MORALE HAZARD

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Abstract

We interpret workers' confidence in their own skills as their morale, and investigate the implication of worker overconfidence on the firm's optimal wage-setting policies. In our model, wage contracts both provide incentives and affect worker morale, by revealing private information of the firm about worker skills. We provide conditions for the non-differentiation wage policy to be profit-maximizing. In numerical examples, worker overconfidence is a necessary condition for the firm to prefer no wage differentiation, so as to preserve some workers' morale; the non-differentiation wage policy itself breeds more worker overconfidence; finally, wage compression is more likely when aggregate productivity is low.

Keywords: Overconfidence, Worker Morale, Wage-setting Policies

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1 Introduction

Most of corporate America abstain from wage differentiation among workers assigned to similar tasks. At the individual firm level, Baker, Gibbs and Holmstrom (1994) studied the wage policy of a large firm and found that individual workers' wages are largely determined by their cohorts (the year of entry into the firm) and their job levels. At the aggregate level, there is substantial evidence of wage compression in the sense that the wage distribution is less dispersed than the underlying distribution of productivity.[1]

In a pioneering survey of wage-setting practices of over three hundred business executives and personnel managers, Bewley (1999) found that wage differentiation among workers assigned to similar tasks are especially avoided when selective wage cuts are involved. Moreover, an overwhelming number of firms believed that selective wage cuts would hurt *worker morale*, so they would lay workers off rather than offer them a lower wage.[2] Since wage differentiation is likely to *boost* the morale of some workers while hurting the morale of others, these executives must mean that the benefit from morale boost to some workers is outweighed by the cost from morale loss to others.

Worker morale is a buzz word in the business world.[3] But what exactly is worker morale? How is morale affected by wage-setting practices, and vice versa? Can firms' concern for worker morale help explain wage compression? How does morale affect incentives? What determines whether a firm should adopt a differentiation or a non-differentiation wage policy? How do aggregate productivity fluctuations affect the benefits and costs of wage differentiation? These are the questions that we address in this paper.

In Bewley's (1999) survey, respondents have many different views of morale. Some emphasize its collective nature. For example, the owner of a small manufacturing company responded that "Morale is having employees feel good about working for the company and respecting it. The employee with good morale likes his work ...". Others have a more individualistic orientation. For example, a general manager of a large company said that "Morale equals motivation." Bewley (1999) summarizes that morale meant "emotional attitudes toward work, co-workers, and the organization. Good morale meant a sense of common purpose consistent with company goals and meant cooperativeness, happiness or tolerance of unpleasantness, and zest for the job." In this paper, we adopt an individualistic approach and interpret a worker's morale as her confidence in

[1]It is well documented that the rate of unemployment is lower among older, more experienced, more educated, and in general, more productive workers. This means that more productive workers are more employable from the firms' viewpoint. As argued in Moscarini (1996), a natural implication of this fact is wage compression, defined as a lower inequality in wages than in individual skills.

[2]Not all firms oppose wage differentiation in the workplace. Former GE Chairman and CEO Jack Welch (2001), for example, believed that strong workforces are built by treating individuals differently: "Some contend that differentiation is nuts – bad for morale. ... Not in my world." (See, however, our footnotes 11 and 15.)

[3]A search using keyword "morale" in Lexis-Nexis Academic Business News Database retrieves more than 1000 documents in the previous six months.

her own ability. This is consistent with some of the above responses but a little narrower, and we believe that it is a relevant view: in our model, a worker with high confidence of her own ability believes that she can "make a difference" (in increasing output and obtaining a high bonus) by exerting effort; thus, in our model, a worker's morale is an intrinsic motivation. A worker has "high morale" when she thinks that her effort has a large impact on output; and conversely, a worker is demoralized when she believes that her costly effort is basically useless.

In our model, a principal (i.e. the firm) hires many agents (i.e. the workers) to produce output. Each worker's output depends on her own effort and ability but not on those of other workers. As is standard, effort is not contractible. The ability of each worker is uncertain. The principal privately observes a performance evaluation of each worker, which is informative about her ability; and workers observe each other's received contract offers. Our first innovation is to consider the effects of relative wage comparisons by workers on the perception that they have about their own skills. Incentive contracts play a signaling besides their traditional allocative role, and affect worker incentives through both channels. The firm can either condition its wage offers on performance evaluations (*differentiation* policy), or conceal its opinion about workers' abilities by offering the same contract to all employees (*non-differentiation* policy).

Any wage differentiation will have two effects. The first is a *sorting effect*, which is beneficial to the firm by allowing it to tailor incentive contracts to each worker's ability. The second is a *morale effect*, which is a double-edged sword: on the one hand, wage differentiation breaks bad news to some workers and depresses their morale; on the other hand, it also breaks good news to other workers and boosts their morale. If effort and ability are complements in production, then workers who suffer a loss in morale are discouraged from exerting effort and hurt the firm's profits; we call this the *negative morale effect*. The other workers are further encouraged and work harder, producing a *positive morale effect* on firm profits. If instead effort and ability are substitutes, negative and positive morale effects switch places: a loss in morale induces the affected workers to try even harder, to compensate the lack of ability; while those who gain morale now believe their natural talent to be sufficient for a good performance at lower effort, and hurt the firm's profits. Either way, one of the two groups of workers optimally reduce effort simply because of the information they acquire. This negative morale effect of wage differentiation is what we term the *Morale Hazard*. The wage-setting policy in our model essentially is an instrument of the firm to manipulate the workers' self-confidence. The difference between wage and cheap-talk methods as means of confidence management is that the former is the only credible instrument since it is costly for the firm to break good news by raising compensations.

Our second innovation is to allow firm and workers to hold different prior beliefs regarding the workers' ability, and this initial disagreement is common knowledge. This assumption is motivated by the findings in psychological research (reviewed in the next section) that people tend to be overconfident about their own possession of any desirable traits, in particular, one's own ability. When workers are initially sufficiently more confident about their ability than the firm, the average morale of the workforce falls when information is revealed by differentiated wage contracts because, in such circumstances, "on average the truth is bad news". While the firm faces a trade-off irrespective of complementarity or substitutability between effort and ability, the case of complementarity interacts interestingly with overconfidence: a loss in morale is followed by an average decline in effort and profits, which may more than compensate the positive morale effect and the sorting effect, thus making wage differentiation undesirable. In numerical examples assuming complementarity, wage differentiation dominates no differentiation when the firm and workers share identical initial beliefs; but as the workers become more overconfident, the non-differentiation policy eventually dominates. We also find a sense in which "overconfidence begets overconfidence". The more overconfident the workforce, and the lower its true average ability, the more likely the firm to offer a pooling contract and to conceal its private information; a majority of workers are spared the bad news and become even more overconfident. Finally, we find that wage compression is more likely when aggregate productivity is low (relative to workers' outside options.)

In our model, wage differentiation among retained employees hurts the morale of some workers, thus a firm may find it too costly to keep such demoralized workers and would rather lay them off altogether. This provides an alternative explanation to the puzzle of "involuntary layoffs": laid-off workers in many cases believe that their relationship with the firm could still generate positive joint surplus (relative to the sum of their outside options) and question why the firm would choose not to keep them at a reduced wage. The standard answer rests on some form of non-transferable utility. Our answer is different: the workers simply over-estimate the surplus from the relationship because of their overconfidence.

The remainder of the paper is structured as follows. Section 2 provides a brief review of the related economic literature on wage compression, and of the large psychological literature underlying our behavioral assumption that workers are overconfident of their own ability or skills; Section 3 presents the model; Section 4 characterizes the optimal wage contract for any belief pair of the worker and the firm, and the profit-maximizing set of contracts offered to the workforce; Section 5 provides examples to illustrate when the non-differentiation wage-setting policy is optimal for the firm; Section 6 presents some discussion of the model and results, as well as some testable implications; Section 7 concludes.

2 Related Literature

2.1 Wage Compression

The economics literature has proposed several explanations for wage compression, ranging from incentives not to sabotage colleagues competing in a tournament (Lazear 1989) to differences in labor supply elasticities as reflected in wages by bargaining in a frictional labor market (Moscarini 1996). In our model, the effect of wage differentiation on worker morale arises because of *relative* wage comparisons. Thus the non-differentiation wage policy is related to the concept of "fairness." Indeed, unfairness by the employer is likely to impact on production through, if any, the resulting loss in worker morale. These ideas are appealing and intuitive from simple introspection, and they form the basis of the prominent efficiency wage theory. Solow (1979), and Akerlof and Yellen (1990) have pioneered the theoretical work on the effects of fairness considerations in wage-setting. Inspired by equity theory in social psychology and social exchange theory in sociology, and supported by ample field evidence, they postulated that worker effort depends not on the offered wage *per se*, but on its divergence from a "fair" reference wage, which depends on what other comparable workers earn. "[..] when people do not get what they deserve, they try to get even." (Akerlof and Yellen 1990)

Instead of assuming it, our model rationalizes the "fair-wage/effort" behavioral hypothesis by showing that, when workers suffer from the often observed human judgement bias, namely self overconfidence, firms may in fact find it optimal to treat workers equally despite different performance measurements. Related to our paper, Rotemberg (2002) studies a model in which employers possess some signals about the workers' productivity, and investigates the effect of the employee's perception of the employer's precision in evaluating individual abilities on income distributions. In his model, "fair" evaluation is akin to "accurate" evaluation. In contrast, in our model, it is the outcome, rather than the precision, of the firm's performance evaluation that is unknown to the worker. Wage compression relative to productivities arises in our model because the wage-setting employer finds it optimal to strategically hide its information about the employees' abilities in order to preserve morale.

Our paper is also related to an emerging economics literature that explores the connection between intrinsic and extrinsic motivations.[4] Benabou and Tirole (2002, Section 2) analyze a mechanism design problem of a principal who is privately informed about the cost of a task that an agent is to perform. They show that, when both base wage and bonus are feasible, the only perfect Bayesian equilibrium satisfying the intuitive criterion of Cho and Kreps (1987) is separating, hence the firm's incentive scheme always reveals its private information to the worker. In our model, the firm simultaneously employs many, in fact, a continuum of, workers. The common candidate

[4]See Kreps (1997) for a review of the related social and industrial psychology and economics literature.

deviation to break a pooling equilibrium is not valid in our context, since any such proposed deviation contract will create an informational externality by affecting the morale of other workers who are not offered the same contract.

Prendergast (1991) also examines strategic information revelation by an informed principal to its workers in the context of a training-promotion problem similar to our model, but with common prior beliefs. He did not show whether and under which conditions the firm prefers a pooling contract.

Bewley (1999, Chapter 21) presents a model in which a worker's realized pace of work is directly affected by her mood (his conceptualization of morale) possibly through some unmodelled physiological process. In contrast, morale in our model is not a direct input of either the production or the utility function of the workers. Instead, a worker's morale affects her incentives by affecting her perception of the effects of her effort.

A similar dilemma of information revelation by a principal is analyzed by Feess, Schieble and Walzl (2001). In their model, an agent spends effort to forecast the quality of a project; and the principal can privately observe an additional informative signal about the project and decides, *ex ante,* whether to disclose his signal to the agent before she exerts effort. In our model, the principal can condition its disclosure decision on the private information realization. Our paper differs from both Benabou and Tirole (2002) and Feess et al. (2001) in that we consider a firm with many workers who observe each other's received offers. Lizzeri, Meyer and Persico (2002) analyze a two-period principal agent or tournament model, where each agent can observe neither her first-period output nor her own ability. They focus on the desirability of performing interim performance evaluation and revealing it to the agents. In their setting, worker ability is only a parameter in the principal's objective function, which affects neither the agent's marginal productivity of effort nor its cost. Therefore, revelation of information on ability does not have the direct impact on the incentive of the agents that we call the "morale effect."

2.2 Psychological Evidence of Overconfidence

Psychological evidence of overconfidence is first and foremost reflected in the *"above median" effect*, whereby well over half of survey respondents typically judge themselves in possession of more desirable attributes than fifty percent of other individuals. In Svenson (1981), 81 American and 80 Swedish students were asked to judge their skill in driving and how safe they were as drivers. It was found that 92.8% of American and 68.7% of Swedish subjects rated themselves as safer than 50% of other drivers. In Larwood and Whittaker (1977), 72 undergraduate management students and 48 presidents of New York state manufacturing firms are asked to rate themselves relative to their classmates or fellow presidents in IQ, likelihood of success, predicted growth in a hypothetical marketing problem, etc. The results indicate an astonishing level of overconfidence: of the 72 students, only 10 felt that they were merely of average intelligence relative to their own classmates and only 2 thought themselves below average; and only 18 of the 72 subjects predicted that their hypothetical firm's sales would be below the industry average. The executive sample also predicted inordinate success, even though more moderate than the students. In Meyer (1975), less than 5% of employees rated themselves below the median.

Psychological evidence of overconfidence is also reflected in the *"fundamental attribution error"* (Aronson 1994), that is, people tend to attribute their successes to ability and skill, but their failure to bad luck or to factors out of their control. Such self-serving biases are bound to lead to overconfidence. Psychologists have gathered a great deal of evidence for the observation that we take credit for the good and deny the bad. For example, students who do well on an exam attribute their performance to ability and effort, whereas those who do poorly attribute it to a poor exam or bad luck (Arkin and Maruyama 1979); gamblers perceive their success as based on skill and their failure as a fluke (Gilovich 1983); when married couples estimate how much of the housework each routinely did, their combined total of housework performed amounts to more than 100 percent - in other words, each person thinks he or she did more work than their partner think he or she did (Ross and Sicoly 1979); two-person teams performing a skilled task accept credit for the good scores but assign most of the blame for the poor scores to their partner (Johnston 1967); when asked to explain why someone else dislikes them, college students take little responsibility for themselves (i.e., there must be something wrong with this other person), but when told that someone else likes them, the students attributed it to their own personality (Cunningham, Starr and Kanouse 1979).[5]

3 The Model

Initial Beliefs. Consider a firm who employs for one period a continuum of workers with unit measure. Workers differ in their ability (or interchangeably, productivity or talent) denoted by a. To simplify the analysis, we assume that a worker's ability $a \in \{a_l, a_h\}$, where $a_h > a_l > 0$. We refer to a_l and a_h respectively as low and high ability. For each worker, neither she nor the firm knows the true value of her ability. The firm has an objective prior belief q_0 that each worker has high ability. We assume that the workers' ability types are independent, hence by Large Numbers q_0 is also the proportion of high ability workers in the firm. In contrast, each worker has a prior belief p_0 that she has high ability. Importantly, we allow the worker and the firm to have heterogeneous beliefs about a, that is, p_0 and q_0 do not have to be equal.[6] In particular, as motivated by the

[5] See Aronson (1994, Chapter 4) for more evidence of self-serving bias. Other authors have also provided similar reviews. See Babcock and Loewenstein (1997), Camerer and Lovallo (1999), Malmendier and Tate (2001), Compte and Postlewaite (2001), among others.

[6] We are aware that heterogenous priors violate Harsanyi's doctrine. See Morris (1995) for a persuasive argument that neither Bayesian decision theory nor standard theories of rationality requires agents to have the same priors.

psychological evidence reviewed in Section 2, we will be interested in the case when the workers are overconfident about their ability relative to the firm's objective assessment, that is, when $p_0 \geq q_0$. The initial beliefs p_0 and q_0 are common knowledge, and when they are not equal, we assume that each party believes the other to be wrong in its assessment of ability.

Production Technology. For simplicity, we assume that each employed worker can produce two levels of output, which are, without loss of generality, normalized to 0 (low output) or y (high output) where $y > 0$. The production technology is stochastic as follows: if a worker with ability $a_j, j \in \{h, l\}$ exerts effort $n \geq 0$, then the probability of high output is

$$\Pr(Y = y | a_j, n) = \pi_j(n), j \in \{h, l\}$$

where $\pi_j' > 0, \pi_j'' < 0$ and satisfy $\pi_j(0) = 0, \lim_{n \to \infty} \pi_j(n) = \bar{\pi}_j \leq 1$, and $\pi_h(n) > \pi_l(n)$ for all $n > 0$. In words, we assume that a worker produces low output unless she exerts positive effort. We assume that output realizations conditional on individual values of a and n are independent across workers.

The cost of effort is independent of the worker's ability and is given by $C(n)$ with $C(0) = 0, C' > 0$ and $C'' \geq 0$. Following the standard assumptions in principal-agent models, we assume that a worker's effort is not observable to the firm, while her output is observable and verifiable by all parties (including possibly the court).

Performance Evaluation. The firm, before beginning (or continuing) the relationship, receives a signal $\theta \in \{l, h\}$ of each individual's ability.[7] The signals are independently and identically drawn as follows:

$$\Pr\{\theta = l | a = a_l\} = \Pr\{\theta = h | a = a_h\} = \alpha,$$

where $\alpha > 1/2$ measures the signal's informativeness of a worker's ability. Importantly, it is assumed that the realization of each worker's signal θ is privately observed by the firm. A worker does not directly observe her performance evaluation θ, but she may possibly infer θ from the wage contract offered by the firm.

Belief Updating and Morale. The firm uses each worker's signal θ to update its belief about her ability. We write q_θ as the firm's posterior belief from Bayes' rule that a worker with signal θ has high ability, that is,

$$q_h = \frac{q_0 \alpha}{q_0 \alpha + (1 - q_0)(1 - \alpha)}, \ q_l = \frac{q_0 (1 - \alpha)}{q_0 (1 - \alpha) + (1 - q_0)\alpha}.$$

A worker does not directly observe her performance evaluation θ, but if she ever infers θ from contract offers (that is, when the firm offers different wage contracts to workers with different

See Morris (1994) and Allen and Gale (1999) for models in which agents have heterogenous prior beliefs.

[7] There are a couple of possible interpretations of the signal θ: it could be the test results of a training period if the relationship is new; or it could be non-output performance evaluation of the worker's ability from the previous periods if the relationship is continuing.

performance evaluations), she will form a posterior belief p_θ regarding her ability by Bayes' rule:

$$p_h = \frac{p_0 \alpha}{p_0 \alpha + (1-p_0)(1-\alpha)},\ p_l = \frac{p_0(1-\alpha)}{p_0(1-\alpha) + (1-p_0)\alpha}.$$

Note that we allow the firm and the workers to agree to disagree on their priors and their subsequent interpretations of the performance evaluations. A worker's confidence about her ability is what we mean by *worker morale* in this paper: p_0 is the worker's initial morale; and p_θ is her morale if she learns her performance evaluation from the wage offers. A worker's morale is hurt after knowing that she receives a low evaluation, and is boosted after knowing of a high evaluation because $p_l < p_0 < p_h$.

Since the firm employs a continuum of workers, it knows beforehand, by Large Numbers, that a measure q_0 of workers has ability a_h, a measure $\alpha q_0 + (1-\alpha)(1-q_0)$ of workers will obtain signal realizations $\theta = h$, and so on. In other words, the firm does not encounter any aggregate surprises. The role of the performance evaluations for the firm is to probabilistically identify and sort workers of ability a_h. After observing the signals, the firm knows again by Large Numbers that a fraction q_h of those workers with high evaluations is indeed of ability a_h; while for the remaining $1-q_h$ fraction of the high evaluations was inaccurate. The firm reasons similarly for workers for whom it has given a poor performance evaluation $\theta = l$.

Preferences. The firm and the workers are risk-neutral. To make the problem interesting, we assume that the workers have *limited liability*, i.e., a worker's total compensation cannot be negative. The outside option of the firm, namely the value of a vacancy, is $V_0 \geq 0$; and the outside option of a worker, namely the value of unemployment, is $U_0 \geq 0$ at the beginning of this relationship. We assume that both U_0 and V_0 are constant. In particular, the worker's outside option U_0 is independent of her morale, which is reasonable only when ability is firm-specific.

Wage Contract. Because the output level is assumed to be either high or low, a contingent wage contract is simply a *two-tier contract* $\{w, \Delta\}$, where w is the *base wage* to be received regardless of the level of output, and Δ is the *bonus* to be paid only when output is high.[8] The firm decides, for each worker, whether to offer her a contract, and if so, the terms of contract. We assume that the firm can credibly commit to any contract, and that each worker can observe *all* wage contracts, or lack thereof, offered by the firm to her colleagues.[9] A worker compares her wage contract with others', makes inference about her performance evaluation privately observed by the firm, and adjust her morale accordingly.

[8]In principle, the firm could condition payments to a worker on the entire distribution of outputs, as in a tournament. However, by Large Numbers, for any set of contract offers the resulting distribution of outputs is known beforehand by each worker, and does not affect her incentives. Hence, our two-tier contract covers this case too.

[9]This assumption might appear unrealistic in many contexts, as it is well-known that many employers consider wage information disclosure taboo, occasionally prohibiting it formally in their labor contracts. However, this kind of secrecy policy is hard to implement in practice. In addition, for those cases where information-sharing is effectively prevented, our analysis maybe useful as a counterfactual, to understand *why* firms have an interest in such confidentiality rules, and workers in breaking them.

The firm can adopt two possible wage-setting policies. The first is a *non-differentiation policy* (or interchangeably, pooling contract policy) under which the firm offers the same wage contract to all workers irrespective of their individual performance evaluations. A worker does not induce any inference on her own ability, thus her morale is maintained at the initial level p_0. The second is a *differentiation policy* (or interchangeably, separating contracts policy) under which different wage contracts are offered according to the individual performance evaluations. A worker infers in equilibrium her individual performance evaluation and updates her morale to p_h or p_l accordingly. We are interested in when the firm will find differentiation or non-differentiation wage-setting policies optimal.

4 Analyzing the Model

In this section, we first characterize the firm's optimal wage contract for a generic belief pair $\{p, q\}$ regarding the worker's ability. The relevant values of p and q may depend on whether differentiation or non-differentiation wage-setting policies are in consideration. Then, we analyze the profit-maximizing set of contracts offered to the workforce.

4.1 Worker's Problem

Suppose that a worker with morale p is offered a wage contract $\{w, \Delta\}$. Then, she chooses whether to accept the employment, and if so, her optimal effort level n^* by solving:

$$\max \left\{ \max_{n \geq 0} \left\{ w + \Delta \left[p\pi_h(n) + (1-p)\pi_l(n) \right] - C(n) \right\}; \quad U_0 \right\},$$

where the inner maximization yields the expected utility from accepting the offer and optimally choosing effort. If we temporarily ignore the non-negativity constraint on n, the inner maximization problem is concave with the necessary and sufficient first order condition:

$$\Delta \left[p\pi_h'(n) + (1-p)\pi_l'(n) \right] = C'(n), \tag{1}$$

which yields a unique optimal level of effort $n^*(\Delta, p)$.

For $n^*(\Delta, p)$ to be positive, the bonus must be large enough. Simple algebra shows that $n^*(\Delta, p) \geq 0$ if and only if

$$\Delta \geq \frac{C'(0)}{p\pi_h'(0) + (1-p)\pi_l'(0)} \equiv \underline{\Delta}$$

If a firm does not find it optimal to provide a bonus at least as high as $\underline{\Delta}$, then such workers with morale p will be laid off. A firm is willing to offer a bonus at least $\underline{\Delta}$ only if $y > \underline{\Delta}$; otherwise, due to our limited liability assumption on the worker, the firm will for sure lose money. To make things interesting, we take this as an assumption:

Assumption. $y > \underline{\Delta}$.

Not surprisingly, the worker's effort level does not depend on the base wage w and is increasing in the level of bonus Δ. The following lemma provides sufficient conditions under which the optimal effort n^* is concave in Δ, a property that will be used later.[10]

Lemma 1 *Sufficient conditions for $n^*(\cdot, p)$ to be concave in Δ are: (i) $C''' \geq 0$, and (ii) $p\pi_h' + (1-p)\pi_l'$ is log-concave.*

4.2 Firm's Problem

We now analyze the firm's choice of the optimal contract to offer to a single worker, given posterior beliefs q and p on her ability. The posteriors are generated by prior beliefs q_0, p_0, performance evaluation outcome θ, and information on θ revealed to the worker by the contract itself (if any).

Given a contract $\{w, \Delta\}$ and the worker's optimal effort choice $n^*(\Delta, p)$, the *perceived* expected utility of a worker with morale p from accepting the wage contract $\{w, \Delta\}$, is

$$U(w, \Delta; p) = w + [p\pi_h(n^*(\Delta, p)) + (1-p)\pi_l(n^*(\Delta, p))]\Delta - C(n^*(\Delta, p));$$

which must exceed the outside option for the contract to be acceptable:

$$U(w, \Delta; p) \geq U_0. \tag{2}$$

The *perceived* expected profit to the firm with belief q from offering a contract $\{w, \Delta\}$ to a worker with morale p is

$$V(w, \Delta; p, q) = [q\pi_h(n^*(\Delta, p)) + (1-q)\pi_l(n^*(\Delta, p))](y - \Delta) - w \tag{3}$$

which must exceed the outside option for the contract to be offered:

$$V(w, \Delta; p, q) \geq V_0. \tag{4}$$

A contract $\{w, \Delta\}$ is (p, q)-*feasible* if it satisfies (2) and (4). The firm's problem is:

$$\begin{aligned} &\max_{\{w \geq 0, \Delta \geq \underline{\Delta}\}} V(w, \Delta; p, q) \\ &\text{s.t. } (2), (4). \end{aligned} \tag{5}$$

The following two lemmas follow from standard arguments:

Lemma 2 *The solution to the firm's problem (5) exists.*

Lemma 3 *In any optimal contract $\{w, \Delta\}$, the worker's participation constraint (2) must bind if $w > 0$, namely if the base wage exceeds the limited-liability floor.*

[10]The proofs of Lemmas 1-3 and 5 are omitted. They are contained in the working paper version of this study and available from the authors upon request.

Note that the worker's participation constraint in general will be slack because of the limited liability assumption. We now establish that when the worker is at least as confident as the firm, a case we focus on, then the firm can without loss of generality only choose a level of bonus and set the base wage to zero:

Proposition 1 *If $p \geq q$, then any optimal contract has a zero base wage: $w = 0$.*

Proof. The proof is by contradiction. If there exists an optimal contract with $w > 0$, then by Lemma 3 the worker's participation constraint (2) must bind. Hence we can solve for w from (2) holding as an equality to obtain

$$w = U_0 - \left[p\pi_h(n^*(\Delta, p)) + (1-p)\,\pi_l(n^*(\Delta, p))\right]\Delta + C(n^*(\Delta, p)).$$

Replacing it into the objective function of the firm in the inner maximization problem of (5), we obtain after some simplification,

$$\max_{\Delta\in[\underline{\Delta},y]}\left\{\begin{array}{c}\Delta\,(p-q)\left[\pi_h(n^*(\Delta,p)) - \pi_l(n^*(\Delta,p))\right] \\ +\left[q\pi_h(n^*(\Delta,p)) + (1-q)\pi_l(n^*(\Delta,p))\right]y - C(n^*(\Delta,p)) - U_0\end{array}\right\}.$$

The first order derivative with respect to Δ is

$$(p-q)\left[\pi_h(n^*(\Delta,p)) - \pi_l(n^*(\Delta,p))\right] + \left[q\pi_h'(n^*(\Delta,p)) + (1-q)\pi_l'(n^*(\Delta,p))\right](y-\Delta)\frac{\partial n^*(\Delta,p)}{\partial\Delta}$$

which is strictly positive whenever $\Delta < y$ and $p \geq q$. Hence the firm maximizes profits by loading maximum incentives and offering $\Delta = y$. But then the firm never obtains any output and pays a positive base wage $w > 0$. Thus the firm's expected profit is negative by offering the optimal wage contract with $w > 0$, violating the firm's participation constraint, that is the firm can do better by offering no contract. A contradiction. ■

Note, by continuity, Proposition 1 should also hold for any $p < q$ as long as it is sufficiently close to q; but, when the worker is sufficiently under-confident, the optimal base wage w may be strictly positive. The reason is as follows: when q is sufficiently larger than p, the firm does not want to offer too high a bonus because it expects the worker to receive it very often. However, the worker disagrees and a low bonus alone may not satisfy her participation constraint. Thus, the firm has to make it up with a strictly positive base wage.

When $p \geq q$, Proposition 1 tells us that the optimal contract will have a zero base wage. Thus we can re-write the firm's problem (5) as

$$\begin{aligned}&\max_{\Delta\in[\underline{\Delta},y]}\left[q\pi_h(n^*(\Delta,p)) + (1-q)\pi_l(n^*(\Delta,p))\right](y-\Delta);\\ &\text{s.t. } (2),\ (4).\end{aligned} \tag{6}$$

To characterize the solution to problem (6), we first ignore the participation constraints (2), (4) and check them *ex post*.

Lemma 4 *If the worker's optimal effort $n^*(\cdot, p)$ is concave in the bonus Δ, then the inner maximization problem in (6) has a unique solution $\Delta^*(p,q) \in (\underline{\Delta}, y)$ which implicitly solves the first-order condition:*

$$0 = \frac{\partial V(0,\Delta;p,q)}{\partial \Delta} = (y-\Delta)\left[q\pi_h'(n^*(\Delta,p)) + (1-q)\pi_l'(n^*(\Delta,p))\right]\frac{\partial n^*(\Delta,p)}{\partial \Delta} - \left[q\pi_h(n^*(\Delta,p)) + (1-q)\pi_l(n^*(\Delta,p))\right]; \tag{7}$$

Proof. Eq. (7) provides the first order condition for the inner problem in (6). Clearly, the second derivative of the value is negative if $n^*(\cdot,p)$ is concave in Δ. Evaluated at $\underline{\Delta}$, the second term of Eq. (7) is zero and thus smaller than the first term; and evaluated at y, the first term is zero and thus larger than the second term. Therefore, there is a unique solution $\Delta^*(p,q) \in (\underline{\Delta}, y)$ to Eq. (7). ∎

It follows from the Implicit Function Theorem applied to Eq. (7) that the candidate optimal bonus $\Delta^*(p,q)$ is increasing in y, the output level in case of success.

So far, we have neglected firm's and worker's participation constraint in problem (6). The solution $\Delta^*(p,q)$ characterized in Lemma 4 will be the actual solution to problem (5) if $\{0, \Delta^*(p,q)\}$ is (p,q)-feasible. Otherwise, we consider two cases: (i) If $\{0, \Delta^*(p,q)\}$ violates the firm's participation constraint (4), then the firm will offer no contract to a worker with morale p, and will lay her off. The reason is simple: by definition, $V(0, \Delta^*(p,q); p, q)$ is the maximum the firm can achieve by choosing Δ when $w = 0$ (recall that we established in Proposition 1 that no optimal contract with $w > 0$ exists). (ii) If the firm's participation constraint (4) holds but the worker's (2) fails at $\{0, \Delta^*(p,q)\}$, namely $U(0, \Delta^*(p,q); p) < U_0$, then the firm will have to consider offering a higher bonus, $\bar{\Delta}(p)$, optimally making the worker just willing to participate:

$$U\left(0, \bar{\Delta}; p\right) = U_0. \tag{8}$$

It is easy to see that $\bar{\Delta}(p) > \Delta^*(p,q)$ because $U(0, \Delta^*(p,q); p) < U_0 = U\left(0, \bar{\Delta}; p\right)$ and $U(0, \Delta; p)$ is increasing in Δ. The firm is willing to offer a contract $\left\{0, \bar{\Delta}(p)\right\}$ if and only if its expected surplus from offering such a contract yields more than its outside option, *i.e.*

$$V\left(0, \bar{\Delta}(p); p, q\right) \geq V_0. \tag{9}$$

We summarize the above discussions in the following:

Proposition 2 *Fix any generic belief pair by a worker and the firm (p,q), with $p \geq q$. Consider bonus levels $\Delta^*(p,q)$ and $\bar{\Delta}(p)$ respectively defined by (7) and (8).*

1. *If $\{0, \Delta^*(p,q)\}$ is (p,q)-feasible, then it is the optimal contract;*

2. *If* $\{0, \Delta^*(p,q)\}$ *is not* (p,q)*-feasible, but* $\{0, \bar{\Delta}(p)\}$ *satisfies (9), then* $\{0, \bar{\Delta}(p)\}$ *is the optimal contract;*

3. *In any other cases, the firm offers no contract to the worker.*

We introduce the following notation to ease exposition below:

$$\tilde{\Delta}(p,q) = \begin{cases} \Delta^*(p,q) & \text{if } \{0, \Delta^*(p,q)\} \text{ is } (p,q)\text{-feasible} \\ \bar{\Delta}(p) & \text{if } \begin{cases} \{0, \Delta^*(p,q)\} \text{ is not } (p,q)\text{-feasible, but} \\ \{0, \bar{\Delta}(p)\} \text{ satisfies (9)} \end{cases} \\ \emptyset & \text{otherwise.} \end{cases} \tag{10}$$

where $\emptyset$ stands for "no contract." Also define $\tilde{V}(p,q)$ as the firm's maximal profit from offering the optimal contract (including the null contract), i.e.,

$$\tilde{V}(p,q) = V\left(0, \tilde{\Delta}(p,q); p, q\right)$$

where $V(0, \emptyset; p, q) \equiv V_0$.

4.3 The Optimal Set of Contracts

Proposition 2 characterizes the firm's optimal contract (or no contract) to a *single* worker for a generic belief pair (p,q) with $p \geq q$. We now characterize the optimal set of contracts offered to *all* workers, taking into account the effect on the workers' beliefs from observing the contract offers.

Non-Differentiation Wage Policy. If the firm offers the same contract $\{0, \Delta\}$ to all workers, they will all maintain their morale at the original level p_0. However, the firm still possesses private information regarding each individual worker. That is, the firm still knows precisely who obtains good performance evaluation $\theta = h$ and who obtains bad performance evaluation $\theta = l$. The firm's optimal bonus under a non-differentiation policy is the solution to the following problem:

$$\begin{aligned} &\max \left\{ \max_{w \geq 0, \Delta \in [\underline{\Delta}, y]} \left\{ \begin{array}{c} [\alpha q_0 + (1-\alpha)(1-q_0)] V(w, \Delta; p_0, q_h) \\ +[\alpha(1-q_0) + (1-\alpha) q_0] V(w, \Delta; p_0, q_l) \end{array} \right\}; \quad V_0 \right\}, \\ &\text{s.t. } (2) \end{aligned} \tag{11}$$

where the first term in the objective function of the inner maximization problem is the firm's expected profit from all the workers who have received $\theta = h$; and the second term is the firm's expected profit from all the worker who have received $\theta = l$. Recall that from the Law of Large Numbers, a measure $\alpha q_0 + (1-\alpha)(1-q_0)$ of workers receive a good performance evaluation, and the remaining $\alpha(1-q_0) + (1-\alpha) q_0$ measure receive a bad evaluation.

Using the linearity of the function $V(w, \Delta; p, q)$ in q [see Eq. (3)] and the martingale property of the firm's belief, we immediately have

$$\begin{aligned} &[\alpha q_0 + (1-\alpha)(1-q_0)] V(w, \Delta; p_0, q_h) + [\alpha(1-q_0) + (1-\alpha) q_0] V(w, \Delta; p_0, q_l) \\ &= [q_0 \pi_h(n^*(\Delta, p_0)) + (1-q_0)\pi_l(n^*(\Delta, p_0))] (y - \Delta) - w. \end{aligned}$$

Replacing this expression in (11), since $p_0 \geq q_0$, Proposition 1 applies and $w = 0$. Thus, problem (11) is identical to problem (6). This implies that under a non-differentiation policy the optimal uniform base wage is zero and the optimal bonus is exactly given by $\tilde{\Delta}(p_0, q_0)$. We thus reach an important conclusion: under a non-differentiation policy, the firm's private performance evaluation is irrelevant, both strategically (by the definition of pooling) and statistically because of the Law of Large Numbers and expected utility. The firm will offer a contract $\tilde{\Delta}(p_0, q_0)$ as described by (10) to all workers, and its expected profit under a non-differentiation policy is $V^P = \tilde{V}(p_0, q_0)$.

Differentiation Wage Policy. If the firm adopts a differentiation wage-setting policy, then the contracts will reveal to each worker the performance evaluation privately observed by the firm. Then $p_0 \geq q_0$ implies $p_\theta \geq q_\theta$, and our contract characterization applies to each worker type. The firm then optimally offers to a worker with evaluation $\theta \in \{l, h\}$ the contract $\left\{0, \tilde{\Delta}\left(p_\theta, q_\theta\right)\right\}$ characterized earlier. Overall, the firm's expected profit under a differentiation policy is

$$V^S = \left[\alpha q_0 + (1-\alpha)(1-q_0)\right] \tilde{V}(p_h, q_h) + \left[\alpha(1-q_0) + (1-\alpha) q_0\right] \tilde{V}(p_l, q_l).$$

Comparing Wage Policies. The difference between the firm's expected profits from the differentiation and non-differentiation policies, under our maintained assumption of weak worker overconfidence $p_0 \geq q_0$, can be usefully decomposed into three components as follows:

$$\begin{aligned} V^S - V^P = & \overbrace{\left[\begin{array}{c} \left[\alpha q_0 + (1-\alpha)(1-q_0)\right]\left[\tilde{V}\left(p_0, q_h\right) - \tilde{V}\left(p_0, q_0\right)\right] \\ + \left[\alpha\left(1-q_0\right) + (1-\alpha) q_0\right]\left[\tilde{V}\left(p_0, q_l\right) - \tilde{V}\left(p_0, q_0\right)\right] \end{array}\right]}^{\text{sorting effect}} \\ & + \overbrace{\left[\alpha q_0 + (1-\alpha)(1-q_0)\right]\left[\tilde{V}(p_h, q_h) - \tilde{V}(p_0, q_h)\right]}^{\text{morale boost effect}} \\ & + \overbrace{\left[\alpha(1-q_0) + (1-\alpha) q_0\right]\left[\tilde{V}(p_l, q_l) - \tilde{V}(p_0, q_l)\right]}^{\text{morale loss effect}}. \end{aligned} \tag{12}$$

The first component is the sorting effect. It captures the gain under a differentiation policy if the firm can tailor workers' contracts according to their individual performance evaluations without, hypothetically, altering the worker's morale. Standard revealed-profit-maximization arguments (see Lemma 5 below) show that the sorting effect is non-negative, thus in favor of differentiation policy. The second component captures the change in profits due to higher morale from informing workers with high performance evaluation of the good news. The last term captures the change in profits due to lower morale from informing workers with low performance evaluation of the bad news. As we will discuss shortly, the impact on firms' profits of each morale effect depends on the complementarity or substitutability of effort and ability in technology. But, one of the two morale effects always reduces the profits of the firm from wage differentiation, thus a trade-off always exists.

In Bayesian decision theory the value function is convex in beliefs by a simple revealed preference argument and the martingale property of beliefs. Thus an informative signal is always beneficial ex-ante. This force is at play here too, and is behind the *sorting effect.* The firm wants to allocate efficiently workers that it perceives having different abilities to different carefully tailored incentive contracts. As indicated above, this effect emerges formally if we fix worker's beliefs and shut down the signaling role of wage offers, thus preserving only the decision-theoretic part of the mechanism design problem.

Lemma 5 *If the worker's participation constraint is slack at the optimal contract, that is if* $\tilde{\Delta}(p_0, q_j) = \Delta^*(p_0, q_j), j \in \{0, h, l\}$, *then the sorting effect is strictly positive for almost all* $q_0 \in (0, 1)$, *and* $p_0 \geq q_0$.

The only instances in which the sorting effect is zero are either: (1) the firm has extreme prior beliefs $q_0 \in \{0, 1\}$; or (2) the worker's participation constraint binds and the optimal contracts under all belief pairs $\{p_0, q_j\}, j \in \{0, h, l\}$ are $\bar{\Delta}(p_0)$; or (3) no contracts are offered under all belief pairs $\{p_0, q_j\}, j \in \{0, h, l\}$.

However, here contract offers have also a signaling role, and exert a *morale effect* by altering the worker's beliefs. In general, it is difficult to determine the net trade-offs of differentiation and non-differentiation policies for any initial pair of beliefs (p_0, q_0). Indeed, we rely on numerical examples in Section 5 to illustrate these trade-offs. However, we have the following two general characterizations for some special cases of initial belief pairs:[11]

Proposition 3 *For any* $q_0 \in (0, 1)$*, the firm's expected profit under a differentiation policy is higher than that under a non-differentiation policy:*

1. *as* $p_0 \to 1$*; or*
2. *when* $p_0 = q_0$*, if and only if* $\tilde{V}(p, p)$ *is convex in* p*.*

Proof. To prove the first statement, note that from Lemma 5, we know that the sorting effect is strictly positive for any $q_0 \in (0, 1)$, but the morale effects are equal to zero when p_0 is equal to 1. The conclusion then follows from continuity.

To prove the "if" part of the second statement, note that if $p_0 = q_0$, we have $p_h = q_h, p_l = q_l$; moreover, p_0 and q_0 are convex combinations of p_h and p_l with weights exactly given by $\alpha q_0 + (1 - \alpha)(1 - q_0)$ and $\alpha(1 - q_0) + (1 - \alpha) q_0$ respectively. The "only if" part obviously follows from the definition of convexity. ■

[11] The first claim of Proposition 3 provides a rationale for Jack Welch's management philosophy mentioned in footnote 2.

Due to the strategic interaction, in general one is unable to establish the convexity of $\tilde{V}(p,p)$, and Proposition 3 is not generally applicable. What Proposition 3 provides is a simple characterization of the superiority of the differentiation policy in terms of convexity of $\tilde{V}(p,p)$ when the firm and workers share identical initial beliefs $p_0 = q_0$. If instead, the workers are overconfident, i.e. $p_0 > q_0$, then convexity no longer suffices. In either case, we show below by parametric examples that a non-differentiation wage-setting policy may indeed dominate a differentiation policy.

A key insight of this paper is that the initial divergence of the levels of confidence between the firm and workers will naturally lead to the changes in magnitudes of the sorting and morale effects. In particular, a moderate level of worker overconfidence is likely to enlarge the morale loss relative to the morale gain and to swamp the sorting effect; when effort and ability are complements, this implies that the non-differentiation policy comes to dominate, as demonstrated in numerical examples in Section 5.

4.4 Complementarity and Substitutability of Ability and Effort

The results that we have obtained so far hold independently of the complementarity or substitutability of effort and ability in production, defined locally by the sign of $\pi_h'(n) - \pi_l'(n)$. Imposing one or the other assumption only changes the interpretation of the results.

We first discuss how complementarity and substitutability of ability and effort fit into our model. Since $\pi_h(0) = \pi_l(0) = 0$, we have $\bar{\pi}_l = \lim_{n\to\infty} \pi_l(n) = \int_0^\infty \pi_l'(n)\,dn$ and $\bar{\pi}_h = \lim_{n\to\infty} \pi_h(n) = \int_0^\infty \pi_h'(n)\,dn$. Therefore, if $\bar{\pi}_l$ is sufficiently smaller than $\bar{\pi}_h$, then it is possible that ability and effort are global complements, i.e. $\pi_h'(n) > \pi_l'(n)$ for all n. Of course, given $\bar{\pi}_l \le \bar{\pi}_h$, it is impossible for ability and effort to be global substitutes. On the other hand, if $\bar{\pi}_l = \bar{\pi}_h$, then evidently ability and effort can not be global complements: if ability and effort are complements in some regions, they must be local substitutes in some other regions. Moreover, if we further assume that $\pi_h'' < \pi_l'' < 0$, then it can be shown that there exists an effort level $\hat{n}$ such that $\pi_h'(\cdot)$ crosses $\pi_l'(\cdot)$ from above at $\hat{n}$. In other words, $\pi_h'(n) > \pi_l'(n)$ if and only if $n < \hat{n}$, where $\hat{n}$ uniquely solves $\pi_h'(\hat{n}) = \pi_l'(\hat{n})$. To induce an effort level $\hat{n}$ from the worker with morale p, the corresponding bonus level $\hat{\Delta}$ must satisfy the firm's first order condition evaluated at $n = \hat{n}$, i.e.,

$$\hat{\Delta}\left[p\pi_h'(\hat{n}) + (1-p)\,\pi_l'(\hat{n})\right] = C'(\hat{n}).$$

By the definition of $\hat{n}$, we have $\pi_h'(\hat{n}) = \pi_l'(\hat{n})$. Thus, $\hat{\Delta} = C'(\hat{n})/\pi_h'(\hat{n})$. If $\hat{\Delta} \ge y$, then the firm will never optimally choose to offer a bonus as high as $\hat{\Delta}$; which in turn guarantees that the effort level induced by the firm under the optimal contract will always be less than $\hat{n}$. Hence the local complements condition will be satisfied in equilibrium.

In our model, when the local complements condition is satisfied at n^*, morale serves an intrinsic motivation; and the strength of the intrinsic motivation is affected by the extrinsic motivation, namely the firm's wage policy. Applying the Implicit Function Theorem to Eq. (1), we obtain that $n^*(\Delta, \cdot)$ increase in p if and only if ability and effort are local complements at $n^*(\Delta, p)$. Also, applying the Implicit Function Theorem to Eq. (7), we obtain that the candidate optimal bonus $\Delta^*(p, q)$ is strictly decreasing in p if $\partial^2 n^*(\Delta, p)/\partial\Delta\partial p \leq 0$, which is not a general property of the optimal effort function $n^*(\Delta, p)$.[12] But on net we have an unambiguous prediction for firm profits, which determine the likelihood of wage compression. Applying the Envelope Theorem to Problem (6), we immediately have:

Proposition 4 *Fix the firm's belief q and assume $p \geq q$. Then the firm's maximum expected profit from hiring a worker with morale p, $V(0, \Delta^*(p, q); p, q)$, is strictly increasing in p if and only if ability and effort are local complements at $n^*(\Delta^*(p, q), p)$.*

Thus, when ability and effort are local complements, it is desirable for the firm to hire overconfident workers because worker overconfidence serves as the intrinsic motivation for higher effort, offsetting the moral hazard inefficiency. In this case, the morale boost effect in Eq. (12) amounts for the firm to a *positive morale effect* on its profits, and the morale loss effect to a *negative morale effect*. The more overconfident workers are to begin with, the more the morale loss dominates the morale gain in the workforce, and the stronger the incentives for the firm not to differentiate wages. The next section shows that this is indeed the case. When ability and effort are local substitutes, all the conclusions are reversed; the negative morale effect is produced by the shirking workers who receive good news and think no longer necessary to work hard to obtain the bonus.

4.5 Involuntary Layoffs

The firm may find it optimal to offer no contract to a particular type of worker, in which case we may say that the worker is fired: $\tilde{\Delta}(p, q) = \emptyset$ and $V(0, \emptyset; p, q) \equiv V_0$. There exist parameter values such the fired worker disagrees with the firm's decision. The disagreement can arise for two reasons. First, the worker may be able to support the relationship and produce profits for the firm in excess of V_0 if she could commit to the first-best effort. This would represent a strict Pareto improvement over the termination of the relationship between the worker and the firm. However, the firm knows that the worker cannot be trusted due to the standard moral hazard problem. Our model also uncovers a new motive for disagreement. Given the worker's optimistic belief, the worker's expectation of the firm's expected profits from offering $\bar{\Delta}(p)$, say, may be larger than V_0 even when taking moral hazard into account; yet the firm's perceived expected profit under its own

[12] It is interesting to contrast with Benabou and Tirole (2002), which make a stark prediction that, in equilibrium, a higher bonus is necessarily associated with bad news in the sense that a principle will offer a higher bonus only when she knows the task to be more difficult. The main reason is that in Benabou and Tirole's model, the agent makes a discrete choice of whether or not to exert effort, which is affected in an additively separable fashion by the principal's private information (if revealed) and the bonus.

belief is smaller than V_0:

$$\begin{aligned}\left[p\pi_h(n^*(\bar{\Delta}(p),p))+(1-p)\pi_l(n^*(\bar{\Delta}(p),p))\right](y-\bar{\Delta}(p)) &> V_0,\\ \left[q\pi_h(n^*(\bar{\Delta}(p),p))+(1-q)\pi_l(n^*(\bar{\Delta}(p),p))\right](y-\bar{\Delta}(p)) &\leq V_0.\end{aligned}$$

In this case the worker suffers a dismissal that from her viewpoint is Pareto dominated and thus unjustified.

Finally, we can reinterpret Bewley (1999)'s finding that a layoff is typically preferred to a selective wage cut because of the negative implications on worker morale. In the perspective of our model, this fact implies that personnel managers consider effort and ability to be complements in production, because they are worried about the effects of morale loss, not of morale gains. This provides a further reason to focus on the complementarity case.

5 Numerical Illustrations

In this section, we present a parametric example of our model by specifying the functions $\pi_h(\cdot), \pi_l(\cdot)$ and $C(\cdot)$, and show that worker overconfidence is an important determinant of whether the firm will favor a non-differentiation wage-setting policy. Alternative parametric specifications of the model that we explored all gave the same qualitative results.

We specify that both the probability functions of high output and effort cost functions are exponential, that is, $\pi_j(n) = 1 - \exp\{-a_j n\}, j \in \{h, l\}$ and $C(n) = \exp\{\lambda n\} - 1$ where $\lambda > 0$. Note that $C(0) = 0$.

The optimal bonus of the firm $\Delta^*(p, q)$ is obtained by numerically solving Eq. (7). Notice that in this exponential model $C''' > 0$ and $p\pi_h' + (1-p)\pi_l'$ is log-concave, so Lemma 1 and in turn Lemma 4 apply, and Eq. (7) is sufficient to pin down the optimal bonus.

5.1 Understanding the Trade-offs

First, we set the outside options of the firm and the worker to zero (i.e., $U_0 = V_0 = 0$) so that the optimal contract in Proposition 2 is given by $\Delta^*(p, q)$ for every pair (p, q). We first show that the non-differentiation policy can dominate the differentiation policy when the worker's initial morale p_0 is sufficiently higher than the firm's initial belief q_0 and, by Proposition 3, sufficiently smaller than 1.

[Figure 1 about here]

Figure 1 depicts the firm's expected profits under the two policies as a function of the worker's initial morale p_0 where other parameter values are set at $q_0 = 1/2, a_l = 1, a_h = 1.5, \lambda = 1, \alpha = 0.9, y = 2$. In this parameter configuration, as well as in those we will explore later in comparative statics exercises, it can be verified that effort and ability are local complements for all feasible effort levels. It is shown in the figure that the non-differential policy provides a higher expected profit for the firm when worker's initial morale p_0 is in an interval $\left(\underline{p}_0^*, \bar{p}_0^*\right)$ where $\underline{p}_0^*$ and $\bar{p}_0^*$ are respectively called the *lower* and *upper (morale) threshold*:[13] for any $\{q_0, \alpha, a_l, a_h, y\}$, they are the two solutions to the following equation in p_0:

$$\tilde{V}(p_0, q_0) = [\alpha q_0 + (1-\alpha)(1-q_0)] \tilde{V}(p_h, q_h) + [\alpha(1-q_0) + (1-\alpha) q_0] \tilde{V}(p_l, q_l).$$

Importantly, the lower threshold for initial worker morale $\underline{p}_0^*$ exceeds the firm's initial belief q_0, and the upper threshold $\bar{p}_0^*$ is less than 1. That is, for the non-differentiation wage policy to dominate the differentiation policy, workers must be sufficiently but not excessively overconfident.

[Figure 2 about here]

Panel A of Figure 2 depicts the lower threshold $\underline{p}_0^*$ as a function of q_0. Note that the lower threshold $\underline{p}_0^*$ increases in the firm's initial belief q_0 and is always higher than q_0 (note that it lies above the dashed 45 degree line). Thus overconfidence of the worker relative to the firm's initial belief is a necessary condition for the firm to adopt non-differentiation wage policy. Indeed, Panel A also suggests that overconfidence "begets" overconfidence. To see this, suppose that the true proportion of high ability workers, q_0, is low. Then Panel A indicates that the firm will be very likely to adopt a non-differentiation policy because the threshold $\underline{p}_0^*$ is also low. This implies that for a given distribution of initial worker beliefs, a firm facing a low quality labor force is more likely to engage in no wage differentiation. Ex post, the majority of the workers receive a poor performance evaluation but never learn it, hence they become even more overconfident in their own ability relative to the firm. Only a minority of the workers lose some of their overconfidence relative to the firm. So the larger the proportion of low ability overconfident workers to begin with, the larger the average reenforcement of overconfidence in equilibrium.

Panel B of Figure 2 depicts the upper threshold $\bar{p}_0^*$ as a function of q_0. The main message is that it is very close to 1 (above 0.9999) for the whole domain of q_0. To summarize, in this exponential example, the non-differentiation wage policy is superior if and only if the worker's initial morale p_0 lies above the lower threshold $\underline{p}_0^*$ and below the upper threshold $\bar{p}_0^*$. The most important fact is that the lower threshold $\underline{p}_0^*$ is higher than q_0, which, together with subsequent graphs, implies that worker overconfidence (but not extreme overconfidence), is a necessary condition for the firm to adopt a non-differentiation policy.

Why does moderate level of worker overconfidence cause the firm to favor the non-differentiation over the differentiation policy? The trade-offs between the two policies can be better understood via the decomposition in expression (12).

[13] In Figure 1, $\bar{p}_0^*$ is indistinguishable from 1 because of precision level. The actual value of $\bar{p}_0^*$ for the figure is 0.999914.

[Figure 3 about here]

Figure 3 shows how the sorting and morale effects change as the worker's initial morale increases. Panel A shows that the sorting effect is strictly positive and increases in p_0. Panel B shows that the positive morale effect decreases in p_0 and approaches zero as p_0 approaches 1. The reason is simple: the morale boost from knowing of a good performance evaluation gets smaller when the worker's initial confidence gets higher (provided that it is higher than $q_0 = 0.5$). Panel C shows that the negative morale effect initially declines and then reverts to zero. Panel D shows the total effects, which implies that the non-differentiation policy dominates the differentiation policy if and only if $p_0 \in \left(\underline{p}_0^*, \bar{p}_0^*\right)$.

[Figure 4 about here]

Why would the negative morale effect start to dominate the other two effects as p_0 increases? From expression (12), we know that two forces shape the relative strength of the negative and positive morale effects as the worker's initial morale p_0 varies. The first force is statistical, namely Bayesian updating; and the second force is due to the curvature of $\tilde{V}$. The second force is relatively unimportant because $\tilde{V}$ is almost linear in p and q. The statistical force is depicted in Figure 4, which shows the ratio of morale boost from knowing of a good signal over the morale loss from knowing a bad signal. This ratio declines to $(1-\alpha)/\alpha = 1/9$ as the worker's initial morale approaches 1. In other words, the morale loss from a bad signal will dominate the morale boost from a good signal as p_0 increases. This explains why the negative morale effect will eventually dominate the positive morale effect as p_0 gets large enough.[14]

5.2 Comparative Statics

The main theoretical and empirical prediction of the model is the range of initial levels of worker confidence in which the firm will prefer a non-differentiation wage policy to a differentiation policy. In the context of this example, it is the interval $\left(\underline{p}_0^*, \bar{p}_0^*\right)$. Since the upper threshold $\bar{p}_0^*$ is invariably extremely close to 1 (above 0.9999), we will conduct comparative statics of the lower threshold $\underline{p}_0^*$ with respect to parameters of the model.

5.2.1 Aggregate Productivity

Aggregate productivity is proxied in our model by the output level y that the worker can achieve by spending effort. In reality, both individual worker productivities y and outside options are likely to rise in an expansion. How does y affect the benefits and costs of wage differentiation? Given the

[14]Note, however, as p_0 goes to 1, both morale effects go to zero, even though the negative morale effect dominates the positive morale effect. Since the sorting effect is always positive, and in fact increases in p_0 for a fixed q_0, the differentiation policy "wins" as p_0 approaches 1.

initial level of worker morale, are firms more willing to engage in non-differentiation wage policy when y is higher (i.e., in a boom)? Figure 5 depicts the non-monotonic relationship between the necessary level of worker overconfidence $\underline{p}_0^*$ and the aggregate productivity shock y.

[Figure 5 about here]

The reason for the non-monotonic relationship between $\underline{p}_0^*$ and y is quite subtle. It can be numerically verified that, for any fixed belief pair (p, q), the sorting effect is increasing and convex in y, the negative morale effect is U-shaped in y, and the positive morale effect is inverted U-shaped in y. The latter two relationships arise because of the curvature of $\tilde{V}$. Overall, the total effects have a U-shaped relationship with y. When the productivity y is small, for any (p, q), the negative morale effect is small because the discouraged workers are not able to produce too much in any case. Thus, in order for the firm to prefer a non-differentiation policy, the workers must be quite overconfident, thus a higher $\underline{p}_0^*$ is necessary. When the productivity y is high, the negative morale effect starts to fall again because the firm offers a higher bonus, so the extrinsic motivation just swamps the intrinsic motivation; the sorting effect starts to pick up fast since the sorting effect is convex in y. Thus again a higher worker overconfidence is needed for the firm to adopt a non-differentiation policy.

The macroeconomic implication of this comparative statics is straightforward. If aggregate productivity is in the increasing region of Figure 5, then the model predicts that the firm is more likely to adopt a non-differentiation wage policy when y is low (in a recession) than when y is high (in a boom). That is, there is more wage compression in a recession. If the firm prefers wage differentiation, then layoffs of workers with poor performance evaluations will be more likely when y is low. As Bewley (1999) found, firms must choose between layoffs and wage rigidity in recessions because selective wage cuts would trigger a loss in morale among those workers, making them no longer employable.

5.2.2 Effects of Other Parameter Changes

We now describe the effects of other parameter changes on the necessary worker overconfidence required for the firm to optimally choose the non-differentiation wage policy. The qualitative effects are rather intuitive. Thus we will only explain these effects and interested readers are referred to our working paper for more quantitative graphs.

First, for any fixed level of firm belief q_0, the level of overconfidence necessary for the non-differentiation policy to dominate, namely $\underline{p}_0^*$, monotonically increases in α, the precision of performance evaluation. The intuition is that, the higher α is, the stronger the sorting effect in favor of differentiation policy; and thus the more worker overconfidence is required for non-differentiation policy to be superior.

Second, worker heterogeneity also affects the firm's optimal wage-setting policy. Worker heterogeneity can be constructed in our numerical example as follows: Keep constant the mean level of worker ability perceived by the firm at $q_0 a_h + (1-q_0)a_l \equiv \bar{a}$ and create mean-preserving increases in worker heterogeneity by varying a_h in the interval $[\bar{a}, \bar{a}/q_0]$ while setting $a_l = (\bar{a} - q_0 a_h) / (1 - q_0)$. In this construction, the larger a_h, the higher heterogeneity in worker ability. Numerical examples show that the necessary level of worker overconfidence required for the non-differentiation policy to be optimal, namely $\underline{p}_0^*$, increases with worker heterogeneity. This arises because the sorting effect becomes stronger as the workers become more heterogeneous. In fact, in the extreme case when $a_l = 0$ and $a_h = \bar{a}/q_0$ (thus low ability worker never produces high output even with effort), $\underline{p}_0^*$ is equal to 1; hence it never pays the firm to adopt the non-differentiation policy.

So far we assumed that outside options U_0 and V_0 are both zero and our basic conclusion is that some worker overconfidence is necessary for the firm to choose the non-differentiation policy. We now discuss the effects of positive outside options. First, our basic conclusion still holds when U_0 is zero but the firm's outside option V_0 is positive and large enough to bind at some optimal contract $\{0, \Delta^*(p_j, q_j)\}, j \in \{0, h, l\}$. To see why, suppose that the firm prefers to differentiate when $p_0 = q_0$ and $U_0 = V_0 = 0$. This implies from our previous calculations that

$$\begin{aligned}
&[\alpha p_0 + (1-\alpha)(1-p_0)] V(0, \Delta^*(p_h, p_h); p_h, p_h) \\
&+ [\alpha(1-p_0) + (1-\alpha) p_0] V(0, \Delta^*(p_l, p_l); p_l, p_l) \\
&> V(0, \Delta^*(p_0, p_0); p_0, p_0),
\end{aligned}$$

since the optimal bonus is given by $\Delta^*(p_j, p_j)$ when $U_0 = 0$. Now consider an increase in the firm's outside option V_0. Because $V(0, \Delta^*(p, p); p, p)$ increases in p (due to the local complementarity of effort and ability in this example), the above inequality implies that, for any $V_0 > 0$,

$$\begin{aligned}
&[\alpha p_0 + (1-\alpha)(1-p_0)] \max\{V(0, \Delta^*(p_h, p_h); p_h, p_h), V_0\} \\
&+ [\alpha(1-p_0) + (1-\alpha) p_0] \max\{V(0, \Delta^*(p_l, p_l); p_l, p_l), V_0\} \\
&\geq \max\{V(0, \Delta^*(p_0, p_0); p_0, p_0), V_0\},
\end{aligned}$$

where the inequality is strict except for the uninteresting case where the firm lays off even the most confident workers after revealing the good news to them. Thus wage differentiation will generally strictly dominate non-differentiation. Therefore, some worker overconfidence is needed for the firm to adopt a non-differentiation wage policy.

We can similarly show that our conclusion holds when the firm's outside option V_0 is zero but workers' outside option U_0 is positive and large enough to bind at some optimal contract $\{0, \Delta^*(p_j, q_j)\}$ where $j \in \{0, h, l\}$. Suppose $p_0 = q_0$. Suppose that U_0 binds only for workers with belief p_l, i.e., the most pessimistic workers who receive bad news in a differentiation contract. The differentiation wage contract must offer $\bar{\Delta}(p_l) > \Delta^*(p_l, p_l)$ to workers with belief p_l. In our numerical example, it can be shown that, if $p_0 = q_0$, then the firm's expected profit is higher under the differentiation wage policy for any $U_0 > 0$ as long as U_0 is in the range where only the participation constraints of workers with belief p_l bind. Thus again some worker overconfidence is necessary for the non-differentiation wage policy to be superior.

6 Discussion and Testable Implications

In this section, we first discuss some of our modelling assumptions and results, and then provide some testable implications of our model.

6.1 Discussion

1. In this paper, a worker's morale does not directly affect the marginal productivity or the marginal cost of effort, a purely psychological channel emphasized by Bewley (1999). Instead, we emphasized an indirect channel: a worker's morale affects her incentives to exert effort through affecting the worker's perceived marginal productivity of effort. However, we believe that our major insight - workers will react asymmetrically to good and bad news when they are moderately overconfident - will lead a firm to prefer a non-differentiation wage policy even if the morale affects marginal productivity or marginal cost of effort directly.
2. We assumed that all workers have identical initial beliefs p_0, and the firm also holds identical initial beliefs q_0 about all the workers. Suppose instead, that there are k different levels of initial belief pairs, $\left(p_0^1, q_0^1\right), ..., \left(p_0^k, q_0^k\right)$, and a large number of workers in each cell. Then as long as these belief pairs are commonly known by all the workers, the optimal wage setting policy derived in the paper can be simply interpreted as the optimal wage policy conditional on a belief pair $\left(p_0^j, q_0^j\right), j = 1, ..., k$. Thus in such a firm with many different belief pairs, workers do observe wage differentiation, but a worker's morale is *only* affected by the wage contract offers received by her co-worker in the same belief pair type, or "reference group" in Akerlof and Yellen (1990)'s language.
3. Our model makes the stark prediction that there is complete wage compression when a firm finds the non-differentiation wage policy to be superior. This strong prediction is due to the simplifying assumptions of the model. First, as we mentioned above, when the firm has different types of workers in terms of initial belief pairs $\left(p_0^j, q_0^j\right), j = 1, ..., k$, the economic forces we highlight in this paper will be consistent with within-firm wage differentiation even when the firm favors the non-differentiation wage policy for any belief pair $\left(p_0^j, q_0^j\right), j = 1, ..., k$. Second, we assumed in the paper a very restricted space for the performance evaluation signals with only $\theta \in \{l, h\}$. As we enrich the signal space to include more performance evaluation outcomes, it is conceivable that the morale concerns emphasized in our paper will lead a firm to favor a semi-pooling wage policy as follows:

the firm reveals extremely good and extremely bad news, but conceal all mediocre news.[15] Under such a wage policy, we will then observe wage compression, but not complete wage equalization, within the firm.

4. A staple of our analysis is that workers are overconfident about their own ability. We then investigate the implication of such overconfidence on the firm's wage setting policy under the assumption that workers process information revealed by the firm's wage contracts in a rational Bayesian fashion. In other words, the workers' bias in our model lies in the prior, not in the information processing. A different approach would be to assume that workers have the correct initial belief, but are biased in their information processing, for example, that they suffer from attribution bias as mentioned in Section 2. When workers suffer from severe attribution bias, the firm will undoubtedly be in favor of a differentiation policy. The reason is obvious: workers who receive bad news will simply attribute it to bad luck and hence will not lower their morale; but workers who receive good news will boost their morale. Of course, any worker who suffers from attribution bias will mostly likely have been overconfident by the time they join the work force. Thus, it seems that worker overconfidence is a natural assumption. When the workers are biased in both their initial beliefs and their information processing, the economic forces emphasized in our model will survive, albeit reduced in strength.

5. In this paper a worker's outside option is unaffected by her confidence level due to the assumption that a worker's ability is firm-specific. This assumption effectively makes labor market competition irrelevant in our setting. An interesting adverse selection issue would arise if workers' ability were general. That is, a firm's wage-setting policy would affect the characteristics of its workforce, much in the same way as the design of an insurance policy would affect the pool of insurees. In such a general equilibrium model, the degree of labor market competition will also affect whether or not it is optimal for the firms to adopt a non-differentiation wage-setting policy. It is conceivable that wage non-differentiation would be more common in less competitive labor markets. We leave the verification of this conjecture for future research.

6.2 Testable Implications

1. One testable implication of our model is that wage differentiation is more prevalent in environments where it is harder to find out the wage offers received by co-workers. One such comparison is offered by public versus private universities. Public universities by law have to publicly disclose the salaries of all professors; while private universities do not have to. Thus the morale ramifications of wage differentiation will be larger in public universities, where we expect to observe less wage

[15] This seems to be the policy of GE under the leadership of Jack Welch. In Chapter 11 of Welch (2001), he said: "Differentiation comes down to sorting out the A, B, and C players." The top 20 percent of the players are identified as the A players and highly rewarded; the bottom 10 percent are identified as the C players and fired; and the middle 70 percent are identified as B players.

differentiation than in private universities.

2. A second testable implication of our model is that wage differentiation is more prevalent if the firm can convincingly use some observable and objective criteria to justify such differentiation. The reason is very simple: wage differentiation based on observable and objective criteria have little effect on worker morale; while wage differentiation under the firm's discretion will convey the firm's private information about the workers and affect their morale. For example, affirmative action laws may impose differential treatment of equally productive workers, which is beyond the control of the firm and is common knowledge. In a "pure" fair-wage model, à la Akerlof and Yellen (1990), workers would still feel the pinch of wage differentiation and alter their effort, even if they were convinced of objective reasons unrelated to their productivities. Of course, one could argue that these objective reasons shape the "reference group" of workers, which is relevant for wage comparisons in the fair-wage model. In providing a micro-foundation of the fair-wage hypothesis, our model suggests operationally how to define the reference group.

3. Our model predicts that, over a wide range of parameters, wage differentiation is directly related to the level of aggregate productivity. The available empirical evidence seems to suggest that income inequality is countercyclical. However, this is far from a robust empirical regularity. More importantly, our prediction concerns wage compression among continuing workers within the same firm. The cyclicality of wage inequality depends to a large extent on composition effects in employment and firms, and the anti-cyclicality of income inequality is certainly generated to some extent by that of unemployment.

7 Conclusion

In this paper, we investigate the implications of worker overconfidence, which is supported by a large body of psychological evidence, on the optimal wage-setting policies of the firms. More specifically, we examine the optimal contract design problem of a principal facing many agents. The principal privately observes individualized performance evaluations of the agents, and decides if it is in its interest to offer them different incentive contracts depending on, thus revealing, their performance evaluations.

We decompose the trade-offs between a differentiation and non-differentiation policy into three components: first, a sorting effect, which allows the firm to tailor individual contracts to her perceived ability, in favor of the differentiation policy; second, a morale boost effect, which means that the morale of the workers with high performance evaluation will be enhanced under a differentiation policy; and third, a morale loss effect, which means that the morale of the workers with low performance evaluation will be hurt under a differentiation policy. We show in numerical examples and conjecture in general that the differentiation policy dominates the non-differentiation policy when the firm and workers share identical initial beliefs. However, worker overconfidence can effectively tilt the balance in favor of the non-differentiation policy. We robustly show in examples that when workers are sufficiently overconfident than the firm, the non-differentiation policy can be the optimal policy. By providing a theoretical link between worker overconfidence and wage-setting practices of the firm, we help explain why firms emphasize against wage disclosure, and abstain from wage differentiation among their workers, as documented by Bewley (1999).

The most interesting extension of the model is to introduce dynamics. As firms accumulate more (private) evidence of the worker, while engaging in non-differentiation wage policy, it is possible that at a certain point, the trade-off may be in favor of differentiation policy. At that time, workers who have accumulated a string of bad performance evaluations, but never told so earlier, will receive the full string of news in one dosage, even being laid off.

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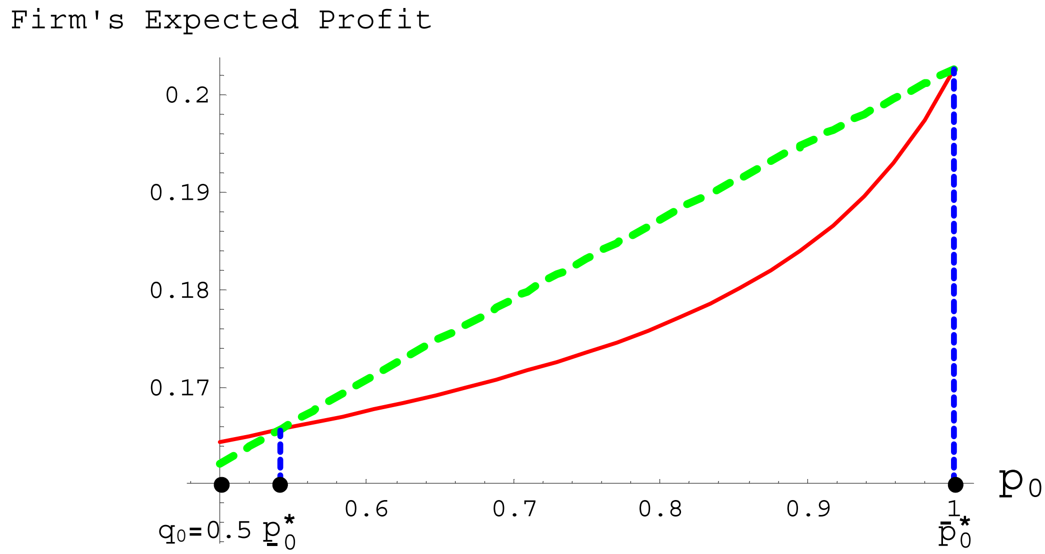

Figure 1: Firm's expected profits under differentiation (solid curve) and non-differentiation (dashed curve) policies as a function of the worker's initial morale p_0. Other parameter values are set at $q_0 = 0.5, a_l = 1, a_h = 1.5, \lambda = 1, \alpha = 0.9, y = 2, U_0 = V_0 = 0$.

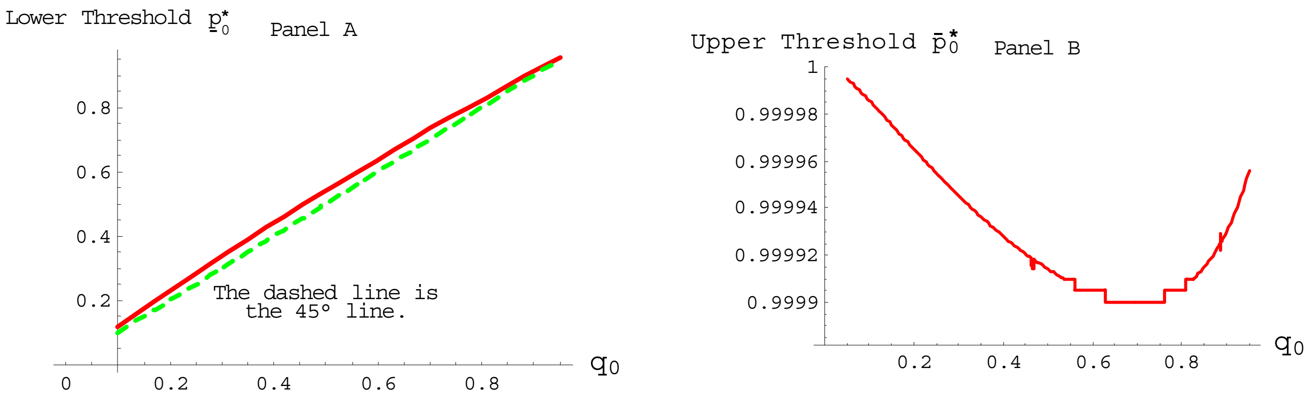

Figure 2: The lower and upper thresholds $\underline{p}_0^*$ and $\bar{p}_0^*$ as a function of q_0 $\alpha = 0.9, a_l = 1, a_h = 1.5, y = 2, U_0 = V_0 = 0$.

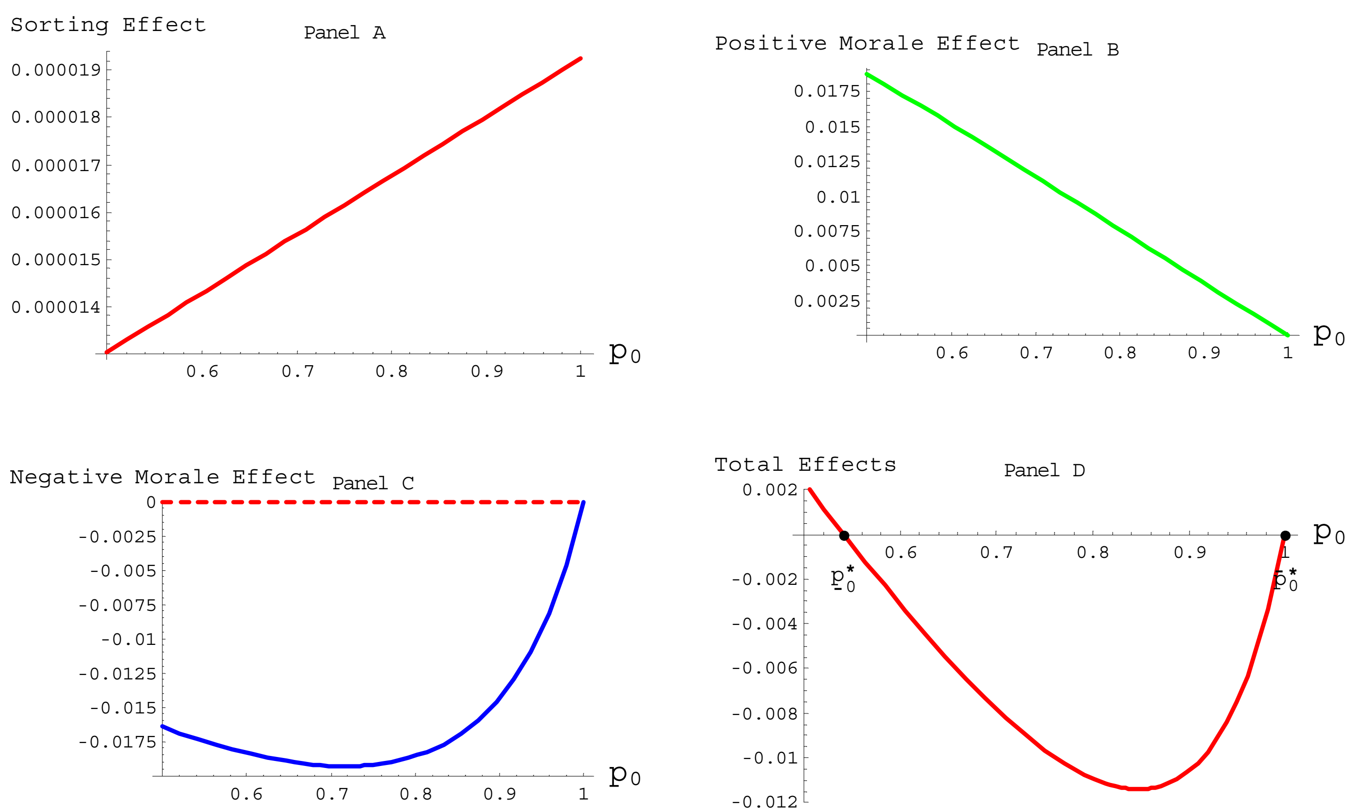

Figure 3: Sorting and morale effects as functions of worker morale p_0 : $q_0 = 0.5, a_l = 1, a_h = 1.5, y = 2, \alpha = 0.9, U_0 = V_0 = 0$.

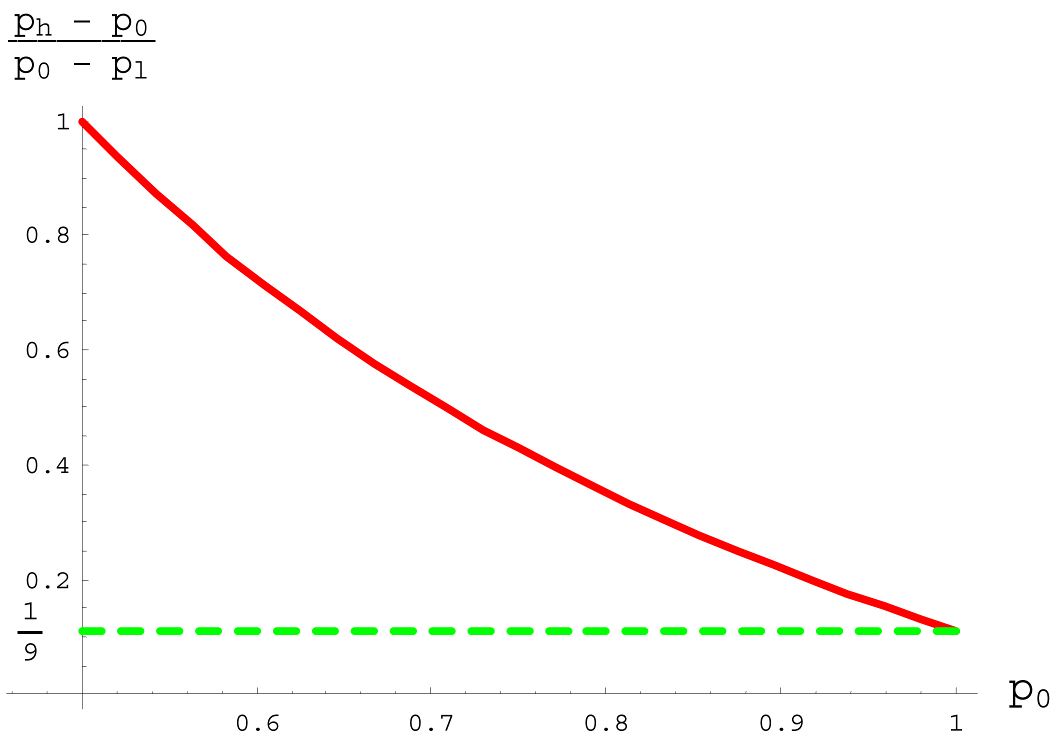

Figure 4: The ratio of morale boost over morale loss as a function of $p_0 : \alpha = 0.9$.

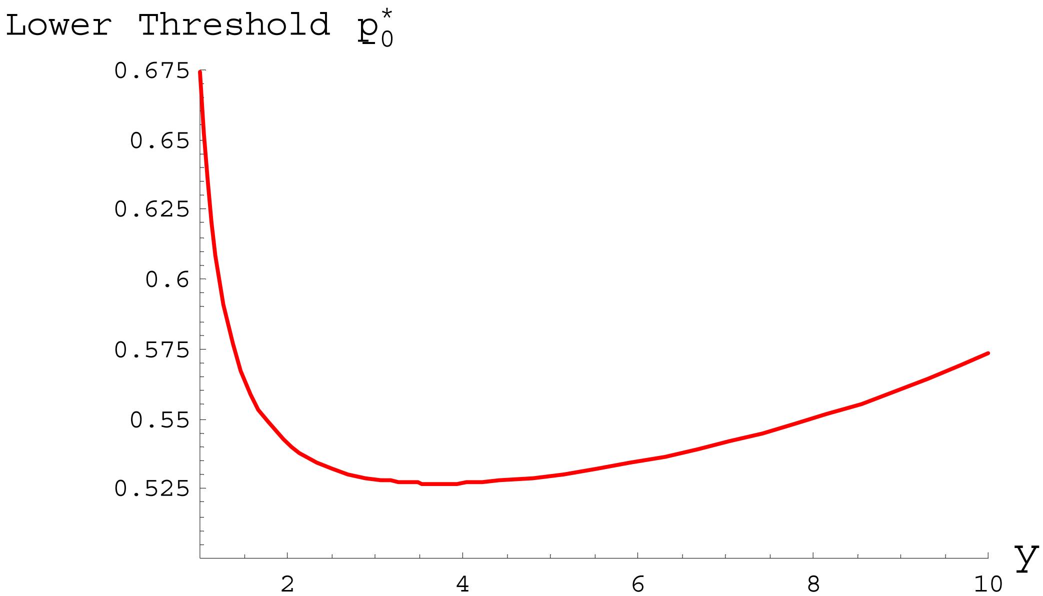

Figure 5: The relationship between $\underline{p}_0^*$ and $y : q_0 = 0.5, a_l = 1, a_h = 1.5, \alpha = 0.9, U_0 = V_0 = 0.$

Appendix: Omitted Proofs

Proof of Lemma 1:

Proof. Note that the firm's expected output is bounded above by qy (when $n = \infty$). Since the firm can guarantee itself $V_0 \geq 0$, the firm will for sure choose $w \leq qy$ and $\Delta \leq y$. We can without loss of generality restrict the firm's feasible contract set to

$$\{(w, \Delta) : w \in [0, qy], \quad \Delta \in [\underline{\Delta}, y], \quad U(w, \Delta; p) \geq U_0\},$$

Since U is continuous in (w, Δ), this is a compact set. Since $V(w, \Delta; p, q)$ is continuous, by Weierstrass Theorem it has a maximum, which we can then compare with V_0. Thus (5) always has a solution. ■

Proof of Lemma 3:

Proof. If $w > 0$ but (2) does not bind, then the firm could slightly reduce w without changing the incentives to exert effort, since n^* depends only on the bonus Δ; and without violating the participation constraint of the worker. This increases the firm's profits, a contradiction to the firm's optimality. ■

Proof of Lemma 2:

Proof. Applying the implicit function theorem to the first order condition (1), we obtain

$$\frac{\partial n^*(\Delta, p)}{\partial \Delta} = \frac{p\pi_h'(n^*) + (1-p)\pi_l'(n^*)}{C''(n^*) - \Delta\left[p\pi_h''(n^*) + (1-p)\pi_l''(n^*)\right]} > 0.$$

Thus,

$$\begin{aligned}\frac{\partial^2 n^*(\Delta, p)}{\partial \Delta^2} &= \frac{[p\pi_h'' + (1-p)\pi_l''] n^{*\prime}}{C''(n^*) - \Delta\left[p\pi_h'' + (1-p)\pi_l''\right]} \\ &- \frac{[p\pi_h' + (1-p)\pi_l'] \left\{ \ \{C''' - \Delta[p\pi_h''' + (1-p)\pi_l''']\} n^{*\prime} - [p\pi_h'' + (1-p)\pi_l''] \ \right\}}{\left\{C'' - \Delta\left[p\pi_h'' + (1-p)\pi_l''\right]\right\}^2},\end{aligned}$$

where, with some abuse of notation, we write $n^{*\prime} \equiv \partial n^*(\Delta, p)/\partial \Delta$. After imposing the common denominator as $\{C'' - \Delta[p\pi_h'' + (1-p)\pi_l'']\}^2$, the numerator is

$$\begin{aligned}&\left[p\pi_h'' + (1-p)\pi_l''\right] n^{*\prime} \left\{C'' - \Delta\left[p\pi_h'' + (1-p)\pi_l''\right]\right\} \\ &- \left[p\pi_h' + (1-p)\pi_l'\right] \left\{\left\{C''' - \Delta\left[p\pi_h''' + (1-p)\pi_l'''\right]\right\} n^{*\prime} - \left[p\pi_h'' + (1-p)\pi_l''\right]\right\} \\ &= n^{*\prime}\left[p\pi_h'' + (1-p)\pi_l''\right]\left\{C'' - \Delta\left[p\pi_h'' + (1-p)\pi_l''\right]\right\} \\ &- n^{*\prime}\left[p\pi_h' + (1-p)\pi_l'\right]\left\{C''' - \Delta\left[p\pi_h''' + (1-p)\pi_l'''\right]\right\} \\ &+ \left[p\pi_h' + (1-p)\pi_l'\right]\left[p\pi_h'' + (1-p)\pi_l''\right]\end{aligned}$$

which is clearly negative when $C''' \geq 0$ and $p\pi_h' + (1-p)\pi_l'$ is log-concave. ■

Proof of Lemma 5.

Proof. Under the assumption,

$$
\begin{aligned}
\tilde{V}(p_0, q_0) &= [q_0\pi_h(n^*(\Delta^*(p_0,q_0),p_0)) + (1-q_0)\pi_l(n^*(\Delta^*(p_0,q_0),p_0))]\,[y-\Delta^*(p_0,q_0)] \\
&= [\alpha q_0 + (1-\alpha)(1-q_0)] \begin{bmatrix} q_h\pi_h(n^*(\Delta^*(p_0,q_0),p_0)) \\ +(1-q_h)\,\pi_l(n^*(\Delta^*(p_0,q_0),p_0)) \end{bmatrix} [y-\Delta^*(p_0,q_0)] \\
&+ [\alpha(1-q_0) + (1-\alpha)\,q_0] \begin{bmatrix} q_l\pi_h(n^*(\Delta^*(p_0,q_0),p_0)) \\ +(1-q_l)\,\pi_l(n^*(\Delta^*(p_0,q_0),p_0)) \end{bmatrix} [y-\Delta^*(p_0,q_0)] \\
&< [\alpha q_0 + (1-\alpha)(1-q_0)] \begin{bmatrix} q_h\pi_h\left(n^*\left(\Delta^*\left(p_0,q_h\right),p_0\right)\right) \\ +(1-q_h)\,\pi_l\left(n^*\left(\Delta^*\left(p_0,q_h\right),p_0\right)\right) \end{bmatrix} [y-\Delta^*\left(p_0,q_h\right)] \\
&+ [\alpha(1-q_0) + (1-\alpha)\,q_0] \begin{bmatrix} q_l\pi_h\left(n^*\left(\Delta^*\left(p_0,q_l\right),p_0\right)\right) \\ +(1-q_l)\,\pi_l\left(n^*\left(\Delta^*\left(p_0,q_l\right),p_0\right)\right) \end{bmatrix} [y-\Delta^*\left(p_0,q_l\right)] \\
&= [\alpha q_0 + (1-\alpha)(1-q_0)]\,\tilde{V}\left(p_0,q_h\right) + [\alpha(1-q_0) + (1-\alpha)\,q_0]\,\tilde{V}\left(p_0,q_l\right),
\end{aligned}
$$

where the second equality follows from the martingale property of Bayes' updating; and the inequality follows from revealed profit maximization of the firm and the fact that the firm's objective function in problem (6) is nonlinear. ■