INFORMATION ACQUISITION
IN INTERDEPENDENT VALUE AUCTIONS

By

Dirk Bergemann, Xianwen Shi and Juuso Välimäki

July 2007

COWLES FOUNDATION DISCUSSION PAPER NO. 1619

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS
YALE UNIVERSITY
Box 208281
New Haven, Connecticut 06520-8281

http://cowles.econ.yale.edu/

Information Acquisition in Interdependent Value Auctions*

Dirk Bergemann[†] Xianwen Shi[‡] Juuso Välimäki[§]

July 20, 2007

Abstract

We consider an auction environment with interdependent values. Each bidder can learn her payoff type through costly information acquisition. We contrast the socially optimal decision to acquire information with the equilibrium solution in which each agent has to privately bear the cost of information acquisition.

In the context of the generalized Vickrey-Clarke-Groves mechanism, we establish that the equilibrium level exceeds the socially optimal level of information with positive interdependence. The individual decisions to acquire information are strategic substitutes. The difference between the equilibrium and the efficient level of information acquisition is increasing in the interdependence of the bidders' valuations and decreasing in the number of informed bidders.

JEL CLASSIFICATION: C72, C73, D43, D83.

KEYWORDS: Vickrey-Clarke-Groves Mechanism, Information Acquisition, Strategic Substitutes, Informational Efficiency.

*We thank the participants of the IESE Conference on Complementarities and Information for helpful comments. We thank the conference discussant, Tim Van Zandt, for many constructive comments that substantially improved the paper. This research is partially supported by NSF Grants #CNS 0428422 and #SES 0518929.

†Department of Economics, Yale University, New Haven, CT 06520-8268, U.S.A., dirk.bergemann@yale.edu

‡Department of Economics, Yale University, New Haven, CT 06520-8268, U.S.A., xianwen.shi@yale.edu

§Department of Economics, Helsinki School of Economics and University of Southampton, 00100 Helsinki, Finland, juuso.valimaki@hse.fi.

1 Introduction

1.1 Motivation

In the vast literature on auctions, surprisingly few papers have focussed explicitly on costly information acquisition. This is somewhat puzzling given the close connections between auctions and price formation processes in competitive markets. Milgrom (1981) uses a multi-unit Vickrey auction model to illustrate the possible coexistence of costly information and efficient information aggregation. The connections to the rational expectations equilibrium have been since worked on extensively but the issue of information acquisition has received a lot less attention. In our view, the questions relating to socially optimal information acquisition remain open for a large class of auction models.

If the auction designer has a utilitarian welfare objective and valuations are independent, it is easy to see that the agents have the correct incentives to acquire information in a socially optimal manner. In the Vickrey auction, individual payoffs, when viewed as functions of own type only, coincide with the sum of payoffs to all players (up to the addition of a constant). As a result, private incentives coincide with those of the planner.

If valuations are interdependent, however, private incentives differ from those of the planner. Maskin (1992) and Bergemann and Välimäki (2002), among others, have shown that given the decisions by other bidders, individual bidders have too strong incentives to acquire information if their valuations are positively dependent. Since these papers consider only individual deviations, it is impossible to compare centralized and decentralized information acquisition decisions. In this paper, we analyze equilibrium information acquisition in a model with binary information decisions where the bidders' true payoff types are initially unknown and each bidder may observe a costly private signal revealing her true payoff type. In a linear model with positive dependence, we show that:

(i) the private value of information exceeds the social value of information everywhere,

(ii) the private value of being informed is decreasing in the number of informed bidders.

Property (i) confirms the results in Bergemann and Välimäki (2002), and property (ii) means that bidders' information decisions are strategic substitutes. Property (ii) insures that the local comparison can be extended to the equilibrium comparison. That is, with positive dependence, more bidders become informed in the equilibria of the information acquisition game than in the social planner's solution.

Our basic model is a single-unit auction where bidders acquire (covertly) information about their true valuations simultaneously prior to the auction stage. Their valuations are assumed to be linear in the signals of both their own and their opponents. A generalized Vickrey-Clarke-Groves (VCG) mechanism is used to allocate the object. We consider pure strategy equilibrium as well as mixed strategy equilibrium of this game of information acquisition. For both types of equilibria, information decisions of bidders are strategic substitutes and the equilibria feature socially *excessive* information acquisition. Moreover, the difference between equilibrium level of information and efficient level of information diminishes as the strength of positive dependence weakens or as the number of informed bidders increases.

Since information acquisition is a binary decision, the pure strategy equilibrium may be asymmetric: in equilibrium some bidders acquire information while others do not. In this case, we show that more bidders become informed in equilibrium than in a planner's solution. For the mixed strategy equilibrium, we focus on the symmetric one. The relevant comparison in the information acquisition game is the problem where the planner chooses the same probability of becoming informed for all bidders. By this choice we can concentrate solely on the informational externalities in the problem rather than the coordination problems arising due to mixing. Again, our results show that the equilibrium probability of information acquisition exceeds the socially optimal probability. In Appendix B, we show that the basic insights of the single-unit auction carry over to a model where multiple objects are sold but bidders have unit demands.

Furthermore, we extend our positive results to a nonlinear model which nests the specification with constant absolute risk aversion as a special case. We confirm both property (i) and (ii). However, it is difficult to obtain similar results in a general nonlinear setting. The main reason is the following. In nonlinear models, the ranking of two agents' valuations, depends in general on the signal realization of a third agent. Thus, the third agent's decision to acquire information produces socially valuable information for the allocation decision between the first two agents. This is possible even in situations where the third agent does not receive the object and the private value of her information is zero. Consequently, this may lead to the reversal of ranking between private value of information and social value of information. That is, property (i) may be violated.

In the paper, we also present an example to show why property (ii) is important for our positive results. In the example, bidders' valuations are positively dependent and individual incentives to gather information are higher than social incentives. But information decisions are strategic complements, and thus violate property (ii). We show that the equilibrium level of information may be insufficient comparing to the social optimal one.

For most of the paper, we focus on the case where payoffs conditional on the allocation depend only on the bidders' true payoff types. This specification corresponds to the case where the types represent specific pieces of independent information about the true value of the object for sale. We also contrast this with a model where the valuations depend on the signals observed by the bidders. This latter model can be seen as a reduced form of a game where the initial auction stage is followed by an investment decision that depends on the reports in the auction stage. For example, the value of a license to operate in several markets will depend on the profitability of each market, which is private information of the firms operating in that market.

For the latter specification, information acquisition decisions remain strategic substitutes but individual incentives to acquire information is socially insufficient. Therefore, property (i) is violated. There are two reasons for considering this alternative model. First of all, it shows that in a model with information acquisition, the results depend delicately on whether payoffs are defined in terms of payoff types (even those that remain unknown to all bidders at the auction stage) or signals. Second, this alternative specification serves as a caveat when thinking about our results' implications for real world auction design.

1.2 Related Literature

Grossman and Stiglitz (1980) propose a standard rational expectations model to address a fundamental issue in economics: How does the market adjust to new information. In their model, ex ante identical and uninformed market traders can acquire information with cost and use it to make a profit by trading a risky asset. The fluctuation of prices makes private information (partially) revealed to uninformed traders. They conjecture that if information is costly, then equilibrium market prices cannot fully reveal private information. Furthermore they argue that costless information is not only sufficient, but also necessary for efficient market. Another important conjecture is that information acquisition decisions are strategic substitutes, the more individuals are informed, the less valuable is the information.

Traders in Grossman and Stiglitz (1980) are assumed to be price-taking. This assumption is criticized in Milgrom (1981) who argues that this assumption underlies the famous impossibility result in Grossman and Stiglitz (1980). On the other hand, Barlevy and Veronesi (2000) show that information acquisition decisions might be strategic complements under a different model specification. They show this by relaxing the standard exponential utility and normal error distribution assumptions but maintaining the price-taking behavior of the agents.

Milgrom (1981) suggests a resolution of the Grossman and Stiglitz paradox by explicitly modelling price formation. A multi-unit Vickrey common value auction with finite number of objects is proposed to illustrate the possible coexistence of costly information and efficient information aggregation. The equilibrium is shown to be an ex post Nash equilibrium. Thus, the bidders' behavior is very similar to the traders' in the rational expectation equilibrium model. The process of price formation, however, is explicitly modelled here and the price-taking assumption is fully justified. He argues that there is no tension in this model between the incentives to gather information and the informational efficiency of prices.

Milgrom (1981) results are extended and his assumptions are relaxed in Pesendorfer and Swinkels (1997) (2000). Pesendorfer and Swinkels (1997) extends Milgrom's common value framework by allowing the number of objects to grow as well as the number of bidders. They show that when both the number of objects and the number of bidders who do not receive an object becomes large, uniform price auctions are informationally efficient. In Pesendorfer and Swinkels (2000), a private component is introduced, so there is a tension between allocation efficiency and information aggregation. When the number of bidders and objects grow proportionally (the ratio of objects to bidders is bounded between 0 and 1), they show that both allocation efficiency and information aggregation are achieved in the limit.

Similar to Pesendorfer and Swinkels (2000), Jackson (2003) studies a uniform price auction with both private and common component, but assumes that information about the common component is costly to obtain. In the presence of information cost, he shows that informational efficiency is not robust even to small information costs.[1]

[1]The result in Jackson (2003) seems to be contradictory to Milgrom (1981). The reason is that they use different notions of informational efficiency. Jackson (2003) requires that the price approximates the true value of the good, while Milgrom (1981) requires that the price fully reveals private information acquired by the market participants. In terms of Jackson's criterion, the level information acquired in Milgrom's model may be insufficient. This suggests that it is important to have an efficiency criterion within a model as in this paper.

All these papers focus on information aggregation in large markets *given* endowed or acquired information. Other equally important questions are: does equilibrium information acquisition coincide with the socially optimal level? What types of models generate excessive information in equilibrium and what types of models lead to insufficient investment in information? These are the focus of the current paper. In the models mentioned above, either the object has pure common value or the common component of the object is the same for everyone, and hence the socially optimal level of information is always zero for allocation purposes. Thus, these specifications are not suitable for our analysis. The specifications contained in the current paper introduce a more general positive dependence into the model, and thus allow us to compare equilibrium level of information to the social efficient level of information.

Another strand of literature has studied the incentives for information acquisition in specific auction formats. Stegeman (1996) shows that first and second price IPV auctions result in the same incentives for information acquisition. Matthews (1984) analyzes information acquisition in affiliated private values auctions and compares the resulting revenues to the auctioneer. Finally, Persico (2000) compares the revenues from first and second price auctions in a general affiliated model where the bidders invest in information prior to participating in the auction. The main focus of these papers is on the revenue generating properties of the various formats. As a result, they say little about the overall level of information acquisition from the social welfare point of view.

This paper is organized as follows. Section 2 sets up the model. Section 3 presents the pure strategy analysis of the single unit auction with a linear payoff structure. Section 4 derives the mixed strategy equilibrium in this environment. Section 5 extends the analysis to nonlinear payoff environments. Section 6 discusses the role of the strategic substitute property and the role of the positive interdependence for the private excess returns from information. Finally, Section 7 concludes. The proofs of all results are delegated to Appendix A, and Appendix B contains the analysis of multi-unit auctions.

2 Model

We consider an auction setting with a single object for sale and I bidders. The true value of the object to bidder i is given by

$$u_i(\theta_i, \theta_{-i}) = \theta_i + \alpha \sum_{j \neq i} \theta_j, \tag{1}$$

where $0 \leq \alpha \leq 1$. The parameter α is a measure of the positive interdependence. If $\alpha = 0$, then the model is one of private values and if $\alpha = 1$, then the model is of pure common values. Each agent i has a quasilinear utility:

$$u_i(\theta) - t_i$$

where $t_i \in \mathbb{R}$ is a monetary transfer.

Initially, each bidder i only knows that the payoff relevant types $\{\theta_j\}_{j=1}^{I}$ are independently drawn from a common distribution F with support $\left[\underline{\theta}, \overline{\theta}\right] \subset \mathbb{R}_+$. The distribution F has an associated density f and a mean valuation:

$$\mu \triangleq \mathbb{E}[\theta_i].$$

Each bidder i can acquire information about her payoff relevant type θ_i at a positive cost $c > 0$. The decision to acquire information is a binary decision. If bidder i acquires information, then she observes the realization of θ_i; otherwise her information is given by the prior distribution F.

We consider the possibility of informational efficiency in an efficient allocation mechanism, namely the generalized Vickrey-Clarke-Groves mechanisms (see Maskin (1992) and Dasgupta and Maskin (2000)). A necessary and sufficient condition for the implementability of the efficient allocation is that $u_i(\theta_i, \theta_{-i})$ satisfies the single crossing property. In the current linear setting, the single crossing condition is equivalent to $\alpha \leq 1$.

We denote by y_i the highest signal among bidders other than i, that is,

$$y_i = \max_{j \neq i} \theta_j.$$

The generalized Vickrey-Clarke-Groves (VCG) mechanism is defined by an allocation rule

$$q_i(\theta_i, \theta_{-i}) = \begin{cases} 1 & \text{if} \quad \theta_i > y_i, \\ 0 & \text{if} \quad \theta_i < y_i, \end{cases} \tag{2}$$

which specifies the probability of winning conditional on the reported type profile θ. In case of a tie among the highest types, the winner is chosen among the tied bidders with equal probability. The associated payment rule

$$t_i(\theta_i, \theta_{-i}) = \begin{cases} u_i(y_i, \theta_{-i}) & \text{if} \quad \theta_i > y_i, \\ 0 & \text{if} \quad \theta_i < y_i, \end{cases} \tag{3}$$

specifies the payment of bidder i. The winning bidder makes a payment which is equal to her value of the object conditional on tying the payoff type of the second highest bidder. Consequently, the equilibrium payoff for a bidder i at type profile θ_i is

$$q_i(\theta) u_i(\theta) - t_i(\theta) = \begin{cases} \theta_i - y_i & \text{if} \quad \theta_i \geq y_i, \\ 0 & \text{if} \quad \theta_i < y_i. \end{cases}$$

In the generalized VCG mechanism, the payment of agent i is independent of the report of agent i conditional on the allocation and the equilibrium is an *ex post* Bayesian Nash equilibrium (see Dasgupta and Maskin (2000)).

With the linear specification of payoff types given by (1), the expected value of an uninformed bidder is the same as that of an informed bidder with a true payoff type $\theta_i = \mu$. The direct revelation mechanism therefore does not have to account for informed and uninformed bidders separately. In section 5 we shall look at environments in which the valuation function is not linear in the types θ_i and then an uninformed agent will have to report her entire distributional information rather than the mean μ only.

3 Pure Strategy Equilibrium

We first establish the socially optimal information policy. Subsequently, we analyze the equilibrium information policies of the agents in the generalized VCG mechanism. Initially we focus our attention on the pure strategy equilibria and consider the mixed strategy equilibrium in the next section.

3.1 Socially Efficient Policy

The bidders are ex ante identical. The socially optimal policy has to weigh the benefits of increasing information against the cost of additional information. The social value of additional information arises from the possibility of identifying an agent with a higher valuation.

As the number of informed agents increases, it becomes increasingly unlikely that an additionally informed agent will have a valuation exceeding the highest valuation among the currently informed agents. The optimal number of informed agents will therefore depend on the prior distribution and the cost of information acquisition. We denote the set of informed agents by $\{1, ..., m\}$ and the remaining set of uninformed agents by $\{m+1, ..., I\}$. The agent m is the *marginally* informed agent. We denote by θ_h the *highest* payoff type among the $(m-1)$ informed bidders, and denote the informed bidder with payoff type θ_h as bidder h.

It is straightforward to characterize the expected social gain of the m-th informed agent. The information of the m-th agent improves the social efficiency if and only if the information is pivotal for the allocation decision. By the single crossing condition, the information of agent m is pivotal if and only if it leads the planner to allocate the object to agent m. If the payoff type of agent m exceeds the types of all other agents, then it has to be larger than the payoff types of all informed agents $\{1, ..., m-1\}$ as well as all uninformed agents $\{m+1, ..., I\}$. Without loss of generality, we may assume that if the object is optimally assigned to an uninformed agent, then it is assigned to agent I.

The expected social gain of informing agent m, denoted by Δ_m^*, is then given by:

$$\Delta_m^* = \mathbb{E}_\theta \left[\left(u_m(\theta) - u_h(\theta) \right) \cdot \mathbf{1}(\theta_m \geq \theta_h \geq \mu) + \left(u_m(\theta) - u_I(\theta) \right) \cdot \mathbf{1}\left(\theta_m \geq \mu > \theta_h\right) \right], \quad (4)$$

where the indicator function $\mathbf{1}(\cdot)$ is defined as

$$\mathbf{1}(A) = \begin{cases} 1 & \text{if} \quad \text{event } A \text{ is true,} \\ 0 & \text{if} \quad \text{event } A \text{ is false.} \end{cases}$$

If bidder m becomes informed, then she may either win against bidder h or bidder I, respectively. The size of the expected gain from the improved allocation is the corresponding difference in the valuations between bidder m and the current winner. We can use the linear structure of the valuation $u_i(\theta)$ to rewrite (4) as:

$$\mathbb{E}_\theta \left[\left((\theta_m - \theta_h) + \alpha(\theta_h - \theta_m) \right) \cdot \mathbf{1}(\theta_m \geq \theta_h \geq \mu) + \left((\theta_m - \mu) + \alpha(\mu - \theta_m) \right) \cdot \mathbf{1}\left(\theta_m \geq \mu > \theta_h\right) \right].$$

With the linear payoff structure, the expected gain from information depends only on θ_m and the highest payoff type among the remaining agents, θ_h or μ. We denote by y_m the highest payoff type among all agents exclusive of m:

$$y_m = \max_{j \neq m} \theta_j = \max\{\theta_h, \mu\}. \quad (5)$$

We can now write Δ_m^* in a more compact form as:

$$\Delta_m^* = (1-\alpha)\,\mathbb{E}_{\theta_m, y_m}\left[(\theta_m - y_m)\cdot \mathbf{1}(\theta_m \geq y_m)\right]. \tag{6}$$

The case of $m = I$ is slightly different. If the last bidder I is becoming informed, then indeed all bidders are informed. The information of bidder I now becomes pivotal in two different circumstances: (i) $\theta_h > \mu$ and (ii) $\theta_h < \mu$. In the first case, bidder I did not get the object without additional information, but may now receive the object. In the second case, bidder I did get the object without information, but may now fail to get the object if her true payoff type turns out to be below θ_h.[2] The expected social gain from information is then given by:

$$\Delta_I^* = \mathbb{E}_\theta\left[(u_I(\theta) - u_h(\theta))\cdot \mathbf{1}\left(\theta_I \geq \theta_h \geq \mu\right) + (u_h(\theta) - u_I(\theta))\cdot \mathbf{1}\left(\mu > \theta_h > \theta_I\right)\right]. \tag{7}$$

By using the linear structure of the model, we can rewrite (7) to obtain:

$$\Delta_I^* = (1-\alpha)\,\mathbb{E}_{\theta_I,\theta_h}\left[(\theta_I - \theta_h)\cdot \mathbf{1}\left(\theta_I \geq \theta_h \geq \mu\right) + (\theta_h - \theta_I)\cdot \mathbf{1}\left(\mu > \theta_h > \theta_I\right)\right]. \tag{8}$$

We denote the socially optimal decision to acquire information for the m-th agent by $s_m^* \in \{0,1\}$. Then we can state the social efficient policy as follows.

Proposition 1 (Social Efficient Policy)

1. *The socially efficient policy s_m^* is given by:*

$$s_m^* = \begin{cases} 0 & \text{if } \Delta_m^* < c, \\ 1 & \text{if } \Delta_m^* \geq c. \end{cases}$$

2. *Δ_m^* is strictly decreasing in m and α for all m.*

The social gain from of an additional informed bidder is positive when the information is pivotal with positive probability. When more bidders are informed, it is less likely that the newly informed bidder has a payoff type higher than those of her opponents. Therefore,

[2] The difference in the analysis of the marginal bidder m for $m < I$ and $m = I$ is only due to our initial choice of selecting the last agent I to be the winning agent among all uninformed agents. Alternatively, the social planner could have randomized the assignment among the uninformed agents (conditional on the uninformed winning). In this case, the complicating effect for agent I would have already appeared for all agents before I.

the gross social gain from information acquisition is decreasing in the number of informed bidders.

With positive dependence, the efficiency loss from misallocation is lower than in an environment with private values. A larger α, which represents a higher positive interdependence, leads to a smaller loss from the misallocation due to the imperfect information. In consequence the social gain from information acquisition is smaller when bidders' valuations are more positively dependent.

3.2 Equilibrium Policy

We now consider the private incentives of the agent to acquire information in the generalized VCG mechanism. We maintain the notation and identify bidder m as the *marginal* bidder to acquire information. We denote the expected private gain of agent m to become informed by $\widehat{\Delta}_m$. As in the socially optimal program, we assume that if an uninformed agent is assigned the object, then it is assigned to the last agent I. If there are multiple uninformed agents, then the resulting monetary transfer will leave agent I indifferent between receiving and not receiving the object. With this convention, it is again useful to treat the case of $m < I$ and $m = I$ separately. The marginal bidder m gains from gathering information if and only if she wins the object with the information. In the generalized VCG mechanism, the m-th bidder's net gain from information is:

$$\widehat{\Delta}_m = \mathbb{E}_\theta \left[\left(u_m \left(\theta_m, \theta_{-m} \right) - u_m \left(y_m, \theta_{-m} \right) \right) \cdot \mathbf{1}(\theta_m \geq y_m) \right],$$

where we defined y_m earlier in (5) as the highest payoff type among all bidders other than m. We can use the linear payoff structure (1) to rewrite $\widehat{\Delta}_m$:

$$\widehat{\Delta}_m = \mathbb{E}_{\theta_m, y_m} \left[\left(\theta_m - y_m \right) \cdot \mathbf{1}(\theta_m \geq y_m) \right]. \tag{9}$$

We compare the private gain with the social gain of information as described by (6) and (9), respectively. If bidder m's information is not pivotal, then both the private and the social value of information about the payoff type θ_m is equal to zero. On the other hand, when bidder m's signal is pivotal, then the private gain from information about θ_m is given $\left(\theta_m - y_m \right)$, but social gain from information is only $\left(1 - \alpha \right) \left(\theta_m - y_m \right)$. The difference between the private and the social gains stem from the requirement of incentive compatibility. If agent m is to report truthfully, then she can only be asked to pay a monetary transfer equal to the lowest possible type at which the planner would be indifferent between assigning and not assigning the object to bidder m. In particular for all payoff types above her pivotal type, the monetary transfer has to stay constant, but her private benefit increases at the rate of 1. In contrast, in the social program, the marginal benefit from an increase in the payoff type of agent m is given by $(1-\alpha)(\theta_m - y_m)$ as a higher payoff type of agent m would already be beneficial at the rate α even if agent m is not assigned the object. Thus the social benefit of an increase in the payoff type of agent m is moderated by the interdependence of the valuations.

As we discussed in the context of the socially efficient policy, the description of the bidder's gain from information is somewhat different in the case of $m = I$. We have:

$$\begin{aligned}\widehat{\Delta}_I &= \mathbb{E}_\theta\left[\left(u_I(\theta_I, \theta_{-I}) - u_I(\theta_h, \theta_{-I})\right)\cdot \mathbf{1}(\theta_I \geq \theta_h \geq \mu)\right] \\ &\quad + \mathbb{E}_\theta\left[\left(u_I(\theta_h, \theta_{-I}) - u_I(\theta_I, \theta_{-I})\right)\cdot \mathbf{1}(\mu > \theta_h > \theta_I)\right],\end{aligned}$$

and using the linear payoff structure we get:

$$\widehat{\Delta}_I = \mathbb{E}_{\theta_I,\theta_h}\left[(\theta_I - \theta_h)\cdot \mathbf{1}(\theta_I \geq \theta_h \geq \mu) + (\theta_h - \theta_I)\cdot \mathbf{1}(\mu > \theta_h > \theta_I)\right]. \tag{10}$$

The first term represents bidder I's gain by winning the object from the informed bidder h when her payoff type is higher than the payoff type of the remaining agents. The second term represents bidder I's gain by avoiding to pay more for the object than it is worth to her given that her true payoff type is lower than θ_h. By analogy, we refer to the equilibrium decision of agent m to acquire information by $\widehat{s}_m \in \{0, 1\}$.

Proposition 2 (Equilibrium Policy)

1. *The equilibrium information policy is given by*

$$\widehat{s}_m = \begin{cases} 0 & \textit{if } \widehat{\Delta}_m < c, \\ 1 & \textit{if } \widehat{\Delta}_m \geq c. \end{cases}$$

2. $\widehat{\Delta}_m$ *is strictly decreasing in* m *and constant in* α *for all* m.

The private decisions of the agents to acquire information are again strategic substitutes: bidder i is less willing to become informed as more of her opponents are informed. When one more opponent gets informed, a bidder's expected gain from information acquisition is reduced in two ways. First, her chance of winning is lower. Second, conditional on winning, her expected gain from winning is lower. Thus, if there are more informed opponents, a bidder's incentives to acquire information are lower. This property ensures that the game of information acquisition has an essentially unique pure strategy equilibrium for any given level of information cost.

3.3 Welfare Comparison

We can now contrast the information decisions in the social and the equilibrium program by comparing the marginal gains of information given by

$$\{\Delta_m^*\}_{m=1}^I \text{ and } \left\{\widehat{\Delta}_m\right\}_{m=1}^I.$$

We use the strategic substitute property to guarantee the uniqueness of the equilibrium.

Proposition 3 (Welfare Analysis)
For all m,

1. *the marginal gains of information satisfy:*
$$\widehat{\Delta}_m \geq \Delta_m^*;$$
2. *the equilibrium information acquisition is (weakly) socially excessive;*
3. *the difference $\widehat{\Delta}_m - \Delta_m^*$ is increasing in α;*
4. *the difference $\widehat{\Delta}_m - \Delta_m^*$ is decreasing in m.*

With positive dependence, the equilibrium gain for a bidder to acquire information is higher than the social gain. This result, together with the strategic substitute property, implies that given any level of information cost, more bidders get informed in equilibrium than desired in the social optimum. The discrepancy between equilibrium and social policy decreases as more bidders are informed and as the positive interdependence weakens.

The statement about the comparison between the private value and the social value of information, $\widehat{\Delta}_m \geq \Delta_m^*$, is weaker than what is actually established in the proof of the proposition. The first part of the above proposition says that *on average* the social value of information is lower than the private value of information. In fact, the social value of information is lower than the private value of information at *every possible profile of payoff* *types.* To see this, notice that when bidder m's signal is not pivotal, both social value and private value of information about θ_m is zero. On the other hand, when bidder m's signal is pivotal, the private gain from information about θ_m is $(\theta_m - y_m)$, but the social gain from information is $(1-\alpha)(\theta_m - y_m)$, which is smaller as long as $\alpha > 0$.

The current results for the single unit auction generalize naturally to the case of multi-unit auctions with the associated generalized VCG mechanism. The precise statements and proofs for the multi-unit case are contained in Appendix B.

4 Mixed Strategy Equilibrium

So far we restricted our attention to the analysis of pure strategy equilibria and we showed that the pure strategy equilibrium is unique up to permutations of the ex ante identical bidders. Similarly, a socially efficient decision is always a deterministic policy. The pure strategy equilibrium is frequently an asymmetric equilibrium in that some bidders acquire and some bidders do not acquire information even though the bidders are ex ante identical. It may be difficult to see how this type of coordination might be achieved in a one-shot game. As a result, it is of interest to also analyze the symmetric mixed strategy equilibria of the game..

The mixed strategy equilibrium leads almost by definition to a socially inefficient information policy since the agents frequently fail to coordinate their decision. In order to facilitate a comparison with a socially optimal solution, we consider the problem of a planner who is restricted to choose an anonymous and hence symmetric information policy across all agents. Under this restricted efficiency criterion we conclude, as in our earlier analysis, that the equilibrium level of information acquisition will be higher than the socially efficient level.

In the current model, the cost of information acquisition is known and identical to all the bidders. It is natural to extend the model to allow for differential costs of information acquisition. If the cost of information acquisition were private information, then we can think of the mixed strategy equilibrium here as the limit equilibrium of a model with private information about the cost of information acquisition. By a standard argument first suggested by Harsanyi (1973), the mixed strategy equilibrium here can then be purified by a model with private information about the cost of information.

We restrict our analysis here to the case of moderate information cost so that the bidders acquire information with a probability strictly between 0 and 1. The case of very low and very high information costs would of course lead to degenerate mixed strategies and the only complication would come from the qualification to weak inequalities rather than equalities in the equilibrium conditions. We denote by $\sigma^* \in (0,1)$ the socially optimal probability of acquiring information, and by $\widehat{\sigma} \in (0,1)$ the equilibrium probability of acquiring information. With slight abuse of notation, we define the expected gain from information function for a representative agent by $\Delta^*(\sigma)$, and we have:

$$\Delta^*(\sigma) \triangleq \sum_{m=1}^{I} \binom{I-1}{m-1} \sigma^{m-1} (1-\sigma)^{I-m} \Delta_m^*.$$

We recall that Δ_m^* is the expected social gain from an additional informed bidder when $(m-1)$ bidders have already acquired information. The expected gain $\Delta^*(\sigma)$ represents the expected social gain from information when the planner is required to choose a single probability σ of acquiring information for all bidders. Similarly, we define $\widehat{\Delta}(\sigma)$ as

$$\widehat{\Delta}(\sigma) \triangleq \sum_{m=1}^{I} \binom{I-1}{m-1} \sigma^{m-1} (1-\sigma)^{I-m} \widehat{\Delta}_m,$$

where $\widehat{\Delta}_m$ is the individual gain for bidder m to acquire information when $(m-1)$ bidders are already informed.

In the symmetric mixed strategy equilibrium, individual bidders must be indifferent between acquiring information and staying uninformed. The expected relative gain from acquiring information must be equal to the cost of information. Thus, the equilibrium policy $\widehat{\sigma}$ must satisfy the following condition:

$$\widehat{\Delta}(\widehat{\sigma}) - c = 0. \tag{11}$$

Similarly, for the socially efficient policy σ^*, the expected social gain from information must be equal to the information cost:

$$\Delta^*(\sigma^*) - c = 0. \tag{12}$$

Proposition 4 (Strategic Substitutes)
The gains from information, $\widehat{\Delta}(\sigma)$ and $\Delta^*(\sigma)$, are strictly decreasing in σ.

The private and the social gain of a bidder from information acquisition are thus decreasing as the probability of other buyers' being informed increases. That is, the decisions to acquire information remain strategic substitutes when we allow for probabilistic policies. It also ensures that there are unique probabilities σ^* and $\widehat{\sigma}$ that satisfy equilibrium conditions (11) and (12), respectively.

Proposition 5 (Excessive Information Acquisition)
For all $\sigma^* \in (0,1)$*:* $\sigma^* < \widehat{\sigma}$.

Thus the bidders have a higher probability of acquiring information in equilibrium than in the social optimum. Proposition 4 and Proposition 5 extend the previous results in the pure strategy analysis to the symmetric mixed strategy equilibrium.

5 Nonlinear Interdependence

We now investigate to what extent the results obtained in the linear payoff model generalizes to nonlinear environments.[3] We introduce a new condition, referred to as no-crossing property, which guarantees that the ranking of any two bidders is unaffected by the payoff type of a third bidder. We then show that the class of constant absolute risk averse utility functions jointly with a linear aggregation model of the payoff types satisfies the no-crossing property.

We consider general nonlinear valuation functions given by:

$$u_i : \left[\underline{\theta}, \overline{\theta}\right]^I \to \mathbb{R}.$$

We maintain a symmetric framework across agents. Specifically we require that for all i and j and all payoff type profiles θ and θ', if θ' is a permutation of θ and $\theta_i = \theta'_j$, then

$$u_i(\theta) = u_j(\theta').$$

We also maintain the single crossing condition in order to guarantee the truthful implementation of the efficient allocation by means of the generalized VCG mechanism:

$$\theta_i \geq \theta_j \Leftrightarrow u_i(\theta) \geq u_j(\theta). \tag{13}$$

Finally, the positive interdependence in the nonlinear setting simply requires that

$$\frac{\partial u_i(\theta)}{\partial \theta_j} > 0, \ \ \forall i, j, \ \forall \theta.$$

[3] We would like to thank our discussant, Tim van Zandt, who asked us to further develop the nonlinear environment and who suggested the no crossing condition.

Clearly, the earlier linear payoff model belongs into the environment considered here. The symmetry assumption is restrictive but natural. The single-crossing property is necessary to implement the efficient allocation. The positive dependence is necessary to guarantee that the individual returns from information exceed the social returns from information. Within this setting, we introduce the no crossing property. Without loss of generality, we label the set of informed bidders by $\{1, ..., m\}$.

Assumption 1 (No Crossing Property)
The valuations $\{u_i(\theta)\}_{i=1}^{I}$ *satisfy the no-crossing property if for all* m *and all* $i, j \neq m$ *:*

$$\exists \theta_m \text{ s.t, } \mathbb{E}[u_i(\theta)|\theta_1, ..., \theta_m] > \mathbb{E}[u_j(\theta)|\theta_1, ..., \theta_m] \Rightarrow \forall \theta_m, \ \mathbb{E}[u_i(\theta)|\theta_1, ..., \theta_m] > \mathbb{E}[u_j(\theta)|\theta_1, ..., \theta_m].$$

It is easy to verify that the earlier linear payoff model satisfies the no-crossing property. The no-crossing property requires that the ranking of the expected payoff of two alternatives, i versus j, is constant across all payoff types θ_m of agent m. If all agents are informed about their payoff type and $m = I$, then the no-crossing property is automatically satisfied by the single crossing condition (13) which simply states that the binary ranking of the alternatives i and j is determined exclusively by their respective payoff types θ_i and θ_j. The no-crossing property condition extends the uniformity of the binary ranking to the comparison of two bidders i and j, where i is informed about her payoff type and j is not informed about her payoff type. If the no-crossing property is violated, then the information of agent m may be socially valuable in determining the allocation between i and j without agent m ever getting the object. But if agent m does not receive the object, then she will have very weak private incentives to acquire information even though it would be socially valuable. In consequence, the ranking between the social incentive and the private incentive to acquire information may be reversed. The no-crossing condition has an important implication.

Lemma 1
If the no-crossing property is satisfied, then for all m *and all* $i, j \neq m$, *if*

$$\mathbb{E}[u_i(\theta)|\theta_1, ..., \theta_{m-1}] > \mathbb{E}[u_j(\theta)|\theta_1, ..., \theta_{m-1}],$$

then

$$\mathbb{E}[u_i(\theta)|\theta_1, ..., \theta_m] > \mathbb{E}[u_j(\theta)|\theta_1, ..., \theta_m].$$

In words, the rank between bidder i and j after bidder m gets informed is the same as the rank between them before bidder m becomes informed. The definition of the marginal gains from information, given by $\widehat{\Delta}_m$ and Δ_m^*, extends in the natural way to the nonlinear environment. The next proposition shows that the private gain from acquiring information will be higher than the social gain from information.

Proposition 6 (Excessive Incentives)
If the no-crossing property is satisfied, then $\widehat{\Delta}_m > \Delta_m^*$ *for all* m.

The following graph illustrates the no-crossing property and the difference between private value and social value of information. The no-crossing property requires that the two curves

$$\mathbb{E}\left[u_i\left(\theta\right)|\theta_1, ..., \theta_m\right] \text{ and } \mathbb{E}\left[u_j\left(\theta\right)|\theta_1, ..., \theta_m\right]$$

do not cross each other for all θ_m, as shown in the graph. In addition, the curve $\mathbb{E}\left[u_m\left(\theta\right)|\theta_1, ..., \theta_m\right]$ crosses both $\mathbb{E}\left[u_i\left(\theta\right)|\theta_1, ..., \theta_m\right]$ and $\mathbb{E}\left[u_j\left(\theta\right)|\theta_1, ..., \theta_m\right]$ only once, which is implied by our single-crossing assumption.

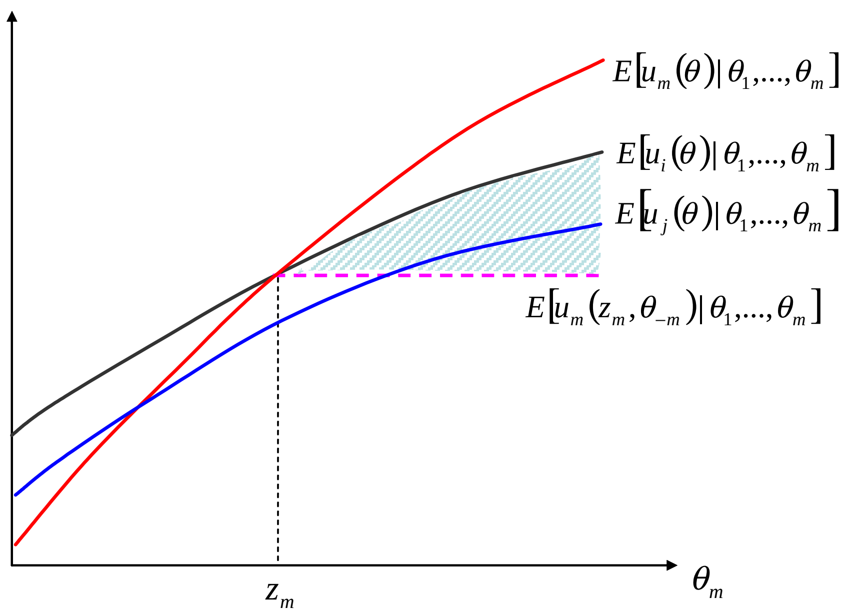

Figure 1: No-crossing property, private gain and social gain from information

Now we can use the graph to illustrate why the private gain from information is higher than the social gain from information. Let's consider the information decision of the marginal bidder m. Suppose bidder i has the highest valuation among bidder m's opponent and let z_m solve the following equation

$$\mathbb{E}\left[u_m\left(\theta\right)|\theta_1, ..., \theta_{m-1}, z_m\right] = \mathbb{E}\left[u_i\left(\theta\right)|\theta_1, ..., \theta_{m-1}, z_m\right].$$

That is, z_m is the cutoff signal that bidder m will overtake bidder i as the winner of the object. In the graph, z_m is the value where the curves $\mathbb{E}\left[u_m\left(\theta\right)|\theta_1,...,\theta_m\right]$ and $\mathbb{E}\left[u_i\left(\theta\right)|\theta_1,...,\theta_m\right]$ cross.

If $\theta_m < z_m$, then the social planner will assign the object to bidder i, and both private value and social value of information about θ_m are zero. On the other hand, if $\theta_m \geq z_m$, the social planner will assign the object to bidder m, and the social gain from information is

$$\mathbb{E}\left[u_m\left(\theta\right)|\theta_1,...,\theta_{m-1},\theta_m\right] - \mathbb{E}\left[u_i\left(\theta\right)|\theta_1,...,\theta_{m-1},\theta_m\right]. \tag{14}$$

In order to induce bidder m to tell the truth, the payment of the generalized VCG mechanism must be independent of θ_m conditional on allocation. In the graph, bidder m's payment conditional on winning is the dashed horizontal line:

$$\mathbb{E}\left[u_m\left(\theta\right)|\theta_1,...,\theta_{m-1},z_m\right] \text{ or } \mathbb{E}\left[u_i\left(\theta\right)|\theta_1,...,\theta_{m-1},z_m\right]$$

Therefore, the private gain from information about θ_m is

$$\mathbb{E}\left[u_m\left(\theta\right)|\theta_1,...,\theta_{m-1},\theta_m\right] - \mathbb{E}\left[u_i\left(\theta\right)|\theta_1,...,\theta_{m-1},z_m\right]. \tag{15}$$

Comparing expression (14) and (15), we can see that, for any realization of $\theta_m \geq z_m$, the private gain exceeds the social gain from information by

$$\mathbb{E}\left[u_i\left(\theta\right)|\theta_1,...,\theta_{m-1},\theta_m\right] - \mathbb{E}\left[u_i\left(\theta\right)|\theta_1,...,\theta_{m-1},z_m\right]$$

which is always positive because $\mathbb{E}\left[u_i\left(\theta\right)|\theta_1,...,\theta_{m-1},\theta_m\right]$ is increasing in θ_m. Integrating the difference with respect to the density of θ_m, we can obtain the expected difference between the private gain and the social gain from information, as indicated by the shaded area in the graph.

Next we demonstrate that the private decisions of the bidders to acquire information will remain strategic substitutes in the nonlinear environment, provided that the no-crossing property is satisfied. In consequence, the pure strategy equilibrium in the nonlinear environment will be unique and the equilibrium will necessarily display excessive information acquisition compared the socially optimal solution.

Proposition 7 (Strategic Substitutes)
For all $m < I$*:* $\widehat{\Delta}_m \leq \widehat{\Delta}_{m-1}$.

The current result is stated and proved only for $m < I$. While we suspect that it will naturally extend to the final bidder, we have not been able to prove this result in the nonlinear setting. A leading example of the no crossing property is the class of constant absolute risk aversion utilities with a linear payoff type structure. Consider the valuation function $u_i(\theta)$ given by:

$$u_i(\theta_i, \theta_{-i}) = 1 - \exp\left[-r\left(\theta_i + \alpha \sum_{j \neq i} \theta_j\right)\right], \tag{16}$$

with a coefficient of absolute risk aversion $r > 0$ and $\alpha \in (0, 1)$. It is easy to verify that the valuation model given by (16) is symmetric, satisfies the single crossing condition and displays positive interdependence. The no-crossing property is satisfied as the payoff types are multiplicatively separable under the exponential utility function and the linear type structure. It is further apparent that the no-crossing condition is valid for any multiplicative separable specification of the valuation function.

If the payoff functions are neither additively nor multiplicatively separable, then the no-crossing property may be violated. This is easily shown with the following example of three bidders, $i \in \{1, 2, 3\}$. The valuation of bidder i is given by:

$$u_i(\theta) = (1 + \theta_i)^2 + (\theta_j + \theta_k)\, \theta_j \theta_k. \tag{17}$$

The payoff types θ_i are assumed to be independently drawn from the uniform distribution on the unit interval. Clearly, the model is symmetric, satisfies the single-crossing property and displays positive interdependence. Within the example given by (17) it is now easy to show that the ranking of an informed agent, say 1 and an uniformed agent, say 3, is affected by the payoff type of agent 2. We omit the calculations.

6 Positive Interdependence

The analysis of the auction environment began with positive interdependence of the payoff types. We established that the private and the social decisions to acquire information are strategic substitutes and that the private returns from information exceed the social returns from information. Consequently, it might be plausible to deduce that the driving force behind the results is the positive interdependence of the payoff types. In this section, we show that positive intrdependence is not sufficient to make information acquisition decisions strategic substitutes. In order to obtain the result on excessive information acquisition, a separate argument for the strategic substitute property is needed.

6.1 Strategic Complements

With positive interdependence, Bergemann and Välimäki (2002) show that the individual bidders will have socially excessive incentives to acquire information given the information decision of the remaining agents. The results in Bergemann and Välimäki (2002) are thus about a local property of the decision of agent i in the sense that the decision of the remaining agents are kept constant. In particular, the characterization of the individual decision may not transfer to the equilibrium decisions of the agents. We now provide an example of positive interdependence with the property that the decisions to acquire information are strategic complements rather than strategic substitutes. Despite the positive interdependence, an equilibrium of the game will display a lower level of information acquisition than the social equilibrium.

Suppose there are two bidders, $i \in \{1, 2\}$, competing for single object. The payoff structure is the linear payoff structure of the previous sections:

$$u_i(\theta_i, \theta_j) = \theta_i + \alpha\theta_j,$$

but we now allow for negative payoff types. The social planner can either allocate the object to bidder 1 or 2 or decided not to allocate the object at all. In particular it is efficient not to assign the object if the expected valuation of both bidders is below zero.

For concreteness, we consider $\alpha = 0.5$ and assume that the payoff types θ_i are independently drawn from the uniform distribution with the support given by $[-5, 1]$. It is now easy to verify that the decisions to acquire information are strategic complements. If bidder j stays uninformed, then it is efficient not to assign the object to bidder i for any realization of her payoff type. In consequence, the value of information for agent i is zero if agent j does not acquire information. On the other hand, if agent j does acquire information, then a positive realization by both agents may lead to the assignment of the object to agent i and hence there is positive value of information.

Therefore, consistent with the analysis of Bergemann and Välimäki (2002), the private gain is weakly higher than the social gain from information. But the analysis of the individual decision of agent i does not necessarily translate into a corresponding equilibrium characterization. For example, we can show by elementary computations that if the cost of information is $c = 1/100$, then the efficient policy requires that both bidders acquire information. However, due to the strategic complementarity, there are now two pure strategy equilibria for the game of information acquisition: one in which both bidders remain uninformed, the other one in which both bidders become informed. It is therefore possible that the two bidders fail to coordinate on the efficient equilibrium and stay uninformed.[4] The key for the failure of the equilibrium characterization is that the private gain from information is increasing in the number of informed bidders, that is, information decisions are strategic complements.

6.2 Privately versus Socially Pivotal Signals

The next class of allocation problems also represent a natural departure from the initial model. There are I agents and the value of the assignment to a bidder is determined by her own payoff type as well as the K highest payoff types among the remaining bidders, with $K < I - 1$. In other words, the agents with the lowest payoff types do not matter. More precisely, the value of the assignment to bidder i is given by:

$$u_i(\theta) = \theta_i + \alpha \sum_{k=1}^{K} y_{ik}, \tag{18}$$

where y_{ik} is the k-th highest signal among the remaining agents exclusive of i. We may interpret the model as one in which an exclusive license is auctioned among I bidders. The license gives the winner the right to operate in her own market and in the K most valuable neighboring markets. The value of the license to operate in K markets depends consequently on the K highest signals.

The difference between this license model and the earlier single unit auction model is easily understood by distinguishing between *privately pivotal* and *socially pivotal* information. In the license auction, an information is privately pivotal if it affects the identity of the winning bidder. Yet, an information can be socially pivotal if it changes the determination of the neighboring markets in which the winner will operate. In the single unit auction model,

[4] We note that the negative payoff types are not necessary to generate the strategic complementarity in the information decision among the agents. A similar result could be generated in an asymmetric three bidder model with positive types in which the information of agent i is relatively more important for agent j than for agent k.

a socially pivotal signal must be a privately pivotal signal as well. Yet, in the current license model, a signal could be socially pivotal but not privately pivotal.

For simplicity we shall assume that the expected value of every market is given by $\mu = 0$ and hence only informed agents can provide efficient licensing agents (and markets). The assignment rule of the associated VCG mechanism is given by:

$$q_i(\theta_i, \theta_{-i}) = \begin{cases} 1 & \text{if} \quad \theta_i > y_{i1}, \\ 0 & \text{if} \quad \theta_i < y_{i1}, \end{cases}$$

and it assigns the object to the bidder with the highest payoff type. The associated payment rule is given by:

$$t_i(\theta_i, \theta_{-i}) = \begin{cases} y_{i1} + \alpha \sum_{k=1}^{K} y_{ik} & \text{if} \quad \theta_i > y_{i1}, \\ 0 & \text{if} \quad \theta_i < y_{i1}. \end{cases}$$

In the case of a tie, the license is randomly assigned among the competing bidders. With this specification, bidders' valuations remain positively interdependent and the decisions to acquire information will be shown to be strategic substitutes. Yet the equilibrium of the information game may feature socially *insufficient* information acquisition.

Now consider the bidders' private incentive to acquire information. As before, let $i = 1, ..., m$ denote the first m bidders who acquire information and let $i = m + 1, ..., I$ denote the remaining bidders who stay uninformed. Since a bidder can gain only by becoming informed and winning the object, the m-th bidder's expected individual gain from information acquisition is simply

$$\mathbb{E}_\theta \left[(u_m(\theta) - t_m(\theta)) \cdot \mathbf{1}\{\theta_m \geq y_{m1}\} \right]. \tag{19}$$

The next proposition shows that the private gain of the m-th bidder from acquiring information is decreasing in m. That is, when more bidders get informed, bidder i has a lower incentive to acquire information and hence the information decisions remain strategic substitutes.

Proposition 8 (Strategic Substitute)
The private value of information $\widehat{\Delta}_m$ is decreasing in m for $K < m < I$.

We now consider the information decision from the social perspective. The social gain from the m-th bidder acquiring information is given by:

$$\mathbb{E}\left[(\theta_m - y_{m1}) \cdot \mathbf{1}\{\theta_m \geq y_{m1}\} + \alpha \sum_{k=1}^{K} \left(\theta_m - y_{m(k+1)}\right) \cdot \mathbf{1}\left\{y_{mk} \leq \theta_m < y_{m(k-1)}\right\} \right]. \tag{20}$$

It is straightforward to see that the social gain from information is always higher than the private gain from information as long as α is strictly positive.

Proposition 9 (Excessive Incentives)
If $\alpha > 0$, then $\widehat{\Delta}_m \leq \Delta^*_m$ for all $m < I$.

Thus, the individual incentive to acquire information is socially insufficient in the current specification. Together with the strategic substitute property established in proposition 8, we can conclude that the equilibrium information acquisition is socially insufficient for some level of information cost c.

7 Conclusion

In a model with positively interdependent valuations, the equilibrium level of information acquisition differs from the socially efficient level. This paper shows that in many specifications of the model, information acquisition is excessive in equilibrium. This opens up a number of new questions. How could a planner correct the incentives? If information acquisition is covert, it is not easy for the planner to change the cost of information and the uninformed bidders can always pretend to be the informed bidders. Participation fees reduce the bidders' expected payoffs, but at the same time they discourage uninformed bidders from participating. This may result in suboptimal allocations. Another possibility could be that the object is assigned at random between the highest bidders if the bids are relatively close. The welfare losses from such a misallocation are small since this happens only when valuations do not differ much. On the other hand, the payments of all winning bids are increased and hence the expected payoffs decrease as a result of such a policy.[5]

Our specification features positive dependence among bidders' valuations. If bidders valuations are negatively dependent, one can show that in our basic specification, information acquisition decisions will remain strategic substitutes but the equilibrium information acquisition will be insufficient comparing to the social optimum.

Finally, in our model, the supply side of the model is fixed. It would be interesting to study information acquisition in large double auctions where the equilibrium price is determined by competing buyers and sellers.

[5]If bidders cannot acquire information prior to participation, Cremer, Spiegel, and Zheng (2003) construct a sequential mechanism that extracts full surplus and induces efficient level of information acquisition.

8 Appendix A: Proofs

This appendix contains the proofs of all the results presented in the main body of the paper. We first state and establish a lemma that will be used in proving both Proposition 1 and 2.

Lemma 2

1. *For all* m,
$$\mathbb{E}_{\theta_m, y_m}\left[(\theta_m - y_m) \cdot \mathbf{1}(\theta_m \geq y_m)\right]$$
is decreasing in m.

2. *For all* m,
$$\mathbb{E}_{\theta_m, \theta_h}\left[(\theta_h - \theta_m) \cdot \mathbf{1}\left(\mu > \theta_h > \theta_m\right) - (\theta_m - \mu) \cdot \mathbf{1}\left(\theta_m \geq \mu > \theta_h\right)\right] < 0.$$

Proof of Lemma 2. We denote by G_m and g_m the cumulative distribution and density of θ_h, respectively. Since bidder h has the highest signal among $(m-1)$ informed bidders, we have the usual order statistics with

$$G_m\left(\theta_h\right) = F^{m-1}\left(\theta_h\right)$$

and

$$g_m\left(\theta_h\right) = (m-1)\, F^{m-2}\left(\theta_h\right) f\left(\theta_h\right).$$

For part 1, we observe that

$$\begin{aligned}
& \mathbb{E}_{\theta_m, y_m}\left[(\theta_m - y_m) \cdot \mathbf{1}(\theta_m \geq y_m)\right] \\
= & \int_{\mu}^{\overline{\theta}} \int_{\mu}^{\theta_m} \left(\theta_m - \theta_h\right) g_m\left(\theta_h\right) f\left(\theta_m\right) d\theta_h d\theta_m + \int_{\mu}^{\overline{\theta}} \int_{\underline{\theta}}^{\mu} \left(\theta_m - \mu\right) g_m\left(\theta_h\right) f\left(\theta_m\right) d\theta_h d\theta_m \\
= & \int_{\mu}^{\overline{\theta}} \left(-\left(\theta_m - \mu\right) F^{m-1}\left(\mu\right) + \int_{\mu}^{\theta_m} F^{m-1}\left(\theta_h\right) d\theta_h \right) f\left(\theta_m\right) d\theta_m + \int_{\mu}^{\overline{\theta}} \left(\theta_m - \mu\right) F^{m-1}\left(\mu\right) f\left(\theta_m\right) d\theta_m \\
= & \int_{\mu}^{\overline{\theta}} \left[\int_{\mu}^{\theta_m} F^{m-1}\left(\theta_h\right) d\theta_h \right] f\left(\theta_m\right) d\theta_m.
\end{aligned}$$

The first equality follows by the definition of y_m, and the second equality is a result of integration by parts. It is easy to see from the last expression that $\mathbb{E}_{\theta_m, y_m}\left[(\theta_m - y_m) \cdot \mathbf{1}(\theta_m \geq y_m)\right]$ is decreasing in m.

For part 2, we note that

$$\begin{aligned}
& \mathbb{E}_{\theta_m,\theta_h}\left[(\theta_h-\theta_m)\cdot\mathbf{1}\left(\mu>\theta_h>\theta_m\right)-(\theta_m-\mu)\cdot\mathbf{1}\left(\theta_m\geq\mu>\theta_h\right)\right] \\
= & \int_{\underline{\theta}}^{\mu}\int_{\underline{\theta}}^{\theta_h}(\theta_h-\theta_m)\,f(\theta_m)\,g_m(\theta_h)\,d\theta_m d\theta_h-\int_{\underline{\theta}}^{\mu}\int_{\mu}^{\overline{\theta}}(\theta_m-\mu)\,f(\theta_m)\,g_m(\theta_h)\,d\theta_m d\theta_h \\
= & \int_{\underline{\theta}}^{\mu}\left[\int_{\underline{\theta}}^{\theta_h}(\theta_h-\theta_m)\,f(\theta_m)\,d\theta_m-\int_{\mu}^{\overline{\theta}}(\theta_m-\mu)\,f(\theta_m)\,d\theta_m\right]g_m(\theta_h)\,d\theta_h \\
< & \int_{\underline{\theta}}^{\mu}\left[\int_{\underline{\theta}}^{\mu}(\mu-\theta_m)\,f(\theta_m)\,d\theta_m-\int_{\mu}^{\overline{\theta}}(\theta_m-\mu)\,f(\theta_m)\,d\theta_m\right]g_m(\theta_h)\,d\theta_h \\
= & \ 0.
\end{aligned}$$

Thus, the proof is complete.■

Proof of Proposition 1. From the social point of view, the m-th bidder should acquire information if and only if the social expected gain Δ_m^* is higher than information cost c. That is,

$$s_m^*=\begin{cases}0 & \text{if} \quad \Delta_m^*<c \\ 1 & \text{if} \quad \Delta_m^*\geq c\end{cases}.$$

The result $\Delta_1^*>\Delta_2^*>....>\Delta_{I-1}^*$ follows from (6) and part 1 of Lemma 2. In order to prove $\Delta_{I-1}^*>\Delta_I^*$, note that

$$\begin{aligned}
\Delta_I^* &= (1-\alpha)\,\mathbb{E}_{\theta_I,\theta_h}\left[(\theta_I-\theta_h)\cdot\mathbf{1}\left(\theta_I\geq\theta_h\geq\mu\right)+(\theta_h-\theta_I)\cdot\mathbf{1}\left(\mu>\theta_h>\theta_I\right)\right] \\
&= (1-\alpha)\,\mathbb{E}_{\theta_I,y_I}\left[(\theta_I-y_I)\cdot\mathbf{1}(\theta_I\geq y_I)\right] \\
&\quad +(1-\alpha)\,\mathbb{E}_{\theta_I,\theta_h}\left[(\theta_h-\theta_I)\cdot\mathbf{1}\left(\mu>\theta_h>\theta_I\right)-(\theta_I-\mu)\cdot\mathbf{1}\left(\theta_I\geq\mu>\theta_h\right)\right] \\
&< (1-\alpha)\,\mathbb{E}_{\theta_I,y_I}\left[(\theta_I-y_I)\cdot\mathbf{1}(\theta_I\geq y_I)\right] \\
&< (1-\alpha)\,\mathbb{E}_{\theta_{I-1},y_{I-1}}\left[(\theta_{I-1}-y_{I-1})\cdot\mathbf{1}(\theta_{I-1}\geq y_{I-1})\right] \\
&= \Delta_{I-1}^*.
\end{aligned}$$

The first inequality follows from part 2 of Lemma 2 and the second inequality follows by part 1 of Lemma 2. The fact that Δ_m^* is decreasing in α follows from expression (6) and (8).■

Proof of Proposition 2. From bidder m's point of view, the optimal information decision is to acquire information if and only if $\widehat{\Delta}_m\geq c$. That is,

$$\widehat{s}_m=\begin{cases}0 & \text{if} \quad \widehat{\Delta}_m<c \\ 1 & \text{if} \quad \widehat{\Delta}_m\geq c\end{cases}.$$

The fact that $\widehat{\Delta}_1 > \widehat{\Delta}_2 > .... > \widehat{\Delta}_{I-1}$ follows from part 1 of Lemma 2. Notice that

$$\begin{aligned}
\widehat{\Delta}_I &= \mathbb{E}_{\theta_I,\theta_h}\left[(\theta_I - \theta_h)\cdot \mathbf{1}\left(\theta_I \geq \theta_h \geq \mu\right) + (\theta_h - \theta_I)\cdot \mathbf{1}\left(\mu > \theta_h > \theta_I\right)\right] \\
&= \mathbb{E}_{\theta_I,y_I}\left[(\theta_I - y_I)\cdot \mathbf{1}(\theta_I \geq y_I)\right] \\
&\quad + \mathbb{E}_{\theta_I,\theta_h}\left[(\theta_h - \theta_I)\cdot \mathbf{1}\left(\mu > \theta_h > \theta_I\right) - (\theta_I - \mu)\cdot \mathbf{1}\left(\theta_I \geq \mu > \theta_h\right)\right] \\
&< \mathbb{E}_{\theta_I,y_I}\left[(\theta_I - y_I)\cdot \mathbf{1}(\theta_I \geq y_I)\right] \\
&< \mathbb{E}_{\theta_{I-1},y_{I-1}}\left[(\theta_{I-1} - y_{I-1})\cdot \mathbf{1}(\theta_{I-1} \geq y_{I-1})\right] \\
&= \widehat{\Delta}_{I-1}.
\end{aligned}$$

The first inequality follows from part 2 of Lemma 2 and the second inequality follows by part 1 of Lemma 2. From expression (9) and (10), it is easy to see $\widehat{\Delta}_m$ is constant in α.■

Proof of Proposition 3. Comparing expression (6)(8) with (9)(10), we have

$$\Delta_m^* = (1-\alpha)\,\widehat{\Delta}_m, \text{ for all } m. \tag{21}$$

Since $0 \leq \alpha \leq 1$, $\Delta_m^* \leq \widehat{\Delta}_m$. This result, together with the fact that both sequences $\{\Delta_m^*\}$ and $\left\{\widehat{\Delta}_m\right\}$ are monotonically decreasing, implies that in equilibrium, (weakly) more bidders are informed in equilibrium than in a socially planner's solution.

From (21), we obtain

$$\widehat{\Delta}_m - \Delta_m^* = \alpha\widehat{\Delta}_m.$$

By Proposition 2, $\widehat{\Delta}_m$ is constant in α and decreasing in m. Therefore, $\widehat{\Delta}_m - \Delta_m^*$ is increasing in α and decreasing in m.■

Proof of Proposition 4. We can rewrite function $\widehat{\Delta}(\sigma)$ as

$$\widehat{\Delta}(\sigma) = (1-\sigma)^{I-1}\,\widehat{\Delta}_1 + \sum_{m=2}^{I}\binom{I-1}{m-1}\sigma^{m-1}(1-\sigma)^{I-m}\,\widehat{\Delta}_m,$$

In order to show $\widehat{\Delta}(\sigma)$ is decreasing in σ, it is sufficient to show $\widehat{\Delta}(\sigma) < 0$.

$$\begin{aligned}
\widehat{\Delta}(\sigma) &= -(I-1)(1-\sigma)^{I-2}\,\widehat{\Delta}_1 + \sum_{m=2}^{I}\binom{I-1}{m-1}\sigma^{m-2}(1-\sigma)^{I-m-1}\left[(m-1)-(I-1)\sigma\right]\widehat{\Delta}_m \\
&= \sum_{m=1}^{I}\binom{I-1}{m-1}\sigma^{m-2}(1-\sigma)^{I-m-1}\left[(m-1)-(I-1)\sigma\right]\widehat{\Delta}_m
\end{aligned}$$

Let $\overline{m}$ be the smallest integer m such that

$$(m-1)-(I-1)\sigma \geq 0.$$

Since $\widehat{\Delta}_1 > \widehat{\Delta}_2 > .... > \widehat{\Delta}_I$ by Proposition 2, we have

$$\begin{aligned}
\widehat{\Delta}(\sigma) &= \sum_{m=1}^{\overline{m}-1} \binom{I-1}{m-1} \sigma^{m-2} (1-\sigma)^{I-m-1} [(m-1)-(I-1)\sigma] \widehat{\Delta}_m \\
&\quad + \sum_{m=\overline{m}}^{I} \binom{I-1}{m-1} \sigma^{m-2} (1-\sigma)^{I-m-1} [(m-1)-(I-1)\sigma] \widehat{\Delta}_m \\
&< \sum_{m=1}^{I} \binom{I-1}{m-1} \sigma^{m-2} (1-\sigma)^{I-m-1} [(m-1)-(I-1)\sigma] \widehat{\Delta}_{\overline{m}} \\
&= \widehat{\Delta}_{\overline{m}} \left[\frac{1}{\sigma(1-\sigma)} \sum_{m=1}^{I} \binom{I-1}{m-1} \sigma^{m-1} (1-\sigma)^{I-m} (m-1) - \frac{I-1}{1-\sigma} \sum_{m=1}^{I} \binom{I-1}{m-1} \sigma^{m-1} (1-\sigma)^{I-m} \right] \\
&= \widehat{\Delta}_{\overline{m}} \left(\frac{I-1}{1-\sigma} - \frac{I-1}{1-\sigma} \right) \\
&= 0.
\end{aligned}$$

The proof for social gain $\Delta^*(\sigma)$ is analogous.■

Proof of Proposition 5. Recall that $\Delta^*_m \leq \widehat{\Delta}_m$ for all m. Therefore, by (12),

$$c = \sum_{m=1}^{I} \binom{I-1}{m-1} (\sigma^*)^{m-1} (1-\sigma^*)^{I-m} \Delta^*_m < \sum_{m=1}^{I} \binom{I-1}{m-1} (\sigma^*)^{m-1} (1-\sigma^*)^{I-m} \widehat{\Delta}_m \quad (22)$$

Comparing to the equilibrium condition about $\widehat{\sigma}$:

$$c = \sum_{m=1}^{I} \binom{I-1}{m-1} \widehat{\sigma}^{m-1} (1-\widehat{\sigma})^{I-m} \widehat{\Delta}_m,$$

we can deduce $\sigma^* < \widehat{\sigma}$ by applying Proposition 4.■

Proof of Lemma 1. The assumption $\mathbb{E}[u_i(\theta)|\theta_1, ..., \theta_{m-1}] > \mathbb{E}[u_k(\theta)|\theta_1, ..., \theta_{m-1}]$ implies that there exists a $\widehat{\theta}_m$ such that

$$\mathbb{E}\left[u_i(\theta)|\theta_1, ..., \theta_{m-1}, \widehat{\theta}_m\right] > \mathbb{E}\left[u_k(\theta)|\theta_1, ..., \theta_{m-1}, \widehat{\theta}_m\right]$$

But by the no-crossing property, we must have

$$\mathbb{E}[u_i(\theta)|\theta_1, ..., \theta_{m-1}, \theta_m] > \mathbb{E}[u_k(\theta)|\theta_1, ..., \theta_{m-1}, \theta_m] \text{ for all } \theta_m.$$

That completes the proof. ■

Proof of Proposition 6. To simplify notation, let J denote the set of informed bidders $\{1, .., m-1\}$ and θ_J denote the vector $\{\theta_j\}_{j \in J}$. We need to show that, for any bidder

$m \notin J$, his private gain is lower than social gain from information about θ_m. Conditional on the realization of θ_J, there are two possible scenarios. First, conditional on θ_J, the social planner awards the item to $i \neq m$. Second, conditional on θ_J, the social planner awards the item to m and bidder i is the runner up. Let z_m be the value at which

$$\mathbb{E}\left[u_m(\theta)\,|\theta_J, \theta_m = z_m\right] = \mathbb{E}\left[u_i(\theta)\,|\theta_J, \theta_m = z_m\right].$$

That is, z_m is the cutoff signal that bidder m will overtake bidder i if bidder m gets signal higher than z_m. By Lemma 1, we need to focus on bidder i and m only.

Case 1: Conditional on θ_J, the social planner awards the item to $i \neq m$.If $\theta_m \leq z_m$, bidder i will retain the object, and both private value and social value of information about θ_m is zero. If $\theta_m > z_m$, then bidder m wins the object and his VCG payment is $\mathbb{E}\left[u_m(\theta)\,|\theta_J, z_m\right]$ or $\mathbb{E}\left[u_i(\theta)\,|\theta_J, z_m\right]$. Therefore, the private gain from information for bidder m is

$$\mathbb{E}\left[u_m(\theta)\,|\theta_J, \theta_m\right] - \mathbb{E}\left[u_i(\theta)\,|\theta_J, z_m\right],$$

which is larger than the social gain from information

$$\mathbb{E}\left[u_m(\theta)\,|\theta_J, \theta_m\right] - \mathbb{E}\left[u_i(\theta)\,|\theta_J, \theta_m\right],$$

because

$$\mathbb{E}\left[u_i(\theta)\,|\theta_J, z_m\right] < \mathbb{E}\left[u_i(\theta)\,|\theta_J, \theta_m\right]$$

by the assumption of positive dependence.

Case 2: Conditional on θ_J, the social planner awards the item to m.

Let bidder i be the second highest bidder when bidder m is informed. If $\theta_m \geq z_m$, bidder m retains the object and both private value and social value of information are zero. If $\theta_m < z_m$, bidder i wins the object. Bidder m's gain from information by avoiding paying more than the worth of the object. His private gain is given by

$$\mathbb{E}\left[u_i(\theta)\,|\theta_J, z_m\right] - \mathbb{E}\left[u_m(\theta)\,|\theta_J, \theta_m\right],$$

which is higher than the social value of information

$$\mathbb{E}\left[u_i(\theta)\,|\theta_J, \theta_m\right] - \mathbb{E}\left[u_m(\theta)\,|\theta_J, \theta_m\right]$$

because

$$\mathbb{E}\left[u_i(\theta)\,|\theta_J, z_m\right] > \mathbb{E}\left[u_i(\theta)\,|\theta_J, \theta_m\right]$$

by the assumption of positive dependence.

Thus, in both cases, the private gain from information is higher than the social gain from information. Therefore, $\widehat{\Delta}_m < \Delta_m^*$ for all m.■

Proof of Proposition 7. Again let J denote the set of informed bidders $\{1,..,m-1\}$ and θ_J denote the vector $\{\theta_j\}_{j\in J}$. Fix J such that $I - \#\{j|j \in J\} \geq 2$, that is, there are at least two uninformed bidders. We need to show that a bidder i's private gain from information when bidder $m \notin J$ is uninformed is lower than his private gain when bidder m becomes informed. Conditional on θ_J only, there are five possible cases.

Case 1: The social planner awards the object to $k \neq \{i, m\}$.

Case 2: The social planner awards the object to m, and the second highest bidder is i.

Case 3: The social planner awards the object to m, and the second highest bidder is $k \neq i$.

Case 4: The social planner awards the object to i, and the second highest bidder is m.

Case 5: The social planner awards the object to i, and the second highest bidder is $k \neq m$.

Given the result of Lemma 1, it is easy to see that for Case 1 and Case 3 bidder m's information decision has no effect on bidder i's expected gain from information. So we only need to analyze the remaining three cases.

Let's first focus on Case 5. In this case, bidder i must be an informed bidder. Let $\theta_{J\setminus i}$ denote the vector of signals of all bidders in J other than i, and let z_i solve

$$\mathbb{E}\left[u_i(\theta)|\theta_{J\setminus i}, \theta_i = z_i\right] = \mathbb{E}\left[u_k(\theta)|\theta_{J\setminus i}, \theta_i = z_i\right]$$

That is, z_i is the cutoff signal that bidder i has higher valuation than bidder k conditional on $\theta_{J\setminus i}$.Then bidder i's expected gain from information is the difference

$$\mathbb{E}\left[u_i(\theta)|\theta_{J\setminus i}, \theta_i\right] - \mathbb{E}\left[u_k(\theta)|\theta_{J\setminus i}, z_i\right].$$

In order to compare bidder i's private gains with and without bidder m's information, it is convenient to introduce a pseudo scenario as a benchmark. We will show that bidder i's expected payoff without bidder m's information is the same as his payoff in the benchmark scenario.

Benchmark: Suppose the social planner has access to bidder m's private information, but only treats it as an exogenous signal. In particular, bidder m cannot be the winner or the runner up based on his information. Then by Lemma 1, bidder i will still win the object and the runner up is still bidder k. However, i's payment will depend on the realization of θ_m. Let $\widehat{z}_i$ solve

$$\mathbb{E}\left[u_i(\theta)\,|\theta_{J\backslash i},\theta_i=\widehat{z}_i,\theta_m\right]=\mathbb{E}\left[u_k(\theta)\,|\theta_{J\backslash i},\theta_i=\widehat{z}_i,\theta_m\right].$$

Then bidder i's expected gain is given by

$$\mathbb{E}\left[u_i(\theta)\,|\theta_{J\backslash i},\theta_i,\theta_m\right]-\mathbb{E}\left[u_k(\theta)\,|\theta_{J\backslash i},\widehat{z}_i,\theta_m\right].$$

We claim that $\widehat{z}_i=z_i$. To see this, suppose $\widehat{z}_i$ be the solution for some θ'_m. That is,

$$\mathbb{E}\left[u_i(\theta)\,|\theta_{J\backslash i},\widehat{z}_i,\theta'_m\right]=\mathbb{E}\left[u_k(\theta)\,|\theta_{J\backslash i},\widehat{z}_i,\theta'_m\right].$$

By no-crossing property assumption, we have for all θ_m,

$$\mathbb{E}\left[u_i(\theta)\,|\theta_{J\backslash i},\widehat{z}_i,\theta_m\right]=\mathbb{E}\left[u_k(\theta)\,|\theta_{J\backslash i},\widehat{z}_i,\theta_m\right].$$

Therefore, $\widehat{z}_i=z_i$. Then it follows from iterated expectation that bidder i's expected payoff with bidder m uninformed is the same as his expected payoff in the benchmark scenario.

Once we establish the payoff equivalence between the case with uninformed bidder m and the benchmark, we can now compare the case with informed bidder m with the benchmark. If

$$\mathbb{E}\left[u_m(\theta)\,|\theta_J,\theta_m\right]>\mathbb{E}\left[u_i(\theta)\,|\theta_J,\theta_m\right],$$

then bidder m wins the object, so the private gain for bidder i goes to zero. Compared to the benchmark, bidder i is worse off. If

$$\mathbb{E}\left[u_i(\theta)\,|\theta_J,\theta_m\right]>\mathbb{E}\left[u_m(\theta)\,|\theta_J,\theta_m\right]>\mathbb{E}\left[u_k(\theta)\,|\theta_J,\theta_m\right],$$

bidder m becomes the second highest bidder, and bidder i payment will based on bidder m's valuations. So bidder i pays more and his payoff decreases. Finally, if

$$\mathbb{E}\left[u_m(\theta)\,|\theta_J,\theta_m\right]<\mathbb{E}\left[u_k(\theta)\,|\theta_J,\theta_m\right],$$

bidder i wins and the runner up is still bidder k, so bidder i's payoff remains the same. To summarize, in Case 5, bidder i's expected gain from information is lower when bidder m gets informed.

For Case 2, bidder m wins and the runner up is i. Since there are at least two uninformed bidders (including m), there must be another uninformed bidder tied with m. So bidder i's payoff will be zero regardless of bidder m's information decision.

For Case 4, bidder i wins and the runner up is bidder m. Again since there are at least two uninformed bidders, there must be another uninformed bidder $j \neq m$. We can redefine the runner up as bidder j, and then we return to Case 5.

Therefore, for all five cases, when bidder m gets informed, bidder i's expected gain from information either stays the same or decreases. The proof is complete.■

Proof of Proposition 8. Note that bidder m acquires information if and only if his expected gain is higher than the information cost. Thus, the private gain $\widehat{\Delta}_m$ is equal to his expected gain.

$$
\begin{aligned}
\widehat{\Delta}_m &= \mathbb{E}_{\theta^m}\left[\left(v_m(\theta) - t_m(\theta)\right) \cdot \mathbf{1}\left\{\theta_m \geq y_{m1}\right\}\right] \\
&= \mathbb{E}_{\theta^m}\left[\left(\theta_m - y_{m1}\right) \cdot \mathbf{1}\left\{\theta_m \geq y_{m1}\right\}\right] \\
&= \int_{\underline{\theta}}^{\bar{\theta}} \left[\int_{\underline{\theta}}^{\theta_m} \left(\theta_m - y_{m1}\right) dF^{m-1}\left(y_{m1}\right)\right] f\left(\theta_m\right) d\theta_m \\
&= \int_{\underline{\theta}}^{\bar{\theta}} \left[\int_{\underline{\theta}}^{\theta_m} F^{m-1}(x)\, dx\right] f\left(\theta_m\right) d\theta_m \\
&= \int_{\underline{\theta}}^{\bar{\theta}} F^{m-1}\left(\theta_m\right)\left(1 - F\left(\theta_m\right)\right) d\theta_m
\end{aligned}
$$

The fourth equality follows from integration by parts. It is easy to see that $\widehat{\Delta}_m$ is decreasing in m.■

Proof of Proposition 9. By definition, the expected social gain Δ_m^* from bidder m becoming informed is

$$
\begin{aligned}
\Delta_m^* &= \mathbb{E}_{\theta^m}\left[\left(\theta_m - y_{m1}\right) \cdot \mathbf{1}\left\{\theta_m \geq y_{m1}\right\} + \sum_{k=1}^{K} \alpha_k \left(\theta_m - y_{m(k+1)}\right) \cdot \mathbf{1}\left\{y_{m(k+1)} \leq \theta_m < y_{mk}\right\}\right] \\
&= \widehat{\Delta}_m + \mathbb{E}_{\theta^m}\left[\sum_{k=1}^{K} \alpha_k \left(\theta_m - y_{m(k+1)}\right) \cdot \mathbf{1}\left\{y_{m(k+1)} \leq \theta_m < y_{mk}\right\}\right] \\
&\geq \widehat{\Delta}_m,
\end{aligned}
$$

as long as $\alpha_k > 0$ for all $k \leq K$.■

9 Appendix B: Multi-Unit Auctions

This Appendix contains the analysis of the multi-unit setting. Suppose there are k homogeneous objects and I bidders with $I \geq 2k$. Each bidder demands at most one object. The value of the object is defined in (1). We will show that most insights obtained in the single-unit setting can be carried through to the multi-unit setting.

As in the single-unit setting, if bidder i gets informed, he observes θ_i. Otherwise, he assigns μ to signal θ_i. Let $i = 1, 2, ..., m$ denote bidders who acquire information, and let $i = m+1, ..., I$ denote bidders who do not. Let θ_h to denote the k-th highest signal among $(m-1)$ informed bidders, and let h denote the bidder with signal θ_h. That is, θ_h is the $(m-k)$-th order statistic among $(m-1)$ random variables independently drawn from distribution F. In addition, y_i is the k-th highest signal among the opponents of bidder i. Finally, we need one new notation. If $m > I - k$, we use θ_υ to denote $(k-(I-m))$-th highest signal among $(m-1)$ informed bidders, and call the informed bidder with θ_υ as bidder υ.

Note that if $\theta_h < \mu$, then at least one object is assigned to uninformed bidders. On the other hand, for $m > I - k$, if $\theta_\nu < \mu$, then every uninformed bidder is assigned one unit.

Similar to (2) and (3), the generalized VCG mechanism $(q_i(\theta_i, \theta_{-i}), t_i(\theta_i, \theta_{-i}))$ is consists of an allocation rule

$$q_i(\theta_i, \theta_{-i}) = \begin{cases} 1 & \text{if} \quad \theta_i \geq y_i \\ 0 & \text{if} \quad \theta_i < y_i \end{cases}$$

that assigns the k objects to bidders with k highest signals, and a payment rule

$$t_i(\theta_i, \theta_{-i}) = \begin{cases} u_i(y_i, \theta_{-i}) & \text{if} \quad \theta_i \geq y_i \\ 0 & \text{if} \quad \theta_i < y_i \end{cases}$$

that specifies the payment of the k winners.

It is useful to classify the multi-unit model into three possible cases.

(1) $m \in [1, k]$. Bidder m is assigned one object if $\theta_m \geq \mu$, and the payoff of bidder m is

$$u_m(\theta_m, \theta_{-m}) - u_m(\mu, \theta_{-m}) = \theta_m - \mu.$$

(2) $m \in [k+1, I-k]$. Bidder m is assigned one object if $\theta_m \geq \max\{\theta_h, \mu\}$. The payoff of bidder m is

$$u_m(\theta_m, \theta_{-m}) - u_m(y_m, \theta_{-m}) = \theta_m - y_m,$$

where $y_m = \max\{\theta_h, \mu\}$.

(3) $m \in [I-k+1, I]$. Bidder m is assigned one object if $\theta_m \geq \max\{\theta_h, \mu\}$ or $\mu > \theta_m \geq \theta_\nu$.The payoff of bidder m is

$$u_m(\theta_m, \theta_{-m}) - u_m(y_m, \theta_{-m}) = \theta_m - y_m,$$

where

$$y_m = \begin{cases} \theta_h & \text{if} \quad \theta_h \geq \mu \\ \mu & \text{if} \quad \theta_m \geq \mu > \theta_h \\ \theta_\nu & \text{if} \quad \mu > \theta_m \geq \theta_\nu \end{cases}.$$

The first two cases correspond to the case $m < I$ in the single-unit setting, while the last case corresponds to the border case $m = I$. Just as in the previous analysis, the last k bidders' incentives to acquire information is slightly different from the first $(I-k)$ bidders. The analysis of the last k bidders is more involved but it provides little additional insights. So we will focus on the analysis of the first $(I-k)$ bidders. That is, throughout this Appendix, we will assume $m \leq I - k$.

As shown in the analysis of single-unit setting, the characterization of the social efficient and equilibrium policy is equivalent to the characterization of the sequences of Δ_m^* and $\widehat{\Delta}_m$.

We start with the social efficient policy. First suppose $m \in [1, k]$. In this case, an informed bidder m improves the efficiency of allocation when $\theta_m \geq \mu$. Without loss of generality, we assume that if at least one objects are assigned to uninformed bidders then bidder I gets one. The expected social gross gain from bidder m being informed is

$$\begin{aligned}
\Delta_m^* &= \mathbb{E}_\theta \left[\left(u_m(\theta_m, \theta_{-m}) - u_I(\mu, \theta_{-I}) \right) \cdot \mathbf{1}(\theta_m \geq \mu) \right] \\
&= \mathbb{E}_\theta \left[\left((\theta_m - \mu) + \alpha(\mu - \theta_m) \right) \cdot \mathbf{1}(\theta_m \geq \mu) \right] \\
&= (1-\alpha)\, \mathbb{E}_{\theta_m} \left[(\theta_m - \mu) \cdot \mathbf{1}(\theta_m \geq \mu) \right]
\end{aligned}$$

Next consider the case where $m \in [k+1, I-k]$. Now an informed bidder m improves allocation efficiency if $\theta_m \geq \max\{\theta_h, \mu\}$. The expected social gross gain from bidder m being informed is

$$\begin{aligned}
\Delta_m^* &= \mathbb{E}_\theta \left[\begin{array}{c} \left(u_m(\theta_m, \theta_{-m}) - u_h(\theta_h, \theta_{-h}) \right) \cdot \mathbf{1}(\theta_m \geq \theta_h \geq \mu) \\ + \left(u_m(\theta_m, \theta_{-m}) - u_I(\theta_I, \theta_{-I}) \right) \cdot \mathbf{1}(\theta_m \geq \mu > \theta_h) \end{array} \right] \\
&= (1-\alpha)\, \mathbb{E}_{\theta_m, \theta_h} \left[(\theta_m - \theta_h) \cdot \mathbf{1}(\theta_m \geq \theta_h \geq \mu) + (\theta_m - \mu) \cdot \mathbf{1}(\theta_m \geq \mu > \theta_h) \right] \\
&= (1-\alpha)\, \mathbb{E}_{\theta_m, y_m} \left[(\theta_m - y_m) \cdot \mathbf{1}(\theta_m \geq y_m) \right],
\end{aligned}$$

where

$$y_m = \max\{\theta_h, \mu\}.$$

Therefore, for $m \leq I-k$, the social gain Δ_m^* is given by

$$\Delta_m^* = \begin{cases} (1-\alpha)\,\mathbb{E}_{\theta_m}\left[(\theta_m - \mu)\cdot \mathbf{1}(\theta_m \geq \mu)\right] & \text{if} \quad m \in [1,k] \\ (1-\alpha)\,\mathbb{E}_{\theta_m, y_m}\left[(\theta_m - y_m)\cdot \mathbf{1}(\theta_m \geq y_m)\right] & \text{if} \quad m \in [k+1, I-k] \end{cases}. \tag{23}$$

Then the social efficient policy is characterized by the following proposition. All proofs are analogous to the proofs in the single-unit setting, and thus omitted.

Proposition 10 (Social Efficient Policy)
For $m \leq I-k$,

1. *The social optimal policy is*

$$s_m^* = \begin{cases} 0 & \text{if} \quad \Delta_m^* < c \\ 1 & \text{if} \quad \Delta_m^* \geq c \end{cases},$$

where Δ_m^* is defined in (23). Furthermore, $\Delta_1^* = ... = \Delta_k^* > \Delta_{k+1}^* > ... > \Delta_{I-k}^*$.

2. *Δ_m^* is decreasing in α.*

Similarly, we can calculate that bidder m's individual gain. For $m \in [1,k]$, the expected individual gain from information is

$$\begin{aligned} \widehat{\Delta}_m &= \mathbb{E}_\theta\left[(u_m(\theta_m, \theta_{-m}) - u_m(\mu, \theta_{-m}))\cdot \mathbf{1}(\theta_m \geq \mu)\right] \\ &= \mathbb{E}_{\theta_m, y_m}\left[(\theta_m - \mu)\cdot \mathbf{1}(\theta_m \geq \mu)\right] \end{aligned}$$

and for $m \in [k, I-k]$, the expected individual gain is

$$\begin{aligned} \widehat{\Delta}_m &= \mathbb{E}_\theta\left[\begin{array}{c} (u_m(\theta_m, \theta_{-m}) - u_m(\theta_h, \theta_{-m}))\cdot \mathbf{1}(\theta_m \geq \theta_h \geq \mu) \\ + (u_m(\theta_m, \theta_{-m}) - u_m(\theta_I, \theta_{-m}))\cdot \mathbf{1}(\theta_m \geq \mu > \theta_h) \end{array}\right] \\ &= \mathbb{E}_{\theta_m, y_m}\left[(\theta_m - y_m)\cdot \mathbf{1}(\theta_m \geq y_m)\right], \end{aligned}$$

where

$$y_m = \max\{\theta_h, \mu\}.$$

Therefore, for $m \leq I-k$, the expected social value of information $\widehat{\Delta}_m$ are defined as follows

$$\widehat{\Delta}_m = \begin{cases} \alpha\mathbb{E}_{\theta_m, y_m}\left[(\theta_m - \mu)\cdot \mathbf{1}(\theta_m \geq \mu)\right] & \text{if} \quad m \in [1,k] \\ \alpha\mathbb{E}_{\theta_m, y_m}\left[(\theta_m - y_m)\cdot \mathbf{1}(\theta_m \geq y_m)\right] & \text{if} \quad m \in [k, I-k] \end{cases}. \tag{24}$$

Then we can characterize the equilibrium policy as follows.

Proposition 11 (Equilibrium Policy)
For all $m \leq I - k$,

1. *The equilibrium information acquisition policy is*

$$\widehat{s}_m = \begin{cases} 0 & if \quad \widehat{\Delta}_m < c \\ 1 & if \quad \widehat{\Delta}_m \geq c \end{cases},$$

   where $\widehat{\Delta}_m$ is defined in (24). Furthermore, $\widehat{\Delta}_1 = ... = \widehat{\Delta}_k > \widehat{\Delta}_{k+1} > ... > \widehat{\Delta}_{I-k}$.

2. *$\widehat{\Delta}_m$ is constant in α.*

Finally, comparing (23) with (24), we can see that the following relation still holds for $m \leq I - k$:

$$\Delta_m^* = (1 - \alpha)\,\widehat{\Delta}_m \leq \widehat{\Delta}_m.$$

Therefore, we have the following results.

Proposition 12 (Welfare Comparison)
For $m \leq I - k$,

1. *The private gain from information $\widehat{\Delta}_m$ is higher than the social gain from information Δ_m^*.*

2. *The difference $\widehat{\Delta}_m - \Delta_m^*$ is increasing in α.*

3. *The difference $\widehat{\Delta}_m - \Delta_m^*$ is decreasing in m.*

Therefore, Proposition 10, 11 and 12 essentially extend Proposition 1, 2 and 3 to the multi-unit setting. Information decisions remain strategic substitutes, the equilibrium of the game of information still features excessive information acquisition, and the difference between individual incentive and social incentive to acquire information diminishes as more bidders get informed.

References

Barlevy, G., and P. Veronesi (2000): "Information Aquisition in Financial Markets," *Review of Economic Studies*, 67, 79–90.

Bergemann, D., and J. Välimäki (2002): "Information Acquisition and Efficient Mechanism Design," *Econometrica*, 70, 1007–1033.

Cremer, J., Y. Spiegel, and C. Zheng (2003): "Optimal Selling Mechanisms with Costly Information Aquisition," Discussion paper, Northwestern University.

Dasgupta, P., and E. Maskin (2000): "Efficient Auctions," *Quarterly Journal of Economics*, 115, 341–388.

Grossman, S., and J. Stiglitz (1980): "On the Impossbility of Informationally Efficient Markets," *American Economic Review*, 70, 393–408.

Harsanyi, J. (1973): "Games with Randomly Disturbed Payoffs: A New Rationale for Mixed-Strategy Equilibrium Points," *International Journal of Game Theory*, 2, 1–23.

Jackson, M. (2003): "Efficiency and Information Aggregation in Auctions with Costly Information," *Review of Economic Design*, 8, 121–41.

Maskin, E. (1992): "Auctions and Privatization," in *Privatization: Symposium in Honor of Herbert Giersch*, ed. by H. Siebert, pp. 115–136. J.C.B. Mohr, Tuebingen.

Matthews, S. (1984): "Information Acquisition in Discriminatory Auctions," in *Bayesian Models in Economic Theory*, ed. by M. Boyer, and R. Kihlstrom, pp. 181–207. North-Holland, Amsterdam.

Milgrom, P. (1981): "Rational Expectations, Information Acquistion and Competitive Bidding," *Econometrica*, 49, 921–943.

Persico, N. (2000): "Information Acquisition in Auctions," *Econometrica*, 68, 135–148.

Pesendorfer, W., and J. Swinkels (1997): "The Loser's Curse and Information Aggregation in Common Value Auctions," *Econometrica*, 65, 1247–1281.

Pesendorfer, W., and J. Swinkels (2000): "Efficiency and Information Aggregation in Auctions," *American Economic Review*, 90, 499–525.

Stegeman, M. (1996): "Participation Costs and Efficient Auctions," *Journal of Economic Theory*, 71, 228–259.