A NOTE ON FAIRNESS, POWER, PROPERTY, AND BEHIND THE VEIL

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Abstract

An Axiomatization for Power and for Equity differ only in the addition of a Behind the Veil Axiom.

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In 1953 Lloyd Shapley [1] proposed as a solution to an n-person game a combinatorial averaging of the marginal productivity of all individuals over their marginal worth to all groups under all orders in which those groups could be formed. This involved considering all $n!$ permutations of the ordering of how players might line up in forming coalitions. This solution, now known as the Shapley Value of an n-person game in coalition form has become a central part of cooperative game theory for over fifty years.

The Shapley value uses as a given the description of a game in coalitional form. For a finite set of players N this is a function v from the set of all coalititons 2^n to the real numbers where $n = |N|$.

The Shapley value requires four axioms. we denote the payoff to player i at the value as $\phi_i(v)$. Remarkably with the specification of four simple axioms an elegant explicit formula is obtained for the value. It is:

$$\phi_i(v) = \frac{1}{n!} \sum_{s=1}^{n} \frac{1}{c(s)} \sum_{\substack{S \ni i \\ |S|=s}} [v(S) - v(S - \{i\})]$$

where $c(s)$ is the number of coalitions of size s containing the designated player i. The symbol $\{i\}$ stands for the set consisting of i.

The four axioms are:

Efficiency:

$$\sum_{i \in N} \phi_i(v) = v(n).$$

Symmetry: If the roles of i and j are symmetric in game v then $\phi_i(v) = \phi_j(v)$.

The dummy property: If i is a dummy player who adds nothing to any coalition $\phi_i(v) = 0$.

Additivity: Consider a game $v + w$ defined by $(v + w)(S) = v(S) + w(S)$ for all S then $\phi(v + w) = \phi(v) + \phi(w)$.

Shapley and Shubik [2] utilized the Shapley value to calculate a power index. Paradoxically, however when one contemplates the axiomatization of fair division or equitable treatment all of the four axioms appear to be attractive. But this appears to lead to the conclusion that the axioms for power and fair division are the same. How can this be reconciled?

Rawls in his book A Theory of Justice [3] attempts to solve the problem of distributive justice utilizing the concept of an initial state of "behind the veil". In this state the distribution of total resources must be agreed upon by all players before any player i knows the position in the game that will be assigned to player i. In essence the specification of the characteristic function can be viewed as an intrinsically given set of property rights that is acceptable as the basis from which to calculate the final distribution. If we accept "behind the veil" as a fifth axiom it trivializes the value, in the sense that given von Neumann–Morgenstern utilities the expected characteristic function is symmetric for all players hence the value outcome is totally symmetric. This addition calls into clear focus the question of the basic treatment of initial property rights both for physical and human capital. With this extra axiom with *ex ante* symmetry in property rights power and equity are the same.

Although the idea of "behind the veil" is clearly counterfactual and biologically unattainable, it lends itself to experimental gaming. In the context of the laboratory it is feasible to require the players to settle upon a division of proceeds prior to knowing what position they will be assigned in an n-person game.

References

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[2] Shapley, L.S. and M. Shubik. 1954. A method for evaluating the distribution of power in a committee system *American Political Science Review* 48: 787–792

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