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Competitive Experimentation with Private Information*

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Abstract

We study a winner-take-all R&D race where firms are privately informed about the uncertain arrival rate of the invention. Due to the interdependent-value nature of the problem, the equilibrium displays a strong herding effect that distinguishes our framework from war-of-attrition models. Nonetheless, equilibrium expenditure in R&D is sub-optimal when the planner is sufficiently impatient. Pessimistic firms prematurely exit the race, so that the expected discounted amount of R&D activity is inefficiently low. This result stands in contrast to the overinvestment in research that is typical of winner-take-all R&D races without private information. We conclude that secrecy in R&D inefficiently slows down the pace of innovation.

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1. Introduction

Research and Development activities produce information −about the promise, feasibility and interim experimental results of a project− that is private to the researcher. There are no incentives to disclose such information, but rather to carefully protect it from industrial espionage.[1] This paper investigates the positive and normative effects of private information on R&D activities. Our main result is that if the social planner is sufficiently impatient, a failure of information aggregation makes aggregate equilibrium expenditure in R&D on average too low with respect to the social optimum. Hence, we conclude that *secrecy inefficiently slows down the pace of innovation.*

The intuition for this result is simple. Different research centers may have different information or beliefs about the feasibility or "promise" of the same lines of research, and may achieve more or less favorable preliminary results. Positive information tends to be concealed, because disclosing it would lure into the race competitors, and possibly imitators. Because "pessimistic" researchers cannot access the private information of more optimistic agents, they give up early in the race or even fail to join it. From the standpoint of a full-informed observer, the more pessimistic firms exit the race too early. Due to borrowing or talent constraints, the more optimistic firms may be limited in the amount of resources they can devote to the project. Thus, the resulting aggregate level of experimentation is too low and research activity is inefficiently postponed.

To the best of our knowledge, these normative results are novel in the theoretical R&D literature. This literature has emphasized different sources of distortions in R&D investment, but its conclusions do not seem to uniformly support the widely held belief that, left to market forces alone, equilibrium investment in R&D would be socially inadequate. As long as innovators can gain full ownership of the social value of their own inventions, the analysis systematically comes to the conclusion that equilibrium R&D investment is socially *excessive*. This is due to duplication costs: in equilibrium each firm over experiments in the attempt of beating competitors, and fails to internalize the negative externality imposed on the losing competitors. This literature has largely developed around the workhorse Poisson model of inventions originally proposed by Reinganum

[1] When private corporations sponsor university research, as a norm they require the faculty and graduate students involved to sign non-disclosure and exclusive-licensing agreements. Cohen et al. (2000) conduct a survey questionnaire administered to 1478 R&D labs in the U.S. manufacturing sector in 1994. They find that firms typically protect the profits due to invention with secrecy and lead time advantages.

(1981, 1982), which we shall also employ in this paper.[2]

The main sources of R&D under-investment previously identified in the literature can be traced back to the hypothesis that innovators cannot fully claim the social value of their inventions.[3] For example, there may be large technological spillovers when one discloses an innovation. Furthermore, patent protection of innovations is often ineffective: Innovations can often be legally imitated or reverse-engineered. Among others, Aghion and Howitt (1992) underline that in a patent system, an innovator can at best gain monopoly profits from her invention, which is less than the whole social surplus, and that she does not internalize the impact of her contribution on future research (see also Horstmann, MacDonald and Slivinski (1985)). Indeed, Mansfield et al. (1977) estimated that the social return of innovations may be twice as large as the private return to the innovator.

Our analysis identifies in secrecy a novel force that may generate equilibrium underinvestment in R&D. This force is present even under the hypothesis that innovators are compensated with the full social value of their innovations. Hence, our work suggests that policies supporting R&D should be adopted even when patent protection is fully effective.

To illustrate these points formally and to explore the role of private information in R&D races, we lay out a simple analytical framework. Two firms challenge each other in a research race with fixed experimentation intensity and winner-take-all termination. The prize can arrive independently to each player still in the race, with identical negatively-exponential arrival processes of underlying unknown parameter (the "promise" of the project). At the beginning of the race, a private signal informs each player of the project's promise.[4] Because of the winner-take-all assumption, the two

[2] In the strategic multi-armed bandit literature (Bolton and Harris 1999, Keller, Rady and Cripps (2004)), equilibrium experimentation is sub-optimal because players cannot conceal their findings from each other. Essentially, under-experimentation consists of the underprovision of a public good. In contrast, we study R&D races where private information is carefully protected. Among other continuous-time models of multi-agent Bayesian learning, see also Bergemann and Valimaki (1997, 2000), and Keller and Rady (2003).

[3] An exception is the analysis of Bergeman and Hege (2001). They study contracts that govern the financing of a research project by an investor. Because the innovator is limitedly liable, and she may diverts the investor's funds, they find that equilibrium funding is inefficiently low. This agency effect is unrelated to the reasons for R&D underinvestment that we identify in this paper.

[4] As an example of how large corporations go about scouting promising projects, Pfizer, currently the largest pharmaceutical company in the world after recently absorbing Warner-Lambert/Parke-Davis, invested in 2002 $5.3 billion in R&D through its specialized arm Pfizer Global Research and Development, which employs 12,500 scientists. In addition to this massive structure, quoting from the company official website, "250 partners in academia and industry strengthen our position on the cutting edge of science and biotechnology by providing access to novel R&D tools and to key data on emerging trends."

In Section 6, we show how to allow also for gradually accumulating private information about the interim results of the research project. Because the results of the analysis are qualitatively the same, we focus the exposition on the firms are unwilling to share their private information. Over time, each firm decides whether to stay in the race, paying a flow cost, or whether to quit. Once a firm has quit, prohibitive sunk costs make re-entry economically infeasible.[5] A technically related model is studied by Keller, Rady and Cripps (2004). However, they do not consider private information.

As time goes by, and the innovation does not arrive, players become pessimistic about the innovation's arrival rate, and eventually quit the race. In the unique symmetric monotonic equilibrium of this game, each firm selects a stopping time increasing in her own private signal, and conditional on whether the opponent is still in the race or not. Due to the interdependent-value nature of the game, the equilibrium displays a rather extreme "winner's curse" property, or more precisely a "survivor's curse". Specifically, if the two private signals are sufficiently close, the firms herd on each other's participation in the race, rationally presuming that the opponents' signal is larger than it actually is. Because of equilibrium monotonicity of stopping times in signals, when a firm quits, the "survivor" discovers the quitter's actual signal and discontinuously revises her beliefs downwards. If the two private signals are sufficiently similar, this negative surprise will make the survivor immediately quit and regret not having quit the race earlier. This "curse" is more extreme than in standard or all-pay interdependent-value auctions and than in wars of attrition models (see Krishna and Morgan (1997)). This fact underlines a key predictive distinction between our R&D game and wars of attrition. Contrary to prior claims that R&D races can be generally represented as wars of attrition, our results show that this is not the case when considering private information and interdependent values.

Our welfare analysis compares the aggregate discounted experimentation costs in equilibrium and in the efficient cooperative solution, where the two firms are joined in a single team and share their private information. The optimal team policy is to run both firms' facilities in parallel, and then to stop them simultaneously when the joint flow cost for experimenting equals the expected joint marginal benefit. In equilibrium, a firm prematurely drops out of the race when her private signal is sufficiently unfavorable relative to joint aggregate information. The survivor then acts

simpler case where private information accrues before starting the race.

[5] In this model, we interpret quitting the R&D race as publicly abandoning a line of research, hence dismissing project-specific facilities and research teams. A temporary suspension is unlikely to be observed by competitors. Although irreversible exit is a strong simplifying assumption, it appears more natural in this context than the opposite assumption of costless re-entry. Indeed, R&D firms are seldom observed re-entering a line of research after dismissing it, possibly because of the large costs involved.

fully informed and remains in the race longer. But the equilibrium discounted expenditure in R&D will be too low with respect to the social optimum, because some experimentation is inefficiently postponed to the future. If instead the firms' signal are very close, then the "survivor's curse" implies that both firms will suboptimally delay their exit from the race, and equilibrium over-experimentation takes place. When averaging out over instances of under-experimentation and over-experimentation, the postponement effect dominates the delayed exit effect as long as the discount rate is sufficiently large, and the equilibrium features suboptimal R&D investment.

For the sake of clarity, we choose to illustrate these points in the simplest possible setup. Our key normative finding seems to be robust to a number of extensions of the model. We allow for private information to accumulate *gradually* over time as research results accrue, and we consider the possibility that the prize arrival to a player induces a positive or negative externality (e.g. technology spillovers, increased competition) on the other player. Section 2 reviews the related theoretical literature, Section 3 lays out the model, Section 4 the unique symmetric monotonic equilibrium, Section 5 illustrates its normative properties, Section 6 illustrates various extensions of the model, Section 7 concludes, an Appendix contains the proofs.

2. Related Literature

Our work is related to several strands of literature, but presents important conceptual differences with respect to each of them.

The key benchmark are continuous-time R&D races modeled either as differential or stopping games. The differential game approach is put forth in Reinganum (1981, 1982). At each moment in time, each player selects an experimentation intensity that affects linearly the arrival rate of the invention, paying a quadratic cost. The simpler approach where each player experiments with fixed intensity until it quits the race can be understood as a stopping game. Choi (1985) adopts this approach to study the case of uncertain λ with commonly known prior. This work is further extended by Malueg and Tsutsui (1999) in a full-fledged differential game à la Reinganum. In these models with symmetric information, equilibrium R&D investment is socially excessive, due to duplication costs.[6] We adopt the stopping game approach to address the effects of private

[6] An alternative approach to modeling R&D competition is the "tug-of-war": firms take turns in making costly steps towards a "finish line." In the absence of uncertainty, these games predict a dramatic preemption effect. Once

information.[7] We find that ex-ante equilibrium R&D investment is inefficiently low.

Our study of private information in R&D races adapts and extends solution techniques from auction theory. But, while our game shares many elements of an all-pay ascending auction, or equivalently of a war of attrition with interdependent values, the models are significantly different. To see this, consider as a benchmark the standard two-player common-value ascending second-price auctions (Milgrom (1981)). In the symmetric monotonic equilibrium, a player with private signal x quits the game at the time $\tau(x)$ when her bid (i.e. the cost) equals the expected value of winning the auctioned good (the benefit) conditional on *both* players holding signal x. In the equilibrium of a common-value war of attrition or all-pay auction, each player leaves the race much *earlier* than this time $\tau(x)$, unless x is very large.[8] Suppose by contradiction that the opponent adopts strategy τ in equilibrium. As the stopping time $\tau(x)$ approaches, the player increasingly believes that the opponent's signal is larger than x, and hence that the race is lost. As the expected benefit of staying in the race vanishes, the player anticipates exit to avoid paying the cost of attrition.

In our game, the random prize arrivals are independent across players. Hence the information that the opponent's signal is larger than x does not imply that the player will lose the race, it only conveys good news on the player's prize arrival rate. As a result, in the monotonic symmetric equilibrium of our game, each player with signal x postpones exit *after* the time when her flow cost equals the expected flow benefit conditional on both players holding signal x. In a sense, the informational spillover that derives from common value and independent arrivals places our game on the opposite side of common-value wars of attritions, with common-value standard auctions in between.[9]

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a firm is known to be ahead in the race, the opponents drop out of the race (Harris and Vickers (1985)). But this preemption effect vanishes if one introduces uncertainty in the duration of each step (Harris and Vickers (1987)).

[7] Lambrecht and Perraudin (2003) and Decamps and Mariotti (2003) study continuous-time stopping games where private information is only of private value. In our game it has interdependent value: Each player's signal is informative on the promise of the project which concerns the R&D race.

[8] Krishna and Morgan (1997) find that the monotonic symmetric equilibrium bidding strategy $b(\cdot)$ of a 2-player war of attrition is such that:

$$b(x) = \int_{-\infty}^{x} v(y,y)\,\lambda(y|y)\,dy,$$

where $v(y,y)$ is the expected value of the auctioned good conditional on both players' signals being equal to y, and $\lambda(y|y)$ is the hazard rate that the opponent's signal equals y given that the player's signal is y. Unless x is large, $\lambda(y|y)$ is small enough that this integral is smaller than $v(x,x)$, the expected value of the good conditional on both players holding signal x.

[9] Coincidentally, however, Bulow, Huang and Klemperer (1999) show that bidders postpone their quitting time

Our work is also related to the literature on information aggregation in timing games. Chamley and Gale (1994) [CG] study a model where each players is privately informed on the "state of the economy" and may irreversibly exercise an investment opportunity at any period. Two key differences distinguish our analysis from CG. First, in CG players face a coordination problem, where either all should invest or none. If they could share their private information, they would do so. In our R&D race, the winner-take-all assumption induces strong incentives to conceal private information.[10] Second, from a substantive viewpoint, in the CG model herding delays investment, as each player would like to wait and see how many opponents choose to invest. But this incentive to wait and see is naturally counteracted in the context of R&D, by the incentive to 'be the first to enter the race' so as to gain some advantage over the opponents. Our model is appropriate to study environments where herding extends experimentation durations and works in favor of overinvestment, so that underinvestment arises from other sources.[11]

More distantly related, Aoki and Reitman (1992) study a two-stage model where firms first may invest to reduce their private-information production cost, and then compete a-la Cournot. In equilibrium, the high-cost firm may choose not to invest and pretend to be a low-cost firm. Unlike them, we study a R&D race with marketable innovations. Firms cannot turn an innovation into profit if the innovation has not even been developed in the first place. Hence there is no reason for bluffing the ownership of inexistent innovations. Indeed, our main result is exactly the opposite: research centers keep high-promise projects secret and this slows down technological progress.

3. The Model

Two players, A and B, play the following stopping game. A prize $b > 0$ arrives to player $i = A, B$ at a random time $t_i \geq 0$, according to a negative exponential process of constant hazard rate $\lambda \geq 0$, c.d.f. $F(t_i|\lambda) = 1 - e^{-\lambda t_i}$ and density $f(t_i|\lambda) = \lambda e^{-\lambda t_i}$. Conditional on the common λ, the two

beyond $\tau(x)$ as in our game, also when they are endowed with a share of the good auctioned off.

[10] Jansen (2004) studies the relation between the degree of appropriability of innovation returns and equilibrium information disclosure. He finds that full appropriability results in secrecy, and that full disclosure takes place only for extreme spillovers.

[11] Gul and Lundholm (1995) study a *continuous*-time coordination timing game that shares many similarities with CG. Again, players would like to exchange information if they were allowed to, and the herding effect goes in the direction of delaying an irreversible decision with random consequences (i.e. delaying investment in CG's terminology).

arrival times t_A, t_B are independent.[12] In order to know t_i and receive the prize, player i must keep paying a flow cost $c > 0$. Stopping payments of such costs implies that the prize is abandoned irreversibly and that t_i will never be learned. We make a *winner-take-all* assumption: the first player to receive the prize ends the game. Costs and prizes are discounted at rate r.

The common hazard rate of arrival of either prize, $\lambda \geq 0$, is drawn by Nature, unobserved by the players, from a distribution $Gamma\,(\alpha, \beta)$:

$$\pi\,(\lambda) = \frac{\alpha^{\beta}}{\Gamma(\beta)} e^{-\alpha\lambda} \lambda^{\beta-1}, \text{ for } \alpha > 0, \beta > 0.$$

Before starting to pay costs, each player i observes a private signal $z_i \leq 0$ distributed according to a negative exponential distribution: for every $Z \leq 0$ and $z = z_i$,

$$H(Z|\lambda) = \Pr(z \leq Z|\lambda) = e^{\lambda Z} \text{ with density } H'(Z|\lambda) = h(Z|\lambda) = \lambda e^{\lambda Z}.$$

The two private signals z_A, z_B are independent, conditionally on λ.

We will refer to a "project" as the possibility of paying c to activate the arrival of a prize. In our model each player has a "project"; based on the realization of the private signal, she decides whether to pursue it or not and, if so, when to stop it irreversibly. The canonical application of the model is as follows. Each of two firms might start an R&D project of the same nature. Before starting the project, each firm observes a private signal on its "promise" λ. This key parameter is common to both projects, because they revolve around the same question, device etc.; but, conditional on the promise, the actual winner is determined by luck and by willingness to continue investing resources in research.

Before beginning the formal study of our model, we single out some properties of belief updating that will be repeatedly used in the analysis.

Belief Updating. Two posterior beliefs play a key role in our analysis. We consider player A's updating, player B's being symmetric. First, suppose that player A is fully informed about the realizations of both players' signals, $z_A = x$, $z_B = y$, and that project A has not delivered a prize

[12] Our model is easily extended to allow for interdependent values or partially public values, instead of pure common value. For example, by letting the arrival rate of invention $i = A, B$ be $\lambda_i = \lambda + \varepsilon_i$, where $\lambda \sim Gamma(\alpha, \beta)$ and $\varepsilon_i \sim Gamma(\alpha, \beta_i)$, with ε_A and ε_B independent: λ measures the "promise" of the project, and the idiosyncratic components ε_i measure the specific promise of player i's approach. The analysis develops similarly, so we focus on the simplest case just for clarity.

by time t and project B by time t'. Conditional on this information, the posterior belief density on λ is

$$\pi\left(\lambda | z_A = x, z_B = y, t_A \geq t, t_B \geq t'\right) = \frac{\pi\left(\lambda\right) h\left(x|\lambda\right) h\left(y|\lambda\right)\left[1 - F\left(t|\lambda\right)\right]\left[1 - F\left(t'|\lambda\right)\right]}{\int_\Lambda \pi\left(\lambda'\right) h\left(x|\lambda'\right) h\left(y|\lambda'\right)\left[1 - F\left(t|\lambda'\right)\right]\left[1 - F\left(t'|\lambda'\right)\right] d\lambda'} \tag{3.1}$$

which is shown in the Appendix to be a distribution $Gamma(\alpha - x - y + t + t', \beta + 2)$. We denote this density by $\pi_{t,t'}(\lambda|x, y)$ and its c.d.f. by $\Pi_{t,t'}(\lambda|x, y)$.

Second, suppose that player A is fully informed, as before, about her own private signal realization $z_A = x$, and knows that project A has not delivered a prize by time t and project B by time t'. But, now, player A only knows about her opponent's signal z_B that it is *not smaller than* y. Conditional on this information, the posterior belief density on λ is:

$$\pi\left(\lambda | z_A = x, z_B \geq y, t_A \geq t, t_B \geq t'\right) = \frac{\pi\left(\lambda\right) h\left(x|\lambda\right)\left[1 - H(y|\lambda)\right]\left[1 - F\left(t|\lambda\right)\right]\left[1 - F\left(t'|\lambda\right)\right]}{\int_\Lambda \pi\left(\lambda'\right) h\left(x|\lambda'\right)\left[1 - H(y|\lambda')\right]\left[1 - F\left(t|\lambda'\right)\right]\left[1 - F\left(t'|\lambda'\right)\right] d\lambda'} \tag{3.2}$$

whose closed-form expression in terms of x, y, t, t' is derived in the Appendix. We denote this density by $\pi_{t,t'}(\lambda|x, y+)$ and its c.d.f. by $\Pi_{t,t'}(\lambda|x, y+)$.

We will show that the key statistic to determine optimal stopping in the model is the expected hazard rate of prize arrival. Conditional on "complete" information (x, y, t, t'), using the properties of the Gamma distribution, this is:

$$E_{t,t'}\left[\lambda | x, y\right] = \int_\Lambda \lambda \pi_{t,t'}(\lambda|x, y) d\lambda = \frac{\beta + 2}{\alpha - x - y + t + t'}$$

clearly increasing in x and y and decreasing in t, t'. Conditional on the partial information that the opponent signal is above y, using the expression for $\pi_{t,t'}(\lambda|x, y+)$ derived in the Appendix:

$$E_{t,t'}\left[\lambda | x, y+\right] = \int_\Lambda \lambda \pi_{t,t'}(\lambda|x, y+) d\lambda = (\beta + 1)\frac{\left(\alpha + t + t' - x\right)^{-\beta-2} - \left(\alpha + t + t' - x - y\right)^{-\beta-2}}{\left(\alpha + t + t' - x\right)^{-\beta-1} - \left(\alpha + t + t' - x - y\right)^{-\beta-1}}.$$

Lemma 1. *The posterior expected hazard rates of arrival of the prize $E_{t,t'}\left[\lambda | x, y\right]$, $E_{t,t'}\left[\lambda | x, y+\right]$, conditional on no prize A arrival by time t, no prize B arrival by time t', the realization of the signal $z_A = x$, and the realization of signal z_B being (respectively) exactly equal to or weakly larger than y, are both strictly decreasing in t and in t', vanish as t grows unbounded, and are strictly increasing in x and in y.*

Finally, knowing that the opponent's private signal realization equals y for sure is bad news compared to knowing only that it is larger than y.

Lemma 2. *For any* t, t', x, *and* $y < 0$, $\Pi_{t,t'}(\lambda|x, y) \prec_{FSD} \Pi_{t,t'}(\lambda|x, y+) \prec_{FSD} \Pi_{t,t'}(\lambda|x, 0)$,

$$E_{t,t'}[\lambda|\, x, y] < E_{t,t'}[\lambda|\, x, y+] < E_{t,t'}[\lambda|\, x, 0].$$

4. Equilibrium Analysis

4.1. Definition

Each player's strategy depends only on her own private signal and on whether her opponent is still in the race. For each player $i = A, B$, a pure strategy in this game is a pair of functions (σ_1, σ_2), describing stopping behavior. The stopping time function given one's own private signal *and* that the opponent is still in the race, is denoted by $\sigma_1 : R_- \to \overline{R}_+$. For any signal x, the strategy σ_1 prescribes that the player stays in the race until time $\sigma_1(x)$ unless observing that the opponent has left the race at any time $\tau < \sigma_1(x)$. The choice of the extended positive real numbers $\overline{R}_+$ as the range of σ_1 is made to allow for the possibility that a player may decide to stay in the game and wait for the prize forever, given some signal realization x, and given that the opponent is not leaving the game either. Note that the stopping time $\sigma_1(x) = 0$ prescribes that the player should not enter the race at all.[13] The stopping strategy as "quitter no. 2", given own private signal and that the opponent has already left the game, is denoted by $\sigma_2 : [\underline{x}^*, 0] \times R_+ \to \overline{R}_+$. Here $\sigma_2(x, \tau) \geq \tau$ describes the player's stopping time when holding signal x and after the opponent has quit at time τ.[14]

We focus on *symmetric* and *monotonic* equilibria, and denote equilibrium strategies as (σ_1^*, σ_2^*).[15] Our monotonicity requirement is that there exist $\underline{x}^*$, $\bar{x}^* \leq 0$ such that $\sigma_1^*(x) = 0$ if $x \leq \underline{x}^*$, $\sigma_1^*(x)$

[13] An alternative interpretation of this model is that each player is initially uninformed, and optimally chooses to enter the race and gather information, which comes in an instantaneous single signal x for simplicity. Given x, she then decides whether to leave the race or continue.

[14] When $\sigma_2(x, \tau) = \tau$, the player leaves the race immediately after seeing that the opponent left at τ. Formally, this continuous stopping time is derived from a finite-time approximation where each period lasts Δ, and $\lim_{\Delta \to 0^+} \sigma_2(x, \tau) = \tau^+$. Metaphorically, in the moment that a player leaves the race, the clock is 'stopped for an instant' and the remaining player is left to choose whether to continue or follow suit. For a general treatment on how to construct stopping time strategies in continuuos time games and on their interpretation, see Simon and Stinchcombe (1989).

[15] As will become evident, the restriction to symmetric equilibria is made mostly for computational ease, and our main welfare prediction of under-experimentation when signals disagree sufficiently holds a fortiori in asymmetric equilibria, were they to exist.

is positive and strictly increasing if $\underline{x}^* < x \leq \bar{x}^*$, and $\sigma_1^*(x) = \infty$ if $x > \bar{x}^*$. If $\bar{x}^* \leq 0$, then $\sigma_1^*(x)$ is finite for any x in the support. We denote by g^* the inverse function of σ_1^* on the domain $[\underline{x}^*, \bar{x}^*]$. We calculate equilibrium strategies by proceeding by backward induction. Starting the analysis from the equilibrium strategy σ_2^* of a player given that the opponent has left the race.

4.2. Equilibrium Play After the Opponent Quits

Suppose that a player (say player B) observes signal y, enters the game at time 0, and then quits first according to the equilibrium strategy σ_1^* at time $\tau = \sigma_1^*(y) > 0$. In a monotonic equilibrium, the remaining player perfectly infers her opponent's private signal $y = g^*(\tau)$, and from that moment acts fully informed. We now calculate her expected value of planning to stop at any given time τ_2.

At time t, conditional on a true value of the prize hazard rate λ, unknown the player, and on the fact that no prize has arrived to date, the expected value of planning to stop a single project at some future date $\tau_2 \geq t$ equals

$$U_{2,t}(\tau_2|\lambda) = \int_t^{\tau_2} \frac{f(s|\lambda)}{1-F(t|\lambda)} \left[\int_t^s (-c)\, e^{-r(v-t)} dv + e^{-r(s-t)} b\right] ds + \frac{1-F(\tau_2|\lambda)}{1-F(t|\lambda)} \int_t^{\tau_2} (-c)\, e^{-r(v-t)} dv$$

The first term is the expected discounted return in case the prize arrives before the project is stopped. Here $f(s|\lambda) / [1-F(t|\lambda)] = \lambda e^{-\lambda(s-t)}$ is the density of the prize arrival time, conditioned on no prize having arrived so far. The second term is the expected discounted return in case the prize does not arrive by the planned quitting time τ_2, premultiplied by the chance that this happens $[1-F(\tau_2|\lambda)] / [1-F(t|\lambda)] = e^{-\lambda(\tau_2-t)}$. The opponent left at time τ; at a subsequent time $t \geq \tau$, the 'survivor' plans to quit at a time $\tau_{2,t}^*(x, g^*(\tau)) \geq t$ such that

$$\tau_{2,t}^*(x, g^*(\tau)) = \arg\max_{\tau_2 \geq t} \left\{ V_{2,t}(\tau_2|x, g^*(\tau), \tau) = \int_\Lambda U_{2,t}(\tau_2|\lambda)\, \pi_{t,\tau}(\lambda|x, g^*(\tau)) d\lambda \right\}.$$

Recall that $\pi_{t,\tau}(\lambda|x, g^*(\tau))$ denotes the density of posterior beliefs conditional on the signal realizations x, on $g^*(\tau)$ being known exactly, and on the facts that neither prize had arrived by time τ, when the first project was stopped, and that the second project running alone did not arrive in $(\tau, t]$ either. By inspection, we see that $V_{2,t}(\tau_2|x, g^*(\tau), \tau)$ is C^2 in τ_2 for every $\tau_2 \geq t$ and every x, τ, t. Hence, the derivation of the optimal stopping time $\tau_{2,t}^*$ can be approached by deriving first and second order conditions, differentiating $V_{2,t}(\tau_2|x, g^*(\tau), \tau)$ with respect to τ_2.[16] Since this type

[16] Alternatively, one may differentiate the value $V_{2,t}(\tau_2|x, g^*(\tau), \tau)$ with respect to current time t and obtain a differential equation for the value, which is the continuous-time Hamilton-Jacobi-Bellman equation for this problem.

of manipulations will be used repeatedly in later omitted proofs, it is instructive to go through them at least once:

$$\begin{aligned}
\frac{dV_{2,t}\left(\tau_2|x,g^*\left(\tau\right),\tau\right)}{d\tau_2} &= \int_\Lambda \frac{d}{d\tau_2}U_{2,t}\left(\tau_2|\lambda\right)\pi_{t,\tau}(\lambda|x,g^*\left(\tau\right))d\lambda \qquad (4.1)\\
= &\int_\Lambda \left[\frac{f\left(\tau_2|\lambda\right)}{1-F\left(t|\lambda\right)}\left(\int_t^{\tau_2}\left(-c\right)e^{-r(v-t)}dv+e^{-r(\tau_2-t)}b\right)-\frac{f\left(\tau_2|\lambda\right)}{1-F\left(t|\lambda\right)}\int_t^{\tau_2}\left(-c\right)e^{-r(v-t)}dv\right.\\
&\left.+\frac{1-F\left(\tau_2|\lambda\right)}{1-F\left(t|\lambda\right)}\left(-c\right)e^{-r(\tau_2-t)}\right]\frac{\pi\left(\lambda\right)h\left(x|\lambda\right)h(g^*\left(\tau\right)|\lambda)\left[1-F\left(t|\lambda\right)\right]\left[1-F\left(\tau|\lambda\right)\right]}{\int_\Lambda\pi\left(\lambda'\right)h\left(x|\lambda'\right)h(g^*\left(\tau\right)|\lambda')\left[1-F\left(t|\lambda'\right)\right]\left[1-F\left(\tau|\lambda'\right)\right]d\lambda'}d\lambda\\
\propto &\int_\Lambda\left[\frac{f\left(\tau_2|\lambda\right)}{1-F\left(\tau_2|\lambda\right)}b-c\right]\frac{\pi\left(\lambda\right)h\left(x|\lambda\right)h(g^*\left(\tau\right)|\lambda)\left[1-F\left(\tau_2|\lambda\right)\right]\left[1-F\left(\tau|\lambda\right)\right]}{\int_\Lambda\pi\left(\lambda'\right)h\left(x|\lambda'\right)h(g^*\left(\tau\right)|\lambda')\left[1-F\left(\tau_2|\lambda'\right)\right]\left[1-F\left(\tau|\lambda'\right)\right]d\lambda'}d\lambda\\
= &\; bE_{\tau_2,\tau}\left[\left.\frac{f\left(\tau_2|\lambda\right)}{1-F\left(\tau_2|\lambda\right)}\right|x,g^*\left(\tau\right)\right]-c=bE_{\tau_2,\tau}\left[\lambda|\,x,g^*\left(\tau\right)\right]-c=b\frac{\beta+2}{\alpha-x-g^*\left(\tau\right)+\tau_2+\tau}-c.
\end{aligned}$$

where in the second line we simplify the first and third terms and we use the expression for $\pi_{t,\tau}(\lambda|x,g^*\left(\tau\right))$ from (3.1), in the fourth line we multiply and divide the integrand by $\left[1-F\left(\tau_2|\lambda\right)\right]$ and the whole expression by a positive renormalizing factor independent of λ, that we omit. Therefore, the first-order condition simply equates the posterior expected hazard rate of prize arrival to the cost/benefit ratio:

$$bE_{\tau_2,\tau}\left[\lambda|\,x,g^*\left(\tau\right)\right]=b\frac{\beta+2}{\alpha-x-g^*\left(\tau\right)+\tau_2+\tau}=c. \qquad (4.2)$$

Intuitively, the marginal cost c of proceeding an extra instant must equal the marginal benefit, which consists of the prize b multiplied by its expected hazard rate conditional on all available information. Due to exponential discounting, this condition is independent of the planning time t. Using the explicit expression for $E_{\tau_2,\tau}\left[\lambda|\,x,g^*\left(\tau\right)\right]$ with respect to the Gamma distribution derived in the Appendix, and rearranging terms, the first-order condition (4.2) yields the following result. The verification of the second-order condition is immediate from equation (4.1).

Proposition 1. ***(Equilibrium Play after the Opponent Quits)*** *In any symmetric monotonic equilibrium, for any $\tau>0$, the optimal stopping time of a player with signal x, after the opponent quits at time τ and reveals her private information $g^*(\tau)$, equals*

$$\sigma_2^*(x,\tau)=\max\{\tau,\frac{b}{c}\left(\beta+2\right)+x+g^*(\tau)-\alpha-\tau\}.$$

The quitting time $\sigma_2^*(x,\tau)$ increases in the signals x and $g^*\left(\tau\right)$, in the benefit/cost ratio b/c and decreases in the time τ the opponent stayed in the race without receiving the prize. Furthermore, it increases in the mean (β/α) and variance $\left(\beta/\alpha^2\right)$ of the prior distribution of λ: the higher the promise of the prize the longer the player is willing to stay in the race, and the higher the variance, the higher is the option value for experimenting.

If a player with signal z fails to join the game, then the remaining player cannot perfectly infer the opponent's signal realization y, because the equilibrium strategy τ_1^* is not invertible for $y \leq \underline{x}^* \equiv g^*(0)$, but only learns that $y \leq \underline{x}^*$. Calculations analogous the previous ones yield the following result.

Proposition 2. ***(Equilibrium Play after the Opponent Fails to Join the Game)*** *In any symmetric monotonic equilibrium, if the opponent fails to join the game, a player with private signal realization x optimally stops at time*

$$\sigma_2^*(x, 0-) = \max\{0, \frac{b}{c}(\beta + 1) + x + \underline{x}^* - \alpha\}.$$

Since entering the game for an arbitrarily small length of time, and then quitting, perfectly reveals own private information y, while not joining the game at all only reveals an upper bound $\underline{x}^*$ to y, there is a natural discontinuity in the equilibrium strategy as a second quitter $\sigma_2^*(x, \tau)$ at $\tau = 0$. In fact, for any $x > \alpha - (\beta + 2) b/c - \underline{x}^*$, so that entering the game for some time is optimal,

$$\lim_{\tau \downarrow 0} \sigma_2^*(x, \tau) = \frac{b}{c}(\beta + 2) + x + \underline{x}^* - \alpha > \max\left\{0, \frac{b}{c}(\beta + 1) + x + \underline{x}^* - \alpha\right\} = \sigma_2^*(x, 0-).$$

For future reference, we let $\underline{x}^{**} \equiv \inf\{x : \sigma_2^*(x, 0) > 0\}$, the lowest signal for which a player is willing to stay in the race, upon seeing that the opponent did not enter the game. To summarize: if $x > \underline{x}^{**}$ then the player enters and stays in for some time no matter what the opponent does; if $\underline{x}^* \leq x \leq \underline{x}^{**}$ then the player enters and quits right away if the opponent failed to join; if $\underline{x}^* < x$ then the player does not enter at all.

4.3. Equilibrium Play Before the Opponent Quits

The most complex part of the equilibrium characterization concerns the earlier phase of the game, when both players are still in the game. Each player must plan an optimal stopping time based on the hypothesis that the opponent will quit later, and on the resulting information about the opponent's private information.

The Value Function. We first determine the value function of a player at any time $t > 0$ for quitting at time $\tau_1 \geq t$, conditional on the facts that opponent has not quit yet at time τ_1 and is adopting a monotonic strategy σ_1^*, with associated inverse g^*. We consider the problem of player A, the other player's calculations being symmetric. The expected value at time t for planning at time t to stop at some time $\tau_1 > t$, conditional on λ, equals to:

$$\begin{aligned}
U_{1,t}(\tau_1|\lambda) = & \int_t^{\tau_1} \frac{f(s|\lambda)}{1-F(t|\lambda)} \frac{1-F(s|\lambda)}{1-F(t|\lambda)} \frac{1-H(g^*(s)|\lambda)}{1-H(g^*(t)|\lambda)} \left[\int_t^s -ce^{-r(v-t)}dv + e^{-r(s-t)}b\right] ds \\
& + \int_t^{\tau_1} \frac{f(s|\lambda)}{1-F(t|\lambda)} \frac{1-F(s|\lambda)}{1-F(t|\lambda)} \frac{1-H(g^*(s)|\lambda)}{1-H(g^*(t)|\lambda)} \left[\int_t^s -ce^{-r(v-t)}dv\right] ds \\
& + \int_t^{\tau_1} \left(\frac{1-F(s|\lambda)}{1-F(t|\lambda)}\right)^2 \frac{h(g^*(s)|\lambda)dg^*(s)/ds}{1-H(g^*(t)|\lambda)} \int_t^s \left[-ce^{-r(v-t)}dv + e^{-r(s-t)}V_{2,s}(\sigma_2^*(x,s)|x,g^*(s))\right] ds \\
& + \left(\frac{1-F(\tau_1|\lambda)}{1-F(t|\lambda)}\right)^2 \frac{1-H(g^*(\tau_1)|\lambda)}{1-H(g^*(t)|\lambda)} \int_t^{\tau_1} -ce^{-r(v-t)}dv. \qquad (4.3)
\end{aligned}$$

Each one of the four lines corresponds to one of the four possible and exhaustive events that can take place at any time s between the current time t and any future date τ_1 at which player A plans to quit. We go through the four lines in order. First player A's prize arrives at $t_A \in [t, \tau_1)$, before (the prize arrives to the rival at time) t_B and before (the opponent quits first at time) τ. Conditional on the true arrival rate λ, the density of this event for $t_A = s \in [t, \tau_1]$ is

$$\frac{f(s|\lambda)}{1-F(t|\lambda)} \frac{1-F(s|\lambda)}{1-F(t|\lambda)} \frac{1-H(g^*(s)|\lambda)}{1-H(g^*(t)|\lambda)}. \qquad (4.4)$$

as shown in the first line of (4.3). In this case, A wins the race, pays costs up to that time t_A and collects the prize b. Second, player B's prize arrives at $t_B \in [t, \tau_1)$, before A's prize arrives at t_A and before A quits at τ_1. As a result, B wins the race at time t_B and player A just pays costs. The density of this event for $t_B = s \in [t, \tau_1]$ is again expressed in line (4.4). Third, player B quits at $\tau \in [t, \tau_1)$ first, i.e. before either prize arrives. Then the signal z_B is revealed to A by inverting $z_B = g^*(\tau)$. The density of this event for $t_A = s \in [t, \tau_1]$ is in the third line of (4.3). Player A pays costs and collects $V_{2,s}(\sigma_2^*(x,s)|x, g^*(s), s)$, the continuation value of going on alone optimally. Fourth and last, nothing happens in the time interval $[t, \tau_1)$: no one quits and no prize arrives. In this case player A quits at τ_1 and just pays costs. The probability of this event is in the fourth line of (4.3).

At any time $t > 0$, given that the opponent adopts a monotonic strategy σ_1^* with inverse g^*, each player chooses the following optimal stopping time as first quitter:

$$\tau_{1,t}^*(x) = \arg\max_{\tau_1 \geq t} \left\{ V_{1,t}\left(\tau_1|x\right) = \int_\Lambda U_{1,t}\left(\tau_1|\lambda\right) \pi_{t,t}(\lambda|x, g^*\left(t\right)+) d\lambda \right\}.$$

The First-Order Condition: Necessity and Sufficiency. In order to find the optimal stopping time of a player before the opponent quits, we differentiate the expected value $V_t\left(\tau_1|x\right)$ with respect to the stopping time τ_1. After substantial manipulations that we omit but make available upon request, we obtain:

$$\begin{aligned} \frac{dV_{1,t}\left(\tau_1|x\right)}{d\tau_1} \quad \propto \quad & -c + E_{\tau_1,\tau_1}\left[\lambda|\, x, g^*\left(\tau_1\right)+\right] b \\ & + V_{2,\tau_1}(\sigma_2^*(x,\tau_1)|x, g^*(\tau_1), \tau_1) E_{\tau_1,\tau_1}\left[\left. \frac{h(g^*\left(\tau_1\right)|\lambda) dg^*\left(\tau_1\right)/d\tau_1}{1 - H(g^*\left(\tau_1\right)|\lambda)} \right| x, g^*\left(\tau_1\right)+ \right]. \end{aligned} \tag{4.5}$$

This marginal value equals minus the flow cost $-c$ plus two flow expected benefit terms, the expected hazard rate $E_{\tau_1,\tau_1}\left[\lambda|\, x, g^*\left(\tau_1\right)+\right]$ of prize arrival times the prize value b, and the expected hazard rate $E_{\tau_1,\tau_1}\left[\left. \frac{h(g^*(\tau_1)|\lambda) dg^*(\tau_1)/d\tau_1}{1 - H(g^*(\tau_1)|\lambda)} \right| x, g^*\left(\tau_1\right)+ \right]$ of the opponent leaving the game times the continuation value $V_{2,\tau_1}(\sigma_2^*(x,\tau_1)|x, g^*(\tau_1), \tau_1)$ of remaining alone. Note that, again, the RHS of (4.5) is independent of the current time t.

Remarkably, *the possibility that the opponent may receive the prize and win the race does not enter the marginal value of waiting to quit the game first.* This surprising property has a simple intuition. The derivative $dV_{1,t}\left(\tau_1|x\right)/d\tau_1$ captures the difference in value at τ_1 between leaving at time $\tau_1 + \Delta\tau_1$ and leaving immediately at τ_1, where $\Delta\tau_1$ is an arbitrarily small period of time. During the period $\Delta\tau_1$, the cost $c\Delta\tau_1$ is paid up-front and sunk, and either one of the two benefits of size b or $V_{2,\tau_1}(\sigma_2^*(x,\tau_1)|x, g(\tau_1), \tau_1)$ may arrive. If nothing arrives or if the opponent wins in the meantime, then either way the payoff is zero. The only effect prize arrival to the opponent is to end to game before $\tau_1 + \Delta\tau_1$ at no further cost, nor benefit to the player. Since the player is anyway ending the game at time $\tau_1 + \Delta\tau_1$, the arrival of the prize to the opponent at any given $t \in (\tau_1, \tau_1 + \Delta\tau_1)$ bears no change in the player's marginal value for waiting as $\Delta\tau_1$ vanishes.

Lemma A.1 in Appendix establishes two "corner properties" of any equilibrium strategy. First, never leaving the game as long as the other player stays in cannot be a best response to itself for any signal realization $x \leq 0$. In other words, there are no symmetric monotonic equilibria where the players remain in the race forever. Conversely, a player stays in the game for a positive amount of time, when her opponent enters the game and stays in, provided that her signal is good enough.

Equilibrium Characterization. The key result of this subsection is a "survivor's curse," that we identify in any monotonic equilibrium of this class of optimal-stopping games of conflicting interests. Suppose that player B adopts a monotonic strategy σ_1 with inverse g. Say that in the event that B remains in the race, player A endowed with signal x plans to quit first at a time τ_1 which satisfies the first-order condition $dV_{1,t}(\tau_1|x)/d\tau_1 = 0$. If B quits first at any time τ earlier than but close enough to τ_1, then A must also immediately leave the race, *regretting not having left earlier.* The intuition behind this result is simple. When player A plans to leave the race at τ_1, she conditions on player B still being in the race and hence on the expectation $E[z_B|z_B \geq g(\tau_1)]$ with respect to B's signal. If in fact B quits first at a time τ close but smaller than τ_1, then A suddenly realizes that B had observed signal $z_B = g(\tau)$, which is much smaller than $E[z_B|z_B \geq g(\tau)]$ and hence smaller also than $E[z_B|z_B \geq g(\tau_1)]$. This induces a sudden pessimistic revision of A's beliefs with respect to the promise of the project; accordingly, A quits immediately after B, regretting her previous over-optimistic expectation of the rival's assessment of the project's feasibility.[17]

Lemma 3. *In any monotonic equilibrium (σ_1^*, σ_2^*), for any signal realization $x \leq 0$, if a player (say A) is planning to quit first at time $\tau_1 > 0$ such that $dV_{1,t}(\tau_1|x)/d\tau_1 = 0$, then for any $\tau < \tau_1$ close enough to τ_1, player A's optimal stopping choice after B quits at τ is to immediately follow suit and gain nil continuation value:*

$$\tau^*_{2,\tau}(x,\tau) = \tau > \frac{b}{c}(\beta+2) + x + g^*(\tau) - \tau - \alpha \quad \textit{and} \quad V_{2,\tau}(\tau^*_{2,\tau}(x,\tau)|x, g^*(\tau), \tau) = 0.$$

Proof. We only need to show that

$$c > bE_{\tau_1,\tau_1}[\lambda|\, x, g^*(\tau_1)] = b\frac{\beta+2}{\alpha - x - g^*(\tau_1) + 2\tau_1},$$

by definition of $\tau^*_{2,\tau}(x,\tau)$ and by continuity of the RHS.

[17] As a mirror image of this survivors's curse, a 'quitter's curse' arises if player B leaves the race at a time τ much earlier than τ_1, the planned stopping time of player A. In this case, player A remains in the race after τ, and player B regrets having left.

Since τ_1 solves the first-order condition $dV_{1,t}(\tau_1|x)/d\tau_1 = 0$, using equation (4.5), we obtain:

$$\begin{aligned} 0 &= bE_{\tau_1,\tau_1}\left[\lambda | \, x, g^*(\tau_1)+\right] \\ &\quad + V_{2,\tau_1}(\sigma_2^*(x,\tau_1)|x, g^*(\tau_1), \tau_1) E_{\tau_1,\tau_1}\left[\left. \frac{h(g^*(\tau_1)|\lambda) dg^*(\tau_1)/d\tau_1}{1-H(g^*(\tau_1)|\lambda)} \right| x, g^*(\tau_1)+\right] - c \\ &\geq bE_{\tau_1,\tau_1}\left[\lambda | \, x, g^*(\tau_1)+\right] - c > bE_{\tau_1,\tau_1}\left[\lambda | \, x, g^*(\tau_1)\right] - c, \end{aligned}$$

where the first inequality follows because $\frac{h(g^*(\tau_1)|\lambda)dg^*(\tau_1)/d\tau_1}{1-H(g^*(\tau_1)|\lambda)} > 0$ and $V_{2,\tau_1}(\tau^*_{2,\tau_1}(x,\tau_1)|x, g^*(\tau_1), \tau_1) \geq 0$, whereas the second inequality follows from Lemma 2: knowing that the opponent's signal is exactly $g^*(\tau_1)$ is bad news with respect to knowing that it is at least $g^*(\tau_1)$. ■

In light of the Lemma 3, for any x, the First-Order Condition $dV_t(\tau_1|x)/d\tau_1 = 0$ can be rewritten simply as:

$$c = bE_{\tau_1,\tau_1}\left[\lambda | x, g^*(\tau_1)+\right], \tag{4.6}$$

which says that a player quits at the time τ_1 when the flow cost c equals the expected flow benefit consisting of the value of the prize b times expected hazard rate λ of arrival of the prize, conditional on own private information x, on the opponent's signal y being larger than $g^*(\tau_1)$, and on neither prize having arrived by time τ_1. In any symmetric monotonic equilibrium (σ_1^*, σ_2^*), symmetry implies that $g^*(\sigma_1^*(x)) = x$, so that Equation (4.6) is further simplified as

$$c = bE_{\tau_1,\tau_1}\left[\lambda | \, x, x+\right] = b(\beta+1)\frac{\left[(\alpha+2\tau_1-x)^{-\beta-2} - (\alpha+2\tau_1-2x)^{-\beta-2}\right]}{\left[(\alpha+2\tau_1-x)^{-\beta-1} - (\alpha+2\tau_1-2x)^{-\beta-1}\right]}. \tag{4.7}$$

By Lemma 1, the right-hand side of this equation is strictly decreasing in τ_1 and strictly increasing in x. Therefore, it has a unique positive and increasing solution $\zeta_1(x)$ for any x such that $bE_{0,0}\left[\lambda | \, x, g^*(0)+\right] = bE_{0,0}\left[\lambda | \, x, \underline{x}^*+\right] \geq c$, i.e. $x \leq \underline{x}^*$, and no solutions otherwise. In particular, $\underline{x}^* = g^*(0)$ is uniquely determined by

$$c = b(\beta+1)\frac{\left[(\alpha-\underline{x}^*)^{-\beta-2} - (\alpha-2\underline{x}^*)^{-\beta-2}\right]}{\left[(\alpha-\underline{x}^*)^{-\beta-1} - (\alpha-2\underline{x}^*)^{-\beta-1}\right]}. \tag{4.8}$$

So far, our analysis has singled out as the unique candidate symmetric monotonic equilibrium stopping strategy the function σ_1^* such that: $\sigma_1^*(x)$ solves the implicit function (4.7) if $bE_{\tau_1,\tau_1}\left[\lambda | \, x, \underline{x}^*+\right] > c$, and $\sigma_1^*(x) = 0$ otherwise. Lemma A.1 in the Appendix has shown that if $bE_{\tau_1,\tau_1}[\lambda|\, x, \underline{x}^*+] > c$ and the opponent adopts the stopping strategy σ_1^*, a player endowed with signal x who is in the race at time t finds it optimal to quit it at time $\sigma_1^*(x) > 0$.

In order to conclude that the stopping strategy σ_1^* is the unique symmetric monotonic equilibrium, we are only left to determine the optimal decision at the very beginning of the game, i.e. at time $t = 0$. We verify in the Appendix (Lemma A.2) that if the opponent enters the game whenever $x > \underline{x}^*$, then it is optimal to enter the race if and only if $x > \underline{x}^*$. Lemma 2 immediately implies that $\underline{x}^{**} > \underline{x}^*$: Upon knowing that the opponent failed to join the race, a player becomes more pessimistic than if the opponent entered for a whatsoever small amount of time. Hence, whenever $\underline{x}^* < x < \underline{x}^{**}$, the player will initially join the race and then exit immediately after upon seeing that the opponent did not join.

The following Proposition summarizes our findings for the equilibrium play σ_1^* before the opponent quits and, together with Propositions 1 and 2 for the strategy σ_2^* after the opponent has quit, fully characterizes the unique symmetric monotonic equilibrium (σ_1^*, σ_2^*). It is important to note that neither the optimal team's strategy, nor the equilibrium strategy depend on the discount rate r. This property will greatly simplify our welfare comparison, where the discount rate will play a major role.

Proposition 3. ***(Equilibrium Play Before the Opponent Has Quit)*** *The unique symmetric monotonic equilibrium stopping time, conditional on a private signal x and on the opponent still being in the game, is*

$$\sigma_1^*(x) = \begin{cases} 0 & \text{if } x < \underline{x}^* \\ \zeta_1(x) & \text{if } x \geq \underline{x}^*, \end{cases}$$

where $\zeta_1(x)$ is the unique increasing solution of (4.7), and $\underline{x}^* < 0$ is the unique root of $\zeta_1(x) = 0$, or equivalently (4.8).

5. Welfare Analysis

5.1. The Cooperative Problem

In order to investigate the welfare properties of the unique symmetric monotonic equilibrium, we compare its outcome to the first-best cooperative solution, in which players join forces in a single team and share their private information. The team operates both projects as a single agent to maximize joint profits. In our model with independent negative-exponential prize arrivals and Gamma prior, the optimal team policy is bang-bang. Specifically, the team keeps both projects active at any time t such that the expected marginal benefit, $2E_{t,t}[\lambda|\, x, y]\, b$ is larger than the marginal cost $2c$, and stops both projects simultaneously at the time t where the costs offset the benefits (note that $E_{t,t}[\lambda|\, x, y] = [\beta+2]/[\alpha-x-y+2t]$ is strictly decreasing in t). This immediately leads to the following Proposition.[18]

Proposition 4. ***(The Team Solution)*** *For every pair of signals x, y on the unobserved promise λ of the two projects, the team optimally stops both projects simultaneously at time*

$$T^{*}(x, y) = \max\left\{0, \frac{1}{2}\left[(\beta+2)\,\frac{b}{c} - \alpha + x + y\right]\right\}.$$

Because this single-agent optimization problem is already well-understood in the dynamic optimization literature from papers studying exponential experimentation models without private information (e.g. Keller, Rady and Cripps (2004)), we omit our formal analysis which is available upon request.[19] The intuition behind the results is as follows. Owing to our model's specification, keeping two projects active instead of one is equivalent to doubling the intensity of experimentation, the experimentation costs and the speed of learning. If there were no uncertainty, the team's optimal policy would be 'bang-bang'. When the marginal trade-off between costs and benefits is positive, the team would pursue the prize at the maximal intensity available, by keeping both projects active. When the marginal trade-off is negative, the team would shut down both projects. Without uncertainty, this trade-off is time invariant, so the two projects stay active either for ever or never. When the hazard rate λ is unknown, the team learns about λ by experimenting. It revises beliefs about λ downwards as time advances and prizes do not arrive. Most importantly, it learns twice faster by keeping both projects active, rather than only one. As the team is impatient

[18] The expression for $T^{*}(x, y)$ coincides with the expression for the stopping strategy of the survivor $\sigma_2^{*}(x, \tau)$ when setting $g(\tau) = y$ and equating $\sigma_2^{*}(x, \tau) = \tau$. In equilibrium, the survivor acts fully informed.

[19] It is easy to show that, after relabeling the variables, the cooperative problem studied by Keller, Rady and Cripps (2004) is equivalent to a N-project version of our team problem where the prior of λ is binary (instead of Gamma, as in our paper) with realizations 0 and $\hat{\lambda} > 0$. In their problem, in fact, the team simultaneously operates N identical bandits. Each bandit comprises a safe arm that yields a known flow payoff, and a risky arm that delivers prizes according to a Poisson distribution of parameter λ. Once one of the risky arms delivers its first prize, all uncertainty about λ is resolved: the team realizes that $\lambda = \hat{\lambda}$ and, in practice, the game ends.

They show that the solution of this problem is such that the team keeps all N risky arms active until the time when the expected marginal benefits of the risky arms coincide with the flow payoffs of the safe arms, and then it simultaneously switches all N risky arms off.

and discounts the future, there is no reason to slow down the speed of learning by proceeding with one project only. Hence, whenever the marginal trade-off between costs and expected benefits is positive, the team chooses to pursue the prize at the maximal intensity available, and it stops both projects when the marginal trade-off becomes negative.

5.2. Interim Welfare Analysis

We now investigate the welfare properties of the unique symmetric monotonic equilibrium, by comparing its outcome to the cooperative solution. We first characterize the welfare properties of equilibrium conditional on the signal realizations x, y. Without loss of generality, we let $x \leq y$, so that player A is more pessimistic than player B. To avoid burdening the exposition with too many subcases, we here focus on the case that $y \geq x > \underline{x}^*$ and $T^*(x, y) > 0$, so that both players and the team join the game. We relegate to the appendix the cases when some agents have such low signals that they do not even enter the race.

When player A quits, she reveals her private signal x to B. If player B's signal y is sufficiently close to x, our "survivor's curse" implies that player B immediately quits the race, regretting not having left before. Clearly, experimentation is excessive because both projects are stopped too late.

Lemma 4. *Suppose that $y \geq x > \underline{x}^*$. If*

$$\sigma_1^*(x) > T^*(x, y) = \frac{1}{2}\left[(\beta + 2)\frac{b}{c} - \alpha + x + y\right], \tag{5.1}$$

then both players' equilibrium stopping times are higher than the efficient common stopping time of both projects:

$$\sigma_2^*(y, \sigma_1^*(x)) = \sigma_1^*(x) > T^*(x, y).$$

Since the function $T^*(x, y)$ is increasing in both x and y, given that $y \geq x$, the inequality in Equation (5.1) holds if x and y are sufficiently close.

A more complex case occurs when x is sufficiently small with respect to y that $\sigma_1^*(x) < T^*(x, y)$. When quitting at time $\sigma_1^*(x)$, player A underestimates the opponent's signal y and quits too soon with respect to the team's solution. Player B stays in the race until time

$$\sigma_2^*(y, \sigma_1^*(x)) = \frac{b}{c}(\beta + 2) + g^*(\sigma_1^*(x)) + y - \alpha - \sigma_1^*(x).$$

In our Gamma-Poisson model, B's equilibrium stopping time $\sigma_2^*(y,\sigma)$ is linear in signal y and in A's exit time σ. As a consequence, player B's expected duration in the race (conditional on no prize arrival) exactly offsets the premature quitting by player A:

$$\sigma_2^*(y,\sigma_1^*(x)) + \sigma_1^*(x) = \frac{b}{c}(\beta+2) + x + y - \alpha = 2T^*(x,y).$$

Although the sum of equilibrium experimentation durations coincides with the team's aggregate duration, a key inefficiency remains. The cooperative solution dictates that the two projects should be *stopped simultaneously.* In equilibrium, *exit is sequential:* Player A exits too soon and her experimentation duration $T^*(x,y) - \sigma_1^*(x)$ is "postponed" by player B after she has completed her optimal duration $T^*(x,y)$. Since the team is impatient, this corresponds to suboptimally slowing down the joint rate of innovation arrival. Hence, experimentation is suboptimal in equilibrium.[20]

To make this statement formal, we decompose the expression for time-0 expected team's welfare, conditional on stopping the first project at time T_1 and the second one at time T_2,

$$W(T_1,T_2|x,y) = B(T_1,T_2|x,y) - K(T_1,T_2|x,y)$$

as the difference between expected discounted rewards

$$B(T_1,T_2|x,y) \equiv b\int_\Lambda \left[\int_0^{T_1} 2f\left(s|\lambda\right)\left(1-F\left(s|\lambda\right)\right)e^{-rs}ds + \left(1-F\left(T_1|\lambda\right)\right)\int_{T_1}^{T_2} f\left(s|\lambda\right)e^{-rs}ds\right]\pi_{0,0}(\lambda|x,y)d\lambda$$

and the expected discounted costs

$$\begin{aligned}
&K(T_1,T_2|x,y) = \int_\Lambda C(T_1,T_2|\lambda)\pi_{0,0}(\lambda|x,y)d\lambda, \text{ with}\\
&C(T_1,T_2|\lambda) \equiv c\left[\int_0^{T_1} 2f\left(s|\lambda\right)\left(1-F\left(s|\lambda\right)\right)\int_0^s 2e^{-rv}dvds + \left(1-F\left(T_1|\lambda\right)\right)^2\int_0^{T_1} 2e^{-rv}dv\right.\\
&\left. + \left(1-F\left(T_1|\lambda\right)\right)\int_{T_1}^{T_2} f\left(s|\lambda\right)\int_{T_1}^s e^{-rv}dvds + \left(1-F\left(T_1|\lambda\right)\right)\left(1-F\left(T_2|\lambda\right)\right)\int_{T_1}^{T_2} e^{-rv}dv\right].
\end{aligned}$$

We prove in the Appendix:

Lemma 5. *If $y > x > \underline{x}^*$, and $0 < \sigma_1^*(x) < T^*(x,y)$, then the time-0 expected present discounted value of the research costs and benefits are smaller in equilibrium than in the team solution:*

[20] This postponement effect extends to (monotonic) asymmetric equilibria, were they to exist. Also in such equilibria firms almost-surely leave the race sequentially, instead of simultaneously. After the most pessimistic firm quits the race, her opponent acts fully informed mimicking the team's optimal solution for a suboptimally constrained termination of the first project.

$K(\sigma_1^*(x), \sigma_2^*(y, \sigma_1^*(x))|x, y) < K(T^*(x, y), T^*(x, y)|x, y)$ *and hence* $B(\sigma_1^*(x), \sigma_2^*(y, \sigma_1^*(x))|x, y) < B(T^*(x, y), T^*(x, y)|x, y)$.

The source of this key result is a failure of information aggregation. Private information can only be revealed credibly by quitting decisions. In any monotonic equilibrium it flows only from the more pessimistic player to her opponent, and never vice versa.

The welfare analysis conditional on the signal pairs (x, y) is summarized in the following Proposition.

Proposition 5. ***(Welfare Analysis Conditional on Signal Realizations)*** *Given the unique symmetric monotonic equilibrium* (σ_1^*, σ_2^*) *and the team's optimal stopping time* $T^*(x, y)$*, there exists an increasing continuous function* $\xi : (\underline{x}^*, 0] \to (\underline{x}^*, 0]$*, implicitly defined by*

$$\sigma_1^*(x) = T^*(x, \xi(x)) = \frac{1}{2}\left[(\beta + 2)\frac{b}{c} - \alpha + x + \xi(x)\right],$$

such that $\xi(x) > x$ *for any* $x < 0$*, and that for any* x, y *with* $y \geq x > \underline{x}^*$ *and* $T^*(x, y) > 0$*,*

1. (Equilibrium Under-Experimentation) *If* $y > \xi(x)$ *(i.e. signals differ sufficiently), then equilibrium experimentation is too low. Both the total time-*0 *expected discounted experimentation costs and the total time-*0 *expected discounted benefit are smaller in equilibrium than in the team's solution.*

2. (Equilibrium Over-Experimentation) *If* $y < \xi(x)$ *(private signals agree sufficiently), then equilibrium experimentation is excessive. The total time-*0 *expected discounted experimentation costs are larger in equilibrium than in the team's solution.*

This concludes the characterization of the normative properties of the equilibrium conditional on the realizations of the private signals. Our interpretation of the results is as follows. If the social planner has any reason to believe that the players disagree in their beliefs about the promise of the project, then she would expect that experimentation is too low in equilibrium. Whereas if the social planner deems that the players' beliefs are in agreement, then she would expect equilibrium over-experimentation.

5.3. Ex-Ante Welfare Analysis

We now conclude the welfare analysis of the unique equilibrium. We determine whether under-experimentation or over-experimentation is to be expected conditional only on prior beliefs about the promise of the project (α and β), and before the players observe their private signals. This is the perspective of a social planner who does not enjoy any informational advantage over any players. We establish that, as long as the social planner is sufficiently impatient and deems the project valuable enough, then she expects under-experimentation in equilibrium.

Intuitively, when the project's expected hazard rate λ is high enough, the social planner is mostly concerned with signal outcomes x, y such that $T^*(x, y) > 0$: it would be optimal that both players enter the game and exit simultaneously. In equilibrium, players jointly over-experiment if their maximum duration $\sigma_1^*(x) = \sigma_2^*(y, \sigma_1^*(x))$ is larger than $T^*(x, y)$, i.e. the more pessimistic player overshoots the team solution. They jointly under-experiment if $\sigma_1^*(x) < T^*(x, y)$ so that more pessimistic player exits too soon, and her shortfall $T^*(x, y) - \sigma_1^*(x)$ is compensated by the opponent but postponed after $T^*(x, y)$. If the social planner is sufficiently impatient, then she is relatively more concerned ex-ante about the risk of premature exit and postponement than about the risk of delayed exit.

Formally, we let

$$D(\alpha, \beta; r) = \int_{\Lambda} \int_{R^2_-} \left[C\left(T^*(x, y), T^*(x, y) | \lambda\right) - C\left(\sigma_1^*(x), \sigma_2^*(y, \sigma_1^*(x)) | \lambda\right) \right] h(x|\lambda) h(y|\lambda) d\pi(\lambda)$$

denote the difference between equilibrium experimentation costs and team costs on the basis of α and β only.

Proposition 6. ***(Unconditional Welfare Properties of the Equilibrium)*** *Given the unique symmetric monotonic equilibrium (σ_1^*, σ_2^*) and the team's solution T^*, if the ex-ante expectation of the project's promise λ is sufficiently large, and if the discount rate r is sufficiently large, then $D(\alpha, \beta; r) < 0$: ex-ante experimentation is too low in equilibrium. If either the project is not valuable ex-ante, or if the social planner is very patient, then $D(\alpha, \beta; r) > 0$ and ex-ante experimentation is too high in equilibrium.*

6. Extensions

Our construction can be extended in several directions. We shall now discuss some of the most interesting and promising ones, to show that, by and large, our conclusions appear to be robust. Formal arguments are available upon request.

Deterministic Accumulation of Knowledge over Time. Our model inherits from differential game models of R&D races the assumption that innovation arrival is governed by a Poisson process of parameter λ. This is equivalent to say that the unknown innovation hazard rate $L(t, \lambda) \equiv f(t|\lambda) / [1 - F(t|\lambda)]$ is constant over time (and equal to λ). Then, it is natural to postulate conjugate Gamma prior and negative exponentially distributed signals. It is not too difficult, however, to extend our equilibrium construction when the innovation hazard rates $L(t, \lambda)$ of unknown parameter λ is not constant over time.[21] The engine of our equilibrium characterization is Theorem A.1 in the Appendix. Its power goes well beyond the Gamma-Poisson specification of our model.

Make the following mild regularity assumptions: the distribution of λ has connected support Λ (with $\underline{\lambda} = \inf \Lambda$ and $\bar{\lambda} = \sup \Lambda$), the hazard rate of the prize $L(t, \lambda)$ is continuous in t, λ, and strictly increasing in λ for every t, with $L(t, \bar{\lambda}) > c/b > L(t, \underline{\lambda})$, and the density $h(x|\lambda)$ of private signals x is differentiable, with support $X \subseteq R$, and $\lim_{x \to \sup X} h(x|\lambda) < \infty$. In this environment, we can prove that our equilibrium characterization presented in Propositions 1 and 3 holds if the following two substantive restrictions are met. First, the signal density $h(x|\lambda)$ is log-supermodular, i.e. the ratio $h'(x|\lambda)/h(x|\lambda)$ is strictly increasing in λ. Second, the expected hazard rates $E_{t,t'}[L(t, \lambda)| x, y]$ and $E_{t,t'}[L(t, \lambda)| x, y+]$ are strictly decreasing in t for any fixed signals x, y and t'; and they both attain the lower bound $L(t, \underline{\lambda})$ in the limit as $t \to \infty$. The first condition is reminiscent of log-supermodularity conditions for pure strategy equilibrium existence derived in Athey (1999). The second condition requires that for any fixed pair of signals x, y, as time goes by and the innovation does not arrive, one becomes more and more pessimistic about the promise of the project, both in the case that one knows precisely both signal realizations x and y, or that one only knows one signal realization to be precisely x while that the other signal realization is larger than y. We

[21] It is also easy to extend our analysis for the possibility that the value of the prize b changes over time, for example growing at the economy's rate.

underline that this is not unduly strong. Among other things, it does not require that the *actual* innovation hazard rate $L(t, \lambda)$ be decreasing over time. As knowledge accumulate throughout the R&D process, $L(t, \lambda)$ is likely to be increasing over time; but it is also quite likely that one becomes less optimistic about its feasibility as time goes by and the innovation does not materialize.

Random Accumulation of Private Information over Time. Knowledge about the project may also arrive randomly as the race unfolds, and therefore remain private. A simple way to capture this phenomenon within our Gamma-Poisson model is to assume that each player i who remains in the race may observe "good news" as time goes by. Each good news is the arrival of a known event at uncertain times, following a Poisson process of parameter $k\lambda$, where $k > 1$. Therefore, the more promising the project, the higher λ and the more frequently good news accrue, typically before the innovation itself because $k > 1$. Ruling out initial private signals for simplicity, all private information at any time t is represented by the number n of good-news accrued to date t. For any t and n, the player's beliefs over λ are again represented by a Gamma distribution. The calculation of the marginal value of waiting to exit is carried out in analogous way as in the present model. The relevant events are (i) the arrival of prize, (ii) the exit of the opponent, (iii) the arrival of good news. Each of these arrivals corresponds to a flow benefit that one weighs against the flow cost of remaining in the race.

Just like in our model, a symmetric monotonic equilibrium strategy consists of two stopping time functions. For number of good news n, the first stopping time function σ_1^* prescribes to leave the race at time $\sigma_1^*(n)$ as long as the opponent is still in the race, whereas given that the opponent left the race at time τ, equilibrium exit is prescribed by stopping time function $\sigma_2^*(n, \tau)$. Monotonicity requires that both these stopping functions are increasing in n. Each player inverts her opponent's good-news index $g^*(\tau)$ upon observing her exit at time τ. If the opponent is still in the race, the only information available is that her good-news index n is at least as large as $g^*(\tau)$.[22] In equilibrium, one player prematurely exits if private information disagrees sufficiently. If the team is impatient enough, equilibrium ex-ante discounted experimentation is suboptimally low. Unlike our model, the equilibrium is fully-separating.

[22]While on the equilibrium path firms may only exit at a countable set of times, there is no practical issue with beliefs off path, because of the assumption of irreversible exit.

Prize Arrival Externalities. While the winner-take-all assumption is a natural benchmark, our framework can also accommodate prize arrival externalities, both negative (e.g. reduced market share, lost competitive edge), or positive (e.g. spillover effects, imitation, innovation dissemination, industry-wide benefits) on the other player. Inspection of the equilibrium equations characterizing the stopping time function σ_1^*, reveals two possibilities. If the externality is borne by a player only while in the race, then positive (negative) externalities exarcerbate (dampen) the herding effect: players postpone (anticipate) their exit because of the benefits (losses) obtained if the prize arrives to the opponent. If instead the externality is borne regardless of whether the player is still in the race or not, then exactly the opposite implication holds: Negative (positive) externalities exacerbate (dampen) herding, because players postpone (anticipate) their exit to reduce the probability that the prize arrives to the opponent (to save on flow cost and shift the burden of experimentation on the opponent).

Ruling out the (fairly implausible) case of negative externalities borne only if in the race, the team optimal solution still consists in running both projects and simultaneously stopping them when marginal benefits (which now internalize externality effects) are offset by flow costs. In fact, when externalities are borne regardless of whether in the race or not, they do not affect the team's stopping policy, and positive externalities borne only if in the race, make the team prize value larger when both projects are open. In equilibrium, one player prematurely exits if private information disagrees sufficiently, and this results in under-experimentation if the team is impatient enough.

7. Conclusion

This paper studies a Poisson R&D race with fixed experimentation intensity and winner-take-all termination, where firms are privately informed about the promise and feasibility of their common line of research. We have calculated the unique symmetric monotonic equilibrium, and compared it with the optimal policy of a team that shares the firm's private information. Due to the interdependent-value nature of the problem, the equilibrium displays a strong herding effect that distinguishes our framework from war-of-attrition models. Earlier models of R&D races without private information predicted equilibrium overinvestment due to duplication costs. Despite our herding effect, we have found that equilibrium expenditure in R&D is sub-optimally low whenever the social planner is sufficiently impatient. This is because the more pessimistic firm prematurely exits the race when information is sufficiently heterogeneous, so that R&D activity is inefficiently postponed. Our conclusions are that private incentives curb information sharing in R&D, and secrecy, in turn, inefficiently slows down the pace of innovation.

A. Appendix. Omitted Proofs.

Posterior Beliefs. We show the expressions for the posterior beliefs in (3.1) and (3.2):

$$\pi_{t,t'}(\lambda|x,y) = \frac{e^{-\alpha\lambda}\lambda^{\beta-1}\lambda e^{\lambda x}\lambda e^{\lambda y}e^{-\lambda t}e^{-\lambda t'}}{\int_\Lambda e^{-\alpha\lambda'}\lambda'^{\beta-1}\lambda' e^{\lambda' x}\lambda' e^{\lambda' y}e^{-\lambda' t}e^{-\lambda' t'}d\lambda'} = \frac{e^{-\lambda(\alpha-x-y+t+t')}\lambda^{\beta+1}}{\Gamma(\beta+2)(\alpha-x-y+t+t')^{-\beta-2}}, \quad \text{(A.1)}$$

$$\pi_{t,t'}(\lambda|x,y+) = \frac{\lambda^\beta\left[e^{-\lambda(\alpha+t+t'-x)} - e^{-\lambda(\alpha+t+t'-x-y)}\right]}{\Gamma(\beta+1)\left[(\alpha+t+t'-x)^{-\beta-1} - (\alpha+t+t'-x-y)^{-\beta-1}\right]}.$$

We begin our proofs with a very useful technical result.

Theorem A.1. *Let $q\left(\lambda,\vec{\theta}\right) : \Lambda\times\Re^n \to \Re_+$, differentiable in $\vec{\theta}$ and integrable in λ with $\int_\Lambda q\left(\lambda,\vec{\theta}\right)d\lambda \in (0,\infty)$. Then the c.d.f. defined by:*

$$\varphi\left(L,\vec{\theta}\right) \equiv \frac{\int_{\underline{\lambda}}^L q\left(\lambda,\vec{\theta}\right)d\lambda}{\int_\Lambda q\left(\lambda',\vec{\theta}\right)d\lambda'}$$

for every $L \in \left(\underline{\lambda},\bar{\lambda}\right)$ is stochastically strictly increasing in every component of $\vec{\theta}$ if $q\left(\lambda,\vec{\theta}\right)$ is log-supermodular in (λ,θ_i), i.e. if $\partial\log q\left(\lambda,\vec{\theta}\right)/\partial\theta_i$ is strictly increasing in λ.

Proof. We want to show that for every $L \in \left(\underline{\lambda},\bar{\lambda}\right)$

$$0 > \frac{\partial}{\partial\theta_i}\frac{\int_{\underline{\lambda}}^L q\left(\lambda,\vec{\theta}\right)d\lambda}{\int_\Lambda q\left(\lambda,\vec{\theta}\right)d\lambda} = \frac{\int_{\underline{\lambda}}^L \frac{\partial q\left(\lambda,\vec{\theta}\right)}{\partial\theta_i}d\lambda}{\int_\Lambda q\left(\lambda,\vec{\theta}\right)d\lambda} - \frac{\int_{\underline{\lambda}}^L q\left(\lambda,\vec{\theta}\right)d\lambda\int_\Lambda \frac{\partial q\left(\lambda,\vec{\theta}\right)}{\partial\theta_i}d\lambda}{\left[\int_\Lambda q\left(\lambda,\vec{\theta}\right)d\lambda\right]^2}$$

using

$$\int\frac{\partial q\left(\lambda,\vec{\theta}\right)}{\partial\theta_i}d\lambda = \int\frac{\partial\log q\left(\lambda,\vec{\theta}\right)}{\partial\theta_i}q\left(\lambda,\vec{\theta}\right)d\lambda$$

the claim reads

$$\int_\Lambda\frac{\partial\log q\left(\lambda,\vec{\theta}\right)}{\partial\theta_i}\frac{q\left(\lambda,\vec{\theta}\right)}{\int_\Lambda q\left(\lambda',\vec{\theta}\right)d\lambda'}d\lambda > \int_{\underline{\lambda}}^L\frac{\partial\log q\left(\lambda,\vec{\theta}\right)}{\partial\theta_i}\frac{q\left(\lambda,\vec{\theta}\right)}{\int_{\underline{\lambda}}^L q\left(\lambda',\vec{\theta}\right)d\lambda'}d\lambda$$

A sufficient condition for the latter inequality is that the RHS be strictly increasing in L. Since the RHS is differentiable in L, it suffices that

$$\begin{aligned} 0 &< \frac{\partial}{\partial L}\left[\int_{\underline{\lambda}}^L\frac{\partial\log q\left(\lambda,\vec{\theta}\right)}{\partial\theta_i}\frac{q\left(\lambda,\vec{\theta}\right)}{\int_{\underline{\lambda}}^L q\left(\lambda',\vec{\theta}\right)d\lambda'}d\lambda\right] \\ &= \frac{\partial\log q\left(L,\vec{\theta}\right)}{\partial\theta_i}\frac{q\left(L,\vec{\theta}\right)}{\int_{\underline{\lambda}}^L q\left(\lambda',\vec{\theta}\right)d\lambda'} - q\left(L,\vec{\theta}\right)\int_{\underline{\lambda}}^L\frac{\partial\log q\left(\lambda,\vec{\theta}\right)}{\partial\theta_i}\frac{q\left(\lambda,\vec{\theta}\right)}{\left[\int_{\underline{\lambda}}^L q\left(\lambda',\vec{\theta}\right)d\lambda'\right]^2}d\lambda \end{aligned}$$

$$\text{or } \frac{\partial \log q\left(L,\vec{\theta}\right)}{\partial \theta_i} > \int_{\underline{\lambda}}^{L} \frac{\partial \log q\left(\lambda,\vec{\theta}\right)}{\partial \theta_i} \frac{q\left(\lambda,\vec{\theta}\right)}{\int_{\underline{\lambda}}^{L} q\left(\lambda',\vec{\theta}\right) d\lambda'} d\lambda.$$

But, since $\int_{\underline{\lambda}}^{L} \frac{q\left(\lambda,\vec{\theta}\right)}{\int_{\underline{\lambda}}^{L} q\left(\lambda',\vec{\theta}\right) d\lambda'} d\lambda = 1$, this follows from the assumption that the LHS is strictly increasing in λ. ■

Proof of Lemma 1. It suffices to show that for every x, y, t, t', the c.d.f. associated with the posterior beliefs $\pi_{t,t'}(\lambda|x, y+)$ are stochastically strictly decreasing in t and in t' and strictly increasing in x and in y. We prove all these results as corollaries of Theorem A.1. Let $\vec{\theta} = (x, y, t, t')$,

$$q\left(\lambda,\vec{\theta}\right) = \pi(\lambda) h(x|\lambda) [1 - H(y|\lambda)] [1 - F(t|\lambda)] \left[1 - F\left(t'|\lambda\right)\right],$$

$$\varphi\left(L,\vec{\theta}\right) = \int_{\underline{\lambda}}^{L} \pi_{t,t'}(\lambda|x, y+) d\lambda.$$

Since the expressions

$$\frac{\partial \log q\left(\lambda,\vec{\theta}\right)}{\partial t} = -\frac{f(t|\lambda)}{1 - F(t|\lambda)} = -\lambda, \quad \frac{\partial \log q\left(\lambda,\vec{\theta}\right)}{\partial t'} = -\frac{f(t'|\lambda)}{1 - F(t'|\lambda)} = -\lambda$$

$$\frac{\partial \log q\left(\lambda,\vec{\theta}\right)}{\partial x} = \frac{h'(x|\lambda)}{h(x|\lambda)} = \lambda, \quad \frac{\partial \log q\left(\lambda,\vec{\theta}\right)}{\partial y} = -\frac{h(y|\lambda)}{1 - H(y|\lambda)} = \frac{\lambda e^{\lambda x}}{1 - e^{\lambda x}}$$

are strictly increasing in λ, all monotonicity results follow from Theorem A.1. The result that $\lim_{t\to\infty} E_{t,t'}[\lambda| x, y+] = 0$ follows from:

$$\lim_{\tau\to\infty} \pi_{\tau,\tau}(\lambda > \varepsilon|x, y+) = \lim_{\tau\to\infty} \frac{\int_{\varepsilon}^{\infty} e^{-\alpha\lambda} \lambda^{\beta-1} \lambda e^{\lambda x} \left(1 - e^{\lambda y}\right) e^{-\lambda 2\tau} d\lambda}{\int_{\varepsilon}^{\infty} e^{-\alpha\lambda'} \lambda'^{\beta-1} \lambda' e^{\lambda' x} \left(1 - e^{\lambda' y}\right) e^{-\lambda 2\tau} d\lambda'} = 0. \blacksquare$$

Proof of Lemma 2. To show the first claim, we need to show that for any $L \in \left(\underline{\lambda}, \bar{\lambda}\right)$,

$$\begin{aligned} & \frac{\int_{\underline{\lambda}}^{L} \pi(\lambda) h(x|\lambda) [1 - H(y|\lambda)] [1 - F(t|\lambda)] [1 - F(t'|\lambda)] d\lambda}{\int_{\Lambda} \pi\left(\lambda'\right) h\left(x|\lambda'\right) \left[1 - H\left(y|\lambda'\right)\right] \left[1 - F\left(t|\lambda'\right)\right] \left[1 - F\left(t'|\lambda'\right)\right] d\lambda'} \\ < \; & \frac{\int_{\underline{\lambda}}^{L} \pi(\lambda) h(x|\lambda) h(y|\lambda) [1 - F(t|\lambda)] [1 - F(t'|\lambda)] d\lambda}{\int_{\Lambda} \pi\left(\lambda'\right) h\left(x|\lambda'\right) h\left(y|\lambda'\right) \left[1 - F\left(t|\lambda'\right)\right] \left[1 - F\left(t'|\lambda'\right)\right] d\lambda'}, \end{aligned}$$

this follows from

$$\begin{aligned} & \int_{\underline{\lambda}}^{L} \frac{h(y|\lambda)}{1 - H(y|\lambda)} \frac{\pi(\lambda) h(x|\lambda) [1 - H(y|\lambda)] [1 - F(t|\lambda)] [1 - F(t'|\lambda)]}{\int_{\underline{\lambda}}^{L} \pi(\lambda) h(x|\lambda) [1 - H(y|\lambda)] [1 - F(t|\lambda)] [1 - F(t'|\lambda)] d\lambda} d\lambda \\ > \; & \int_{\Lambda} \frac{h(y|\lambda)}{1 - H(y|\lambda)} \frac{\pi(\lambda) h(x|\lambda) [1 - H(y|\lambda)] [1 - F(t|\lambda)] [1 - F(t'|\lambda)]}{\int_{\Lambda} \pi(\lambda) h(x|\lambda) [1 - H(y|\lambda)] [1 - F(t|\lambda)] [1 - F(t'|\lambda)] d\lambda} d\lambda, \end{aligned}$$

where the inequality follows because

$$\frac{h(y|\lambda)}{1-H(y|\lambda)}=\frac{\lambda e^{\lambda y}}{1-e^{\lambda y}},\quad\text{and } \frac{\partial}{\partial\lambda}\left(\frac{\lambda e^{\lambda y}}{1-e^{\lambda y}}\right)=e^{\lambda y}\frac{\left(1-e^{\lambda y}+\lambda y\right)}{\left(1-e^{\lambda y}\right)^{2}}<0\text{ for any }\lambda\text{ and }y.$$

The proof of the second inequality is analogous. ■

Proof of Proposition 2. We consider the problem of player A who contemplates stopping second at time τ^A, player B's problem being symmetric. We compute

$$\begin{aligned}
& E\left[\lambda|\, z_A=x, z_B\leq y, t_A\geq t, t_B\geq t'\right] \\
= & \int_\Lambda \lambda\frac{\pi(\lambda)\,h(x|\lambda)\,H(y|\lambda)\,[1-F(t|\lambda)]\,[1-F(t'|\lambda)]}{\int_\Lambda \pi(\lambda')\,h(x|\lambda')\,H(y|\lambda')\,[1-F(t|\lambda')]\,[1-F(t'|\lambda')]\,d\lambda'}d\lambda \\
= & \int_\Lambda \frac{\lambda^{\beta+1}e^{-\lambda(\alpha+t+t'-x-y)}}{\Gamma(\beta+1)\,(\alpha+t+t'-x-y)^{-\beta-1}}d\lambda=\frac{\beta+1}{\alpha-x-y+t+t'}.
\end{aligned}$$

If the first-order condition has a positive solution, this is unique and a global maximum of the value function. Therefore, the desired stopping time is the maximum of 0 and the solution τ^A to $c=bE\left[\lambda|\, z_A=x, z_B\leq g(0), t_A\geq\tau^A, t_B\geq 0)\right]$. ■

Lemma A.1 *Suppose that player B plays a monotonic strategy σ_1 with inverse g. For any signal realization x observed by player A, there exists a time $\bar{\tau}>0$ large enough that player A's marginal value of waiting $V'_{1,t}(\tau_1|x)$ is negative for any $\tau_1\geq\bar{\tau}$. For any signal realization x such that $bE_{0,0}[\lambda|\,x,g(0)+]>c$, there exists $\underline{\tau}>0$ small enough that $V'_{1,t}(\tau_1|x)>0$ for any $t\leq\tau_1\leq\underline{\tau}$.*

Proof. From Proposition 1, we have

$$\sigma_2^*(x,\tau_1)=\max\{\tau_1,\frac{b}{c}(\beta+2)+x+g(\tau_1)-\alpha-\tau_1\}.$$

Since $g(\tau_1)\leq 0$, it follows that $\frac{b}{c}(\beta+2)+x+g(\tau_1)-\alpha-\tau_1\leq\frac{b}{c}(\beta+2)+x-\alpha-\tau_1$. For any x, there is $\bar{\tau}$ large enough such that for any $\tau_1\geq\bar{\tau}$, $\frac{b}{c}(\beta+2)+x-\alpha-\tau_1\leq\tau_1$; hence $\sigma_2^*(x,\tau_1)=\tau_1$, $V_{2,\tau_1}(\sigma_2^*(x,\tau_1)|x,g(\tau_1),\tau_1)=0$ and

$$\begin{aligned}
V'_{1,t}(\tau_1|x) \quad\propto\quad & bE_{\tau_1,\tau_1}[\lambda|\,x,g(\tau_1)+] \\
& +V_{2,\tau_1}(\sigma_2^*(x,\tau_1)|x,g(\tau_1),\tau_1)E_{\tau_1,\tau_1}\left[\left.\frac{h(g(\tau_1)|\lambda)g'(\tau_1)}{1-H(g(\tau_1)|\lambda)}\right|x,g(\tau_1)+\right]-c \\
= \quad & bE_{\tau_1,\tau_1}[\lambda|\,x,g(\tau_1)+]-c\leq bE_{\tau_1,\tau_1}[\lambda|\,x,0]-c=b\frac{\beta+2}{\alpha-x+2\tau_1}-c,
\end{aligned}$$

where the inequality follows by Lemma 2, and $[\beta+2]\,/\,[\alpha-x+2\tau_1]\to 0$ for $\tau_1\to 0$.

Since

$$\begin{aligned}
V'_{1,t}(\tau_1|x) \quad=\quad & bE_{\tau_1,\tau_1}[\lambda|\,x,g(\tau_1)+] \\
& +V_{2,\tau_1}(\sigma_2^*(x,\tau_1)|x,g(\tau_1),\tau_1)E_{\tau_1,\tau_1}\left[\left.\frac{h(g(\tau_1)|\lambda)g'(\tau_1)}{1-H(g(\tau_1)|\lambda)}\right|x,g(\tau_1)+\right]-c \\
\geq\quad & bE_{\tau_1,\tau_1}[\lambda|\,x,g(\tau_1)+]-c,
\end{aligned}$$

for any x such that $bE_{0,0}\left[\lambda|\, x, g\left(0\right)+\right] > c$, and any $t \leq \tau_1$ small enough, it must be that $V_{1,t}'\left(\tau_1|x\right) > 0$ by continuity of $V_{1,t}'$. ■

Lemma A.2 *Suppose that player B plays the first-quitter stopping strategy σ_1^*. If player A holds a signal $x \leq \underline{x}^*$, then at time 0 she optimally chooses not to enter the game. Whereas if $x > \underline{x}^*$, then player A enters the game at time 0, and optimally selects the stopping time $\sigma_1^*\left(x\right)$.*

Proof. Consider the choice at time $t = 0$ of player A with a signal x. If she chooses not to enter the game and set $\tau_1 = 0$, her payoff is $V_{1,0}\left(0|x\right) = 0$. If she chooses to enter the game, and set $\tau_1 > 0$, then she will observe whether B enters the game or not. This allows us to write A's expected payoff for playing any $\tau_1 > 0$ as:

$$V_{1,0}\left(\tau_1|x\right) = \Pr\left(z_B \leq \underline{x}|x\right) \lim_{t\downarrow 0} V_{2,t}(\sigma_2^*(x,0)|z_A = x, z_B \leq \underline{x}, t_B \geq 0) + \left[1 - \Pr\left(z_B \leq \underline{x}|x\right)\right] \lim_{t\downarrow 0} V_{1,t}\left(\tau_1|x\right)$$

Suppose first that x is such that $bE_{\tau_1,\tau_1}\left[\lambda|\, x, \underline{x}+\right] \leq c$; and hence that the equilibrium prescription is $\tau_{11}\left(x\right) = 0$. Since this implies that for any $\tau_1 \geq t > 0$, $V_{1,t}'\left(\tau_1|x\right) < 0$; it follows that for any $\tau_1 > 0$, $\lim_{t\downarrow 0} V_{1,t}'\left(\tau_1|x\right) < 0$. Since

$$b\frac{\beta + 1}{\alpha - x - \underline{x} + 2\tau_1} < b\frac{\beta + 2}{\alpha - x - \underline{x} + 2\tau_1} = bE_{\tau_1,\tau_1}\left[\lambda|\, x, \underline{x}\right] < bE_{\tau_1,\tau_1}\left[\lambda|\, x, \underline{x}+\right],$$

where the last inequality is by from Lemma 2, it follows that $\sigma_2^*(x,0) = 0$ by Proposition 2, and hence $\lim_{t\downarrow 0} V_{2,t}(\sigma_2^*(x,0)|z_A = x, z_B \leq \underline{x}, t_B \geq 0) = 0$. So the player optimally chooses to follow the equilibrium prescription $\sigma_1^*\left(x\right) = 0$. Second, suppose that x is such that $bE_{\tau_1,\tau_1}\left[\lambda|\, x, \underline{x}+\right] > c$; the player will comply with the equilibrium prescription $\sigma_1^*\left(x\right) > 0$ because $\lim_{t\downarrow 0} V_{2,t}(\sigma_2^*(x,0)|z_A = x, z_B \leq \underline{x}, t_B \geq 0) \geq 0$ and for any τ_1 small enough $\lim_{t\downarrow 0} V_{1,t}'\left(\tau_1|x\right) > 0$. ■

Proof of Lemma 5. Consider the time-0 team's expected costs for adopting stopping times $T_1 \leq T_2$ (such that $T_1 + T_2 = 2T^*$) and conditional on any λ:

$$\begin{aligned} C\left(T_1, T_2|\lambda\right) &= c\Bigg[\int_0^{T_1} 2f\left(s|\lambda\right)\left(1 - F\left(s|\lambda\right)\right)\int_0^s 2e^{-rv}dvds + \left(1 - F\left(T_1|\lambda\right)\right)^2\int_0^{T_1} 2e^{-rv}dv \\ &\quad + \left(1 - F\left(T_1|\lambda\right)\right)\int_{T_1}^{T_2} f\left(s|\lambda\right)\int_{T_1}^s e^{-rv}dvds + \left(1 - F\left(T_1|\lambda\right)\right)\left(1 - F\left(T_2|\lambda\right)\right)\int_{T_1}^{T_2} e^{-rv}dv\Bigg] \end{aligned} \tag{A.2}$$

$$\begin{aligned} &= c\Bigg[\int_0^{T_1} 2\lambda e^{-\lambda s}e^{-\lambda s}\int_0^s 2e^{-rv}dvds + e^{-2\lambda T_1}\left(\int_0^{T_1} 2e^{-rv}dv\right) \\ &\quad + e^{-\lambda T_1}\int_{T_1}^{T_2} \lambda e^{-\lambda s}\int_{T_1}^s e^{-rv}dvds + e^{-\lambda T_1}e^{-\lambda T_2}\int_{T_1}^{T_2} e^{-rv}dv\Bigg] \\ &= c\Bigg[-\frac{4\lambda}{\left(2\lambda + r\right)r}\left(1 - e^{-(2\lambda + r)T_1}\right) + \frac{2}{r}\left(e^{-2\lambda T_1} - e^{-(2\lambda + r)T_1}\right) + \frac{1}{r}\left(e^{-\lambda T_2 - (r + \lambda)T_1} - e^{-(\lambda + r)T_2 - \lambda T_1}\right) \\ &\quad + 2\frac{1 - e^{-2\lambda T_1}}{r} + \frac{1}{r}\left(e^{-\lambda T_1}\left(e^{-(\lambda + r)T_1} - e^{-\lambda T_2 - rT_1}\right) - e^{-\lambda T_1}\frac{\lambda}{\lambda + r}\left(e^{-(\lambda + r)T_1} - e^{-(\lambda + r)T_2}\right)\right)\Bigg] \end{aligned}$$

multiplying by r/c, simplifying terms and substituting $2T^* - T_1$ for T_2,

$$\frac{rC\left(T_1, 2T^* - T_1|\lambda\right)}{c} = \frac{2r}{2\lambda + r} - e^{-(2\lambda+r)T_1}\frac{r^2}{(\lambda+r)(2\lambda+r)} - \frac{r}{\lambda+r}e^{-(\lambda+r)2T^*+rT_1}$$

Taking a derivative with respect to T_1,

$$\frac{\partial C\left(T_1, 2T^* - T_1|\lambda\right)}{\partial T_1} \propto e^{-(2\lambda+r)T_1}\frac{r^2}{\lambda+r} - \frac{r^2}{\lambda+r}e^{-(\lambda+r)2T^*+rT_1} = e^{-(2\lambda+r)T_1}\frac{r^2}{\lambda+r}\left(1 - e^{-(\lambda+r)2(T^*-T_1)}\right).$$

This quantity is strictly positive if and only if $T_1 < T^*$, thereby implying that if $T_1 + T_2 = 2T^*$, then $C\left(T_1, T_2|\lambda\right)$ is maximized by setting $T_1 = T^* = T_2$.

If $y \geq x > \underline{x}$ and $\sigma_1^*(x) < T^*(x,y)$, then, since $\sigma_2^*\left(y, \sigma_1^*\left(x\right)\right) = 2T^*(x,y) - \sigma_1^*(x)$,

$$C\left(\sigma_1^*\left(x\right), \sigma_2^*\left(y, \sigma_1^*\left(x\right)\right)|\lambda\right) < C\left(T^*(x,y), T^*(x,y)|\lambda\right) \text{ for every } \lambda,$$

and hence

$$\int_\Lambda C\left(\sigma_1^*\left(x\right), \sigma_2^*\left(y, \sigma_1^*\left(x\right)\right)|\lambda\right)\pi_{0,0}(\lambda|x,y)d\lambda < \int_\Lambda C\left(T^*(x,y), T^*(x,y)|\lambda\right)\pi_{0,0}(\lambda|x,y)d\lambda.$$

This inequality, together with the inequality

$$W\left(\sigma_1^*\left(x\right), \sigma_2^*\left(y, \sigma_1^*\left(x\right)\right)|x,y\right) < W\left(T^*(x,y), T^*(x,y)|x,y\right)$$

implied by the team's optimal stopping times $T_1 = T_2 = T^*(x,y)$, shows that

$$B\left(\sigma_1^*\left(x\right), \sigma_2^*\left(y, \sigma_1^*\left(x\right)\right)|x,y\right) < B\left(T^*(x,y), T^*(x,y)|x,y\right). \blacksquare$$

Completion of Interim Welfare Analysis. To avoid triviality, we assume that $\alpha < (\beta + 2)\, b/c$; if not $T^*(x,y) = \max\{0, [(\beta+2)b/c + x + y - \alpha]/2\} = 0$ and $\sigma_1^*\left(x\right) = \sigma_2^*\left(x, \tau\right) = \sigma_1^*(y) = \sigma_2(y,\tau) = 0$ for any signals $(x,y) \in R_-^2$ and time $\tau \in R_+$: neither the team, nor any of the two players ever participate in the game. The subcases that remain to consider are as follows.

First, $\sigma_1^*\left(x\right) > T^*(x,y) = 0$, i.e. $x + y < \alpha - (\beta + 2)\,\frac{b}{c}$ and $y \geq x > \underline{x}^*$: both players enter the game whereas the team would not participate. This may happen with positive probability because $2\underline{x}^* < \alpha - (\beta + 2)\,\frac{b}{c}$.

Second, when player A fails to join the race, player B only learns that $x \leq \underline{x}^*$ but cannot precisely figure out x. Hence she quits at:

$$\sigma_2^*(y, 0) = \frac{b}{c}\left(\beta + 1\right) + y + \underline{x}^* - \alpha.$$

Player B stays in the game more than a negligible amount of time if and only if $y > \underline{x}^{**}$ where simple algebraic manipulations imply that $T^*\left(\underline{x}^*, \underline{x}^{**}\right) > 0$. Hence, when neither player stays in the race for more than an instant (i.e. $x \leq \underline{x}^*$, $y \leq \underline{x}^{**}$), there are two possibilities: if $T^*(x,y) = 0$, the outcome is trivially efficient because also the team would not participate, whereas if $T^*(x,y) > 0$

we have a strong but somewhat special instance of under-experimentation in which the team would pursue the project and the players give up immediately.

When $x \leq \underline{x}^*$ but $y > \underline{x}^{**}$, only player B joins, and remains for a significant amount of time in the race: Under-experimentation occurs if (i) $\sigma_2^*(y,0) < 2T^*(x,y)$, i.e. $x > \underline{x}^* - \frac{b}{c}$, using their respective expressions derived earlier. Due to imperfect equilibrium information transmission, when the most pessimistic player does not even enter the race both players end up stopping too soon in equilibrium. When (ii) $\sigma_2^*(y,0) > 2T^*(x,y)$, i.e. $x < \underline{x}^* - \frac{b}{c}$, and $T^*(x,y) > 0$, by the same reasoning as in Lemma 5, the equilibrium features under-experimentation if $\sigma_2^*(y,0)$ is close enough to $2T^*(x,y)$ (x is large enough, but still below $\underline{x}^* - \frac{b}{c}$) and equilibrium over-experimentation otherwise. When (iii) $T^*(x,y) = 0$, instead, the players should not enter the race in the first place, and a somewhat special instance of over-experimentation arises.

Proof of Proposition 6. For any signal pair (x,y), we let $z = x+y$, $\mathcal{H}(z)$ denote the unconditional distribution of z, and $\mathcal{H}(x|z)$ the conditional distribution of x given z. From the convolution formula, the density of z conditional on the arrival rate λ equals

$$\int_z^0 \lambda e^{\lambda(z-x)} \lambda e^{\lambda x} dx = \lambda^2 e^{\lambda z} \int_z^0 dx = -z\lambda^2 e^{\lambda z}$$

(recall $y \leq 0$, so $z \leq x$). The c.d.f. is then

$$\mathcal{H}(z) = \int_0^\infty -z\lambda^2 e^{\lambda z} \frac{\alpha^\beta}{\Gamma(\beta)} e^{-\alpha\lambda} \lambda^{\beta-1} d\lambda = -z \frac{\alpha^\beta}{(\alpha - z)^{\beta+2}} \beta(\beta+1).$$

Next, notice from their expressions in (resp.) (B.2) and (A.1) that both the team's optimal stopping time $T^*(x,y)$ and the posterior beliefs about the arrival rate after observing both signals x,y but before beginning the race $\pi_{0,0}(\lambda|x,y)$ depend on x,y only through their sum $z = x+y$. Therefore, with a slight abuse of notation we let $T^*(z) = T^*(x, z-x)$ for any $0 \geq x \geq z$, and $\pi_{0,0}(\lambda|z) = \pi_{0,0}(\lambda|x,y)$.

Without loss of generality normalize $c = 1$ and let $\mathcal{C}(x,z|\lambda) = C(\sigma_1^*(x), \sigma_2^*(z-x, \sigma_1^*(x))|\lambda)$ denote the total expected discounted cost of experimentation given λ and $x \geq z$ in equilibrium, and $\mathcal{C}^*(z|\lambda) = C(T^*(z), T^*(z)|\lambda)$ the analogous magnitude given λ and z when following the team's optimal policy. Conditioning on the sum of signals z, then on x and finally on λ, the difference in expected PDV of experimentation costs is:

$$D(\alpha,\beta;r) = \int_{-\infty}^0 \int_z^0 \int_\Lambda [\mathcal{C}(x,z|\lambda) - \mathcal{C}^*(z|\lambda)]\, d\Pi_{0,0}(\lambda|z)\, d\mathcal{H}(x|z)\, d\mathcal{H}(z).$$

Split this integral in two parts, one for $T^*(z) > 0$, i.e. $z > \alpha - (\beta+2)\frac{b}{c}$, and the residual. This second piece, conditional on the team not experimenting, is

$$\int_{-\infty}^{\alpha-(\beta+2)\frac{b}{c}} \int_z^0 \int_\Lambda [\mathcal{C}(x,z|\lambda) - \mathcal{C}^*(z|\lambda)]\, d\Pi_{0,0}(\lambda|z)\, d\mathcal{H}(x|z)\, d\mathcal{H}(z).$$

The probability of this second event equals

$$\Pr\left(z \leq \alpha - (\beta+2)\frac{b}{c}\right) = \min\left\langle \left[(\beta+2)\frac{b}{c} - \alpha\right] \frac{\alpha^{\beta}}{(\alpha - z)^{\beta+2}} \beta(\beta+1), 1 \right\rangle$$

This chance converges to zero both for $\alpha \to 0$ (by simple inspection) and for $\beta \to \infty$: in fact, $\alpha / (\alpha - z) < 1$ for any $z < 0$, and for large β the geometric term $\alpha^{\beta} / (\alpha - z)^{\beta+2} = \alpha^{-2} [\alpha / (\alpha - z)]^{\beta+2}$ vanishes faster than the exploding polynomial terms in β.

Therefore, to study the sign of $D(\alpha, \beta; r)$ as $\beta/\alpha \to \infty$ we can concentrate on the term conditional on the team experimenting.

$$\begin{aligned}
\mathcal{D}(\alpha, \beta, z; r) &\equiv \int_z^0 \int_\Lambda \left[\mathcal{C}(x, z|\lambda) - \mathcal{C}^*(z|\lambda)\right] d\mathcal{H}(x|z)\, d\Pi_{0,0}(\lambda|z) \\
&= 2\int_\Lambda -\left\{ \int_z^{\tilde{\xi}(z)} \left[\mathcal{C}^*(z|\lambda) - \mathcal{C}(x, z|\lambda)\right] d\mathcal{H}(x|z) + \int_{\tilde{\xi}(z)}^{z/2} \left[\mathcal{C}(x, z|\lambda) - \mathcal{C}^*(z|\lambda)\right] d\mathcal{H}(x|z) \right\} d\Pi_{0,0}(\lambda|z),
\end{aligned}$$

where we use the symmetry between x and $y = z - x$ (which makes both $\mathcal{C}(x, z|\lambda)$ and $d\mathcal{H}(x|z)$ symmetric in x and $z - x$), and calculate only the case that $x < y = z - x$ (which entails $x \leq z/2$); and where the function $\tilde{\xi} : (\alpha - (\beta+2)\frac{b}{c}, 0] \to (\alpha - (\beta+2)\frac{b}{c}, 0]$ is implicitly defined by $\sigma_1^*(\tilde{\xi}(z)) = T^*(z)$, so that $\sigma_1^*(x) > (<) T^*(z)$ if $x > (<) \tilde{\xi}(z)$ and $x \leq z/2$ and the first player certainly over- (under-) experiments (since $\xi(x) > x$, we know that $\tilde{\xi}(z) < z/2$). Conditional on λ and given discounting, we know from Lemma 5 that this implies that equilibrium features over- (under-) experimentation and $\mathcal{C}(x, z|\lambda) > (<) \mathcal{C}^*(z|\lambda)$. Hence both inner integrals are strictly positive, and their difference as written above has an ambiguous sign. The question we address now is which effect dominates on average.

Now, if $\tilde{\xi}(z) < x \leq z/2$, then $\sigma_1^*(x) > T^*(z)$. Because $\sigma_2^*(y, \sigma_1^*(x)) = \max\{\sigma_1^*(x), 2T^*(x, y) - \sigma_1^*(x)\}$, we can use the calculations (A.2) in the proof of Lemma 5 and, simplifying notation by omitting the dependence on x and z in $\sigma_1^*(x)$ and $T^*(z)$, obtain

$$\mathcal{C}(x, z|\lambda) - \mathcal{C}^*(z|\lambda) = \frac{2}{2\lambda + r}\left(e^{-(2\lambda+r)T^*} - e^{-(2\lambda+r)\sigma_1^*}\right).$$

If instead $x < \tilde{\xi}(z) \leq z/2$, then $\sigma_1^*(x) < T^*(z)$, and, again simplifying notation

$$\begin{aligned}
\mathcal{C}^*(z|\lambda) - \mathcal{C}(x, z|\lambda) &= \frac{2}{2\lambda + r}\left(e^{-(2\lambda+r)\sigma_1^*} - e^{-(2\lambda+r)T}\right) \\
&\quad - \frac{1}{r}\left[e^{-(2\lambda+r)\sigma_1^*} - \frac{\lambda}{\lambda + r}\left(e^{-2\lambda\sigma_1} - e^{-\lambda 2T^*}\right) - e^{-2\lambda T^* - r\left(2T^* - \sigma_1^*\right)}\right].
\end{aligned}$$

For any fixed z, and λ, pick an arbitrary pair of signals (x, u) such that $z \leq u < \tilde{\xi}(z) < x \leq z/2$. We study the ratio:

$$\begin{aligned}
\frac{\mathcal{C}^*(z|\lambda) - \mathcal{C}(x, u|\lambda)}{\mathcal{C}(x, z|\lambda) - \mathcal{C}^*(z|\lambda)} &= \frac{\frac{2}{2\lambda+r}\left(e^{-(2\lambda+r)\sigma_1^*(u)} - e^{-(2\lambda+r)T^*}\right)}{\frac{2}{2\lambda+r}\left(e^{-(2\lambda+r)T^*} - e^{-(2\lambda+r)\sigma_1^*(x)}\right)} \\
&\quad - \frac{\frac{1}{r}\left[e^{-(2\lambda+r)\sigma_1^*(u)} - e^{-2\lambda T^* - r\left(2T^* - \sigma_1^*(u)\right)} - \frac{\lambda}{\lambda+r}\left(e^{-2\lambda\sigma_1^*(u)} - e^{-\lambda 2T^*}\right)\right]}{\frac{2}{2\lambda+r}\left(e^{-(2\lambda+r)T^*} - e^{-(2\lambda+r)\sigma_1^*(x)}\right)}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{e^{-(2\lambda+r)\sigma_1^*(u)} - e^{-(2\lambda+r)T^*}}{e^{-(2\lambda+r)T^*} - e^{-(2\lambda+r)\sigma_1^*(x)}} - \frac{e^{-(2\lambda+r)\sigma_1^*(u)} - e^{-2\lambda T^* - r\left(2T^* - \sigma_1^*(u)\right)}}{\frac{2r}{2\lambda+r}\left(e^{-(2\lambda+r)T^*} - e^{-(2\lambda+r)\sigma_1^*(x)}\right)} + \frac{\frac{\lambda}{\lambda+r}\left(e^{-2\lambda\sigma_1^*(u)} - e^{-2\lambda T^*}\right)}{\frac{2r}{2\lambda+r}\left(e^{-(2\lambda+r)T^*} - e^{-(2\lambda+r)\sigma_1^*(x)}\right)} \\
&\geq \frac{e^{-(2\lambda+r)\sigma_1^*(u)} - e^{-(2\lambda+r)T^*}}{e^{-(2\lambda+r)T^*} - e^{-(2\lambda+r)\sigma_1^*(x)}} - \frac{e^{-(2\lambda+r)\sigma_1^*(u)} - e^{-2\lambda T^* - r\left(2T^* - \sigma_1^*(u)\right)}}{\frac{2r}{2\lambda+r}\left(e^{-(2\lambda+r)T^*} - e^{-(2\lambda+r)\sigma_1^*(x)}\right)}.
\end{aligned}
$$

Where the inequality follows because $\sigma_1^*(x) > T^* > \sigma_1^*(u)$.

As the social planner becomes more and more impatient:

$$
\begin{aligned}
&\lim_{r\to\infty}\left[\frac{e^{-(2\lambda+r)\sigma_1^*(u)} - e^{-(2\lambda+r)T^*}}{e^{-(2\lambda+r)T^*} - e^{-(2\lambda+r)\sigma_1^*(x)}} - \left(\frac{2\lambda+r}{2r}\right)\frac{e^{-2\lambda\sigma_1^*(u)}e^{-r\sigma_1^*(u)} - e^{-2\lambda T^*}e^{-r\left(2T^* - \sigma_1^*(u)\right)}}{\left(e^{-(2\lambda+r)T^*} - e^{-(2\lambda+r)\sigma_1^*(x)}\right)}\right] \\
=\;& \lim_{r\to\infty}\left[\frac{e^{-(2\lambda+r)\sigma_1^*(u)}}{e^{-(2\lambda+r)T^*}}\frac{1 - e^{-(2\lambda+r)(T^* - \sigma_1^*(u))} - \frac{2\lambda+r}{2r}\left(1 - e^{-2(\lambda+r)(T^* - \sigma_1^*(u))}\right)}{1 - e^{-(2\lambda+r)(\sigma_1^*(x) - T^*)}}\right] \\
=\;& \frac{1}{0^+}\frac{1 - 0 - \frac{1}{2}(1-0)}{1-0} = +\infty,
\end{aligned}
$$

because $\lim_{r\to\infty} e^{-(2\lambda+r)(T^* - \sigma_1^*(u))}$ as $T^* > \sigma_1^*(u)$.

Since the argument holds for any arbitrary $z > \alpha - (\beta+2)\frac{b}{c}$, any λ and any arbitrary pair of signals (x,u) such that $u < \tilde{\xi}(z) < x$, this concludes that, for any z,

$$\mathcal{D}(\alpha,\beta,z;r) < 0 \text{ for } r \text{ large enough.} \quad \blacksquare$$

Derivation of Equation (4.5).

$$
\begin{aligned}
\frac{dV_{1,t}(\tau_1|x)}{d\tau_1} =& \int_\Lambda \Bigg[\frac{f(\tau_1|\lambda)}{1-F(t|\lambda)}\frac{1-F(\tau_1|\lambda)}{1-F(t|\lambda)}\frac{1-H(g^*(\tau_1)|\lambda)}{1-H(g^*(t)|\lambda)}\left(\int_t^{\tau_1}(-2c)\,e^{-r(v-t)}dv + e^{-r(\tau_1-t)}b\right) \\
&+ \left(\frac{1-F(\tau_1|\lambda)}{1-F(t|\lambda)}\right)^2\frac{h(g^*(\tau_1)|\lambda)dg^*(\tau_1)/d\tau_1}{1-H(g^*(t)|\lambda)}\left[\int_t^{\tau_1}(-c)\,e^{-r(v-t)}dv + e^{-r(\tau_1-t)}V_{2,\tau_1}(\sigma_2^*(x,\tau_1)|x,g^*(\tau_1),\tau_1)\right] \\
&+ \frac{2(1-F(\tau_1|\lambda))f(\tau_1|\lambda)[1-H(g^*(\tau_1)|\lambda)] + [1-F(\tau_1|\lambda)]^2 h(g^*(\tau_1)|\lambda)dg^*(\tau_1)/d\tau_1}{[1-F(t|\lambda)]^2[1-H(g^*(t)|\lambda)]}\int_t^{\tau_1}ce^{-r(v-t)}dv \\
&+ \left(\frac{1-F(\tau_1|\lambda)}{1-F(t|\lambda)}\right)^2\frac{[1-H(g^*(\tau_1)|\lambda)]}{[1-H(g^*(t)|\lambda)]}(-c)\,e^{-r(\tau_1-t)}\Bigg]\pi_{t,t}(\lambda|x,g^*(t)+)\,d\lambda
\end{aligned}
$$

$$
\begin{aligned}
=\;& \int_\Lambda \Bigg[\frac{f(\tau_1|\lambda)}{1-F(t|\lambda)}\frac{1-F(\tau_1|\lambda)}{1-F(t|\lambda)}\frac{1-H(g^*(\tau_1)|\lambda)}{1-H(g^*(t)|\lambda)}e^{-r(\tau_1-t)}b \\
&+ \left(\frac{1-F(\tau_1|\lambda)}{1-F(t|\lambda)}\right)^2\frac{h(g^*(\tau_1)|\lambda)dg^*(\tau_1)/d\tau_1}{1-H(g^*(t)|\lambda)}e^{-r(\tau_1-t)}V_{2,\tau_1}(\tau_2^*(x,\tau_1)|x,g^*(\tau_1),\tau_1) \\
&+ \left(\frac{1-F(\tau_1|\lambda)}{1-F(t|\lambda)}\right)^2\frac{[1-H(g^*(\tau_1)|\lambda)]}{[1-H(g^*(t)|\lambda)]}(-c)\,e^{-r(\tau_1-t)}\Bigg]\pi_{t,t}(\lambda|x,g^*(t)+)\,d\lambda
\end{aligned}
$$

$$= \Sigma_{t,t}(\tau_1, x, g^*(\tau_1)+) \int_\Lambda \left\{ \frac{f(\tau_1|\lambda)}{1-F(\tau_1|\lambda)} b \right.$$
$$\left. + \frac{h(g^*(\tau_1)|\lambda) dg^*(\tau_1)/d\tau_1}{1-H(g^*(\tau_1)|\lambda)} V_{2,\tau_1}(\sigma_2^*(x,\tau_1)|x, g^*(\tau_1), \tau_1)) - c \right\} \pi_{\tau_1,\tau_1}(\lambda|x, g^*(\tau_1)+) d\lambda$$

$$\propto bE_{\tau_1,\tau_1}[\lambda|\, x, g^*(\tau_1)+] + V_{2,\tau_1}(\sigma_2^*(x,\tau_1)|x, g^*(\tau_1), \tau_1) E_{\tau_1,\tau_1}\left[\frac{h(g^*(\tau_1)|\lambda) dg^*(\tau_1)/d\tau_1}{1-H(g^*(\tau_1)|\lambda)} \middle| x, g^*(\tau_1)+ \right] - c$$

where we introduce a normalizing factor

$$\Sigma_{t,t}(\tau_1, x, g^*(\tau_1)+) \equiv e^{-r(\tau_1-t)} \frac{\int_\Lambda \pi(\lambda) h(x|\lambda) [1-H(g^*(\tau_1)|\lambda)] [1-F(\tau_1|\lambda)]^2 d\lambda}{\int_\Lambda \pi(\lambda) h(x|\lambda) [1-H(g^*(t)|\lambda)] [1-F(t|\lambda)]^2 d\lambda'} > 0.$$

B. Appendix. Analysis of the Team Problem.

In the first-best solution, players join forces in a single team, sharing four pieces of information: the two signal realizations x, y, that project A has not delivered a prize by time t and project B by time t'. In principle the team may choose to irreversibly stop projects in sequence. Hence, we study the problem by backward induction, in the same manner as in the equilibrium analysis. We first consider the case in which one of the two projects (project "1") has been irreversibly stopped at time $T_1 \geq 0$, and the other (project "2") is still ongoing at time $T_2 \geq T_1$. This will allow us later to solve backwards the problem where both projects are still ongoing. Project 1 may be indifferently either project A or B, as they have identical statistical properties.

We first determine the optimal stopping time of the second project, $T_{2,t}^*(x, y, T_1)$ for any signal pair x, y, current calendar time t, and time T_1 when the first project was stopped. Evidently, this problem is analogous to the equilibrium problem of a single player with signal x after her opponent left the race. Due to equilibrium monotonicity, the player is informed of her opponent's signal y. Because the team cannot reactivate the first project, it maximizes the same payoffs as the remaining player in the race. Thus the analysis is analogous to the analysis leading to Proposition 1.

At time t, conditional on a true value of λ, the expected value of planning to stop a single project at $T_2 \geq t$ equals

$$Q_{2,t}(T_2|\lambda) = \int_t^{T_2} \frac{f(s|\lambda)}{1-F(t|\lambda)} \left[\int_t^s (-c) e^{-r(v-t)} dv + e^{-r(s-t)} b \right] ds + \frac{1-F(T_2|\lambda)}{1-F(t|\lambda)} \int_t^{T_2} (-c) e^{-r(v-t)} dv$$

where the subscript "2" denotes the relevance of this value for the second project, the first project being already off line. The team, having stopped the first project at calendar time T_1, plans at a subsequent time $t \geq T_1$ to stop the second project at an even further time $T_{2,t}^*(x, y, T_1) \geq t$ such that

$$T_{2,t}^*(x, y, T_1) = \arg\max_{T_2 \geq t} \left\{ W_{2,t}(T_2|x, y, T_1) = \int_\Lambda Q_{2,t}(T_2|\lambda) \pi_{t,T_1}(\lambda|x, y) d\lambda \right\}. \quad \text{(B.1)}$$

To find the optimal T_2 we differentiate the expected value function (B.1). The same manipulations as in Equation (4.1) yield:

$$\frac{dW_{2,t}\left(T_2|x,y,T_1\right)}{dT_2} \propto bE_{T_2,T_1}\left[\lambda|\,x,y\right] - c = b\frac{\beta+2}{\alpha-x-y+T_1+T_2} - c.$$

This leads to the following result.

Lemma B.1. *For every pair of signals x, y, if the team has stopped the first project at time T_1, the optimal stopping time of the second project is*

$$T_2^*(x,y,T_1) = \max\{T_1, T_2(x,y,T_1) \equiv \frac{b}{c}(\beta+2)+x+y-\alpha-T_1\},$$

$$\text{where } T_1 \geq T_2(x,y,T_1) \text{ if and only if } T_1 \geq \frac{1}{2}\left[\frac{b}{c}(\beta+2)+x+y-\alpha\right] \equiv T^*(x,y). \tag{B.2}$$

As a result, if $T_1 \geq T^*(x,y)$, i.e. the team carries on both projects for a long enough time, then it must optimally stop them simultaneously: $T_2^*(x,y,T_1) = T_1$.

We may now calculate by backward induction the optimal stopping time of the first project $T_1^*(x,y)$. Specifically, we will show that the team's optimal policy always prescribes to stop both projects simultaneously, so that $T_1^*(x,y) = T^*(x,y) = T_2^*(x,y,T_1^*(x,y))$.

The team plans at time t a stopping time

$$T_{1,t}^*(x,y) = \arg\max_{T_1 \geq t}\left\{W_{1,t}(T_1|x,y) = \int_\Lambda Q_{1,t}\left(T_1|\lambda\right)\pi_{t,t}(\lambda|x,y)d\lambda\right\}$$

where

$$\begin{aligned} Q_{1,t}\left(T_1|\lambda\right) &= \int_t^{T_1}\frac{2f\left(s|\lambda\right)\left(1-F\left(s|\lambda\right)\right)}{\left(1-F\left(t|\lambda\right)\right)^2}\left[\int_t^s\left(-2c\right)e^{-r(v-t)}dv + e^{-r(s-t)}b\right]ds \\ &\quad + \frac{\left(1-F\left(T_1|\lambda\right)\right)^2}{\left(1-F\left(t|\lambda\right)\right)^2}\left[\int_t^{T_1}\left(-2c\right)e^{-r(v-t)}dv + e^{-r(T_1-t)}Q_{2,T_1}\left(T_2^*(x,y,T_1)|\lambda\right)\right] \end{aligned}$$

is the expected value of stopping at time T_1 conditional on a known value of λ. Here $2f\left(s|\lambda\right)\left(1-F\left(s|\lambda\right)\right)$ is the density of arrival of the prize to either one of the two projects, namely the derivative of the corresponding c.d.f. $1-\left(1-F\left(s|\lambda\right)\right)^2$, the first line collects payoffs in case the prize arrives while both projects run together (before T_1), and the second line the cost of running two projects fruitlessly until T_1 and then collecting the payoff of continuing optimally with one project from that moment forward.

To find $T_{1,t}^*(x,y)$, as for the second project we differentiate the value function $W_{1,t}$ with respect to T_1.

Lemma B.2. *For every pair of signals x, y,*

$$\frac{dW_{1,t}(T_1|x,y)}{dT_1} \propto -c + E_{T_1,T_1}[\lambda(b - Q_{2,T_1}(T_2^*(x,y,T_1)|\lambda))|x,y]. \tag{B.3}$$

Proof. First we expand the derivative as in the analysis determining $T_2^*(x,y,T_1)$:

$$\frac{dW_{1,t}(T_1|x,y)}{dT_1} = \int_\Lambda \frac{d}{dT_1} Q_t\left(T_1|\lambda\right)\pi_{t,t}(\lambda|x,y)d\lambda$$

$$\begin{aligned}
= & \int_\Lambda \left\{ \frac{2f\left(T_1|\lambda\right)\left(1-F\left(T_1|\lambda\right)\right)}{\left(1-F\left(t|\lambda\right)\right)^2}\left[\int_t^{T_1}(-2c)\,e^{-r(v-t)}dv + e^{-r(T_1-t)}b\right]\right. \\
& -\frac{2f\left(T_1|\lambda\right)\left(1-F\left(T_1|\lambda\right)\right)}{\left(1-F\left(t|\lambda\right)\right)^2}\left[\int_t^{T_1}(-2c)\,e^{-r(v-t)}dv + e^{-r(T_1-t)}W_{2,T_1}\left(T_2^*(x,y,T_1)|x,y,T_1\right)\right] \\
& \left. +\frac{\left(1-F\left(T_1|\lambda\right)\right)^2}{\left(1-F\left(t|\lambda\right)\right)^2}\left[(-2c)\,e^{-r(T_1-t)} + \frac{d}{dT_1}\left(e^{-r(T_1-t)}W_{2,T_1}\left(T_2^*(x,y,T_1)|x,y,T_1\right)\right)\right]\right\}\pi_{t,t}(\lambda|x,y)d\lambda
\end{aligned}$$

simplifying the term in $2c$ and collecting the discounting term

$$\begin{aligned}
= \; & e^{-r(T_1-t)}\int_\Lambda \left\{ \frac{2f\left(T_1|\lambda\right)\left(1-F\left(T_1|\lambda\right)\right)}{\left(1-F\left(t|\lambda\right)\right)^2}\left[b - W_{2,T_1}\left(T_2^*(x,y,T_1)|x,y,T_1\right)\right] + \frac{\left(1-F\left(T_1|\lambda\right)\right)^2}{\left(1-F\left(t|\lambda\right)\right)^2}\cdot\right. \\
& \left.\cdot\left[-2c - rW_{2,T_1}\left(T_2^*(x,y,T_1)|x,y,T_1\right) + \frac{d}{dT_1}W_{2,T_1}\left(T_2^*(x,y,T_1)|x,y,T_1\right)\right]\right\}\pi_{t,t}(\lambda|x,y)d\lambda
\end{aligned}$$

using the definitions of $\pi_{t,t}(\lambda|x,y)$, $\pi_{T_1,T_1}(\lambda|x,y)$

$$\frac{d}{dT_1}W_{2,T_1}\left(T_2^*(x,y,T_1)|x,y,T_1\right) \propto \int_\Lambda \left[\frac{f\left(T_1|\lambda\right)}{\left(1-F\left(T_1|\lambda\right)\right)}\left(b - W_{2,T_1}\left(T_2^*(x,y,T_1)|x,y,T_1\right)\right)\right]\pi_{T_1,T_1}(\lambda|x,y)$$

$$+\int_\Lambda \left[-\frac{r}{2}W_{2,T_1}\left(T_2^*(x,y,T_1)|x,y,T_1\right) + \frac{1}{2}\frac{d}{dT_1}W_{2,T_1}\left(T_2^*(x,y,T_1)|x,y,T_1\right) - c\right]\pi_{T_1,T_1}(\lambda|x,y)d\lambda$$

$$\propto bE_{T_1,T_1}\left[\lambda|\,x,y\right] - c - W_{2,T_1}\left(T_2^*(x,y,T_1)|x,y,T_1\right)\left(\frac{r}{2}+E_{T_1,T_1}\left[\lambda|\,x,y\right]\right) + \frac{1}{2}\frac{d}{dT_1}W_{2,T_1}\left(T_2^*(x,y,T_1)|x,y,T_1\right). \tag{$*$}$$

Using the expression for $W_{2,T_1}\left(T_2^*(x,y,T_1)|x,y,T_1\right)$, and the envelope theorem to ignore the effect of T_1 on this value through $T_2^*(x,y,T_1)$,

$$\frac{d}{dT_1}W_{2,T_1}\left(T_2^*(x,y,T_1)|x,y,T_1\right) = \frac{d}{dT_1}E_{T_1,T_1}\left[Q_{2,T_1}\left(T_2^*(x,y,T_1)|\lambda\right)|x,y\right]$$

$$\begin{aligned}
= \; & \frac{d}{dT_1}\int_\Lambda \left[\int_{T_1}^{T_2^*(x,y,T_1)}\frac{f\left(s|\lambda\right)}{1-F\left(T_1|\lambda\right)}\left[\int_{T_1}^{s}(-c)\,e^{-r(v-T_1)}dv + e^{-r(s-T_1)}b\right]ds\right. \\
& \left. + \frac{1-F\left(T_2^*(x,y,T_1)|\lambda\right)}{1-F\left(T_1|\lambda\right)}\int_{T_1}^{T_2^*(x,y,T_1)}(-c)\,e^{-r(v-T_1)}dv\right]\pi_{T_1,T_1}(\lambda|x,y)d\lambda
\end{aligned}$$

$$
\begin{aligned}
= \; & E_{T_1,T_1}\left[-\lambda b+\frac{1-F\left(T_2^*(x,y,T_1)|\lambda\right)}{1-F\left(T_1|\lambda\right)}c+c\int_{T_1}^{T_2^*(x,y,T_1)}\frac{f\left(s|\lambda\right)}{1-F\left(T_1|\lambda\right)}|x,y\right] \\
& +rE_{T_1,T_1}\left[\int_{T_1}^{T_2^*(x,y,T_1)}\frac{f\left(s|\lambda\right)}{1-F\left(T_1|\lambda\right)}\left[\int_{T_1}^{s}\left(-c\right)e^{-r(v-T_1)}dv+e^{-r(s-T_1)}b\right]ds|x,y\right] \\
& +rE_{T_1,T_1}\left[-c\frac{1-F\left(T_2^*(x,y,T_1)|\lambda\right)}{1-F\left(T_1|\lambda\right)}\int_{T_1}^{T_2^*(x,y,T_1)}e^{-r(v-T_1)}dv|x,y\right] \\
& +E_{T_1,T_1}\left[\lambda Q_{2,T_1}\left(T_2^*(x,y,T_1)|\lambda\right)|x,y\right]+\int_{\Lambda}Q_{2,T_1}\left(T_2^*(x,y,T_1)|\lambda\right)\frac{d\pi_{T_1,T_1}(\lambda|x,y)}{dT_1}d\lambda
\end{aligned}
$$

$$
\begin{aligned}
= \; & E_{T_1,T_1}\left[-\lambda b+c|x,y\right]+rW_{2,T_1}\left(T_2^*(x,y,T_1)|x,y,T_1\right) \\
& +E_{T_1,T_1}\left[\lambda Q_{2,T_1}\left(T_2^*(x,y,T_1)|\lambda\right)|x,y\right]+\int_{\Lambda}Q_{2,T_1}\left(T_2^*(x,y,T_1)|\lambda\right)\frac{d\pi_{T_1,T_1}(\lambda|x,y)}{dT_1}d\lambda
\end{aligned}
$$

Replacing this expression for $dW_{2,T_1}\left(T_2^*(x,y,T_1)|x,y,T_1\right)/dT_1$ into $(*)$ and collecting terms

$$
\begin{aligned}
& \frac{dW_{1,t}(T_1|x,y)}{dT_1}\propto\frac{1}{2}\left\{bE_{T_1,T_1}\left[\lambda|\,x,y\right]-c\right\}-E_{T_1,T_1}\left[\lambda|\,x,y\right]E_{T_1,T_1}\left[Q_{2,T_1}\left(T_2^*(x,y,T_1)|\lambda\right)|x,y\right] \\
& +\frac{1}{2}E_{T_1,T_1}\left[\lambda Q_{2,T_1}\left(T_2^*(x,y,T_1)|\lambda\right)|x,y\right]+\frac{1}{2}\int_{\Lambda}Q_{2,T_1}\left(T_2^*(x,y,T_1)|\lambda\right)\frac{d\pi_{T_1,T_1}(\lambda|x,y)}{dT_1}d\lambda.**
\end{aligned}
$$

(B.4)

Using the expression for $\pi_{T_1,T_1}(\lambda|x,y)$ and after some manipulations

$$
\frac{d\pi_{T_1,T_1}(\lambda|x,y)}{dT_1}=-2\lambda\pi_{T_1,T_1}(\lambda|x,y)+2\pi_{T_1,T_1}(\lambda|x,y)E_{T_1,T_1}\left[\lambda|x,y\right]
$$

and replacing into (B.4)

$$
\begin{aligned}
\frac{dW_{1,t}(T_1|x,y)}{dT_1} \propto \; & \frac{1}{2}\left\{bE_{T_1,T_1}\left[\lambda|\,x,y\right]-c\right\}-E_{T_1,T_1}\left[\lambda|\,x,y\right]E_{T_1,T_1}\left[Q_{2,T_1}\left(T_2^*(x,y,T_1)|\lambda\right)|x,y\right] \\
& +\frac{1}{2}E_{T_1,T_1}\left[\lambda Q_{2,T_1}\left(T_2^*(x,y,T_1)|\lambda\right)|x,y\right]-E_{T_1,T_1}\left[\lambda Q_{2,T_1}\left(T_2^*(x,y,T_1)|\lambda\right)|x,y\right] \\
& +E_{T_1,T_1}\left[\lambda|\,x,y\right]E_{T_1,T_1}\left[Q_{2,T_1}\left(T_2^*(x,y,T_1)|\lambda\right)|x,y\right] \\
\\
\propto \; & \frac{1}{2}\left\{bE_{T_1,T_1}\left[\lambda|\,x,y\right]-c-E_{T_1,T_1}\left[\lambda Q_{2,T_1}\left(T_2^*(x,y,T_1)|\lambda\right)|x,y\right]\right\} \\
\propto \; & -c+E_{T_1,T_1}\left[\lambda\left(b-Q_{2,T_1}(T_2^*(x,y,T_1)|\lambda)\right)|x,y\right]. \blacksquare
\end{aligned}
$$

Intuitively, by delaying the stopping time T_1 of the first project an extra instant, the team pays the flow cost c and receives the following expected marginal benefit: at hazard rate λ, the prize arrives and the benefit b is obtained, but on the other hand the continuation value

$Q_{2,T_1}(T_2^*(x,y,T_1)|\lambda)$ of proceeding with only one project is lost.[23] If both projects are stopped simultaneously, namely if T_1 is such that $T_2^*(x,y,T_1) = T_1$, then clearly the continuation value of the second project alone is zero: $Q_{2,T_1}(T_2^*(x,y,T_1)|\lambda) = 0$. In this case the familiar expression $dW_{1,t}(T_1|x,y)/dT_1 \propto (bE_{T_1,T_1}[\lambda|\, x,y] - c)$ obtains.

Next, we show that the optimal stopping time $T_1^*(x,y)$ of the first project cannot exceed the magnitude $T^*(x,y)$ defined in (B.2). In fact, proceed by contradiction. By Lemma B.1, if $T_1 > T^*(x,y)$, then $T_2^*(x,y,T_1) = T_1$, i.e. the team stops the second project at the same time as the first one, so the continuation value is zero for every value of λ: $Q_{2,T_1}(T_2^*(x,y,T_1)|\lambda) = 0$. Thus

$$\frac{dW_{1,t}(T_1|x,y)}{dT_1} \propto -c + bE_{T_1,T_1}[\lambda|x,y] < -c + bE_{T^*(x,y),T^*(x,y)}[\lambda|x,y] = 0$$

where the first proportionality follows from (B.3), the inequality from the assumption $T_1 > T^*(x,y)$ and the monotonicity of the expected hazard rate, and the last equality from the definition of $T^*(x,y)$ as the solution to the first-order condition for the team. But then $T_1 = T_1^*(x,y)$ violates a necessary first-order condition and cannot be optimal.

If parameters are such that $T^*(x,y) = 0$, then using $0 \le T_1^*(x,y) \le T^*(x,y) = 0$ we get $T_2^*(x,y,T_1^*(x,y)) = T_1^*(x,y) = 0$ and no project is ever started. But, in the case that $T^*(x,y) > 0$, the key question is still whether it is best for the team to stop the two projects simultaneously at time $T^*(x,y)$, or to stop them sequentially, so that $T_1^*(x,y) < T_2^*(x,y,T_1^*(x,y))$. By Lemma B.1 sequential stopping requires that $T_1^*(x,y) < T^*(x,y)$; the following Lemma shows that this inequality is in fact impossible.

Lemma B.3. *For every pair of signals x,y such that $T^*(x,y) > 0$, the optimal stopping time $T_1^*(x,y)$ of the first project cannot be strictly smaller than $T^*(x,y)$.*

Proof. First we calculate

$$\begin{aligned} Q_{2,t}(T|\lambda) &= \int_t^T \frac{f(s|\lambda)}{1-F(t|\lambda)}\left[\int_t^s -ce^{-r(v-t)}dv + e^{-r(s-t)}b\right]ds + \frac{1-F(T|\lambda)}{1-F(t|\lambda)}\int_t^T -ce^{-r(v-t)}dv. \\ &= \int_t^T \lambda e^{-\lambda(s-t)}\left[\int_t^s -ce^{-r(v-t)}dv + e^{-r(s-t)}b\right]ds - e^{-\lambda(T-t)}\int_t^T -ce^{-r(v-t)}dv \\ &= (\lambda b - c)\frac{1-e^{-(\lambda+r)(T-t)}}{\lambda+r}. \end{aligned}$$

Using this expression in (B.3), for every $T_1 \in (t, T^*(x,y)]$ we obtain:

$$\frac{dW_{1,t}(T_1|x,y)}{dT_1} \propto -c + E_{T_1,T_1}\left[\lambda\left(b - (\lambda b - c)\frac{1-e^{-(\lambda+r)\left(T_2^*(x,y,T_1)-T_1\right)}}{\lambda+r}\right)|x,y\right]$$

[23] Notice that $U_{2,T_1}(T_2^*(x,y,T_1)|\lambda) < b$, as in the continuation the impatient team can earn at most b, and not immediately a.s. Furthermore, the hazard rate λ of the prize and the continuation value conditional on λ, namely $U_{2,T_1}(T_2^*(x,y,T_1)|\lambda)$, are multiplied *within* the posterior expectation: the team cares about their covariance induced by the common dependence on λ.

using $c = E_{T_1,T_1}[c|x,y]$, some algebra and $T^*(x,y) > 0$,

$$
\begin{aligned}
&= E_{T_1,T_1}\left[(\lambda b - c)\frac{r + \lambda e^{-(\lambda+r)\left(T_2^*(x,y,T_1)-T_1\right)}}{\lambda + r}|x,y\right] \\
&= \int_\Lambda (\lambda b - c)\frac{r + \lambda e^{-(\lambda+r)\frac{b}{c}(\beta+2)+x+y-\alpha-2T_1}}{r+\lambda}\frac{e^{-\lambda(\alpha-x-y+2T_1)}\lambda^{\beta+1}}{\Gamma(\beta+2)(\alpha-x-y+2T_1)^{-\beta-2}}d\lambda \\
&\propto S(T_1) \equiv \int_\Lambda \left(\lambda\frac{b}{c} - 1\right)\frac{r + \lambda e^{-(\lambda+r)\left(\frac{b}{c}(\beta+2)+x+y-\alpha-2T_1\right)}}{\lambda+r}e^{-\lambda(\alpha-x-y+2T_1)}\lambda^{\beta+1}d\lambda,
\end{aligned}
$$

where note that $S(T^*(x,y)) = 0$ because if $T_1 = T^*(x,y)$ then $T_2^*(x,y,T_1) - T_1 = 0$.

Differentiating $S(T_1)$ we obtain:

$$
\begin{aligned}
S'(T_1) &= \frac{d}{dT_1}\left[\int_\Lambda \left(\lambda\frac{b}{c} - 1\right)\frac{r + \lambda e^{-(\lambda+r)\left(\frac{b}{c}(\beta+2)+x+y-\alpha-2T_1\right)}}{\lambda+r}e^{-\lambda(\alpha-x-y+2T_1)}\lambda^{\beta+1}d\lambda\right] \\
&= \int_\Lambda \left(\lambda\frac{b}{c} - 1\right)\frac{-2r\lambda + 2r\lambda e^{-(\lambda+r)\left(\frac{b}{c}(\beta+2)+x+y-\alpha-2T_1\right)}}{\lambda+r}e^{-\lambda(\alpha-x-y+2T_1)}\lambda^{\beta+1}d\lambda \\
&= 2r\int_\Lambda \left(\lambda\frac{b}{c} - 1\right)\frac{r + \lambda e^{-(\lambda+r)\left(\frac{b}{c}(\beta+2)+x+y-\alpha-2T_1\right)}}{\lambda+r}e^{-\lambda(\alpha-x-y+2T_1)}\lambda^{\beta+1}d\lambda \\
&\quad +2r\int_\Lambda \left(\lambda\frac{b}{c} - 1\right)\frac{-\lambda - r}{\lambda+r}e^{-\lambda(\alpha-x-y+2T_1)}\lambda^{\beta+1}d\lambda \\
&= 2rS(T_1) - 2r\int_\Lambda \left(\lambda\frac{b}{c} - 1\right)e^{-\lambda(\alpha-x-y+2T_1)}\lambda^{\beta+1}d\lambda \\
&= 2rS(T_1) - 2r\frac{\Gamma(\beta+2)}{(\alpha-x-y+2T_1)^{\beta+2}}\left[\frac{(\beta+2)}{\alpha-x-y+2T_1}\frac{b}{c} - 1\right].
\end{aligned}
$$

So, finally we obtain the differential equation

$$
S'(T_1) = 2rS(T_1) - 2r\frac{\Gamma(\beta+2)}{(\alpha-x-y+2T_1)^{\beta+3}}2(T^*(x,y) - T_1) \tag{B.5}
$$

where clearly $(\alpha - x - y + 2T_1)^{\beta+3} > 0$. We use (B.5) to prove the claim. First, we exclude an optimal interior (positive) $T_1^*(x,y) \in (0, T^*(x,y))$. By contradiction: if there is an optimal interior $T_1 = T_1^*(x,y) \in (0, T^*(x,y))$, it must satisfy the NFOC

$$
\frac{dW_{1,t}(T_1|x,y)}{dT_1} = 0 \Rightarrow S(T_1) = 0.
$$

But then, $S(T_1^*(x,y)) = 0$ and $T_1^*(x,y) < T^*(x,y)$ in (B.5) together imply $S'(T_1^*(x,y)) < 0$. By continuity there exists $\varepsilon > 0$ such that $S(T_1^*(x,y) + \varepsilon) < 0$. Then by smoothness of S (many times differentiable) and $S(T^*(x,y)) = 0$ there exists a $T_1' \in (T_1^*(x,y), T^*(x,y))$ such that $S'(T_1') = 0 > S(T_1')$; but from (B.5) $T_1' < T^*(x,y)$ and $S(T_1') < 0$ imply $S'(T_1') < 0$ a contradiction. Second, we exclude a corner solution at $T_1 = T_1^*(x,y) = 0$. For this, we would require

$$
\frac{dW_{1,t}(0|x,y)}{dT_1} \leq 0 \Rightarrow S(0) \leq 0
$$

which, together with $T^*(x,y) > 0 = T_1^*(x,y)$ and (B.5), imply $S'(0) < 0$. But then the function $S(t)$ keeps declining at increasing rate from the initial value $S(0) \leq 0$ (formally the differential equation for S is exploding downward) as t rises from 0, contradicting $S(T^*(x,y)) = 0$. ■

This completes the analysis of the team problem. The results are summarized in the following Proposition.

Proposition B.1 (The Team Solution). *For every pair of signals x, y on the unobserved promise λ of the two projects, the team optimally stops both projects simultaneously at time*

$$T_1^*(x,y) = T_2^*(x,y,T_1^*(x,y)) = \max\{0, \frac{1}{2}[(\beta+2)\frac{b}{c} + x + y - \alpha]\}.$$

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