46.66]	7.65 [17.83]
Exit: M&A	3.78 [5.20]	4.93 [11.07]	7.06 [13.23]	7.28 [48.71]	5.12 [17.50]
Exit: Down	4.93 [11.93]	8.10 [16.96]	8.87 [14.19]	6.37 [19.50]	7.46 [27.96]
Alive	7.36 [10.48]	9.77 [17.59]	8.36 [12.41]	10.08 [27.53]	9.19 [17.15]
Total	5.33 [10.25]	7.69 [15.69]	8.88 [14.91]	7.46 [27.96]	7.34 [17.02]
Panel D: Ratio of investment to post-money valuation: average [standard deviation]					
Exit: IPO	0.30 [0.21]	0.25 [0.18]	0.25 [0.17]	0.31 [0.23]	0.27 [0.19]
Exit: M&A	0.34 [0.20]	0.27 [0.17]	0.30 [0.18]	0.30 [0.20]	0.29 [0.18]
Exit: Down	0.32 [0.19]	0.28 [0.17]	0.30 [0.17]	0.28 [0.23]	0.29 [0.18]
Alive	0.33 [0.20]	0.30 [0.18]	0.29 [0.17]	0.26 [0.22]	0.31 [0.19]
Total	0.32 [0.20]	0.28 [0.18]	0.29 [0.17]	0.28 [0.22]	0.29 [0.18]

Table 3 Firm Characteristics: IPO/M&A vs Down Firms

This table summarizes the post-money valuation (million $ in log) of the first round V_1, the investment volume (million $ in log) in the first round I_1, the ratio of investment volume to post-money valuation in the first round (in log)) I_1/V_1, the ratio of investment volume in the first round to the duration between first and the second round (in log) I_1/D_1, and the abnormal return per month (gross return in log) from the first round to the second round AR_2, for IPO/M&A firms and Down firms. Down firms are firms with documented and estimated exits being Down. The reported numbers are means, standard deviations (in parenthesis) and the number of observations used to calculate the means and standard deviations (in brackets). This table also reports the *t*-statistics and corresponding *p*-values for testing the hypotheses of identical means between IPO/M&A and Down firms.

	IPO/M&A firms	Down firms	Difference t-tests
V_1	2.289	2.485	T statistic: -4.887
	(1.176)	(1.172)	P value: 0.000
	[1,432]	[2,142]	
I_1	0.725	0.772	T statistic: -1.764
	(1.465)	(1.624)	P value: 0.078
	[4,457]	[10,650]	
I_1/V_1	-1.239	-1.265	T statistic: 0.963
	(0.796)	(0.828)	P value: 0.336
	[1,430]	[2,140]	
I_1/D_1	-1.637	-1.423	T statistic: -6.824
	(1.764)	(1.741)	P value: 0.000
	[4.457]	[10.650]	
AR_2	0.287	-0.339	T statistic: 25.335
	(0.386)	(0.828)	P value: 0.000
	[891]	[1,551]	

Table 4 Probit Analysis of Firm Exits

This table reports the results of a probit analysis regarding the determinants of the exit types of VC-backed firms. For firm i, $value_{i,1}$ is the post-money valuation at round 1, $AR_{i,2}$ is the average monthly abnormal return (gross return in log) between round 1 and 2, $flow_{i,1}$ is the investment flow for round 1 (investment volume in round 1 divided by the number of months between round 1 and round 2). Exit classifications are as in Table 2. The regressions also include a vector of dummies for the business status (start up, in development, and shipping and profitable) at the time of its first financing round, and for the industry (IT, health, and retail) of the firm. *Heteroskedasticity-robust* standard deviations are in parentheses. *** denotes significance at the 1% level, ** at the 5% level, and * at 10% level.

	IPO and M&A (1) Vs Down (0)	IPO (1) Vs Down (0)	M&A (1) Vs Down (0)	IPO (1) Vs M&A (0)
$log(value_{i,1})$	-0.012	0.024	**-0.127	**0.129
	(0.030)	(0.032)	(0.052)	(0.055)
$AR_{i,2}$	***0.809	***0.778	***0.610	0.240
	(0.049)	(0.050)	(0.068)	(0.149)
$log(flow_{i,1})$	-0.011	-0.008	*-0.101	0.034
	(0.016)	(0.016)	(0.056)	(0.049)
Sample size	2,440	890	1,771	2,219

Table 5 Determinants of Exit Values of Successful Firms

This table reports the regression results regarding the determinants of the exit values for IPO and M&A firms.

$$log(exit.value_i) = \alpha + \beta_1 log(value_{i,1}) + \beta_2 AR_{i,2} + \beta_3 log(flow_{i,1}) + \rho' dummy_i + \varepsilon_i$$

In the above equation, for firm i, $exit.value_i$ is the exit value (IPO market value minus capital raised from IPO or post M&A value minus capital infused), $value_{i,1}$ is the post-money valuation at round 1, $AR_{i,2}$ is the average monthly abnormal return (gross return in log) between round 1 and 2, $flow_{i,1}$ is the investment flow for round 1 (investment volume in round 1 divided by the number of months between round 1 and round 2). The vector $dummy_i$ contains dummies for the business status (start up, in development, and shipping and profitable) at the time of its first financing round, and for the industry (IT, health, and retail) of the firm. *Heteroskedasticity-robust* standard deviations are in parentheses. *** denotes significance at the 1% level, ** at the 5% level, and * at 10% level.

	Regression 1	Regression 2	Regression 3	Regression 4
$log(value_{i,1})$	***0.200 (0.038)			***0.190 (0.044)
$AR_{i,2}$		***0.660 (0.114)		***0.699 (0.115)
$log(flow_{i,1})$			***0.037 (0.006)	0.009 (0.017)
Sample Size	807	658	1,817	657
R2	0.12	0.15	0.08	0.18

Table 6 Tests on Pareto Distribution of Failure Risk

Panel A of this table reports the number of all pre-exit non mezzanine rounds that lead to another round or an exit, the number of them that lead to another round or a successful exit (surviving rounds), and the survival probability (surviving rounds divided by all pre-exit rounds) for different round types. Round types are determined according to the round status reported in the data (left column) and according to their sequence number in the round sequence of each company (right column). Panel A also reports the means and medians of post-financing gross returns per month (in log) for surviving rounds (surviving returns) and for all non-exit rounds (unconditional returns). Panel B reports results of probit analyses regarding the determinants of survivals. In the analyses, the dependant binary variable equals 1 for surviving rounds and 0 for failing rounds. Explanatory variables are a dummy that equals 1 for all early and late rounds (left column) or all 2nd and 3rd rounds (right column), a dummy that equals 1 for all late rounds (left column) or all 3rd rounds (right column), industry dummies, and dummies for the months when the rounds were raised. *** denotes significance at the 1% level, ** at the 5% level, and * at 10% level.

Panel A: Survival summary

Variables	Round Status			Round Sequence		
	First/Seed	Early	Late	1st	2nd	3rd
Total rounds	16,665	16,488	10,611	17,488	10,180	6,424
Surviving rounds	11,852	12,542	8,547	12,224	7,786	5,104
Survival probability	0.711	0.761	0.805	0.699	0.764	0.795
Surviving return mean	0.086	0.062	0.057	17,488	10,180	6,424
Surviving return median	0.056	0.039	0.028	12,224	7,786	5,104
Unconditional return mean	-0.459	-0.413	-0.315	0.699	0.764	0.795
Unconditional return median	0.022	0.015	0.015	0.699	0.764	0.795

Panel B: Probit analysis of survivals

Variables	Dummy for early and late	Dummy for late	Dummy for 2nd and 3rd	Dummy for 3rd
Coefficient	***0.208 (0.016)	***0.136 (0.019)	***0.256 (0.018)	***0.128 (0.024)

Table 7 Tests on Risk Neutrality (Constant Expected Returns)

Panel A of this table reports the *t*-statistics and corresponding *p*-values (in parentheses) for the equality of means of unconditional gross returns (post-financing gross returns for all non-exit rounds) per month (in log) between rounds. Panel B reports the results for Wilcoxon signed-rank tests for the equality of the medians of unconditional gross returns. Both tests use all return observations for each type of round. The return observations are not in pairs, and the numbers of return observations from different types of rounds are not necessarily equal. Negative *t*-statistics indicate that earlier rounds have lower means. *** denotes significance at the 1% level, ** at the 5% level, and * at 10% level.

Panel A. Mean equality tests for unconditional returns

	Early	Late		2nd Round	3rd Round
First/Seed	**-2.015 (0.044)	**-2.027 (0.043)	1st Round	***-4.831 (0.000)	***-6.120 (0.000)
Early	-	-1.388 (0.165)	2nd Round	-	**-2.026 (0.043)

Panel B. Median equality tests for unconditional returns

	Early	Late		2nd Round	3rd Round
First/Seed	*0.0555	0.399	1st Round	**0.021	***0.004
Early	-	**0.045	2nd Round	-	0.504

Table 8 Firm-matched Investment Volume, Investment Flow across Rounds

Panel A reports the means, medians and standard deviations of investment volume (million $) and investment flow (million $ per month) for individual rounds according to round types. This table only includes rounds from firms that have all three round statuses (left column) or at least three rounds (right column) before exit. Round types are determined according to round status (left column) and round sequence (right column) as in Table 7. Panel B reports tests for the equality of the means and medians for investment volume and flow across rounds for the firms. We first subtract the tested variable for later rounds from earlier rounds, and then use *t*-tests to test the null hypothesis that the means of the differences are positive (Wilcoxon signed-rank tests to test the null hypothesis that the medians of the differences are positive). Negative *t*-statistics indicate that earlier rounds have lower means. The reported numbers for the mean equality tests are *t*-statistics, and the reported numbers for the median equality tests are *p*-values. *** denotes significance at the 1% level, ** at the 5% level, and * at 10% level.

Panel A Summary statistics

Round Status				Round Order			
Variables	First	Early	Late	Variables	1st Round	2nd Round	3rd Round
Investment volume							
# of firms		3,336		# of firms		6,464	
mean	4.745	7.746	10.040	mean	4.751	6.653	8.806
median	2.195	4.000	4.815	median	2.000	3.000	3.750
Std. dev.	15.778	13.239	15.832	Std. dev.	15.217	15.251	16.936
Investment flow							
# of firms		3,049		# of firms		6,103	
mean	0.759	1.279	1.928	mean	0.802	1.059	1.452
median	0.242	0.444	0.603	median	0.227	0.333	0.404
Std. dev.	2.418	3.860	8.395	Std. dev.	3.292	2.893	4.921

Panel B. Equality tests

	Early	Late		2nd Round	3rd Round
Investment volume mean tests					
First/Seed	***-10.713	***-8.335	1st Round	***-8.764	***-9.552
Early	-	***-15.202	2nd Round	-	***-15.560
Investment volume median tests					
	Early	Late		2nd Round	3rd Round
First/Seed	***0	***0	1st Round	***0	***0
Early	-	***0	2nd Round	-	***0
Investment flow mean tests					
	Early	Late		2nd Round	3rd Round
First/Seed	***-7.218	***-4.001	1st Round	***-5.202	***-5.840
Early	-	***-7.499	2nd Round	-	***-9.105
Investment flow median tests					
	Early	Late		2nd Round	3rd Round
First/Seed	***0	***0	1st Round	***0	***0
Early	-	***0	2nd Round	-	***0

Table 9 Determinants of Firms' First Round Investment Flow

This table reports the regression results regarding the determinants of firms' first round investment flow.

$$log\left(flow_{i,1}\right) = \alpha + \beta_1 log\left(value_{i,1}\right) + \beta_2 log(exit.value_i) + \rho' dummy_i + \varepsilon_i$$

In the above equation, for firm i, $flow_{i,1}$ is the investment flow for round 1 (investment volume in round 1 divided by the number of months between round 1 and round 2), $value_{i,1}$ is the post-money valuation of the firm at round 1, $exit.value_i$ is the exit value of the firm (IPO market value minus capital raised from IPO, or post M&A value minus capital infused, or $1 for firms going out of business). The vector $dummy_i$ contains dummies for the business status (start up, in development, and in production) at the time of the first financing round, and for the industry (IT, health, and retail) of the firm. *Heteroskedasticity-robust* standard deviations are in parentheses. *** denotes significance at the 1% level, ** at the 5% level, and * at 10% level.

	Unconditional regressions (all firms)			Conditional on successful exits (IPO and M&A firms)		
	Regression 1	Regression 2	Regression 3	Regression 1	Regression 2	Regression 3
$value_{i,1}$	***0.855 (0.014)		***0.840 (0.016)	***0.801 (0.027)		***0.752 (0.0350
$exit.value_i$		-0.001 (0.002)	-0.003 (0.002)		***0.330 (0.030)	***0.136 (0.032)
Sample size	4,122	12,467	2,946	1,429	1,817	806
R2	0.51	0.04	0.52	0.43	0.11	0.47

Table 10 Determinants of Round Investment Flow

This table reports the regression results regarding the determinants of the investment flow (investment volume divided by the duration between current and next rounds) for all non-exit rounds.

$$log(flow_{i,l}) = \alpha + \beta_1 AR_{i,l} + \beta_2 AR_{i,l}^2 + \beta_3 log(value_{i,l-1}) + \beta_4 log(flow_{i,l-1}) + \beta_5 log(sunkcost_{i,l-1}) + \rho' dummy_{i,l} + \varepsilon_{i,l}$$

In the above equation, for firm i, $flow_{i,l}$ is the investment flow for round l (investment volume in round l divided by the number of months between round l and round $l+1$), $AR_{i,l}$ is the average monthly abnormal return (gross return in log) between round $l-1$ and l, $value_{i,l-1}$ is the post-money valuation of the firm (million $) at round $l-1$, $sunkcost_{i,l-1}$ is the sum of all investment volume (million $) from the first round to round $l-1$. The vector $dummy_{i,l}$ contains dummies for the business status (start up, in development, and in production) at the time of the first financing round, for mezzanine rounds, and for the industry (IT, health, and retail) of the firm. *Heteroskedasticity-robust* standard deviations are in parentheses. *** denotes significance at the 1% level, ** at the 5% level, and * at 10% level.

	Regression 1	Regression 2	Regression 3	Regression 4	Regression 5
$AR_{i,l}$	***0.953 (0.096)	***0.868 (0.085)	***0.242 (0.088)	***0.769 (0.086)	**0.531 (0.088)
$AR_{i,l}^2$	***-0.264 (0.044)	***-0.279 (0.039)	***-0.181 (0.039)	***-0.234 (0.039)	**-0.225 (0.038)
$log(value_{i,l-1})$		***0.571 (0.017)			***0.253 (0.027)
$log(flow_{i,l-1})$			***0.525 (0.015)		***0.258 (0.025)
$log(sunkcost_{i,l-1})$				***0.547 (0.016)	0.144 (0.031)
Sample size	4,408	4,408	4,406	4,334	4,334
R2	0.07	0.27	0.27	0.27	0.31

Table 11 Determinants of Pre-exit Rounds for Successful Firms

This table reports the regression results regarding the determinants of the number of rounds before exit for all IPO and M&A firms.

$$log(rounds_i) = \alpha + \beta_1 log\left(\frac{exit.value_i}{value_{i,1}}\right) + \beta_2 AR_{i,2} + \beta_3 AR_{i,2}^2 + \beta_4 log(exit.value_i) + \rho' dummy_i + \varepsilon_i$$

In the above equation, for firm i, $rounds_i$ is the number of financing rounds before exit, $exit.value_i$ is the exit value (IPO market value minus capital raised from IPO or post M&A value minus capital infused), $value_{i,1}$ is the post-money valuation at round 1, $AR_{i,2}$ is the average monthly abnormal return (gross return in log) between round 1 and 2. The vector $dummy_i$ contains dummies for the business status (start up, in development, and in production) at the time of the first financing round, and for the industry (IT, health, and retail) of the firm. *Heteroskedasticity-robust* standard deviations are in parentheses. *** denotes significance at the 1% level, ** at the 5% level, and * at 10% level.

	Regression 1	Regression 2	Regression 3	Regression 4
$log\left(\frac{exit.value_i}{value_{i,1}}\right)$	***0.360 (0.046)	***0.455 (0.048)	***0.525 (0.061)	***0.626 (0.063)
$AR_{i,2}$		***-0.985 (0.325)		***-0.899 (0.321)
$AR_{i,2}^2$		**0.302 (0.120)		**0.287 (0.119)
$log(exit.value_i)$			***-0.302 (0.076)	***-0.325 (0.079)
Sample size	807	658	807	658
R2	0.21	0.24	0.22	0.25

Table 12 Determinants of Round Investment Volume

This table reports the regression results regarding the determinants of the investment volume for all non-exit rounds.

$$log(investment_{i,l}) = \alpha + \beta_1 AR_{i,l} + \beta_2 AR_{i,l}^2 + \beta_3 log(value_{i,l-1}) + \beta_4 log(investment_{i,l-1})$$
$$+\beta_5 log(sunkcost_{i,l-1}) + \rho' dummy_{i,l} + \varepsilon_{i,l}$$

In the above equation, for firm i, $investment_{i,l}$ is the investment volume (raised capital in million $) in round l, $AR_{i,l}$ is the average monthly abnormal return (gross return in log) between round $l-1$ and l, $value_{i,l-1}$ is the post-money valuation of the firm (million $) at round $l-1$, $sunkcost_{i,l-1}$ is the sum of all investment volume (million $) from the first round to round $l-1$. The vector $dummy_{i,l}$ contains dummies for the business status (start up, in development, and in production) at the time of the first financing round, for mezzanine rounds, and for the industry (IT, health, and retail) of the firm. *Heteroskedasticity-robust* standard deviations are in parentheses. *** denotes significance at the 1% level, ** at the 5% level, and * at 10% level.

	Regression 1	Regression 2	Regression 3	Regression 4	Regression 5
$AR_{i,l}$	***0.556 (0.080)	***0.433 (0.071)	***0.314 (0.069)	**0.335 (0.072)	***0.340 (0.069)
$AR_{i,l}^2$	***-0.179 (0.035)	***-0.172 (0.031)	***-0.111 (0.030)	***-0.132 (0.031)	***-0.123 (0.030)
$log(value_{i,l-1})$		***0.486 (0.015)			***0.217 (0.023)
$log(investment_{i,l-1})$			***0.539 (0.014)		***0.443 (0.025)
$log(sunkcost_{i,l-1})$				***0.462 (0.014)	**-0.059 (0.029)
Sample size	4,962	4,602	4,599	4,518	4,518
R2	0.04	0.23	0.28	0.23	0.33

Table 13 Determinants of Pre-exit Duration for Successful Firms

This table reports the regression results regarding the determinants of the duration from the first round to exit for IPO and M&A firms.

$$log(duration_i) = \alpha + \beta_1 AR_{i,2} + \beta_2 AR_{i,2}^2 + \beta_3 log(value_{i,1}) + \beta_4 log\left(\frac{investment_{i,1}}{value_{i,1}}\right) + \beta_5 log\left(\frac{exit.value_i}{value_{i,1}}\right) + \rho' dummy_i + \varepsilon_i$$

In the above equation, for firm i, $duration_i$ is the number of months from the first round to the exit, $AR_{i,2}$ is the average monthly abnormal return (gross return in log) between round 1 and 2, $value_{i,1}$ is the post-money valuation at round 1, $investment_{i,1}$ is the investment volume (million $) in round 1, $exit.value_i$ is the exit value (IPO market value minus capital raised from IPO or post M&A value minus capital infused). The vector $dummy_i$ contains dummies for the business status (start up, in development, and in production) at the time of the first financing round, and for the industry (IT, health, and retail) of the firm. *Heteroskedasticity-robust* standard deviations are in parentheses. *** denotes significance at the 1% level, ** at the 5% level, and * at 10% level.

	Regression 1	Regression 2	Regression 3	Regression 4	Regression 5
$AR_{i,1}$	***-1.061 (0.099)	***-1.150 (0.088)	***-1.067 (0.098)	***-1.500 (0.111)	***-1.429 (0.103)
$AR_{i,1}^2$	***0.271 (0.040)	***0.287 (0.036)	***0.274 (0.040)	***0.372 (0.041)	***0.360 (0.038)
$log(value_{i,1})$		***-0.284 (0.018)			***-0.260 (0.027)
$log\left(\frac{investment_{i,1}}{value_{i,1}}\right)$			***0.083 (0.031)		-0.015 (0.031)
$log\left(\frac{exit.value_i}{value_{i,1}}\right)$				***0.188 (0.017)	***0.067 (0.019)
Sample size	890	890	889	658	657
R2	0.25	0.41	0.25	0.040	0.49

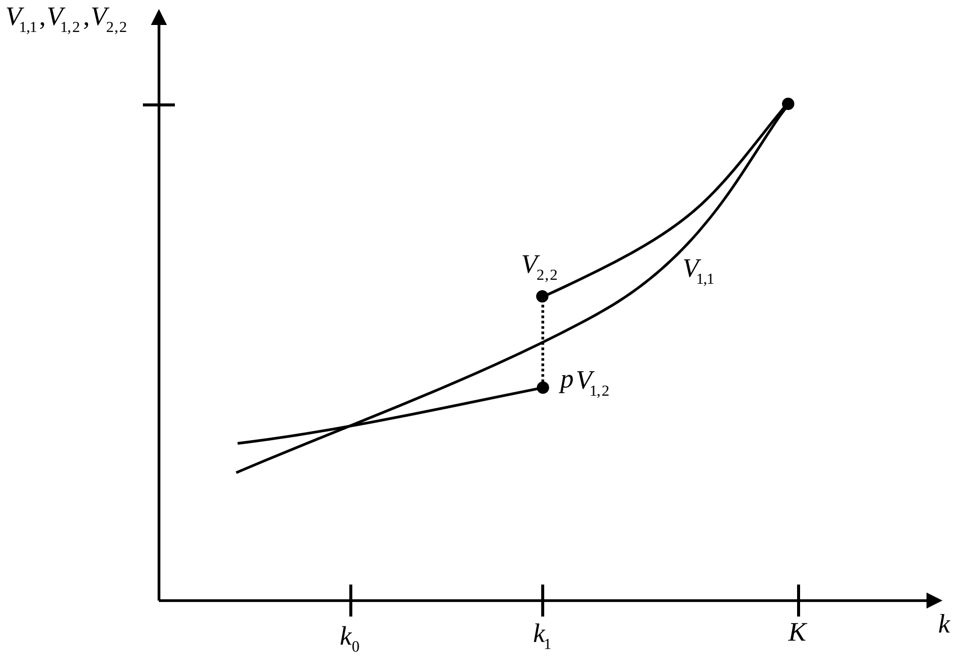

Figure 1: Staging and Valuation

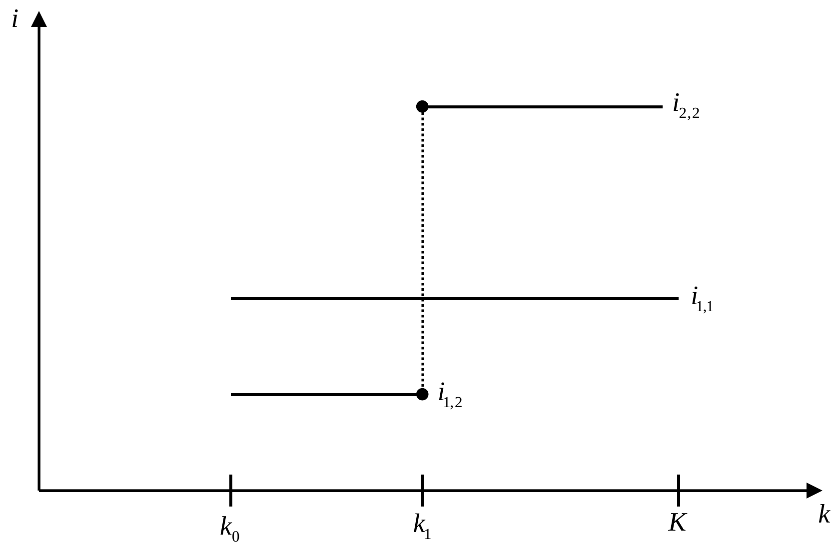

Figure 2: Staging and Investment Flow