INDETERMINACY OF CITIZEN-CANDIDATE EQUILIBRIUM

By

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"Indeterminacy of citizen-candidate equilibrium"

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1. <u>Introduction</u>

For many years, the Downsian model of political equilibrium was the only (formal) model, and it suffered from two severe limitations: it portrayed political parties, or candidates, as being concerned only with winning elections (as opposed to implementing particular policies), and it possessed equilibria only (roughly speaking) when the policy space was unidimensional, and preferences were single-peaked. In the mid-1990's two pairs of authors proposed, independently, a new model of political equilibrium which remedied both of these problems. The model was originally proposed by Osborne and Slivinski (1996), who dubbed it the 'citizen-candidate model', and later formulated to apply to environments with multi-dimensional policy spaces by Besley and Coate (1997).

In the citizen-candidate (CC) model, citizens have preferences over a policy space. Each citizen considers whether or not to stand for election. If a candidate enters the contest, she pays a cost, and if she wins, she enjoys a benefit from holding office, as well as deriving utility from implementing the policy upon which she ran. It is assumed that , if a candidate stands for election, she must announce her ideal policy; to do otherwise would not be credible in this one-shot game. The model generally possesses pure-strategy equilibria with a small set of candidates, even when the policy space is

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multi-dimensional. Thus, both limitations of the Downs model are overcome, because candidates are explicitly 'ideological,' as well as caring about the spoils of office, and equilibria exist.

In this note, I study the set of CC equilibria in what might be called the canonical spatial model, where citizens have preferences defined on a Euclidean policy space, and where, except for the utility derived from winning office, all citizens care only about policies, not about the personalities of candidates. (This is thus a special case of the Besley-Coate (BC) environment.) More precisely, the model is 'canonical' in that there is a continuum of citizen types (this being the standard way of formalizing the notion that polities are large and citizens are very heterogeneous), and they are distributed according to a continuous probability measure on the space of types.

The authors who proposed the CC model worked in an environment of certainty: uncertainty as to the electoral outcome existed only when there were voters who were indifferent between announced policies or when the plurality winner was non-unique. I want to avoid the knife-edge peculiarities of electoral models with certainty, and so I will introduce uncertainty in electoral outcomes in a standard way.

I will study the set of 2-candidate CC equilibria in the canonical model. Such an equilibrium is characterized by a pair of policies. Let T be the policy space; then the manifold of equilibria is a subset of T x T. I will argue that the canonical model suffers from an indeterminacy of equilibrium, in the sense that the equilibrium manifold can comprise an open set in T x T. In other words, the dimensionality of the equilibrium manifold is the same size as the dimensionality of T x T.

It is probably too much to ask, of a concept of political equilibrium for multi-dimensional policy spaces, that its political equilibria (1) always exist and (2) be locally unique (i.e., the dimension of the equilibrium manifold be zero). The concept of party-unanimity Nash equilibrium that I have proposed (Roemer [2001]) generally exists, and is always a two-dimensional manifold in T x T, regardless of the dimension of T. But it is probably the case that an equilibrium concept is insufficiently restrictive if it allows the equilibrium manifold to be of the 'same size', in the sense of dimension, as the parent space.

2. A canonical spatial model

There is a continuum of citizen types, $h \in H$, distributed according to a probability measure F on H. There is a policy space $T \subset \mathbf{R}^n$, some $n > 1$, that we take to be a compact, convex set. The von Neumann- Morgenstern utility of type h on T is given by the vNM utility function $v(\cdot, h): T \to \mathbf{R}$.

We assume that there is a representative agent for each type; we denote the representative agent for type h also by h. The set of representative agents plays the game described in Besley and Coate (1997). If a candidate enters the electoral contest, he pays a cost of δ; if a candidate wins the election, he enjoys a utility supplement of β. Except for these two numbers, all citizens are entirely policy oriented; that is, a citizen of type h derives a utility of $v(t, h)$ from the implementation of t, regardless who the implementor is.

We introduce uncertainty in voting as follows. Given any two policies $t^1, t^2 \in T$, define the set of types who prefer t^1 to t^2 as:

$$\Omega(t^1, t^2) = \{h \mid v(t^1, h) > v(t^2, h)\}.$$

We assume that v and F are such that, if two policies are distinct, then only a set of types of measure zero is indifferent between them. It therefore follows that, if all citizens vote their preferences, the fraction who would vote for t^1 in a contest between t^1 and t^2 would be $F(\Omega(t^1, t^2))$. In a two-candidate election, uncertainty would only exist as to the outcome if this fraction were exactly one-half. We now say, however, that this *predicted vote share* is subject to a margin of error, and so there is only a *probability* that t^1 wins the election, which is an increasing function of the predicted vote share. To avoid complexities that add nothing to the argument, I will assume henceforth that, in any election (among any number of candidates), the probability of victory for a candidate is *equal* to predicted vote share of her policy.

We assume, following BC, that candidates must announce their ideal policies. We denote the ideal policy of type h by t^h.

Thus, the data of the model are $(T, H, F, v, \beta, \delta)$.

We now state the definition of a pure strategy 2-candidate CC equilibrium. It is a Nash equilibrium played by the set of type- representatives, where each chooses either to enter or not to enter the electoral contest, and each wishes to maximize her expected utility. (Readers are referred to BC for more detail.)

There are two sets of inequalities that characterize an equilibrium. The first (inequalities (4) of BC) state that the expected utility of each candidate will weakly decrease, if he contemplated not to run; and the second says that the expected utility of any type who is not a candidate would weakly decrease, if he contemplated to run (inequalities (5) of BC).

Let (h_1,h_2) be a 2-candidate CC equilibrium. Then these two sets of conditions

may be written as:

$$\left.\begin{array}{l} \pi_1(v(t^{h_1},h_1)+\beta)+\pi_2 v(t^{h_2},h_1)-\delta \geq v(t^{h_2},h_1) \\ \pi_1 v(t^{h_1},h_2)+\pi_2(v(t^{h_2},h_2)+\beta)-\delta \geq v(t^{h_1},h_2) \end{array}\right\} \quad (A)$$

and

$$\forall h \notin\{h_1,h_2\} \quad \pi_1 v(t^{h_1},h)+\pi_2 v(t^{h_2},h) \geq \pi_1^*(h)v(t^{h_1},h)+\pi_2^*(h)v(t^{h_2},h)+\pi_h^*(h)v(t^h,h)-\delta \quad (B)$$

where π_1 and π_2 are the probabilities that h_1 and h_2 win in the contest between them, and

$\pi_i^*(h)$ is the probability that h_i wins, for $i=1,2$ in the 3-candidate election among

$\{h_1,h_2,h\}$, and $\pi_h^*(h)$ is the probability that h wins in that election.

We now specialize to the case of Euclidean preferences, and take T to be a

compact, convex set in $\mathbf{R}^2$. The set of types is $H=T$, and the utility function is

$$v(t,h)=-d_{th}$$

where d_{th} means the distance from t to h. F can be any continuous probability measure

on H. Denote, for easier reading, $d_{12} \equiv d_{h_1 h_2}$. Of course, the ideal policy of h is $t=h$.

Using the facts that $\pi_1 = 1-\pi_2$ and $d_{hh}=0$, we can rewrite the two inequalities of

(A) as:

$$\text{for } i=1,2 \quad \pi_i(\beta+d_{12}) \geq \delta \quad (A')$$

while inequalities (B) may be written as:

$$\text{for all } h \notin\{h_1,h_2\} \quad \delta \geq \beta\pi_h^*(h)+(\pi_1-\pi_1^*(h))d_{1h}+(\pi_2-\pi_2^*(h))d_{2h}. \quad (B')$$

3. An example with a large set of equilibria

Because preferences are Euclidean, the set $\Omega(t^1, t^2)$ is easy to characterize: it is the set of types that lie on the same side of the perpendicular bisector of the line segment joining t^1 and t^2 as t^1 lies. In a 3-candidate race among $\{h_1, h_2, h\}$, the set of types preferring h to the other two policies is $\Omega(h, h_1) \cap \Omega(h, h_2)$. Thus, in the 3-candidate race, the probabilities of victory are, by our convention:

$$\pi_1^*(h) = F(\Omega(h_1, h) \cap \Omega(h_1, h_2))$$
$$\pi_2^*(h) = F(\Omega(h_2, h) \cap \Omega(h_2, h_1))$$
$$\pi_h^*(h) = F(\Omega(h, h_1) \cap \Omega(h, h_2))$$

Now choose any pair of types $\{h_1, h_2\}$ such that:

$$\pi_1 = \pi_2 = \frac{1}{2} \qquad (1)$$

and $\qquad m \equiv \sup_{h \notin \{h_1, h_2\}} \pi_h^*(h) < \frac{1}{2}. \qquad (2)$

The second inequality says that no other type can expect to win at least half the votes, should she enter the race with the first two: these conditions are easy to satisfy. Essentially, they require that h_1 and h_2 be not too similar and not too extreme.

Denote:

$$M \equiv \sup_{h \notin \{h_1, h_2\}} (-\frac{d_{12}}{2} + (\frac{1}{2} - \pi_1^*(h))d_{1h} + (\frac{1}{2} - \pi_2^*(h))d_{2h});$$

M is finite because T is compact. Now choose β large enough (and positive) so that:

$$\beta(\frac{1}{2} - m) > M \qquad (3)$$

which is surely possible by virtue of (2), and choose $\varepsilon > 0$ small enough so that:

$$\beta(\frac{1}{2} - m) - \varepsilon > M \qquad (4a)$$

$$\frac{1}{2}(\beta + d_{12}) - \varepsilon > 0 \qquad (4b)$$

which is surely possible by virtue of (3). Now define:

$$\delta = \frac{1}{2}(\beta + d_{12}) - \varepsilon \qquad (5)$$

which is positive by (4b).

I now claim that $\{h_1, h_2\}$ is a CC 2-candidate equilibrium for the polity $(T, H, F, v, \beta, \delta)$. We need to verify inequalities (A') and (B'). Inequalities (A') both become

$$\frac{1}{2}(\beta + d_{12}) \geq \delta$$

which is true in virtue of (4b) and (5). Indeed, since $\varepsilon > 0$, both inequalities in (A') are strict. Inequalities (B') become:

$$\frac{1}{2}(\beta + d_{12}) - \varepsilon \geq \beta\pi_h^*(h) + (\frac{1}{2} - \pi_1^*(h))d_{1h} + (\frac{1}{2} - \pi_2^*(h))d_{2h}$$

which can be written:

$$\forall h \notin \{h_1, h_2\} \quad \beta(\frac{1}{2} - \pi_h^*(h)) - \varepsilon \geq -\frac{d_{12}}{2} + (\frac{1}{2} - \pi_1^*(h))d_{1h} + (\frac{1}{2} - \pi_2^*(h))d_{2h}$$

which is implied by :

$$\beta(\frac{1}{2} - m) - \varepsilon > M,$$

our inequality (4a). Indeed, since (4a) is a strict inequality, we have that the continuum of inequalities (B') are *boundedly strict*:   that is, for the whole continuum $h \in H \setminus \{h_1, h_2\}$, the inequalities hold at least with some fixed, positive tolerance.

Because, given our choice of (β, δ), the inequalities in (A') are strict, and the inequalities in (B') are boundedly strict, it follows by the continuity of v and F, that there is an open neighborhood of (h_1, h_2) in H everyone of whose points comprises a 2-candidate CC equilibrium for the polity $(T, H, F, v, \beta, \delta)$.   (It is necessary to note that the

probability function is continuous except at points where two candidates propose the same policy. But that never happens in these equilibria.)   This follows because, in a small open neighborhood of (h_1, h_2), the inequalities (A') and (B') will continue to hold.

4. Conclusion

I have shown that, in a canonical model of electoral politics – the Euclidean spatial model with a continuum of types – the dimensionality of the manifold of 2-candidate citizen-candidate equilibria is the same size as the dimensionality of the cross-product of the policy space with itself, which is the space in which the equilibria live. I contend that this means that the citizen-candidate equilibrium concept is insufficiently restrictive.

There are other examples where equilibrium concepts deliver very large sets of equilibria: one thinks of the core of a finite exchange economy, which is also, typically, of the same dimension as the space in which the core allocations live. For that reason, the core is an insufficiently restrictive equilibrium concept for a finite economy. Indeed, the core shrinks to a small set of allocations in continuum economies. That does not seem to be case with the citizen-candidate concept.