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FIAT MONEY AND THE EFFICIENT FINANCING OF THE FLOAT, PRODUCTION AND CONSUMPTION. PART I: THE FLOAT

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Abstract

The basic distinction in the optimization conditions between the general equilibrium model of a T period exchange economy and a strategic market game process model is between a set of equations homogeneous of order zero and a set of nonhomogeneous equations. The latter have an amount M of outside or fiat money added to the system. If there is an outside bank willing to lend or accept deposits at an interest rate $\rho > 0$ at the end of time T the initial amount of money M will have been consumed in interest payments to the outside bank. The price level is fully determined and in an economy where all assets are traded, the float is financed efficiently, otherwise there is a price wedge between buying and selling prices.

Key words: Fiat money, float, strategic market games.

1 Introduction

Four models of trade are examined. They are a general equilibrium model of exchange; then three market models which are defined in detail below, they are: a sell–all strategic market game with complete markets; a sell–all market game with only spot markets and a bid–offer strategic market game with complete markets. In the first model the introduction of fiat money and a money rate of interest is irrelevant. In the other three models the existence of fiat money as an asset plays an essential role when there is a positive rate of interest. When the rate of interest is zero and there is a positive initial supply of fiat money as an asset, only pathological noncooperative equilibria exist where either there is unbounded borrowing or no trade.

The results here suggest that for an economy in which there is any amount of money tied up in transactions lags, if the rate of interest is not zero then a nonzero amount of fiat held as an asset with no offsetting debt against it may be used to finance the float and in the instance of a "sell all" economy trade will be Pareto optimal and would be the same as in a general equilibrium economy if the money is distributed in proportion to ownership claims; otherwise any other distribution results in a lump sum redistribution and a role for an inside money market and financiers appears.

2 Four Models of Trade

We will illustrate our observations with an economy involving n types of trader each trading in m goods for T periods. For simplicity we begin with only consumer perishables. Production or inventorying of durable consumables introduce further strategic freedom in the models and are treated separately.

Consider n types of trader, where a trader i has the utility function

$$\varphi^i(x^i_{11}, ..., x^i_{mT}).$$

Trader i has an initial endowment of $(a^i_{11}, ..., a^i_{mT})$.

Model 1: The General Equilibrium Model

The first order optimization conditions for this general equilibrium exchange economy model are given by:

$$\text{maximize } G_i = \varphi^i(x^i_{11}, ...x^i_{mT}) + \lambda^i \sum_{j=1}^{m} \sum_{t=1}^{T} p_{jt}(a^i_{jt} - x^i_{jt})$$

$$\frac{\partial G_i}{\partial x_{jt}} = 0 \Rightarrow \frac{\partial \varphi^i}{\partial x^i_{jt}} - \lambda^i p_{jt} = 0, \text{ or} \tag{1}$$

$$\frac{\frac{\partial \varphi^i}{\partial x^i_{jt}}}{p_{jt}} = \lambda^i \text{ for } i = 1, ..., n,\ j = 1, ..., m \text{ and } t = 1, ..., T. \tag{2}$$

$\partial G^i/\partial \lambda^i = 0$ yields

$$\sum_{j=1}^{m}\sum_{t=1}^{T} p_{jt}(a_{jt}^i - x_{jt}^i) = 0 \text{ for } i = 1, ..., n. \tag{3}$$

We obtain nmT equations from (2), n equations from (3) and a further mT equations from

$$\sum_{i=1}^{n}(a_{jt}^i - x_{jt}^i) = 0 \text{ for } j = 1, ..., m \text{ and } t = 1, ..., T \tag{4}$$

yielding $nmT + mT + n$ conditions to determine

nmT	variables	x_{jt}^i
mT	variables	p_{jt}
n	variables	λ^i

but with perishables there is an arbitrary scaling of price each period.

Under well known reasonable conditions (Debreu, 1959) at least one set of relative prices which clear all markets efficiently exist and absolute prices can be scaled arbitrarily in each period.

We will illustrate our general observations with a simple example involving two types of trader each trading in two goods.

Consider two types of trader, all with the same utility function

$$\varphi(x, y) = xy.$$

Traders of type 1 have an initial endowment of $(a, 0)$ and type 2 have an initial endowment of $(0, a)$.

The general equilibrium model for this example is trivially easy. By consideration of symmetry it is easy to observe that the equilibrium endowments will be $\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)$ and the final utility for all agents is $a^2/4$. Nevertheless as we will need most of the notation, the full formal notation is developed.

Let p_j = the price of the jth commodity $j = 1, 2$; x^i = the consumption of 1 by i; and y^i = the consumption of 2 by i.

The general equilibrium exchange problem is defined as:

$$\begin{array}{ll} \text{maximize} & x^1 y^1 \\ \text{subject to} & (a - x^1)p_1 - y^1 p_2 = 0 \text{ for trader } 1 \end{array}$$

$$\begin{array}{ll} \text{maximize} & x^2 y^2 \\ \text{subject to} & -x^2 p_1 + (a - y^2)p_2 = 0 \end{array}$$

where as the price normalization is arbitrary we may set $p_1 = 1$ thus

$$p_2 = \frac{x^2}{a - y^2} = \frac{a - x^1}{y^1}$$

which in this simple instance yields

$$p_2 = \frac{a/2}{a - a/2} = 1.$$

We may optimize the two Lagrangians

$$\begin{aligned} G_1 &= x^1 y^1 + \lambda_1\{(a - x^1)p_1 - y^1 p_2\} \\ G_2 &= x^2 y^2 + \lambda_2\{-x^2 p_1 + (a - y^2)p_2\} \end{aligned}$$

where $x^1 + x^2 = a$, $y^1 + y^2 = a$ and where by normalization $p_1 = 1$.

First order conditions yield

$$\frac{\partial}{\partial x^1} \qquad y^1 = \lambda_1 \tag{5}$$

$$\frac{\partial}{\partial y^1} \qquad x^1 = \lambda_1 p_2 \tag{6}$$

$$\frac{\partial}{\partial \lambda_1} \qquad a - x^1 = y^1 p_2 \tag{7}$$

$$\frac{\partial}{\partial x^2} \qquad y^2 = \lambda_2 \tag{8}$$

$$\frac{\partial}{\partial y^2} \qquad x^2 = \lambda_2 p_2 \tag{9}$$

$$\frac{\partial}{\partial \lambda_2} \qquad x^2 = (a - y^2) p_2 \tag{10}$$

and

$$x^1 + x^2 = a \tag{11}$$

$$y^1 + y^2 = a. \tag{12}$$

The variables are x^1, x^2, y^1, y^2, p_2, λ_1 and λ_2. It is easy to check that

$$x^1 = x^2 = y^1 = y^2 = a/2,\ p_1 = p_2 = 1 \text{ and } \lambda_1 = \lambda_2 = a/2$$

provides the solution.

We now remodel this simple situation in three different ways as strategic market games where individuals utilize a fiat money to buy but receive their incomes from any sales at the start of the next period.

A strategic market game is a full process model of trade. It includes price formation and requires a mechanism which defines all potential outcomes regardless of optimality conditions, i.e., what happens for every combination of feasible strategies.

We consider the sell-all game (see Shapley and Shubik, 1977) in two versions. Model 2 is the sell–all game with all futures markets and Model 3 is the sell–all game with only spot markets. Model 4 is the bid–offer game (see Dubey and Shubik, 1978) with all futures markets.

The modeling requirements of a process model require a host of details such as which markets exist, how they are cleared; what the actual length of time a period is meant to represent; how many times does a market meet during a period (or how long is it open). These details turn out to be critical in discussing the velocity of money , the minimal length of time for which an interest rate is paid and the amount of money required to run an economy efficiently. However for the purposes of this discussion which is aimed at producing a minimal model to illustrate the role of fiat money in financing the float, several radical simplifications are made which can all be relaxed later to study the other phenomena noted.

We assume that all trade is paid for in cash. Individuals offer goods for sale and bid cash to buy the goods. The market price is formed by the amount of cash chasing the amount of goods offered. Individuals are paid after they have sold their goods. Even if payments were made a fraction of a second after the market has cleared it is an empirical question as to whether all individuals can sweep their incoming funds into an interest bearing account until the next time they need the cash.

In the sell–all models we assume that all real assets of all individuals must be put up for sale at some market which meets only once each period (say a 24 hour day). The minimal time for which interest is earned by an individual is one day, thus cash utilized for a transaction today or income received from today does not earn interest for its day of use (the argument will still hold even if the length of the period is any finite Δt, but not zero as in the Arrow–Debreu model). One may argue that it is unreal to require that a farmer sell all of his milk and buy back that which the family consumes. As a first approximation it may be a better approximation to reality than the general model in which individuals buy and sell all commodities.. In fact most people are not traders. They do not shift from buying to selling coffee when its price goes up a dollar.

The fourth model studied is the bid–offer model in which we permit individuals to act as traders in all commodities.

Model 2: The Sell–All Game with All Markets

By all markets we mean that every futures market exists. Thus all trades for all T periods may be regarded as taking place simultaneously. At period 1 there are mT markets available. There is a market for the spot delivery of good j and for its delivery at every period in the future. In essence, as in the general equilibrium model, individual action is represented in strategic form with the game collapsed into one period.

We consider a strategic market game with n types of trader,[1] m goods and T time periods.

The endowment of an agent of type i is $(a^i_{11}, ..., a^i_{mT}, M^i)$ where $(M^1, M^2, ..., M^n)$ is the distribution of fiat. Suppose that there is an outside bank which stands ready to accept deposits or lend at a rate of interest ρ.

Let d^i = the amount borrowed $(+)$ or deposited $(-)$ by i.

[1] Without going into the technical details we assume that each type of traders is representative of a mass of similar small traders thus we can assume that they regarded themselves as so insignificant that they have no influence over price. See, for example, Dubey and Shapley, 1994.

Let b^i_{jt} = the amount bid by i in market jt.

$$b^i_{jt} \geq 0 \text{ and } \sum_{j=1}^{m} \sum_{t=1}^{T} b^i_{jt} \leq M^i + d^i.$$

A strategy by an agent of type i is given by

$$(d^i, b^i_{11}, ..., b^i_{MT}).$$

The extensive form of the game is indicated in Figure 1.

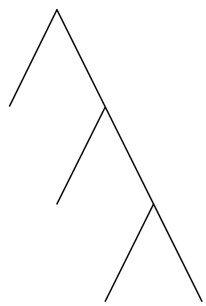

Figure 1

With no uncertainty and all futures markets, although goods will be delivered for T periods there is no need for market activity after the first period, hence there is no need for money for trade beyond the first period.

The maximization problem of an individual of type i can be stated as:

$$\max_{d^i, b^i_{jt}} \quad \varphi^i(x^i_{11}, ..., x^i_{mT})$$
$$\text{subject to} \quad \sum_{j=1}^{m} \sum_{t=1}^{T} [p_{jt}(a^i_{jt} - x^i_{jt}) + M^i - \rho d^i] \geq 0.$$

If we rule out bankruptcy[2] and observe that money is worthless after the last trading period then the constraint will be satisfied precisely. Thus we can consider

$$\text{maximize } G = \varphi^i\left(\frac{b^i_{11}}{p_{11}}, ..., \frac{b^i_{mT}}{p_{mT}}\right) + \lambda^i \left[\sum_{j=1}^{m} \sum_{t=1}^{T} p_{jt}(a^i_{jt} - x^i_{jt}) + M^i - \rho d^i\right]$$
$$\text{where } x^i_{jt} = \frac{b^i_{jt}}{p_{jt}}.$$

It is straightforward to observe that there will be a solution with $M^i > 0$ and $\rho > 0$. Furthermore suppose that we have solved model 1 and have a competitive equilibrium set of prices. Given this information we are able to calculate the wealth

[2] In several other publications the important feature of active bankruptcy has been dealt with; see Shubik and Wilson, 1977; Dubey, Geanakoplos and Shubik, 1988.

of all individuals. Let the wealth of an individual i be w^i. If the initial money supply is given out in proportion to the w^i the solution will coincide with that of the CE. This is illustrated in the simple example calculated below.

We consider the game where each individual has M units of fiat, Type 1 has $(a, 0)$, Type 2 has $(0, a)$. All traders are required to sell all and to bid to buy. If they wish they may borrow from an outside bank. A strategy by a trader α is to bid (b_1^α, b_2^α) where if $b_1^\alpha + b_2^\alpha > M$ the trader automatically borrows, if the bids are less than M he saves.[3]

An individual of Type 1 attempts to

$$\max_{0 \leq b_1, 0 \leq b_2} \left(\frac{b_1^1}{p_1}\right)\left(\frac{b_2^1}{p_2}\right) + \lambda_1[ap_1 + (M - b_1^1 - b_2^1)(1+\rho)]; \tag{13}$$

similarly for Type 2

$$p_1 = \frac{b_1^1 + b_1^2}{a},\ p_2 = \frac{b_2^1 + b_2^2}{a} \text{ where } b^i = b_1^i + b_2^i.$$

For $\rho > 0$ a symmetric solution yields

$$p_1 = p_2 = \frac{M(1+\rho)}{\rho a},\ b_j^i = \frac{M(1+\rho)}{2\rho},\ x_j^i = \frac{a}{2},\ d^i = \frac{M}{\rho} \text{ and } \lambda = \frac{\rho a^2}{2M(1+\rho)}.$$

For $\rho = 0$ price is unbounded with $M > 0$.

For $\rho \to \infty$, $p \to M/a$, $b \to M/2$.

The change in the rate of interest only lowers the price level while leaving the distribution of trade unchanged. The borrowing at a positive interest rate consumes the cash in the economy by the end of the game.

If the initial money were not distributed in proportion to wealth as suggested above, for any arbitrary distribution of wealth there will nevertheless be efficient allocation of resources, but the initial holdings of fiat influence the outcome.

Model 3: The Bid–Offer Game with All Markets

In this discussion the word "float" is used in the broad sense to include not only banking transactions in limbo, but any gap in time, even a fraction of a second, in the course of transactions where the strategic resources are unavailable to all parties. This may appear to be a minor institutional annoyance in developing a theory of equilibrium price but the sequential feature of trade and the finiteness of transactions time appear to be critical in the study of process.

Before we contrast the sell–all model with all futures markets with the sell–all model with spot markets only; we contrast the sell–all model with the bid–offer model. The difference between the former and the latter is that in the former all goods pass through the markets; in the latter individuals may consume their own inventories and send only a fraction to the market.

[3] Alternatively we could have the trader borrow d^i and bid so that $b_1^i + b_2^i \leq M + d^i$. This distinguishes a loan from a credit line.

The purpose of this comparison is to illustrate, by example, two points. They are the differences in the price level for the same amount of money and the differences in the strategic worth of money to the individuals.

A strategy by a trader α of type 1 is to offer an amount of the first good q^α for sale and bid an amount b^α for the second good. A strategy for a trader γ of type 2 is to offer an amount r^γ of the second good for sale and bid an amount d^γ for the first good.

The optimization for a trader of type 1 is[4]

$$\max_{\substack{0\leq b\\ 0\leq q\leq a}} (a-q)\left(\frac{b}{p_2}\right) + \min \mu_1[0,(qp_1-(b-M)(1+\rho)] \tag{14}$$

where, as this is a game of strategy μ_1 is the unit penalty for default in failure to pay back one's loan.

The optimization for a trader of type 2 is

$$\max_{\substack{0\leq d\\ 0\leq r\leq a}} \left(\frac{d}{p_1}\right)(a-r) + \min \mu_2[0,(-(d-M)(1+\rho)+rp_2] \tag{15}$$

where

$$p_1 = \frac{\int b^\alpha}{\int r^\gamma} \tag{16}$$

$$p_2 = \frac{\int d^\gamma}{\int q^\alpha}. \tag{17}$$

Suppose that we select the bankruptcy penalties to be high. If penalties are high enough that all agents at equilibrium repay their loans we can treat (14) and (15) as though they were optimizations where we interpret the Lagrangians as default penalties

$$\max_{\substack{0\leq b\\ 0\leq q\leq a}} (a-q)\left(\frac{b}{p_2}\right) + \mu_1(qp_1-(b-M)(1+\rho)) \tag{18}$$

$$\max_{\substack{0\leq d\\ 0\leq r\leq a}} \left(\frac{d}{p_1}\right)(a-r) + \mu_2(-(d-M)(1+\rho)+rp_2) \tag{19}$$

where p_1 and p_2 are formed as in (16) and (17). First order conditions yield:

$$\frac{\partial}{\partial q} \qquad \frac{b}{p_2} = \mu_1 p_1 \tag{20}$$

$$\frac{\partial}{\partial b} \qquad \frac{a-q}{p_2} = \mu_1(1+\rho) \tag{21}$$

$$\frac{\partial}{\partial q} \qquad \frac{d}{p_1} = \mu_2 p_2 \tag{22}$$

[4] As we consider a type symmetric equilibrium we dispense with the α and γ denoting a specific agent from a continuum of small agents except for the integrals in defining price.

$$\frac{\partial}{\partial b} \qquad \frac{a-r}{p_1} = \mu_2(1+\rho) \tag{23}$$

$$\frac{\partial}{\partial \mu} \qquad qp = (b-M)(1+\rho). \tag{24}$$

From (18) and (19) we observe that no symmetric solution can exist except for $\rho = 0$. If one did then $b = d$, $r = q$ and $p_1 = p_2 = p$. This implies $p = b/q$ hence if the budget were to balance

$$qp = b(1+\rho) \text{ or } q\frac{b}{q} = b(1+\rho)$$

which holds only when $\rho = 0$.

When $\rho > 0$ we conjecture, then establish the existence of a type symmetric equilibrium (TSNE). If such an equilibrium exists then $b = d$, $r = q$, $p_1 = p_2$, $\mu_1 = \mu_2 = \mu$.

We obtain

$$q = \frac{a}{2+\rho},\ b = \frac{M(1+\rho)}{\rho}, \text{ and } p = \frac{M(1+\rho)(2+\rho)}{\rho a}$$

This yields as final endowments for a trader:

$$\text{for type 1} \quad \left(a-q, \frac{b}{p}\right) = \left(a - \frac{a}{2+\rho}, \frac{a}{2+\rho}\right) = \left(\left(\frac{1+\rho}{2+\rho}\right)a, \frac{a}{2+\rho}\right)$$

$$\text{for type 2} \quad \left(\frac{a}{2+\rho}, \left(\frac{1+\rho}{2+\rho}\right)a\right)$$

This displays the well known inefficiency "wedge" in the difference between buying and selling prices.

We obtain that for $\rho = 0$ the distribution is $(\frac{a}{2}\ \frac{a}{2})$ and $(\frac{a}{2}\ \frac{a}{2})$ but p is unbounded.

As ρ increases the wedge between buying and selling is increased.

Table 1 compares the sell–all model with bid–offer for an even distribution of money with $\rho = 1$, $M = 1$, $a = 1$.

	Borrow	Bid	p_1^1	p_2^1	p_2^1	p_2^2	x_1^1	x_2^1	x_1^2	x_2^2	Offer
Sell–all	1	2	1	1	1	1	1	1	1	1	1
Bid–offer	1	2	3	6	6	3	2/3	1/3	1/3	2/3	1/3

Table 1

The extra notation reflects $p_1^1 =$ the selling price of good 1 to trader 1, while $p_2^1=$ the buying price of good 2 to trader 1.

Model 4: The Sell–All Game with Spot Markets Only

If there are only spot markets then trade must take place every period. The optimization can be planned as:

$$\max \varphi^i \left(\frac{b_{11}^i}{p_{11}}, ..., \frac{b_{mT}^i}{p_{mT}}\right)$$

where p_{jt} are spot prices and

$$\begin{aligned} d_1^i &= M^i - \sum_{j=1}^m b_{j1}^i \\ d_2^i &= \sum_{j=1}^m p_{j1}a_{j1}^i - d_1^i(1+\rho) - \sum_{j=1}^m b_{j2}^i \\ &\vdots \\ d_{T+1}^i &= \sum_{j=1}^m p_{jT}a_{jT}^i - d_T^i(1+\rho) = 0 \end{aligned} \tag{25}$$

In model 2 when all futures markets are present all markets are only active for the first period and if M is the total amount of outside money at the start the level of (futures) prices is given by:

$$\sum_{j=1}^m \sum_{t=1}^T p_{jt}a_{jt}^i = \frac{M(1+\rho)}{\rho}. \tag{26}$$

The net money holdings at the start of each period over time are

$$(M, 0, ..., 0).$$

In model 4 the money holdings will be dependent on the profile of the inflow of the real assets. Contrasting the constraints on the optimization in model 2 with model 4 in each instance there is only one binding constraint. In equations (25) only the last imposes a constraint as credit is unlimited for the others.

A simple example illustrates the upper and lower bounds on prices and market activity depending on the use of futures or spot markets. Consider a sell–all economy at its simplest. It lasts for T periods, all agents have a utility function of the form:

$$\sum_{t=1}^T \beta^{t-1}\varphi(x_t),\ 0 < \beta \leq 1.$$

Each agent has ownership to one unit of input at time t of a perishable. $(1, 1, ..., 1)$ describes the ownership claims. The equilibrium solution is trivially autarchic.. Each consumes his own resources. But suppose all must buy through the markets and each is given M units of money at the start together with the possibility of borrowing for one period, or depositing at an interest rate of ρ.

We consider three examples for illustration.

β	ρ	T	Markets	Active market periods	Price level	First period borrowing	Last period borrowing
1	ε	large	full	1	$\frac{1+\varepsilon}{\varepsilon T}$	$1/\varepsilon$	$1/\varepsilon$
1/2	1	large	full	1	1	1	1
1/2	1	large	spot	T	$1+\frac{1}{2^T-1}$	$\frac{1}{2^T-1}$	$1/\left(1-\frac{1}{2^T}\right)$

Table 2

Price level is indicated by the spot market price at the first period.

3 General Comment

In the examples above cash could be invested and earn the rate ρ, but if used in trade it earns nothing. If we imagined a Gesell style of money with coupons with the rate $\rho^* < \rho$ attached to the cash the analysis still goes through.

If we consider an economy with n types of trader, T periods and any number of commodities with the sell–all requirement each period, then for any regular preferences the above observations hold. The basic observation is merely that one can treat any durable or storable which can last τ periods as up to τ tradable commodities. Then one observes cash flows. Let I_t be the income in period t and E_t be the expenditure. For general equilibrium we have $\sum_{t=1}^{T}(I_t - E_t) = 0$; for sell all

$$\sum_{t=1}^{T}((1+\rho)^{t-1}I_t - (1+\rho)^t E_t) + M(1+\rho)^t = 0.$$

The sell–all model has the property that it emphasizes the monetary control system. For example if there were some agents with money but with no real resources whatsoever they would participate in the economy. Furthermore with a distribution of money other than one in which all use all of their cash to finance trade (or the float) those with surplus money relative to the competitive valuation of their resources may become bankers or lenders in a money market.

The general model suggested here can be extended to the infinite horizon. But for any finite horizon borrowing will not be stationary even though inputs are stationary, as the cash must be consumed by the end. As the horizon lengthens stationarity is approached.

When the rate of interest is zero and $M > 0$ there is a singularity in the system. The equations cannot be solved as the cash cannot be consumed. It is a "hot potato" as no one wishes to have valueless cash after the end of the T period economy. With $\rho > 0$ debt takes care of this.[5]

4 Time, Dynamics the Price Level and the Breaking of Symmetry

The building of a process model and the financing of the float may appear at first glance to be a minor adjustment over the general equilibrium analogue. However it suggests several critical differences. The introduction of money enables us to formulate easily a full process model which sets up the necessary conditions to study dynamics. The existence of an outside or central bank combined with the lack of intrinsic value of the fiat permits the economy to become cash consuming as long as there is some cash in the economy which does not have an offsetting debt against it. All other financial instruments appear in pairs as is shown in double entry bookkeeping. The fiat money supply may be varied by changing the level of the debt. But what is suggested here is that at the start there must be some fiat that remains as

[5] This has been observed in Shubik (1980), Dubey and Geanakoplos (1992) and Shubik (1996).

an asset after all debts are canceled. Thus it will be the only financial instrument for which there is no offset. It is like a commodity money in that aspect. But because it is intrinsically valueless it is consumed by the end of the game. This however means that there is no longer the scaling indeterminacy in the price level that is present in a general equilibrium economy.

Most modern economies are highly (but not perfectly monetized). If the economy were fully monetized (i.e., all assets pass through the market each period: the sell-all model) then not only does the presence of a positive amount of fiat money finance the float but it does so efficiently.

In any finite horizon economy borrowing cannot be stationary. However as the horizon becomes longer for a given set of stationary inputs and fixed initial amount of money the rate of interest required to consume it approaches zero in the overlapping generations stationary state (or it is β in the dynasty or infinite life model). The solution depends on bankruptcy conditions being such that strategic bankruptcy does not pay. The amount of money required is $M > 0$. But because units are arbitrary, without any loss of generality we can select the amount of money for which there is no offsetting asset to be such that $M = 1$ and adjust the bankruptcy penalty in terms of the monetary unit so that it is sufficient to discourage bankruptcy.

In many of the models of a monetary economy (for example, Dubey and Geanakoplos 1992; Hool, 1976; Bewley, 1980; Grandmont and Younes, 1973; Lucas; 1980 Shubik, 1973, 1996) there is a transactions lag of one period. The length of the period is usually not specified. All that matters is that it is not zero. Given that it is some finite t, if there is an outside bank which will borrow or lend at a positive rate of interest the individuals will consume the cash in financing the float. This is not seen clearly in the infinite horizon because there is a singularity when $\rho = 0$ and a full stationary state without the presence of outside money is feasible. The float is financed at no cost, full symmetry exists with no financial instrument without an offsetting item against it, the model becomes essentially timeless and the price level is no longer determined. Aristotle's view of money as essentially barren is borne out in a stationary state in an OLG model with $\rho = 0$.

As markets become more efficient and global, and in some instances are open continuously 24 hours a day, it may appear that continuous time models provide the insights into economics. Although transmission speeds may approach the speed of light, many production and consumption processes require sizeable amounts of time. Furthermore the key elements in finance are evaluation, perception and risk assessment. These take time and, all other things being equal, there is a tradeoff between accuracy and evaluation time. The general equilibrium price system provided deep insights into the static properties of production and exchange and no insight into the role of money and finance in guiding the economy.

The introduction of money and financial institutions is a first step towards a viable economic dynamics where the emphasis switches more towards problems in perception, evaluation and induction rather than an examination of equilibrium conditions. The model presented here deals only with consumption and exchange. In Part 2 the analysis will be extended to production and finance. By paying attention to the distinctive finite timing differences among production processes, consumption processes; and the frequency of trade in markets for goods and services and financial markets it becomes possible to construct a formal model needed to estimate how much money is required to finance the float, the working capital and capital stock of an economy. In doing this the one period aspects of a cash in advance constraint become irrelevant.

The appropriate process models are a mixture of continuous time with discrete events imposed on the essentially continuous financial markets. The key feature is that fiat money is frequently useful in the dynamic process of running markets and forming prices where it provides an individually anonymous symbol of trust. Its presence as an asset breaks the symmetry imposed by the study of trade at equilibrium and provides the extra degree of freedom in the economic system permitting price formation regardless of equilibrium conditions. The lack of value of the paper is a virtue in removing the static indeterminacy in price that is present in the moneyless general equilibrium model. The amount of money required to run an economy depends on the physical facts of the time lags in transactions, production and timing differences in the consumers' income and consumption. Details as suggested by a study such as Orr (1970) are critical to this evaluation.

5 A Comment on Time, Endogenous and Exogenous Uncertainty

If error is increased as humans try to perform, at least some of their activities faster, then under any reasonable measure of optimality the appropriate model of economic activity will have discrete time decisions in an economy with an endogenously generated random component.

As soon as individuals try to adjust even for their endogenously generated uncertainty they may require precautionary reserves as they may be unable to repay debt. The size of reserves will depend, not only on preferences and resources, but on the level of perception, bankruptcy, default and settlement rules.

If there were also exogenous uncertainty present without complete markets money would also be held for speculation reserves. Furthermore with uncertainty both the perception and risk aversion aspects of borrowing and lending become relevant. Much of credit granting can (and is) provided by an inside "money market" or banking system which does not require an outside (and essentially risk neutral government bank). But risk assessment may lead to credit restriction and the intertemporal equations describing cash flows, borrowing and lending period-by-period in Model 4 become potential constraints on the optimization.

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