THEORY OF MONEY AND FINANCIAL INSTITUTIONS: A SUMMARY OF A GAME THEORETIC APPROACH

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Abstract

A game theoretic approach to the theory of money and financial institution is given utilizing both the strategic and coalitional forms for describing the economy. The economy is first modeled as a strategic market game, then the strategic form is used to calculate several cooperative forms that differ from each other in their utilization of money and credit and their treatment of threats. It is shown that there are natural upper and lower bounds to the monetary needs of an economy, but even in the extreme structures the concept of "enough money" can be defined usefully, and for large economies the games obtained from the lower and upper bounds have cores that approach the same limit that is an efficient price system. The role of disequilibrium is then discussed.

JEL Classification: C71, C72, D53, E40

Keywords: Money, prices, core, threat, market game, strategic market game.

1. INTRODUCTION

This paper presents a non-technical sketch of a game theoretic approach to the theory of money and financial institutions developing the thesis that both the institutional aspects of the financial system and the mathematical analytics are needed to provide an adequate understanding of the role of finance in economic theory. The game theoretic details are given in a series of publications, alone or with various joint authors over the last fifty years.

No theorems are given here, but the models, concepts and results are stressed. In particular, the appropriate level of abstraction is stressed and interpreted in game theoretic terms. The level of abstraction is highly dependent on the question at hand. Too high a level of abstraction may guarantee the question cannot be answered as key details have been omitted. Too low a level of abstraction leaves one with a morass of detail that may easily block analysis. The selection of the right level of abstraction still is somewhat of an art form, not unlike the provision of context given by a sensitive historian.

The theory of games provides the methods for strategic analysis. The major roles of finance in the economy are evaluative and strategic, thus game theory methods are natural to its study. Finance, together with markets, provides the evaluation and control mechanisms needed to control and run a complex economy. The role of finance is to control the dynamics of an economy. In equilibrium its role can easily be unseen as it disappears as a paper or symbolic shadow of the physical production and consumption processes it guides.

2. PRIMITIVE CONCEPTS OF A FORMAL GAME

Much of formal game theory utilizes one of three forms of description as the primitive concept accepted from which the formal economic model is developed. The three forms are: The extensive form of the game; the strategic form and the coalitional form. Each can be utilized without reference to the other, but the level of abstraction clearly flows in one direction.

The extensive form is clearly process oriented and essentially institutional. All moves and information conditions are spelled out. The rules of the game are given in detail. It is a matter of a modeling decision as to whether to include items such as verbal statements as part of the formal moves of the game. Usually they are excluded because it is extremely difficult to reduce them to formal mathematics, but in a limited way it can be done.

The strategic form can be derived from the extensive form by utilizing the construction of a set of strategies derived from the extensive form. The game theoretic concept of a strategy is a complete book of instructions that could be given by a player to a proxy player to play on his behalf. The instructions must cover every contingency feasible in the game. Even a casual examination of the extensive form of a game as simple as tic-tac-toe or Checkers has billions of strategies. In elementary expositions of game theory the 2×2 matrix game is often utilized. The hyperinflation of the number of strategies is glossed over in the exploration of the non-cooperative equilibria of the game in strategic form. In actual institutions, consultation, delegation and aggregation are used to execute strategies.

The conceptual leap from the extensive form to the strategic form is achieved in one bound having to apply no more than logic. No extra considerations of modeling are called for. This is not so when we proceed from the strategic representation of a game to the coalitional representation. In multilateral bargaining and diplomatic negotiations the coalitional form is a natural starting place. We ask the straightforward question "What can a coalition of size S achieve if it refuses to cooperate? " Unfortunately this question is not fully formulated. It does not specify what we expect the influence to be of the other set of $N - S$ participants. They could ignore S, or could carry out sanctions or threats that are of significance to S's welfare. This is the well known problem of specifying the threat structure when describing a game in coalitional form. The solution lies far less in the mathematics than in the perceptive modeling of the problem at hand. Those who are more interested in the mathematics of solutions to games in coalitional form than in the mapping from the physical phenomenon to the coalitional representation (often referred to as the characteristic function) accept the coalitional form as a primitive concept without argument or concern as to where it came from.

The progression to higher degrees of abstraction as we proceed from the extensive form to the strategic form and then to the coalitional form can be verified when it is observed that there is a one-to-many mapping from the coalitional form to the strategic form and from the strategic form to the extensive form, but not vice-versa.

3. EXCHANGE MODELS AND THE LEVEL OF ABSTRACTION

A prime concern here is to compare the general equilibrium model of economic exchange with game theoretic approaches to the same problems. Two basic game theoretic approaches are adopted. They are strategic market games based on the strategic form and solved utilizing the noncooperative equilibrium and market games based on the coalitional form and solved primarily for the core of the game in coalitional form.

Without going into detail, some basic observations can be made concerning the relationship among market games, modified market games, strategic market games and the competitive equilibrium exchange economy.

The mathematical model of the Walrasian economy was a masterpiece of parsimony, but the existence proof of an efficient price system was obtained at the cost of providing a model that could not be played as an experimental game without providing considerable extra detail to convert it into a fully defined process model. The omission of price formation was an abstraction that was sufficient for an existence proof but gave an essentially dynamic and strategic process an appearance of timelessness. The general equilibrium system is presented as a model with a high level of abstraction, well defined for the examination of some of its static properties, but not adequately defined for the study of process or the role of information. In game theoretic terms, the model presented is an incompletely formulated game in strategic form. It is as though all time has been collapsed into a single move.

The two game theoretic approaches to modeling a multi-person economy both provide completely well defined games. The strategic market game utilizes the extensive or strategic form and is considerably more institutional than the general equilibrium model. It offers details concerning price formation. An immediate criticism that can be leveled against it is that going down to the level of detail required to specify price mechanisms destroys the generality of the model and produces a welter of special cases from which little of general interest can be deduced. This however, may be a fact of life whose influence is modified when competition is sufficiently large.

Market games in contrast with strategic market games are at a higher level of abstraction than the general equilibrium system. They utilize the coalitional form as the given or primitive structure. Market games have been studied in two forms, usually referred to as side-payment (SP) and no side-payment games (NSP). The distinction can most easily seen in terms of the existence or non-existence of a special monetary commodity we can call Marshallian money, i.e., a special extra commodity that enters the preferences of all individuals as a linear separable term in their utility functions. Another way of expressing this is that the utility functions of all individuals are quasi-concave. If an economy has enough Marshallian money the individuals can jointly maximize their welfare and adjust their division of proceeds on a planar Pareto optimal surface.

The NSP game has no magic transferable utility, but in a game theoretic model of an exchange economy with many small agents it is as if the unit of account serves as a transferable utility for each individual locally. To a small individual the unit of account acts as though he has perfect liquidity, but for the system as a whole the unit of account nets to zero. The game theoretic literature on the λ-transfer provides the mathematical basis for the link between individual liquidity and global illiquidity (Shubik, 1984).

4. THE SOLUTIONS ADOPTED

The different models employ different solution concepts. In essence, a formal game is described by its rules. These in turn enable the investigator to describe the complete set of outcomes from all plays of the game (sometimes referred to in the study of stochastic processes as the complete state space). A solution is nothing more than some device for selecting a subset of outcomes from the total set of outcomes. A solution designed to provide a specific prediction will select a single outcome.

4.1. The Competitive Equilibrium

The competitive equilibrium solution stresses individual optimization where each agent acts given all market prices. How these prices are formed is dealt with by the assumption of rational expectations where the implicit assumption is that the expectations of all individuals are mutually consistent. As is well known, (Debreu, 1959) generically the competitive equilibrium solution is a finite point set, i.e., the solution is not unique. There can be many equilibria each with different implications concerning the distribution of wealth in exchange. If the general equilibrium model as formulated by Arrow and Debreu (1954) and Debreu (1959) is adhered to, the conditions for the uniqueness of a competitive equilibrium pose a mathematically difficult problem that has not yet been resolved. However if the problem is enlarged and bankruptcy conditions are considered there is a straightforward unique solution (see Qin and Shubik 2006).

The general equilibrium system is usually presented as stressing the virtues of competition. Yet this is not correctly reflected in the mathematics unless a continuum of agents is considered or a limiting process that reflects "many agents" is defined. The proof presented by Debreu in his otherwise precise book holds true for $n = 1$, where n is the number of agents. This is hardly a description of competition.

4.2. Strategic Market Games

Strategic market games provide full process models. Phrased differently, any strategic market game is sufficiently specified that it is possible to program a playable game that can be used for experimentation. Unfortunately, the need to be specific about items such as price formation opens the model builder up to a plethora of special cases. How can we deal with the overwhelming number of cases that appear to exist when we look at the institutional realities? There are ways to do so. We may attempt to define minimal institutions and we can also consider models that in some appropriate way reflect upper and lower bounds on relevant processes.

The concept of minimal institution can be made rigorous in terms of game theory modeling. An illustration serves to indicate how. The simplest price formation mechanism is one where some individuals bid quantities of money and others simultaneously offer quantities of some goods. The strategic and extensive form of the game can be made to coincide. There is only one information set per player and a move and a strategy coincide (Shubik, 1972). It is shown elsewhere that there are three elementary mechanisms for forming price. The first is known as the sell-all market, where all individuals are required to put up all of their physical assets for sale. This is a tax collector's dream, evaluating all wealth at market. The agents bid amounts of money in order to buy and they receive the income from their sales later (Shapley and Shubik, 1977). In any market the move of an individual is one-dimensional, consisting of a bid. The second market is the buy-sell market, where quantities of the good are offered for sale and quantities of money are bid to buy all goods for sale (Dubey and Shubik, 1978). The third model is the simultaneous move double auction. Here the strategy of each individual has four dimensions in each market. She can name a personal price and quantity at which she is willing to sell and similarly a price and quantity to buy.

A further market model has been developed by Sorin (1996) that captures the possibility that each individual issues his own currency. It can be interpreted in terms of international trade or an economy where "everyone is a banker. " The integrating factor in the model is that as soon as all individuals have bid their own currencies a centralized efficient clearing house calculates the exchange rate required to balance all books and informs all so that the goods shipped to each bidder reflect the bidder's claims accurately by being adjusted for the correct exchange rates. In actuality, incomplete information and imperfect reputation make this ideal market hard to obtain.

In each model the solution utilized is the noncooperative equilibrium (Nash, 1951). Because all of the models are "one-shot games " where the strategic form supplies the same information as the extensive form game there are no qualifications about the nature of a noncooperative equilibrium, and although the noncooperative equilibria are not unique they generically form a point set. This does not hold once the extensive and strategic forms differ. Concepts such as the perfect equilibrium (Selten, 1975) are required to distinguish among those equilibria which do or do not reflect threats and do or do not make use of history. Even the simple definition of equilibrium manifests some difficulties (Shubik, 1996).

4.3. Market Games

There are many solution concepts associated with a game in coalitional form. The characteristic function itself can be regarded as a "presolution," in the sense that it already reflects how the modeler has handled the threats that must be reflected in the coalitional form.

The various cooperative solutions that have been suggested each reflect a particular normative property. In particular, the comments here are confined to two solutions. They are the core and the value of a game. The term imputation is used to describe the vector of payoffs indicating the amounts that each individual obtains at any outcome from an *n*-person game. The definition of the core as a solution is the set of imputations that are not dominated by each other or any other imputations. Essentially they are the outcomes that no coalition has the power to argue about by threatening not to cooperate by going into business by itself. There is a large literature dating even from pre-game theory, starting with Edgeworth (1881), showing that the competitive equilibria lie within the core and that under the appropriate conditions a limiting process can be established that shows the convergence of the core to the competitive equilibria (Edgeworth, 1881; Shubik, 1959; Scarf, 1973 and others).

The value of a game, in various forms, may be attributed to Nash (1953), Shapley (1953) and Harsanyi (1959). It attaches a single evaluation or a finite point set of valuations to any game. The value is defined by a set of axioms. From which, for a sidpayment game an explicit formula for any game can be deduced. In essence, each individual obtains his combinatoric marginal value productivity; i.e., imagine that an individual enters every feasible coalition of size s in every one of the orderings feasible. We calculate the a priori probabilities of each of these events and credit the individual with the overall expected productive contribution of the individual being measured.

5. RESULTS FROM GAME THEORY RELATING TO COMPETITIVE EQUILIBRIA

One of the motivations in the utilization of market games was to explore the limiting features of different solutions and to see if they offered different insights into the concept of competitive equilibrium. The conventional properties from welfare theory are that the CE is efficient and that if the appropriate price system is given it provides a signal that permits individual decentralized optimization. The core adds the feature of the CE being unchallenged by any coalitions in a cooperative game. The convergence of the value, under the appropriate conditions to the CE provides justification for the consideration of fair division axioms. The lack of uniqueness of the CEs and the no-sidepayment value raise questions concerning whether the society outside of the economy selects among the non-unique welfare weightings.

Missing from the list of solutions above, who in the limit can be related to the competitive equilibrium is the Cournot model of competition. Yet a glance at the original work of Cournot (1838) clearly shows a well defined precursor to the Nash non-cooperative equilibrium together with an example of the convergence of the noncooperative equilibrium under replication.

Having utilized cooperative game theory to provide a "shopping list " of mass market properties for the price system approached via cooperative games, a reasonable question to ask was why is the Cournot model not on the list? In 1970 I addressed the question as to why we did not have a closed economy model that included the Cournot market structure and linked it to general equilibrium, much in the same way that Cournot had already done for the open model.

Unlike the cooperative models that were in coalitional form and did not take price as a primitive concept, the Cournot model had to be in strategic form with an explicit mechanism for price formation.

The critical distinction between what Cournot had done and what needed to be done was that the open, or partial equilibrium model had to be embedded in a closed general equilibrium framework. This required facing up to a problem involving counting the numbers of degrees of freedom in the strategy sets of the individuals in the game being built. The approach adopted was to try to build an extremely simple symmetric game preserving the intrinsic symmetries of all strategy sets. The idea was to consider an economy with n agents, each one being a monopolist, and then replicating the number of agents in each type utilizing the simplest method of price mechanism.

On the first attempt to try to construct an intrinsically symmetric game, a difficulty was encountered. If each individual has only one good and price is formed by the amount of one good chasing another, then there are only $n - 1$ prices to be formed. Unless all individuals have the relatively complicated strategy of bidding in all pairs of markets or unless an extra commodity (real or virtual) is introduced that is used as a means of payment for all others one cannot maintain the symmetry of the strategy sets (see Shubik and Smith, 2006; Smith and Shubik, 2005). I chose to endow all individuals with an *n* + 1st commodity to be used a means of payment at n market posts where one of the *n* commodities could be exchanged directly for the *n* + 1st commodity. This game preserved the symmetry that would have been lost had a commodity belonging to a single type been selected as a money. The nature of the price formation mechanism of the game naturally introduced cash-flow constraints. In short, it provided a simple model for a closed simple Cournot market with trade intermediated by markets run with money. In this first version the money was a physically undistinguished commodity (Shubik, 1972; Shapley and Shubik, 1977; Shapley, 1976). In subsequent models many different properties of the monetary commodity have been considered (see for example Shubik and Tsomocos, 2002). These appear in the preference conditions, the initial conditions, the methods of creation and destruction and rules of operation and default.

With the appearance of cash flow constraints due to the logic of cash payment in price formation, it is natural that with a commodity money prices are not homogeneous of order zero and that the concept of "enough money " can be well-defined. If there is not enough money the shadow price of the constraint will appear in addition to the price of the money as a commodity. This additional price can be diminished by the explicit introduction of a credit structure with the appropriate instruments. Thus an array of banking and credit arrangements can be modeled that depend on the structure of the markets and society.

5.1. Enough Money, Sidepayment and No-Sidepayment Games

It is of some interest to observe that the condition that there is enough Marshallian money in a sell-all economy is precisely the condition required to convert an NSP market game into an SP market game. As the amount of "trading utiles " (or an ideal good that enters all utility functions as a linear term) is increased the Pareto surface (in payoff space) moves outward and a planar segment is progressively enlarged. As this is happening the initial no-trade position is also moving out. Projections from this position define the individually rational segment of the Pareto surface. The point at which the individually rational segment of the Pareto surface is completely planar is the point at which there is enough money or perfect liquidity (see also Dubey and Neyman, 1984).

5.2. An Aside on Modeling, Mathematics and Uniqueness

A key challenge in economic theorizing is the appreciation of the tradeoff between mathematical tractability and the realism and relevance of the model to be analyzed. Sometimes, by adding extra features to a model one makes the analysis far simpler; sometimes the reverse is true. In particular it is noted here that the addition of bankruptcy or default conditions to a specific strategic market game can be utilized to select a unique equilibrium point with the extra properties that it is the unique minimal cash flow equilibrium, if all solutions are normalized so that each is deemed to have the same initial wealth. The default conditions and bankruptcy laws of a society are part of the legal-socio-political context and are not merely determined by pure economics. In an *n*-person game model these extra conditions can be mathematized as *n* extra constraints on the optimization and it can be shown (Qin and Shubik, 2005) that this is sufficient or the selection of a unique equilibrium that requires the minimal use of cash, i.e., it minimizes the need for a trust-substitute in trade.

The uniqueness conditions posed for the pure exchange model of general equilibrium are far harder to study than the enlarged problem and may be less realistic and relevant than the institutionally richer problem; even though they offer a difficult and challenging problem to the pure mathematician.

6. THE RECONCILIATION OF MARKET GAMES AND STRATEGIC MARKET GAMES

Market games have been studied at a level of abstraction higher than that of general equilibrium. Process does not appear in the mathematical formulation. All that matters in the analysis is the valuation of the payoffs to all coalitions. Strategic market games, in contrast are not as abstract as the general equilibrium models. The rules of the game can be interpreted as defining elemental institutions. They are the carriers of process. Under certain circumstances it can be shown that the details about the institutions do not matter. Dubey, MasColell and Shubik (1980) were able to show that for a large number of agents a large class of mechanisms will guide the system to the same set of outcomes. When numbers are few, mechanisms clearly matter.

A reasonable question to ask is can there be a way to link market games with strategic market games, yet nevertheless preserving a high level of abstraction and generality in the new class of games being created. Qin, Shapley and Shimomura (2006) considered a modified market game where an added condition in the form of a "law of one price " is imposed as a condition requiring that all transfers of wealth must pass through a market with a price. However in doing so they evaluated the coalitional form of the resultant game by assuming (as did Edgeworth) that a coalition S is independent of the coalition $N - S$. This assumption, empirically may be quite good when $N - S$ is large and S is small, has the effect of suppressing the possibility that the actions of $N - S$ could threaten or otherwise influence the welfare of S via the interlinkage of the markets. It is suspect when $N - S$ and S are roughly the same size. It can be dispensed with by linking certain strategic market games directly with associated market games. In other words the market game is derived directly from the strategic market game thereby linking the strategic and coalitional forms.

The way to forge a useful connection between market games and strategic market games involves an admixture of both institutional and mathematical considerations. The direct connection of the coalitional form to a strategic form of a game involves considerable specification and can be regarded as somewhat special and institutional. However by selecting two different ways to achieve this goal, where the two ways can be interpreted as representing, in an appropriate sense, upper and lower bounds, this can be done. Given that it is done, a further step remains. This involves showing that the lower and upper bounding characteristic functions have cores that approach the same limit as the number of agents becomes large.[1]

The basic argument is that these upper and lower bounds have a natural interpretation in a society's need for money or a trust substitute. The lower bound is an economy that requires no money. This is implicitly reflected in the complete irrelevance of cash flow constraints to the general

[1] This involves obtaining two sequences of characteristic functions depending on the number of agents and solving the sequence of games for their cores.

equilibrium model. The upper bound is an economy where all wealth has to be monetized every period. This is the sell-all model. In terms of cash requirements every other economy lies in between these two bounds. The more complicated and realistic price mechanisms require extensive form representation by a game tree of many plies and their reduction to strategic form produces games with enormous strategy sets. These reflect the institutional detail. In theory, any of the myriads of lengthy extensive forms required to model the variety of institutional forms that exist in any economy can be specified. In practice, in order to do so a deep *ad hoc* study of the specific institution at hand is called for. The mere knowledge that this extensive form can be reduced to a strategic form tells us little more than it is a well defined strategic market game whose cash flow requirements lie between the lower and upper bounds already noted.

At this stage of our knowledge of institutional dynamics, possibly the best model and solution concept to be utilized in the study of economic dynamics, is an institutional study bolstered with a well written historical narrative, supplemented with whatever quantifiable statistics can be produced and supported analytically by the type of modeling found in operations research or game theoretic industrial organization.

The above observations could be interpreted almost as a counsel of despair saying that everything is an institutional special case. This is not so. The details and use of the model depend on the questions being asked. The understanding of the specific dynamics of the steel industry in order to run a profitable firm requires knowing many detailed facts about the steel industry. If the question concerns how much financial control the financial system has on the steel industry far less detail is called for.

A purpose of this work is to understand the control and perception roles of the financial institutions in a modern economy. The goal here is to convey an understanding without being trapped in the many technical and mathematical details required to illustrate the relationship between competition and financial control. The key point being made is that financial control comes through an ability to manipulate cash flow constraints. This ability is limited by a host of institutional, socio psychological and political conditions. The understanding requires both institutional and mathematical features. The structure of the cash flow constraints is heavily determined by institutional factors. The control of the constraints calls for perception and purpose in guidance and action on the constraints.

The theory offered here provides the next critical step that was not given by general equilibrium theory. It shows where and how money and financial institutions fit as a control mechanism over a process oriented economy. It also shows that as the number of competitors becomes large the ideal of a general equilibrium system can be established by considering either the core of a market game or the noncooperative equilibria of a strategic market game. In fact, many parts of most economies do not approach the limit but remain oligopolistic and for them specific institutional forms matter considerably. However the economic models which require no financing or the most financing are mathematically far simpler than the intermediate models, if it is feasible to show that they each achieve the same equilibrium limits, this tells us that the overall structure of the static properties of the general equilibrium system can be extended to economies with a financial control mechanism, but the tightness of the control depends on numbers and institutions. When numbers are large, institutions matter less.

7. OBSERVATIONS ON UNCERTAINTY AND TIME

The discussion above has hardly touched upon uncertainty and multistage models. Yet it is well known that much of modern finance deals with incomplete markets where the presence of uncertainty through time plays a critical role. An excellent summary of the state of investigation of economies with incomplete markets has been provided by McGill and Quinzi (1996). The essence of modern finance is how to design instruments and institutions to optimize the allocation of funds across future uncertain states. The work discussed here is complementary with both the body of economic theory of incomplete markets and the growing literature applying stochastic processes to practical problems in finance such as the pricing of options. The former may deal with an overall closed economic system but primarily stresses equilibrium and hardly deals with process and institutions; the latter deals almost exclusively with partial equilibrium phenomena or with the investigation of dynamics in highly specific models such as those based on studies of mass data from the New York or London stock exchanges (Farmer *et al.*, 2005). The critical argument here is that conceptually the understanding of the role of money involves the appreciation of trust, evaluation and perception. Although they almost always do involve exogenous uncertainty, the logic of trade is such that endogenous uncertainty is sufficient to require the existence of monetary, financial, market and contractual enforcement mechanisms for any economy. An old American quip: "In God we trust, all others pay cash," offers a succinct summary.

Human memory is fragile and multistage trade, even without exogenous uncertainty, requires contractual instruments that are enforced by the society. Even if all individuals are impeccably honest, if their memories are not perfect, records are called for. The importance of accounting is often overlooked in economic theorizing. Without its development modern economies would not be feasible. Little items such as the abandoning of Roman for Arabic numerals totally changed the operational usefulness of business accounting (Pacioli, 1498; Shubik, 1995).

8. AN ASIDE ON THE COMBINATORICS OF SEARCH AND THE ANTHROPOLOGY OF MONETARY TRADE

There is a substantial literature both in economic anthropology dealing with early monetary phenomena (see for example Einzig, 1944) and in the economic combinatorics of pre-market trade, starting at least as early as Jevons' (1875) discussion of the failure of the double coincidence of wants. This literature includes Starr (1975), Kiyotaki and Wright (1989), Bak, Norrelykke and Shubik (1999) and many others. It presents interesting insights into the combinatorics of search, bargaining and trade and possibly offers some interesting analogies with anthropological observations. It is a subfield of mathematical, experimental, socio-psychological and anthropological interest, but if anything, it illustrates the probability that there were many ways and many reasons for the emergence of money, credit and the other mechanisms of monetary control.

9. THE MATHEMATICS OF ECONOMIC STATICS AND FINANCIAL DYNAMICS

9.1. Equilibrium Economics

The predominant mode of analysis in the study of the overall price system has been devoted to the study of equilibrium. Mathematically, the problems are cast as a set of equations where the optimization of each individual is subject to a single constraint that amounts to "balancing the books at the end of time."

From a game theoretic point of view the general equilibrium system is presented in strategic form. All of time is collapsed into a single move. From the viewpoint of modeling the economy it is as though the economy had achieved the utopian state of Saint Thomas More. There is no need for money or financial instruments in Utopia or in the general equilibrium economy.

Even were we to get no more concrete than the abstract optimization problem posed by a price guided production and consumption economy, as is evinced by the work of Scarf (1973) the computational problems involved in calculating the equilibrium prices are considerable.

There is a considerable literature on economic dynamics ranging from the careful mathematical structures of Ramsay (1928) and von Neumann (1937, 1945) to the work on business cycles and to the mathematical dynamic models as formulated by Grandmont (1987) and others.

9.2. Disequilibrium Finance

In the modeling of the financial system that controls the economy, cash flow constraints play an important role. An economy with a financial control system imposed over the real assets manifests its control structure in a mass of cash flow constraints. The manifestation of these constraints depends directly and indirectly on the credit structure. An economy without credit, with a fixed transactions technology and commodity money such as gold can only change its money supply via gold-mining or importation or exportation or it can stretch its supply by changes in the velocity. It has no central bank or commercial banks. The money-lenders are capitalists with gold to lend.

In contrast, a system with a government fiscal authority, a central bank and commercial banks has the ability to change the money supply. Mathematically financial control amounts to being able to manipulate the effectiveness of cash flow constraints. This can be illustrated as follows. Consider an economy where individuals are required to trade using cash or bank money. If it is in competitive equilibrium all cash flow constraints are met. Now suppose that the economy is perturbed by a change in tastes, endowments or the possibility of the introduction of a new product with an uncertain effective demand. The projected money demand and cash flow constraints will be changed. Some constraints will become binding, such as those in the industry contemplating the production of the new product. Other constraints may become slack such as those in an industry whose demand has diminished due to a change in tastes.

Slack constraints may offer the opportunity for money lending with cash flowing from the depressed area to the growing part of the economy. This may not be enough, but the ability of the banking system consisting of the commercial banks and central bank to create more money enables them to relax selectively the constraints on the system. The selection of which restraints to relax becomes a matter of investment judgment. The banking system, government and private holders of capital are all in the position to evaluate which constraints they choose to relax. The task of the financial system is not merely to supply the trust-substitute pills to the economy to relax the cash flow constraints, but to allocate funds among competing constraints. In order to do so it must serve as a perception and financial decision device over the economy.

The dynamics that triggers the actions of the financial systems comes from many sources. It is consistent with scenarios such as that sketched by Schumpeter (1911, 1961); the large literature on business cycles (see Minsky, 1987) or in more mathematical form has been suggested in a shift in expectations caused by exogenous events such as sunspots (see Cass and Shell 1983).

9.3. Where Does Macro-economics Fit into This Scheme?

Economic thought in the past century has been favored with the presence of many great macro-economic practitioners. Keynes is arguably the most distinguished, and an individual such as Tobin is a worthy successor. In spite of the brilliance of these individuals there has been a tension between the micro-economic theorists and the macro-economic practitioners, whether they call themselves theorists or not. The two sets of individuals operate at different levels of abstraction. The nature of macro-economic dynamics demands an *ad hoc* approach. Policy advice and power are interlinked. The topic under examination consists of the relatively ephemeral institutions of today, how they are changing and how they can be controlled. The deeper invariants of the system are of academic interest to the good macro-economist who should be perfectly happy to use a temporary construct such as the Philips curve or any other phenomenon that has the decency to have a half-life of at least five years.

Keynes was possibly the most distinguished applied economist and advocate of all times. He, possibly deliberately, gave up a great career as a deep formal economic theorist to pursue the economic aspects of the political and social goals of his time. Even today the tug between the socially important immediate needs of a society and the longer term development of solid economic theory is enormous. The need for good applied economic advice to navigate the quick sands of political, institutional and technological change here and now calls for great minds. But advocacy for today's institutional fix and the development of an economic theory based on the invariant properties behind an economic science are not the same. It is not a matter of virtue versus vice. They serve different social functions.

10. THE LIQUIDITY AND VOLATILITY OF THE FINANCIAL SYSTEM

It has been noted that the financial system appears to be far more volatile than the economic system, in general. So it should be. It has the role of the sensory, guiding and control mechanism of the economy. Like the control mechanism of the human body it is small relative to the mass it controls. Furthermore, although its energy consumption is small relative to the economy as a whole, it is nevertheless substantial and markets, brokers and trading mechanisms run on margins far smaller than the firms in the physical economy. Because of the enormous volume, even with low margins the rewards of finance can be considerable. But because of the low margins, a slight change in costs, due to changes in technology, taxes, law or other factors, can cause a substantial swing from one transactions method or financing technique to another. In manufacturing a swing of a few basis points rarely makes much difference to a decision, in a transaction technology it does.

A basic property of a system designed to perceive and control is that its timing constants will be far shorter than those of the body or organization it is designed to control. Flexibility and speed are critical to its function. Yet it is here that the clash between the viewpoints of finance and economics can lead to a mismodeling of the critical factors of function. The economist seeking to explore and analyze equilibrium properties may easily assign zero costs to the transactions technology. In doing so it becomes easy to obtain results that show enormous indeterminacy in methods of financing (Geanakoplos and MasCollel, 1989). But the difference between low transactions costs and no transactions cost is the difference between being able to appreciate the role of finance in the economy and obliterating its key features. As soon as a cost is attached to the operation of every process the excess of degrees of freedom in the finance guided economy is removed. How to estimate the actual costs is an empirical and conceptual problem of considerable size as many of the costs involve network phenomena and zones of increasing returns to scale, caused by stochastic elements, (Arthur, 1989) set-up costs and other phenomena.

11. ON MATHEMATICAL INSTITUTIONAL ECONOMICS

Many years ago, when first attempting to study economic dynamics, I coined the phrase "Mathematical Institutional Economics" (Shubik, 1959). My reason for utilizing this phrase is that I was trying to extend results on static non-cooperative equilibrium oligopoly studies based on Cournot, Edgeworth and Bertrand to multistage models of oligopolistic competition. In trying to do so, I found myself buried in a mountain of special cases, all of which appeared to be relevant to some particular situation. At that time I did not fully appreciate that the apparent conflict between the detailed micro-economic study of economic institutions and abstract economic theory did not need to be a conflict. Economics, as it is currently taught has not faced up to the type of libraries and compendia of models accepted by operations researchers applying inventory theory. Any form of economic dynamics, be it the design of an inventory policy for a supermarket or a prediction of the development of the steel industry over the next two or five years calls for much *ad hoc* detail. A brief contemplation of the curse of dimensions in economic analysis makes this evident. When an actual economic institution is examined any group of competent practitioners can list hundreds of different variables or other factors which bear some relevance to the institution and its motion through time. Even with large scale simulation capabilities, a selection of the subset of variables most appropriate to the questions at hand is called for.

The transactions technology of any society is a matter of institutional fact, replete with hosts of special cases and details. Abstractly the structure emerges as a set of constraints on a multi-agent optimization problem. The evaluation of the importance of the constraints is essentially the task of the entrepreneurs, financiers and government of the society. The adjustment of these constraints amounts to a game primarily among the entrepreneurs, financiers and government.

The appropriate appreciation of institutional fact is a necessary condition that precedes answering applied questions about the economy. The mere knowledge of the institutions is not enough. If analytical thought is to be given to policy the appropriate mathematical models from theory that exists at a higher level of abstraction must be hand-tailored to fit close enough to the facts to make their mathematical analysis relevant and useful. In actuality, few theorists and few practitioners are psychologically able to bridge the gap between over-abstraction and practice.

12. FINANCIAL DYNAMICS AND NEVER ENOUGH MONEY

It has been noted above in Section 9.2, that a rigorous mathematical definition can be given to the concept of enough money when the equilibrium conditions are being studied. As has already been noted, there is enough money, well distributed if the shadow prices of all cash flow constraints is zero. Furthermore, a game theoretic interpretation can be given that links a no-sidepayment game with a sidepayment game via an ideal or Marshallian money. The tidy micro-economic package of economic equilibrium can accept the financial structure as an unneeded icing on the cake inasmuch as the equilibrium conditions are precisely the conditions that bring about the minimal need and utilization of the financial structure.

In disequilibrium there is almost always never enough money. The constraints are either slack or binding and the binding constraints are where perceived growth is being constrained.

13. IS THERE AN IDEAL CURRENCY AND DOES IT MATTER?

The Grail is in the seeking and not the cup.

In human striving the search for the Holy Grail provides much motivation. The mystic cup, the fountain of youth or the shining city are always luring us over the horizon. So it is with the search for the ideal currency.

The ideal currency would be the Marshallian money in sufficient supply that can convert a no-side-payment game into a side-payment game. It would provide perfect liquidity. In this imperfect world over several thousand years the predominant commodity of choice that comes close has been gold. But it has many drawbacks including high costs of extraction and erratic and insufficient supplies.

Even more perfect than gold would be a Utopian world where all individuals trusted each other completely, each had an impeccable reputation and all had perfectly accurate memories. Even in this world, unlike in the fantasies of Saint Thomas More, good recordkeeping would facilitate fair play.

As more stable societies have evolved, since the late 17th century, the trend has been towards a paper or purely abstract money. It is part of an evolutionary process which apparently heads towards a limit point of Utopian perfection but never attains it. Transactions costs are part of physical reality and even a paper money or cipher money and credit system requires the existence and costs of bookkeeping, contract law, default laws and the enforcement of the courts.

Currently there are of the order of 200 national currencies in existence and the move towards a single international currency is on the way. Barring major catastrophe a multi-layered organization of central banks with an empowered world central bank and a single major international currency is to be expected eventually. Ricardo (1817) writing at a time when the transactions and transportation economy was both considerably slower and smaller than today opted for gold as a currency essentially because he viewed the choice between gold and paper as one involving choosing between trust in politicians and trust in gold. Hopefully as reasonably stable and moderately honest administrations grow, the possibilities for symbolic currency grow along with them. As the nation sates become willing to surrender parts of their political power to international bodies the possibilities for regional and then a fully international currency grow.

We live in an imperfect world with politicians, social pressure groups and economic entrepreneurs all engaged in forms of trading and arbitrage in parts of the socio-political economic structure replete with complex externalities. A little careful analysis indicates that in spite of the loose analogy and metaphorical references to "the market for votes ", no efficient market for votes exists (Shubik and van der Heyden, 1978). Voting mechanisms are not markets. They were designed to work specifically in area where markets do not work. In general, when voting systems are considered game theoretically they are modeled as games whose core solution is empty thereby ruling out the hope that an efficient price for votes can be found.

Weaving between political and socio-economic reality and the idealizations of normative economic theory the message is clear. The concept of the perfect money is mathematically straightforward. Unfortunately, it does not seem to have a good enough physical counterpart in the world as we know it. Gold is about as close as we get, especially when we add specific physical properties to the list. These are often left out in idealized studies. They include features such as fungibility, durability, cognizability, and the feature that gold is non-tarnishing. Although heavy, gold is still an item to utilize when skipping borders in a world with political turbulence. But the ideal money in a stable society is an abstraction supported by law and custom and their acceptance and enforcement. The natural institution for the support of a nationally-acceptable money is the government who is, essentially, the only institution known to every adult functioning member of society. The higher is its reputation, the better a chance it has to support a stable symbolic currency and a financial system that is needed for the control of the economy.

14. SYNTHESIS AND ANALYSIS

This essay has been offered in the spirit of jigsaw puzzle solving. Markets and money are together a deep social systemic phenomenon. The economic and financial institutions of all societies are products of their time and place. Many different bits and pieces must be melded together before the overall pattern emerges. There is no simple causality. The theme of finance is flexibility in order to maintain effective control and its manifestation is in the ease in which it changes form.

At a high level of abstractions all economies have agents that include producers, consumers, transactors and financiers. They are all imbedded in polities that have government, law, and politicians. The agents have preferences, and ownership claims. Modern economies have legal persons who are players, yet are not human, but are owned collectively by humans. The problems posed by the governance and strategic control of the corporation remain with us to this day. Even so, the economic aspects of individual and joint optimization, the basic concepts of information and strategy can all be formulated at a high level of abstraction. The properties of the "unrealistic models " constructed require high levels of analysis whether they are Edgeworth's (1881) or Nash's (1953) formulation of bargaining; Shapley's formulation of the core and value; Arrow and

Debreu's (1952) or von Neumann's (1937) price governed economy, or Ramsey's (1928) infinite horizon model of saving.

The need to run the economy of here and now requires a deep individual and comparative understanding of the institutions of today. The understanding of the invariant properties of an economic system is of considerable service in framing the right analysis of the basic functions of any institution. The two are complementary but require completely different skills and world views. Perhaps one of the more perceptive exemplars of clash between abstraction and institutional and historical understanding was presented in Herman Hesse's Magister Ludi. The masters of an abstract game retreated further and further from the world around them requiring more and more intellect and technique and less and less sensory awareness of the world in which their cage of abstraction was embedded.

In the understanding of a useful theory of money and financial institutions, the need is not to choose between the richness of institutions and the elegance of mathematical and logical abstractions; but to choose both. This requires adjusting each to the other. Furthermore, the clash between institutional and mathematical studies in economics presents a false dichotomy furthered by the emphasis on equilibrium studies. Equilibrium analysis can often be utilized to present a false view of generality. Equilibrium studies can be formulated with abstractions that remove almost all traces of institutions. The institutions are carriers of process and the carriers of process can only be ignored in a search for existence proofs of points of equilibrium.

The approach adopted here is resolutely process oriented. In any attempt to model economic activity as a game in extensive form institutions appear implicitly as part of the rules of the game. Furthermore, it is possible to define and analyze minimal institutions. The concept of a minimal institution together with being able to define structures that require the most and least amounts of money required by organizations enables us to see how the financial control mechanism of an economy fits over the economy as a whole. The simplified upper and lower bounds can be analyzed and both are consistent with the overall equilibrium properties of the competitive price system. The complexities of economic reality, however, lie in between and any appreciation of the dynamics of these institutions calls for detailed knowledge of the institutions and their societies modified by the application of the analytical results obtained from the parsimonious abstractions of economic theory.

15. REPRISE

The ideal money is a perfect substitute for trust.

The general equilibrium system of Walras and his successors was a great feat of abstraction, but in its progressive refinements and abstraction it emerged as a timeless pre-institutional representation of the economic world that did not even need to include money and financial institutions in its structure.

Any attempt to turn this structure into a playable game calls for carriers of process; but the carriers of process are the institutions of the economy.

Among the myriads of models available, it is possible to select minimally complex institutions that require, as a lower bound no money whatsoever, and a model that requires the maximum money that calls for the monetization of all nonmonetary wealth each period.

The lower and upper bounding models can be mathematized and analyzed at a high level of abstraction. If there is (as can be defined precisely) enough money available and appropriately distributed the upper bounding model will give the same static equilibrium economic production and consumption results as the lower bounding model when there are many agents in the economy.

Economic reality lies between these bounds and even a casual attempt to model the more complex process calls for institutional detail. This detail, no matter how complex, is reflected in the cash flow conditions. The degree to which the cash flow conditions act as constraints is determined by the monetary and credit institutions of each society.

In a stationary equilibrium the shadow prices of the constraints are zero and trade is ideally, costlessly financed by either money or credit (the mix depending on a blend of reputation, information and trust in the economy under consideration). In fact, costs are non-trivial and the levels of trust and enforcement call for difficult *ad hoc* evaluations.

The *strategic market game* models and the associated modified *market game* models provide two ways to attach the web of money and financial institutions to the essentially non-monetary *general equilibrium model*.

The concept of "enough money, well-distributed" is a mathematical abstraction that both links side-payment and NSP cooperative market games and provides the zero shadow price conditions on the cash-flow constraints in the noncooperative strategic market games.

Except for the bounding extreme abstraction of the "sell-all" strategic market game the actuality of enough money is determined by the institutional structure of the economy.

The presence of the monetary and financial system in the economy in stationary equilibrium is hardly discernable.

In disequilibrium the financial system appears as the control mechanism for the society. The agents who control the money and credit structure may influence those constraints that are either slack or have positive shadow prices.

The decision to change the intensity of the constraints lies heavily with the lenders and their judgment of risk. The twin roles of the lenders are as risk evaluators and financial resource allocators.

Finance is strategic. The methods of game theory provide the appropriate structure and precision to go beyond general equilibrium in providing complete process models.

The complete process models set up the conditions needed for the next stages in the development of finance. They are institutional, behavioral and analytical. The institutions are reflected in the rules of the game; the behavioral assumptions are reflected in the solution concepts considered. Both of these require empirical inputs. They are needed to justify not only the assumptions about structure but where the distributions of uncertainty and behavioral predictions come from prior to analysis.

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