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Abstract

This paper explores the general equilibrium impact of social security portfolio diversification into private securities, either through the trust fund or private accounts. The analysis depends critically on heterogeneities in saving, production, assets, and taxes. Limited diversification weakly increases interest rates, reduces the expected return on short-term investment (and the equity premium), decreases safe investment, increases risky investment and increases a suitably weighted social welfare function. However, the effects on aggregate investment, long-term capital values, and the utility of young savers hinges on assumptions about technology. Aggregate investment and long-term asset values can move in opposite directions.

Keywords: Social security, privatization, diversification

JEL Classification: H55

Introduction

Policy discussions of social security portfolio diversification into equities have concentrated on the consequences for retirement benefits and the budget viability of the system, ignoring general equilibrium repercussions (and sometimes even claiming there would be none). In contrast, we analyze the general equilibrium ramifications for prices, for utility levels, and for investment. We show that these ramifications can be substantial and paradoxical when part of the population does not adjust its private savings portfolio in response to a change in its social security portfolio. We also show how critically they depend on heterogeneity in saving, in production, in assets, and in taxes.¹

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¹For policy discussions, see, e.g., Advisory Council on Social Security (1997). For examples of claims that make sense in a representative agent model but are not adequate once heterogeneity is recognized, see Financial Economists' Roundtable (1998), and Greenspan (1997). For a discussion of privatization in general, see Diamond (1999). For another analysis of portfolio diversification where general equilibrium effects matter, see Abel (2001). Our paper differs from that of Abel in that we attribute the lack of portfolio diversification of some workers to a lack of savings, while Abel focuses on a fixed cost of portfolio diversification. This difference implies a different response to social security portfolio diversification, with Abel finding an income effect lowering investment from consumers who stop paying the fixed cost and stop investing in the stock market because of the change in social security portfolio. Moreover, Abel assumes an aggregate production function, leaving no role for direct choice about the riskiness of aggregate production.

Among the elderly, Social Security income is distributed very differently than private pension and asset income.² For the bottom quintile of the income distribution, 81% of income comes from Social Security, while only 6% is from pensions plus income from assets. For the top quintile, 23% comes from Social Security, while 46% is from pensions and assets — dramatically different percentages. Similarly, there are great differences in saving and investing among current workers. Among all those who were paying social security taxes in 1995, fully 59% held no stock, either directly or through pension plans. Even among those between 45 and 54 years of age, 50% held no stock, directly or indirectly.³ These differences have important implications for diversification proposals.

We represent this heterogeneity in saving behavior by supposing that there are two types of representative agents, one of which does no saving (except through social security) and the other of which saves and selects a portfolio (and, for simplicity is assumed not to be covered by social security). We refer to the two types of agents as workers and savers.

Our analysis includes heterogeneity in production. We suppose that there are two short-term technologies, which produce safe and risky output.⁴ We also assume there are two long-term real assets, called safe land and risky land, which produce safe and risky output in perpetuity. Distinguishing between safe and risky output allows portfolio diversification to increase production in one technology sector and reduce it in the other, thereby changing the riskiness of aggregate output. Including long-term real assets allows changes in the equilibrium prices of land to redistribute wealth between generations. Wealth redistribution is interesting for its own sake, but also because of its effect on investment.

Social security diversification is likely to change the rate of interest, requiring higher taxes to pay the higher coupons on government bonds. But the increased income tax burden may fall on households in different proportions than the increased interest income from holding government bonds. This, in turn, will have feedback effects on the equilibrium interest rate.

Our analysis integrates all these different effects, showing the impact of a small portfolio diversification on equilibrium, given assumptions on utility and technology. In order to keep the analysis simple, social security is modeled as an unfunded pay-as-you-go system together with a defined contribution system.⁵ Social security diversification occurs when the asset mix in the defined contribution system is suddenly shifted from bonds toward equities, and then maintained at the higher equity level forever after. Many proposed reforms of the U.S. social security system roughly fit this model. For example, if workers were suddenly given discretionary accounts, some of them with little or no outside saving would choose to invest part of their accounts in equities, and then our analysis would apply.⁶ The differences between defined benefit and defined contribution systems as distributors of rate-of-return risk have been explored in OLG models with a single representative agent per generation.⁷ This paper is meant to complement those studies.

The paper begins in Sections 2-4 by laying out the model. Five assumptions are spelled out. We suppose that the demands by savers for consumption when young, and

²See Mitchell and Moore (1997), Social Security Administration (1996).

³Quoted in Geanakoplos, Mitchell, and Zeldes (1999). See Kennickell, Starr-McCluer, and Sunden (1997) and Ameriks and Zeldes (in progress).

⁴With two types of production, the aggregate uncertainty in production varies with the mix of the two types. This is a simple way to allow aggregate uncertainty to be endogenous.

⁵Equivalently we could think of it as a partially-funded defined benefits system that adjusts benefits in response to asset returns in the same way.

⁶Since we suppose that all workers have the same utility, it makes no difference whether social security accounts are personal or are managed by a trust fund, provided that they choose the same asset mix and benefits are adjusted in this way.

⁷See, for example, Bohn (1997, 1998, 1999), Diamond, (1997). We are not aware of other equilibrium studies considering portfolio diversification.

for safe and risky consumption when old, are normal; that increases in government bond interest payments raise the payments on government bonds held by social security more than they raise workers' income taxes (and thus raise savers' taxes more than they raise savers' income on the government bonds they hold); that the level of risky investment does not affect the relative outputs across states (short-term risky production is along a ray in state space); that the output from both short-term and long-term risky production is independently and identically distributed each period; and that workers' wages are not stochastic.

Under these five assumptions, (a little) diversification can generate a Pareto gain and necessarily raises a suitable social welfare function.⁸ In a weak sense, it increases risky investment, decreases safe investment, raises interest rates, lowers expected returns on short-term risky securities, and reduces the equity premium. Aggregate investment might rise or fall, aggregate land prices might go up or down, and welfare of old and young savers might go up or down. The possibility that the direction of change of some variables depends on technology has not appeared in the social security reform debate.

To isolate each possibility and illuminate its cause, we build our general model gradually. In the first model (Section 5), we suppose that risky and safe production each consist of a single linear activity, so that technology determines the rates of return on both safe and risky assets. Diversification cannot change equilibrium prices, and thus has no effect on the utility of savers. On the other hand, diversification does change the level and riskiness of social security benefits, raising the expected utility of workers if they prefer some stocks to an all-bond portfolio.⁹ This effect persists in all of the models considered. Moreover, diversification in this model raises risky investment, lowers safe investment, leaves aggregate investment unchanged and so raises expected output.

In the second model (Section 6), we retain the risky activity but suppose that there is no safe investment in the original equilibrium. Diversification now raises the safe interest rate, requiring an increase in taxation to pay the government bond interest. This creates welfare effects (from changing returns and changing taxes) in addition to the direct welfare impact of the change in portfolio on the social security benefits. Diversification raises risky investment, and so aggregate investment. In Section 7 we add infinitely-lived assets to the model of Section 6. Diversification still raises the safe interest rate. Now, a rise in the safe interest rate lowers the value of infinitely-lived assets, hurting the savers holding these assets and benefitting young savers in the future, creating a feedback leading to further investment in the future. This case illustrates the possibility that the purchase of stock and sale of bonds by the social security system might paradoxically lower stock values, even under plausible circumstances.

To clarify the dependence of some of the results on technological assumptions, we consider a model (Section 9) where there is a perfectly safe linear technology, but no risky technology. The safe linear technology fixes the equilibrium government bond interest rate. In this case the price of infinitely-lived risky assets goes up after diversification, generating an intergenerational redistribution in the opposite direction to that of the

⁸Diversification from a point of zero exposure to equities raises the sum total of weighted utility in the economy if household utilities are weighted so that the expected marginal utility of a dollar for sure is the same for every saver and every retired worker. Since workers do not save, their marginal rates of substitution across time are not proportional to interest rates, and it is impossible to require marginal utilities for young workers to match up as well, for all time periods. But even if they do not match up in the welfare function, the welfare function must be increased by diversification.

⁹This conclusion does not rely on an assumption that workers are more risk tolerant than savers. Presumably they are not. The welfare gains come from the superior risk-sharing social security diversification permits when there are workers who do not have savings to invest on their own, and when, in the absence of social security diversification, social security benefits have a low correlation with stock returns. This point was made in Geanakoplos, Mitchell, and Zeldes (1999), who also tried to quantify the welfare gain in a special quadratic example.

previous model. Moreover, diversification now lowers safe investment, and so aggregate investment. This case highlights the possibility that social security diversification might raise the price of equities, benefitting the old savers but reducing the equity returns for all future savers, making them worse off. This drop in welfare might lead them to reduce savings, thereby reducing future investment and output.

Section 10 has a general nonlinear model of production that includes all the previous linear models as special cases. Section 11 discusses more general defined benefit systems. Section 12 has concluding remarks. Proofs of some Propositions are in the Appendix.

1 Technology

We analyze the equilibrium of a stochastic overlapping-generations economy, where each generation lives for two periods. There is one perishable consumption good in each date-event, which can either be eaten or invested using a productive technology. Young consumers have nonstochastic endowments, which can be interpreted as earnings from inelastically supplied labor with a technology that is linear and nonstochastic in labor.

At each date-event there are two short-term investment opportunities which transform the single perishable consumption good into (safe or stochastic amounts of) consumption goods in the next period. In Sections 1–9, we assume a linear short-term technology to avoid the complications from feedback of investment levels on rates of return to productive investments. The safe investment produces $R_0 k_0$ in the period following an investment of k_0 , with no durability in the capital, where $R_0 > 1$ is a constant. (Thus we are assuming a positive safe rate of return.) The risky investment produces Rk in the period following an investment of k , also with no durability in the capital, where $R > 0$ is a random variable. For convenience, we assume the risky returns to be independently and identically distributed each period. Each of these technologies may or may not be used in equilibrium, depending on rates of return.¹⁰ In Section 7 we add to the model two types of infinitely-lived assets, called land, yielding safe and risky outputs in perpetuity. In Section 10 we generalize the model to allow for nonlinear production.

2 Consumers

To bring out the difference between social security covered workers and wealth holders, we assume there are workers who do not save and savers who are not covered by social security; that is, two representative agents in each birth cohort.¹¹ We assume that each worker receives w in the first period, with each saver receiving W .

We assume no population growth and normalize the population so that there is a unit measure of (identical) savers and a measure of size n of (identical) workers. The representative saver maximizes expected lifetime utility of consumption, taking prices as given. Expected lifetime utility, V , is equal to $U_1[C_1] + E\{U_2[C_2]\}$, where C_1 is consumption when young, and C_2 is consumption when old, and with U_i increasing, concave and twice continuously differentiable. In the model without land, the savers divide exogenous first period wealth, W , among consumption and (up to) three tangible assets — government bonds, B , and two types of physical capital: k_0 , which is the safe asset, and k , which is

¹⁰When a technology is not in use, we suppose that the marginal utility of beginning to use it is strictly less than the marginal cost of beginning to use it. For completeness, we mention that there are (knife-edge) regimes with a technology that is not in use but with the marginal benefit of beginning to use it exactly equal to the marginal cost of beginning to use it. Generically, these regimes will not be observed.

¹¹Feldstein (1985) makes a two-types assumption in his classification of agents as rational and myopic. Having savers covered by social security would complicate the notation without changing the analysis.

the risky asset. In addition, the savers pay income taxes, T , in the second period.¹² Thus, we denote expected utility maximization for the representative saver by:

$$\begin{aligned} V &= \max U_1[C_1] + E\{U_2[C_2]\} & (1) \\ \text{s.t. } & W = C_1 + B + k_0 + k \\ & C_2 = (1+r)B + R_0k_0 + Rk - T, \end{aligned}$$

where the rate of return, R , is random, but taxes are not, as of the time of first-period decisions. If the safe real asset is held in equilibrium, then $1+r$ is equal to R_0 , since the government bond and the safe real asset are perfect substitutes.

Consumer choice can also be viewed in terms of three (composite) consumer goods — first-period consumption and safe and risky second-period consumption, which we denote as C_1 , J and K .¹³ It is therefore convenient to imagine that there is a safe financial asset promising one unit of safe consumption and also a risky financial asset promising one unit of risky consumption R , so that J and K can be bought directly.¹⁴ With first-period consumption as numeraire, we denote the price of second-period risky consumption as p_K . The price of one unit of second-period safe consumption is denoted by p_J . When the risky investment is undertaken in equilibrium, $k > 0$, then we must have $p_K = 1$. When safe investment is undertaken in equilibrium, $k_0 > 0$, then we must have $p_J = 1/R_0$. Whether or not real safe investment is undertaken in equilibrium, p_J is always equal to $1/(1+r)$, as long as the supply of government bonds to savers is positive. We now restate the consumer choice problem as:

$$\begin{aligned} V &= \max U_1[C_1] + E\{U_2[J + RK]\} = \max V^*(C_1, J, K) & (2) \\ \text{s.t. } & C_1 + p_J J + p_K K = I = W - p_J T. \end{aligned}$$

Demands for all three consumer goods are functions of the prices of second-period safe and risky consumptions and of net lifetime wealth. We denote them by $C^*[p_J, p_K, I]$, $K^*[p_J, p_K, I]$ and $J^*[p_J, p_K, I]$.

Throughout the paper, we *assume* the function $V^*(C_1, J, K)$ is such that all three of first-period consumption, and safe and risky second-period consumptions are normal goods. The normality of the three goods in turn *guarantees* that all three goods are Hicksian substitutes (given the intertemporally additive structure of preferences described in (2)). Moreover, a sufficient condition for normality of all three goods is that second period utility displays decreasing absolute risk aversion (DARA) and increasing relative risk aversion (IRRA). (For proofs of these assertions, see Aura, Diamond, Geanakoplos, (2002).)¹⁵

In contrast, we model workers, who also have two-period lifetimes, as nonsavers. Each worker earns a wage, w , in the first period (with inelastically supplied labor), pays payroll

¹²Taxes are used to pay interest on government bonds. By collecting taxes in the second period of life, they are paid back to the same cohort they are collected from. Collecting taxes in the first period instead would be equivalent to changing the level of government debt outstanding.

¹³Since all trading and production opportunities can be written in terms of these composite commodities, analysis of equilibrium can be done in these terms. Written in this form, the usual properties of compensated demands hold for the vector of consumptions. On the properties of compensated demands in the presence of uncertainty, see Diamond and Yaari (1972) and Fischer (1972). Moreover, analysis can be done in this form without the assumption of expected utility.

¹⁴When risky real investment is being undertaken, we can interpret the risky financial asset as shares in the output of a risky firm. When there is no real investment being undertaken, then this risky financial asset is like a contingent futures contract. An investor can acquire the right to future risky consumption by buying the risky financial asset (i.e., the shares or the futures contract) without having to actually undertake any risky investment, provided that he can find somebody who is willing to sell the asset.

¹⁵For the reader interested in non-expected utility maximization, we must assume that V^* is such that all three goods are normal, *and* that all pairs of goods are Hicksian substitutes.

taxes t_w in the first period, and consumes $w - t_w$. In the second period, workers consume social security benefits, b , which may be random, less income taxes t . We denote workers' lifetime utility by v and note that it satisfies:

$$v = u_1[c_1] + E\{u_2[c_2]\} = u_1[w - t_w] + E\{u_2[b - t]\}. \quad (3)$$

We distinguish two sources of taxes since the first-period payroll tax will be used for social security, while the second-period income tax will be used to pay part of the interest on the government debt outstanding.

The lack of randomness in income for young workers, w , guarantees a lack of randomness in the pay-as-you-go component of the financing of social security benefits for contemporaneous old workers, as we shall see in the next section.

3 Government and the Social Security System

It is assumed that each period the government rolls over one-period debt with a value of G . The interest payments on this debt are financed by income taxes on older workers and older savers, with the principal rolled over to preserve the debt outstanding.

$$T_t + nt_t = r_{t-1}G, \quad (4)$$

where taxes collected in period t are used to pay interest at rate r_{t-1} on debt issued in period $t - 1$. We assume that taxes are divided between savers and nonsavers in the proportions a and $1 - a$, $0 < a < 1$. Using the relationship between r and p_J , $p_J = 1/(1+r)$, we have the period and steady-state relations:

$$\begin{aligned} T_t &= ar_{t-1}G; \quad t_t = (1 - a)r_{t-1}G/n; \\ T &= arG = a(1 - p_J)G/p_J; \\ t &= (1 - a)rG/n = (1 - a)(1 - p_J)G/np_J. \end{aligned} \quad (5)$$

Note that income taxes in units of first-period consumption equal $a(1 - p_J)G$.

We model the social security system as a combination of a pay-as-you-go system together with a defined contribution system without worker choice of portfolio. Equivalently, we can think of the system as a partially-funded defined benefit system where the stochastic returns on stocks are fully used in determining that period's retirement benefits. The social security trust fund holds the value F of government debt, and the value $p_K K^f$ of risky assets (possibly equal to zero at the outset). Denoting the total value of the trust fund by F_0 , and supposing the trust fund holds only short-term assets, the trust fund budget set in any period is:

$$F + p_K K^f = F_0. \quad (6)$$

Given the need to maintain the trust fund portfolio, F and $p_K K^f$, and given constant payroll taxes t_w , and a stationary population, social security benefits satisfy the period and steady-state relations:

$$\begin{aligned} b_t &= t_w + ((1 + r_{t-1})F_{t-1} - F_t + (RK_{t-1}^f - p_K t K_t^f))/n \\ b &= t_w + (rF + (R - p_K)K^f)/n. \end{aligned} \quad (7)$$

Thus the expected utility of workers, v , given in (3), satisfies the steady-state relations:

$$v = u_1[w - t_w] + E\{u_2[t_w - t + (rF + (R - p_K)K^f)/n]\}. \quad (8)$$

Observe from equation (7) that all the variations in risky asset payoffs held by the trust fund are passed through directly to the current retirees. There is no risk sharing across generations, as there could be in a defined benefits plan, either by spreading return risks across several cohorts or by varying the payroll tax rate. The wage and the payroll tax rate are assumed to be constant over time; the retirement benefits, however are free to vary, and will do so if the rates of return earned on the trust fund holdings vary. Similarly, the second-period income tax will change if the interest rate on government debt changes.

A crucial part of our analysis is that if $K^f = 0$, young workers at time $t - 1$ can look forward with certainty to the social security benefits they will receive when they are old at time t . The return r_{t-1} they will get from the trust fund bond investment is already locked in. Furthermore, they can perfectly predict the pay-as-you-go portion of their benefits, since, in stationary equilibrium, wages of the young at time t are nonrandom. In reality, of course, future real wages cannot be predicted with certainty. In our judgment, however, they are substantially less random than stock returns.

4 Stationary Equilibrium without Land

In stationary equilibrium, prices and young savers' consumption and asset holdings are constant through time and across states of nature. All that varies is output, consumption of the old savers and old workers, and social security benefits. With a single commodity, and stationary and independent productivity shocks, stationary equilibrium will exist. Since wages do not vary, a new steady state is reached starting with the generation born immediately after a permanent policy change.

When savers undertake risky investment, p_K is equal to one. Stationary equilibrium in the model with short-term risky production, but without land, is then defined by prices and quantities (r, C_1, C_2, B, k_0, k) such that given r and taxes T , the choices (C_1, C_2, B, k_0, k) solve the savers' optimization problem (1), and such that savers' demand for government bonds equals the supply available to savers:

$$B = G - F = G - F_0 + p_K K^f. \quad (9)$$

If safe real investment is undertaken in equilibrium, then the interest rate on government bonds equals the return on safe investment. If not, then the government interest rate is determined by market clearance with no additional supply of safe assets.¹⁶

Alternatively, we can write the market clearing conditions in terms of the consumption demands C^* , J^* , K^* introduced in (2). This separates the consumption and savings decisions of the savers from the production decisions of firms. From now on we interpret k_0 to be the safe production chosen by the firms, and we interpret k as the risky production chosen by the firms. Savers' demand for safe second-period consumption must equal the supply of safe second period consumption to savers, which is equal to the total principal and interest payments of government bonds, less what is held by the social security system, plus the level of safe production, less what is needed to pay taxes.

$$\begin{aligned} J^*(p_J, p_K, W - a(1 - p_J)G) &= (G - F_0 + p_K K^f)/p_J + R_0 k_0 - a(1 - p_J)G/p_J \\ &= B/p_J + R_0 k_0 - a(1 - p_J)G/p_J, \end{aligned} \quad (10)$$

¹⁶Since the savers are both the demanders and the suppliers of real investment, the investment markets automatically clear if savers solve (1). The consumption market automatically clears as well, once the bond market clears. To check this, we can verify that supply of consumption equals demand,

$$W + nw + (R_0 - 1)k_0 + (R - 1)k + (R - 1)K^f = C_1 + C_2 + nc_1 + nc_2.$$

The reader can verify that after substituting for C_1 and C_2 from (1), c_1 and c_2 from (3), taxes from (5), and benefits from (7), this equation reduces to (9).

where $a(1 - p_J)G$ is the present discounted value (PDV) of taxes. Using the same variables, we can write market clearing in the risky good market as

$$K^*(p_J, p_K, W - a(1 - p_J)G) = k - K^f. \quad (11)$$

The supply of risky second period consumption to savers is equal to risky production, less what is held by the social security system.¹⁷

Market clearance could occur with or without each type of production, depending on rates of return. Profit maximization of the firms gives

$$\begin{aligned} p_J &= 1/(1 + r) = 1/R_0 && \text{if } k_0 > 0 \\ &\leq 1/R_0 && \text{if } k_0 = 0 \\ p_k &= 1 && \text{if } k > 0 \\ &\leq 1 && \text{if } k = 0 \end{aligned} \quad (12)$$

Stationary equilibrium is now defined as a vector (p_J, p_K, k_0, k) such that (10)–(12) hold. The condition defining the savers' holdings of government bonds B , given by (9) still holds, and we continue to use B as a convenient shorthand for the RHS of (9). But equation (9) will not be treated as an independent equation, since it follows from (10)–(12) given the saver's budget constraint in (2) and trust fund budget constraint in (6).

Depending on whether $k_0 > 0$ or $k_0 = 0$, and whether $k = 0$ or $k > 0$, equilibrium can be one of four different types, or regimes.¹⁸ The effect of social security diversification depends crucially on which regime the original equilibrium is in.¹⁹ In each case, we analyze the effect on equilibrium of a change in trust fund investment in risky assets: $d(p_K K^f) = -dF > 0$. Since we are mainly interested in the case where $K^f = 0$, when $d(p_K K^f) = dp_K K^f + p_K dK^f = p_K dK^f$, in what follows we shall take K^f as the exogenous variable, and we shall compute the equilibrium comparative statics by totally differentiating the equilibrium equations with respect to K^f . Since wages do not vary, the economy achieves stationary equilibrium in a single period after a change in the portfolio allocation of the trust fund. If the unanticipated change comes at some date t , then generations born at date t and after will consume as in the new steady state, and generations born at date $t - 2$ and before will consume as in the original steady state. The generation born at date $t - 1$ will consume as if it made consumption and asset choices when young in the original equilibrium, but was then forced to pay taxes and liquidate assets at date t at the new steady state prices.

5 Social Security Diversification with Both Safe and Risky Investment

In this section, we assume the economy is such that in equilibrium both physical assets are held, $k_0 > 0$, $k > 0$. In this setting, technology determines prices. That is, the interest rate on government bonds equals the (exogenously fixed) rate of return on the safe asset, and the price of the risky consumption good equals 1, the cost of the risky

¹⁷To complete the picture we could explicitly model the production decisions of the firms to maximize profit:

$$\max[p_J R_0 k_0 - k_0] + \max[p_K k - k].$$

¹⁸As mentioned above, we shall restrict attention to economies that have no knife-edge stationary equilibria in which $k_0 = 0$ and $p_J = 1/R_0$, or in which $k = 0$ and $p_K = 1$.

¹⁹Furthermore, small changes in the trust fund create small changes in equilibrium. Equilibrium before and after social security diversification will therefore be of the same type.

physical asset. Since prices do not change when the trust fund alters its portfolio, savers are left unaffected. (With the interest rate unchanged, second-period taxes do not change, so the budget set of savers is indeed unaffected.) With unchanging prices, savers demand the same combination of all three consumption goods — first-period, second-period safe and second-period risky consumptions. Thus, if the trust fund sells some bonds and uses the money to invest in risky production, savers are indifferent to buying the extra bonds and reducing their investment in safe production by the same value, thereby maintaining the equilibrium. Thus aggregate risky investment goes up, aggregate safe investment goes down and aggregate investment is unchanged. Since the expected return on risky investment must exceed the return on safe investment (for both to be held by risk-averse savers), expected aggregate output goes up.²⁰

If the trust fund initially has only a small amount of the risky asset, this policy is a (weak) Pareto gain — savers are not affected and workers gain since the workers are not risk averse for the first bit of investment in risky assets. To see this, consider the change in worker expected utility, (8), (noting that interest rates r and therefore taxes t are unaffected). Assuming that K^f is zero, an increase in K^f and matching decrease in F affects expected utility as

$$\begin{aligned} dv/dK^f &= E\{u'_2[c_2](R - 1 - r)\}/n \\ &= u'_2[c_2]E\{(R - 1 - r)\}/n > 0. \end{aligned} \tag{13}$$

The last equality is obtained by noting that second period consumption of workers is certain, hence so is second period marginal utility, so it may be brought outside of the expectation operator. The final inequality follows from the excess expected risky return (see footnote 21). Thus we have shown

Proposition 1. *Suppose both the safe and risky assets are held in stationary equilibrium (in positive quantities). Then, increased trust fund investment in risky assets increases risky investment, decreases safe investment, leaves aggregate investment unchanged and increases expected output.*

If the trust fund initially held no risky assets, then the diversification will lead to a weak Pareto improvement, increasing the utility of all workers (except the old at the time the policy is implemented, who are unaffected) and leaving the utility of all savers unchanged.

The equity premium is defined as the difference between the expected return on the risky investment and the return on the safe investment, $E\{R\} - (1 + r)$. Since the equity premium must be consistent with the portfolio choice of risk-averse savers (who hold a strictly positive quantity of risky assets by hypothesis), it must be positive in equilibrium. As long as the equity premium is positive, there is an expected utility gain to workers from diversification in a model where they bear no other risk.

The crucial step in this argument is the paradoxical claim that workers are *more* risk tolerant *on the margin* than savers. One might suspect that savers are more risk tolerant than workers, all else being equal. That is, it may well be that the worker utility u is a concave transformation of the saver utility U , thereby displaying more risk aversion at any level of consumption. And workers have lower incomes on average than savers, which also makes them more risk averse if there is decreasing absolute risk aversion. But all else

²⁰To see this, note that with $p_K = 1$ and both assets held, the savers' first order condition is:

$$U'_1[C_1] = E\{U'_2[C_2](1 + r)\} = E\{U'_2[C_2]R\};$$

Since C_2 and R are perfectly correlated, $U'_2[C_2]$ and R are negatively correlated. Hence the equality of expectations can only hold if $E\{R\} > 1 + r = R_0$.

is not equal. The savers hold the entire risky capital stock of the nation, while the workers hold none (if K^f is zero). Our proof that welfare rises after social security diversification only needed that both u and U are differentiable, and that workers are not exposed to any stock market risk or other risks correlated with stock market risk.

In reality, workers' retirement income is not completely statistically independent of stock returns. Social security benefits are connected by an explicit formula to real wages. Over career horizons as long as forty years, there is considerable covariance between real wages and stock returns. The question then becomes, how big is worker exposure to stocks, how big is the equity premium, and how risk averse are workers? Addressing this question in detail is beyond the scope of this paper.²¹ Our judgment is that after properly calibrating the stock exposure implicit in aggregate wages, one would come to the conclusion that the average worker is significantly less exposed to stock returns than savers. At the point where the trust fund holds no stock, it seems very likely to us that the average worker would be better off by some investing in equity. The converse would hold only if it would be optimal for such a worker just starting to save, to hold a portfolio with no stocks at all.

However, what is best for the average worker may not be best for every worker. Though our model has assumed that all workers are identical, in reality some workers may be far more risk averse, so that for them any additional stock exposure may be bad, preventing social security diversification from being a Pareto gain (Pestieau and Posen, 1999). However, in considering a more general setting with heterogeneous workers, the reader should bear in mind that the lowest income workers would be protected by the safety net (SSI).

By the same logic used in the proof of Proposition 1, further increases in social security risky asset holdings would also be weak Pareto gains until the optimal portfolio for workers was reached, unless the saver's holdings of the safe real investment reached zero first.²² In considering the optimal level of social security diversification, we note that since social security benefits become more correlated with stock returns as diversification increases, the welfare benefits to further diversification decline. The proof of Proposition 1 is thus an argument for limited diversification.

The welfare gain from social security diversification in Proposition 1 is not related to any "unexplained" excess return of stocks over bonds. If savers were also covered by social security, Proposition 1 would show no (ex ante) gain from social security diversification for the typical saver, despite the equity premium (except if some savers were 100% in stocks in their portfolio and wanted some of social security to be in stock as well). For every dollar in the social security trust fund that is shifted to equity, the welfare gains described in Proposition 1 apply to that fraction of each dollar that goes to support the benefits of workers with little financial wealth who do not borrow, and are therefore holding no stocks.²³ If there were no such workers, as is the case in a representative agent model with only rational savers, a small enough change in trust fund portfolio policy would have no effects at all (Smetters, 1997 and Bohn, 1997, 1998).

The welfare gains to nonsavers from social security diversification holds in all of our

²¹If the workers' and savers' utilities u and U display similar risk aversion, and both display increasing relative risk aversion, then the poorer workers should have a higher fraction of their wealth invested in stocks than the richer savers.

²²The Social Security program is a response to the inadequacy of retirement saving by many workers. Just as mandating savings can raise utility for many workers, adjusting the portfolio can add to utility.

²³Some workers are unable to hold stocks because they have not saved enough. Others do not hold stocks even though they could. Some of the latter may not be optimizing and would also gain from the diversification, while some may not have been willing to bear the cost of learning about stocks and would also gain since they do not have to pay a cost if investment is done centrally. However, workers so risk averse that they should hold no stocks would lose from diversification, as noted above. Some workers may mistakenly be overinvested in stocks and would also lose from trust fund diversification if they do not reduce their stockholdings in response to trust fund investment.

subsequent models. In later models the technology is not perfectly elastic. Then the change in social security portfolio will lead savers to alter their final consumption, forcing equilibrium prices to change, in directions we investigate in the following models. Since workers do not save, the effect of social security diversification on asset prices and taxes is independent of their (marginal) risk tolerance.

6 Social Security Diversification with only Risky Investment

While the riskiness of aggregate output is plausibly endogenous, as in the bilinear model above, it is unrealistic to suppose that technology fixes the returns on all assets, independent of preferences. We now consider an economy with the expected return on risky investment given by technology, but without any safe investments, implying that the return on government bonds is endogenous. That is, in equilibrium: $k_0 = 0$ and $k > 0$. Given the constant marginal returns to risky investments, equilibrium requires $p_K = 1$. The interest rate on government debt is determined by the supply and demand for bonds, with market clearance given in equation (10). Combining the budget constraint in (2) with (5), (9), and (10) reduces market clearing to a single equation in a single variable $p_J = 1/(1+r)$:

$$B = G - F_0 + K^f = W - C^*[p_J, 1, W - a(1 - p_J)G] - K^*[p_J, 1, W - a(1 - p_J)G]. \quad (14)$$

That is, government bonds not held by social security equal the wealth of savers less what they spend on first-period consumption and what they invest in risky production. Note that with G , F_0 , and W all fixed, the response of aggregate investment, $K^* + K^f$, to portfolio policy is minus the response of the consumption of savers, C^* . To analyze the effect of diversification on the interest rate, we differentiate (14) with respect to K^f . Since diversification increases the supply of bonds available to savers, we would expect it to lower p_J and raise r . But the situation is a bit more complicated. Any change in the interest rate (with no change in gross debt outstanding) requires a change in income taxes to cover interest costs. Thus, to sign the change in the interest rate, we use a further assumption on the share of taxes paid by savers, beyond our assumptions on demand functions. By changing the interest rate and therefore also taxes, social security diversification causes a redistribution between savers and workers, in addition to the gain to workers noted in Section 5. To analyze all these changes, we first derive the changes in utilities from an arbitrary change in the interest rate and taxes, and from this relation we deduce that under our assumptions the interest rate does indeed increase.

6.1 Income Taxes, the Interest Rate, and Welfare

A change in the price of safe second-period consumption affects the utility of savers in two ways. One is the change in the cost of safe second-period consumption, which, by the envelope theorem, is $-U'_1 J^*$. The second is the change in the PDV of taxes paid. Using the market clearing condition for J^* given by (9) and (10), the change in utility to savers from a change in p_J is

$$\begin{aligned} \partial V / \partial p_J &= -U'_1 \{J^* + d[a(1 - p_J)G]\} / dp_J \\ &= -U'_1 \{[B - a(1 - p_J)G] / p_J - aG\} \\ &= -U'_1 [B - aG] / p_J \\ &= -E\{U'_2\} [B - aG] / p_J^2, \end{aligned} \quad (15)$$

where we have used the FOC for consumer choice in the last step.

A change in the interest rate on government bonds causes a redistribution of income between taxpayers and interest recipients. Hence, if the shares of marginal second-period taxes paid by savers, a , and workers, $1 - a$, do not match their shares in the holding of government debt (directly by savers and indirectly through social security for workers), $B \neq aG$, a change in the interest rate affects utilities. Recognizing that p_J is the only endogenous price, it follows from (15)²⁴ that the response of expected utility to trust fund portfolio diversification satisfies:

$$\begin{aligned} dV/dK^f &= -E\{U'_2\}\{(B - aG)/p_J^2\}\{dp_J/dK^f\} \\ &= E\{U'_2\}(B - aG)\{dr/dK^f\}. \end{aligned} \quad (16)$$

Workers are affected by trust fund investment as in section 5 and by the impact of the interest rate on benefits and taxes. Differentiating expected lifetime utility of workers (8) with respect to $dK^f = -dF$, noting from (5) that $dt/dr = (1 - a)G/n$ and from (9) that $F = G - B$, we have:

$$dv/dK^f = E\{u'_2(R - 1 - r)\}/n - E\{u'_2\}(B - aG)(dr/dK^f)/n. \quad (17)$$

As was the case in the regime with both investments, the expected utility of workers increases from bearing some risk if they were bearing none before diversification. In addition, the effect on workers from the change in the interest rate has the opposite sign from its effect on savers, as can be seen by comparing (16) and (17). If a is equal to B/G , then this effect is zero and workers only gain from improved risky investment. When $a = B/G$, we have a (weak) Pareto gain, as in the case with both investments.²⁵

Denoting the social evaluation of the marginal utility of second-period safe consumption of a worker relative to that of a saver by m , the impact on a social welfare function for each cohort in steady state (SWF) equals the weighted sum of individual impacts:

$$\begin{aligned} dSWF/dK^f &= m\{ndv/dK^f\} + 1\{dV/dK^f\} \\ &= mE\{u'_2(R - r - 1)\} - E\{mu'_2 - U'_2\}(B - aG)dr/dK^f. \end{aligned} \quad (18)$$

Thus there is a direct utility gain from improved risk bearing and a redistributive term, which vanishes if a equals B/G . If there is an income distribution change ($a \neq B/G$), its effect depends on the direction of transfer and the sign of $E\{mu'_2 - U'_2\}$. In particular, if other policy tools result in a level of m so that one unit of second-period safe consumption gives the same expected marginal social welfare to every old agent, then the redistribution term drops out and total weighted utility is increased by diversification.

Whether $B < aG$ depends on the size of the trust fund and how tax policy responds to increased interest costs. If $F = 0$, then $B = G > aG$, and an increase in interest rates helps savers, because for every extra dollar in interest receipts received, they pay only $a < 1$ dollars extra in taxes. On the other hand, if the trust fund holds all the government bonds, then $B = 0 < aG$, and savers lose from an increase in interest rates.²⁶

²⁴and, $dp_J/dr = -p_J^2$

²⁵Actually, if $a = B/G$, then savers obtain a second order benefit from trust fund diversification, assuming dr/dK^f is not 0.

²⁶To consider who in reality is a receiver of government bond interest payments net of the taxes levied to pay for them, we need to consider which taxes are raised if interest costs are higher. If it is just the income tax increased, then low income people are not taxed at all. However, if the earned income tax credit is altered along with the income tax (violating our assumption that it is taxes on older workers that are adjusted), then the impact is throughout the income distribution. A realistic case would indeed consider tax changes on young workers and savers as well as on the old. That would have additional effects, which we are not analyzing.

In reality, the social security trust fund pays benefits to nonsavers and savers. But because of the redistributive nature of the social security benefit rules, nonsavers have a claim on benefits that exceeds their share of payroll taxes. At the end of 2001, the trust fund was \$1.2 trillion (OASI plus DI), and increasing rapidly. Once the trust fund is big relative to the outstanding stock of government bonds, interest rate increases can be expected to help workers and hurt savers.

6.2 Interest Rate and Aggregate Investment

Increasing the supply of government bonds available to the savers will raise the interest rate if demand slopes down, provided that the indirect effects of the interest rate on demand (through income taxation) do not offset the direct effect. The proof is straightforward and also shows that aggregate investment rises.²⁷ However, the rise in aggregate investment smaller than the increase in trust fund investment in risky assets.

We can state these results (which are proved formally in the appendix) as:

Proposition 2. *Suppose that risky investments, but not safe investments, are undertaken in equilibrium and that $B \leq aG$. Then, increased trust fund investment in risky assets increases the interest rate on government debt, and increase aggregate real investment, though by less than the quantity of the trust fund purchases.*

If the trust fund initially held no risky assets, then the diversification increases the expected utility of all workers (except the old at the time the policy is implemented, who are unaffected) and increases the weighted sum of utility of all workers and savers, weighted so that the marginal social utility of second-period consumption is the same for all. If in addition, $B = aG$, then trust fund diversification does not affect the utility of young savers in every generation (up to first order, increasing it up to second order). If instead, $B < aG$, then diversification lowers the utility of young savers in every generation.

Complementing this proposition, we note that if an increase in the government bond interest rate redistributes wealth from workers to savers, $B > aG$, then the interest rate may rise or fall.²⁸

6.3 Extending the Model to More Investment Technologies

In the bilinear model of Section 5, social security diversification resulted in more investment in the risky technology and less in the safe technology, and so riskier investment and output. This possibility of riskier aggregate investment is necessarily missing in models with just one technology, such as we just considered in Section 6. Yet, firms do make investment choices that affect the riskiness of their output. One way to model such choices would be to assume that each firm has two distinct risky technologies (identical across firms) and chooses the overall riskiness of its output by the mix of the two technologies (that is, an activity analysis model). By the Modigliani–Miller theorem, it would not matter which firm made which production choice as long as the aggregate of investment

²⁷From (15) we see that reducing p_J (i.e. increasing r) lowers the welfare of savers if $B < aG$. Since C and K are normal and Hicksian substitutes for J , a reduction in p_J lowers demands C^* and K^* , and thus by (14) raises the demand for B . This confirms that demand for B slopes down, so diversification raises r . The fall in C proves that aggregate investment rises, as noted after (14).

²⁸So far, we have considered two different models with perfectly elastic and perfectly inelastic supplies of safe assets. We could consider an intermediate model with a downward-sloping demand by foreigners for government debt. This would give a change in the equilibrium interest rate that was between the two cases analyzed. In this case, the increase in the interest rate on government debt would involve increased payments abroad as well as transfers from taxpayers to trust fund beneficiaries.

in each technology were the same. It might be interesting to see whether trust fund diversification into equities (buying the same fraction of each firm) led firms to choose riskier output. We do not pursue this question here.

7 Adding Infinitely-Lived Assets

In the models above, no asset lasts more than one period. Thus a change in the interest rate does not redistribute wealth across generations. To consider intergenerational redistribution, we now add two infinitely-lived assets. A change in policy that changes the prices of the long-lived assets will redistribute wealth between the old, who own the assets at the time the policy is implemented, and all future generations who buy them. We assume fixed quantities of both types of infinitely-lived assets, referred to as safe land and risky land. Each unit of safe land provides one unit of consumption, independent of the state of nature, in each period.²⁹ Each unit of risky land produces the same (realized) output as one unit of the (contemporaneous) risky investment.³⁰ We denote the supplies of the two assets by L_0 and L , and their prices by p_0 and p . Because of the stationary structure of the economy, in stationary equilibrium, these prices are constant over time. The effects of trust fund diversification are modified by the presence of land, but not drastically changed. As before, the analysis depends on which short-term investments are undertaken in the original stationary equilibrium. The case where both are undertaken, $k_0 > 0$ and $k > 0$, is exactly like the case without land since no prices change, and it is not repeated here. In the rest of Section 7, we reformulate the definition of equilibrium to include land.

In reality, the stock market is made up of both short-term and long-term investments, and so the effect of social security diversification on stock market prices involves both short-term and long-term asset price changes.

7.1 Land and the Dynamic Asset Span

Land (of either type) lasts forever and gives new output forever and is sold each period by the older generation to the younger one. Given a stationary economy, and the assumption that land output is independent and identically distributed each period, the price of land (just *after* the realization of output) is constant across time, and across realizations of output.³¹ The one-period gross return from purchasing land is equal to its dividend that period, plus a constant capital value. The one-period returns on either type of infinitely-lived land are therefore (endogenous) convex combinations of the returns on risky short-term investments and the safe return on government bonds. Thus we can incorporate land into our model without introducing a new risk characteristic. We do not need to reformulate the choice problem of savers in terms of the three composite consumer goods.

If a young saver buys one unit of safe land, it costs p_0 and yields safe consumption in the second period (from output and resale) of $1 + p_0$. Since this is a perfect substitute for buying $(1 + p_0)/(1 + r)$ units of the government bond, we have

$$p_0 = (1 + p_0)/(1 + r), \tag{19}$$

²⁹This might be a fixed number of government consols, the interest on which is financed by taxation on successive generations.

³⁰When the return to risky investment is independent of the level of investment (as assumed in all sections other than Section 10), there is no distinction between land that provides output and land that provides capital input.

³¹The iid assumption implies that there is never any “news” about future returns, so land values never change.

or

$$p_0 = 1/r = p_J/(1 - p_J). \quad (20)$$

Similarly, by spending p on risky land, the consumer gets the risky dividend that can be purchased at a price of p_K , (by investing in the short-lived risky asset) and the ability to sell the asset at price p , which has a current value of $p/(1 + r)$. Thus, by arbitrage, we have the equilibrium price of risky land satisfying:

$$p = p_K + p/(1 + r) = p_K + pp_J \quad (21)$$

or

$$p = (1 + r)p_K/r = p_K/(1 - p_J). \quad (22)$$

Thus the prices of both kinds of land are determined by the price of the short-term assets. Both land prices increase with the price of second-period safe consumption; equivalently, land prices decrease when the interest rate rises. The total value of land is

$$P = p_0L_0 + pL = L_0p_J/(1 - p_J) + Lp_K/(1 - p_J). \quad (23)$$

The supply of safe consumption because of land is the output from safe land plus the proceeds from the sale of all land, $L_0 + P$, which can also be written as $(L_0 + p_KL)/(1 - p_J)$. We note for later use that

$$dP/dp_J = (P + L_0)/(1 - p_J). \quad (24)$$

To keep the analysis simple, we shall continue to suppose that the trust fund holds only short-term assets.³²

7.2 Stationary Equilibrium with Land

We now define equilibrium in terms of the three goods C^* , J^* , K^* , as we did earlier. The presence of land does not change the expected utility maximization problem of savers given in (2) nor their demand curves. The presence of land does change the supplies of consumption. The supply of safe consumption now includes the output of safe land and the proceeds from the sale of both kinds of land. The supply of risky consumption increases by the output of risky land. Thus, equilibrium conditions (10) and (11) become

$$J^*(p_J, p_K, W - a(1 - p_J)G) = B/p_J + R_0k_0 - a(1 - p_J)G/p_J + L_0 + P, \quad (25)$$

$$K^*(p_J, p_K, W - a(1 - p_J)G) = k - K^f + L. \quad (26)$$

8 Social Security Diversification with Risky Investment and Land

We suppose the economy with land is such that in stationary equilibrium there is only risky production, $k_0 = 0$ but $k > 0$. Therefore, $p_K = 1$. As in Section 6, equilibrium

³²Any equilibrium where the trust fund holds L^f acres of risky land is equivalent to an equilibrium in which the trust fund holds L^f units of the risky asset and $L^f pp_J$ bonds. (The dividends are the same, and by (21) the bond payoffs pL^f can be used each period to repurchase the same portfolio.) In particular, starting from a portfolio exclusively in short-term assets, a trust fund purchase of L^f acres of risky land (obtained by selling bonds) would have precisely the same effect as the purchase of L^f units of the risky short-term asset (obtained by selling bonds). However, the effect of a further trust fund purchase of land will differ in the two cases because it will change asset prices, giving a different capital gain to the two portfolios.

boils down to the market for safe consumption. Rewriting (25) using the saver's budget constraint (2), and (5), (9), and (23), we have:

$$G - F_0 + K^f + p_J(L_0 + L)/(1 - p_J) = W - C^*[p_J, 1, W - a(1 - p_J)G] - K^*[p_J, 1, W - a(1 - p_J)G]. \quad (27)$$

This differs from (14) in Section 6 by the addition of the last term on the LHS, representing the supply of safe consumption from the presence of land. As before, we have a single equation in a single variable, p_J . And, with the same assumptions as before, we will again find that diversification causes p_J to go down, equivalent to the interest rate going up.

From (26), aggregate investment in short-lived production, k , is trust fund demand for real investment, K^f , plus the demand of savers for risky consumption, K^* , minus the portion of that demand that is satisfied by purchasing risky land, L . Thus, using (27), aggregate investment in risky short-term assets, can be written:

$$K^f + K^* - L = W - (G - F_0) - C^*[p_J, 1, W - a(1 - p_J)G] - P. \quad (28)$$

That is, short-term investment equals the wealth of savers less what they spend on consumption, on bonds and on land. With W , G , and F_0 all fixed, the response of aggregate investment in short-term assets to social security diversification is minus the sum of the response of consumption of savers, C^* , and the change in the value of total land, P . With the same assumptions as before, we will again find that aggregate investment goes up.

8.1 Expected Utility

In the previous models without land, old savers were not affected by social security diversification. But with the introduction of land they have something to sell, whose value might be affected by social security diversification. For example, if land prices go down in value (as the interest rate rises), the old savers at the time of the trust fund diversification lose, *ceteris paribus*. Young savers gain, as do savers in every succeeding generation.

We begin with old savers at the time of implementation of the policy change. They are affected only by the change in the value of the land that they hold:

$$dV_{\text{old}}/dK^f = U'_2[dP/dp_J]\{dp_J/dK^f\}. \quad (29)$$

Since the new stationary equilibrium is achieved immediately after the trust fund purchases, young savers at the time of the purchases are affected exactly the same way as all future savers. This differs from the setting without land, (16), only in that the demand for safe consumption (25) is met through land as well as bonds, in contrast to (10).

$$dV/dK^f = -E\{U'_2\}[(B - aG) + p_J(L_0 + P)]/p_J^2\{dp_J/dK^f\}. \quad (30)$$

The first term reflects the within-cohort redistribution between savers and workers as a consequence of different shares in government bonds and in the taxes to pay the interest on the bonds. The second term reflects the across-cohort redistribution from changes in the price of the safe consumption that is purchased from the previous generation by buying land. The formula is expressed in terms of the value of land. Using (24), the change in expected utility can be expressed in terms of the change in land value:

$$dV/dK^f = -E\{U'_2\}[(B - aG) + p_J(1 - p_J)dP/dp_J]/p_J^2\{dp_J/dK^f\}. \quad (31)$$

Now it is clear that changing land values does affect young savers. But if social security is diversified at time 1, and land prices fall, the young at time 1 do not gain by the whole drop in land prices, since the resale value of the land when they get old also falls.

Notice that the expected utility of young savers can increase or decrease, depending on the balance of redistributions between savers and workers, and redistributions between old savers and young savers. If the value of all land, P , exceeds the total of all government bond promises $G/p_J = G(1+r)$, then social security diversification must improve the welfare of young savers (assuming $dp_J/dK^f < 0$), even though it creates a redistribution from young savers to young workers if $B < aG$. Evidently young savers gain more from old savers than they lose to young workers. A similar conclusion holds if the value of safe land p_0L_0 is greater than the value of government bonds G outstanding, as can be seen from (30), and the equation $p_J(L_0 + P) = p_0(L_0 + L)$.

Equation (17) quantifying the effect of trust fund diversification on workers in the risky linear case without land applies without change in this risky linear case with land. As before, the increased exposure to risky stock and the rise in interest rates make workers better off, assuming $B - aG < 0$ and $dp_J/dK^f < 0$. The analysis of a social welfare function is also unchanged.

8.2 Interest Rate, Investment and Land Values

From (20) and (22), the value of safe land and of risky land each move in the same direction as p_J , that is in the opposite direction of the change in interest rates. Thus assuming that diversification raises r , as it did when there was no land, it follows that trust fund purchases of risky short-term investments *reduce* the price of risky land (and also the price of safe land). It is a remarkable, and unanticipated, property of the current model that the increase in demand for risky land reduces its price! It is often claimed that if social security bought stocks, it would raise the value of the stock market. This conclusion is seen to be more delicate than it sounds. Since the interest rate increases, it is not so surprising after all to find a tendency for stock prices to decline, for stock prices depend on discounting future returns. When technology fixes the return on short-term risky investments, the interest rate effect is the only one impinging on stock prices. We pursue this question of land values further in the next sections.

It remains to show that trust fund purchases of risky investment do indeed increase the interest rate on government debt. An increase in the supply of government bonds available to the savers will raise the interest rate if demand slopes down. The proof requires confirming that the indirect effects of the interest rate on demand (through income taxation and the change in land values) do not offset the direct effect. However, the situation is much subtler than it was without land in section 6 because the welfare of young savers might go up or down. Yet the effect on interest rates and aggregate investment is unambiguous, as we show in the appendix.

Proposition 3. *Suppose there is both safe and risky land, that risky investments, but not safe investments, are undertaken in equilibrium and that $B \leq aG$. Then, increased trust fund investment in risky assets increases the interest rate on government debt, and increases aggregate real investment. The increase in aggregate investment may be larger or smaller than the trust fund purchase of risky investment. Moreover, the prices of both kinds of land fall. The total value of land therefore falls, though $|dP/dK^f| < 1/p_J$.*

If the trust fund initially held no risky assets, then the diversification increases the expected utility of all workers (except the old at the time the policy is implemented, who are unaffected). Old savers at the time the policy is implemented are hurt. If, in addition, $(aG - B) < p_0(L_0 + L)$, then all other savers are helped. If, however, $(aG - B) > p_0(L_0 + L)$, then all other savers are also hurt. Nevertheless, the diversification increases the weighted sum of utility of all workers and savers, weighted so that the marginal social utility of second-period consumption is the same for all.

From differentiation of the equilibrium condition, we note that the presence of long-lived assets decreases the sensitivity of interest rates to trust fund diversification, and thus decreases the size of the interest rate increase. We might interpret the quantitative part of Proposition 3 as follows. Given that a period in this model represents something like 30 years, and that the real interest rate has historically been about 2.3% per annum, a crude estimate of p_J is about $\frac{1}{2}$. A \$500 billion transfer of trust fund assets from bonds into stock, maintained there forever, must lower land prices, but could not lower land prices by more than \$1 trillion. Moreover, the level of real investment could increase by more than \$500 billion.

9 Social Security Diversification with only Safe Investment and Land

To show that social security diversification could have other effects, depending on the technology, we turn to an economy with one-period safe real investments but no one-period risky investments undertaken in equilibrium, $k_0 > 0$ and $k = 0$. Many of our preceding results are now reversed. While perhaps extreme, this case illustrates some of the effects of trust fund diversification if there are rapidly diminishing returns to risky investment, so that trust fund diversification reduces the risky investment opportunities available to savers. The interest rate on government bonds is technologically determined by the return on safe real one-period assets, $p_J = 1/(1+r) = 1/R_0$, and the price of safe land is technologically determined as well. The price p_K of risky consumption, and the price p of risky land, depend on the evaluation of risky consumption by savers. To reflect this endogeneity, we analyze the effects of an increase in the value of trust fund holdings of risky investments, $p_K K^f$, rather than the quantity K^f . Starting with $K^f = 0$, the two analyses are the same (since $K^f dp_K = 0$).

We could envision someone supplying a short-term risky financial asset promising R without any short-term real risky production. A seller of this asset could simply deliver out of land dividends, without producing any risky output. Thus we can define social security diversification exactly as before, namely as the sale of bonds and the purchase of short-term risky securities. Recall that starting from a position with no risky securities, trust fund purchases of short-term risky assets in exchange for bonds has exactly the same effect as the trust fund purchase of risky land in exchange for bonds.

In the absence of risky short term production, the only source of risky consumption is risky land, and each acre of risky land provides one unit of risky consumption. (Hence K^* can be interpreted as the savers' demand for risky land.) Market clearing in the market for risky consumption, (29), now reduces to:

$$K^*(1/R_0, p_K, W - a(1 - 1/R_0)G) = L - K^f. \quad (32)$$

Thus we again have a single equation in a single variable. But now it is p_K instead of p_J .

9.1 Expected Utility, the Price of Risky Consumption, the Price of Land, and Investment

With the interest rate on government bonds fixed by the return on riskless investments, taxes do not change after diversification. Savers' utility changes only on account of a change in price of risky consumption. From the envelope theorem (or Roy's identity) and (32) we have:

$$dV/d(p_K K^f) = -U'_1 \{L - K^f\} dp_K / d(p_K K^f). \quad (33)$$

Thus the expected utility of savers moves in the opposite direction from the price of risky investment. With risky second-period consumption being a normal good, the demand is downward sloping and the price p_K must rise to clear the market in response to an increase in trust fund demand for risky consumption. The result is that all savers, starting from the young at the time of social security diversification, lose.

This raises an interesting point for the current privatization debate. Many of today's young are clamoring for diversification on the grounds that stocks earn higher returns than bonds. But any rational young saver should already be investing so much of his wealth in stock that he is indifferent on the margin between further investments in stocks and bonds. Thus if prices did not change, the direct effect of social security diversification should be irrelevant to a young saver (even supposing he is covered by social security). However, if the extra demand for risky assets raises p_K (equivalently, if it lowers the expected return savers can get over their lives), then equation (33) shows that it reduces their welfare, provided that the riskless rate does not also change.

With young savers worse off, and saving less, aggregate real investment also drops, as described in the next proposition (proved in the appendix).

Proposition 4. *Suppose there is both safe and risky land and that safe investments, but not risky investments, are undertaken in equilibrium. Then, increased trust fund investment in risky assets increases the price of risky consumption, and decreases safe and so aggregate real investment. Moreover, social security diversification raises the price of risky land, leaving the price of riskless land unchanged. Therefore the total value of land rises.*

If the trust fund initially held no risky assets, then the diversification raises the expected utility of all workers (except the old at the time the policy is implemented, who are unaffected). Old savers at the time the policy is implemented are also helped. All young and future savers lose utility as a result of the policy. Nevertheless, trust fund purchases of risky investment increase the weighted sum of utility of all workers and savers, weighted so that the marginal social utility of second-period consumption is the same for all.

10 A concave technology

The analysis so far was made simpler by the presence of at most one endogenous rate of return. The other rate of return and wages were fixed by technology independent of production decisions. When there are no short-term production possibilities or when both technologies are strictly concave, rather than linear, then we need to solve simultaneously two equilibrium equations in two unknown rates of return (wages remaining exogenous). This makes the underlying economic factors harder to see. But we can still carry out the analysis, as we now show.³³

We suppose that the safe technology takes the form $f(k_0)$ and that the risky technology takes the form $g(k)R$, where f and g are twice differentiable and concave and R is stochastic. We suppose the productive sectors of the economy are owned entirely by the

³³A further generalization of the model would have been to introduce labor as a nonseparable input to production. If labor were applied at the same time as capital, for example, at planting time, before uncertainty is resolved, there would be little additional complication. But if labor is applied to production after uncertainty is resolved, for example, at harvest time (so that the capital of one generation combines with the labor of the next), then labor income becomes state dependent and there would be no steady state equilibrium (though perhaps a Markov equilibrium). One could also allow for distinct models of land, depending on whether ownership of land ensures a given level of (possibly stochastic) output each period, or whether the ownership of land provides a given level of capital input to production each period. When the marginal product of capital was given, the two approaches were the same.

savers.³⁴ Each saver receives a rent or profit from ownership of technology, in addition to his wage, as income.³⁵ This model includes the previous models as special cases.

Generalizing the model does reveal what general qualitative properties persist across all the equilibrium regimes studied above. We find that social security diversification always raises the riskless interest rate, and lowers the expected short-term risky return. It decreases safe investment and increases risky investment. Its effect on total investment and total land value could go either way.

To describe equilibrium in terms of budget set (2) and the variables C^* , J^* , K^* requires recognition of the return from owning technologies as part of the definition of income, I . We begin with the productive sector, which is assumed to maximize profits, taking prices as given. Let

$$\Pi(p_J, p_K) = \max[p_J f(k_0) - k_0] + \max[p_K g(k) - k] \quad (34)$$

Income for the savers is now defined as

$$I(p_J, p_K) = W - a(1 - p_J)G + \Pi(p_J, p_K). \quad (35)$$

With this definition of income, we can define savers' demands C^* , J^* , K^* from budget set (2) as before. Stationary equilibrium is now described by a vector (p_J, p_K, k, k_0) satisfying the market clearance conditions.

$$J^*(p_J, p_K, I(p_J, p_K)) + a(1 - p_J)G/p_J = (G - F_0 + p_K K^f)/p_J + L_0 + P + f(k_0) \quad (36)$$

$$K^*(p_J, p_K, I(p_J, p_K)) = L - K^f + g(k) \quad (37)$$

$$p_J = 1/(1 + r) = \begin{cases} 1/f'(k_0) & \text{if } k_0 > 0 \\ \leq 1/f'(k_0) & \text{if } k_0 = 0 \end{cases} \quad (38)$$

$$p_K = \begin{cases} 1/g'(k) & \text{if } k > 0 \\ \leq 1/g'(k) & \text{if } k = 0 \end{cases} \quad (39)$$

$$P = L_0 p_J / (1 - p_J) + L p_K / (1 - p_J). \quad (40)$$

We confine our attention to “regular economies” that satisfy two restrictions. The first is that if in any equilibrium, either safe or risky investment is not undertaken, then the corresponding price/marginal product condition in (38) or (39) is a strict inequality. The second restriction is that at every equilibrium, if we linearize the five equations (36)–(40), and then differentiate with respect to the five variables (p_J, p_K, k, k_0, P) , we get an invertible matrix. Since nearly every economy is regular, there is almost no loss of generality in looking only at regular economies.³⁶

As in the linear model, there are four equilibrium regimes depending on whether risky or safe investment is undertaken. Nevertheless, since all four of these regimes are consistent with the hypothesis that the economy is regular, we can handle all the cases as part of the same analysis, which proceeds by contradiction. We conclude that the effects of social security diversification on short-term prices and investment can be generalized from the special cases of the linear model to the more general concave model of this section.

³⁴That is, each of the unit measure of savers owns access to these technologies in terms of own capital input. Since each saver will invest the same amount, we can do the analysis in terms of aggregates.

³⁵This modeling approach differs from that with an externality that could result in the same aggregate output function, but without the separation of returns between the return on capital inputs and the return on ownership of technology. This alternative approach would give a larger return to trust fund investment in capital since there would not be an increase in the return to savers from owning technology.

³⁶That is, if we endow savers with a very small amount s of safe consumption and a very small amount r of risky consumption in their old age, then almost any choice of s and r (precisely, all except for a measure zero set of choices) will give a regular economy.

Proposition 5. *Suppose we have a regular economy with concave short-term production technology and land and $B \leq aG$. Then, increased trust fund investment in risky assets (weakly) raises p_K and k and (weakly) lowers p_J and k_0 .*

If the trust fund initially held no risky assets, then the diversification raises the expected utility of all workers (except the old at the time the policy is implemented, who are unaffected). If it also raises the price of land, then it helps old savers and hurts all young and future savers. If on the other hand, it lowers the total value of land, then it hurts all old savers and may help or hurt young and future savers. Nevertheless, trust fund purchases of risky investment increase the weighted sum of utility of all workers and savers, weighted so that the marginal social utility of second-period consumption is the same for all.

Proposition 5 includes the relevant portions of earlier Propositions as special cases. Furthermore, since in the risky linear case and in the safe linear case one of the prices p_K or p_J is fixed by the technology, Proposition 5 and the formulas for land prices yield the land price results from earlier propositions as well. On the other hand, the proof is indirect, and proceeds by finding a contradiction, yielding less insight than the explicit formulas derived in the earlier propositions.

Proposition 5 is qualitative, so we cannot use it in general to sign the effect of social security diversification on aggregate investment, for that involves comparing the magnitudes of the effects on safe and on risky investment, or on total land value.

11 Defined Benefits

We have restricted attention so far to a defined contribution social security system for analytical convenience, and to make the point that even there, social security diversification in moderation brings potential welfare gains. We show now that at least for our central risky linear case, we can readily incorporate a defined benefit structure without changing the comparative statics conclusions.

In our defined contribution social security system, we supposed that the trust fund maintained a constant value F invested in government bonds, and a constant value κ invested in risky securities, distributing any surplus as changes in benefits to the contemporaneous old. If the fund acted only to maintain F , distributing a part of the surplus over F as benefits to the contemporaneous old, and investing the rest in risky equities, then over time the benefits and the size of the trust fund investment in risky equities would change.

Leaving tax rates fixed, the level of benefits would adapt to the level of the trust fund, thereby rising with the return on the portfolio, as does a defined contribution system, but not rising dollar-for-dollar, as the returns got spread over future cohorts. In this way the benefits of a cohort would depend on the realized returns over a longer period of time. With a sensible benefit rule, and a plausible stochastic process for the return on capital, this would raise expected utility for future generations, measured as of the time of implementation of the policy, since a diversified social security system could spread the return risk over many generations. Thus the gain from a diversified portfolio becomes larger with a good policy for determining benefits.

In any “defined benefits system” with risky investments, benefits (and or taxes) must be changed, depending on the returns of the risky investments. The point is to smooth benefits, while recognizing the need to satisfy a nonnegativity constraint should there be a prolonged period of low returns. In the presence of random returns on a nontrivial portion of the trust fund, it is necessary to recognize the probability that the portion of the trust fund invested in risky assets would become negative if both benefits and taxes were

unchanged. Thus, every “defined benefit system” must be changed from time to time. The policies that determine such change need to be modeled in order to consider the value of smoothing that comes from defined benefits. In a model with randomness in other aspects of the economic and demographic environment as well, the change in uncertainty from a diversified portfolio would not be such a salient change in the system.³⁷

In the risky linear case the variations described above in the trust fund holding of risky securities and in social security benefits have no effect on any equilibrium price. The extra money invested in risky securities is absorbed by an increase in risky production, with no effect on p_K . The environment of savers is thus exactly the same as it was in Section 8. The same comparative statics conclusions on prices and quantities for the defined contribution system of Section 8 would therefore apply to the defined benefits system described here, no matter what the precise benefit rule.

12 Concluding Remarks

Some proponents of social security diversification say it would help young savers because stocks have traditionally earned a higher return than social security is projected to yield in the future. They have been rightly criticized for sometimes forgetting about the unfunded liability embodied in social security commitments to today’s old, and for ignoring the riskiness of stock returns.³⁸ Naturally our model recognizes both of these considerations, and not surprisingly it shows that the equity premium would fall after diversification. Our analysis also makes clear that the welfare of young savers depends on at least two more considerations. First, their income taxes will rise (to pay the higher interest on government bonds). Second, the assets they buy and sell will change in value. Young savers, being net buyers of long-term assets, will lose utility if land prices rise. Unless long-term capital values go down substantially after diversification, if their taxes rise by more than their interest receipts, young (and future) savers will be made worse off by social security diversification. On the other hand, today’s old savers will be made better off if long-term capital values rise.

Proponents of social security diversification also sometimes argue that it will stimulate aggregate investment. We find that it does stimulate risky investment, but it also decreases safe investment. The effect on aggregate investment depends critically on technological assumptions. Investment is driven by the savings of the young. A rise in long term capital values, which reduces their welfare, tends to reduce their savings, and thus aggregate investments.

In the risky linear technology model, to which we devote the most attention, social security diversification *lowers* long term capital values and increases aggregate investment. In the safe linear model, it raises long term capital values and lowers aggregate investment. The common sense conclusion that trust fund diversification would (if it did anything at all) increase real investment and increase stock market value is thus seen to be questionable. In both simple models one or the other of the common sense predictions is reversed.

We have also shown that changing the trust fund portfolio policy away from 100% government debt raises total welfare (suitably defined), as well as causing welfare redistributions among household types.

The paper assumed that the technology is iid. This leaves out the effect of news about future technologies on current asset prices. This would be an interesting extension.

³⁷If a defined contribution system is to fulfill its social purpose, it will also need periodic change in response to economic and demographic developments.

³⁸This line of criticism is developed in Geanakoplos, Mitchell, and Zeldes (1998, 1999).

Presumably this would make asset returns riskier and add to the social value of sharing risks more widely and so strengthen the case for investment in equities. The paper assumed that labor is a separable input from capital. Allowing changes in investment to change wages would have created another interesting redistribution.

There are four points to make relative to the current policy debate.³⁹ First, contrary to some assertions, the heterogeneity of the population implies that trust fund portfolio choice does have real effects on the economy. Second, while it is appropriate to be concerned about the risk associated with a change in portfolio policy, it seems to us unlikely that workers are so risk averse that a portfolio completely invested in Treasury bonds is optimal. This point is reinforced by the ability of the government to spread risk over successive cohorts since Social Security is a defined benefit system. That is, if a defined benefit system is well-run, there is a stronger case for trust fund investment in private securities than in the models analyzed here which assumed a defined contribution system. Third, the marginal social benefit to diversification declines as the level of diversification increases (exposing workers to more risk), which puts a limit on the amount of socially desirable diversification. Fourthly, the models considered here have substituted equity investment for bond investment, holding constant the level of funding of Social Security. Many proposals for investment in stocks, whether through the trust funds or through individual accounts, use stock investments as a reason to increase or decrease the financing of Social Security (at least in the short run) relative to what might be proposed without such investment (e.g., see Smetters, 1997). Such a change involves intergenerational redistribution that has been not incorporated in the analysis in this paper. Our analysis does apply, however, to proposals that would substitute a portfolio change for cuts in future benefits.

13 Appendix

Proof of Proposition 2. Equilibrium is given by (14). Differentiating (14) gives:

$$dp_J/dK^f = -(d\{C^*[p_J, 1, W - a(1 - p_J)G] + K^*[p_J, 1, W - a(1 - p_J)G]\}/dp_J)^{-1}. \quad (41)$$

With $I = W - (1 - p_J)aG$ as net wealth, and V as utility, and using the Slutsky equation⁴⁰, and the size of the income effect given by (15), we have

$$dp_J/dK^f = -1/(C_p^c[p_J, 1, V] + K_p^c[p_J, 1, V] + ((C_I^*[p_J, 1, I] + K_I^*[p_J, 1, I])(-[B - aG]/p_J))) < 0, \quad (42)$$

where subscripts p and I refer to partial derivatives with respect to p_J and income I , and superscript c means compensated demand. To see that this expression is less than zero, first note that compensated changes keep expected utility constant and marginal utilities are proportional to prices, implying that $0 = C_p^c + p_K K_p^c + p_J J_p^c = C_p^c + K_p^c + p_J J_p^c$. Since compensated own effects are always negative, $J_p^c < 0$, it follows that $C_p^c + K_p^c > 0$. Since both C and K are normal goods, and $B - aG \leq 0$, the denominator of (42) is positive. Note that this analysis holds for any $K^f \geq 0$ consistent with equilibrium with positive risky and zero safe investment.

To consider the impact of changing social security portfolio policy on aggregate investment, we need only determine its effect on the consumption of young savers, since the consumption of young workers does not change. From (11) we know (using Slutsky and

³⁹For more discussion of diversification, see Munnell and Balduzzi (1998) and Diamond (1999).

⁴⁰ $dC^*[p_J, 1, W - (1 - p_J)aG]/dp_J$ equals $C_p^c + C_I^* (dV/dp_J)/U'_1[C_1]$ is the change in income that would give the same utility at the old prices as given by the new prices and the new income.

the income effect term from (15)) that

$$\begin{aligned}
dk/dK^f &= d\{K^* + K^f\}/dK^f = -(dC^*[p_J, 1, W - a(1 - p_J)G]/dp_J)(dp_J/dK^f) \quad (43) \\
&= -\{C_p^c[p_J, 1, V] + C_I^*[p_J, 1, I](-[B - aG]/p_J)\}(dp_J/dK^f) \\
&= \{C_p^c - C_I^*[B - aG]/p_J\}/\{C_p^c + K_p^c - (C_I^* + K_I^*)[B - aG]/p_J\} > 0.
\end{aligned}$$

We already saw that the denominator is positive. Also $C_p^c[p_J, 1, V] > 0$, since J and C are Hicksian substitutes if J is normal. Furthermore, $C_I^* > 0$, since C is normal. Thus if $B \leq aG$, substitution and income effects go the same way. Hence trust fund diversification lowers C^* , thereby raising total risky investment $K^* + K^f$. Our analysis also shows that $dk/dK^f < 1$, when all the goods are normal, for then $K_p^c > 0$ and $K_I^* > 0$, and the denominator of (43) is larger than the numerator.

Without land, old savers and old workers at the time diversification is first implemented are not affected. Given a rise in the interest rate, equations (16), (17), and (18) demonstrate the utility gains. \blacksquare

Proof of Proposition 3. Equilibrium is given by (27) or (28). Differentiating (28), using the Slutsky equation, using the impact of a price change on V , (30), and using the derivative of P from (24), we have:

$$\begin{aligned}
dp_J/dK^f &= -(d(C^* + K^* + P)/dp_J)^{-1} \quad (44) \\
&= -1/\{C_p^c + K_p^c + (C_I^* + K_I^*)(- (B - aG)/p_J - L_0 - P) + dP/dp_J\} \\
&= -1/\{\{C_p^c + K_p^c\} - \{(C_I^* + K_I^*)(B - aG)/p_J\} + \{-(C_I^* + K_I^*)(L_0 + P) + dP/dp_J\}\} \\
&= -1/\{\{C_p^c + K_p^c\} - \{(C_I^* + K_I^*)(B - aG)/p_J\} + \{(L_0 + P)(1/(1 - p_J) - (C_I^* + K_I^*))\}\}.
\end{aligned}$$

The first terms in the denominator of the last line of (44) are the compensated demands for first-period consumption and risky second-period consumption with respect to the price of safe second-period consumption and have a positive sum, as noted in the proof of Proposition 2. The next term reflects the redistribution between savers and workers and is positive if savers have a larger share in taxes than in bonds ($B \leq aG$) and have normal demands. The final term reflects the intergenerational redistribution between old savers when the policy is implemented and later cohorts. It is also positive when the demand for safe second-period consumption is normal (which implies that $C_I^* + K_I^* = 1 - p_J J_I^* < 1$) and the price of second period consumption, p_J , is between zero and one ($r > 0$), as we have assumed. Thus $dp_J/dK^f < 0$, as was the case without land.

The falls in the prices of land follows from (20), (22) and the fall in p_J . We can get further information about the size of dP/dK^f by multiplying out the terms in (44). In the third line of (44), define x to solve $-(C_I^* + K_I^*)(L_0 + P) + dP/dp_J = p_J\{dP/dp_J\} + x$. Using (24), and the fact that $(C_I^* + K_I^*) < 1$, we know that $x > 0$. Now multiplying out the terms in (44), and using the fact that the rest of the terms in the denominator of (44) are positive and the fact that $dp_J/dK^f < 0$, gives $-p_J dP/dK^f < 1$.

As noted in (28), the level of investment in short-term production possibilities, $K^f + K^* - L$, is equal to the endowment of young savers less their first-period consumption, less the amount spent on purchasing land, less the unified net debt of the government. Note that L is constant. Hence the change in short-term risky investment is given by the change in the RHS of equation (28). Differentiating (28), using the Slutsky equation with the income effect from (30), and the derivative of P from (24), gives

$$\begin{aligned}
dk/dK^f &= d\{K^* + K^f\}/dK^f = -[d(C^* + P)/dp_J][dp_J/dK^f] \quad (45) \\
&= -\{C_p^c + (C_I^*)(- (B - aG)/p_J + L_0 + P) + dP/dp_J\}[dp_J/dK^f] \\
&= -\{\{C_p^c - (C_I^*)(B - aG)/p_J\} + \{-(C_I^*)(L_0 + P) + dP/dp_J\}\}[dp_J/dK^f] \\
&= -\{\{C_p^c\} - \{C_I^*(B - aG)/p_J\} + \{(L_0 + P)(1/(1 - p_J) - C_I^*)\}\}[dp_J/dK^f] > 0.
\end{aligned}$$

The first term C_p^c in the last line is the compensated cross elasticity of first-period consumption with respect to the price of second-period safe consumption. If the demand for riskless second-period consumption is normal, then C^* and J^* are Hicksian substitutes and this term is positive (Aura, Diamond, and Geanakoplos, 2002). With normality of demand for first-period consumption and redistribution from savers to workers ($B \leq aG$) the second term is positive. The third term is also positive when the demands for second-period safe and risky consumption are normal ($C_I^* < 1$) and the price of second-period consumption, p_J , is between zero and one ($r > 0$), as we have assumed. Multiplying by the minus sign in front and by $dp_J/dK^f < 0$ gives a positive number.

Replacing $[dp_J/dK^f]$ in the first line of (45) by the first line of (44), we get $dk/dK^f = [d(C^* + P)/dp_J]/[d(C^* + K^* + P)/dp_J]$. From this we see that whether dk/dK^f is above or below one depends on the sign of dK^*/dp_J , which is the sign of $K_p^c - K_I^* \{[(B - aG)/p_J] + (L_0 + P)\}$. The compensated derivative, K_p^c , is positive since the two assets are Hicksian substitutes. Thus, we see that if $-(B - aG) > p_J(L_0 + P)$, then $dk/dK^f < 1$. But if $-(B - aG) < p_J(L_0 + P)$, and K_p^c is small, then dk/dK^f could be greater than 1. In such a case, the drop in the value of land gives such a big positive income boost to young savers, who are buyers of the land, that they increase their holdings of risky assets K^* even while the competing return on safe assets has gone up.

Given the rise in the interest rate, the utility results follow from the equations above. We note that the SWF needed for the last conclusion has weights:

$$SWF = mnv_{\text{old}}^0 + V_{\text{old}}^0 + \sum_1^\infty \delta(t)[mnv^t + V^t]$$

The superscript t refers to the generation of birth, and we suppose the diversification takes place at time $t = 1$. The weight m is chosen, as before, so that starting from the original equilibrium, an additional dollar gives the same marginal social utility whether it is given to an old saver or an old worker from the same generation. Finally we suppose $\delta(t) = 1/(1+r)^{t-1}$, where r is the interest rate prevailing in the original equilibrium. This also preserves the property that a simple redistribution has no impact on social welfare. To calculate the effect of social security diversification on social welfare, the utility gains must be added across all generations. Using (29) and (30), and recognizing that $(1 - p_J)$ is equal to $r/(1+r)$, the change in total land value does not affect social welfare. The sum of all savers' utility gains from the fall in land prices, discounted by the equilibrium interest rate, exactly balances the change in utility of the old from the generation in retirement at the time the policy was implemented. ■

Proof of Proposition 4. Define $\kappa = p_K K^f$ and consider changes in its value. Differentiating the equilibrium condition (32), and using the Slutsky equation and the income effect from (33), we have:

$$\begin{aligned} dp_K/d\kappa &= -(p_K \{dK^* [p_J, p_K, W - p_J T(p_J)] / dp_K - K^f / p_K\})^{-1} \\ &= -1 / \{p_K K_{p_K}^c - p_K K_I^* [L - K^f]\} - K^f > 0. \end{aligned} \quad (46)$$

Since compensated own price effects are always negative, and K^* is a normal good, all the terms in the denominator are negative. Multiplied by the negative sign outside, we get the claimed result. From the connection between p_K and the price of risky land, (22), we conclude that the price of risky land also rises. The price of safe land does not change.

Next, we show that aggregate investment declines after trust fund diversification. Rearranging the equilibrium condition (25), using budget set (2), and market clearance for risky consumption (32) gives:

$$R_0 k_0 = J^*(p_J, p_K, W - aG(1 - 1/R_0)) + aG(R_0 - 1) - (L_0 + P) - R_0(G - F_0 + p_K K^f) \quad (47)$$

$$\begin{aligned}
&= R_0(W - C^*(1/R_0, p_K, W - aG(R_0 - 1)/R_0) - p_K K^*) - (L_0 + P) - R_0(G - F_0 + p_K K^f) \\
&= R_0(W - C^*(1/R_0, p_K, W - aG(R_0 - 1)/R_0)) - (L_0 + P) - R_0(G - F_0 + p_K L).
\end{aligned}$$

Dividing by R_0 , differentiating with respect to κ , noting from (23) that $\partial P/\partial p_K = L/(1 - p_J) = LR_0/(R_0 - 1)$, and then using the Slutsky equation with the income effect derived in (33), we have:

$$\begin{aligned}
dk_0/d\kappa &= -(dC^*/dp_K + L/(R_0 - 1) + L)(dp_K/d\kappa) \\
&= -(C_{p_K}^c - C_I^*[L - K^f] + LR_0/(R_0 - 1))(dp_K/d\kappa) \\
&= -(C_{p_K}^c + L[(R_0/(R_0 - 1)) - C_I^*] + C_I^* K^f)(dp_K/d\kappa) < 0.
\end{aligned} \tag{48}$$

To see that the derivative $dk_0/d\kappa$ is negative, note first that since K is normal, C and K are Hicksian substitutes, so $dC^c/dp_K > 0$. By normality of K and J , $C_I^* < 1$. Since $R_0/(R_0 - 1) > 1$, the second term is positive. Finally, since C^* is normal and the trust fund holdings of risky consumption are nonnegative, the last term is positive as well. Thus the sum in parenthesis is positive, and since $dp_K/d\kappa > 0$, safe investment declines.

With a rise in the price of land, the utility of old savers rises when the policy is implemented, since the value of the land they are holding rises. In turn, this lowers the expected utility of young savers and those in future cohorts. Starting from a trust fund invested exclusively in bonds, the expected utility of workers is increased by diversification in the same way as in Proposition 1. ■

Proof of Proposition 5. Stationary equilibrium is described by a vector (p_J, p_K, k, k_0) satisfying the definitions of profit and income, (34), (35), and the equilibrium conditions (36)–(40). (If condition (38) or (39) is a strict inequality, then we drop the corresponding production input level, fixing it at 0, and also drop the corresponding equation, and look at the remaining 4×4 or 3×3 matrix.)

We analyze the effect on equilibrium of a change in trust fund investment in risky assets: $d\kappa = -dF > 0$, where κ is the value invested in risky consumption, $p_K K^f$.⁴¹

We begin as usual by considering welfare effects. Trust fund portfolio diversification will change the prices of consumer goods and land in equilibrium. The price changes will also affect taxes paid. The changes in individual utilities are derived from these changes by the envelope theorem and the market clearance relations.

The old savers at the time of implementation of the policy change hold all the land in the economy, which they sell to the next generation, so

$$\begin{aligned}
\partial V_{\text{old}}/\partial p_J &= U_2'[dP/dp_J]; \quad \partial V_{\text{old}}/\partial p_K = U_2'[dP/dp_K] \\
dV_{\text{old}}/d\kappa &= U_2'[dP/dp_J](dp_J/d\kappa) + U_2'[dP/dp_K](dp_K/d\kappa).
\end{aligned} \tag{49}$$

Their utility rises or falls upon implementation of the policy change as total land value rises or falls.

Since the new stationary equilibrium is achieved immediately after the trust fund purchases, young savers at the time of the purchases are affected exactly the same way as all future savers. Using the envelope theorem and market clearing equations (36) for safe consumption and (37) for risky consumption, and the definitions of income taxes (5) and profit (34), we can derive the response of expected utilities to changes in prices.

$$\begin{aligned}
\partial V/\partial p_J &= U_1'\{-J^* + d\Pi/dp_J - d[a(1 - p_J)G]/dp_J\} \\
&= U_1'\{-\{B - a(1 - p_J)G\}/p_J + L_0 + P\} - d[a(1 - p_J)G]/dp_J
\end{aligned} \tag{50}$$

⁴¹ Furthermore, small changes in the trust fund create small changes in equilibrium. Equilibrium before and after social security diversification will therefore be of the same type.

$$\begin{aligned}
&= -U'_1\{[B - aG]/p_J + L_0 + P\} \\
\partial V/\partial p_K &= U'_1\{-K^* + d\Pi/dp_K\} = -U'_1\{L - K^f\} \\
dV/d\kappa &= (\partial V/\partial p_J)(dp_J/d\kappa) + (\partial V/\partial p_K)(dp_K/d\kappa) \\
&= -U'_1\{[B - aG]/p_J + L_0 + P\}(dp_J/d\kappa) - U'_1\{L - K^f\}(dp_K/d\kappa).
\end{aligned} \tag{51}$$

It is interesting that safe output $f(k_0)$ and risky output $g(k)$ both cancel out of (50) and (51), respectively. The reason is that the firms effectively trade only with the savers. If the savers own the technology, then there is no welfare effect from price changes via production. If the output becomes more valuable, increasing the profit for the firm, then it becomes more expensive for the savers to buy from the firm.

From the equations for land value and its derivative, (23) and (24), (50) and (51) can also be written as:

$$\begin{aligned}
\partial V/\partial p_J &= -U'_1\{[B - aG]/p_J + (1 - p_J)dP/dp_J\} \\
\partial V/\partial p_K &= -U'_1\{(1 - p_J)dP/dp_K - K^f\}
\end{aligned} \tag{52}$$

Assuming that the trust fund does not hold long-term assets, worker utility is not affected by land prices. Hence we have:

$$\begin{aligned}
dv/d\kappa &= E\{u'_2\{((R/p_K) - 1 - r)/n - dt/d\kappa + (F_0 - \kappa)(dr/d\kappa)/n - (RK^f/p_k)(dp_K/d\kappa)/n\}\} \\
&= E\{u'_2\{((R/p_K) - 1 - r + (F - (1 - a)G)(dr/d\kappa) - (RK^f/p_k)(dp_K/d\kappa))\}/n \\
&= E\{u'_2((R/p_K) - 1 - r)\}/n - E\{u'_2\}(B - aG)(dr/d\kappa)/n - E\{u'_2(RK^f/p_k)\}(dp_K/d\kappa)/n.
\end{aligned} \tag{53}$$

We consider a weighted sum of utilities, denoted SWF:

$$SWF = mnv_{\text{old}}^0 + V_{\text{old}}^0 + \sum_1^\infty \delta(t)[mnv^t + V^t]$$

with the weights as described in the proof of Proposition 3.

We now turn to the proof of the first part of Proposition 5, which is indirect and proceeds by finding a contradiction.

Recalling that J^* is the demand for safe consumption, define J as the supply of safe consumption, net of tax. Rewriting (36), we get

$$J^*(p_J, p_K, I(p_J, p_K)) = J \equiv [(1 - a)G - F_0 + \kappa]/p_J + L_0 + P + aG + f(k_0). \tag{54}$$

Recalling that K^* is the demand for risky consumption, define K as the supply of risky consumption. Similarly rewriting (37), we get

$$K^*(p_J, p_K, I(p_J, p_K)) = K \equiv L + g(k) - \kappa/p_K \tag{55}$$

Multiplying J^* from (54) by p_J and K^* from (55) by p_K and adding, and then using the budget set (2), and the definition of I and P from (35) and (40), and the identity $P = p_J L_0 + p_K L + p_J P$, we get

$$C^*[p_J, p_K, I(p_J, p_K)] = C \equiv W - [G - F_0] - P - k - k_0 \tag{56}$$

We think of C^* as the demand for current consumption, and C as the supply.

The strategy of proof consists of differentiating the three equations $J^* = J$, $K^* = K$, and $C^* = C$ derived in (54)–(56), and the equation (40) defining P , as well as the identity $\kappa = p_K K$, with respect to the six variables $d\kappa$, dp_J , dp_K , dP , dk_0 , dk . From the regularity hypothesis, we know that the change in equilibrium values will indeed be differentiable.

We will now prove that when $dk > 0$, equilibrium can be restored only if $dp_J \leq 0$, $dk_0 \leq 0$, $dp_K \geq 0$, $dk \geq 0$. We use (38) or (39) only to conclude that dp_J and dk_0 have the same sign, as do dp_K and dk . Thus the proof applies to every equilibrium regime. We obtain

$$dJ = -\{[(1-a)G - F_0 + \kappa]/p_J^2\}dp_J + [1/p_J]d\kappa + dP + f'(k_0)dk_0 \quad (57)$$

$$dK = g'(k)dk + [\kappa/p_K^2]dp_K - [1/p_K]d\kappa \quad (58)$$

$$dC = -dP - dk - dk_0. \quad (59)$$

We also use the welfare effect from (52)

$$dV = -U'_1\{[B - aG]/p_J\}dp_J + (1 - p_J)dP - K^f dp_K. \quad (60)$$

If $dp_J = 0 = dp_K$, then savers' demands do not change, so from (57) we must have $dk_0 < 0$, and from (58) $dk > 0$ and we are done. Thus we assume that not both $dp_J = 0 = dp_K$, and we rule out three cases by contradiction.

Case 1 Suppose $dp_J \geq 0$, $dk_0 \geq 0$, $dp_K \geq 0$, $dk \geq 0$. Since we cannot have both $dp_J = 0$ and $dp_K = 0$, in fact at least one price strictly rises. Then from (40), $dP > 0$. From (59), this implies that

$$dC^* = dC < 0. \quad (61)$$

Adding dJ and dK from (57) and (58), we have that

$$\begin{aligned} p_J dJ + p_K dK &= -\{[(1-a)G - F_0 + \kappa]/p_J\}dp_J + d\kappa + p_J[dP + f'(k_0)dk_0] \\ &\quad + p_K g'(k)dk + [\kappa/p_K]dp_K - d\kappa \\ &= -\{[(1-a)G - F_0 + \kappa]/p_J\}dp_J + p_J[dP + f'(k_0)dk_0] + p_K g'(k)dk + [\kappa/p_K]dp_K > 0. \end{aligned} \quad (62)$$

provided that $B - aG = [(1-a)G - F_0 + \kappa] \leq 0$. Observe next that, given that $dp_J \geq 0$, $dp_K \geq 0$, the compensated change

$$dC^c(p_J, p_K, U) \geq 0, \quad (63)$$

since all three goods are Hicksian substitutes (see Aura, Diamond, Geanakoplos, 2002). To maintain constant utility, the compensated change

$$p_J dJ^c(p_J, p_K, U) + p_K dK^c(p_J, p_K, U) \leq 0. \quad (64)$$

Since by (61), $dC^* < 0$, and by (63), $dC^c(p_J, p_K, U) \geq 0$, it follows, since C^* is normal, that welfare of the savers must have gone down. Since J^* and K^* are also normal, it follows from (64) that

$$p_J dJ^* + p_K dK^* < 0. \quad (65)$$

But (62) and (65) contradict each other, since $dJ = dJ^*$ and $dK = dK^*$.

Before proceeding to case 2, we remark that we could have deduced (64) directly from the fact that savers' utility is additively separable between consumption when young and when old. Similarly (65) holds for the same reason. Thus case 1 does not really need the hypothesis of normality.

Case 2 Suppose $dp_J \leq 0$, $dk_0 \leq 0$, $dp_K \leq 0$, $dk \leq 0$. We get the same contradiction as in case 1, with all the signs reversed.

Case 3 Suppose $dp_J \geq 0$, $dk_0 \geq 0$, $dp_K \leq 0$, $dk \leq 0$. Since J^* and K^* are Hicksian substitutes, we have

$$\begin{aligned} dK^* &\geq (K_I^*)dI, \\ dJ^* &\leq (J_I^*)dI, \end{aligned} \tag{66}$$

where dI is the change in wealth which would cause the same change in welfare (at constant prices) as caused by the price changes. From (57) we deduce (assuming that $B - aG \leq 0$) that

$$dJ^* = dJ \geq dP, \tag{67}$$

and from (58), we know that

$$dK^* = dK = g'(k)dk + [\kappa/p_K^2]dp_K - [1/p_K]d\kappa < [\kappa/p_K^2]dp_K \leq 0. \tag{68}$$

Suppose welfare for the savers went up, or stayed the same, $dI \geq 0$. Then since K^* is a normal good, we know from (66) that $dK^* \geq 0$, a contradiction to (68). Alternatively, suppose that welfare for the savers went down, $dI < 0$, and the total value of land went up or stayed the same, $dP \geq 0$. Then from (66) $dJ^* < 0$. But from (67) we have that $dJ^* = dJ \geq 0$, contradiction. The only remaining possibility is that $dI < 0$ and $dP < 0$. But from $dP < 0$ and the welfare effect described in (60) we deduce that

$$dI = dV/U'_1 > K^f dp_K. \tag{69}$$

From the fact that all goods are normal, so that $0 < (K_I^*) < 1/p_K$, and $dp_K \leq 0$, and $dI < 0$, it then follows from (66) and (69) that

$$dK^* \geq (K_I^*)dI > dI/p_K > K^f dp_K/p_K = (\kappa/(p_K)^2)dp_K. \tag{70}$$

But now (68) and (70) are contradictory.

Using profit maximization for the first time, we cannot get any of the mixed cases, such as where $dp_K > 0$ and $dk < 0$, so the theorem follows after eliminating the above three cases.

The welfare part of proposition 5 follows by combining the welfare implications derived at the beginning of the proof with $dp_J \leq 0$ and $dp_K \geq 0$. ■

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