Estimating Exchange Rate Equations Using Estimated Expectations

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Estimating Exchange Rate Equations Using Estimated Expectations

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Abstract

This paper takes a somewhat different approach from the recent literature in estimating exchange rate equations. It assumes uncovered interest rate parity and models how expectations are formed. Agents are assumed to base their expectations of future interest rates and prices, which are needed in the determination of the exchange rate, on predictions from a ten equation VAR model. The overall model is estimated by FIML under model consistent expectations. The model generally does better than the random walk model, and its properties are consistent with observed effects on exchange rates from surprise interest rate and price announcements. Also, the focus on expectations is consistent with the large observed short run variability of exchange rates.

1 Introduction

Exchange rate equations are not the pride of open economy macroeconomics. Although some results since the classic paper of Meese and Rogoff (1983) suggest that exchange rate equations can beat a random walk, the evidence is mixed and

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the general view still seems pessimistic.[1] This paper takes a somewhat different approach from the recent literature in estimating exchange rate equations. It assumes uncovered interest rate parity and models how expectations are formed. Agents are assumed to base their expectations on predictions from a ten equation VAR model, where the expectations are constrained to be model consistent in the manner discussed in Section 2. The overall model can be used to make predictions of the spot exchange rate, which can then be compared to predictions, for example, from the random walk model. It will be seen that the model generally does better than the random walk model.

The model is presented in Section 2; estimation is discussed in Section 3; and the estimates and prediction comparisons are presented in Section 4. Section 5 examines some of the properties of the model and compares these to the effects on exchange rates from surprise interest rate and price announcements. Four exchange rates are examined, all relative to the U.S. dollar: the Canadian dollar, the Japanese yen, the German mark, and the Australian dollar. The variables and notation used in this paper are presented in Table 1. Variables with an asterisk are U.S. variables.

[1]See, for example, Engel, Mark, and West (2007, p. 1), who provide pessimistic quotes from Samo and Taylor (2002), Bacchetta and van Wincoop (2006), and Evans and Lyons (2002). Engle, Mark, and West (2007) find some positive results for monetary exchange-rate models estimated by panel techniques, although they end their paper on a very cautious note.

Table 1
The Data Used

Raw Data: Canada, Japan, Germany, Australia		
S	=	Spot exchange rate, end of period, home currency per U.S. dollar (EE).
F	=	Three-month forward exchange rate, home currency per U.S. dollar (F).
R	=	Three-month interest rate, annual rate, percentage points (RS).
P	=	GDP deflator (PY).
Y	=	Real GDP (Y).
YS	=	Trend value of Y (YS).
PM	=	Import price deflator (PM).
CA	=	Current account (S).
Raw Data: United States		
R^*	=	Three-month Treasury bill rate, annual rate, percentage points (RS).
P^*	=	GDP deflator ($GDPD$).
Y^*	=	Real GDP ($GDPR$).
YS^*	=	Potential output (YS).
U^*	=	Civilian unemployment rate, percent (UR).
PM^*	=	Import price deflator (PIM).
CA^*	=	Current account (S_{US}).
Variables in the Model		
s	=	$\log S$
r	=	$\log(1 + R/400)$
p	=	$\log P$
u	=	$(YS - Y)/YS$
z	=	$\log PM$
b	=	$\log(1 + CA/(P \cdot YS))$
r^*	=	$\log(1 + R^*/400)$
p^*	=	$\log P^*$
u^*	=	UR^*
z^*	=	$\log PM^*$
b^*	=	$\log(1 + CA^*/(P^* \cdot YS^*))$

• The variable names in parentheses for Canada, Japan, Germany, and Australia are the variables in Table B.2 in Fair (2004). The variable names in parentheses for the United States are the variables in Table A.2 in Fair (2004) except for S_{US}, which is in Table B.5 in Fair (2004).
• The estimation periods are 1972:2–2004:3 for Canada and Japan, 1972:2–2004:4 for Australia, and 1972:2–1998:4 for Germany.

2 The Model

The Exchange Rate Equation

The uncovered interest rate parity condition is:[2]

$$S_t = q_t S^e_{t+1} \tag{1}$$

where (assuming that the data are quarterly and that the foreign country is the United States) S_t is the spot exchange rate, home currency per U.S. dollar (so an increase in S_t is a depreciation of the home currency), at the end of quarter t, S^e_{t+1} is the expected value of the spot exchange rate for the end of quarter $t+1$ made at the end of quarter t, and q_t is the relative interest rate variable. q_t equals $(1+R^*_t)/(1+R_t)$, where R_t is the three-month home-country interest rate observed at the end of quarter t and R^*_t is the three-month U.S. interest rate observed at the end of quarter t.

Solving equation (1) forward $m-1$ quarters yields:

$$S_t = q_t q^e_{t+1} \cdots q^e_{t+m-1} S^e_{t+m} \tag{2}$$

Assuming that agents in fact solve equation (1) forward $m-1$ times and that they have expectations of $q_{t+1} \cdots q_{t+m-1}$ and an expectation of S_{t+m}, equation (2) determines the exchange rate for quarter t. In the empirical work various values of m were tried to see which led to the best fit.

[2] Uncovered interest rate parity is often rejected in empirical work. Cochrane (2001, pp. 430–434) has a nice review and update of this work. Slightly more positive results than those reviewed by Cochrane are discussed in Bekaert, Wei, and Xing (2007). In this paper uncovered interest rate parity is assumed but not tested. It is the case for the data used in this paper that equation (1) holds almost exactly when S^e_{t+1} is replaced by the forward rate for quarter $t+1$ observed at the end of quarter t, F_{t+1} in Table 1. This paper is assuming in effect that F_{t+1} correctly measures the expected future spot rate for quarter $t+1$. The data on F are not needed in this paper.

Regarding S^e_{t+m} in equation (2), it is assumed that agents expect purchasing power parity (PPP) to hold in the long run, where S^*_{t+n} will be used to denote the expected long run value of the exchange rate, and that agents expect there is a gradual adjustment to PPP. In particular, it is assumed that

$$S^e_{t+m} = S^{*\lambda}_{t+n} S^{(1-\lambda)}_{t-1} e^{\mu_{1t+1}}, \quad 0 < \lambda \leq 1 \tag{3}$$

The error term, μ_{1t+1}, reflects all the factors that affect S^e_{t+m} aside from S^*_{t+n} and S_{t-1}. The PPP assumption is

$$S^*_{t+n} = \alpha \rho^e_{t+n} \tag{4}$$

where ρ^e_{t+n} is the expected relative price level for quarter $t+n$. ρ equals P/P^*, where P is the price level of the home country and P^* is the price level of the United States. Equation (4) states that agents take (at the end of quarter t) the long run (PPP) value of the exchange rate for quarter $t+n$ to be proportional to the expected relative price level for quarter $t+n$. As was the case for m, in the empirical work various values of n were tried to see which led to the best fit.

In order to complete the above specification, a model is needed of how expectations of the future relative interest rates and the future relative price level are formed. Agents are assumed to use the VAR equations discussed below for this purpose. To summarize, then, there are three assumptions about agents in the model other than the assumption that they use the VAR equations. The first is that they solve equation (1) forward $m-1$ quarters—equation (2). If, for example, m is 3, then given q_t (which is observed at the end of quarter t), given expectations for q_{t+1} and q_{t+2}, and given an expectation for S_{t+3}, the spot rate for quarter t must be the expected rate for quarter $t+3$ times the product of the three relative interest rates if uncovered interest rate parity holds. The second assumption is that agents take the long run value of the exchange rate to be proportional to the expected relative price level for quarter $t+n$—equation (4). The third assumption is that agents expect there is a gradual adjustment to PPP—equation (3)—where λ is the speed of adjustment.

Combining equations (2)–(4) yields;

$$\log S_t = \log q_t + \log q^e_{t+1} + \cdots + \log q^e_{t+m-1} + \lambda \log \alpha + \lambda \log \rho^e_{t+n} + (1-\lambda) \log S_{t-1} + \mu_{1t+1} \tag{5}$$

Using the notation in Table 1 and letting $\beta = \lambda \log \alpha$, equation (5) is

$$s_t = \log q_t + \log q^e_{t+1} + \cdots + \log q^e_{t+m-1} + \beta + \lambda \log \rho^e_{t+n} + (1-\lambda) s_{t-1} + \mu_{1t+1} \tag{6}$$

The VAR Equations

In order to estimate equation (6), expected values of q are needed for quarters $t+1$ through $t+m-1$ and of ρ for quarter $t+n$. Agents are assumed to use the following VAR equations to generate these expectations. There are five variables per country: the three-month interest rate, the price level, a gap variable, the import price level, and the current account as a percent of GDP. The variables are listed in Table 1. The right hand side variables in each equation include a constant term, a time trend, s_{t-1}, s_{t-2}, and two lags of each of the ten variables:

$$\begin{aligned} r_{t+1} &= f_2(cnst, t, s_{t-1}, s_{t-2}, r_t, r_{t-1}, p_t, p_{t-1}, u_t, u_{t-1}, z_t, z_{t-1}, b_t, b_{t-1}, \\ &\quad r^*_t, r^*_{t-1}, p^*_t, p^*_{t-1}, u^*_t, u^*_{t-1}, z^*_t, z^*_{t-1}, b^*_t, b^*_{t-1}) + \mu_{2t+1} && (7) \\ p_{t+1} &= f_3(\ldots) + \mu_{3t+1} && (8) \\ u_{t+1} &= f_4(\ldots) + \mu_{4t+1} && (9) \end{aligned}$$

$$
\begin{aligned}
z_{t+1} &= f_5(\ldots) + \mu_{5t+1} && (10)\\
b_{t+1} &= f_6(\ldots) + \mu_{6t+1} && (11)\\
r^*_{t+1} &= f_7(\ldots) + \mu_{7t+1} && (12)\\
p^*_{t+1} &= f_8(\ldots) + \mu_{8t+1} && (13)\\
u^*_{t+1} &= f_9(\ldots) + \mu_{9t+1} && (14)\\
z^*_{t+1} &= f_{10}(\ldots) + \mu_{10t+1} && (15)\\
b^*_{t+1} &= f_{11}(\ldots) + \mu_{11t+1} && (16)
\end{aligned}
$$

The f_i functions are assumed to be linear, $cnst$ denotes the constant term, and t denotes the time trend.

The present approach does not depend on this particular VAR model. The model that one is after is not necessarily the model that best approximates the economy. One wants the model that best approximates what agents actually use in forming their expectations, and agents may use something simpler than the best model of the economy. In future work it may be interesting to experiment with other models. As discussed in the Conclusion, the model used need not be linear and can include model-consistent future expectations as explanatory variables.

The present VAR model assumes that agents use data for each country on 1) the short term interest rate, 2) the domestic price level, 3) a measure of demand pressure (unemployment rate for the United States and output gap for the other countries), 4) a cost shock variable as measured by the price of imports, and 5) a current account variable. All variables are assumed to be trend stationary, and a time trend is added to the equations because the domestic price level and the price of imports have trends.[3]

[3]If some of the variables are not trend stationary, the estimated asymptotic standard errors may be poor approximations to the true standard errors. One way to examine the accuracy of asymptotic distributions is to use a bootstrap procedure, and an example of this is in Fair (2004, Chapter 9).

The overall model is closed with the two identities:

$$\log q_t = r_t^* - r_t \tag{17}$$

$$\log \rho_t = p_t - p_t^* \tag{18}$$

Solution

The complete model consists of equations (6)–(18). The timing implicit in the model is the following. All expectations for quarters $t+1$ and beyond are formed at the end of quarter t. All variables with a subscript t are assumed to be observed at the end of quarter t except for s_t. In particular, note that q_t is assumed to be known at the end of quarter t. The VAR equations generate predictions for quarters $t+1$ and beyond, based on information through quarter t. Also, certainty equivalence is assumed in moving from equations (2)–(4) to equation (6). Some results are presented in Fair (2004, Chapter 10) that suggest that this assumption may not be a bad approximation in macroeconometric work, but no direct test of this assumption is made in this paper. If, however, one begins with equation (6), the model (6)–(18) is linear in expectations, and so the certainty equivalence assumption is not needed.

The model (6)–(18) cannot be solved in the usual way—quarter by quarter—because of the expected future values in equation (6). Because the equations are linear in logs, they can be solved by linear techniques. They can also, however, be solved by the extended path (EP) method in Fair and Taylor (1990), and this method has been used in the computation of the FIML estimates below. Although

The results in this chapter suggest that for the kind of macro time series variables examined in this paper the estimated asymptotic standard errors are fairly accurate.

the EP method is computationally more expensive, it can handle nonlinear models, and so in future work there is no need to restrict the model to be linear.

Given a set of coefficient estimates and assuming zero values for all current and future error terms, the model can be solved for quarter $t+1$ by the EP method as follows. First, guess a path for the future interest rate and price expectations, where in the present case the first expectation that is relevant for the price levels are for quarter $t+n$. Assume that the path extends to quarter $t+k$, where k is considerably larger than n. Then take these expectations as fixed and solve the model dynamically in the usual way through quarter $t+k-n+1$. (The solution values for each quarter can be obtained by iteration using the Gauss-Seidel method.) The solution values through quarter $t+k-n+1$ of the interest rate and price variables can be taken as new guesses, and the model can be solved again taking these guesses as given. Preliminary convergence is reached when the solution values from one iteration to the next are within some prescribed tolerance level. This convergence is only preliminary because the initial guesses for quarters $t+k-n+2$ through $t+k$ have not been changed. The next step is to increase k by one and solve again. Overall convergence is reached when increasing k by one more has a small effect (i.e., within some prescribed tolerance level) on the solution values for quarter $t+1$. There is no guarantee that convergence will be achieved, although for the work in this paper convergence was always achieved.

Note that the solution value of s for quarter t affects the solution values from the VAR equations for quarters $t+2$ and beyond (because s_{t-1} and s_{t-2} appear in the equations), which in turn affects s_t through the future predicted values of the interest rates and price levels. When overall convergence is achieved, the solution value of s_t is model consistent in that it is consistent with the predicted future values of the interest rates and price levels.

In computing root mean squared errors in Section 4 and in examining the properties of the model in Section 5, predictions of s are generated for quarters beyond t. These predictions are based only on information available at the end of t. In fact, in the process of solving the model for t, predictions for $t + 1$ and beyond are needed using the EP method. If k in the solution method is taken to be large enough, then the predictions for $t + 1$ and beyond that are used to compute the predictions for t can be used without further calculations. The main point to realize is that no information beyond the end of quarter t is used for the future predictions.

Discussion

The "fundamental" that is driving the exchange rate is the expected future relative price level. Agents expect there to be PPP in the long run. They use the VAR equations to form expectations of the future price levels of the two countries. Anything that changes these expectations changes the expected long run value of the exchange rate, which changes the spot rate. Expectations of future interest rates also affect the spot rate through equation (2).

This way of modeling exchange rate determination is different from traditional exchange rate models. In "asset" models of exchange rate determination of the kind examined by Meese and Rogoff (1983), the exchange rate is on the left hand side and relative money supply, real output, and interest rate variables are on the right hand side. In a more general version of this model examined in Engel, Mark, and West (2007, equation (7), p. 6) the exchange rate is on the left hand side and various current and expected future macroeconomic variables are on the right hand side. Many macroeconomic variables thus directly affect the exchange rate in these models. In the model in this paper many macroeconomic variables also affect the exchange rate, but they do so by affecting agents expectations of the future interest rates and price levels through the VAR equations (or, more generally, through whatever model the agents are assumed to use). The focus here is on estimating how agents form their expectations.

3 Estimation

There are 240 coefficients to estimate in the VAR equations and 2 coefficients to estimate in equation (6), β and λ. Under the assumption that the errors terms $\mu_{1t+1}, \ldots, \mu_{11t+1}$ are jointly normally distributed with zero means and some covariance matrix Σ, the 242 coefficients can be estimated by full information maximum likelihood (FIML). The FIML estimation of nonlinear models with rational expectations is discussed in Fair and Taylor (FT) (1990), and the procedure presented in this paper has been used to obtain the FIML estimates. This procedure handles very general problems, although it is computationally intensive. It is roughly as follows. First, for a given set of coefficients the model can be solved for each quarter of the estimation period using the EP method. If there are T observations, then there are T overall solutions using the EP method. Each overall solution for a quarter is conditional on all actual lagged values. In other words, the solution for quarter t is based on information through quarter $t-1$. Once the model has been solved for the T quarters for the given set of coefficients, the value of the likelihood function can be computed because the error terms can be computed. One maximum likelihood function evaluation thus requires T uses of the EP method, where each use requires solving the model for many quarters into the future.

Once a procedure is available for computing the value of the likelihood function for a given set of coefficients, the estimation problem can be turned over to a nonlinear maximization algorithm. These algorithms search over sets of coefficients to find the set that maximizes the objective function. For the FIML estimation of large models the algorithm that I have found best to use is the Parke (1982) algorithm, and it has been used for the work below.

In the FIML estimation of rational expectations models most of the solution time is spent solving the model. The extra calculations that, say, the Parke algorithm takes once the value of the likelihood function has been computed are trivial. Therefore, one way to estimate the computational cost is to count the number of times the equations are "passed through" using the Gauss-Seidel method. One pass through is simply calculating once the left hand side values of the equations for a given set of right hand side values for a single quarter. The number of "pass throughs" required for a typical estimation problem is discussed in the next section.

Finally, note that actual data are not needed beyond the end of the estimation period, because no simulation begins after the end. Any solution values that are needed beyond the end of the estimation period are computed dynamically.

4 The Results

Computational Issues

The sources for the data are presented in Table 1. The estimation periods begin in 1972:2, which is roughly the beginning of flexible exchange rates. The estimation period ends in 2004:3 for Canada and Japan, 2004:4 for Australia, and 1998:4 for Germany. For Germany 1998:4 was the last quarter before the introduction of the euro. There are thus 130 observations for Canada and Japan, 131 observations for Australia, and 107 observations for Germany. The covariance matrix of the error terms for each country, Σ, is 11×11. The covariance matrix of the coefficient estimates, which will be denoted V, is 242×242. V is the inverse of the matrix of the second derivatives of the log of the likelihood function.

Consider the estimation problem for Canada. (The estimation problems for the other countries are similar.) There are 242 coefficients to estimate and 130 observations. The estimates were obtained using the Fair-Parke (FP) (2003) program, a FORTRAN program. This program uses the EP method to solve the model and the Parke algorithm to compute the FIML estimates. The number of iterations that the Parke algorithm takes to converge depends on the starting point and the tolerance level. A typical run to estimate the 242 coefficients takes about 300 iterations, which requires about 280,000 evaluations of the likelihood function. (These evaluations include those needed to compute the covariance matrix V numerically.) The number of "pass throughs" for 280,000 evaluations is about 9.75 billion. The time taken for this many pass throughs on a computer with a 2.7 Ghz chip is about

27 hours.

The full estimation problem thus requires about a day of computer time, which makes it costly to search, say, over different values of m and n. In addition, the numerical computation of V does not always result in a positive definite matrix, due probably to tolerance issues and rounding errors. An alternative procedure that was followed for much of the estimation was to fix the coefficients in the VAR equations except the constant terms at their OLS estimates and estimate only β, α, and the ten constants. No restrictions were placed on Σ. This procedure will be called "restricted" estimation. It always resulted in a positive definite matrix for the restricted V, which is 12×12. It is obviously must faster, taking usually less than an hour of computer time on the 2.7 Ghz chip computer. Unless stated otherwise, the estimation discussed below is the restricted estimation.

Coefficient Estimates

In the first stage of the estimation work for each country values of n of 5, 9, and 13 were tried and values of m of 2, 3, and 4 were tried. The value of the likelihood function was recorded for each estimation. For all four countries $m = 2$ resulted in the largest likelihood function value. This means that agents are estimated to solve equation (1) forward only one quarter. For all but Canada $n = 9$ resulted in the largest likelihood function value. For Canada the maximum was at $n = 5$. The likelihood functions were well behaved in the sense that the maximum for n was independent of the value used for m and the maximum for m was independent of the value used for n.

Table 2
Coefficient Estimates for Equation (6)

	Restricted Estimation				**Full Estimation**	
	$\hat{\beta}$	$\hat{\lambda}$	χ^2	p-value	$\hat{\beta}$	$\hat{\lambda}$
Canada	.020	.055	1.710	.191	.021	.061
	(5.09)	(3.20)				
Japan	-.080	.032	8.105	.004	-.055	.020
	(-2.04)	(1.64)				
Germany	-.018	.092	0.017	.896	-.017	.082
	(-2.86)	(3.20)				
Australia	.039	.064	22.607	.000	.031	.043
	(3,84)	(2.62)				

- n is 5 for Canada and 9 for the other countries.
- m is 2 for all four countries.
- t-statistics are in parentheses.
- The χ^2 test is of the restriction that the coefficients of ρ^e_{t+n} and s_{t-1} in equation (6) sum to one.
- The estimation periods are 1972:2–2004:3 for Canada and Japan, 1972:2–2004:4 for Australia, and 1972:2–1998:4 for Germany.
- See Table 1 for a description of the data.

The estimates are presented in Table 2. They are based on $m = 2$ for all four countries and $n = 5$ for Canada and $n = 9$ for the other three. Only the estimates for β and λ are presented. The χ^2 test in the table is a test of the hypothesis that the coefficients of $\log \rho^e_{t+n}$ and s_{t-1} sum to one.[4] Coefficient estimates from the full estimation are also presented in Table 2. They do not have t-statistics because the estimates of the full V were unreliable.

The estimates of λ in Table 2 are significant at conventional levels except for Japan, where the estimate is .032 and the t-statistic is 1.64. The estimates are small, ranging from .032 to .092, and thus show a slow adjustment to PPP. The

[4] If L is the value of the log of the likelihood function in the restricted case and L^* is the value in the unrestricted case, $2(L^* - L)$ is distributed as χ^2 with one degree of freedom. This test simply requires reestimating the model without the summation restriction imposed.

summation restriction is rejected for Japan and Australia, but not for Canada and Germany. The full estimation estimate of λ is larger than the restricted estimation estimate for Canada and smaller for the others. In general the full and restricted estimates are fairly close. The estimation results are thus supportive of the model in the sense that the estimates of λ are significant or nearly significant and the estimates are similar in size across countries.

Root Mean Squared Errors

Once the model is estimated, it can be used to make predictions of the exchange rate, and these can be compared to predictions from the random walk model. Various root mean squared errors (RMSEs) are presented in Table 3. The prediction period considered is 1990:1–2004:4 (60 quarters) for Australia, 1990:1–2004:3 (59 quarters) for Canada and Japan, and 1990:1–1998:4 (36 quarters) for Germany. Four sets of results per country are presented in Table 3. The first uses the full estimates in Table 2 (along with the full estimates of the VAR coefficients). The second uses the restricted estimates in Table 2 (along with the OLS estimates of the VAR coefficients except for the constant terms, which are not restricted). The RMSEs for these two sets are within sample because the prediction periods are within the estimation periods.

The RMSEs for the third set are outside sample. They are based on rolling estimates. For the first estimates the estimation period ended in 1989:4 (all estimation periods begin in 1972:2). These estimates were used for the prediction period that began in 1990:1. For the second estimates the estimation period ended in 1990:1,

Table 3
Root Mean Squared Errors
Percentage Points

		Quarters ahead		
		1	4	8
Canada	Within sample—full	2.65	4.99	6.63
	Within sample–restricted	2.72	5.42	7.51
	Outside sample	2.80	5.86	9.65
	Random walk	2.81	6.24	9.29
Japan	Within sample—full	6.03	9.73	12.42
	Within sample—restricted	6.05	10.16	13.94
	Outside sample	6.29	12.32	20.26
	Random walk	6.25	11.83	18.57
Germany	Within sample—full	5.69	9.24	9.99
	Within sample—restricted	5.67	9.23	10.11
	Outside sample	5.77	9.26	11.64
	Random walk	5.92	10.16	12.70
Australia	Within sample—full	4.85	9.94	13.79
	Within sample—restricted	4.79	9.73	13.17
	Outside sample	4.94	10.56	16.12
	Random walk	5.04	11.53	18.03

- The prediction periods are 1990.1–2004:3 for Canada and Japan, 1990.1–2004:4 for Australia, and 1990.1–1998:4 for Germany.
- Number of observations for one-, four-, and eight-quarters ahead: Canada and Japan, 59, 56, 52, Australia, 60, 57, 53, Germany, 36, 33, 29.

and they were used for the prediction period that began in 1990:2. For the last estimates the estimation period ended in 2004:3 for Australia, 2004:2 for Canada and Japan, and 1998:3 for Germany. The calculations for these RMSEs required estimating the model 60 times for Australia, 59 times for Canada and Japan, and 36 times for Germany. All these estimates were restricted FIML estimates. Full estimation would have required many months.

The random walk model is $S_t = S_{t-1}$. The fourth set of RMSEs in Table 3 is for this model. The predictions from the random walk model are all in effect outside sample since there are no coefficients estimated.

One-quarter-ahead, four-quarter-ahead, and eight-quarter-ahead RMSEs are presented in Table 3. For Canada and Japan there are 59 one-quarter-ahead predictions, 56 four-quarter-ahead predictions, and 52 eight-quarter-ahead predictions. For Australia, the respective numbers are 60, 57, and 53, and for Germany they are 36, 33, and 29.

Comparing within sample—full to within sample—restricted, the full RMSEs are slightly smaller for Canada and Japan, about the same for Germany, and slightly larger for Australia. In general the full and restricted RMSEs are fairly close, and so the predictions of the model are not sensitive to full versus restricted estimation.

All the within sample RMSEs are noticeably smaller than the random walk RMSEs. The outside sample RMSEs are smaller than the random walk RMSEs for Germany and Australia and larger for Japan. For Canada the outside sample RMSE is the same for one-quarter-ahead, smaller for four-quarters-ahead, and larger for eight-quarters-ahead. The overall RMSE results are thus somewhat in favor of the present model over the random walk model. It is also possible that the results would be even more favorable to the present model if full estimation were feasible. In other words, given that the within sample—full RMSEs for Canada and Japan are smaller than the within sample—restricted RMSEs, it may be that Canada and Japan would also have lower outside sample RMSEs than the random walk model had the rolling estimates been done using full estimation.

5 Properties of the Model

Two Experiments

Two experiments were performed per country using the restricted estimates in Table 2. First, the model was solved for s for the period beginning in $t = 1992{:}1$ with all the current and future error terms set to zero. Call this the "base" solution. Then for the first experiment the error term in equation (7)—the equation determining the interest rate for the home country—was taken to be .005 in $t+1 = 1992{:}2$ and zero otherwise. All other error terms were still set to zero. The model was then solved again for s for the period beginning in $t = 1992{:}1$. Call this the "r shock" solution. This shock is roughly a .5 percentage point shock to the interest rate. For the second experiment the error term in equation (8)—the equation determining the price level for the home country—was taken to be .01 in $t+1 = 1992{:}2$ and zero otherwise. All other error terms were still set to zero. The model was then solved again for s for the period beginning in $t = 1992{:}1$. Call this the "p shock" solution. This shock is roughly a one percentage point shock to the price level.

The results are presented in Table 4.[5] Each value in the table is the predicted value from the shocked solution minus the predicted value from the base solution times 100 (to put the values in percentage points). The variables are s, $\log q$, and $\log \rho$. Values are presented for 8 quarters for s, for 9 quarters for $\log q$, and for 17 quarters for $\log \rho$ (13 quarters for Canada). Remember that the predicted value for s for a given quarter depends on the predicted value of $\log q$ for one quarter ahead

[5]Because the model is linear (in logs), the results in Table 4 do not depend on the particular starting quarter, 1992:1, used. Any quarter will give the same results.

Table 4
Effects of an Interest Rate Shock and a Price Shock
Values in Percentage Points

			Canada			
		r **shock**			p **shock**	
Quarter	s	$\log q$	$\log \rho$	s	$\log q$	$\log \rho$
1992:1	-0.52	0.00	0.00	0.04	0.00	0.00
1992:2	-1.41	-0.50	0.00	0.06	0.00	1.00
1992:3	-2.00	-0.40	-0.10	0.01	-0.02	1.06
1992:4	-2.31	-0.24	-0.36	-0.06	-0.05	0.95
1993:1	-2.46	-0.16	-0.42	-0.11	-0.05	0.84
1993:2	-2.51	-0.10	-0.38	-0.13	-0.03	0.75
1993:3	-2.50	-0.07	-0.34	-0.15	-0.02	0.66
1993:4	-2.45	-0.05	-0.29	-0.19	-0.02	0.58
1994:1		-0.03	-0.26		-0.04	0.50
1994:2			-0.25			0.43
1994:3			-0.26			0.37
1994:4			-0.28			0.32
1995:1			-0.30			0.27
			Australia			
		r **shock**			p **shock**	
Quarter	s	$\log q$	$\log \rho$	s	$\log q$	$\log \rho$
1992:1	-0.44	0.00	0.00	0.08	0.00	0.00
1992:2	-1.23	-0.50	0.00	0.12	0.00	1.00
1992:3	-1.68	-0.37	0.18	0.05	-0.04	1.01
1992:4	-1.86	-0.21	0.42	-0.11	-0.10	1.03
1993:1	-1.93	-0.14	0.54	-0.29	-0.13	1.06
1993:2	-1.95	-0.11	0.61	-0.45	-0.13	1.12
1993:3	-1.95	-0.09	0.67	-0.56	-0.11	1.19
1993:4	-1.90	-0.07	0.74	-0.62	-0.09	1.25
1994:1		-0.05	0.81		-0.07	1.27
1994:2			0.88			1.27
1994:3			0.94			1.23
1994:4			0.96			1.17
1995:1			0.95			1.10
1995:2			0.90			1.02
1995:3			0.82			0.94
1995:4			0.72			0.87
1996:1			0.61			0.80

Table 4 (continued)
Effects of an Interest Rate Shock and a Price Shock
Values in Percentage Points

Japan

	r **shock**			p **shock**		
Quarter	s	$\log q$	$\log \rho$	s	$\log q$	$\log \rho$
1992:1	-0.49	0.00	0.00	0.00	0.00	0.00
1992:2	-1.57	-0.50	0.00	-0.04	0.00	1.00
1992:3	-2.61	-0.62	0.16	-0.15	-0.03	0.96
1992:4	-3.38	-0.49	0.36	-0.32	-0.07	0.89
1993:1	-3.96	-0.39	0.45	-0.49	-0.09	0.77
1993:2	-4.37	-0.32	0.42	-0.61	-0.07	0.63
1993:3	-4.63	-0.25	0.38	-0.66	-0.04	0.47
1993:4	-4.76	-0.19	0.36	-0.65	-0.01	0.30
1994:1		-0.13	0.36		0.02	0.14
1994:2			0.40			-0.02
1994:3			0.48			-0.17
1994:4			0.60			-0.31
1995:1			0.75			-0.43
1995:2			0.90			-0.52
1995:3			1.03			-0.60
1995:4			1.13			-0.65
1996:1			1.18			-0.68

Germany

	r **shock**			p **shock**		
Quarter	s	$\log q$	$\log \rho$	s	$\log q$	$\log \rho$
1992:1	-0.46	0.00	0.00	-0.002	0.00	0.00
1992:2	-1.31	-0.50	0.00	-0.08	0.00	1.00
1992:3	-1.96	-0.42	0.08	-0.13	-0.08	0.53
1992:4	-2.47	-0.37	0.22	-0.08	0.02	0.59
1993:1	-2.84	-0.33	0.34	0.01	0.02	0.45
1993:2	-3.07	-0.27	0.40	0.15	0.06	0.33
1993:3	-3.18	-0.22	0.45	0.31	0.07	0.22
1993:4	-3.17	-0.16	0.45	0.46	0.08	0.12
1994:1		-0.09	0.44		0.07	0.03
1994:2			0.41			-0.02
1994:3			0.35			-0.05
1994:4			0.27			-0.05
1995:1			0.17			-0.03
1995:2			0.06			0.02
1995:3			-0.07			0.10
1995:4			-0.20			0.19
1996:1			-0.33			0.29

- Interest rate shock was 0.5 percentage points.
- Price shock was 1.0 percentage points.
- s = log of the spot exchange rate.
- $\rho = (1 + R^*)/(1 + R)$, where R is home country's interest rate and R^* is U.S. interest rate.
- $q = P/P^*$, where P is home country's price level and P^* is U.S. price level.

and the predicted value of $\log \rho$ for 9 quarters ahead (5 quarters for Canada)—equation (6).

The results for the r shock experiment are similar across countries. The exchange rate appreciates—from 0.44 percent after one quarter for Australia to 0.52 percent for Canada and from 1.90 percent after eight quarters for Australia to 4.76 percent for Japan. The relative interest rate variable, $\log q$, decreases, which means that the home country's interest rate increases relative to the U.S. interest rate. This decrease in $\log q$ is, of course, what is driving the appreciation. For Canada the relative price level variable, $\log \rho$, decreases. This is what one might expect the VAR equations to show, namely that an increase in Canada's interest rate relative to the U.S. interest rate and the corresponding appreciation of the exchange rate lead to the price level in Canada decreasing relative to the price level in the United Sates. The fall in $\log \rho$ in turn leads to the Canadian dollar appreciating more than otherwise. For the other countries, however, $\log \rho$ increases (except for Germany for 1995:3 and 1995:4), which, other things being equal, mitigates the appreciation. The VAR equations have not been constrained in any way, and it turns out that for Australia, Japan, and Germany they have the property that an increase in the home country's relative interest rate leads to an increase in its relative price level. The main result of this experiment, however, is that the interest rate effect dominates the price effect and so there is an appreciation following a positive interest rate shock.

The results for the p shock experiment vary somewhat across countries. Consider Canada and Australia first. For both countries there is an initial depreciation of the exchange rate and then an appreciation beginning four quarters out. For both countries the relative price level increases, and this has a depreciating effect on the exchange rate. However, the price shock also leads to a decrease in $\log q$ (an increase in the home country's interest rate relative to the U.S. interest rate), and this has an appreciating effect on the exchange rate. The net effect is that beginning four quarters out there is an appreciation of the exchange rate. The interest rate effect thus dominates the price effect beginning four quarters out.

For Japan the exchange rate appreciates in all quarters. One reason for this is that $\log q$ decreases (until 1994:1), which has an appreciating effect. In addition, the price level decreases beginning ten quarters out, which also has an appreciating effect on the exchange rate from the second quarter on. So for Japan the interest rate effect and the price effect are working in the same direction because the VAR equations predict that the initial increase in the Japanese relative price level is reversed ten quarters out.

For Germany the exchange rate appreciates for the first four quarters and depreciates after that. Ten through thirteen quarters out the relative price level decreases, which has an appreciating effect on the exchange rate for quarters two through five. Then beginning in 1995:2 the relative price level increases, which has a depreciating effect on the exchange rate for quarters five on. $\log q$ decreases for quarters three on, which has a depreciating effect on the exchange rate. The interest rate effect and the price effect are thus working in the same direction for quarters five on for Germany.

To summarize, the results of the r shock are easy to describe. The increase in the home country's interest rate relative to the U.S. interest rate leads to an appreciation. The relative price level may increase or decrease, but if it does increase, the depreciating effect from the increase is not large enough to offset the appreciating effect from the interest rate increase. The results of the p shock are more complicated. For two countries there is an initial depreciation, and for the other two there is an initial appreciation. From five quarters on there is an appreciation for three countries and a depreciation for the other. The results depend on what the VAR equations predict for the relative interest rate and relative price level, and these predictions are different across countries. It is clear for Canada and Australia, however, that the interest rate effect dominates the price effect after three quarters in that the exchange rate appreciates even though the relative price level increases.

Comparison to Surprise Announcement Effects

The properties just described are consistent with the responses of exchange rates to surprise announcements about interest rates and prices. In Fair (2003) I searched, using tick data on stock and bond prices and exchange rates, for announcements and events that led to large changes in prices within five minutes. The period examined was 1982–2000, and news wires were used for the searches. 221 announcements and events were found that led to large five minute changes in at least one of the five variables examined. The five variables were the S&P 500 stock price index, the 30-year U.S. Treasury bond price, and three exchange rates. The three exchange rates were the U.S. dollar relative to the Deutsche mark or euro, the Japanese yen, and the British pound.

Consider a Fed announcement about its choice for the federal funds rate that led to a large change in the bond price in absolute value within five minutes after the announcement. The above properties suggest that if the bond price increased (a decrease in the U.S. bond rate), the U.S. dollar should depreciate. This is a negative interest rate shock. Conversely, if the bond price decreased, the U.S. dollar should appreciate, a positive interest rate shock. Table 3 in Fair (2003) lists all 221 announcements and events and their five minute effects. There are 11 relevant federal funds announcements in this table. Of these 11, 8 showed the dollar appreciating against all three currencies when the interest rate rose and depreciating against all three currencies when the interest rate fell, as expected from the model.[6]

Consider now price announcements. In Table 3 in Fair (2003) there are 19 surprise price announcements—either the consumer price index or the producer price index—that led to large changes in the bond rate and the exchange rates. In all cases a positive price surprise led to an increase in the bond rate and a negative price surprise to a decrease. The interest rate effect and the price effect are thus working in opposite directions regarding exchange rate changes. One would thus expect from the properties of the model that the effect on the exchange rates could go either way, with perhaps the interest rate effect dominating more often. This is in fact the case. Of the 19 announcements, 5 had the price effect dominating and 14 had the interest rate effect dominating.[7]

[6]The 8 events are 29, 85, 90, 157, 180, 192, 197, and 216. The 3 others are 127, 175, and 207. (Fair (2003), Table 3, pp. 323–324.)

[7]The 5 price dominating announcements are 124, 125, 142, 161, and 210. The 14 interest rate dominating announcements are 57, 59, 61, 64, 69, 72, 83, 92, 93, 108, 115, 120, 148, and 155. (Fair (2003), Table 3, pp. 324–325.)

6 Conclusion

This paper models exchange rate determination by assuming 1) uncovered interest rate parity, 2) agents solve the parity condition forward (one quarter in the empirical results), 3) agents expect there is a gradual adjustment to PPP, 4) agents take the long run (PPP) value of the exchange rate to be proportional to a predicted future relative price level, and 5) agents use a VAR model to form their predictions. The model is estimated by FIML under model consistent expectations. The estimates in Table 2 show a significant relative price variable and slow adjustment to PPP. The root mean squared errors in Table 3 are generally supportive of the model: the model usually beats the random walk model. The properties of the model in Table 4 show more important interest rate effects than price effects and are broadly consistent with the effects of surprise interest rate and price announcements on exchange rates.

The model differs from traditional exchange rate models, where an exchange rate is on the left hand side of an equation and various current and possibly expected future macroeconomic variables are on the right hand side. Instead, the exchange rate equation—equation (6)—has on the right hand side only current and expected future relative interest rates, an expected future relative price, and the lagged exchange rate. Macroeconomic variables affect the exchange rate through the VAR equations, which are used to generate predictions of future interest rates and prices.

In future work it will be interesting to see how the model in this paper compares to models in the literature other than the random walk model. The fact that it does well relative to the random walk model suggests that it may do well relative to other models since few models do as well as the random walk model. Also, in future work it will be interesting to experiment with models other than the VAR model used here. No searching was done in this study over alternative models. The VAR equations were specified at the beginning of this study and never changed. As noted above, in future work there is no reason to limit the model to be a VAR model or to be linear. One could even specify the model to have expected future values on the right hand side and force the expectations to be model consistent. The FIML estimation procedure already takes into account expected future variables on the right hand side and forces them to be model consistent, and so no extra work is involved using a more complicated model. Again, as noted above, the model that one is after is the model that best approximates what agents actually use, not necessarily the actual economy.

Finally, the stress in this paper on expectations driving exchange rates is consistent with the large observed short run variability of exchange rates. Anything that affects expectations of future interest rates and prices affects the current exchange rate. In the model expectations are generated using the VAR equations, but in practice there are undoubtedly many things not accounted for in the VAR equations that affect expectations. It is thus not surprising from the perspective of the model that exchange rates are volatile.

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