# COWLES FOUNDATION FOR RESEARCH IN ECONOMICS AT YALE UNIVERSITY 

Box 2125, Yale Station
New Haven, Connecticut 06520

COWLES FOUNDATION DISCUSSION PAPER NO. 1282

Note: Cowles Foundation Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. Requests for single copies of a Paper will be filled by the Cowles Foundation within the limits of the supply. References in publications to Discussion Papers (other than mere acknowledgment by a writer that he has access to such unpublished material) should be cleared with the author to protect the tentative character of these papers.

## ALTERNATIVE METHODS FOR MEASURING PRODUCTIVITY GROWTH

William D. Nordhaus

November 2000

# Alternative Methods for Measuring Productivity Growth 

William D. Nordhaus ${ }^{1}$<br>November 14, 2000


#### Abstract

The present study is a contribution to the theory of the measurement of productivity growth. First, it examines the welfare-theoretic basis for measuring productivity growth and shows that the ideal welfare-theoretic measure is a chain index of productivity growth rates of different sectors which uses current output weights. Second, it lays out a technique for decomposing productivity growth which separates aggregate productivity growth into three factors - the pure productivity effect, the effect of changing shares, and the effect of different productivity levels. Finally, it shows how to apply the theoretically correct measure of productivity growth and indicates which of the three different components should be included in a welfareoriented measure of productivity growth. The study concludes that none of the measures generally used to measure productivity growth is consistent with the theoretically correct measure.


Measuring productivity growth has been a growth industry within economics for half a century. Over this period, there have been substantial changes and improvements in the construction of the underlying data and methods. Particularly notable are improvements in measuring output and prices and in implementing improved indexes, notably the use of "superlative" price and output measures by government statistical agencies. ${ }^{2}$

Productivity growth is usually taken to be an obvious index of welfare. Paul Krugman put it succinctly, "Productivity isn't everything, but in the long run it is almost everything." ${ }^{3}$ The link between productivity growth and economic welfare is actually not obvious. There has, however, been surprisingly little attention to the construction of productivity measures.

The present paper is part of a larger study which is devoted to analytical and empirical questions in productivity measurement. ${ }^{4}$ The present paper makes three contributions to understanding the measurement of productivity. First, it examines the welfare-theoretic basis for measuring productivity growth. Second, it lays out a technique for decomposing productivity growth

[^0]which divides aggregate productivity trends into three factors that contribute to the growth in economy-wide productivity. Finally, it discusses the appropriate way to apply the ideal welfaretheoretic measure in practice.

The major practical result of this study is that current measures of productivity growth are generally inappropriate from the point of view of reflecting economic welfare. We propose an alternative measure of productivity growth, the chain-weighted index of sectoral productivity growth rates, which better approximates the ideal index.

## I. Welfare Aspects of Productivity Measures

We begin with the question of the ideal approach to measuring productivity growth. We approach this issue using the tools of index number theory. ${ }^{5}$ For simplicity, we assume that all output is devoted to consumption goods and that consumption goods are immediately used up (i.e., there are no durable goods). ${ }^{6}$ We further assume that the appropriate measure of real income is a smooth utility function of the form

$$
\begin{equation*}
\mathrm{U}_{\mathrm{t}}=\boldsymbol{U}\left(\mathrm{C}_{10}, \mathrm{C}_{2 \mathrm{t}}, \ldots, \mathrm{C}_{\mathrm{nt}}\right) \tag{1}
\end{equation*}
$$

where $\mathrm{C}_{\mathrm{it}}$ is the flow of services from consumption goods at time t , and there are n goods, $\mathrm{i}=1, \ldots \mathrm{n}$. We do not assume any particular form for $\boldsymbol{U}$, but we do assume that the utility function is homothetic. Under this assumption, we can construct Divisia indexes of real income changes by taking the weighted average growth of individual components.

It will be convenient to simplify by assuming that each good is produced by primary factors alone, so $\mathrm{C}_{\mathrm{it}}=\boldsymbol{F}_{i}\left(\mathrm{~S}_{\mathrm{it}}\right)$, where $\boldsymbol{F}_{i}$ is a constant returns to scale production function for industry i and $S_{\text {it }}$ is a scalar index of inputs into the industry i (for example, $S$ might be a Cobb-Douglas function of the relevant inputs). If $\mathrm{A}_{\mathrm{it}}$ is total factor productivity in sector i , we can then write the production function as $\mathrm{C}_{\mathrm{it}}=\mathrm{A}_{\mathrm{it}} \mathrm{S}_{\mathrm{it}}$.

For this discussion, we assume that the economy is characterized by perfect competition, that all factors are priced at their marginal products, and that all goods are priced at their marginal costs. This assumption removes influences of imperfect competition and the distortions that may arise from indirect taxation. Finally, we assume that households have identical utility functions and endowments.

In addition, we make three simplifying normalizations. First, each household supplies one unit of the composite input, $S$. Second, we normalize the price of the composite input $S$ to be unity. These normalizations imply that each household has 1 unit of income. Third, we assume that initial price and level of productivity are equal to 1 . Under these assumptions, the price of each good is given by:

[^1]We now consider the expenditure function, $\boldsymbol{V}$, that comes from maximizing the utility function in (1) subject to the budget constraint:

$$
\begin{equation*}
\mathrm{E}_{\mathrm{t}}=\boldsymbol{V}\left(\mathrm{P}_{1 \mathrm{t}}, \mathrm{P}_{2 \mathrm{t}}, \ldots, \mathrm{P}_{\mathrm{nt}}, \mathrm{U}_{\mathrm{t}}\right) \tag{3}
\end{equation*}
$$

where $E_{t}$ is expenditure. Note that the income term has been suppressed because we normalize income to be unity. Differentiating (3) with respect to time yields:

$$
\mathrm{dE} / \mathrm{dt}=\left(\partial \boldsymbol{V} / \partial \mathrm{P}_{1 \mathrm{t}}\right)\left(\mathrm{dP}_{1 \mathrm{t}} / \mathrm{dt}\right)+\left(\partial \boldsymbol{V} / \partial \mathrm{P}_{2 \mathrm{t}}\right)\left(\mathrm{dP}_{2 \mathrm{t}} / \mathrm{dt}\right)+\cdots+\left(\partial \boldsymbol{V} / \partial \mathrm{P}_{\mathrm{nt}}\right)\left(\mathrm{dP}_{\mathrm{nt}} / \mathrm{dt}\right)
$$

Using the properties of the expenditure function, we have

$$
\mathrm{dE}_{\mathrm{l}} / \mathrm{dt}=\mathrm{C}_{1 \mathrm{t}} \mathrm{dP}_{1 \mathrm{t}} / \mathrm{dt}^{2}+\mathrm{C}_{2 \mathrm{t}} \mathrm{dP}_{2 \mathrm{t}} / \mathrm{dt}+\cdots+\mathrm{C}_{\mathrm{nt}} \mathrm{dP}_{\mathrm{nt}} / \mathrm{dt}
$$

Dividing by $E_{t}$ and multiplying and dividing each term on the right hand side by the relevant $P_{i t}$ yields

$$
\begin{equation*}
[\mathrm{dE} / \mathrm{dt}] / \mathrm{E}_{\mathrm{t}}=\mathrm{C}_{1 \mathrm{t}} \mathrm{P}_{1 \mathrm{t}}\left[\left(\mathrm{dP} \mathrm{P}_{1 \mathrm{t}} / \mathrm{dt}\right) \mathrm{P}_{1 \mathrm{t}}\right] / \mathrm{E}_{\mathrm{t}}+\mathrm{C}_{2 \mathrm{t}} \mathrm{P}_{2 \mathrm{t}}\left[\left(\mathrm{dP}_{2 \mathrm{t}} / \mathrm{dt}\right) \mathrm{P}_{2 \mathrm{t}}\right] / \mathrm{E}_{\mathrm{t}}+\cdots+\mathrm{C}_{2 \mathrm{t}} \mathrm{P}_{\mathrm{nt}}\left[\left(\mathrm{dP}_{\mathrm{nt}} / \mathrm{dt}\right) \mathrm{P}_{\mathrm{nt}}\right] / \mathrm{E}_{\mathrm{t}} \tag{4}
\end{equation*}
$$

or using (2):

$$
\begin{equation*}
\mathrm{g}\left(\mathrm{E}_{\mathrm{t}}\right)=-\left[\sigma_{1 \mathrm{t}} \mathrm{~g}\left(\mathrm{~A}_{1 \mathrm{t}}\right)+\sigma_{2 \mathrm{t}} \mathrm{~g}\left(\mathrm{~A}_{2 \mathrm{t}}\right)+\cdots+\sigma_{\mathrm{nt}} \mathrm{~g}\left(\mathrm{~A}_{\mathrm{nt}}\right)\right] \tag{5}
\end{equation*}
$$

where $\sigma_{i t}=\mathrm{C}_{\mathrm{it}} \mathrm{P}_{\mathrm{it}} / \mathrm{E}_{\mathrm{t}}=$ the share of good i in total nominal spending at time t .
We now proceed to determine the growth in real income due to changes in the total factor productivities in different industries. Defining real income as $R_{t}$, the growth in real income can be calculated as the growth in $R_{t}$ over time. We use the notation that $g\left(R_{t}\right)=\Delta R_{t} / R_{t-1}=$ the rate of growth of $\mathrm{R}_{\mathrm{t}}$. Since (5) represents the decline in total expenditure or income necessary to attain a constant utility, by homotheticity the growth in real income that can be attained with the actual consumption shares, productivity levels, and prices is therefore:

$$
\begin{equation*}
\mathrm{g}\left(\mathrm{R}_{\mathrm{t}}\right)=\sigma_{1 \mathrm{t}} \mathrm{~g}\left(\mathrm{~A}_{1 \mathrm{t}}\right)+\sigma_{2 \mathrm{t}} \mathrm{~g}\left(\mathrm{~A}_{2 \mathrm{t}}\right)+\cdots+\sigma_{\mathrm{nt}} \mathrm{~g}\left(\mathrm{~A}_{\mathrm{n} t}\right) \tag{6}
\end{equation*}
$$

In words, the growth rate of real income or real output is the chain-weighted index of sector-level productivity growths. The weights in the index are the current nominal shares of each good in total nominal consumption. With discrete time, equation (6) should be calculated as an equation in growth rates using Fisher or other superlative weights.

We now see how equation (6) applies to the question of the ideal welfare-theoretic measure of productivity in an economy with many sectors experiencing varying rates of productivity growth. The major result is that the ideal measure of productivity growth is a weighted average of the productivity growth rates of different sectors. This formula is very similar to that currently used in constructing superlative indexes of prices and output. The important point is that the indexes used in the appropriate measure are chain indexes of productivity growth rather than differences in the growth rates or indexes of output and inputs.

## II. Decomposing Actual Productivity Growth into its Components

In this section, we turn to the question of how productivity growth is actually measured. It will be convenient to begin with aggregate measures of productivity growth and break them into their major components. We will see that the welfare analysis of the previous section fits very neatly into this decomposition.

## Productivity Accounting

Consider aggregates of output $\left(\mathrm{X}_{\mathrm{t}}\right)$, composite inputs $\left(\mathrm{S}_{\mathrm{t}}\right)$, and total factor productivity $\left(\mathrm{A}_{t}=\mathrm{X}_{\mathrm{t}} / \mathrm{S}_{t}\right)$. These aggregates are the sum (or chained indexes) of industry output, inputs, and productivity ( $\mathrm{X}_{\mathrm{it}}, \mathrm{S}_{\mathrm{it}}$, and $\mathrm{A}_{\mathrm{it}}$ ). We can rewrite these as built up from industry values ( $\mathrm{i}=1, \ldots, \mathrm{~N}$ ) as follows: ${ }^{7}$

$$
\begin{aligned}
A_{t} & =X_{t} / X_{t}=\left(\sum_{i} X_{i t}\right) /\left(\sum_{j} S_{j t}\right) \\
& =\sum_{i}\left[\left(X_{i t} / S_{i t}\right)\left(S_{i t} /\left(\sum_{j} S_{j t}\right)\right]\right.
\end{aligned}
$$

or

$$
\begin{equation*}
A_{t}=\sum_{i} A_{i t} W_{i t} \tag{7}
\end{equation*}
$$

where $\mathrm{w}_{\mathrm{it}}=$ share of total inputs devoted to industry i , that is, $\mathrm{w}_{\mathrm{it}}=\mathrm{S}_{\mathrm{it}} /\left(\sum_{\mathrm{j}} \mathrm{S}_{\mathrm{jt}}\right)$. Note that in the ideal case of perfect competition and constant returns to scale, with no indirect taxes, the share of inputs is also the share of nominal outputs.

We can calculate the change in total factor productivity as follows:

$$
\begin{aligned}
\Delta \mathrm{A}_{\mathrm{t}} & =\sum_{\mathrm{i}} \mathrm{~A}_{\mathrm{it}} \mathrm{w}_{\mathrm{it}}-\sum_{\mathrm{i}} \mathrm{~A}_{\mathrm{it}-1} \mathrm{w}_{\mathrm{it}-1} \\
& =\sum_{\mathrm{i}} \mathrm{~A}_{\mathrm{it}} \mathrm{w}_{\mathrm{it}}-\sum_{\mathrm{i}} \mathrm{~A}_{\mathrm{it}-1} \mathrm{w}_{\mathrm{it}}+\sum_{\mathrm{i}} \mathrm{~A}_{\mathrm{it}-1} \mathrm{w}_{\mathrm{it}}-\sum_{\mathrm{i}} \mathrm{~A}_{\mathrm{it}-1} \mathrm{w}_{\mathrm{it}-1}
\end{aligned}
$$

or

$$
\Delta \mathrm{A}_{\mathrm{t}}=\sum_{\mathrm{i}} \mathrm{w}_{\mathrm{it}} \Delta \mathrm{~A}_{\mathrm{it}}+\sum_{\mathrm{i}} \mathrm{~A}_{\mathrm{it}-1} \Delta \mathrm{w}_{\mathrm{it}}
$$

Now dividing by $\mathrm{A}_{\mathrm{t}-1}$, we have

$$
\Delta \mathrm{A}_{\mathrm{t}} / \mathrm{A}_{\mathrm{t}-1}=\sum_{\mathrm{i}} \mathrm{w}_{\mathrm{it}}\left(\Delta \mathrm{~A}_{\mathrm{it}} / \mathrm{A}_{\mathrm{it}-1}\right)\left(\mathrm{A}_{\mathrm{it}-1} / \mathrm{A}_{\mathrm{t}-1}\right)+\sum_{\mathrm{i}}\left(\mathrm{~A}_{\mathrm{it}-1} / \mathrm{A}_{\mathrm{t}-1}\right) \Delta \mathrm{w}_{\mathrm{it}}
$$

Define productivity relatives as $\mathrm{R}_{\mathrm{it}}=\mathrm{A}_{\mathrm{it}} / \mathrm{A}_{\mathrm{t}}$. This leads to

$$
\Delta \mathrm{A}_{\mathrm{t}} / \mathrm{A}_{\mathrm{t}-1}=\sum_{\mathrm{i}} \mathrm{w}_{\mathrm{it}}\left(\Delta \mathrm{~A}_{\mathrm{it}} / \mathrm{A}_{\mathrm{it}-1}\right) \mathrm{R}_{\mathrm{it}-1}+\sum_{\mathrm{i}}\left(\mathrm{~A}_{\mathrm{it}-1} / \mathrm{A}_{\mathrm{t}-1}\right) \Delta \mathrm{w}_{\mathrm{it}}
$$

${ }^{7}$ This treatment ignores the discrepancy between chained real GDP or inputs and the sum of the industry chained real outputs or inputs.

We now define $\mathrm{s}_{\mathrm{it}}=\mathrm{w}_{\mathrm{it}} \mathrm{R}_{\mathrm{it}-1}=\mathrm{S}_{\mathrm{it}} / \mathrm{S}_{\mathrm{t}} \mathrm{A}_{\mathrm{it}-1} / \mathrm{A}_{\mathrm{t}-1}=\left(\mathrm{S}_{\mathrm{it}} / \mathrm{S}_{\mathrm{t}}\right)\left(\mathrm{X}_{\mathrm{it}-1} / \mathrm{X}_{\mathrm{t}-1}\right) /\left(\mathrm{S}_{\mathrm{it}-1} / \mathrm{S}_{\mathrm{t}-1}\right)$. For smooth time series and small time steps, $\mathrm{s}_{\mathrm{it}} \cong \mathrm{X}_{\mathrm{it}} / \mathrm{X}_{\mathrm{t}}=\sigma_{1 \mathrm{t}}$.

$$
\begin{equation*}
g\left(A_{t}\right)=\sum_{i} s_{i t} g\left(A_{i t}\right)+\sum_{i} R_{i t-1} \Delta w_{i t} \tag{8}
\end{equation*}
$$

Finally, add and subtract $\sum_{\mathrm{i}} \mathrm{s}_{\mathrm{it}} \mathrm{g}\left(\mathrm{A}_{\mathrm{it}}\right)$ from equation (8), where k is the base year. This yields the final equation:

$$
\begin{equation*}
g\left(A_{t}\right)=\sum_{i} s_{i k} g\left(A_{i t}\right)+\sum_{i}\left(s_{i t}-s_{i k}\right) g\left(A_{i t}\right)+\sum_{i} R_{i t-1} \Delta w_{i t} \tag{9}
\end{equation*}
$$

As long as all series are smooth series and with small time steps, this becomes

$$
\begin{equation*}
g\left(A_{t}\right)=\sum_{i} \sigma_{i k} g\left(A_{i t}\right)+\sum_{i}\left(\sigma_{i t}-\sigma_{i k}\right) g\left(A_{i t}\right)+\sum_{i} R_{i t-1} \Delta w_{i t} . \tag{9'}
\end{equation*}
$$

## Interpretation

Equations (9) allows an interesting interpretation of the trend in aggregate productivity growth. This equation shows that the aggregate can be broken into three components: a pure (fixedweight) productivity term which uses fixed base-year expenditure or output weights, a term that reflects the difference between current nominal output weights and base-year weights, and a third term which reflects the interaction between changing weights and relative productivity levels in different sectors. For convenience, we will designate these three terms as follows.

Pure Productivity Effect. The first term on the right hand side of equation (9) is a fixedweighted average of the productivity growth rates of different sectors. More precisely, this measures the sum of the growth rates of different industries weighted by base year nominal output shares of each industry. Another way of interpreting the pure productivity effect is as the productivity effect if there were no change in output composition among industries.

The Baumol effect. The second term captures the interaction between the differences in productivity growth and the changing shares of different industries over time. This effect has been emphasized by William Baumol in his work on unbalanced growth. ${ }^{8}$ According to Baumol, those industries which have relatively slow output growth generally are accompanied by relatively slow productivity growth (services being a generic example and live performances of Mozart string quartette being a much-cited specific example).

Denison Effect. The third term captures the effect of changing shares of employment on aggregate productivity. This is the Denison effect, after Edward Denison who pointed out that the movement from low-productivity-level agriculture to high-productivity-level industry would raise productivity even if the productivity growth rates in the two industries were the same. Denison showed that this effect was an important component of overall productivity growth. ${ }^{9}$ The Denison

[^2]effect is the sum of the changes in output shares of different industries weighted by their relative productivity levels.

## Appropriate Treatment of the Different Effects

A major question in measuring productivity growth concerns the appropriate construction of indexes. Which of the three components of equation (9) or (9') should be included if our productivity measures are to be a useful measure of welfare?

For this discussion, we turn as an application to changes in labor productivity - that is, we interpret the variable $S$ as labor hours worked. The question then becomes what is the ideal measure of labor productivity? This question can be answered by comparing measured productivity growth in equation (9') with the ideal productivity growth measure shown in equation (5). Abstracting from differences in timing that lead to differences between $\sigma_{i t}$ and $\mathrm{s}_{\mathrm{it}}$, a comparison of the two equations shows that the ideal index of productivity growth from a welfare-theoretic perspective includes the first two terms in (9) or (9') but excludes the third term.

This implies that the pure productivity effect and Baumol effect should be included in a welfare-oriented measure of productivity growth. The reason for the pure productivity effect is intuitive. Additionally, the Baumol effect reflects the impact of changing expenditure shares on the overall productivity measure. If spending is primarily devoted to sectors that have low productivity growth, then this implies that our economic welfare will indeed be growing relatively slowly.

This approach also indicates that the Denison effect should normally be excluded from an ideal productivity index. To understand the reason for its exclusion requires some discussion of the potential sources of the Denison effect. Recall that the Denison effect arises primarily because of differences in the levels of productivity by industry. We can identify three major reasons for differences. The first is that differences in productivity levels reflect differences in inputs which are not captured by our productivity measures. For this first case, the Denison effect should be excluded from a welfare-oriented measure of productivity because interindustry shifts produce spurious changes in productivity growth. If in fact the levels of total factor productivity are equal in all industries, then the Denison effect would by construction be zero. ${ }^{10}$

A second reason for differences in productivity levels would arise because of differences in indirect taxation. A third reason would arise from disequilibrium in input or output markets, because of slow migration of labor from farming to industry, or because of market power. If the second and third reasons were the major source of differences in productivity levels, then the treatment is more complex and some or all of the Denison effect would be appropriately included in a welfare-theoretic measure of productivity growth. As in other cases where tax wedges or other distortions apply, the appropriate treatment will usually be somewhere between inclusion and exclusion depending upon the relevant elasticities.

An examination of the actual patterns of labor productivity across sectors suggests that the differences in productivity arise primarily because of the first reason, differences in inputs which are not captured by our productivity measures. In 1998, the level of labor productivity (gross output per hour worked) differed by more than a factor of 100 across major industries. One major source of

[^3]difference comes from differences in capital intensity or in labor skills across industries. For example, the highest gross output per person employed in 1998 at the two-digit level was in nonfarm housing services, with a productivity level 34 times that of the overall economy. The high productivity arose because this sector is essentially entirely imputed rents. Other high productivity ratios are found in capital-intensive sectors such as pipelines, oil and gas extraction, and telephone services. Similarly, high productivity levels are found in sectors with high human capital such as security and commodity brokers. At the other end of the spectrum are industries with low-skilled workers, such as private households, personal services, and apparel.

The second and third sources of productivity differences appear less significant today. The industry which gave rise to the Denison effect, farming, had a productivity ratio of 90 percent of the total economy in 1998. This suggests that disequilibrium in labor migration patterns is a relatively unimportant source of productivity differences today. Moreover, there are few two-digit industries where indirect taxes are a large share of gross output. The major case is tobacco products, where indirect taxes were 60 percent of gross output in 1998. Among one-digit industries, the ratio of indirect business taxes to gross output in 1998 ranged from a low of 2 percent in construction to a high of 21 percent in wholesale trade. While these differences are not trivial, they are much smaller than the differences in productivity due to differing capital intensities or labor qualities.

We can illustrate the problems discussed here using a numerical example. Table 1 illustrates how the Denison effect can provide misleading estimates of productivity growth if not properly calculated. It shows an economy with two industries with differing levels of productivity. Industry 1 is a high productivity sector while industry 2 is a low productivity sector. The last three lines in the table show three different ways of calculating productivity growth. Line 20 shows the preferred measure of productivity growth from equation (9), which includes the pure productivity effect and the Baumol effect. Line 21 shows the difference of growth rates methodology, which is the difference of growth rates approach used by the BLS and many scholars. Line 22 shows the simplest aggregate measure of total output per total hour.

For the example in Table 1, both the aggregate and the difference of growth rates methodologies give misleading results because they include the Denison effect. Because the high productivity industry has a rising share of nominal output, aggregate productivity rises at 7.25 percent (line 22). However, because hours are declining in the low productivity industry, the difference of growth rates approach shows a very low productivity growth rate of 0.52 percent (line 21). The correct number given in line 20 (including only the pure productivity and Baumol effects) shows an intermediate productivity growth rate of 2.75 percent.

Table 2 shows an example with a strong Baumol effect. Here, the initial levels of productivity in the two sectors are equal, but industry 1 shows a declining share of output along with strong productivity growth while industry 2 is technologically stagnant with strong demand growth. In this case, the Baumol effect is strong ( -3.33 percentage points in the last column of line 17). Moreover, the difference of growth rates method again seriously understates the true productivity growth rate.

## Application to aggregate U.S. data

We can illustrate the procedures using actual data on labor productivity for the United States. These data are derived from two companion papers on the subject. The first companion paper
presents a new data set on aggregate and industrial productivity derived from income-side data. ${ }^{11}$ The second companion paper applies the concepts in this paper and the data in the first paper to estimating productivity growth and the role of the new economy in the recent productivity upsurge. ${ }^{12}$

Figure 1 and Table 3, which are drawn from the second paper, show a comparison of two measures of labor productivity for the overall economy over the period 1978-1998. The series called "ideal measure" is the welfare-theoretic index derived in a manner defined in equation (6) above. This shows the rate of productivity growth that best measures the growth in average living standards. The measure labeled "GDI productivity" is the growth of total labor productivity, measured as income-side GDP per hour worked.

The results show a significant difference between the two concepts. The ideal or welfaretheoretic measure is higher than standard labor productivity in every subperiod. The differences are relatively small in the most recent period, but they are substantial in earlier periods. On average, the ideal or welfare-theoretic measure over the entire period was 0.21 percentage points per year higher than total income-side productivity growth. In the second companion paper, we show that this difference is exactly equal to the Denison effect over the period, as is predicted in the discussion of the decomposition of productivity growth above.

## Current Approaches to Constructing Productivity Measures

Given the vast literature on productivity growth, there is surprisingly little discussion of the welfare-theoretic interpretation of alternative measures. There is of course a vast literature on the construction of ideal indexes of prices and output. One of the few studies to apply these to productivity is by Caves, Christensen, and Diewert. ${ }^{13}$ This study does not address the issue of differing levels of productivity in different industries, however. One important study of the relationship between alternative measures of productivity and welfare theory was by Baumol and Wolff, which recommended the use of what they called a "deflated index of total factor productivity." ${ }^{14}$ This index is constructed by deflating nominal labor productivity by the an economy-wide average of the real wage. While this formula does correct for the Denison effect, it does not appear to identify the need for a chain index to measure welfare improvements of productivity growth.

Early applied analyses of total factor productivity and labor productivity did not analyze the appropriate index from a welfare-theoretic point of view. ${ }^{15}$ Alternative approaches were used in

[^4]${ }^{12}$ See William D. Nordhaus, "Productivity Growth and the New Economy," November 13, 2000, available at www.econ.yale.edu/~nordhaus/homepage/ write_new_economy.htm.
${ }^{13}$ Douglas W. Caves, Laurits R. Christensen, and W. Erwin Diewert, "The Economic Theory of Index Numbers and the Measurement of Input, Output, and Productivity," Econometrica, Vol. 50, no. 6, Nov. 1982, pp. 1393-1414.

[^5]these studies, but the central approaches were generally ones which weighted productivity growth by fixed output weights. This approach is clearly inappropriate and can give misleading results if productivity growth differs by sector.

The work of Jorgenson and Griliches and later work by them and co-authors derive measures of productivity growth from general transformation functions. ${ }^{16}$ This approach led to the suggestion, pioneered by Griliches and Jorgenson, that productivity growth be estimated as the difference between Divisia indexes of output growth and input growth. This approach, which we call the "difference of growth rates" approach, will be close to the ideal approach if productivity levels in different industries are close, but it will not in general provide the correct result from a welfare theoretic point of view if the Denison effect is present. (See Tables 1 and 2.)

The Bureau of Labor Statistics currently uses the difference of growth rates approach in its productivity measures. ${ }^{17}$ The output measures are currently chain indexes of output using Fisher or Tornqvist weights. Labor inputs are either hours or weighted hours at work. Measures of productivity are calculated as the differences between the growth in output and the growth of inputs. The BLS measures therefore suffer from the deficiency that it includes the Denison effect. More generally, it is not a chain index of productivity growth, which is the preferred measure.

In summary, none of the current approaches to estimating productivity growth appear to be well-grounded in welfare economics. Rather, assuming that differences in productivity growth are due to differences in unmeasured inputs, the appropriate measure would be a chain index of productivity growth of different sectors weighted by current expenditure or current-value inputs shares. In terms of the decomposition in equation (9), the appropriate measure would be total productivity growth after removing the Denison effect. Alternatively, the appropriate measure would be the pure productivity effect plus the Baumol effect. It is useful to note, that none of the current measures of productivity follow the appropriate procedure for measuring productivity growth.

Brookings Papers on Economic Activity, 1982, No. 2, pp. 423-54; and Edward F. Denison, Accounting for Slower Growth: the United States in the 1970s, Washington, Brookings, 1979.
${ }^{16}$ The major studies are usefully summarized in Dale W. Jorgenson, Productivity: Volume I, Postwar U.S. Economic Growth, MIT Press, Cambridge, MA., 1999. See especially Chapter 3, "The Explanation of Productivity Change," with Zvi Griliches.

[^6]Table 1
Example of Alternative Measures of Productivity Growth Showing Strong Denison Effect

|  |  | Period | Industry 1 | $\begin{aligned} & \text { Industry } \\ & 2 \end{aligned}$ | Fisher growth rate | Total | share of industry 1 | share of industry 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Output |  |  |  |  |  |  |  |
| 2 |  | 1 | 100.00 | 100.00 |  | 200.00 | 50.00\% | 50.00\% |
| 3 |  | 2 | 105.00 | 90.00 | -2.310\% | 195.00 | 53.85\% | 46.15\% |
| 4 | Hours |  |  |  |  |  |  |  |
| 5 |  | 1 | 10.00 | 100.00 |  | 110.00 | 9.09\% | 90.91\% |
| 6 |  | 2 | 10.00 | 90.00 | -9.494\% | 100.00 | 10.00\% | 90.00\% |
| 7 | Output per hour |  |  |  |  |  |  |  |
| 8 |  | 1 | 10.00 | 1.00 |  | 1.82 | 550.00\% | -450.00\% |
| 9 |  | 2 | 10.50 | 1.00 | 7.250\% | 1.95 | 538.46\% | -438.46\% |
| 10 | Productivity Relatives |  |  |  |  |  |  |  |
| 11 |  | 1 | 5.500 | 0.550 |  |  |  |  |
| 12 |  | 2 | 5.385 | 0.513 |  |  |  |  |
| 13 |  |  |  |  |  |  |  |  |
| 14 | Productivity growth | 2 | 5.00\% | 0.00\% | 7.250\% |  |  |  |


|  |  | Base period <br> is second <br> period | Base period <br> is first <br> period |
| :--- | :--- | :--- | :--- |
| 15 |  | $2.75 \%$ | $2.50 \%$ |
| 16 | Pure productivity effect | $\mathbf{0 . 0 0 \%}$ | $\mathbf{0 . 2 5 \%}$ |
| 17 | Baumol Effect | $\mathbf{4 . 5 0 \%}$ | $\mathbf{4 . 5 0 \%}$ |
| 18 | Denison Effect | $\mathbf{7 . 2 5 \%}$ | $\mathbf{7 . 2 5 \%}$ |
| 19 | Sum | $\mathbf{2 . 7 5 \%}$ |  |
| 20 | Pure productivity plus Baumol effect | $\mathbf{0 . 5 2 \%}$ |  |
| 21 | Difference in growth rates method | $\mathbf{7 . 2 5 \%}$ |  |

Table 2
Example of Alternative Measures of Productivity Growth

|  |  | Period | Industry 1 | $\begin{gathered} \text { Industry } \\ 2 \end{gathered}$ | Fisher growth rate | Total | share of industry 1 | share of industry 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Output |  |  |  |  |  |  |  |
| 2 |  | 1 | 100.00 | 100.00 |  | 200.00 | 50.00\% | 50.00\% |
| 3 |  | 2 | 70.00 | 140.00 | 8.990\% | 210.00 | 33.33\% | 66.67\% |
| 4 | Hours |  |  |  |  |  |  |  |
| 5 |  | 1 | 100.00 | 100.00 |  | 200.00 | 50.00\% | 50.00\% |
| 6 |  | 2 | 60.00 | 140.00 | 5.886\% | 200.00 | 30.00\% | 70.00\% |
| 7 | Output per hour |  |  |  |  |  |  |  |
| 8 |  | 1 | 1.00 | 1.00 |  | 1.00 | 100.00\% | 0.00\% |
| 9 |  | 2 | 1.17 | 1.00 | 5.000\% | 1.05 | 111.11\% | -11.11\% |
| 10 | Productivity Relatives |  |  |  |  |  |  |  |
| 11 |  | 1 | 1.000 | 1.000 |  |  |  |  |
| 12 |  | 2 | 1.111 | 0.952 |  |  |  |  |
| 13 |  |  |  |  |  |  |  |  |
| 14 | Productivity growth | 2 | 16.67\% | 0.00\% | 5.000\% |  |  |  |
| 15 |  |  |  |  |  | Base period is second period | Base period is first period |  |
| 16 | Pure productivity effect |  |  |  |  | 5.00\% | 8.33\% |  |
| 17 | Baumol Effect |  |  |  |  | 0.00\% | -3.33\% |  |
| 18 | Denison Effect |  |  |  |  | 0.00\% | 0.00\% |  |
| 19 | Sum |  |  |  |  | 5.00\% | 5.00\% |  |
| 20 | Pure productivity plus B | umol eff |  |  |  | 5.00\% |  |  |
| 21 | Difference in growth rate | method |  |  |  | 3.79\% |  |  |
| 22 | Aggregate productivity g | rowth |  |  |  | 5.00\% |  |  |

## Showing Strong Baumol Effect

## Figure 1

Alternative Measures of Labor Productivity for Overall Economy, 1978-98


Source: Revised industry 114000a: Tables: Chart 14.

Table 3
Alternative Measures of Labor Productivity for Overall Economy, 1978-98

|  | $1978-89$ | $1990-95$ | $1996-98$ | $1978-98$ |
| :--- | ---: | ---: | ---: | ---: |
|  |  |  |  |  |
| Ideal Measure | $1.38 \%$ | $1.26 \%$ | $2.39 \%$ | $1.44 \%$ |
| GDI Productivity | $1.15 \%$ | $0.95 \%$ | $2.32 \%$ | $1.23 \%$ |

Source: Revised industry 114000: BasicData


[^0]:    ${ }^{1}$ I am grateful for helpful comments by Erwin Diewert.
    ${ }^{2}$ A discussion of the use of Fisher indexes in the national income and product accounts is found at Survey of Current Business, Vol. 72, April 1992, pp. 49-52 and J. Steven Landefeld and Robert P. Parker, "BEA's Chain Indexes, Time Series, and Measures of Long-Term Economic Growth," Survey of Current Business, Vol. 77, May 1997, p. 58-68.
    ${ }^{3}$ Paul Krugman, The Age of Diminished Expectations, MIT Press, Cambridge, Mass., 1990, p. 9.
    ${ }^{4}$ See William D. Nordhaus, "Alternative Methods for Measuring Productivity Growth," November 6, 2000 and William D. Nordhaus, "Productivity Growth and the New Economy," November 13, 2000. Both papers are available at www.econ.yale.edu/~nordhaus/homepage/write_new_economy.htm.

[^1]:    ${ }^{5}$ There are many excellent references to the theory of index numbers. A succinct formal statement is W. Erwin Diewert, "Index Numbers," in J. Eatwell, M. Milgate, and P. Newman, eds., The New Palgrave: A Dictionary of Economics, Vol.1, London: The Macmillan Press, 1987, pp. 690-696.
    ${ }^{6}$ Durable goods and investment can be added by using the approach introduced by Martin Weitzman, "On the Welfare Significance of National Product in a Dynamic Economy, Quarterly Journal of Economics, Vol. 90, 1976, pp.156-162.

[^2]:    ${ }^{8}$ See William J. Baumol, "Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis," The American Economic Review, Vol. 57, No. 3, June 1967, pp. 415-426. This was updated and revised in William J. Baumol, Sue Anne Batey Blackman, and Edward N. Wolff, "Unbalanced Growth Revisited: Asymptotic Stagnancy and New Evidence," The American Economic Review, vol. 75, no. 4, September 1985, pp. 806-817.
    ${ }^{9}$ A number of studies found this syndrome. See particularly his studies of postwar Europe in Why Growth Rates Differ, Brooking, Washington, DC, 1962.

[^3]:    ${ }^{10}$ This points to an important reason for constructing complete measures of inputs. In principle, if all differences in productivity levels are due solely to differences in quality and quantity of all inputs, a complete and accurate accounting system would show that total factor productivities were equal in all industries. At this point, the Denison effect [the third term in equations (9) or ( $\left.9^{\prime}\right)$ ] would be calculated to be zero.

[^4]:    ${ }^{11}$ See William D. Nordhaus, "Alternative Methods for Measuring Productivity Growth," November 6,2000, available at
    www.econ.yale.edu/~nordhaus/homepage/ write_new_economy.htm.

[^5]:    ${ }^{14}$ William J. Baumol and Edward N. Wolff, "On Interindustry Differences in Absolute Productivity,"The Journal of Political Economy, Vol. 92, No. 6, Dec., 1984), pp. 1017-1034.
    ${ }^{15}$ See William D. Nordhaus, "The Recent Productivity Slowdown," Brookings Papers on Economic Activity, 1972, no. 3, pp. 493-536; Martin N. Baily, "The Productivity Growth Slowdown by Industry,

[^6]:    ${ }^{17}$ See Kent Kunze, Mary Jablonski, and Virginia Klarquist, "BLS Modernizes Industry Labor Productivity Program," Monthly Labor Review, vol. 118, no. 7, July 1995, pp. 3-12 and John Duke and Lisa Usher, "BLS Completes Major Expansion of Industry Productivity Series," Monthly Labor Review, vol. 121, no. 9, September 1998, pp. 35-51.

