THREE MINIMAL MARKET INSTITUTIONS WITH HUMAN AND ALGORITHMIC AGENTS: THEORY AND EXPERIMENTAL EVIDENCE

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Three Minimal Market Institutions with Human and Algorithmic Agents: Theory and Experimental Evidence[1]

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Abstract

We define and examine the performance of three minimal strategic market games (sell-all, buy-sell, and double auction) in laboratory relative to the predictions of theory. Unlike open or partial equilibrium settings of most other experiments, these closed exchange economies have limited amounts of cash to facilitate transactions, and include feedback. General equilibrium theory, since it abstracts away from market mechanisms and has no role for money or credit, makes no predictions about how the paths of convergence to the competitive equilibrium may differ across alternative mechanisms. Introduction of markets and money as carriers of process creates the possibility of motion. The laboratory data reveal different paths, and different levels of allocative efficiency in the three settings. The results suggest that abstracting away from all institutional details does not help understand dynamic aspects of market behavior. For example, the oligopoly effect of feedback from buying an endowed good is missed. Inclusion of mechanism differences into theory may enhance our understanding of important aspects of markets and money and help link conventional equilibrium analysis with dynamics.

Keywords: strategic market games, laboratory experiments, minimally intelligent agents,

adaptive learning agents, general equilibrium.

JEL Codes: C92, D43, D51, D58, L13

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Three Minimal Market Institutions with Human and Algorithmic Agents: Theory and Experimental Evidence

1. MINIMAL MARKET INSTITUTIONS

In this paper we define three minimal market institutions, and compare their theoretical properties to the outcomes observed in laboratory experiments with human agents and with simple algorithmic agents. These mechanisms are stripped of details and retain only the basic features necessary to be trading games playable in laboratory. Three price formation mechanisms considered here, listed by the nature of the strategy sets in a single market for each trader, are:

1. The sell-all model (strategy set of dimension 1);
2. The buy-sell model (strategy set of dimension 2)[2];
3. The simultaneous double auction model (strategy set of dimension 2 or 4).

These mechanisms utilize a commodity money for trade, and are described in Section 2. We find that non-cooperative and competitive general equilibrium solutions provide reasonable but imperfect static benchmarks to organize the laboratory observations. In absence of a widely accepted dynamic learning or disequilibrium theory, we compare the market outcomes of trading by profit-motivated humans to the outcomes of two simple computer simulations using minimally intelligent and adaptive learning algorithms as traders. The properties of even these minimal market mechanisms diverge when the number of traders is small. This differentiation raises questions of the appropriate level of specificity/generality for useful study of market mechanisms, to which we return in the final section of the paper.

The development of general competitive and non-cooperative equilibrium models has been followed during the recent decades by documentation of the properties of specific market institutions in game theory, industrial organization, experimental gaming, and experimental economics. The present study is an attempt to fill a gap that remained next to the abstract Walrasian end of the spectrum which is bereft of all institutional details.

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[2] Generically the dimensionality of the strategy set of the buy-sell model is two per market—the number of owned units of the good offered for sale and units of money bid to buy that good. In the laboratory implementation reported here, each individual was endowed with only one of the two goods, thus reducing the strategy set to dimension one per market—the number of owned units of one good offered for sale and the units of money bid to buy the other good.

Partial equilibrium exchange markets have been modeled as games in strategic form solved for their non-cooperative equilibria starting with Cournot 1838 (1897), Bertrand 1883, and Edgeworth 1925, followed by many others. Nash 1951, provided the full generalization of the concept of a non-cooperative equilibrium and Dubey 1982, Dubey and Shubik 1978, 1980, Quint and Shubik 2005, Shapley 1995, Shapley and Shubik. 1977, Shubik 1973, Sorin 1996 and several others extended the analysis to closed economies. There is also a related partial equilibrium literature introducing uncertainty into auction and double auction models as is evinced by the work of Vickery 1961, Griesmer, Levitan and Shubik 1967, Milgrom and Weber 1982, Satterthwaite and Williams 1989.

There are two other relevant literatures: one in macro-economics stressing rational expectations (exemplified by Lucas, 1987, 1988, Lucas and Sargent 1981) and the other in mathematical finance mostly on competitive partial equilibrium open models dealing explicitly with money, transactions costs, and the constraints on cash flows. All approaches broadly involve money, markets and financial institutions. There has been considerable gaming activity on bargaining, bidding and on the emergence of competitive prices in some simple markets with little stress on the explicit role of money (Marimon, Spear and Sunder 1993, Lim, Prescott and Sunder 1994, and Marimon and Sunder 1993, 1994, 1995). Our paper presents gaming with a role for money; two other papers include credit and other financial instruments in addition to money (Huber et al. 2008a, 2008b).

Experiments that examine the properties of markets and competition (Smith 1982, Plott 1982) show that markets with only a few independent individual traders often yield outcomes in close neighborhood of competitive equilibrium predictions. Most experimentation has involved trade in a single market. In the spirit of general equilibrium, we consider two markets. We formulate experimentally playable strategic market games where the trade is mediated by money, but the overall system is closed.

The remainder of the paper is structured as follows: In Section 2 the three minimal institutions are described. Section 3 gives the general and non-cooperative equilibrium predictions for each institution which serve as static benchmarks for comparing the experimental data. Section 4 describes two dynamic benchmarks—minimally intelligent[3]

[3] Since Gode and Sunder's "zero intelligence" agents originally defined for double auctions had to be modified to operate in broader classes of market environments, we changed the label to "minimally intelligent."

and adaptive learning algorithmic traders. Section 5 describes the experimental setup we used to implement these markets in the laboratory with human traders. The results are presented in Section 6, followed by some concluding remarks.

2. THREE MINIMAL MARKET GAMES

We examine three mechanisms which are minimal in the following sense. In order to reflect an exchange economy with money we need at least two commodities in addition to money whose special properties we wish to explore. A game cannot have less than one information set and less than one move per player. If they move simultaneously there will be one information set. Further, price should be at least generically sensitive to, i.e., be a function of, bids and offers. In the sell-all game, the money bid for each commodity is the single move in each market, and calculation of price as the ratio of the sum of money bid and total available quantity of the commodity is the simplest price function. If the mechanism is to satisfy an additional requirement that agents either buy or sell (and possibly do both) in the market for each commodity, we get the buy-sell as the minimal mechanism; the strategy set still has dimension 1 although it consists of the quantity of endowed good offered for sale in one market, and the quantity of money bid in the other market. Finally, the requirement that individuals be able to specify their price and quantity limits leads to a double sealed bid as the minimal mechanism with a four dimensional strategy set, although we use sequential double auction in this paper because its properties have been studied extensively in the experimental gaming literature. It differs from the double auction sealed bid in the number of information sets.

2.1 Definitions

Money

In each market game two commodities are traded and one more instrument is used as a means of payment (money). This money is introduced as a linear term in the subjects' utility functions.[4]

[4] For a detailed justification for this assumption see Shubik (1999).

Bids

(1) *A money bid*: A trader i bids an amount of money b^i_j for the j^{th} commodity. The trader has no reserve price and accepts the market price. This allows a simple quantity bid for a mechanism similar to Cournot's 1897. The market clearing mechanism gives the trader i quantity $x^i_j = b^i_j / p_j$ of good j where p_j is the market price that is formed collectively by individual bids and offers.

(2) *A price-quantity bid*: Suppose that a trader i instead of offering an amount of money to buy a good j, bids a personal unit price p^i_j he is willing to pay to buy up to an amount q^i_j of the good. It is reasonable to expect that he is willing to buy q^i_j or less at a price less than or equal to p^i_j. There is an implicit limit in this bid inasmuch as $q^i_j\, p^i_j$ must be less than or equal to the individual's credit line plus cash. Since we do not consider a credit mechanism in the three market institutions considered here, $q^i_j\, p^i_j$ cannot exceed the available cash. Minor variations of these bids consider any upper or lower bounds on prices or quantities acceptable to the bidder.

Offers

Analogously, there are two simple forms of offers.

(1) *A non-contingent offer to sell*: Suppose that an individual i owns a^i_j units of good j and wishes to sell some of it. The simplest strategy is for her to offer $q^i_j \leq a^i_j$ units for sale at the market-determined price.

A somewhat more complex action, but still not involving any more information and confined to a single move is:

(2) *The price-quantity offer:* Suppose that a trader i is willing to sell up to an amount q^i_j ($\leq a^i_j$) of good j at unit price p^i_j. It is reasonable to expect that she is willing to sell q^i_j or less for a price more than or equal to p^i_j.

We use observable acts to buy (bids) and sell (offers) as the building blocks to construct three simple market games. Simplifying them any further will prevent any trading. The first two market games involve a single move by every agent, taken simultaneously. The third, double auction, involves sequential multiple moves by various players. Each game can be generalized to multiple plays.

Consider n individuals where i has an endowment a^i_j of good j ($j = 1, \ldots, m$) and an endowment M^i of money. Suppose there are m markets, one for each good j where it can be exchanged for money. A plausible restriction on the market mechanism is that all trades in a given market take place at the same time and the same price. This requires that $p^i_j = p_j$ for $i = 1, \dots n$.

In general, we cannot assume that bids in one market are independent of bids in the others. There is at least a cash or credit budget constraint that links actions across markets.

2.2 The Sell-All Model

This is the simplest of the three models. Consider n traders trading in $m+1$ goods, where the $m+1^{st}$ good has a special operational role, in addition to its possible utility in consumption. Each trader i has an initial bundle of goods and money $a^i = (a^i_1, ..., a^i_m, M^i)$, where $a^i \geq 0$ for all $j = 1, ..., m+1$ and $a^i_{m+1} = M^i$, and the utility $u^i = u^i(x_1, ..., x_m, x_{m+1})$, where u^i need not actually depend directly on x_{m+1}; a fiat money is not excluded.

In order to keep strategies simple, let us suppose that the traders are required to offer for sale *all* of their holdings of the first m goods. Instead of owning their initial bundle of endowments outright; the traders own a *claim on the proceeds* when the bundle is sold at the prevailing market price.

Suppose there is one trading post for each of the first m commodities, where the total supplies $(a_1, ..., a_m)$ are deposited for sale "on consignment," so to speak. Each trader i submits bids by allocating amounts b^i_j of his endowment m^i of the $m+1^{st}$ commodity among the m trading posts, $j = 1,..., m$. There are a number of possible rules governing the permitted range of bids. In the simplest case, with no credit of any kind, the limits on b^i_j are given by:

$$\sum_{j=1}^{m} b^i_j \leq M^i, \text{ and } b^i_j \geq 0, \; j = 1, \dots, m.$$

An interpretation of this spending limit is that the traders are required to pay *cash in advance* for their purchases. The prices are formed from the simultaneously submitted bids of all buyers; we define

$$p_j = b_j / a_j, j = 1, ..., m.$$

Thus, bids precede prices. Traders allocate their budgets fiscally, committing specific quantities of their means of payment to the purchase of each good without definite knowledge of what the per-unit price will be (and how many units of each good their bid will get them). At an equilibrium this will not matter, as prices will be what the traders expect them to be. In a multi-period context, moreover, the traders will know the previous prices and may expect that variations in individual behavior in a mass market will not change prices by much. But any deviation from expectations will result in changing the quantities of goods received, and not in the quantities of cash spent. In a mass market, the difference between the outcomes from allocating a portion of one's budget for purchase of a certain good, and from a decision to buy a specific amount at an unspecified price, will not be too different.

The prices in the model are determined so that they exactly balance the books at each trading post. The amount x^i_j of the j^{th} good that the i^{th} trader receives in return for his bid b^i_j is

$$x^i_j = \begin{cases} b^i_j / p_j \text{ if } p_j > 0, \; j = 1, \ldots, m, \\ 0 \quad \text{if } p_j = 0, \; j = 1, \ldots, m. \end{cases}$$

His final balance of the $m+1^{st}$ good, taking account of his sales as well as his purchases, is

$$x^i_{m+1} = a^i_{m+1} - \sum_{j=1}^{m} b^i_j + \sum_{j=1}^{m} a^i_j \; p_j \, .$$

2.3 The Buy-Sell Model

Subjects face a more complex task in the buy-sell model: instead of one money bid in each of the two markets in sell-all, they submit the quantity of their endowed good they wish to sell, and the money bid for the other good they want to buy. Thus they enter only one number in each market but these numbers are in different dimensions (goods and money). Since moves are simultaneous, there are no contingencies in this market either. Physical quantities of goods are submitted for sale and quantities of money are submitted for purchases, and the markets are cleared. The mechanism does not permit any underemployment of resources.[5] The amount x^i_j of the j^{th} good that the i^{th} trader receives in return for his bid b^i_j is:

$$x^i_j = \begin{cases} b^i_j / p_j \text{ if } p_j > 0, \; j = 1, \ldots, m, \\ 0 \quad \text{if } p_j = 0, \; j = 1, \ldots, m. \end{cases}$$

However price is somewhat different as it depends on the quantities of each good offered for sale (and not on the endowment of each good):

$$p_j = b_j / q_j, j = 1, \ldots, m \, .$$

[5] Except when there is no bid or offer, in which instance all resources are returned to their owners. If they are ripe tomatoes, the owner is in trouble.

His final amount of the $m+1^{st}$ good, taking account of trader i's sales as well as his purchases, is

$$x^i_{m+1} = a^i_{m+1} - \sum_{j=1}^{m} b^i_j + \sum_{j=1}^{m} q^i_j \; p_j \; .$$

2.4 The Sequential Bid-Offer or Double Auction Model

Any trader is free to submit a bid in either market to buy one unit at or below a specified price, and an ask to sell one unit at or above a specified price as long as he has the money (to buy) or good (to sell). The computer screen shows all outstanding bids in descending order and all outstanding asks in ascending order. Traders are free to accept the lowest outstanding bid or the highest outstanding ask and consummate a trade. If the highest bid and lowest ask cross, a trade is automatically recorded at that price.

The double auction model doubles the size of the strategy set, changing price into a strategic variable from a mere outcome of the quantity strategies in the sell-all and buy-sell models. In each of the m markets, an individual's strategy has four components $(p, q; p^*, q^*)$ where the first pair of numbers is interpreted as an offer to sell amount q or less for a price p or more, and the next pair is a bid to buy amount q^* or less at a price p^* or less.

From the viewpoint of both game theory and experimental gaming the number of decisions in a double auction is more than in the other two markets. Imposing a condition that one can either buy or sell, but not both, is a possible theoretical simplification. In practice, however, an individual can be a buyer or a seller or a trader. Most consumers are buyers and most producers are sellers of specific commodities or services; a trader can be active on both sides of the market.

In these games the terminal amount of money ($M - b + pa$) held by each individual was added to their dollar payoffs. This served to fix the price level that the transactions would be expected to approach towards the end. The observed divergence between these predicted and realized prices in some cases was considerable, and is discussed later.

3. GENERAL AND NON-COOPERATIVE EQUILIBRIA

The non-cooperative equilibrium (NCE) solution is a fairly natural game theoretic way to approach these games without any direct communication. A non-cooperative equilibrium satisfies the existence of mutually consistent expectations. If each predicts that the other will play his strategy associated with a non-cooperative equilibrium the actions of all will be self-confirming. No one acting individually will have an incentive to deviate from this equilibrium. This could be called an outcome consistent with "rational expectations," but as the outcome may be neither unique nor generically optimal, the label of "rational" is best avoided.

The competitive general equilibrium (CGE) solution is defined as the set of prices that clear all markets efficiently. In general, the mathematical structure of NCE and CGE differ. However, it can be shown in theory that, as the number of traders in a market increases, under reasonable conditions, the NCE approaches the CGE. In symmetric markets without face-to-face communication experimentation can verify that with as few as 5-10 traders on each side, the outcomes approximate the CGE, and any differences between the two can be explained by the NCE.

3.1. The Non-Cooperative Equilibrium in the Sell-All Market

Sell-all is the simplest model and for experimental purposes we keep the payoff structure simple to explain to subjects untutored in economics or mathematics:

$$\alpha\sqrt{xy} + M - b + pa$$

where α is an appropriately chosen parameter (explained in the discussion of the game), the square root of *xy* is a simple Cobb-Douglas utility function whose range of values is furnished in a coarse-grid table in order to ease the computational burden. The linear term ($M - b + pa$) is the residual amount of money (initial endowment less the amount of money bid plus earnings from selling a units at price p).[6]

The mathematical solutions of this model under different constraints are given in Appendix B. Table 1 shows the NCE for markets with 2, 3, 4, 5, 10 and many traders on each side for the parameter values used in the experiment.

(Insert Table 1 about here)

3.2. The Non-cooperative Equilibrium in the Buy-Sell Market

The basic difference between the sell-all and the buy-sell model lies in the freedom subjects have to control the amount of goods to sell in the market for the endowed good (see Table 2). The general formulae for the NCE are given in Appendix B.

(Insert Table 2 about here)

3.3. The Non-cooperative Equilibrium in the Double-Auction Model

[6] The utilization of a money with a Marshallian or constant marginal utility can be interpreted in terms of a known expectation of the worth of future purchasing power. In this context any change in price level can be attributed to error and learning the equilibrium of the actual game is stationary. This device provides an easy and logically consistent way in an experimental game to provide terminal conditions.

For simplicity, the bid-offer market is modeled as a simultaneous sealed bid auction. The clearing method for the one-shot game is simplicity itself. Bids are assembled in a down-sloping histogram and offers in an up-sloping histogram. Market price is formed where the two lines intersect.[7]

The double auction used in the experiment is a continuous process where bids and offers flow in sequentially and a trade takes place whenever they match or cross. We use this continuous double auction rather than the simultaneous sealed bid auction so traders can learn from the order-book and from past prices.

Two individuals on each side of the market are sufficient for the competitive equilibrium to be a NCE. A simple example considering optimal response is sufficient to show this. Suppose that there are two individuals each of two types. All have the payoff function given above, but individuals of type 1 and 2 have endowments of $(a, 0, M)$ and $(0, a, M)$, respectively, where the first component is the endowment of the first good, the second the endowment of the second good and the third the endowment of money. Suppose $M > a/2$ and $\alpha = 2$ (the parameter in the payoff function), a trader of type 1 offers to sell $a/2$ or less of good 1 at a price of 1 or more and to buy up to $a/2$ of good 2 at a price of 1 or less, it can be verified that this is an equilibrium outcome and the price of both goods is 1 ($p_1 = p_2 = 1$). [8]

There is a considerable amount of experimental evidence that in a single market the double auction mechanism yields efficient allocations. In their single-commodity double auctions, Gode and Sunder (1993 and 1997) found that it requires negligible skills or intelligence from traders for the market outcome to lie in close proximity of the competitive equilibrium. However, we consider two markets for two commodities; whether the complementarities between the two make a difference remains open. Obviously the task of trading on two markets simultaneously is markedly more demanding that trading on a single-commodity market.

In their one-shot versions, the three games are the simplest price formation mechanisms that can be constructed, involving the maximum of one (sell-all and buy-sell) and four (double auction) strategic variables. They can all be analyzed for their

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[7] It is necessary to take care of several cases; see Dubey and Shubik (1980) or Dubey (1982).

[8] From a strictly technical game theoretic point of view there is a continuum of non-cooperative equilibria, all with the same efficiency that are consistent with the competitive equilibrium outcome.

NCE. Unlike most other market experiments, these are general equilibrium full feedback models, not partial equilibrium constructs.

The non-cooperative model of the general equilibrium in theory, generates an asymmetry in actions when there are few agents, as can be seen in the sell-all model where a seller obtains an oligopolistic income from buying a commodity to which he has ownership claims (as contrasted with buying a commodity he does not have). This asymmetry is the largest in the buy-sell game, the next largest in the sell-all game and the smallest in the double auction (see tables 1 and 2 for numerical examples for 5+5 traders).

Paradoxically, because MI agents (see Section 4 below) ignore their oligopolistic influence the theoretical prediction is that in all markets the price should be as close or closer to the competitive equilibrium than with oligopolistic human traders, but because of the random action there should be a variation in payoffs that is not present in the equilibrium analysis of the three games.

The speed of learning and the variation among players is not predicted by the static non-cooperative or general equilibrium theories. Many learning theories have been proposed and in the next section, we consider one non-learning and one simple learning algorithm. We only conjecture that as human subjects learn, variations in the outcomes of markets will diminish in the later periods (replications) of the game.

4. DYNAMIC BENCHMARKS

Richness of the data sets generated from market experiments with human subjects is not captured in the static NCE and CGE benchmarks. Unfortunately, there is no generally accepted disequilibrium theory of dynamic learning. We compare the results obtained from markets populated by profit-motivated human traders with the results from markets populated by two different kinds of simple algorithmic traders described in this section: the non-learning minimal intelligence (MI) benchmark (after Gode and Sunder 1993's zero-intelligence or ZI, see footnote 3), and adaptive learning agents (AL).

1. Minimally Intelligent (MI) Traders.

In sell-all markets, given the money endowment of M, each agent picks an uniformly distributed random number between 0-M as its total money bid (for A and B combined). A second uniformly distributed random variable z between 0 and 1 is drawn to define the share of this money bid invested in A with (1-z) invested in B.

In the buy-sell market, each trader offers to sell a randomly chosen quantity of the endowed good (from uniform distribution between 0-*a*) and bids a randomly chosen quantity of money for the other good (from uniform distribution between 0-*M*).

In double auctions, with equal probability and independently, one trader is picked, one of the two markets is picked, and either bid or ask is picked. Given the trader's current holdings of the two goods and cash, computer calculates the opportunity set (the maximum amount of bid the trader can make without diminishing its net payoff), and draws a random number between the current bid and this calculated upper limit (if the latter is more than the former) and submits it as a bid from this trader. In case of asks, the computer calculates the minimum amount of ask the trader can submit without diminishing its net payoff and submits a random number between this calculated lower limit and the current ask (if the latter is above the former), as the ask.[9] Higher bids replace lower ones as market bids, and lower asks replaced higher ones as market asks. Whenever market bids and market asks cross, a transaction is recorded at the price equal to the bid or ask, depending on which of the two was submitted earlier (see Appendix C).

2. Adaptive Learning (AL) Traders.

The adaptive learning (AL) algorithmic traders are a modification of the MI traders described in the preceding paragraphs. In sell-all and buy-sell markets, each AL trader keeps track of the past decisions which yielded the highest payoff and uses an adaptive learning parameter λ (set to 0.5 in the simulations) to adjust the most recent decision towards this "historical best" decision. The bid for the next period is then λ times the "historical best" decision plus (1- λ) times new random variables (as in MI).[10] In double auction algorithm starts period 1 with a "price aspiration" of money/goods in the endowed quantities and uses each observed transaction price to adjust this aspiration by λ(transaction price –price aspiration). In addition to the constraints described above in description of MI traders, AL traders use this price aspiration as an additional constraint, not bidding above and not asking below this level. We consciously chose learning algorithms where the agents only look at their own earnings and their own decisions; market variables are not considered.

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[9] This means that bids are randomly distributed ~U(Current Bid, $((100/0.5)\ (((c_A+1)c_B)^{0.5} - (c_Ac_B)^{0.5}\ )$; asks are randomly distributed ~U($(100/0.5)\ (-((c_B-1)c_A)^{0.5} + (c_Ac_B)^{0.5}\ )$, Current Ask). After each transaction, current bid is set to 0 and current ask is set to the initial cash balance of 4,000.

[10] With λ=0 the AL-simulation would be the same as the MI-simulation as then no learning would take place.

The paths of markets populated by these two kinds of artificial players should serve as much as a warning as benchmarks. Rigid rule gaming in cleaned up abstract laboratory conditions contrasts sharply with the battlefield conditions of phenomena of substantive interest. Under the conditions chosen here, there is a unique analytical interior perfect non-cooperative equilibrium. In such situations, it is not difficult to find many dynamic procedures such as hill-climbing, optimal response, exponential lag weighted forecasting or adaptive forecasting rules that work well on a reasonably smooth terrain with a unique joint maximum. Kumar and Shubik, 2004 note that one can take an example such as the well known Scarf model of global instability with a unique equilibrium point and easily find a control process that gives contrary results.

The large body of work that applies dynamic programming microeconomic methods to problems of macroeconomics tells us little about learning and disequilibrium behavior. Our human and algorithmic games merely yield an empirical picture of the markets populated by various kinds of traders. It is easy to fit many process models to the data ex post. We, too, could try to fit some plausible rules of behavior to the observed data. The gains from such an exercise being doubtful; we refrain from doing so.

5. THE EXPERIMENTAL SETUP

We conducted and report on two independent sessions for each of the three market games considered in this paper. In each session, programmed in Z-tree software (see Fischbacher, 2007), the participants traded two goods—labeled A and B—for money. Each session had ten participants, five of them were endowed with some units of A and none of B, while the other five had some units of B and none of A.[11] All had the same starting endowment of money. Each session consisted of ten or twenty independent rounds of trading. Subjects' "consumption" at the end of each round was accumulated in a "bank account" with the experimenter. No goods balances were carried over from one round to the next, and each subject was re-endowed with the ownership claims to goods A or B at the beginning of each round. In all treatments money is carried over to the following round (see descriptions of specific treatments below and in Table 3).

(Insert Table 3 about here)

5.1 Sell-All Call Market

[11] In addition, we conducted one session with 20 participants, of whom 10 were of each type.

In Treatment 1 (sell-all Market) the initial endowments were 200 or 0 units of A, 0 or 200 units of B, and 6,000 in cash. All units of A and B were sold automatically at a price derived from the set of bids submitted by the traders. In other words, subjects did not have to decide on the number of units they wished to sell; all their holdings of goods were sold at the prevailing market price. Consequently, they had ownership claim to the revenue from selling 200 units of the good they were endowed with. The only decision participants had to make was how much of their money endowment they wished to bid to buy good A and how much to bid to buy good B. (see Appendix A for instructions and the 'trading screen'). Each sell-all market was repeated for 20 periods.

As outlined above the unit prices of A and B are calculated as the respective sums of money bid for the respective good by all traders divided by the total units of each goods for sale. With 6,000 units of money endowment per trader there is more than enough money to reach general equilibrium at prices of 20 per unit of A and B. At general equilibrium traders would spend 2,000 on each good and keep 2,000 of their money endowment unspent. However, in a thin market with only a few traders, deviating from general equilibrium spending level may make sense to traders. When a trader spends more on the good he is endowed with, he raises its price and therefore his revenue from selling a part of his endowment of this good. Apart from the general equilibrium, there also exists a non-cooperative equilibrium in which traders spend 2213.4 on the good they own, 1810.6 on the other good, and keep 1976.0 unspent. Prices are slightly higher at 20.12 for both goods in this equilibrium. We conducted two runs of this treatment.

5.2 Buy-Sell Call Market

Unlike in Treatment 1, traders in this treatment directly control the goods they are endowed with, and decide how many, if any, units they wish to sell (in Treatment 1 all units were sold automatically). Again half of the traders are endowed with 200 units of A and none of B, while the other half are endowed with 200 units of B and none of A. Each trader has an initial endowment of 4,000 units of money at the beginning of the first round of the session. Money balances are carried over from one round to the next. Each buy-sell market was repeated for 20 periods.

Traders make two decisions: The amount of their money to buy the good they do not own, and the number of units to sell out of the 200 units of the good they own.

Prices for A and B are calculated by dividing the total investment for the respective good by the number of units put up for sale. Competitive equilibrium prices and conditions are the same as in Treatment 1. Final holdings of goods are (100,100) each (prices are 20/20, each trader spends 2,000 for the good he does not own, and sells 100 units of the good he owns). At the non-cooperative equilibrium with 5 traders on each side of the market traders of type 1 offer 78.05 units of the second good for sale and bid 1560.97 units of money for the first good. Traders of type 2 do the opposite (see Table 2). Final endowments are (78.05, 121.95) for traders of type 1 and (121.95, 78.05) for traders of type 2. Prices are 20/20.

5.3 Double Auction Market

Treatment 3 features a double auction market where participants can trade goods A and B in a continuous market for several periods. We simplify trading by considering only transactions for one unit at a time. To reduce the number of transactions needed to reach equilibrium levels, initial endowments of A and B are reduced from to (20/0, 0/20), so traders own 20 units of a good rather than 200. Each period lasts for 180 seconds to give the participants enough time to allow participants to trade ten units of goods they do not yet own, and by selling ten units of the good they are endowed with, required to reach equilibrium. Traders are endowed with 4,000 units of money, which is more than enough for trading.

Competitive equilibrium and non-cooperative equilibrium prices coincide for the closed double auction model as was shown by Dubey (1982)[12]. They are 100 for each good. The first run of the double auction market was repeated for 10 periods, the second run for 11 periods.

In the double auction experiments we allow market as well as limit orders. All orders are executed according to price and then time priority. Market orders have priority over limit orders in the order book. This means market orders are always executed instantaneously. Again holdings of money and goods are carried over from one round to the next.

Participants receive current information about their cash and stock holdings, their wealth, and their transactions within the current period on the screen. In the centre of the screen they see the open order books and they have the opportunity to post limit or

[12] The results for the non-cooperative equilibrium are delicately dependent on the formulation of details of the game; see Shubik (1959), Wilson (1978), and Schmeidler (1980). In some models it is possible that there is no pure strategy non-cooperative equilibrium, in others there may be a multiplicity of equilibria with the same value.

market orders. On the left side of the screen transaction prices of the round are charted against time.

6. EXPERIMENTAL RESULTS

We use seven aspects of market outcomes—allocative efficiency, level and volatility of prices, symmetry of allocation across the two goods, money balances (except in double auctions where it is undefined), cross-trader dispersion of earnings, and trading volume—to assess the behavior of three market mechanisms relative to three static (autarky, competitive general equilibrium, and non-cooperative equilibrium), and two dynamic (markets populated by minimally intelligent or MI, and adaptive learning or AL agents) benchmarks. In addition, we examine the velocity of money, and kurtosis and autocorrelation of returns in double auctions.

Allocative efficiency is measured by the total earnings of all traders as a fraction of the total earnings in competitive equilibrium. The behavior of transaction prices is measured by market clearing prices for sell-all and buy-sell markets, and by average transaction prices (averaged across transactions within one period) in the double auction markets. Symmetry of allocation is the ratio of consumption of good *A* and *B* (= min (c_A/c_B, c_B/c_A)). Given the parameters chosen for these experiments, goods A and B should be allocated symmetrically at the competitive equilibrium, which has the symmetry measure of 1. Autarkic symmetry is 0. Money balances refer to the percentage of initial money left unspent after buying decisions are made (and before the proceeds of any sales are received) in sell-all and buy-sell markets.

We report these four performance measures relative to the three abovementioned static benchmarks summarized in Table 4. Under autarky, efficiency and symmetry are 0, prices are undefined, and money balance is 100 percent. The competitive general equilibrium allocations are 100 units each of good *A* and *B* in sell-all and buy-sell markets, and 10 units of each good in the double auctions, yielding a symmetry measure of 1 in all cases. Prices are 20 in sell-all and buy-sell markets, and 100 in the double auction markets.

(Insert Table 4 about here)

The third benchmark for market performance is non-cooperative equilibrium for 10 traders (five endowed with good *A* and five endowed with good *B*). Application of theory to the parameters of these markets yields bids of 2214 and 1811 for the owned and the not-owned good respectively, and final holdings of 110 and 90 units in the sell-all model. In buy-sell model non-cooperative equilibrium requires selling 78 of the 200 units of the owned good and buying 78 with a bid of 1561 units of money. In the double auction traders should keep 11 of their 20 units of the good they are endowed with and buy 9 of the other. Unspent money balance is 32.92 percent in sell-all, 60.98 percent unspent in buy-sell, and not defined in the double auction. The resulting measures for symmetry are 0.82 in sell-all, 0.64 in buy-sell, and 0.82 in double auction. Prices are 20.12 in sell-all, 20 in buy-sell, and 100 in double auction.

Finally, we compare the results obtained from human traders in these three markets against computer simulations of markets populated with minimal intelligence (MI) and adaptive learning (AL) algorithmic agents described in Section 4. These computer simulations provide dynamic bases of comparison for markets populated by profit motivated human traders. We simulate each of the three market structures 1,000 times with specified algorithmic traders.

Each market statistic observed over the 1,000 replications is sorted into quitiles for each period. Bands in shades of gray in the background of Figures 1 to 9 show the distribution of the performance of the markets under the specified trading algorithms. Note that in Figures 1, 4 and 7, the double-auction simulations have zero dispersion and the quintiles collapse the bands of gray to zero width.

6.1 Allocative Efficiency

Allocative efficiency of the markets is measured each period by the average amount earned by traders as a percentage of the competitive general equilibrium amount (1,000 points). Six panels of Figure 1 show the time series of efficiency in two replications of each of the three market games; in the left column of panels the human market data are charted against the background of quintiles of efficiency statistics from 1,000 replications of markets populated by MI algorithmic traders and the right column has quintiles from AL algorithmic traders in the background. The autarky (efficiency = 0) benchmark is not included in the chart. The solid black line of competitive general equilibrium (efficiency = 100) frames the charts at the top and the non-cooperative equilibrium efficiency (for 5+5 = 10 players) is shown in a dotted line slightly below. In the following paragraphs we compare the efficiencies observed for specific market games against these benchmarks across the three market games.

(Insert Figure 1 about here)

The first obvious observation is that the allocative efficiency of all six sessions of three market games is much closer to the predictions of competitive equilibrium (100 percent for all three mechanisms), and non-cooperative equilibrium (99.5, 97.6, and 99.5 percent for the three mechanisms) but far from the autarky prediction of zero.

A second observation is that there are differences in the efficiency of the three mechanisms: the average efficiency is the highest for sell-all (97.7 and 97.6 for the two human sessions), followed by buy-sell (91.4 and 94.6 for the two human sessions) and the lowest for double auctions (87.8 and 93.2 for the two human sessions). Most experimental gaming results from double auction markets tend to report higher efficiencies (close to 100 percent). However, virtually all such experiments have been conducted in single market partial equilibrium settings.[13] With human subjects, the efficiency dominance of double auction observed for single-commodity markets is not preserved in general equilibrium settings in the presence of complementarities across two or more markets. If the values across the markets were not complementary, we expect the efficiencies to be higher.

Third, the gray bands of quintiles of 1,000 replications of markets run with two kinds of algorithmic agents form the background of the six panels. Efficiency of sell-all markets with human traders is much greater than the median (= 81.5 percent on average across periods) and about equal to the maximum (= 97.8 percent on average across periods) achieved with MI traders. AL traders learn rapidly in the first three periods achieving a higher median efficiency (= 94.9 percent on average for periods 4-20) which is still lower than the efficiency of markets with human traders (97.7 and 97.6 percent respectively in the two runs).

Efficiency of buy-sell markets with human traders is greater than the median (= 87.2 percent on average across periods) and generally lies in the top quintile achieved with MI traders. Again, AL traders learn rapidly in the first three periods achieving a higher median efficiency (= 95.9 percent on average for periods 11-20) which is equal or higher than the efficiency of markets with human traders.

Since double auction markets with algorithmic traders (MI as well as AL described above in Section 6.0) always achieve the upper bound of 100 in efficiency, the quintile bands in the two bottom panels of Figure 1 collapse to a line that coincides with

[13] See Gode, Spear and Sunder (2004) for an exception.

the CGE efficiency which is higher than the efficiency achieved by double auction markets populated by human traders.

The first panel of Figure 8 shows the efficiencies observed in the buy-sell market with 20 (10+10) human traders. The average efficiency across the 20 periods is 98.1 percent, as compared to 91.4 and 94.6 percent respectively in the two sessions with 10 (5+5) traders. Median efficiency with AL traders gradually rises to about 96 percent. The data are consistent with the conjecture that the market outcomes approach GCE as the number of trader increases.

(Insert Figure 2 about here)

6.2 Price Levels

Each of the six panels in Figure 2 charts the observed prices (or average prices in DA) for goods A and B in two sessions of one of the three market games. The continuous horizontal line marks the CGE prices (20 in sell-all and buy-sell and 100 in DA). The corresponding NCE prices are 20.12, 20, and 100, and therefore do not show as a separate line in these charts. The data are charted against the background of gray quintile bands of prices from MI simulations in the left column; the same data are charted again in the right column of panels against the background of quintiles from AL simulations.

In both sessions of the sell-all market, prices of good A as well as B are close to the competitive general equilibrium prediction of 20 (and NCE prediction of 20.12). Across-periods average prices of A and B are 18.92 and 20.90 respectively in one run and 21.52 and 20.49 in the other. The price support selected for MI simulations predicts a price of 15, and the quintile bands of simulated MI price series for both goods are centred around 15. However, the simulated AL price series start out centred at 15 but adjust to the neighbourhood of CGE-NCE prediction of 20 during the first 10 periods and stay there. Also, note that the dispersion of AL prices is smaller than for MI prices. Apparently, even the simple sell-all market mechanism provides enough discipline and structure to adjust the price near the equilibrium level with the simple adaptive learning algorithm used here.

(Insert Figure 2 about here)

Prices of goods A and B in the two sessions of buy-sell markets are qualitatively different from the results of the sell-all markets and across the two sessions. In the first session, the prices of goods A and B are distributed around 10 (cross-period average of 11.32 for A and 9.24 for B) which is about one half of the CGE and NCE prediction of

20. In the second session, prices of both goods lie much closer to the CGE price of 20 (cross-period average of 19.69 for A and 16.34 for B). The quintile bands of simulated MI price series for both goods are centred at the CGE-NCE prediction of 20. However, this is not true of the AL price series; these bands start out being centred at the CGE-NCE predication of 20, but quickly move down in the first three periods to settle around 17. Compared to the sell-all markets, both human as well as algorithmic prices in buy-sell markets exhibit a greater dispersion.

In the first session of the double auction market, range of prices (235-275 for an average of 261 for A and 246 for B) lay far above the CGE and NCE prediction of 100. In the second double auction session, these prices are lower (in the 155-265 range for an average of 225 for A and 170 for B), but are still significantly above the CGE-NCE prediction of 100. It is remarkable that these large deviations from equilibrium prices result in only a relatively small drop in the allocative efficiency of these auctions. As pointed out by Gode and Sunder (1993), the allocations (and therefore the efficiency) properties of the markets tend to be more robust than the prices.

A possible explanation for the divergence between the predicted price level and the actual price level in some of the games is that in spite of the theoretical power of backward induction in games of finite duration, the terminal conditions are not taken into account until close to the end of the session, a topic to which we return in the concluding section of the paper.

The mean transaction prices in the double auction simulations with MI traders are about 146, considerably above the CGE-NCE price of 100. To shed some light on the price dynamics in the double-auction the first panel of Figure 3 shows the within-period path of average transaction prices starting in high 200's and converging tightly to the close neighbourhood of the CGE-NCE price of 100 in the later part of every trading period. Since the prices converge from above, the average of all transactions in period is about 146 in spite of convergence to 100 at the end of the period. In contrast, the DA markets with human traders exhibit no such tendency and most transactions are distributed around the period mean. The mean of transaction prices in the double auction simulations with AL traders is about 70, which is considerably below the CGE-NCE price of 100. Again, as seen in the second panel of Figure 3, the price paths converge tightly to the close neighbourhood of the CGE-NCE prediction of 100 in the later part of every period. However, in this case, the prices converge from below, yielding a lower average of 70 in spite of convergence to equilibrium at the end of the period.

(Insert Figure 3 about here)

Table 5 shows that the double auction markets have active trading. Each period was allowed to run for three minutes and we ran 10 and 11 periods in runs 5 and 6 respectively for a total duration of 30 and 33 minutes, and 994 and 1,114 transactions respectively. This translates into about one transaction every two seconds and 20 transactions per trader per period on average in both runs. Recall that 20 (18) transactions per trader are necessary to reach CGE (NCE), with each trader buying 10 (9) units of the good he does not have and selling 10 (9) units of the good he is endowed with. However, as we saw in discussion of efficiency and symmetry, while the total number of transactions was close to CGE prediction, their distribution across traders showed greater variation. Beside, some traders bought as well as sold within the same market.

(Insert Table 5 about here)

Finally, the second panel of Figure 8 shows the prices observed in the buy-sell market with 20 (10+10) human traders. The average price across the 20 periods is 16.45 for A and 16.40 for B, as compared to 11.3/9.2 and 19.9/16.3 respectively in the two sessions with 10 (5+5) traders. Prices in human markets are close to the median of AL simulations. The price data do not show any marked tendency to be closer to GCE price as the number of trader increases from 10 to 20.

6.3 Symmetry

Figure 4 shows the asymmetry introduced by the oligopoly effect: in sell-all markets a trader gains an advantage from spending more for the good he is endowed with because of a feedback effect on income as his own bids influence the prices of the two goods. As shown in Table 4, CGE symmetry is 100 for all three markets, and NCE symmetry is 0.82, 0.64, and 0.82 for sell-all, buy-sell, and DA respectively. We see that observed symmetry is highest in the two sell-all markets (0.76 and 0.71 on average), lower in the two buy-sell (0.6 and 0.71 on average), and lowest in the double auction setting (0.53 and 0.6 on average). On the whole, the data are organized better by NCE than CGE. The lower the symmetry the lower the average earnings, because skewed investment leads to lower earnings in the earning functions used in these experiments.

The MI simulation quintiles of symmetry in sell-all as well as buy-sell markets with median of about 0.4 lie well below the human data and the static benchmarks. The

AL simulation quintiles rise rapidly during the first 4 periods to a median of about 0.6 and stabilize there. Since in MI simulations of double auction traders are allowed to trade indefinitely, all traders do so until their holdings are perfectly symmetrical (and their payoffs reach the individual maximum). Accordingly, the quintile bands in the bottom two panels of Figure 4 collapse to 1.0 and therefore are not visible.

(Insert Figure 4 about here)

The third panel of Figure 8 shows the symmetry of holdings observed in the buy-sell market with 20 (10+10) human traders. The average symmetry across the 20 periods is 0.81, as compared to 0.60 and 0.71 in the two sessions with 10 (5+5) traders. In AL simulations, the median of symmetry approaches approximately 0.65. The data are consistent with the conjecture that the market outcomes approach GCE as the number of traders increases.

6.4 Price Volatility

In sell-all and buy-sell markets we only have a single price per commodity per period. Figure 2 shows the time series of prices for the three markets. The F-value of 56.78 from Levene-test on equality of variances on period-to-period log price changes is significant at $p < 0.001$ which suggests that the dispersion of log prices changes in sell-all markets are significantly lower than in buy-sell markets.

With their sequence of transactions and intra-period returns, double auctions offer far more data. In two lab runs with human traders, the standard deviation of intra-period returns has median of 12.8 and 18.2 percent. The median is a comparable 21 percent in AL simulations, but much higher at 83 percent in MI simulations. As several recent experimental papers have shown, prices in DA markets are often sticky, i.e., move less than what would be justified by changes in fundamental values (see Kirchler, 2009 and Noussair and Powell, 2008). In light of these studies the relatively low volatility of markets populated by humans is not surprising.

6.5 Money Balances

The payoff functions were parameterized so that beyond a certain level we would expect that individuals would prefer to hold back on spending additional cash. Figure 5 compares actual money balances (money left unspent) against the benchmarks of competitive equilibrium (33.3 percent in sell-all, 50 percent in buy-sell), non-cooperative equilibrium (32.9 percent in sell-all and 61 percent in buy-sell), and autarky (100 percent in both). For sell-all, the MI simulations (the top left panel) have a steady state distribution with a median of about 50 percent of the money not spent while the AL simulations this median starts with 50 percent but drops over the 20 periods to the neighborhood of 33.3 percent which is CGE prediction. For buy-sell, the MI simulations have a median around the CGE predication of 50 percent but AL simulations show a higher median of about 56 percent. Since money balances remain unchanged in double auction, they are not shown.

We find that money balances in the sell-all markets with profit-motivated human traders are close (33.63 and 29.99) to the CGE level of 33.33 percent, while in the buy-sell markets traders kept more (73.72 and 60.47 percent) of their money than CGE prediction of 50 percent, but closer to the non-cooperative equilibrium of 60.98 percent. As a consequence prices in all sell-all markets are close to CGE-levels of 20, but are much lower in buy-sell markets. Our conjecture about this finding is that traders in buy-sell markets were much more aware of their influence on prices of other people's goods, than they were in the sell-all market.

(Insert Figure 5 about here)

The fourth panel of Figure 8 shows the unspent money holdings observed in the buy-sell market with 20 (10+10) human traders. The average unspent money across the 20 periods is 62 percent of the initial endowment, as compared to 74 and 60 percent respectively in the two sessions with 10 (5+5) traders. The median of AL simulations is approximately 56 percent. The money holdings data do not show a marked tendency to be closer to GCE prediction of 50 percent as the number of trader increases from 10 to 20.

6.6 Cross-sectional Standard Deviation of Individual Traders' Earnings

The cross-sectional standard deviation of individual traders' period earnings for the 10 (5+5) trader sessions is shown in Figure 6. The CGE and NCE static benchmarks have no dispersion. Standard deviation is 15 and 16.5 percent of the CGE earnings in the two sell-all markets. Compared to the simulated MI median of about 60 percent and AL median of about 28 percent, the human sell-all markets yield lower dispersion.

In the laboratory buy-sell markets (7 to 61 percent with median 34 percent) and double auctions (15 to 62 percent with median 30 percent) the standard deviation is higher than in sell-all markets (6 to 39 percent with median 13 percent).

Turning to the simulations, in buy-sell, the MI simulated median of dispersion is higher at about 51 percent and the AL simulated median is lower at about 28 percent. In

DA, the MI simulated median dispersion is lower at about 16.5 percent and AL simulated median is even lower at about 12.5 percent. In other words, the cross-sectional dispersion of earnings is higher than simulated earnings in 3 out of 6 comparisons.

Except in the first few periods of the AL simulations, there is no evidence that the dispersion of earnings declines through the replications over the periods of a session. In contrast, in the only 20 (10+10) trader session we ran for buy-sell market (see bottom left panel of Figure 8), the cross sectional standard deviation is much lower (an average of 11 percent) and declines steadily from approximately 24 percent in the first period to about 5 percent in the 20th period. Standard deviation of earnings in AL simulations is about twice as large as in human session. It seems reasonable to conclude that there are no significant differences among the standard deviation of earnings across the three mechanisms, and no consistent tendency of the standard deviation to decrease over replications.

(Insert Figure 6 about here)

6.7 Trading Volume as a Percent of CGE Volume

Observed trading volume as a percent of CGE volume is shown in Figure 7. In the top panels, presenting the sell-all markets we see that trading volume in the lab sessions was always somewhat lower (86.6 and 83.0 percent in runs 1 and 2 respectively) than required to reach CGE (100 percent). The volume is slightly higher in the buy-sell sessions (105.2 and 88.8 percent), although it is highly variable. In the double auctions (68.4 and 80.9 percent) volume is lowest on average but seems to increase over time.

The median volume of MI as well as AL simulations is slightly above the CGE level of 100 percent in sell-all markets and about equal to the CGE level in buy-sell markets. In double auctions, all simulated quintiles collapse to CGE level of 100 percent and the bands do not show up in the figure. Both the (5+5) trader double auctions as well as the (10+10) trader buy-sell market (see the last panel of Figure 8) exhibit a tendency for the trading volume to increase over periods of a session. No such tendency is present in the (5+5) trader sell-all and buy-sell markets.

(Insert Figure 7 about here)

6.8 Velocity and Quantity theory

Sell-all and buy-sell games do not allow much leeway for variations in velocity of money. Except for being able to hoard there is no strategic component to timing of trades. As the market meets only once per period and the quantity of money is well defined, in essence, the quantity theory of money holds by definition. In contrast, double auction allows the opportunity for money to turn over many times through trading within the same period.

Obtaining operationally tight definitions of money, its velocity and the endogenous variations in velocity is a theoretical challenge. Without detailed microstructure, the concept of the velocity of money is not operational. To define velocity, one needs a clear understanding of what is meant by money; a measure of its quantity; and an operational descriptions of the individuals' trading opportunity sets and strategies.

Our gaming set up assigns operational meaning to all of them albeit in a limited way. There is only one means of payment in the game. In the double auction market, in each period there is an implicitly defined minimal trading grid size, the minimal time for a trade to be offered and completed. The individuals have the strategic choice as to when to bid and thus influence velocity.

Table 5 shows the velocity (turnover) of both, money and goods. During the ten 180-second trading periods with 10 traders, 200 units of goods generated a volume of 994 (turnover rate of 5.0) in Run 1 and 1114 (turnover rate of 5.6) in run 2. The median turnover rates in simulated markets are slightly higher (5.46 For MI and 6.08 for AL) since all possible units get traded in these markets.

Total money stock of 40,000 was used to make gross payments/receipts of 252,363 (turnover of 6.3) in Run 1 and 214,716 (turnover of 5.4) in Run 2. In other words, each unit of money changed hands about six times during each session, and each unit of goods was traded more than five times. Because of the continuous trading in single units of goods, the total amount money needed to facilitate this trading was much less than what we provided. At the prices we observed (the maximum was 500) one can argue that 5,000 units of money should have been enough to move from initial endowment to CGE position in single unit transactions by traders if they alternate between selling an endowed unit and buying a unit of the other good.[14] Also, note that in simulated markets which had lower prices, turnover rates for money are also much lower (4.02 for MI and 2.18 for AL). The reason for the lower turnover rates for money are the

[14] We have not yet conducted an experiment to verify whether providing a smaller amount of money will affect its velocity.

lower prices in the simulation – e.g. prices in the AL-simulation were on average around 70, while in the lab runs they were consistently above 200.

There is no straight forward way of translating this velocity observed in the laboratory to natural economies; these data would be useful in comparative studies of alternative mechanisms in laboratory environments.

6.9 Kurtosis and Autocorrelation

Financial market returns are known to exhibit (1) excess kurtosis (fat tails relative to Gaussian distribution), (2) no significant autocorrelation of returns, and (3) significant autocorrelations of simple derivatives of returns, e.g. absolute or squared returns. The last finding hints at volatility clustering, as a significant autocorrelation of absolute returns shows that large price changes are more likely to occur after other price changes (e.g. Mandelbrot 1963a,b, Plott and Sunder 1982, Bouchaud and Potters 2001, Plerou et al. 1999, Cont 1997, 2001, and Voit 2003).

In the data generated from the double auction markets we find excess kurtosis (8.9 for good A and 9.0 for good B in Run 1, and 28.8 and 11.7 respectively in Run 2). These numbers are comparable to the excess kurtosis in the range of 5 to 20 found in stock market returns (variations depending on time horizon and whether tick data or daily closing prices are used).

We also calculated kurtosis for our MI- and AL-simulations. In the MI-simulations median kurtosis was 19, with a minimum of -2 and a maximum of 162. This is consistent with earlier findings that simple double-auction markets regularly produce excess kurtosis. Somewhat puzzling we found that median kurtosis in the 1000 AL-simulation runs was -2.9 with a maximum of 9 and a minimum of -3. Here 95 percent of the runs exhibit negative kurtosis. Most price changes were in the same range and large outliers are missing in these runs, obviously resulting from the learning mechanism (updating of price aspiration) we used.

Figure 9 presents data on the autocorrelation functions for 20 lags of retuns (top panel) and absolute returns (bottom panel) in the double auction. In the four series of laboratory returns we have no significant autocorrelations of returns after lag 1, which is consistent with the price series being random walks. The negative lag 1 autocorrelation is a well-known result of bounce between bids and asks in the double-auction mechanism. Since the simulations also involve bids and asks we observe the same negative autocorrelation for lag 1 but barely any significant values after that.

The autocorrelation function of absolute returns, however, is consistently outside the significance bounds for both goods in Run 1 (but not that long in Run 2), suggesting the possibility—but no certainty—of volatility clustering in these laboratory markets. For the simulations we find almost no persistence in autocorrelation in absolute returns and thus no volatility clustering.

(Insert Figure 9 about here)

6.10 Ranking of the Three Market Mechanisms

Competitive and non-cooperative equilibria are defined for the abstract end of the institutional spectrum of price formation processes. As we discussed in the introduction, the three minimal market institutions examined here can be located ordinally right next to this abstract end of the spectrum of market institutions. In the preceding section, we have presented the performance of the three mechanisms using various measures of performance. Table 6 presents the ordinal rankings of the three mechanisms with respect to their distance from the abstract end along these six dimensions on the basis of how they perform when they are populated by human, and MI and AL algorithmic traders.

(Insert Table 6 about here)

While there are some deviations in six specific measures of performance, it is clear that, on the whole, when these mechanisms are populated by profit-motivated human traders, the outcomes of the sell-all mechanism is the closest to the CGE as well as NCE predictions, followed by buy-sell and double auction in that order. This ordinal ranking of correspondence to the predictions of the abstract models matches the ranking of specificity (i.e., additional assumptions) needed to define each market mechanism. Perhaps it is not surprising that the increasing specificity of market mechanisms adds some distance between their performance and the abstract benchmark. To what degree this process will continue with additional specificity remains to be explored.

When profit-motivated human traders are replaced by MI and AL algorithmic traders, the quintile bands from 1,000 replications of simulated markets suggest that the rankings of their outcomes change: DA is closest to CGE predications (followed by buy-sell and sell-all) and buy-sell is closest to NCE (followed by sell-all and DA). In other words, none of the three mechanisms dominates the other two in its proximity to the predications of the static models.

7. Concluding Remarks

We report the performance of three minimal market mechanisms which are closed, full feedback models with explicit price-formation mechanisms and trade involving some form of money. The experiment reveals that (1) the non-cooperative and general competitive equilibrium models provide a reasonable anchor to locate most but not all the observed outcomes of the three market mechanisms; (2) there is some evidence that outcomes tend to get closer to GCE predictions as the number of players increases; (3) unlike well known results from many partial equilibrium double auctions, prices and allocations in our double auctions with full feedback reveal significant and apparently persistent deviations from CGE predictions; (4) the outcome paths from the three market mechanisms exhibit significant and persistent differences among them; and (5) since the dynamics of markets populated by profit-motivated human subjects is at least partially captured in markets when humans subjects are replaced by simple algorithmic traders, the importance of market structures in determining their outcomes is reinforced.

The comparison of market mechanisms populated by profit-motivated human traders with the frequency distribution (quintiles) of those populated by MI (minimally intelligent) and AL (adaptive learning) agents is a methodological innovation of this paper. In contrast, the earlier work in experimental gaming focuses on comparison with static predications of various equilibrium models alone. Presenting experimental results jointly with the frequency distribution of simulations allows–in our opinion–a better understanding of both the experimental results and the simulations.

The study of these three minimal mechanisms raises a basic issue about the level of specificity/generality at which one should identify the properties of (market) mechanisms. For example, on one hand, the double auction is an obvious—and extreme—abstraction from the complex rules and design of, say, the New York Stock Exchange. If each article in its rulebook and each feature of its design of a market helps define and determine its properties, every detail matters, and nothing can be abstracted away in the study of market mechanisms. Considered in their full details, no two markets are alike, and the study of market mechanism would constitute a voluminous encyclopedia with little generality and therefore little scientific content.

On the other hand, the competitive general equilibrium models of markets abstract away the details of trading mechanisms until they are reduced to become identical. The power, and the limitations, of these models arise from their generality.

This paper takes only a small step away from total generality by considering three minimally specific trading mechanisms—sell all, buy-sell, and double auction—in forms which are still highly abstract relative to what we see in the world of trade and commerce. We find that introducing even a small amount of specificity differentiates the paths markets take towards the general equilibrium prediction. It seems reasonable to conjecture that additional specificity in market mechanisms may reveal further differentiation in their properties, albeit at a diminishing rate and importance.

If the properties of mechanisms depend on the level of specificity/generality at which we study them, what is the appropriate level for their use? This question is not unique to economics and is shared with other sciences. Boyle's Law (pressure x absolute temperature = a constant) for gases, and Ohm's Law (voltage / current = a constant) for electricity are so powerful and simple in their generality, and yet must be modified to specific gases and circuits in most practical applications. A science consists of a spectrum of laws that extend from most general approximations at one end to increasingly specific details at the other where it blends into engineering. The appropriate level of detail and specificity can be determined only from the question sought to be answered through the investigation.

As social institutions, mass market mechanisms may have evolved to minimize the importance of individual social psychological factors and the experiments presented here support this observation. They also suggest that the non-cooperative equilibrium approach is more fundamental than the competitive equilibrium, with the former encompassing the latter as a special limiting case. Furthermore the former requires the full specification of price formation mechanisms and the simplest of such mechanisms are studied here.

An important question, both in theory and in experimentation has been raised here in the treatment of terminal value of money to the experimental subjects. Theory requires that terminal or "salvage value" conditions be imposed if the game has a finite termination. Furthermore in many formal economic models a discount factor plays an important role. Yet our runs indicated that for the most part human players pay little attention to terminal conditions until close to the very end. In further experimentation it appears to be highly desirable to devise an appropriate control to study this phenomenon.

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Table 1: Non-cooperative Equilibria for Sell-all Model

(Parameter values used in the laboratory experiments: a =200; M = 6,000; α = 10)

Number of Agents	Price(1)	Price(2)	Quantity(1)	Quantity(2)	Unspent money	Payoff
2	21.14	21.14	0.6277a	0.3723a	0.2953M	0.4834 αa
3	20.40	20.40	0.5838a	0.4162a	0.3200M	0.4929 αa
4	20.20	20.20	0.5626a	0.4374a	0.3267M	0.4961 αa
5	20.12	20.12	0.5501a	0.4499a	0.3293M	0.4975 αa
10	20.03	20.03	0.5250a	0.4750a	0.3323M	0.4994 αa
Many	20.00	20.00	0.5000a	0.5000a	0.3333M	0.5000 αa

Table 2: Non-cooperative Equilibria in Buy-sell Market

(Parameter values used in the laboratory experiments: a =200; M = 4,000; α = 10)

No. of Agents	Price(1)	Price(2)	Quantity(1)	Quantity(2)	Unspent money	Payoff
2	20.00	20.00	0.8000a	0.2000a	0.8000M	0.4000 αa
3	20.00	20.00	0.6923a	0.3077a	0.6923M	0.4615 αa
4	20.00	20.00	0.6400a	0.3600a	0.6400M	0.4800 αa
5	20.00	20.00	0.6098a	0.3902a	0.6098M	0.4878 αa
10	20.00	20.00	0.5525a	0.4475a	0.5525M	0.4972 αa
Many	20.00	20.00	0.5000a	0.5000a	0.5000M	0.5000 αa

Table 3: Design Parameters for Six Sessions of Three Market Games

Runs	Market Game	Endowments of Individuals			Money carried over?	Payoff function
		Good A	Good B	Money		
1+2	Sell-All	200 for 5 traders; 0 for 5 others	0 for 5 traders; 200 for 5 others	6,000	Yes	$10(c_A c_B)^{0.5}$ each period +0.25 final money bal.
3+4	Buy-Sell	200 for 5 traders; 0 for 5 others	0 for 5 traders; 200 for 5 others	4,000	Yes	$10(c_A c_B)^{0.5}$ each period +0.5 final money bal.
5+6	Double Auction	20 for 5 traders; 0 for 5 others	0 for 5 traders; 20 for 5 others	4,000	Yes	$100(c_A c_B)^{0.5}$ +0.5 final money bal.

Table 4 Equilibrium Predictions for the Three Market Games

Runs	Market Game	Autarky	General Equilibrium	Non-cooperative Equilibrium
1+2	Sell-All	$P_A = P_B =$ NA $X_A = 200$ or 0 $X_B = 200$ or 0 Net money =0 Points = 0	$P_A = P_B = 20$ $X_A = X_B = 100$ Net money =0 Points = 1,000	$P_A = P_B = 20.12$ $X_{own} = 110$ $X_{other} = 90$ Net money = 0 Points = 995
3+4	Buy-Sell	$P_A = P_B =$ NA $X_A = 200$ or 0 $X_B = 200$ or 0 Net money =0 Points = 0	$P_A = P_B = 20$ $X_A = X_B = 100$ Net money =0 Points = 1,000	$P_A = P_B = 20$ $X_{own} = 122$ $X_{other} = 78$ Net money = 0 Points = 976
5+6	Double Auction	$P_A = P_B =$ NA $X_A = 20$ or 0 $X_B = 20$ or 0 Net money =0 Points = 0	$P_A = P_B = 100$ $X_A = 20$ or 0 $X_B = 20$ or 0 Net money =0 Points = 1,000	$P_A = P_B = 100$ $X_A = 11$ $X_B = 9$ Net money = 0 Points = 995

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Table 5: Market data on the two double auction markets

		Goods in market	Money in market	Goods traded	Money paid	Turnover stocks	Turnover money	Transactions/ trader/period
Human	Run 5	200	40,000	994	252,362	5.0	6.3	19.9
	Run 6	200	40,000	1,114	214,716	5.6	5.4	20.3
Simulations	MI	200	40,000	1,092	160,747	5.5*	4.0*	21.8
	AL	200	40,000	1,216	87,309	6.1*	2.2*	24.3

*Median over 1,000 replications of the market.

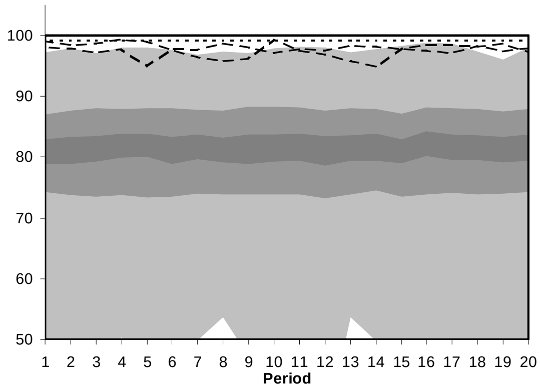
Table 6: Ranking of Three Market Mechanisms on the Basis of Distance from CGE and NCE Benchmarks

Mkt. Mech-anism	Traders	Alloc. Effic.	Money Balances	Symmetry	Prices	Dispersion Of Profits	Trading Volume	Ave. Rank	Ave. Rank for Mech.
Competitive General Equilibrium									
Sell-All	**Human**	1	1	1	1	1	2	1.17	2.08
	MI	3	2	3	2	3	3	2.67	
	AL	3	1	3	2	2.5	3	2.4	
Buy-Sell	**Human**	2	2	2	2	3	1	2.00	1.97
	MI	2	1	2	1	2	2	1.67	
	AL	2	2	2	3	2.5	2	2.25	
DA	**Human**	3	NA.	3	3	2	3	2.80	1.73
	MI	1	NA	1	3	1	1	1.4	
	AL	1	NA	1	1	1	1	1	
Non-Cooperative Equilibrium									
Sell-All	**Human**	1	1	2	1	1	1	1.17	2.06
	MI	3	2	3	2	3	3	2.67	
	AL	3	1	2.5	2	2.5	3	2.33	
Buy-Sell	**Human**	2	2	1	2	3	2	2.00	1.70
	MI	2	1	2	1	2	2	2.00	
	AL	2	2	1	3	2.5	2	2.1	
DA	**Human**	3	NA	3	3	2	3	2.80	2.17
	MI	1	NA	1	3	1	1	1.40	
	AL	1	NA	2.5	1	1	1	1.30	

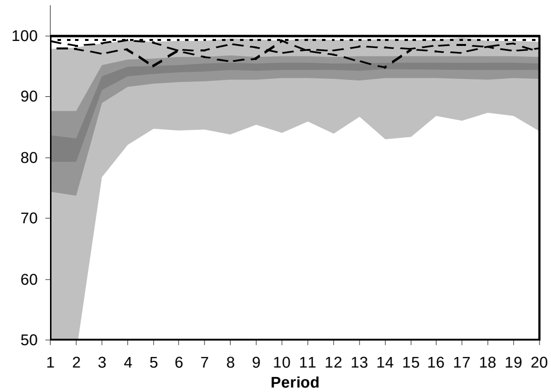
Figure 1: Efficiency of Allocations (Average Earnings)

5th quintile | 4th quintile | 3rd quintile | 2nd quintile | 1st quintile | GE | NCE | Humans

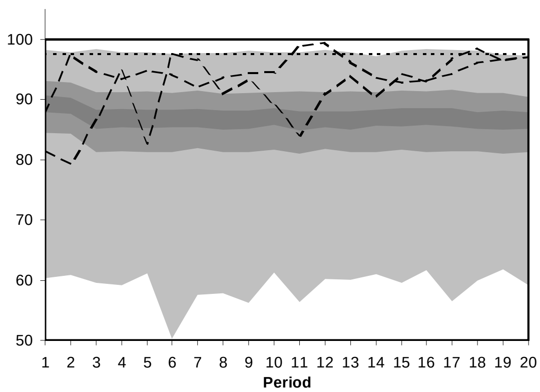
Sell-All (two lab runs plus MI in background)



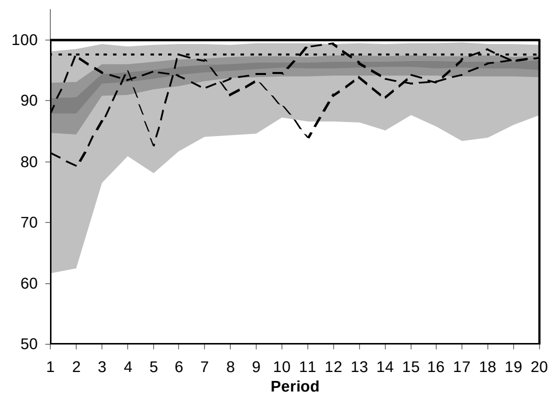
Sell-All (two lab runs plus AL in background)



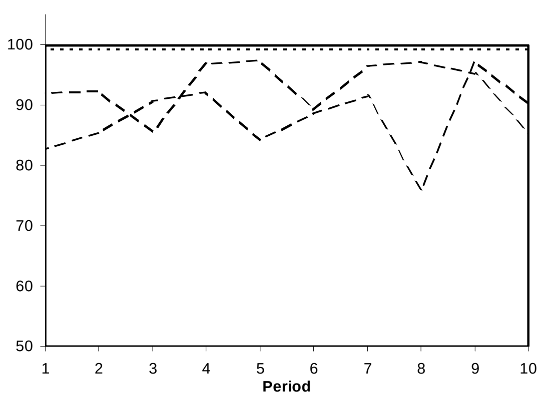
Buy-Sell (two lab runs plus MI background)



Buy-Sell (two lab runs plus AL in background)



Double Auction (two lab runs plus MI)



Double Auction (two lab runs plus AL)

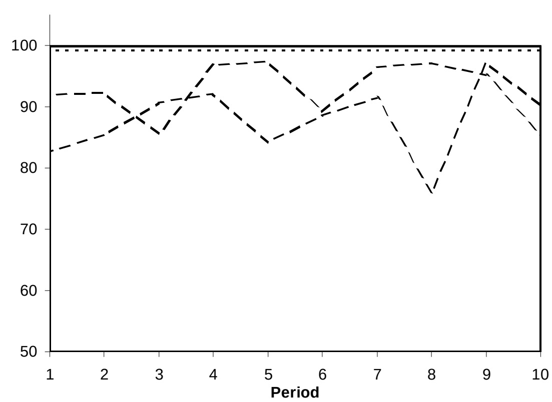


Figure 2: Average Transaction Prices of Goods A and B in the Lab and in the MI and AL Simulations (quintiles of distribution of 1000 runs)

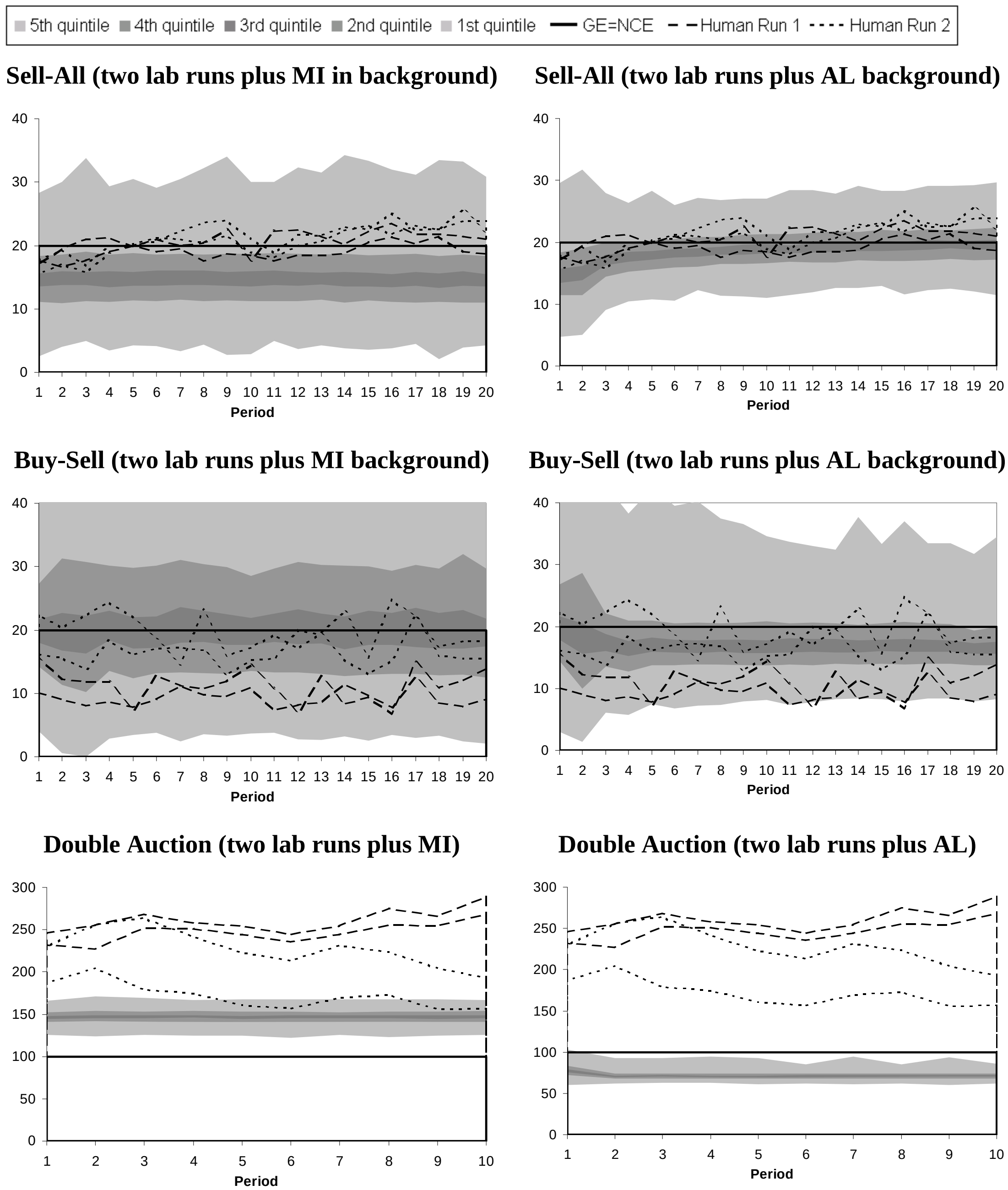
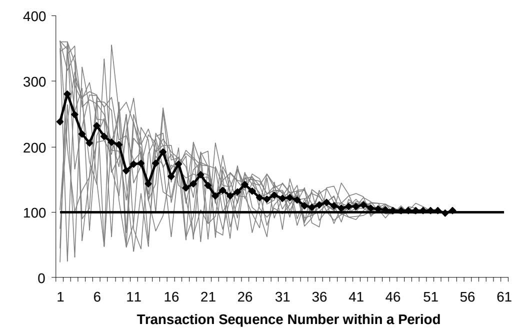


Figure 3: Double Auction Transaction Price Paths within individual Trading Periods with MI Traders (grey lines show individual runs, the black line with diamonds the average)

Double Auction (two lab runs plus MI)



Double Auction (two lab runs plus AL)

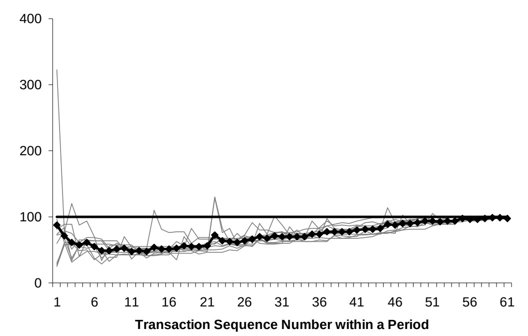
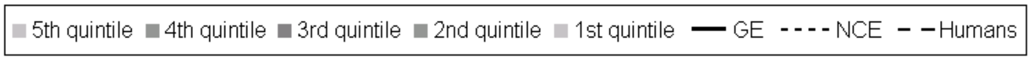
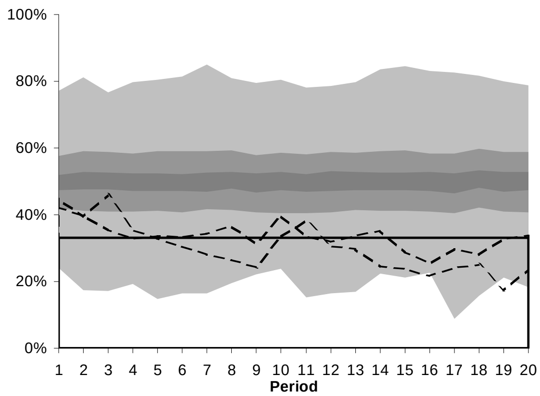


Figure 4: Symmetry of Allocations

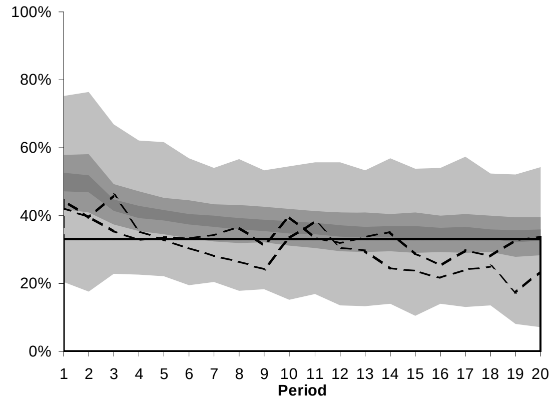
Figure 5: Unspent Money as Percentage of Initial Endowment



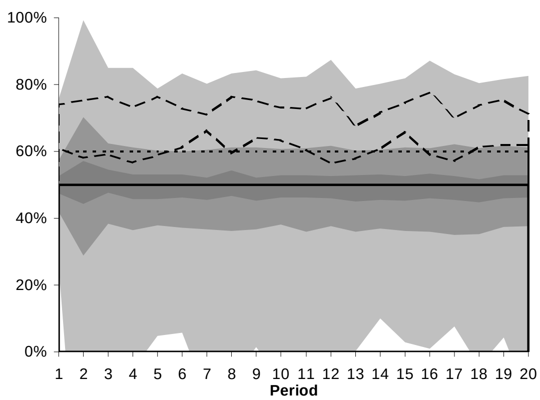
Sell-All (two lab runs plus MI in background)



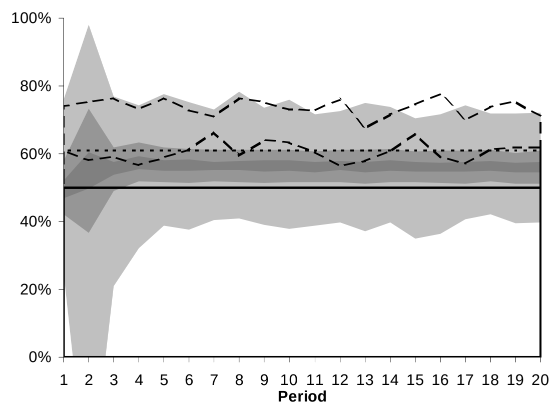
Sell-All (two lab runs plus AL in background)



Buy-Sell (two lab runs plus MI background)



Buy-Sell (two lab runs plus AL in background)



Double Auction (two lab runs plus MI)

not applicable

Double Auction (two lab runs plus AL)

not applicable

Figure 6: Standard Deviation of Individual Earnings per Period

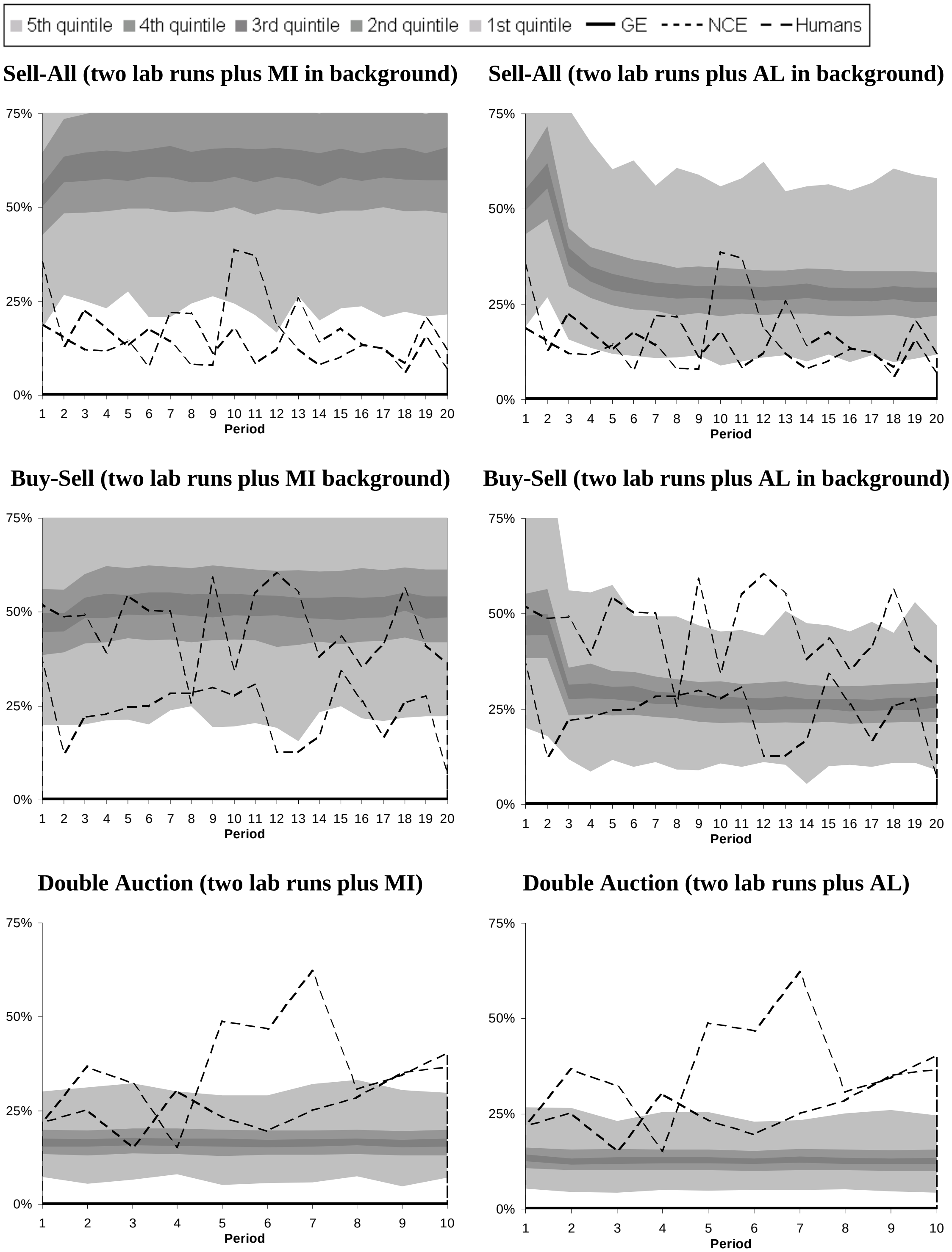


Figure 7: Goods traded as Percentage of Trade needed to achieve GCE

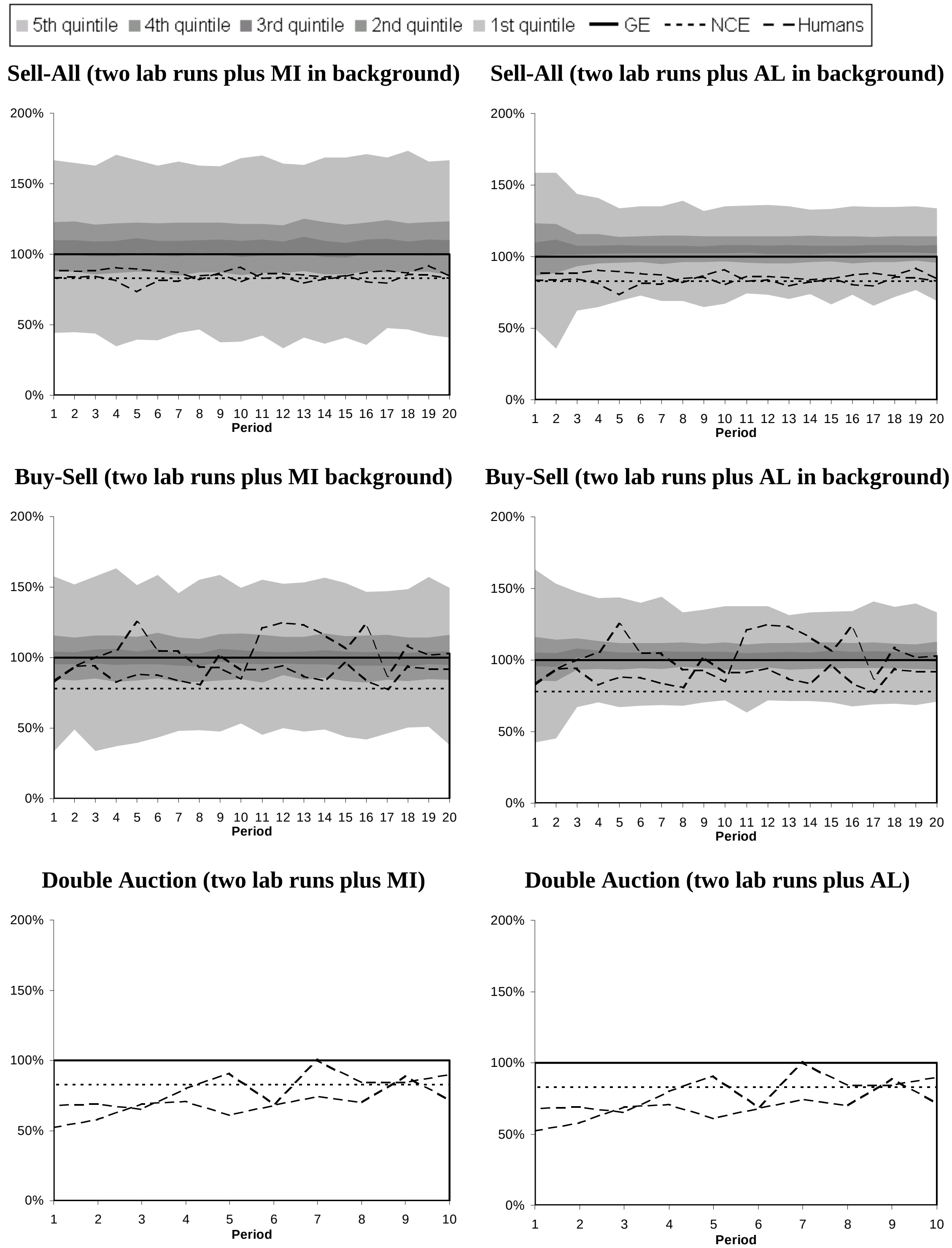


Figure 8: Overview Data for Buy-Sell Market with n=20 traders
(1 lab run + AL background)

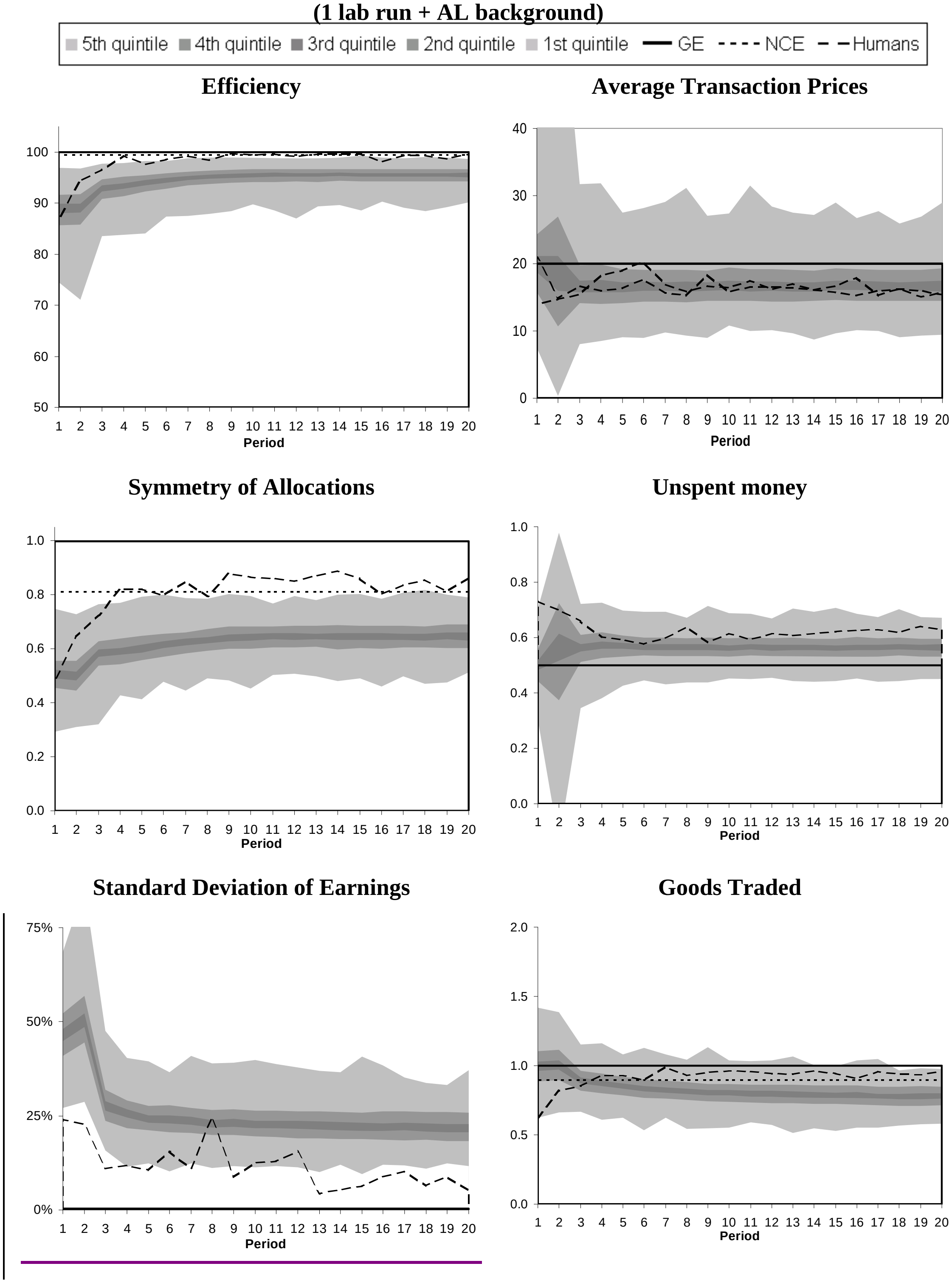
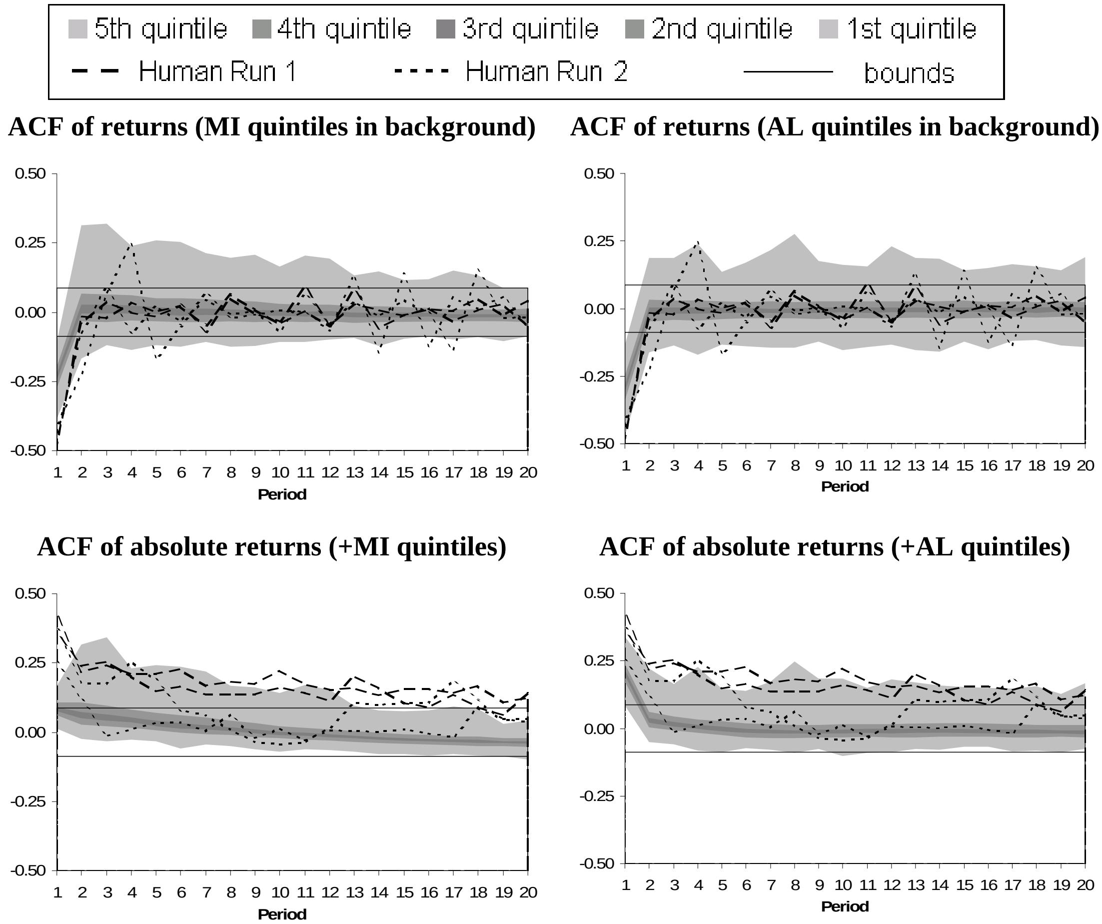


Figure 9: Autocorrelation functions of returns and absolute returns in the Double Auction (human and simulation runs)



Appendix A: Experimental instructions

Market Game 1: Sell-All (with money carried over), Sessions 1 and 2

This is an experiment in market decision making. The instructions are simple, and if you follow them carefully and make good decisions, you will earn more money, which will be paid to you at the end of the session.

This session consists of several periods and has 10 participants. At the beginning of **each** period, five of the participants will receive as income the proceeds from selling 200 units of good A, for which they have ownership claim. The other five are entitled to the proceeds from selling 200 units of good B. In addition you will get 6,000 units of money at the start of the experiment. Depending on how many goods A and B you buy and on the proceeds from selling your goods this amount will change from period to period.

During each period we shall conduct a market in which the price per unit of A and B will be determined. All units of A and B will be sold at this price, and you can buy units of A and B at this price. The following paragraph describes how the price per unit of A and B will be determined.

In each period, you are asked to enter the amount of cash you are willing to pay to buy good A, and the amount you are willing to pay to buy good B (see the center of Screen 1). The sum of these two amounts cannot exceed your current holdings of money at the beginning of the period.

The computer will calculate the sum of the amounts offered by all participants for good A. (= Sum_A). It will also calculate the total number of units of A available for sale (n_A, which will be 1,000 if we have five participants each with ownership claim to 200 units of good A). The computer then calculates the price of A, $P_A = Sum_A/n_A$.

If you offered to pay b_A to buy good A, you will get b_A/P_A units of good A.

The same procedure is carried out for good B.

Your final money balance will be your money at the beginning of the period plus the money from the sales of your initial entitlement to proceeds from A or B less the amount you pay to buy A and B:

New money holdings = Money at start of period + P_A*#A + P_B*#B – b_A – b_B

With #A and #B being either 200 or zero.

The number of units of A and B you buy (and consume), will determine the number of points you earn for the period:

Points earned = $10 * (b_A/P_A * b_B/P_B)^{0.5}$

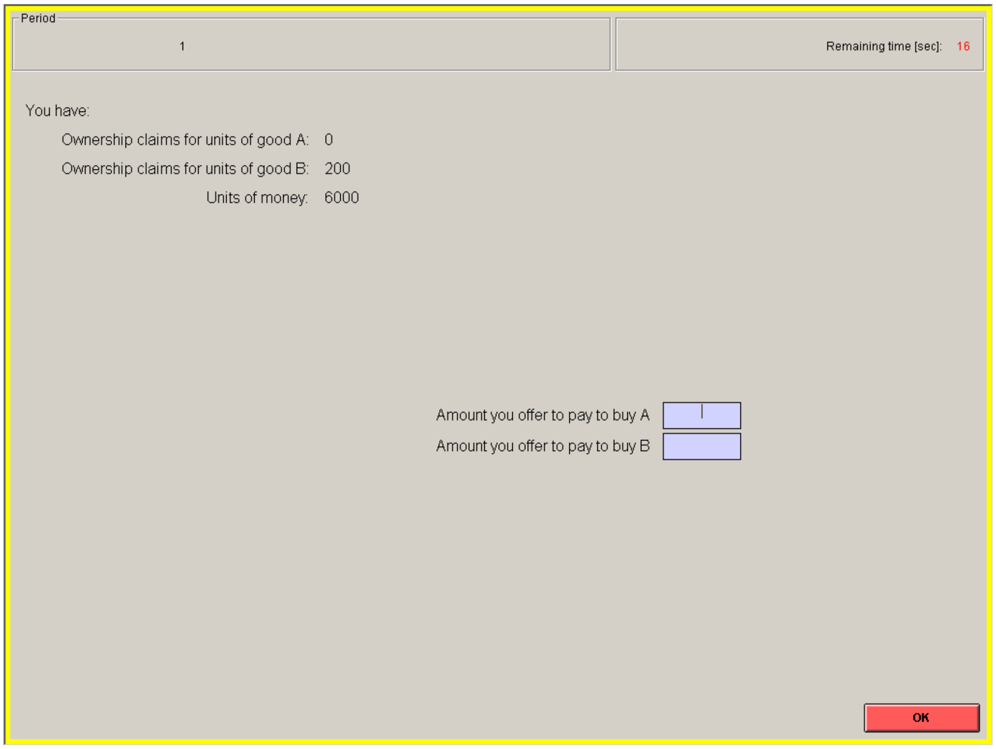
Example: If you buy 100 units of A and 100 units of B in the market you earn

$10 * (100 * 100)^{0.5} = 1,000$ points.

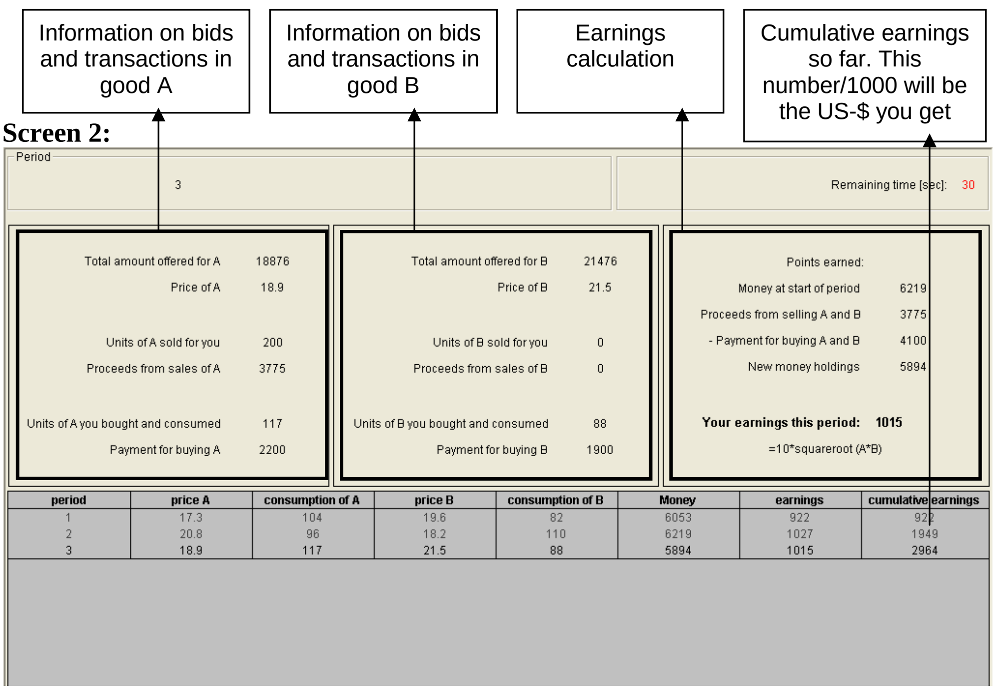
Your money holdings will only be relevant in the last period. At this time the starting endowment of 6,000 units of money will be deducted from your final money holdings. The net holdings, positive or negative, will be divided by 4 and this number will be added to your total points earned.

Screen 2 shows the example of calculations for Period 3. There are 10 participants in the market, and half of them have 200 units of A, the other half 200 units of B. Here we see a subject entitled to proceeds from 200 units of good A.

Screen 1:



Screen 2:



The earnings of each period (shown in the last column in the lower part of Screen 2) will be added up at the end of session. At the end they will be converted into real Dollars at the rate of 1,000 points = 1 US$, and this amount will be paid out to you.

How to calculate the points you earn:

Points earned = 10 * $(b_A/P_A * b_B/P_B)^{0.5}$

To give you an understanding for the formula the following table might be useful. It shows the resulting points from different combinations of goods A and B. It is obvious, that more goods mean more points, however, to get more goods you usually have to pay more, thereby reducing your money balance, which will limit your ability to buy in later periods.

		Units of good B you buy and consume										
Units of A you buy and con-sume		**0**	**25**	**50**	**75**	**100**	**125**	**150**	**175**	**200**	**225**	**250**
	0	0	0	0	0	0	0	0	0	0	0	0
	25	0	250	354	433	500	559	612	661	707	750	791
	50	0	354	500	612	707	791	866	935	1000	1061	1118
	75	0	433	612	750	866	968	1061	1146	1225	1299	1369
	100	0	500	707	866	1000	1118	1225	1323	1414	1500	1581
	125	0	559	791	968	1118	1250	1369	1479	1581	1677	1768
	150	0	612	866	1061	1225	1369	1500	1620	1732	1837	1936
	175	0	661	935	1146	1323	1479	1620	1750	1871	1984	2092
	200	0	707	1000	1225	1414	1581	1732	1871	2000	2121	2236
	225	0	750	1061	1299	1500	1677	1837	1984	2121	2250	2372
	250	0	791	1118	1369	1581	1768	1936	2092	2236	2372	2500

Examples:

1) If you buy 50 units of good A and 75 units of good B and both prices are 20, then your points from consuming the goods are 612. Your net change in money is 200 (A or B) * 20 = 4,000 minus 50 * 20 – 75 * 20 = 1,500, so you have 1,500 more to spend or save in the next period.
2) If you buy 150 units of good A and 125 units of good B and both prices are 20, then your points from consuming the goods are 1369. Your net cash balance is 200 (A or B) * 20 = 4,000 minus 150 * 20 – 125 * 20 = -1,500, so you have 1,500 less to spend or save in the next period.

Market Game 2: Buy-Sell (with money carried over), Sessions 3 and 4

This is an experiment in market decision making. The instructions are simple, and if you follow them carefully and make good decisions, you will earn more money, which will be paid to you at the end of the session.

This session consists of several periods and has 10 participants. At the beginning of each period, five of the participants will receive ownership claim to 200 units of good A, and the other five will receive ownership claim to 200 units of good B. In addition each participant will get 4,000 units of money at the start of period 1 of the experiment.

Each participant is free to sell any or all the goods he/she owns for units of money. The amount of your money balance will change depending on the proceeds from selling your goods, and how many units of goods A and B you buy, and this balance will be carried over from period to period.

During each period we shall conduct a market in which the price per unit of A and B will be determined. All units of A and B will be sold at this price, and you can buy units of A and B at this price. The following paragraphs describe how the price per unit of A and B will be determined.

In each period, you are asked to enter the cash you are willing to pay to buy the good you do not own (say A), and the number of units of the good you own that you are willing to sell (say B) (see the center of Screen 1). The cash you **bid to buy cannot exceed your money balance at the beginning of the current period**, and the units you **offer to sell cannot exceed your ownership claim of that good (200)**.

The computer will calculate the sum of the amounts of **money** offered by all participants for good A. (= Sum_A). It will also calculate the total number of units of **A offered** for sale (q_A), and determine the price of A, $P_A = Sum_A/q_A$.

If you offered to pay b_A to buy good A, you will get to buy b_A/P_A units of good A. The same procedure is carried out for good B to arrive at the price $P_B = Sum_B/q_B$ and the number of units you buy $= b_B/P_B$.

The amount of money you pay to buy one good is subtracted, and the proceeds from the sale of the other good are added, to your initial money balance of 4,000, in order to arrive at your final money balance.

Both goods are perishable and must be either sold or consumed in the current period. The number of units of A and B you own at the end of the period, c_A and c_B (unsold units of owned good and purchased units of the other good) will be consumed and determine the number of points you earn for the period:

Points earned = $10 * (c_A * c_B)^{0.5}$

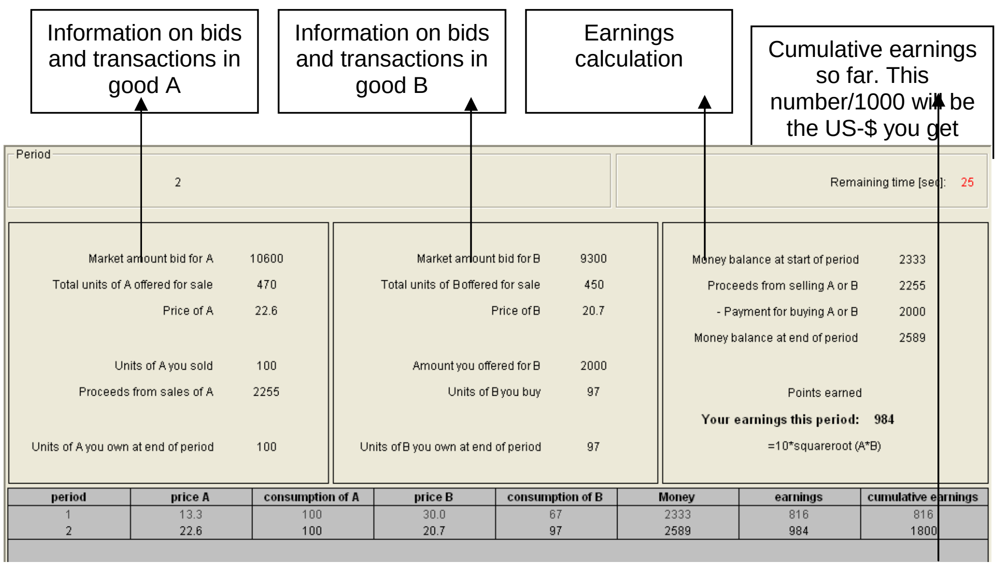
Example: If you sell 75 units of A and buy 90 units of B in the market you earn

$10 * ((200-75) * 90)^{0.5} = 1,061$ *points.*

Your cash balance holdings will help determine the points you earn only in the last period. At this time the starting endowment of 4,000 units of money will be deducted from your final money holdings. The net holdings (which may be negative) will be divided by 2 and this number will be added to (or subtracted from) your total points earned.

Screen 1:

Screen 2 shows an example of calculations for Period 2. There are 10 participants in the market, and half of them have 200 units of A, the other half 200 units of B. Here we see a subject starting with 200 units of good A.



The earnings of each period (shown in the last column in the lower part of Screen 2) will be added up at the end of session. At the end they will be converted into real Dollars at the rate of 1,000 points = 1 US$ and this amount will be paid out to you.

How to calculate the points you earn:

The points earn each period are calculated with the following formula:

$$\text{Points earned} = 10 * (c_A * c_B)^{0.5}$$

The following table may be useful to understand this relationship. It shows the resulting points from different combinations of goods A and B. Consuming more goods means more points. However, to **consume** more goods now you usually have to buy more and sell less, reducing your cash balance carried into the future.

		Units of good B you **keep** and consume										
Units of A you buy and con-sume		**0**	**25**	**50**	**75**	**100**	**125**	**150**	**175**	**200**	**225**	**250**
	0	0	0	0	0	0	0	0	0	0	0	0
	25	0	250	354	433	500	559	612	661	707	750	791
	50	0	354	500	612	707	791	866	935	1000	1061	1118
	75	0	433	612	750	866	968	1061	1146	1225	1299	1369
	100	0	500	707	866	1000	1118	1225	1323	1414	1500	1581
	125	0	559	791	968	1118	1250	1369	1479	1581	1677	1768
	150	0	612	866	1061	1225	1369	1500	1620	1732	1837	1936
	175	0	661	935	1146	1323	1479	1620	1750	1871	1984	2092
	200	0	707	1000	1225	1414	1581	1732	1871	2000	2121	2236
	225	0	750	1061	1299	1500	1677	1837	1984	2121	2250	2372
	250	0	791	1118	1369	1581	1768	1936	2092	2236	2372	2500

Examples:

1) If you sell 150 units of good A at a price of 25 (keeping 50) and buy 125 units of good B at a price of 22, you earn 612 (= 50*125) points from consuming the goods in the current period, and your net cash balance carried over to the following period changes by +1,000 (= 150 * 25 – 125 *22). You have 1,000 in cash to spend in the future.
2) If you buy 150 units of good A and sell 75 units of good B (keeping 125) and both prices are 20, then your points from consuming the goods are 1369. Your net cash balance changes by -1,500 (= -150 * 20 + 75* 20), so you have 1,500 less to spend in the future.

Market Game 3: Double Auction (money not carried over), Sessions 5 and 6

This is an experiment in market decision making. The instructions are simple, and if you follow them carefully and make good decisions, you will earn more money, which will be paid to you at the end of the session.

This session consists of several periods and has 10 participants. At the beginning of each period, five of the participants will receive 20 units of good A, and the other five will receive 20 units of good B. In addition each participant will get 4,000 units of money at the start of period 1 of the experiment (see top of Screen 1).

Each participant is free to sell any or all the goods he/she owns, or buy more units for money. The amount of your money balance will change depending on the proceeds from selling or buying goods A and B, and this balance will be carried over from period to period.

During each period we shall conduct a market in which t A and B will be traded in a double auction. The following paragraphs describe how A and B can be traded.

Trading

See Screen 1. There is a chart of transaction prices on the left, followed by two columns to trade Good A and two columns to trade Good B.

You can **buy or sell one unit** of either good in each transaction. You can buy goods in one of two ways:

(1) Enter a bid price in the light blue box above the red **BID** button on your screen, click on this red button, and wait for some trader to accept your bid (i.e., sell to you at your bid price); or

(2) Click on the red **BUY** button to buy one unit of the good at the price listed at the top of the ASK column above this red button.

Similarly, you can **sell one unit** of either good in one of two ways:

(1) Enter an ask price in the light blue box above the red **ASK** button on your screen, click on this red button, and wait for someone else to accept your ask (i.e., buy from you at your ask price); or

(2) Click on the **SELL** red button to sell one unit of a good at the price listed at the top of the BID column above this red button.

You may enter as many bids and asks as you wish. A new bid (to buy) is allowed only if you have sufficient amount of cash on hand in case all your outstanding bids are accepted (to prevent your cash holdings from dropping below zero). A new ask (to sell) is allowed if you have sufficient units of goods to sell in case all your asks are accepted (to prevent your units of goods from falling below zero).

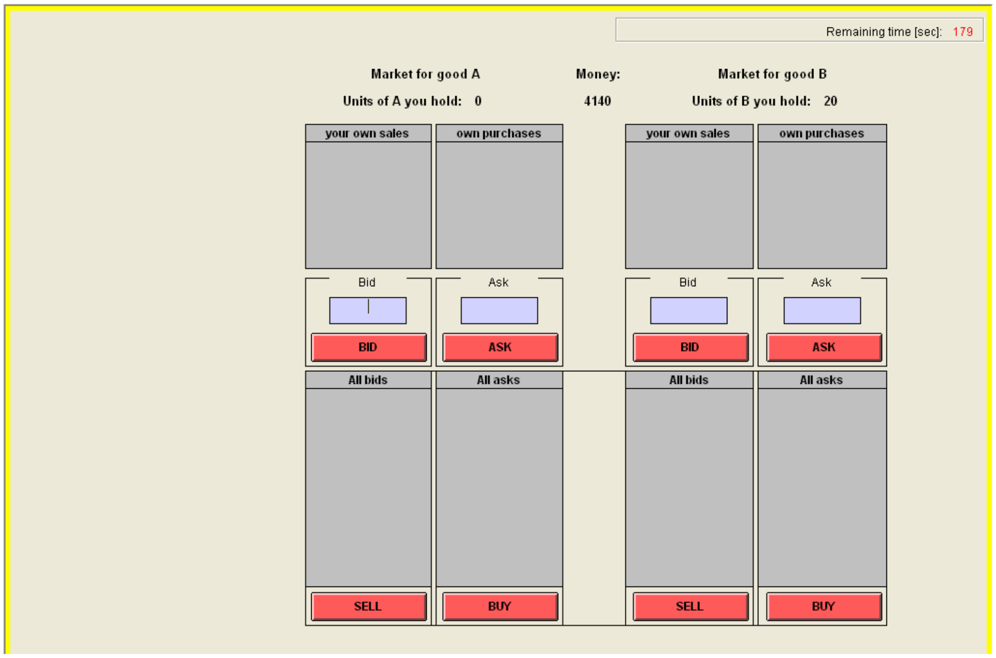
Both goods are perishable and must be either sold or consumed in the current period. The number of units of A and B you own at the end of the period, c_A and c_B will be consumed and determine the number of points you earn for the period:

Points earned = $100 * (c_A * c_B)^{0.5}$

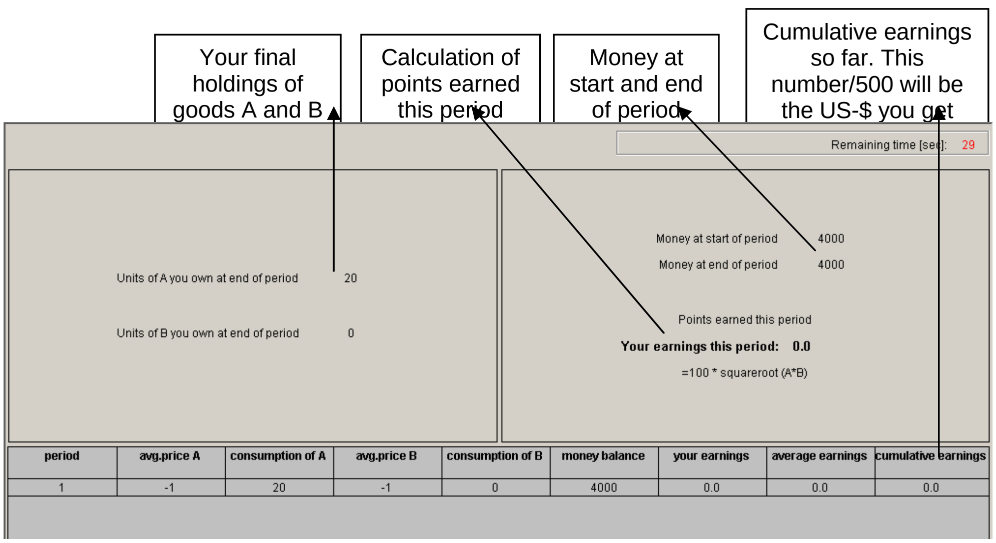
Example: If you sell own 7 units of A and 12 units of B at the end of period, you earn $100 * (7 * 12)^{0.5} = 916.5$ *points.*

Your cash balance holdings will help determine the points you earn only in the last period. At this time the starting endowment of 4,000 units of money will be deducted from your final money holdings. The net holdings (which may be negative) will be divided by 2 and this number will be added to (or subtracted from) your total points earned.

Screen 1



Screen 2 shows an example of calculations for Period 2.



The earnings of each period (shown in the last column in the lower part of Screen 2) will be added up at the end of session. At the end they will be converted into real Dollars at the rate of 500 points = 1 US$ and this amount will be paid out to you.

How to calculate the points you earn:
The points earned each period are calculated with the following formula:

$$\text{Points earned} = 100 * (c_A * c_B)^{0.5}$$

The following table may be useful to understand this relationship. It shows the resulting points from different combinations of goods A and B. Consuming more goods means more points. However, to **consume** more goods now you usually have to buy more and sell less, reducing your cash balance carried into the future.

		Units of good B you consume										
		0	**1**	**2**	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**
Units of A you consume	**0**	0	0	0	0	0	0	0	0	0	0	0
	1	0	100	141	224	316	387	447	500	548	592	632
	2	0	141	200	316	447	548	632	707	775	837	894
	5	0	224	316	500	707	866	1000	1118	1225	1323	1414
	10	0	316	447	707	1000	1225	1414	1581	1732	1871	2000
	15	0	387	548	866	1225	1500	1732	1936	2121	2291	2449
	20	0	447	632	1000	1414	1732	2000	2236	2449	2646	2828
	25	0	500	707	1118	1581	1936	2236	2500	2739	2958	3162
	30	0	548	775	1225	1732	2121	2449	2739	3000	3240	3464
	35	0	592	837	1323	1871	2291	2646	2958	3240	3500	3742
	40	0	632	894	1414	2000	2449	2828	3162	3464	3742	4000

Example: If you sell 15 units of good A (keeping 5) and buy 12 units of good B you earn 775 (= $100*(5 * 12)^{0.5}$) points from consuming the goods in the current period.

Appendix B:

Notation
b^i_j= the bid of individual i *(i=1,…,n)* in market j *(j=1,2)*
A = utility function scaling parameter, the same for each trader
p_j= price of commodity j
m= initial money holding of each trader
(a,0)= initial holding of goods of type 1
(0,a)= initial holdings of goods of type 2.

Calculations for Sell-All

An individual i initially endowed with good j wishes to maximize his payoff function which is of the form:

$$\prod^i = A\sqrt{\frac{b_1^i b_2^i}{p_1 p_2}} + (m - b_1^i - b_2^i + p_j a)$$

The calculation for the sell-all model requires to solution of the two equations derived for each trader from the first order conditions on the bidding in the two goods markets. By symmetry we need only be concerned with one type of trader.

We obtain the equation

$$\frac{b_2}{b_1}\left(\frac{(n-1)b_1 + nb_2}{nb_1 + (n-1)b_2}\right) = \frac{n}{n-1}$$

As n becomes large this yields $b_1 = b_2$**.** Substituting in for b_1 in terms of b_2 we can calculate Table 1.

Calculations for buy-sell

The payoff function for Player 1 in the buy-sell market is given by

$$\prod = A\sqrt{\frac{b_1^1}{p_2}(a - q_2^1)} + (m - b_1^1 + p_2 q_2^1)$$

And similarly for Player 2;

where q_j^i is the amount of good j offered for sale by individual i in market j

We obtain from individual maximization of these equations the following values

$$b = \frac{Aa(n-1)^2}{2(n^2+n-1)}$$

$$q = \frac{a(n-1)^2}{2(n^2+n-1)}$$

These are utilized to calculate Table 2.

Appendix C: Algorithm Used for Double Auction with Minimally Intelligent (MI) and Adaptive Learning (AL) Traders

1. Total number of traders = n
Endowment: $E_A/E_B/M$
Current balances at any point of time during trading: $c_A/c_B/m$
Adaptive learning parameter: $\lambda = 0.5$ (set $\lambda = 0$ for no learning, i.e., MI algorithm)
Set initial price aspiration = total money endowment/total goods endowment

2. Randomly pick one of the n traders in the market with equal probability (with replacement); For the chosen trader, randomly pick one of the two markets with equal probability (with replacement).

3. For the chosen market, randomly pick bid or ask with equal probability (with replacement)

> 3a. If bid is picked for the chosen trader for the chosen market A:
> Calculate $d = (100/2)\ (((c_A+1)c_B)^{0.5} - (c_A c_B)^{0.5}\ )$. Pick a uniform random number U ~ (current bid, min (d, price aspiration), and submit it as a bid for A.
>
> 3b. If bid is picked for the chosen trader for the chosen market B:
> Calculate $d = (100/2)\ (((c_B+1)c_A)^{0.5} - (c_A c_B)^{0.5}\ )$. Pick a uniform random number U ~ (current bid, min (d, price aspiration), and submit it as a bid for B.
>
> 3c. If ask is picked for the chosen trader for the chosen market A:
> Calculate $e = (100/2)\ ((-(c_A-1)c_B)^{0.5} + (c_A c_B)^{0.5}\ )$. Pick a uniform random number U ~ (max(e, price aspiration), current ask), and submit it as an ask for A.
>
> 3d. If ask is picked for the chosen trader for the chosen market B:
> Calculate $e = (100/2)\ ((-(c_B-1)c_A)^{0.5} + (c_A c_B)^{0.5}\ )$. Pick a uniform random number U ~ (max(e, price aspiration), current ask), and submit it as an ask for B.

4. If the new bid is higher than the current bid, it becomes the current bid; if the new ask is lower than the current ask, it becomes the current ask.

5. Whenever current bid and current ask cross, record a transaction at price equal to current bid or current ask (whichever was submitted earlier). Adaptively adjust new price aspiration = existing price aspiration + λ * (transaction price – existing price aspiration).

6. Let the simulation run for 25,000 iterations to complete a period. At the end of the period, Use the final c_A , c_B, and m for calculating earnings of each trader.

7. Repeat over the specified number of periods to complete the market.

8. Repeat over the specified number of replications of the market.