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November 2006

COWLES FOUNDATION DISCUSSION PAPER NO. 1589

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS
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Outsourcing Induced by Strategic Competition*

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October 31, 2006

Abstract

We show that intermediate goods can be sourced to firms on the "outside" (that do not compete in the final product market), even when there are no economies of scale or cost advantages for these firms. What drives the phenomenon is that "inside" firms, by accepting such orders, incur the disadvantage of becoming Stackelberg followers in the ensuing competition to sell the final product. Thus they have incentive to quote high provider prices to ward off future competitors, driving the latter to source outside.

Keywords: Intermediate goods, outsourcing, Cournot duopoly, Stackelberg duopoly

JEL Classification: D41, L11, L13

*It is a pleasure to thank Warren Sanderson and Michael Zweig for helpful conversations.

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1 Introduction

One of the principal concerns of any firm is to configure the supply of intermediate goods essential to its production. Of late, with the liberalization of trade and the lowering of barriers to entry, supply chain configurations have assumed global proportions. Indeed, in several industries, it has become the trend for firms to cut across national boundaries and outsource their supplies "offshore", provided the economic lure is strong enough. Many diverse factors influence firms' decisions. First, of course, there is the immediate cost of procuring the goods which—other things being equal—firms invariably seek to minimize. Then there is the question of risk: a firm may be unwilling to commit itself to a single party and instead spread its orders among others, even if they happen to be costlier, in order to ensure a steady flow of inputs. Sometimes a firm may tie up with a broad spectrum of suppliers so as to increase its access to the latest technological innovation, which could be forthcoming from any one of them. There can arise situations when a firm is impelled to select suppliers that will be strategic allies in its endeavor to penetrate newly emerging markets. For the analyses of these and other factors, and how they impinge on firms' decisions, see, e.g., Jarillo (1993), Spiegel (1993), Vidal and Goetschalkx (1997), Domberger (1998), Aggarwal (2003), Shy and Stenbacka (2003), Chen et al. (2004).

One intriguing possibility that has been alluded to, but not much explored, is that *strategic incentives* may arise in an oligopoly which outweigh other considerations and play the pivotal role in firms' selection of suppliers. Instances of this are presented by Jarillo and Domberger, of which we recount only two.

The first case comes from Germany. AEG[1] used to be a traditional supplier to both BMW[2] and Mercedes Benz. At some point, with a view to vertical integration, Mercedes Benz acquired AEG. This caused BMW to look for a different supplier, despite the inevitable extra costs of the switch (see p. 67, Jarillo, 1993).

The second case involves General Electric (GE) in the United States. In the early 1980's, GE investigated the possibility of outsourcing its lower brand microwave ovens from outside, since these had become too costly to manufacture at its factory in Maryland. Discussions were first held with, and even trial orders given to, Matsushita which happened to be a major rival of GE and also the world leader for this product in terms of both volume and technology. But ultimately GE turned to Samsung, then a small company with little experience in microwaves. The strategy entailed additional costs, such as sending American engineers to Korea, but it worked well for GE (see, pp. 84-86, Jarillo, 1993; and also Case Study 6.2, p. 108, Domberger, 1998).

Such case studies clearly point to the need for a game-theoretic analysis. In this paper we bring to light a scenario in which the outsourcing patterns emerge out of the strategic competition between firms. We find that it is typically *not* the case that a firm will outsource supplies to its rivals. There are two distinct reasons for this. The first is based on increasing returns to scale: if a firm places a sizeable order with its rival, it significantly lowers the rival's costs on account of the increasing returns, and this stands to its detriment in the ensuing competition on the final product. Thus the firm is led to outsource to others who

[1] Allgemeine Deutsche Electricitätsgesellschaft

[2] Bayerische Motoren Werke (or, Bavarian Motor Works)

may be costlier but, being out of the final product market, do not pose the threat of future competition. The second reason is more subtle and persists even in the case of constant returns to scale (i.e., linear costs)—indeed, it comes to the fore in this case. It is the main focus of this paper.

To be precise, suppose there are many firms $\mathcal{N}$ competing in the market for a final product α. Intermediate goods η are critical to the production of α, but only *some* of the firms $\mathcal{I} \subset \mathcal{N}$ have the competence to manufacture η at reasonable cost. The other firms $\mathcal{J} \equiv \mathcal{N} \backslash \mathcal{I}$ must obtain η from elsewhere. One possibility is to outsource η to their rivals in $\mathcal{I}$. But there is also a fringe of firms $\mathcal{O}$ on the "outside" which can manufacture η. What distinguishes $\mathcal{O}$ from $\mathcal{I}$ is that no firm in $\mathcal{O}$ can enter the market for the final product α. (This could be because it lacks the technology to convert η to α, or else faces high set-up costs—and, possibly, other barriers to entry—in the market α.[3]) To keep matters simple, we consider a purely linear model, i.e., in which the costs of production for both η and α are linear; as is the market demand for α.

Our main result is that, in this scenario, strategic considerations can come into play that will cause the firms in $\mathcal{J}$ to outsource η (outside) to $\mathcal{O}$ rather than (inside) to $\mathcal{I}$, *even if the costs of manufacturing η are higher in $\mathcal{O}$ than in $\mathcal{I}$*, so long as they are not much higher.

The intuition goes roughly as follows and is best seen with just three firms. Suppose (i) $\mathcal{I}$ and $\mathcal{J}$ are Cournot duopolists which compete in the market for the final product α; (ii) $\mathcal{I}$ and $\mathcal{O}$ can produce the intermediate good η, but $\mathcal{J}$ cannot; and (iii) $\mathcal{O}$ cannot enter the market for α. Thus $\mathcal{J}$ is confronted with the decision of how much η to outsource to $\mathcal{I}$ and how much to $\mathcal{O}$, all of which it will convert to α. It turns out that the optimal course of action for $\mathcal{J}$ is to outsource *exclusively* to either $\mathcal{I}$ or $\mathcal{O}$, never to both. Now if $\mathcal{J}$ outsources to $\mathcal{I}$, then $\mathcal{I}$ immediately knows the amount outsourced. This has the effect of establishing $\mathcal{J}$ as leader in the Stackelberg game that ensues in the market for α, in which $\mathcal{I}$ is forced to become the follower. In contrast, if $\mathcal{J}$ outsources to $\mathcal{O}$ then—thanks to the sanctity of the secrecy clause[4]—$\mathcal{I}$ will only know that $\mathcal{J}$ has struck a deal with $\mathcal{O}$ but not the quantity that $\mathcal{J}$ has ousourced. Thus $\mathcal{I}$ and $\mathcal{J}$ will remain Cournot duopolists in the ensuing game on market α.

If costs for manufacturing η do not vary too much between $\mathcal{I}$ and $\mathcal{O}$, then $\mathcal{I}$ will earn less as a Stackelberg follower than as a Cournot duopolist. This will tempt $\mathcal{I}$ to push $\mathcal{J}$ towards $\mathcal{O}$ by quoting so high a price for the intermediate good η that, in spite of the premium that $\mathcal{J}$ is willing to pay for the privilege of being the leader, $\mathcal{J}$ prefers to go to $\mathcal{O}$. The temptation can only be resisted if it is feasible for $\mathcal{I}$ to provide η at such an exorbitant price that it can recoup as provider what it loses as follower. But such an exorbitant price can be undercut by $\mathcal{O}$, as long as $\mathcal{O}$'s costs are not too much higher than $\mathcal{I}$'s. The upshot is that in any

[3]In particular, think of the following set-up. The market for α is concentrated in the "developed world". The firms in $\mathcal{O}$, on the other hand, are located offshore in the "developing world" and can manufacture η but lack the (advanced) technology for converting η to α. Even if some of them were to make the technological breakthrough, they would face not just the standard set-up costs for penetrating the market α, but further barriers to entry that pertain to foreign firms. This international setting perhaps makes our hypothesis of an outside fringe $\mathcal{O}$ more viable. But we do not need it, and all we formally postulate is the existence of this fringe.

[4]The secrecy clause is crucial to our analysis. It can be upheld on the simple ground that it is routinely seen in practice (see, e.g., Ravenhill, 2003; Clarkslegal and Kochhar, 2005). But, as we argue in Section 6, there are a variety of settings in which it can be shown to hold endogenously in equilibrium.

subgame-perfect Nash equilibrium[5] (SPNE) of the game, $\mathcal{J}$ will outsource to $\mathcal{O}$.

To complete the intuitive argument, we must still show that $\mathcal{J}$'s outsourcing orders will be exclusive. If $\mathcal{J}$ intends to produce no more than its Cournot quantity of α, then its rival $\mathcal{I}$'s output of α is invariant of who $\mathcal{J}$ outsources η to, and so $\mathcal{J}$ would do best to outsource η from whichever of $\mathcal{I}$ or $\mathcal{O}$ is charging the lower price. On the other hand, if $\mathcal{J}$ intends to produce more than its Cournot quantity, then it is best for $\mathcal{J}$ to fully take advantage of its leadership and outsource the Stackelberg amount to $\mathcal{I}$.

The actual argument is more intricate and the exact result is presented in Section 3. As was said, there are no economies of scale or cost advantages for the outside firm $\mathcal{O}$. In fact, we suppose that $\mathcal{O}$ has a higher cost than $\mathcal{I}$ for manufacturing η. Our main result states that, if $\mathcal{O}$'s cost does not exceed a well-defined threshold, $\mathcal{J}$ will outsource to $\mathcal{O}$ in any SPNE.

Worthy of note is the fact that it is *not* $\mathcal{J}$ who has the "primary" strategic incentive to outsource to $\mathcal{O}$. This incentive resides with $\mathcal{I}$ who is anxious to ward off $\mathcal{J}$ and force $\mathcal{J}$ to turn to $\mathcal{O}$. The anxiety gets played out when $\mathcal{O}$ does not have a severe cost disadvantage compared to $\mathcal{I}$. Otherwise, $\mathcal{I}$ is happy to strike a deal with $\mathcal{J}$ since it can get high provider prices that compensate it for becoming a follower. Which subgame gets played between $\mathcal{I}$ and $\mathcal{J}$ on market α—Cournot or Stackelberg—is thus not apriori fixed, but *endogenous* to equilibrium. This is all the more striking since, in our overall game, the option is open for firm $\mathcal{J}$ to outsource to both $\mathcal{I}$ and $\mathcal{O}$ and to thus bring any "mixture" of the Stackelberg and Cournot games into play. The logic of the SPNE rules out mixing and shows that only one of the two pure games will occur along the equilibrium play.

It should also be mentioned that our game involves simultaneous moves at various junctures, first at the very start, when firms $\mathcal{I}$ and $\mathcal{O}$ independently quote prices at which they are willing to supply η, and later in those subgames which follow after $\mathcal{J}$'s decision to outsource positive amounts of η to $\mathcal{O}$. Thus we are far from having perfect information in our game, and it is not a priori clear that SPNE will even exist in *pure* strategies. We prove that, in fact, there is a continuum of pure strategy SPNE, across which the outputs of the firms differ, but the outsourcing pattern is nevertheless invariant.

Economies of scale can easily be incorporated into our model. But then, as was said, a new strategic consideration arises, though it does not affect the tenor of our results (see Section 5.1 and, for full details, Chen & Dubey, 2005). The primary strategic incentive to outsource to $\mathcal{O}$ can shift from $\mathcal{I}$ to $\mathcal{J}$. For now $\mathcal{J}$ must worry that if it outsources η to $\mathcal{I}$, then $\mathcal{I}$ will develop a cost advantage on account of economies of scale. In other words, $\mathcal{I}$ will be able to manufacture η for itself at an average cost that is significantly lower than what it charges to $\mathcal{J}$. This might outweigh any leadership advantage that $\mathcal{J}$ obtains by going to $\mathcal{I}$. So, foreseeing a competitor in $\mathcal{I}$ that is fierce inspite of being a follower, $\mathcal{J}$ would prefer to outsource to $\mathcal{O}$ as long as $\mathcal{O}$'s price is not too much above $\mathcal{I}$'s. This, in turn, will happen if $\mathcal{O}$'s costs are not significantly higher than $\mathcal{I}$'s. But then, if $\mathcal{J}$ is outsourcing to $\mathcal{O}$, economies of scale can drive $\mathcal{I}$ to outsource to $\mathcal{O}$ as well!

These two strategic considerations, the first impelling $\mathcal{I}$ to push $\mathcal{J}$ towards $\mathcal{O}$ and the second impelling $\mathcal{J}$ to turn away from $\mathcal{I}$ on its own and to seek out $\mathcal{O}$, are intermingled in the presence of economies of scale. It is hard to disentangle them and say precisely when one

[5] *Throughout* we confine ourselves to pure strategies.

fades out, leaving spotlight on the other. But by eliminating economies of scale altogether, we are here able to focus on just the first scenario, wherein the game turns essentially on the informational content of the strategies.

Our analysis indicates that firms which position themselves on the "outside", by *not* entering the market for the final product, are more likely to attract orders for intermediate goods. There is some evidence that this can happen in practice. By the mid-1980's (see Ravenhill, 2003), US companies in the electronics industry were looking "to diversify their sources of supply" in order to fare better against their Japanese competitors. Malaysia and Singapore made a strong bid to get the US business. A key feature of the government policies of both nations was that "they were not attempting to promote national champions in the electronic industry", but the objective was rather "to build a complementary supply base, not to create local rivals that might displace foreign producers". Their success in becoming major supply hubs for electronic components is well documented. Of course it is true that they had the advantage of low-cost skilled labor. But what we wish to underscore is their deliberate and well-publicized *abstention* from markets for the final products. According to our analysis, the abstention by itself gave Malaysia and Singapore a competitive edge: even if their costs were to rise and exceed those in Japan, US firms would still favor them as suppliers, since the Japanese firms are entrenched rivals on the final product.

In conclusion, let us mention that there is considerable literature on endogenous Stackelberg leadership.[6] The paper most closely related to ours,[7] and inviting immediate comparison, is Baake, Oechssler and Schenk (1999). They consider a duopoly model to examine what they call "cross-supplies" within an industry—in our parlance, this is the phenomenon that a firm outsources to its rival. The "endogenous Stackelberg effect" is indeed pointed out by them: firm A, upon accepting the order outsourced by its rival B, automatically becomes a Stackelberg follower in the ensuing game on the final markets. But there are set-up costs of production in their model, and provided these costs are high enough, A can charge B a sufficiently high price so as to be compensated for being a follower. The upshot is that cross-supplies can be sustained in SPNE.

There are several points of difference between their model and ours. First, their argument relies crucially on the presence of sufficiently strong economies of scale (set-up costs). If these are absent or weak, there is no outsourcing in SPNE in their model. In contrast, in our model, outsourcing occurs *purely* on account of the endogenous Stackelberg effect (recall that we have constant returns to scale[8]). Second, outsourcing occurs only in *some* of their SPNE: there always coexist other SPNE where it does not occur. In our model, the outsourcing is invariant across *all* SPNE. In short, they show that outsourcing *can* occur, while we show that it *must*. Third, it is critical for their result that there be no outside suppliers.[9] Such suppliers would generate competition that would make it infeasible for A to charge a high price to B, invalidating their result. In our model, the situation is different. We allow for both kinds of suppliers: those that are inside as potential rivals and others that

[6]E.g., Hamilton and Slutsky (1990), Robson (1990), Mailath (1993), Pal (1993), van Damme and Hurkens (1999)—in all of which the timing of entry by firms is viewed as strategic.

[7]We are grateful to an anonymous referee for bringing it to our attention

[8]Though, as was said, outsourcing is further boosted by economies of scale in our model.

[9]Recall that these are suppliers who are not present as rivals in the final product market.

are outside. It turns out that increasing the number of either type leaves our result intact (see Section 5). Finally—and this, to our mind, is the most salient difference—the economic phenomena depicted in Baake et al. and here are different, indeed almost complementary. In Baake et al., the issue is to figure out when a firm will outsource to its rival. Here we consider precisely the opposite scenario and pinpoint conditions under which a firm will turn away from its rival and outsource instead to an outsider, even if the outsider happens to have a costlier technology.[10] The fact that both models take cognizance of the endogenous Stackelberg effect is a technical—albeit interesting—point. What is significant is that this effect is embedded in disparate models and utilized to explain complementary economic phenomena.

The paper is organized as follows. We present the model in Section 2, stripped down to its bare minimum, and with just three firms. The main result is stated in Section 3 and its proof is in Section 4. In Section 5, we indicate how our result is robust to various extensions of the model. Finally, in Section 6 we give an intuitive justification for the presence of the secrecy clause in the contract between firm $\mathcal{J}$ and firm $\mathcal{O}$.

2 The Model

For ease of notation, we substitute 0, 1, 2 for $\mathcal{O}$, $\mathcal{I}$, $\mathcal{J}$. As was said, firms 1 and 2 are duopolists in the market for a final good α. An intermediate good η is required to produce α. Firm 1 can manufacture η, but 2 cannot. There is an "outside" firm 0 which can also manufacture η. What distinguishes 0 from 1 is that 0 cannot enter the market for the final good α. Firm 0's sole means of profit is the manufacture of good η for the "inside" firms 1 and 2.

The inverse market demand for good α is given by $P = \max\{0, a - Q\}$, where Q denotes the total quantity of α produced by firms 1 and 2, and P denotes the price of α. The constant marginal cost of production of good η is c_0 for 0 and c_1 for 1. Furthermore both 1 and 2 can convert x units of good η into x units of good α at the (for simplicity) same constant marginal cost, which w.l.o.g we normalize to zero. We assume

$$0 < c_1 < c_0 < (a + c_1)/2 \tag{1}$$

The condition $c_1 < c_0$ gives a cost disadvantage to the outside firm 0 and loads the dice against good η being sourced to it. The inequality $c_0 < (a + c_1)/2$ prevents 1 from automatically becoming a monopolist in the market for good α.

The extensive form game between the three firms is completely specified by the parameters c_1, c_0, a and so we shall denote it $\Gamma(c_0, c_1, a)$. It is played as follows. For $i \in \{0, 1\}$ and $j \in \{1, 2\}$, put

$$q_j^i \equiv \text{ quantity of good } \eta \text{ outsourced by firm } j \text{ to firm } i$$

[10]Our analysis thus suggests that the current widespread trend of outsourcing to offshore locations can well persist for strategic reasons, even if offshore costs were to rise, so long as the offshore companies abstain from the final product markets of their clients.

(and put $q \equiv \{q_j^i\}_{j=1,2}^{i=0,1}$). In the first stage of the game, firms 0 and 1 simultaneously and publicly announce prices p_0 and p_1 at which they are ready to provide good η. Seeing these prices, firm 2 then chooses q_2^0, q_2^1. Firm 1 observes q_2^1 but not q_2^0, since q_2^0 is part of the secret contract between 0 and 2. Finally[11] firm 1, also knowing the prices, decides how much q_1^1 to produce on its own and how much q_1^0 to outsource to 0, making sure that $x_1(q) \equiv q_1^1 + q_1^0 - q_2^1 \geq 0$ so that it is able to honor its commitment to supply q_2^1 units of η to 2. Denote $x_2(q) \equiv q_2^0 + q_2^1$. Thus $x_1(q)$ and $x_2(q)$ are the outputs produced by 1 and 2 in the market α.

It remains to describe the payoffs of the three firms at the terminal nodes of the game tree. Any such node is specified by $p \equiv (p_0, p_1)$ and $q = \{q_j^i\}_{j=1,2}^{i=0,1}$. The payoff to firm i is $\Pi_i(p, q)$ where

$$\Pi_0(p,q) = p_0(q_1^0 + q_2^0) - c_0(q_1^0 + q_2^0)$$

$$\Pi_1(p,q) = (a - x_1(q) - x_2(q))x_1(q) + p_1 q_2^1 - p_0 q_1^0 - c_1 q_1^1$$

$$\Pi_2(p,q) = (a - x_1(q) - x_2(q))x_2(q) - p_0 q_2^0 - p_1 q_2^1$$

This completes the description of the game $\Gamma(c_0, c_1, a)$.

3 The Main Result

By an SPNE of $\Gamma(c_0, c_1, a)$, we shall mean a *subgame perfect Nash equilibrium in pure strategies* of the game $\Gamma(c_0, c_1, a)$.

Our main result asserts that, if the the cost disadvantage of the outside firm 0 is not too significant (i.e. $c_0 - c_1$ is not too large), then 2 will outsource good η to 0 in any SPNE.

Put

$$c^* = \frac{13}{14}c_1 + \frac{1}{14}a$$

and observe that (1) implies

$$c_1 < c^* < \frac{a + c_1}{2}.$$

Our result is summarized in Figure 1 below, in which c_0 is varied on the horizontal axis, holding a and c_1 fixed (and is even allowed to fall below c_1).

Notice that the interval (c_1, c^*) is of particular interest because here firm 0 has a cost disadvantage compared to firm 1, yet 2 outsources η to 0 rather than from 1. Strategic considerations dominate firms' behavior here. Below this interval, when $c_0 \leq c_1$, 0 has a cost advantage over 1 and so 2 even more readily outsourced to 0; in fact, for small enough c_0, *both* 1 and 2 outsource to 0. We shall ignore this easy case where firm 0 becomes additionally attractive on account of its lower cost. To keep strategic incentives in the foreground, we shall suppose throughout that $c_0 > c_1$.

[11] We *could* have supposed that firm 1 must place its order with 0 before finding out the quantity q_2^1. This would alter the game somewhat but not our conclusion (Theorem 1 in Section 3 will hold without any change). But the timing that we have given seems more natural to us. There is a fundamental asymmetry of information between firms 1 and 2. Firm 1 always has the option of waiting to see how much q_2^1 firm 2 will outsource to it before approaching 0 to outsource its own q_0^1. In contrast, firm 2 can never know whether firm 1 has gone to 0 or not, so it cannot plan to wait until 1 has outsourced to 0 before placing its order with 1.

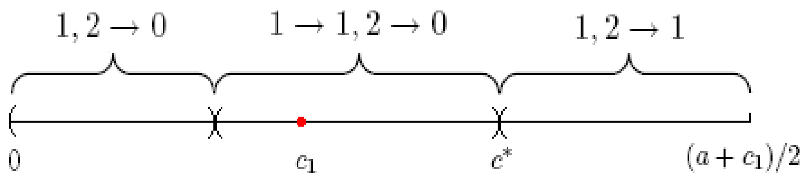

Figure 1: The Outsourcing Pattern ($j \to i \equiv j$ outsources η exclusively to i)

For a precise statement of our result, define the function $\tau : [c_1, (a+c_1)/2] \to R_+$ by

$$\tau(p_0) = \frac{(3-2\sqrt{2})(a+c_1)}{6} + \frac{2\sqrt{2}p_0}{3}; \tag{2}$$

and, for any interval $[u, v] \subseteq [c_1, (a+c_1)/2]$, define

$$(\text{Graph } \tau)[u, v] \equiv \{(p_0, \tau(p_0)) | p_0 \in [u, v]\}$$

and abbreviate

$$\text{Graph } \tau \equiv (\text{Graph } \tau)[c_1, (a+c_1)/2]$$

Since $\tau(c_1) > c_1$ and $\tau((a+c_1)/2) = (a+c_1)/2$, Graph τ is a straight line contained in the square $[c_1, (a+c_1)/2]^2$ (see Figure 2).

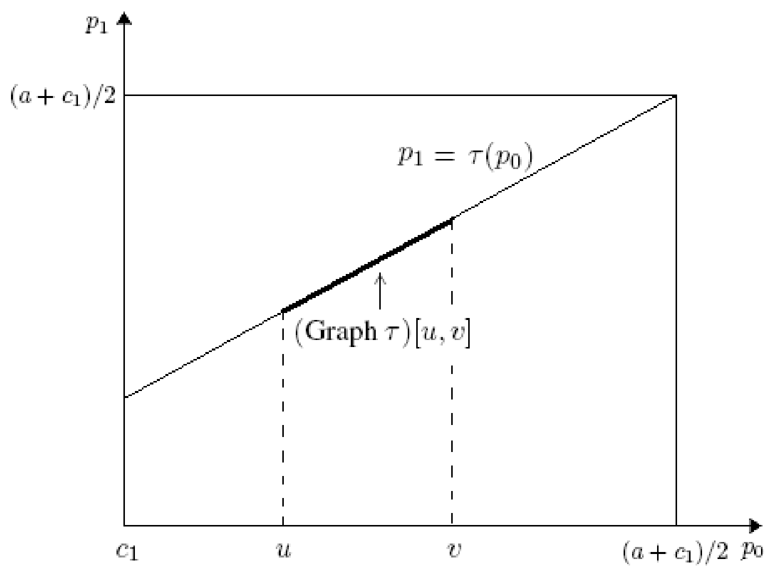

Figure 2: (Graph $\tau)[u, v]$

We are now ready to state our main result.

Theorem. (I) *In any SPNE of* $\Gamma(c_0, c_1, a)$*, firm* 1 *never outsources to firm* 0*, i.e.*, $q_1^0 = 0$.

(II) *If* $c_0 \in (c_1, c^*)$, *there is a continuum of SPNE of* $\Gamma(c_0, c_1, a)$, *indexed by supplier prices* $(p_0, p_1) \in$ *(Graph* τ*)*$[c_0, c^*]$; *and, in every SPNE, firm* 2 *outsources* η *to the outside firm* 0.

(III) *If* $c_0 \in (c^*, (a+c_1)/2)$, *there is a continuum of SPNE of* $\Gamma(c_0, c_1, a)$, *indexed by supplier prices* $(p_0, p_1) \in$ *(Graph* τ*)*$[c_1, c_0]$; *and, in every SPNE, firm* 2 *outsources* η *to the inside firm* 1.

(IV) *Finally, if* $c_0 = c^*$, *there are two SPNE of* $\Gamma(c_0, c_1, a)$ *with the same provider prices* $(p_0, p_1) = (c^*, \tau(c^*))$, *but firm* 2 *outsources* η *to* 0 *in the first SPNE and to* 1 *in the second SPNE.*

4 Proof

4.1 Preparatory Lemmas

Throughout the lemmas below, c_1 and a are fixed and (recall) $c_0 \in (c_1, (a+c_1)/2)$.

Lemma 1. *In any SPNE of* $\Gamma(c_0, c_1, a)$, *we must have* $p_0 \geq c_1$.

Proof. Suppose $p_0 < c_1$. Since $c_1 < c_0$ (by assumption, see (1)), firm 0 makes $(p_1 - c_0) < 0$ dollars per unit of the total outsourced order $q_1^0 + q_2^0$ that it receives. If it could be shown that $q_1^0 + q_2^0 > 0$, there would be an immediate contradiction, because firm 0 can in fact ensure zero payoff by deviating from p_0 to some sufficiently high p_0' (any p_0' higher than the maximum price a in market α will do), at which price neither firm will outsource anything to it.

To complete the proof, we now show that $q_1^0 + q_2^0 > 0$.

Let $q_2^0 = 0$ (otherwise we are done). If 2 produces a positive amount, it must outsource to 1, i.e., $q_2^1 > 0$. Then, since $p_0 < c_1$, 1 will pass on this order to 0, i.e., $q_1^0 > 0$.

If 2 produces nothing then, as is easily verified, firm 1 will make a positive sale of α, i.e., $q_1^0 + q_1^1 > 0$. But the cost of producing $q_1^0 + q_1^1$ is $p_0 q_1^0 + c_1 q_1^1$. Since $p_0 < c_1$, optimality requires that $q_1^1 = 0$, so we conclude that $q_1^0 > 0$. ■

In view of Lemma 1, we will *assume* $p_0 \geq c_1$ *throughout the rest of this section.*

Let $G(p_0, p_1, q_2^1)$ denote the subgame between 1 and 2, after (p_0, p_1) and q_2^1 are announced. In this subgame, 1 and 2 simultaneously choose (q_1^0, q_1^1) and q_2^0, with $q_1^0 + q_1^1 \geq q_2^1$. Denote $z \equiv q_1^0 + q_1^1$. If $p_0 > c_1$ then, in order to produce z, it is a strictly dominant strategy for firm 1 to set $q_1^0 = 0$ and $q_1^1 = z$ (i.e., to produce all of z at the lower cost c_1); if $p_0 = c_1$, then firm 1 is indifferent on the split of z. In either case, firm 1 procures η at cost c_1.

We may suppress η and think of $G(p_0, p_1, q_2^1)$ as a game involving only good α, in which 1 produces $x_1 \equiv z - q_2^1 \equiv q_1^0 + q_1^1 - q_2^1$ at cost c_1 and 2 produces q_2^0 at cost p_0; and in which 2 has an "endowment" q_2^1 procured before entering the game at price p_1. The payoffs of 1 and 2 in $G(p_0, p_1, q_2^1)$ are given by

$$\Pi_1(x_1, q_2^0) = (a - q_2^1 - x_1 - q_2^0)x_1 - c_1 x_1 + (p_1 - c_1)q_2^1$$

$$\Pi_2(x_1, q_2^0) = (a - q_2^1 - x_1 - q_2^0)(q_2^1 + q_2^0) - p_0 q_2^0 - p_1 q_2^1$$

(The terms $(p_1 - c_1)q_2^1$ and $p_1 q_2^1$, involving good η, can be viewed as constants that are given from the past, before the game $G(p_0, p_1, q_2^1)$ is played.)

Lemma 2. *$G(p_0, p_1, q_2^1)$ has a unique NE.*

Proof. Let $(q_1^{\mathcal{C}}(p_0), q_2^{\mathcal{C}}(p_0))$ denote the quantities of firms 1 and 2 in the unique NE of the Cournot game $G(p_0, p_1, 0)$. As is well-known

$$(q_1^{\mathcal{C}}(p_0), q_2^{\mathcal{C}}(p_0)) = \begin{cases} ((a - 2c_1 + p_0)/3, (a + c_1 - 2p_0)/3) & \text{if } p_0 \le (a + c_1)/2 \\ ((a - c_1)/2, 0) & \text{if } p_0 \ge (a + c_1)/2 \end{cases} \tag{3}$$

Let $[y]_+ \equiv \max\{0, y\}$ for any $y \in R$. It is easy to check that the NE of $G(p_0, p_1, q_2^1)$ is unique and, indeed as follows.

(i) if $0 \le q_2^1 \le [q_2^{\mathcal{C}}(p_0)]_+$, then 2 produces $q_2^{\mathcal{C}}(p_0) - q_2^1$ and 1 produces $q_1^{\mathcal{C}}(p_0)$ (as before);

(ii) if $[q_2^{\mathcal{C}}(p_0)]_+ < q_2^1 \le a - c_1$, then 2 produces zero and 1 produces $(a - c_1 - q_2^1)/2$;

(iii) if $q_2^1 > a - c_1$, then both produce zero. ■

Lemma 3. *Suppose $p_0 \ge (a + c_1)/2$. Then the NE of $G(p_0, p_1, q_2^1)$ is invariant of p_0. Hence w.l.o.g. we may restrict $p_0 \le (a + c_1)/2$.*

Proof. If $p_0 \ge (a + c_1)/2$, then $q_2^{\mathcal{C}}(p_0) = 0$ by (3) and then (from (i), (ii), (iii) in the proof of Lemma 2) $q_2^0 = 0$. Since $c_1 < (a + c_1)/2$, we have $p_0 > c_1$ and hence $q_1^0 = 0$ as well. So firm 0 receives no order from anyone when $p_0 \ge (a + c_1)/2$. The lemma follows. ■

We now move one step back in the game tree of $\Gamma(c_0, c_1, a)$ and denote by $G(p_0, p_1)$ the game that ensues after the simultaneous announcement of p_0 and p_1. In looking for SPNE of $G(p_0, p_1)$, it suffices to consider the problem in which firm 2 chooses q_2^1 and then the unique NE of $G(p_0, p_1, q_2^1)$ is played.

First imagine two games between firms 1 and 2 in the market α. In both games, the inverse demand for α is fixed at $P = \max\{0, a - Q\}$ and the (constant, marginal) cost of firm 1 (to produce α) is fixed at c_1. The constant marginal cost $c \in [c_1, (a + c_1)/2]$ of firm 2 (to produce α) is considered variable and hence the game depends on c. Let $\mathcal{S}^{21}(c)$ be the Stackelberg duopoly with 2 as the leader and 1 the follower and let $\mathcal{C}(c)$ be the Cournot duopoly between 1 and 2. These games have unique SPNE.[12] Let $f(c)$ and $\ell(c)$ denote the profits of 1 (follower) and 2 (leader) in the SPNE of $\mathcal{S}^{21}(c)$. Let $\kappa_1(c)$ and $\kappa_2(c)$ denote the corresponding profits in $\mathcal{C}(c)$. Finally, let $q_1^{\mathcal{S}}(c)$ and $q_2^{\mathcal{S}}(c)$ denote the output produced by 1 and 2 in the SPNE of $\mathcal{S}^{21}(c)$ (and recall $q_1^{\mathcal{C}}(c)$ and $q_2^{\mathcal{C}}(c)$ are the corresponding outputs in $\mathcal{C}(c)$). It is well known that

$$(q_1^{\mathcal{S}}(c), q_2^{\mathcal{S}}(c)) = \begin{cases} (0, (a - c)/2) & \text{if } c \le [2c_1 - a]_+, \\ (0,\ a - c_1) & \text{if } [2c_1 - a]_+ \le c \le [(3c_1 - a)/2]_+, \\ ((a - 3c_1 + 2c)/4,\ (a + c_1 - 2c)/2) & \text{if } [(3c_1 - a)/2]_+ \le c \le (a + c_1)/2, \\ ((a - c_1)/2,\ 0) & \text{if } c \ge (a + c_1)/2. \end{cases} \tag{4}$$

[12] In the Cournot game $\mathcal{C}(c)$, SPNE is just NE.

Lemma 4 below characterizes the SPNE of $G(p_0, p_1)$ as (p_0, p_1) varies. To state it, we need to partition the price space $[c_1, (a+c_1)/2] \times [0, \infty)$ of (p_0, p_1) into four regions $R_{\mathcal{M}}$, $R_{\mathcal{S}}$, $R_{\mathcal{C}}$ and Graph τ (see Figure 3). Recall that $[y]_+ \equiv \max\{0, y\}$ for any $y \in R$, and put

$$R_{\mathcal{M}} = \{(p_0, p_1) \in R^2_+ | c_1 \leq p_0 \leq (a+c_1)/2, 0 \leq p_1 \leq [(3c_1 - a)/2]_+\}$$

$$R_{\mathcal{S}} = \{(p_0, p_1) \in R^2_+ | c_1 \leq p_0 \leq (a+c_1)/2, [(3c_1 - a)/2]_+ < p_1 < \tau(p_0)\}$$

$$R_{\mathcal{C}} = \{(p_0, p_1) \in R^2_+ | c_1 \leq p_0 \leq (a+c_1)/2, p_1 > \tau(p_0)\}$$

Also, let us use the phrase "in SPNE" to mean "in the play induced by the SPNE". We are now ready to state Lemma 4.

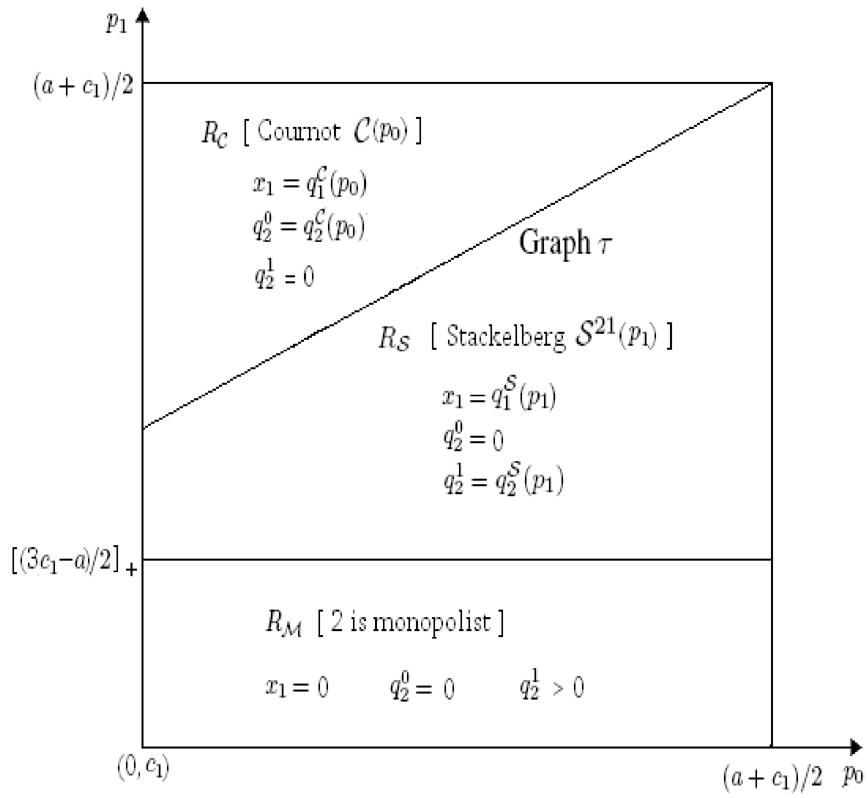

Figure 3: SPNE of $G(p_0, p_1)$

Lemma 4. (Figure 3) *Suppose* $p_0 \leq (a+c_1)/2$. *Let* $x_1(q) \equiv q_1^0 + q_1^1 - q_2^1$ *and* $x_2(q) \equiv q_2^0 + q_2^1$ *be the quantities sold by firms* 1 *and* 2 *in the market* α.

(i) *In any SPNE of* $G(p_0, p_1)$, $q_2^0 q_2^1 = 0$ *and w.l.o.g.* $q_1^0 = 0$ (*so that* $x_1(q) = q_1^1 - q_2^1$ *and* $x_2(q) = max\{q_2^0, q_2^1\}$).

(ii) *If* $(p_0, p_1) \in R_{\mathcal{M}}$, $G(p_0, p_1)$ *has a unique SPNE with* $q_2^0 = 0$, $x_2(q) = q_2^1 > 0$ *and* $x_1(q) = 0$. *Firms* 0, 1, 2 *have zero, negative, positive payoffs respectively and firm* 2 *is a monopolist.*

(iii) *If* $(p_0, p_1) \in R_{\mathcal{S}}$, $G(p_0, p_1)$ *has a unique SPNE in which* $q_2^0 = 0$ *and the ensuing game is* $\mathcal{S}^{21}(p_1)$ *where* $x_2(q) = q_2^1 = q_2^{\mathcal{S}}(p_1)$ *and* $x_1(q) = q_1^{\mathcal{S}}(p_1)$. *Firms* 0, 1 *and* 2 *earn zero,* $F(p_1) \equiv f(p_1) + (p_1 - c_1)q_2^{\mathcal{S}}(p_1)$ *and* $\ell(p_1)$ *respectively.*

(iv) *If* $(p_0, p_1) \in R_{\mathcal{C}}$, $G(p_0, p_1)$ *has a unique SPNE in which* $q_2^1 = 0$ *and the ensuing game is* $\mathcal{C}(p_0)$ *where* $x_2(q) = q_2^0 = q_2^{\mathcal{C}}(p_0)$ *and* $x_1(q) = q_1^1 = q_1^{\mathcal{C}}(p_0)$. *Firms* 0, 1 *and* 2 *earn* $(p_0 - c_0)q_2^{\mathcal{C}}(p_0)$, $\kappa_1(p_0)$ *and* $\kappa_2(p_0)$ *respectively.*

(v) *If* $(p_0, p_1) \in$ *Graph* τ, $G(p_0, p_1)$ *has exactly two SPNE. In the first SPNE,* $q_2^0 = 0$ *and the ensuing game is* $\mathcal{S}^{21}(p_1)$ *where* $x_2(q) = q_2^1 = q_2^{\mathcal{S}}(p_1)$ *and* $x_1(q) = q_1^{\mathcal{S}}(p_1)$; *firms* 0, 1 *and* 2 *earn zero,* $F(p_1) \equiv f(p_1) + (p_1 - c_1)q_2^{\mathcal{S}}(p_0)$ *and* $\ell(p_1)$ *respectively. In the second SPNE,* $q_2^1 = 0$ *and the ensuing game is* $\mathcal{C}(p_0)$ *where* $x_1(q) = q_1^1 = q_1^{\mathcal{C}}(p_0)$ *and* $x_2(q) = q_2^0 = q_1^{\mathcal{C}}(p_0)$; *firms* 0, 1 *and* 2 *earn* $(p_0 - c_0)q_2^{\mathcal{C}}(p_0)$, $\kappa_1(p_0)$ *and* $\kappa_2(p_0)$ *respectively.*

Proof. We first argue that w.l.o.g. $q_1^0 = 0$. Recall $p_0 \geq c_1$. If $p_0 > c_1$, it is obvious that $q_1^0 = 0$. If $p_0 = c_1$, there is an irrelevant multiplicity of optimal choices for firm 1: it is indifferent between all pairs (q_1^0, q_1^1) such that $q_1^0 + q_1^1$ is a given constant z. But no matter how 1 breaks the tie, this has no effect on the rest of the game, i.e., on the choice (q_2^0, q_2^1) of firm 2, or on the price of α, or on the payoffs of 1 and 2. Thus we may take[13] $q_1^0 = 0$ and $q_1^1 = z$.

The rest of the proof is again a matter of straightforward calculation. From (i), (ii), (iii) in the proof of Lemma 2, we can compute the payoffs $\Pi_i(p_0, p_1, q_2^1)$ at the terminal node of the game $\Gamma(c_0, c_1, a)$ that is reached by the unique NE of the subgame $G(p_0, p_1, q_2^1)$. (Note that Π_1 and Π_2 include the sunk cost $p_1 q_2^1$ incurred by 2 and concomitant gain $(p_1 - c_1)q_2^1$ of 1, prior to reaching the node (p_0, p_1, q_2^1) in Γ.) These are as follows (recalling the Cournot quantities $q_i^{\mathcal{C}}(p_0)$ from (3)).

$$\Pi_2(p_0, p_1, q_2^1) = \begin{cases} (q_2^{\mathcal{C}}(p_0))^2 + (p_0 - p_1)q_2^1 & \text{if } 0 \leq q_2^1 \leq q_2^{\mathcal{C}}(p_0) \\ (a + c_1 - 2p_1 - q_2^1)q_2^1/2 & \text{if } q_2^{\mathcal{C}}(p_0) < q_2^1 \leq a - c_1 \\ (a - q_2^1)q_2^1 - p_1 q_2^1 & \text{if } a - c_1 < q_2^1 < a \\ -p_1 q_2^1 & \text{if } q_2^1 \geq a \end{cases} \tag{5}$$

$$\Pi_1(p_0, p_1, q_2^1) = \begin{cases} (q_1^{\mathcal{C}}(p_0))^2 + (p_1 - c_1)q_2^1 & \text{if } 0 \leq q_2^1 \leq q_2^{\mathcal{C}}(p_0) \\ (a - c_1 - q_2^1)^2/4 + (p_1 - c_1)q_2^1 & \text{if } q_2^{\mathcal{C}}(p_0) < q_2^1 \leq a - c_1 \\ (p_1 - c_1)q_2^1 & \text{if } q_2^1 > a - c_1 \end{cases} \tag{6}$$

[13] To be very formal, when $p_0 = c_1$, the choices $(q_1^0, q_1^1) \in R_+^2$ of firm 1 may be partitioned into "equivalence classes" $\Lambda(z)$, $z \in R_+$, where $\Lambda(z) = \{(q_1^0, q_1^1) \in R_+^2 | q_1^0 + q_1^1 = z\}$. The game $G(p_0, p_1)$, and in particular the set of its SPNE, is unaffected by which element firm 1 picks in $\Lambda(z)$. In other words, when $p_0 = c_1$, firm 1 may be viewed as choosing only z (and it is irrelevant which point in $\Lambda(z)$ it actually picks to "effect" z).

Furthermore note that when we go a step back to the root of the tree $\Gamma(c_0, c_1, a)$, it can in fact never happen in any SPNE of $\Gamma(c_0, c_1, a)$ that $p_0 = c_1$ and that $q_1^0 > 0$ (see Lemma 7).

Next we move one step back in the game tree Γ and consider the maximization problem faced by firm 2 at the start of the game $G(p_0, p_1)$. The set of its optimal choices is

$$\beta(p_0, p_1) = \arg\max_{q_2^1 \geq 0} \Pi_2(p_0, p_1, q_2^1)$$

It can be verified, using (5), that

$$\beta(p_0, p_1) = \begin{cases} \{q_2^{\mathcal{S}}(p_1)\} \;\text{ if } p_1 < \tau(p_0) \\ \{0, q_2^{\mathcal{S}}(p_1)\} \;\text{ if } p_1 = \tau(p_0) \\ \{0\} \;\text{ if } p_1 > \tau(p_0) \end{cases} \tag{7}$$

The lemma follows from (i), (ii), (iii) (in the proof of Lemma 2) and (7). ■

Lemma 5. *Suppose* $p_1 \geq (a + c_1)/2$*. Then the SPNE of* $G(p_0, p_1)$ *are invariant of* p_1*. Hence w.l.o.g. we may restrict* $p_1 \leq (a + c_1)/2$*.*

Proof. When $p_1 \geq (a + c_1)/2$, we are in the region $R_{\mathcal{C}}$. So $q_2^1 = 0$ by (iv) of Lemma 4, proving the result. ■

Let us recall (from Lemma 4) the payoff $F(p_1)$ of firm 1, when 1 is the follower in $\mathcal{S}^{21}(p_1)$ and charges p_1 to firm 2, i.e., $F(p_1) \equiv f(p_1) + (p_1 - c_1)q_2^{\mathcal{S}}(p_0)$.

Lemma 6. *F is strictly increasing on* $[c_1, (a + c_1)/2]$.

Proof. Straightforward computation, using (4). ■

Lemma 7. *In any SPNE of* $\Gamma(c_0, c_1, a)$*, the following hold*:

(i) $p_1 > [(3c_1 - a)/2]_+$

(ii) $p_0 < (a + c_1)/2$

(iii) $q_2^0 + q_2^1 > 0$

(iv) *if* $p_0 < c_0$*, then* $q_1^0 = q_2^0 = 0$.

(v) *if* $q_2^1 > 0$*, then* $(p_0, p_1) \in$ *(Graph* τ*)*$[c_1, c_0]$.

Proof. (i) Suppose $p_1 \leq [(3c_1 - a)/2]_+$. Then we are in the region $R_{\mathcal{M}}$ and, by (ii) of Lemma 4 (see Figure 3) $q_2^1 > 0$. Since $0 < c_1 < a$ (by (1)), we have $[(3c_1 - a)/2]_+ < c_1$ and so $p_1 < c_1$. Thus firm 1's payoff is $(p_1 - c_1)q_1^1 < 0$. But 1 can deviate and set a sufficiently high price (any price above a will do) to ensure that firm 2 does not outsource to it, and thus 1 can earn a non-negative payoff, a contradiction.

(ii) By Lemma 3, we may suppose $p_0 \leq (a+c_1)/2$. So if the claim is false, $p_0 = (a+c_1)/2$. By Lemma 5 and (i) above, $p_1 \in ([(3c_1 - a)/2]_+, (a + c_1)/2]$. If $p_1 < (a + c_1)/2$, then $(p_0, p_1) \in R_{\mathcal{S}}$ and so, by (iii) of Lemma 4, firm 1 gets payoff $F(p_1)$. Since F is strictly increasing, we must have $p_1 = (a+c_1)/2$ (otherwise 1 can improve its payoff by increasing p_1). We conclude that $p_0 = p_1 = (a + c_1)/2$. Then $(p_0, p_1) \in$ Graph τ and, by (v) of Lemma 4, there are two possible SPNE of $G(p_0, p_1)$. No matter which prevails, $q_2^0 = 0$ (by (3) and (4)) and hence firm 0 gets zero payoff. Let 0 deviate by changing p_0 to p_0' where $c_0 < p_0' < (a+c_1)/2$. But $(p_0', p_1) \in R_{\mathcal{C}}$ and $q_2^0 > 0$ (by Lemma 4 and (3)), so firm 0 earns positive payoff as a result of the deviation, a contradiction.

(iii) Denote $I_0 \equiv [c_1, (a+c_1)/2)$ and $I_1 \equiv ([(3c_1-a)/2]_+, (a+c_1)/2]$. Then we have $(p_0, p_1) \in I_0 \times I_1$ by Lemma 5 and (i) and (ii) above.

First consider $p_1 = (a+c_1)/2$. Then $(p_0, p_1) \in R_{\mathcal{C}}$ for any $p_0 \in I_0$ and in any SPNE of $G(p_0, p_1)$, we have $q_2^0 = q_2^{\mathcal{C}}(p_0)$ by Lemma 4. Since $q_2^{\mathcal{C}}(p_0) > 0$ for $p_0 \in I_0$ (by (3)), we have $q_2^0 > 0$.

Next consider $p_1 \in I_1 \backslash \{(a+c_1)/2\}$. Then it follows from Lemma 4 that in any SPNE of $G(p_0, p_1)$, $q_2^0 + q_2^1$ equals either $q_2^{\mathcal{C}}(p_0)$ or $q_2^{\mathcal{S}}(p_1)$. Since $q_2^{\mathcal{C}} p_0) > 0$ for $p_0 \in I_0$ (by (3)) and $q_2^{\mathcal{S}}(p_1) > 0$ for $p_1 \in I_1 \backslash \{(a+c_1)/2\}$ (by (4)), the result follows.

(iv) When $p_0 < c_0$, firm 0 gets $(p_0 - c_0) < 0$ dollars for every unit that is outsourced to it. If $q_1^0 + q_2^0 > 0$, then 0 gets negative payoff. But 0 can deviate and set a sufficiently high price to ensure that no firm outsources to it and 0 can thus guarantee zero payoff, a contradiction.

(v) By (i) and (ii) above and by Lemma 5, $c_1 \leq p_0 < (a+c_1)/2$ and $[(3c_1-a)/2]_+ < p_1 \leq (a+c_1)/2$. But then, by Lemma 4, $q_2^1 > 0$ implies $(p_0, p_1) \in [R_{\mathcal{S}} \cup$ Graph $\tau]$. If $(p_0, p_1) \in R_{\mathcal{S}}$, i.e., $p_1 < \tau(p_0)$, then (again by Lemma 4) firm 1 earns $F(p_1)$. Since F is strictly increasing (by Lemma 6), 1 can improve its payoff by raising its price to $p_1 + \varepsilon < \tau(p_0)$, a contradiction. This proves $(p_0, p_1) \in$ Graph τ.

It remains to show that $p_0 \leq c_0$. Suppose $p_0 > c_0$. Then since $c_0 > c_1$ by assumption, we have $p_0 > c_1$ which immediately implies that $q_1^0 = 0$. Since $q_2^1 > 0$, we also have $q_2^0 = 0$ by (i) of Lemma 4. So firm 0 gets no order and earns zero payoff. Let firm 0 reduce p_0 to $p_0 - \varepsilon > c_0$. Since $(p_0, p_1) \in$ Graph τ as shown in the previous paragraph, $(p_0 - \varepsilon, p_1) \in R_{\mathcal{C}}$ (see Figure 3) and so firm 2 will outsource a positive amount to 0 after 0's deviation (by Lemma 4 and (3)). Thus 0 earns a positive payoff after its deviation, a contradiction. ■

Recall the Stackelberg and Cournot duopoly games, $\mathcal{S}^{21}(c)$ and $\mathcal{C}(c)$ in which the cost of firm 1 is fixed at c_1 while that of its rival firm 2 is a variable c. The function $\kappa_1(c)$ simply gives the standard Cournot profit of firm 1. In contrast, $F(c) = f(c) + (c - c_1) q_2^{\mathcal{S}}(c)$ lumps together the profit $f(c)$ that 1 makes as the follower in $\mathcal{S}^{21}(c)$ as well as the revenue $(c - c_1) q_2^{\mathcal{S}}(c)$ that 1 earns by supplying 2 its Stackelberg-leader output $q_2^{\mathcal{S}}(c)$ at price c.

The following lemma compares F and κ_1. First define

$$\tilde{c} = 55c_1/62 + 7a/62 \tag{8}$$

and observe that $c_1 < \tilde{c} < (a+c_1)/2$ by (1).

Lemma 8. (Figure 4) κ_1 *is strictly increasing on* $[c_1, (a+c_1)/2]$. *Moreover,* $F < \kappa_1$ *on* $[c_1, \tilde{c})$, $F > \kappa_1$ *on* $(\tilde{c}, (a+c_1)/2)$, $F(\tilde{c}) = \kappa_1(\tilde{c})$ *and* $F((a+c_1)/2) = \kappa_1((a+c_1)/2)$.
Proof. Straightforward computation using the explicit formulae for κ_1 and F that follow from (3) and (4). ■

For any c, we shall define $\lambda(c)$ to be the minimum cost of firm 2 at which 1 is willing to switch from the Cournot game $\mathcal{C}(c)$ to being follower in the Stackelberg game $\mathcal{S}^{21}(\lambda(c))$. Precisely

$$\lambda : [0, \tilde{c}] \to [0, \tilde{c}]$$

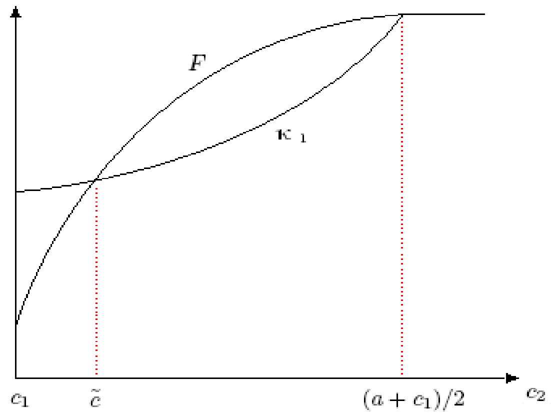

Figure 4: The Functions κ_1 and F

is given by

$$\lambda \equiv F^{-1} \circ \kappa_1.$$

The function λ is well-defined, strictly increasing and $\lambda(\tilde{c}) = \tilde{c}$.

Lemma 9. (Figure 5) *Let $c \in [c_1, \tilde{c}]$. Then $\kappa_1(c) = F(\lambda(c))$, $F(y) < \kappa_1(c_2)$ for $y < \lambda(c)$ and $F(y) > \kappa_1(c)$ for $y > \lambda(c)$.*

Proof. The proof follows from lemmas 6, 8 and the definition of λ. ■

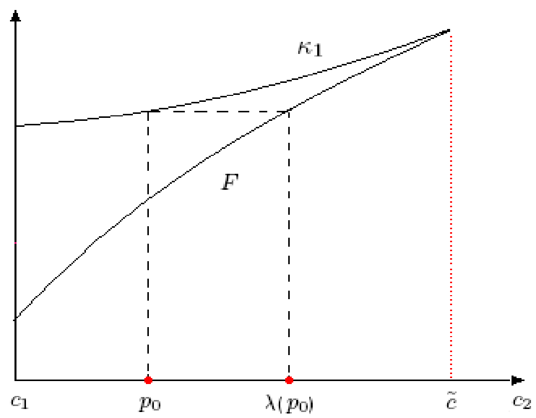

Figure 5: The Function λ

The next lemma compares the functions τ and λ. Define

$$c^* = 13c_1/14 + a/14 \tag{9}$$

and observe from (1) and (8) that

$$c_1 < c^* < \tilde{c}. \tag{10}$$

Lemma 10. (Figure 6) *Let* $c \in [c_1, \tilde{c}]$. *Then* $\lambda(c^*) = \tau(c^*)$, $\tau(c) < \lambda(c)$ *for* $c \in [c_1, c^*)$, $\tau(c) > \lambda(c)$ *for* $c_2 \in (c^*, \tilde{c}]$.

Proof. Straightforward computation using the explicit formula for τ in (2) and the explicit formulae for κ_1 and F that follow from (3) and (4). ■

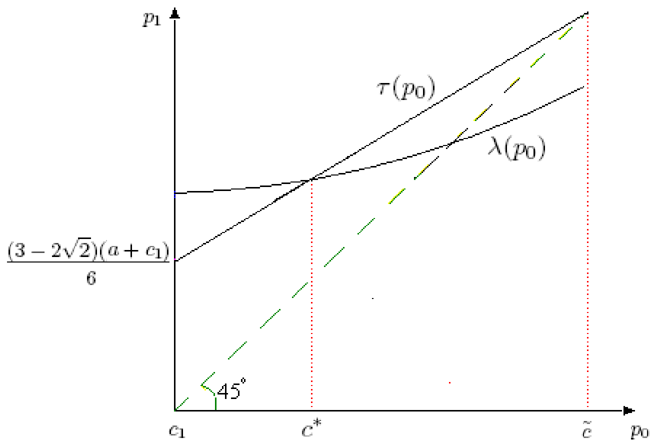

Figure 6: $\tau(p_0)$ and $\lambda(p_0)$

Lemma 11. $(p_0 - c_0)q_2^{\mathcal{C}}(p_0)$ *is increasing in* p_0 *for* $p_0 \in [c_1, \tilde{c}]$.

Proof. A simple calculation shows that $(p_0 - c_0)q_2^{\mathcal{C}}(p_0) = (p_0 - c_0)(a + c_1 - 2p_0)/3$ for $p_0 \in [c_1, (a + c_1)/2]$ from which the result follows. ■

Lemma 12. *In any SPNE of* $\Gamma(c_0, c_1, a)$, *if* $q_2^0 > 0$, *then* $(p_0, p_1) \in (Graph\tau)[c_1, c^*]$.

Proof. In step 1 we show that $p_0 \in [c_1, c^*]$ and in step 2 we show that $p_1 = \tau(p_0)$.

Step 1: By Lemma 1, we have $p_0 \geq c_1$. So it suffices to show that $p_0 \leq c^*$.

Since $q_2^0 > 0$ we have, by (iv) and (v) of Lemma 4, that $(p_0, p_1) \in [R_{\mathcal{C}} \cup$ Graph $\tau]$ and that the payoff of firm 1 is $\kappa_1(p_0)$. In what follows, we show that if $p_0 > c^*$, firm 1 can earn more than $\kappa_1(p_0)$ by setting a price $p_1' < \tau(p_0)$, a contradiction establishing step 1.

Recall from (10) that $c_1 < c^* < \tilde{c}$. First suppose that $\tilde{c} \leq p_0 < (a + c_1)/2$. By (ii) of Lemma 7, we must have $p_0 < (a + c_1)/2$. Let firm 1 change p_1 to $p_1' \equiv \tau(p_0) - \varepsilon > 0$. Then $(p_0, p_1') \in R_{\mathcal{S}}$ and, by (iii) of Lemma 4, 1's payoff is $F(p_1')$. Since $\tau(p_0) > p_0$ for $p_0 < (a + c_1)/2$, we have $p_1' > p_0$ for small enough ε, implying that $F(p_1') > F(p_0)$ (since F is strictly increasing—see Lemma 6). Since $p_0 \geq \tilde{c}$, it follows from Lemma 8 that $F(p_0) \geq \kappa_1(p_0)$. Hence $F(p_1') > \kappa_1(p_0)$, showing that firm 1 has made a gainful deviation, a contradiction. So we must have $p_0 < \tilde{c}$.

Now suppose that $c^* < p_0 < \tilde{c}$. Then $\lambda(p_0) < \tau(p_0)$ by Lemma 10. Let firm 1 change p_1 to $p_1' \equiv \lambda(p_0) + \varepsilon$ where ε is small enough to ensure that $\lambda(p_0) + \varepsilon < \tau(p_0)$. Then $(p_0, p_1') \in R_{\mathcal{S}}$ and 1 gets the payoff $F(p_1')$ (by (iii) of Lemma 4). Since F is strictly increasing, $F(p_1') > F(\lambda(p_0))$. By the definition of λ, we have $F(\lambda(p_0)) = \kappa_1(p_0)$. Hence

$F(p_1') > \kappa_1(p_0)$, showing that firm 1 has made a gainful deviation, a contradiction. This proves that $p_0 \in [c_1, c^*]$.

Step 2: Since $q_2^0 > 0$, we must have $(p_0, p_1) \in [R_{\mathcal{C}} \cup \text{ Graph } \tau]$ and the payoff of firm 0 is $(p_0 - c_0)q_2^{\mathcal{C}}(p_0)$ (by (iv) and (v) of Lemma 4).

By Lemma 5, we may suppose that $p_1 \leq (a + c_1)/2$. We have already shown that $p_0 \in [c_1, c^*]$. Since $c^* < (a + c_1)/2$, we have $\tau(p_0) < (a + c_1)/2$. If $(p_0, p_1) \in R_{\mathcal{C}}$, then $p_1 \in (\tau(p_0), (a+c_1)/2]$. Let 0 deviate and set a price $p_0' \equiv p_0 + \varepsilon < \tilde{c}$ where ε is sufficiently small to ensure that $p_0' < \tilde{c}$ and $p_1 > \tau(p_0')$. Then $(p_0', p_1) \in R_{\mathcal{C}}$ and firm 0 will earn $(p_0' - c_0)q_2^{\mathcal{C}}(p_0')$ after the deviation. Then by Lemma 11, it follows that $(p_0' - c_0)q_2^{\mathcal{C}}(p_0') > (p_0 - c_0)q_2^{\mathcal{C}}(p_0)$. This shows that when $(p_0, p_1) \in R_{\mathcal{C}}$, firm 0 can make a gainful deviation. Hence we must have $(p_0, p_1) \in \text{ Graph } \tau$ which, together with $p_0 \in [c_1, c^*]$, proves that $(p_0, p_1) \in (\text{Graph } \tau)[c_1, c^*]$. ■

4.2 Proof of the Theorem

Proof of (I) This has been proved as (i) of Lemma 4 and (iv) if Lemma 7.

Proof of (II) Consider $c_0 < c^*$. First we show that $q_2^1 = 0$ in any SPNE. For if $q_2^1 > 0$, we must have $(p_0, p_1) \in (\text{Graph } \tau)[c_1, c_0]$ by (v) of Lemma 7. Then $p_1 = \tau(p_0) < (a + c_1)/2$ and, by (v) of Lemma 4, firm 1 gets payoff $F(p_1) = F(\tau(p_0))$. Let 1 deviate and choose $p_1' \in (\tau(p_0), (a+c_1)/2]$. Then $(p_0, p_1') \in R_{\mathcal{C}}$ and, by (iv) of Lemma 4, 1 gets payoff $\kappa_1(p_0)$. Since $p_0 \leq c_0 < c^*$, Lemma 10 implies that $\tau(p_0) < \lambda(p_0)$. By the strict monotonicity of F (Lemma 6), it follows that $F(\tau(p_0)) < F(\lambda(p_0))$. By the definition of λ, $F(\lambda(p_0)) = \kappa_1(p_0)$. Since $F(p_1) = F(\tau(p_0))$, we conclude that $\kappa_1(p_0) > F(p_1)$, showing that firm 1 has improved after the deviation, a contradiction.

By (iii) of Lemma 7, $q_2^0 + q_2^1 > 0$. We have just shown that $q_2^1 = 0$. Hence we must have $q_2^0 > 0$ in any SPNE. Then it follows from Lemma 12 that $(p_0, p_1) \in (\text{Graph } \tau)[c_1, c^*]$ in any SPNE. By (iv) of Lemma 7, we must have $p_0 \geq c_0$. Since $c_0 < c^*$, the interval $[c_0, c^*]$ is non-empty, hence $(p_0, p_1) \in (\text{Graph } \tau)[c_0, c^*]$.

It remains to show that for any $(p_0, p_1) \in (\text{Graph } \tau)[c_0, c^*]$ we do get an SPNE with $q_2^0 > 0$.

First consider firm 2. Since $(p_0, p_1) \in \text{Graph } \tau$ we see (by (v) of Lemma 4) that firm 2 has exactly two optimal choices, which involve exclusive orders from either 0 or 1. Since it is already choosing the former, it cannot profit by a unilateral deviation.

Next consider firm 0. Its payoff is $(p_0 - c_0)q_2^{\mathcal{C}}(p_0)$, which is non-negative since $p_0 \geq c_0$. But $(p_0, p_1) \in \text{Graph } \tau$, i.e., $p_1 = \tau(p_0)$. If 0 reduces its price from p_0 to $p_0' < c_1$, then (since $c_1 < c_0$), 0 gets at most zero payoff. If 0 reduces its price from p_0 to $p_0' \geq c_1$, then $\tau(p_0') < \tau(p_0) = p_1$. So $(p_0, p_1) \in R_{\mathcal{C}}$ implying (by (iv) of Lemma 4) that 0 will get $(p_0' - c_0)q_2^{\mathcal{C}}(p_0')$. Observe that $(p_0' - c_0)q_2^{\mathcal{C}}(p_0') < (p_0 - c_0)q_2^{\mathcal{C}}(p_0)$ (by Lemma 11 and the fact that $c^* < \tilde{c}$), so again the deviation is not gainful. If 0 increases its price from p_0 to p_0', then $\tau(p_0') > \tau(p_0) = p_1$ and $(p_0, p_1) \in R_{\mathcal{S}}$. Then, by (iii) of Lemma 4, 0 gets zero payoff, again gaining nothing.

Finally consider firm 1. Its payoff is $\kappa_1(p_0)$. Recall that $p_1 = \tau(p_0)$. If 1 raises its price to $p'_1 > p_1 = \tau(p_0)$, then $(p_0, p'_1) \in R_{\mathcal{C}}$ and, by (iv) of Lemma 4, 1 will still get $\kappa_1(p_0)$. If 1 lowers its price to $p'_1 < p_1 = \tau(p_0)$, then $(p_0, p'_1) \in R_{\mathcal{S}}$ and, by (iii) of Lemma 4, 1 will get $F(p'_1)$. By the strict monotonicity of F, we have $F(p'_1) < F(p_1) = F(\tau(p_0))$. Since $p_0 \le c^*$, we have $\tau(p_0) \le \lambda(p_0)$ (Lemma 10), so that $F(\tau(p_0)) \le F(\lambda(p_0))$. By the definition of λ, $F(\lambda(p_0)) = \kappa_1(p_0)$. Hence we conclude that $F(p'_1) < \kappa_1(p_0)$, showing that 1 cannot improve by any unilateral deviation. This completes the proof of part (II).

Proof of (III) Consider $c^* < c_0 < (a + c_1)/2$. If $q_2^0 > 0$, then (a) $p_0 \ge c_0$ (by (iv) of Lemma 7) and (b) $p_0 \le c^*$ (by Lemma 12). Since $c^* < c_0$, both (a) and (b) cannot hold. So we must have $q_2^0 = 0$. Since $q_2^0 + q_2^1 > 0$ (by (iii) of Lemma 7), we conclude that $q_2^1 > 0$. Then, by (v) of Lemma 7, it follows that $(p_0, p_1) \in (\text{Graph } \tau)[c_1, c_0]$.

It remains to show that for any $(p_0, p_1) \in (\text{Graph } \tau)[c_1, c_0]$ we do get an SPNE with $q_2^0 > 0$.

First consider firm 2. We can argue exactly as in the proof of (I) that it cannot make a gainful unilateral deviation.

Next consider firm 0. Its payoff is zero. Since $p_0 \in [c_1, c_0]$, by lowering its price to $p'_0 < p_0 \le c_0$, it can get at most zero payoff. Since $(p_0, p_1) \in \text{Graph } \tau$, $p_1 = \tau(p_0)$. If 0 it raises its price to $p'_0 > p_0$, then $\tau(p'_0) > \tau(p_0) = p_1$. Hence $(p'_0, p_1) \in R_{\mathcal{S}}$ and, by (iii) of Lemma 4, 0 continues to get zero payoff.

Finally consider firm 1. Its payoff is $F(p_1) = F(\tau(p_0))$. If 1 lowers its price to $p'_1 < p_1 = \tau(p_0)$, then $(p_0, p'_1) \in R_{\mathcal{S}}$ and, by (iii) of Lemma 4, 1 gets $F(p'_1)$. By the monotonicity of F, $F(p'_1) < F(p_1)$ and so 1 does not profit. If 1 raises its price to $p'_1 > p_1 = \tau(p_0)$, then $(p_0, p'_1) \in R_{\mathcal{C}}$ and, by (iv) of Lemma 4, 1 gets $\kappa_1(p_0)$. Consider two cases. If $p_0 \ge \tilde{c}$, we have $F(p_0) \ge \kappa_1(p_0)$ by Lemma 8. Since $p_0 < (a + c_1)/2$, we have $\tau(p_0) > p_0$ (see Figure 2) so that $F(\tau(p_0)) > F(p_0)$. Hence $F(\tau(p_0)) > \kappa_1(p_0)$, so 1 does not gain. If $p_0 < \tilde{c}$, we have $\tau(p_0) > \lambda(p_0)$ by Lemma 10, so $F(\tau(p_0)) > F(\lambda(p_0))$. By the definition of λ, $F(\lambda(p_0)) = \kappa_1(p_0)$ and we have $F(\tau(p_0)) > \kappa_1(p_0)$, so once again 1 does not gain. This completes the proof of part (III).

Proof of (IV). The argument is as in parts (II) and (III), hence omitted. ■

5 Variations of the model

Our model can be varied in many ways, but the essential theme remains intact: if $\mathcal{O}$'s costs are not much higher than $\mathcal{I}$'s, $\mathcal{J}$ will outsource to $\mathcal{O}$. The overall analysis follows the outline of the proof of Theorem 1, but the details can get more complicated, and we omit them here.

5.1 Economies of scale

Keeping the rest of the model fixed as before, now suppose that there are increasing, instead of constant, returns to scale in the manufacture of the intermediate good η, i.e., the average cost $c_i(q)$ of manufacturing q units of η falls (as q rises) for both $i = 0, 1$. For simplicity, suppose $c_i(q)$ falls linearly and that $c_0(q) = \lambda c_1(q)$ for some positive scalar[14] λ. It can then be shown that there exists a threshold $\lambda^* > 1$ such that if $\lambda < \lambda^*$:

(i) firm 2 outsources to firm 0 in any SPNE,

(ii) both firms 1 and 2 outsource to firm 0 in any SPNE when economies of scale are not too small.

This result is established in Chen and Dubey (2005). (We already gave the intuition for it in the introduction.)[15]

5.2 Multiple firms of each type

Suppose there are n_0, n_1, n_2 replicas of firms 0, 1, 2. The timing of moves is assumed to be as before, with the understanding that all replicas of a firm move simultaneously wherever that firm had moved in the original game. Restricting attention to type-symmetric SPNE, Theorem 1 again remains intact with a lower threshold.

5.3 Only Outside Suppliers

The strategic incentives that we have analyzed can arise in other contexts. Suppose, for instance, that 1 and 2 both need to outsource the supply of the intermediate good η to outsiders $\mathcal{O} = \{O_1, O_2, \ldots\}$. If 2 goes first to $\mathcal{O}$ and 1 knows which O_i has received 2's order, then 1 will have incentive to outsource to some O_j that is distinct from O_i, even if O_j's costs are higher than O_i's, so long as they are not much higher. For if 1 went to O_i, it might have to infer the size of 2's orders and thus be obliged to become a Stackelberg follower (e.g., because O_i has limited capacity and can attend to 1's order only after fully servicing the prior order of 2). Alternatively, even if 1 does not know who 2 has outsourced to, or indeed if 2 has outsourced at all, it may be safer for 1 to spread its order among several firms in $\mathcal{O}$ so that it minimizes the probability of becoming 2's follower. We leave the precise modeling and analysis of such situations for future research.

6 The Secrecy Clause

It is crucial to our analysis that the quantity outsourced by 2 to 0 cannot be observed by 1. This is not an unrealistic assumption. Many contracts, in practice, do incorporate a confidentiality or secrecy clause (see, e.g., Ravenhill, 2003; Clarkslegal and Kochhar, 2005).

But the secrecy clause can often be *deduced* to hold endogenously in equilibrium (in appropriately "enlarged" games).

[14] Thus $c_1(q) = \max\{0, c - bq\}$ and $c_0(q) = \lambda \max\{0, c - bq\}$ for positive scalars b, c, λ.

[15] It is needed here that the economies of scale be not too pronounced, otherwise pure strategy SPNE may fail to exist. More precisely, for the average cost function $c_1(q)$ =max$\{0, c - bq\}$, it is assumed that $0 < b < c/2a$ to guarantee (i) the existence of pure strategy SPNE and (ii) in equilibrium, the quantity produced entails positive marginal cost.

Indeed suppose that the quantity q outsourced by 2 to 0 can be made "public" (and hence observable by 1) or else kept "secret" between 2 and 0. We argue that a public contract can never occur (be active) at an SPNE, as long as the game provides sufficient "strategic freedom" to its various players. For suppose it did occur : 1 knew that 2 buys q units of η from 0 at price p_0. Thus 1 is a Stackelberg follower in the final market α, regardless of whom 2 chooses to outsource η to. It would be better for 1 to quote a lower price $p_0 - \varepsilon$ for η. This would be certain to lure 2 to outsource to 1. But $p_0 \geq c_0$, since 0 could not be making losses at the presumed SPNE; hence $p_0 - \varepsilon > c_1$ for small enough ε (recall $c_0 > c_1$). By manoeuvering 2's order to itself, firm 1 thus earns a significant profit on the manufacture of η. It does lose a little on the market for α, because 2 has a lower cost $p_0 - \varepsilon$ of η (compared to the p_0 earlier), but the loss is of the order of ε. Thus 1 has made a profitable unilateral deviation, contradicting that we were at an SPNE.

Note that our argument relies on the fact that 1 has the strategic freedom to "counter" the public contract. If, furthermore, 0 also has the freedom to reject the public contract and counter it with a secret contract, then—foreseeing the above deviation by firm 1—firm 0 will only opt for secret contracts.

The most simple instance of such an enlarged game is obtained by inserting an initial binary move by 0 at the start of our game Γ. This represents a declaration by 0 as to whether its offer to 2 is by way of a public or a secret contract. The game Γ follows 0's declaration. It is easy to verify that any SPNE of the enlarged game must have 0 choosing "secret", followed by an SPNE of Γ. Of course, more complicated enlarged games can be thought of. For example, after the simultaneous announcement of p_0 and p_1 in our game Γ, suppose firm 2 has the option to choose "Public q" or "Secret q" in the event that it goes to 0, followed by "Accept" or "Reject" by 0. Clearly 1 finds out q only if "Public q" and "Accept" are chosen. On the other hand, if 0 chooses "Reject" we (still having to complete the definition of the enlarged game) could suppose that 2's order of η is automatically directed to 1. This game is more complex to analyze, but our argument above still applies and shows that a public contract will never be played out in any SPNE.

We thus see that the secrecy clause can often emerge endogenously from strategic considerations, even though—for simplicity—we postulated it in our model. It has been pointed out already by Clarkslegal and Kochhar that the firm placing orders (firm 2 in our model) may demand secrecy in order to protect sensitive information from leaking out to its rivals and destroying its competitive advantage. Our analysis reveals that the firm *taking* the orders (i.e., firm 0) may also—for more subtle strategic reasons—have a vested interest in maintaining the secrecy clause.

References

Aggarwal, V.K. (Ed.), 2003. *Winning in Asia, US Style: Market and Nonmarket strategies of Success*, Palgrave Macmillan.

Baake, P., Oechssler, J. and Schenk, C., 1999. Explaining Cross-Supplies. *Journal of Economics*, 70, 37-60.

Chen, Y. and Dubey, P., 2005. Outsourcing under Strategic Competition and Increasing

Returns to Scale. Mimeo, Center for Game Theory in Economics, SUNY at Stony Brook.

Chen, Y., Ishikawa, J. and Yu, Z., 2004. Trade Liberalization and Strategic Outsourcing. *Journal of International Economics*, 63, 419-436.

Clarkslegal and Kochhar, 2005. *Outsourcing fact sheet*.
<http://www.clarkslegal.com/download/pdf/OUT_fs_english_indian_contracts.pdf>

Domberger, S. 1998. *The Contracting Organization: A Strategic Guide to Outsourcing*. Oxford University Press.

Hamilton, J. and Slutsky, S., 1990. Endogenous Timing in Duopoly Games: Stackelberg or Cournot Equilibria. *Games and Economic Behavior*, 2, 29-46.

Jarillo, J.C. 1993. *Strategic Networks: Creating Borderless Organization*. Butterworth-Heinmann.

Mailath, G.J. 1993. Endogenous Sequencing of Firm Decisions. *Journal of Economic Theory*, 59, 169-182.

Pal, D. 1996. Endogenous Stackelberg Equilibria with Identical Firms. *Games and Economic Behavior*, 12, 81-94.

Ravenhill, J. 2003. Market and Nonmarket Strategies of the U.S. Electronics Industry, in Aggarwal (2003), 206-235.

Robson, A., 1990. Duopoly with Endogenous Strategic Timing: Stackelberg Regained. *International Economic Review*, 31, 263-274.

Shy, O. and Stenbacka, R., 2003. Strategic Outsourcing. *Journal of Economic Behavior and Organization*, 50, 203-224.

Spiegel, Y., 1993. Horizontal Subcontracting. *The RAND Journal of Economics*, 24, 570-590.

van Damme, E. and Hurkens, S. 1999. Endogenous Stackelberg Leadership. *Games and Economic Behavior*, 28, 105-129.

Vidal, C.J. and Goetschalkx M. 1997. Strategic Production-Distribution Models: A Critical Review with Emphasis on Global Supply Chain Models. *European Journal of Operational Research*, 98, 1-18.