COWLES FOUNDATION FOR RESEARCH IN ECONOMICS
AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut 06520

COWLES FOUNDATION DISCUSSION PAPER NO. 1112

TIME AND MONEY

Martin Shubik

January 1996

TIME AND MONEY

Martin Shubik*

ABSTRACT: General equilibrium is timeless, and without outside money, the price system is homogeneous of order zero. Some finite horizon strategic market game models are considered with an initial issue of fiat money held as an asset. For any arbitrary finite horizon, the solution is time-dependent. In the infinite horizon, time disappears with the initial issue of fiat money present as circulating capital in the fully stationary state and the price level is determined.

1 AN APPROACH TO THE THEORY OF MONEY AND FINANCIAL INSTITUTIONS

In the past twenty-five years my basic research has been directed primarily toward the development of a theory of money and financial institutions. This paper is devoted to an essentially nontechnical presentation of the key elements of the approach together with some of the key results. I also indicate the basic shift in paradigm away from equilibrium towards the study of process, learning and evolution, which is transforming economic analysis.

The basic approach involves a combination of game theory and experimental gaming. The economy must be modeled and considered as a "playable game," i.e., a fully defined process model whose rules can be sufficiently comprehended that the game can be played.

The central feature of a theory of money and financial institutions is the minimization of the need for trust. The key role of financial institution is information processing, aggregation and disaggregation, and evaluation. However, the interpretation of data is critical. It is not what the numbers are, but what they mean. Thus heuristic phrases such as the old banking maxim of "character, competence and collateral" can be operationalized.

In this paper I deal only with exchange economies with completely stationary or cyclical inputs. I believe that the basic points made here hold for growth models with some adjustments concerning the boundedness conditions on the money supply being related to the size of growth.

---

* Much of the work noted here has been the result of joint collaborations over many years. My main collaborators have been Lloyd Shapley, Pradeep Dubey, John Geanakoplos, Yannis Karatzas, William Sudderth as well as several others. Specific acknowledgements are given in my forthcoming volume (Shubik, 1996).

1.1 On time and general equilibrium

The finite general equilibrium model of Arrow and Debreu (1954) and others has been regarded as one of the key works of microeconomic theory as it explains in both an elegant and abstract manner the importance of the price system in the running of a decentralized efficient economy. Unfortunately, in the masterful development of the mathematics to study equilibrium, all of the richness and context required to study process was suppressed. It is an actor poor, context poor, preinstitutional, timeless abstraction devoted to illustrating the equilibrium properties of the price system. A detailed critique of the nature of the abstraction, its uses and limitations is discussed elsewhere (Shubik, 1996, Ch 4). In spite of its drawbacks, the approach here is deeply indebted to the general equilibrium analysis inasmuch as it presented the first coherent, consistent closed model of the economy. Because of its concern only with equilibrium, the techniques used were able to avoid the fussy details, concerning information, bankruptcy, division by zero or unboundedness in strategy sets in the formation of prices. These details cannot be avoided when a process model is built with enough care to define the system for all points in the set of feasible outcomes. It is in these details that the institutions of the economy lie. They are part of the rules of the game and act as the carriers of process. Once they are well-defined, the possibility for their modification becomes evident and the transition from an equilibrium economics to an evolutionary economics becomes clear.

1.2 The games within the game

Before we limit our concern to the more or less purely economic aspects of a theory of money and financial institutions it is desirable to have an appreciation for the broader context within which economic activity is embedded

In going from current static microeconomic analysis toward a process analysis a key distinction must be made between "the rules of the game" or its formal structure and the motivation and behavior we consider in attempting to define a solution to the game under examination. A solution, at its most abstract and general, is nothing more than a prescription or transformation that carries some state $S(t)$ of the system being studied into a state $S(t+1)$. Nothing is necessarily implied about optimization or equilibrium in this definition of solution. An operations researcher may invent a payoff function or a microeconomist may assume the existence of a utility function, and if, as a good first order approximation, the interaction of other individuals or institutions, can be ignored they take as their solution concept some form of maximization of these functions. In much of macroeconomic analysis, instead of formulating an explicit optimization some form of behavioral conditions are postulated and they enable the analyst to update the system and study the time series generated.

It is possible to connect optimizing behavior with an evolving environment by formalizing overlapping generations (OLG) games with government, other legal persons of indefinite lives, and real persons.[1] A player in a game of strategy is characterized by a strategy set and a payoff function. A solution of a game involves the specification of some sort of choice rule usually based on some operation by the player on his strategy set. This operation may involve the player's attempt to maximize his payoff function. But to describe a game of strategy it is not necessary that an individual who is a strategic player has a utility function. Instead any player, such as government or an institution, could be equipped with a *choice rule* that instructs the player as to what to do in all circumstances. Even though the player may have an infinite life the decision rule whereby it advances into the future may be based on a finite set of arguments. An appropriate way to incorporate government and corporations into an overlapping generations model is to introduce them as infinitely lived strategic players without utility functions but with a decision or choice rule determined by the real persons. An alternative way of modeling government is as a mechanism whose move is determined as the outcome of a game played between a set of politicians and bureaucrats where the politicians are elected every so often by the overlapping generations of real persons and the politicians appoint the bureaucrats, but for longer terms than themselves.

Figure 1 suggests the scheme for an overlapping generations economy containing three types of agent. They are: (1) the young, (2) the old, and (3) the government. The first two are the real or natural persons, who can be regarded as many in number and possibly are most conveniently represented by a continuum of agents. The third is the government and may be represented by a single large atom.

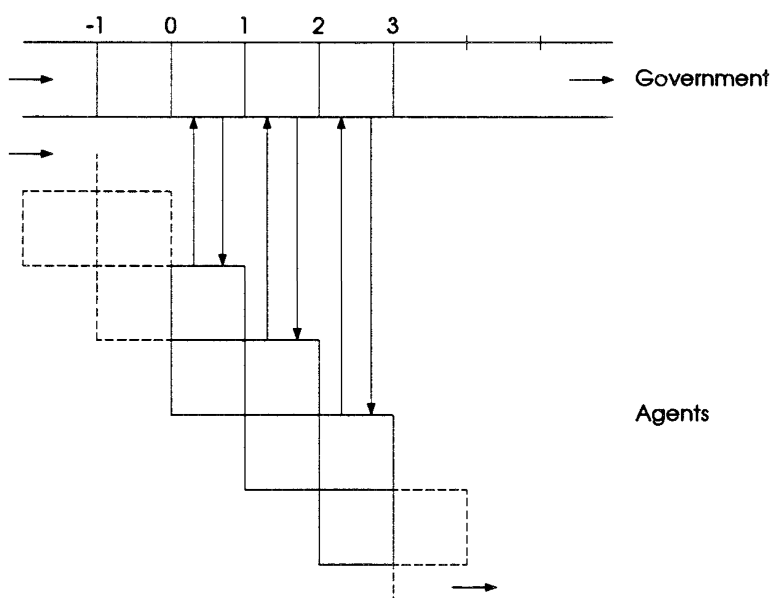

FIGURE 1: The Games within the Game

[1]See Woodford and Muller (1988) for a treatment of an economic model with both finitely and infinitely lived agents.

At point of time $t = 0$ there are two generations alive, those born at time $t = -1$ and $t = 0$. Government as well as the live agents must select its move at $t = 0$. A reasonable restriction on its choice rule is that it depends at most on all of the decisions of all of the natural persons alive at $t = 0$.

At time t there are two generations alive, those born at $t-1$ and those born at t. They are concerned with maximizing their welfare.

We may consider that an individual born at t has a set of strategies S_t. Part of any strategy is an action, message or instruction that he sends to the government and part is an instruction that will depend upon the state of the system at the start of $t+1$.

Heuristically the actions of the government at time t might depend on cultural, historical, social, political and economic factors. If we choose to model the agents as nonsocial, nonhistorical individualistic local maximizers we then have the most unfavorable and simplest set of conditions to examine the possibility that there may be structures where the needs of future generations could be served by the selfish behavior of the living.

One takes as given an institution called government whose strategy set and decision rule are given. We then consider the outcome of a game played *in the context* of these rules. Having considered politico-economic or economic efficiency within the context of the given institution we may then wish to ask a separate question. That is how did the institution evolve in the first place and how is its structure modified.

Government behavior can be considered in terms of both moves and strategy, but the more appropriate words are choice or action and policy or plan designed to set the environment for the real persons. We may regard a government at any particular point in time as carrying out two activities. It sets taxes, subsidies, fixes the money supply, controls the central bank rate of interest and manages the national debt, as well as taking action on many items of social welfare entailing public goods and services. The government also announces or otherwise indicates policy. Thus, for example, it indicates that it has a program or plan over the next few years to raise taxes and to lower the public debt.

Empirically we may observe that most governments, at any point in time, appear to have some form of plan, but the plan is continually updated, modified or otherwise changed. There is often a considerable skepticism held by the public concerning the plausibility that the stated plans of the government at time t will be consistent with its actions at time $t+1$. Yet in human affairs, even if policy changes in the future there is a perceived need that it be announced in the present, if only to provide some indication of the gap between promise and fulfillment.

When one looks at governmental planning and policy, at least in any explicit form, it does not appear to be particularly long term. One suspects that it stretches at most no longer than between administrations. In a democratic society composed of individuals mainly concerned with their own welfare, it is reasonable to accept as a comfortable upper bound on the length of any announced plan the lifetime of the youngest strategically active individual.

The device of the "game within the game" allows for a reconciliation of local optimization with global evolution. The small live players behave by trying to optimize locally and taking, as a first order approximation, their environment as given; their actions change the environment, but usually on a time scale too slow to influence their daily concerns.

1.3 Quantitative and qualitative results

In my forthcoming book (Shubik, 1996) the detailed description of strategic market games together with the considerable notation and many examples and calculations are presented. The mathematical proofs are to be found in a series of papers with colleagues and are due primarily to them.

As the purpose of the exposition here is to sketch the ideas without delving into the notation, proofs or details, we concentrate on the qualitative aspects of the work rather than becoming enmeshed in the quantitative detail.

1.4 Four basic modeling principles

The basic approach adopted here has been dominated by four basic methodological rules, which merit discussion. They are:

[1] The insistence on the construction of well defined process models.

[2] The specification of laws of conservation.

[3] The understanding of the role of symmetry.

[4] The understanding of the invariant properties of the system.

A brief elaboration of these points is now given.

[1] Economic dynamics are notoriously difficult. Broad intuitive insights have enabled economists such as Marx and Keynes broadly to sketch global grand dynamics, which have caught the imagination of governments and the masses and have had long term consequences on policy. When the more cautious attempts have been made to measure phenomena, such as the propensity to consume or the Phillips curve, however, the grand dynamics appear to melt into a welter of special cases and context dependent incidents. In contrast, the highly mathematical and apparently precise formulation of general equilibrium has no dynamics as it avoids process modeling. A first step in trying to overcome the gap between the two sets of theory is to provide a process basis for the general equilibrium models. But in attempting to do so, phenomena such as thin markets, inactive markets, hyperinflation and panics appear to be intimately connected with mathematical details such as division by zero. An insistence on well-defining

the payoffs associated with every point in the state space provides an elementary check against basic error in the description of dynamics.

[2] The writings on the theory of money have involved considerable vagueness concerning the definitions of money, its various properties, and near substitutes. The attitude adopted here is to avoid the type of debate concerning "what is money, really?" and instead consider that all forms of money and credit instruments have a physical existence. We may imagine them as colored chips, and devote our concern to describing how they are created, how they are destroyed, and how they move through the system. For example, we consider fiat money as blue chips, bank money as green chips, clearinghouse credits as black chips and IOU notes as red chips. This approach avoids much of the mystery associated with processes such as bankruptcy. Red chips (IOU notes) may be destroyed, but blue chips (fiat money) are conserved. In many of the models investigated, in static equilibrium it is useful to assume that the quantity of fiat in the hands of the public plus the quantity of unissued fiat is a constant, or in the case of growth, that the new fiat the government is allowed to create is related to the growth.

[3] All financial instruments except the original issue of fiat are created in pairs or are clearly offset against a physical item such as when shares in a new corporation are issued directly against the stockholder contributions of physical resources. Double entry bookkeeping, is in essence, an exercise in symmetry. All resources are offset. To make the books fully balance always, the accountants have developed a residual category called ownership, which is debited against any asset without an offset. The initial issue of cash is such an asset. The presence of fiat money with no offsetting debt is somewhat nonsymmetric. The restoration of full symmetry and the disappearance of time appear to be intimately related.

[4] The task of formal model building in the social sciences in general and economics in particular is filled with difficulties involving the interpretation of the robustness and the relevance of the model. In particular, even when the formal model is built it is necessary to be specific about the solution concept to be adopted and to be in a position to know under what variations of the basic assumptions the results still hold. For example, the price system in many economic models involving an environment without exogenous uncertainty is invariant under ordinal transformations of utility functions. The pure strategy Nash equilibria of a system are invariant under transformations that preserve strategic domination, and these are even more general than the ordinal transformations of utility functions. In contrast, for an economy with exogenous uncertainty the results of the analysis are invariant only under cardinal transformations. Welfare considerations may call for both cardinality and comparability. Before policy comments can be made the claimed invariant properties need to be specified.

1.5 The key models

There are 12 ($3\times2\times2$) basic models that must be examined to appreciate the basic features of an economy utilizing fiat money as a means of exchange. These basic models involve the elementary descriptions of the trading and finance structure of the economy treated to study their behavior when there is no exogenous uncertainty present and then when there are stochastic elements. All models must be also considered in a finite horizon version and then when there is an infinite horizon. The three basic models consist of: (1) an economy that uses only fiat money for transactions; all agents have a nonzero supply and there are no credit markets; (2) an economy with an initial issue of fiat, but with an outside bank, which stands ready to borrow and lend at an exogenously set rate of interest; (3) an economy with an initial issue of fiat and an inside money market for borrowing and lending where the endogenous rate of interest is formed by the competitive forces of supply of fiat and the demand for loans.

Figure 2 shows the 12 basic models that are considered:

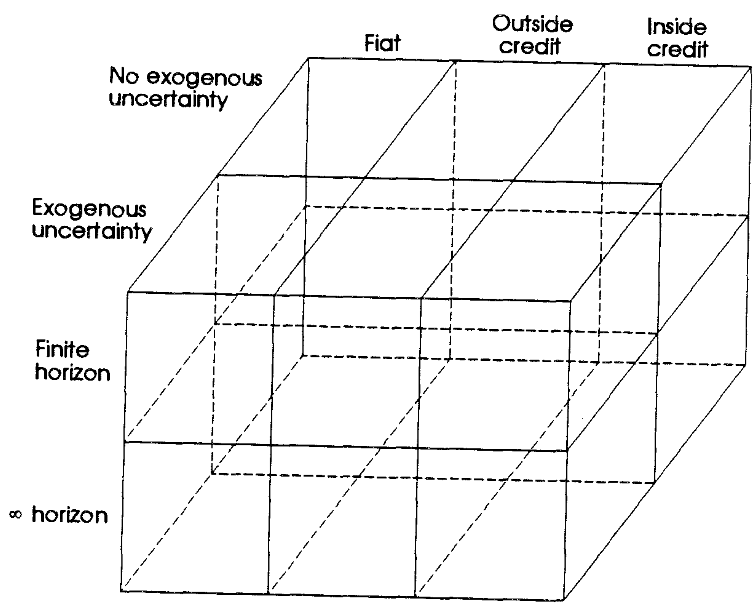

FIGURE 2: 12 Basic Models

The results noted here have been obtained for players who conceivably could live forever; but with little difficulty they can be replaced by overlapping generations, and the results are substantially similar for some settings. But with OLG one has to become specific about the birth-death processes assumed and the rules of inheritance.

2 THE NONSTOCHASTIC STATIONARY STATE

In the beginning there was the big bank and fiat money was created. The economy went forward through time with an evergrowing borrowing until by the end of time all fiat money had been consumed; all motion stopped, time disappeared and perfect symmetry was restored to the economy.

On the cosmology of fiat money

This somewhat mystical comment above is illustrated by two relatively simple examples and contains the first of the three basic points concerning the relationship between a monetary and a nonmonetary general equilibrium economy. It can be rephrased a little less poetically as follows.

The introduction of fiat money (and near monies, such as bank money) as a means of payment separates the timing of income and expenditures. It thereby provides the prerequisite degree of freedom needed to permit a trading dynamics. In a finite horizon economy money is worthless at the end of time.

As observed by Dubey and Geanakoplos (1990) if there is an outside bank in a finite horizon economy where income is received after sales, then by the standard finite horizon backward induction argument from dynamic programming all individuals expecting income to come in at $T+1$ will try to arrange to be in debt by precisely the income they are expecting to receive in $T+1$. There are two extensions of the finite horizon model which are of importance. The first comes from the consideration of game theoretic models as contrasted with non strategic general equilibrium models. If individuals are permitted to borrow and there is any point in the state space of the game where an individual is unable to repay his debt, the bankruptcy and reorganization rules must be specified in order to resolve the individual's inability to repay. If at the end of the game there are resources of any sort (such as durables) left over then the rules must specify how they are to be evaluated at the end of the game.

If, for example, as is often done we utilize a utility function of the form:

$$\Phi = \sum_{t=1}^{T} \beta^{t-1} \varphi(x_t)$$

from a game theoretic strategic point of view we must add $T+2$ extra terms, one for each period indicating the bankruptcy penalty to be levied for an individual who fails to pay back a debt due at time t and an extra one at the day of settlement, period $T+1$ to indicate, what if any value is attributed to left over assets. Instead of the payoff function above we may modify the payoff to:

$$\Phi = \sum_{t=1}^{T} \beta^{t-1} \left[\varphi(x_t) + \mu_1 \min[\text{Debt}_t, 0]\right] + \beta^T \mu_1 \min[\text{Debt}_{T+1}, 0] + \mu_2 \max[\text{Asset}_{T+1}, 0] .$$

The first of the two terms involving a minimum is the period by period penalty for going bankrupt. If we were to introduce assets directly there would also be information concerning the loss of assets and garnishing of income — but at the highest level of simplicity this term indicates that going bankrupt has unpleasant consequences. The last two terms indicate the cost of bankruptcy at the end of the game and the value of assets left over. The μs can be regarded as simple parameters or complex expressions. For simplicity they can be regarded as simple scalars.

A term involving β^T is in front of this last bankruptcy term indicating that for a large T it will be extremely weak verging on insignificant. This enables us to make a distinction between new fiat or outside money that might be introduced as a short term loan to be paid back after a single period and the initial supply of fiat which can be regarded either as an asset, or as a loan which need only be paid back at the end of the game. But for a game with a long duration the distinction between the two last alternatives can be made arbitrarily small. A bankruptcy penalty discounted far into the future is hardly a penalty. An immediate corollary that follows from our addition of a bankruptcy penalty to the payoff function is that *a debt which never has to be paid back is an asset.*

The term showing the value attached to the assets left over at the end is sometimes referred to in dynamic programming as the salvage value term. An alternative economic interpretation is that it portrays expectations.

We now are confronted with a paradox. If we consider the infinite horizon we do not see the extra terminal conditions involving bankruptcy and the worth of left over assets. The approach which can cover this difficulty is to study the limiting behavior of the finite horizon problem and ask whether the limiting behavior appears to approach the behavior at the limit. Fortunately this appears to be the case.

2.1 The finite horizon

We now consider an extremely simple model for purposes of illustration. Suppose that we consider a simple T period economy where each individual tries to maximize his utility function as noted above. Each individual begins with one unit[2] of fiat money and has an ownership claim each period to the income from the sale of one unit of the only consumer perishable, all of which is put up for sale each period.

A strategy of an individual α is a vector of the form

$$b^\alpha = (b_1^\alpha,\ b_2^\alpha,\ \dots\ )$$

---

[2]If I were being precise in the notation I would distinguish between a density and a finite measure. The agents are assumed to be nonatomic hence their resources are in terms of densities. The central bank is atomic. As the examples are extremely simple the meaning should be clear from context.

Price is formed by the amount of money chasing the amount of goods. This is:

$$p_t = \int b_t^{\alpha}/1$$

Each individual obtains as income at the start of period $t+1$ the amount p_t which equals his ownership claim from the amount of good he has sold. For the simple example posed above there is an elementary simple solution. All bid all of their money all of the time. This amounts to:

$$b_t^{\alpha} = 1 \quad \forall\ \alpha,\ t$$

$$p_t = p = 1 \quad \forall\ t$$

$$\text{Each obtains: } \sum_{t=0}^{T} \beta^t \varphi(1)\ .$$

In this set up there is an "end of the world" pathology as all traders obtain a perfectly worthless income at the start of period $T+1$ from the proceeds from the forced sale of the resource at the last period T.

If we had started all individuals with different amounts of fiat, the strategy would be more complex, although for a linear utility function all would have spent everything during the first period and the stationarity would settle in by the second period (see Shubik and Whitt, 1973).

We now introduce an outside bank with reserves of $B = 1$. The economy is otherwise the same as before except that the individuals can deposit or borrow from the outside bank. The outside bank sets the rate of interest exogenously. We select $1+\rho = 1/\beta$, show the solution for this rate, and show that it has an extra property when we go to the infinite horizon that may be deemed to be desirable. The property is that in a fully stationary state for the physical economy, prices are also stationary, i.e., there is no inflation or deflation.

Suppose that at equilibrium the price is p each period. We assume that at time t an individual will borrow or lend (+/–) an amount Δ_t thus we have a set of $T+1$ equations:

$$1 + \Delta_1 = p$$

$$1 - \rho\Delta_1 + \Delta_2 = p$$

$$\vdots$$

$$1 - \rho\sum_{J=1}^{t-1} \Delta_j + \Delta_t = p$$

$$\vdots$$

$$(1+\rho)\Delta_T = p\ .$$

The solution to these for Δ and p yield

$$\Delta_t = \frac{(1+\rho)^{t-1}}{(1+\rho)^T - 1} \quad \text{and} \quad p = 1 - \frac{(1+\rho)^T}{(1+\rho)^T - 1} .$$

We observe that as $T \to \infty$, $\Delta_T \to \beta$ and $p \to 1$. The system approaches a full stationary state and the need for bank reserves diminishes. For any finite first sequence of periods the amount borrowed can be made arbitrarily close to zero and the two solutions without and with the outside bank approach each other.

2.2 The dual solutions: Fiat with or without the outside bank

In the examples given above for a simple trading economy, the amount of good traded each period is the same. The introduction of the outside bank seems to be an economically irrelevant artifact. Suppose instead the economy were cyclical. For illustration we select a periodicity of 3. Instead of having $Q = 1$ be the amount of good for sale each period we may consider an economy where the amount of good for sale is :

$$Q_{3t+1} = Q_1;\ Q_{3t+2} = Q_2;\ Q_{3t+3} = Q_3 \ \ \textit{for} \ \ t = 1, 2, \ldots .$$

If we now consider an example analogous to the economy in Section 2.1 without a bank, but where all have an initial issue of fiat, prices can be adjusted from period to period only by hoarding. For example suppose the amounts of good were 1, 2, 3 repeated cyclically and the utility functions were:

$$\sum_{t=1}^{3T} \beta^{t-1} x_t^{\alpha} .$$

Spot prices in general equilibrium should be the same each period, but as there is a fixed amount of money in the system, this can only be achieved by hoarding. In general for a k-cycle with arbitrary equilibrium points we must consider 2^k Lagrangian multipliers to determine when the individual should hoard or spend all (Quint and Shubik, 1995a,b,c). In this simple example, if β is small enough the present will always dominate the future, thus all will always spend all and the spot prices will be 1, 1/2, 1/3. At the other extreme if $\beta = 1$ the spot prices would be 1/3, 1/3, 1/3 and in the second and third periods 1/3 and then 2/3 of the money supply would be hoarded. In general, prices will be $\max[1/3\beta^2, 1]$, $\max[1/3\beta, 2/3]$, 1/3. If, instead of an economy with fiat and no borrowing and lending, we introduce an outside bank and an exogenous rate of interest the solution changes. In particular, there can be no hoarding as a nonatomic individual can always improve by depositing in the bank rather than hoarding.[3]

Suppose we set $\beta = 4/5$, then in an economy without loans the spot prices will be 25/48, 20/48 and 16/48 and the hoarding will be 23/48 in the first period, 8/48 in the second and 0 in the third. Each individual will consume (1, 2, 3, 1, 2, 3, ...) which is the general equilibrium solution. The total amount of money in the system is 1, hence $M + B = 1 + 0 = 1$, where M is the amount held by the individuals and B is the amount held by the central bank. Here, as there is no central bank the amount it holds is 0.

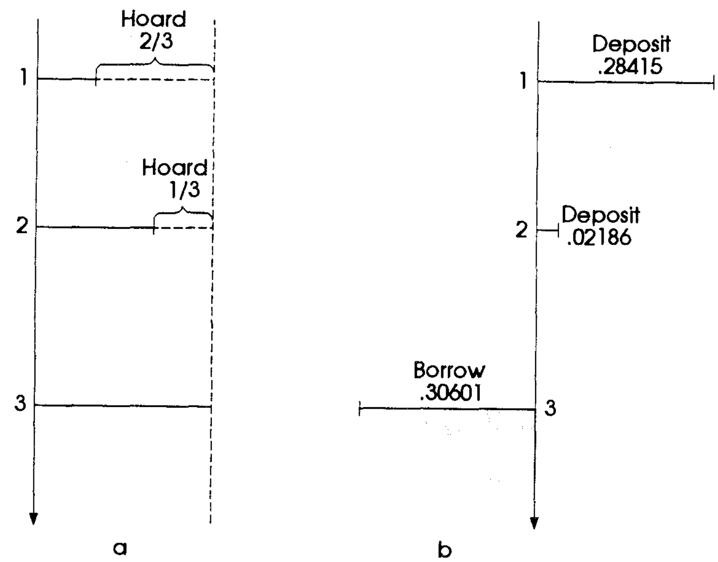

FIGURE 3: Hoarding or Depositing and Borrowing

We now consider the same economy with the addition of a central bank which is prepared to borrow and lend at a given fixed rate of interest. If $1+\rho = 1/\beta$ this gives $\rho = 25\%$ or 1/4. We may select for comparison an economy with $M + B = 1$ and the same spot price in the third period. If we do this then all the spot prices will be 1/3, 1/3, 1/3, ... and the inital money distribution will be $M = .693989$ and $B = .306011$. In the first period the traders will deposit $x = .284153$ (52/183), in the second period they deposit $y = .021858$ (4/183) and in the third period they borrow $z = .306011$. We note that $x+y = z$. The cash needs each period are 1/3, 2/3 and 1. Figure 3a shows the hoarding and Figure 3b shows the depositing and borrowing for the three periods. The vertical arrow represents time. In the third period

[3]Surprisingly in the sell-all model (which is probably a reasonable approximation of the economy Pareto optimality is achieved in a multidimensional economy because the inneviency or "wedge in prices" caused by a difference between buying and selling is not encountered. In a model involving bid-offer where individuals do not have to sell their assets gneral equilibrium efficiency will be lost. Empirically it is my guess that this loss is both realistic and not very large relative to other asspects of the economy.

the bank reserves are depleted to zero, which is the equivalent to the no hoarding in the third period without the bank.

Empirically we do not see the lining up of relative prices brought about by massive hoarding or intertemporal loans achieved by A staying out of the market while B buys. Instead we see an economy where there is borrowing and lending and a rate of interest. Although in a stationary analysis the two possibilites appear to be available as substitutes, in an actual dynamic economy the economy with borrowing and lending and an interest rate appears to require far less coodrination and information than the other.

2.3 The disappearance of time and money

There are two basic points to be gleaned from the discussion in Sections 2.1 and 2.2. They are the relationship between time and the presence of fiat money, and the duality between hoarding and borrowing and lending.

OBSERVATION 1: The presence of an initial issue of fiat which is an asset (or the equivalent, an indefinitely long debt) creates an overall system which is slightly nonsymmetric in the sense that this money is the only financial asset without a counterclaim against it. When there is an outside bank present, the inside economy over time consumes this initial input of fiat and it flows into the bank as a net interest payment. If we limit the investigation to any finite horizon, no matter how long, by backward induction the terminal constraint is tight, when there is no borrowing or lending and no salvage value for left over assets. This, in essence, fixes a price level.[4] When there is borrowing and lending, the bankruptcy conditions must be introduced to complete the definition of the model.

If the initial issue of fiat is regarded as a debt to be paid back to the government at the end of the game, then the possibility for the existence of a continuum of prices will depend on the magnitude of the bankruptcy penalty.

As long as the system is finite, it remains slightly nonsymmetric and the borrowing and lending is not perfectly stationary. When the system is infinite, the system becomes completely symmetric and the amount of fiat money held jointly by the economic agents and the outside bank is conserved. This amount is the circulating capital of the economy. In a world with no transactions needs whatsoever there is no need for circulating capital.

The system becomes fully stationary or timeless in the sense that optimal strategies are based only upon a non-time indexed state of the system.

---

[4]There may still be some indeterminacy caused by constraints in the form of inequalities and by alternative trading and inventory possibilities. See Qunt and Shubik (1995a,b,c).

OBSERVATION 2: In Sections 2.1 and 2.2 we considered two economies with a fixed input repeated each period where the first economy had no bank and the second had an outside bank. They clearly differed for the finite horizon, but converged to the same solution for the infinite horizon. This convergence could raise questions as to the need for an outside bank. When we consider a cyclical economy with differing inputs over the cycle, even in the infinite horizon the two economies differ. They give, for the appropriate initial conditions, the same economic solution, but give different prices and are computationally distinct. In the economy without an outside bank relative prices are adjusted by hoarding. For a sufficiently high β all money will be utilized, generically only once per cycle, for the remaining period some part of it will be in hoard.[5] The money holdings are almost always a slack variable and price depends on cash flow inequalities as sketched in Figure 3 above. When a bank exists and charges a positive rate of interest, then money can never be a slack variable. Borrowing and lending from the outside bank allows the money to flow into and out of the real goods economy and the books balance at the end of every cycle. In the infinite horizon the circulating capital needed to balance the books every cycle remains in transit between the bank and the agents.

The two different solutions are duals, where in one the amount of money is adjusted via hoarding and in the other it is adjusted by the appropriate level of interest which permits a different form of overall conservation with money flowing into and out of the central bank.

2.4 An aside on the money market

Our discussion has concentrated on the contrast between an economy using fiat money for transactions with and without an outside bank. There are two basic needs for borrowing money, one is to finance transactions and the other to finance intertemporal consumption or the purchase of capital stock whose returns will come in the future. The first financing needs may call for variation in the money supply, while the second could be handled by individuals who have surplus money at period t lending directly to those who are short. It is straightforward to construct a model with a money market with the endogenous formation of the rate of interest or a model with both a money market and an outside bank where the endogenous and exogenous interest rates interact (KSS, 1995 or Shubik, 1996). The interest rate is formed by the amount of IOU notes offered in exchange for the amount of fiat put up for loan. The money market turns out to be a case intermediates between the model with no loans whatsoever, and the model with an outside bank. The endogenous rate of interest will either be positive or it will be zero

[5]We may interpret the hoarding as an adjustmnt of the velocity of money, or the number of times it turns over per period. In the models with an outside bank, the money not in the hands of the outside banks turns over preciesly once per period. In the models without the bank the money in the hands of the agents has a velocity of less than one.

and hoarding will take place. The money market alone cannot adjust the money supply seamlessly to the variations needed for the appropriate relative prices.

OBSERVATON 3: The efficient equating of the supply and demand for money in a competitive loan market via a well defined positive rate of interest is in general not possible. The belief that it is possible can only be based on the extension of trade in money substitutes which one might wish to consider as money. An inside money market trades fiat for debt, it does not create fiat.

3 INCOMPLETE MARKETS AND STOCHASTIC ELEMENTS

In Section 2 we observed a duality between nonstochastic economies with and without an outside bank. When there is a stochastic element present the duality is by no means as clear. In particular in a nonstochastic economy with loans the role of bankruptcy is to prevent strategic default. At least around equilibrium there should be no active bankruptcy if the default penalty is sufficiently harsh. This is no longer true when there is stochastic uncertainty present

3.1 Fiat money as insurance

The occurrence of hoarding in the model of an exchange economy utilizing money, but without loans illustrated in Section 2 serves to enable the individuals to vary relative prices. A new reason to hold fiat appears when there is exogenous uncertainty present in the market. The fiat saved serves as insurance against a low income. It provides precautionary reserves against disaster. Alternatively it provides immediate liquidity to take advantage of opportunities, KSS (1994) provides an example where in equilibrium the wealth distribution of a population of agents with the same preferences is such that a part of the monetary wealth of the economy is always in hoard.

3.2 Institutions and equilibrium

In attempting to establish the existence of equilibrium in strategic market games a persistent set of difficulties keep appearing. At first glance they appear to be minor and seem to represent some form of casual extra institutional observation which should be omitted in early modeling by the theorist trained in abstraction and desiring high levels of generality. Then can be added later after the broad outlines of a general theory have been established. In the development of an understanding of the properties of a price system this was the approach adopted. A rigorous, frictionless and timeless theory of price was developed in a series of models abstracting out the role of process.

In attempting to build a complete closed process model of an exchange economy utilizing fiat money the annoying details which could be avoided in the nonprocess analysis appear to be of more importance than just minor frictions or quaint institutional additions to an otherwise adequate theory. They raise basic questions concerning the role of financial institutions as the carriers of process. In particular if we model the economy as a strategic game and try to well-define this game for every point in the set of attainable outcomes we must be specific as to what happens if only one side of a market is active or if neither side of a market is active. If the interest rate is formed by the amount of IOU notes offered in exchange for an amount of fiat, what happens if no fiat is offered, does the rate become infinite, or is the market inactive? Can individuals borrow unlimited amounts of fiat or create unbounded amounts of debt? A way to handle these difficulties is to invent agents and institutions such as the specialist stock trader whose role is to "make a market" for thinly traded stock, or a government agricultural agency who steps into the market and buys wheat when the price goes too low and reverses position with inventories when the price is too high. In some instances the agency might even burn the wheat after having "maintained an orderly market." We may invent an ideal central bank which steps in on the appropriate side of the market if the rate of interest is either too high or too low. KSS (1995a,b) by using devices such as those described above are able to establish the existence of equilibrium in strategic market game models of the economy both with secured lending and with active bankruptcy. Paradoxically at the equilibrium the regulatory mechanism is inactive, thus it is not seen, but it would appear decisively in the dynamics. The apparatus is needed to make sure that there are no pathologies which could develop if the system were to achieve a feasible state which might be far from equilibrium. Even with the presence of the appropriate institutional apparatus there is no guarantee that the system started far from equilibrium would ever achieve equilibrium. All that is guaranteed is that the system cannot completely blow up.

OBSERVATION 4: At the least, the necessary conditions for the existence of an equilibrium in an economy with money and credit call for the presence of institutions and rules that preserve orderly markets and bounded variations in prices and interest rates. Mathematically these amount to making sure that certain payoff functions are compact.

Our experience with the problems in establishing the existence of equilibrium in the competitive models of the economy with secured and unsecured lending suggests that financial institutions and laws are not a mere institutional curiosity for the institutional economist but logical necessities for the economic system if one expects equilibrium to be feasible. Furthermore, even though at equilibrium they are apparently inactive, they are needed.

3.3 Panics and power laws

In Section 3.2 a sketch of the reasons for the need for institutions and appropriate rules such as credit constraints and bankruptcy laws has been given. Our original concern has been with the existence of equilibrium in the financial and product markets. But it is reasonable to ask the reverse question along with several other obvious companion questions.

The basic question is what happens to the system if the regulatory rules are not sufficient to guarantee the existence of equilibrium? An immediate companion question is empirical. Are the institutions and laws of society such that they are always sufficient to guarantee the existence of equilibrium?

I conjecture that under fairly reasonable and general circumstances the type of power law suggested by Mandelbrot (1963, 1967) associated with financial panics may arise in the models postulated here when the system starts in disequilibrium and the laws and regulatory agencies are not adequate.

I further suggest that there are more or less natural reasons to suspect that the institutional safeguards against the possibilities for the development of "economic earthquakes" are rarely if ever perfect. The observations here are, for the most part, consonant with the discussion given by Minsky (1986) on the financial system as well as the approach of Arthur (1994), Ijiri and Simon (1977), and others in their approach to simulating economic dynamics. In essence, many of the credit conditions have a crawling peg built into them in the way that expectations are permitted to influence the granting of credit. The presence of leverage combined with the possibility for chain-reaction bankruptcy can trigger disasters of any magnitude if the credit limits are functions of inflated or deflated recent asset prices and expected prices. It is often when the economy has heated up that the credit restrictions become lax and in the downward spiral when more credit is needed to prevent the slide, the credit tightens.

The physical scientist acquainted with a power law such as that for the probability of the occurrence of an earthquake of some given magnitude knows that there is no physical way known to humans to truncate the fat tail of the distribution. The situation with the economic earthquake is somewhat different. Beyond a certain force of the economic Richter scale the stock markets close and bank holidays are declared. The economy reorganizes and the law suits stretch out for many years.

4 INSTITUTIONS AND EVOLUTION

The time for going beyond the essentially pre-institutional timeless models of general equilibrium has clearly arrived in economic theory. The ability to compute and to simulate has changed by many orders of magnitude in the last few years. These new abilities permit an exploration of dynamics that was recently impossible. But the suggestion made here is that the natural first careful extensions beyond general equilibrium theory can be made by formulating well defined, even though highly simplified process models. In the act of fully defining these models the institutions are invented as a matter of logical necessity. In the attempt to establish the conditions for the existence of equilibrium one is also able to establish the conditions for the lack of equilibrium. Even casual empiricism is sufficient to indicate that the possibility is high that the institutions are not strong enough to avoid panics, breaks and failure of smooth process. Thus the conflicting view presented by conventional general equilibrium theory and its offspring, conventional finance, and the work suggesting that there can be basic instabilities are not necessarily at odds, but represent different arrays of institutional control or lack of control.

The economy is embedded in the polity and in the body of laws of the society. The disequilibriating shocks in the system trigger the changes in the rules governing the environment in which it operates.

4.1 The needed institutions and instruments

There is a minimal set of basic institutions and instruments needed before one is in a position to construct a viable theory of money. The actors, institutions and instruments used here are individual agents and a central bank, a goods market for price formation, a money market for endogenous interest rate formation and an outside exogenous interest rate. Also needed are bankruptcy conditions and a terminal evaluation of assets left over at the end of any finite game. These can quite naturally be interpreted as expectations. Fiat money and short term IOU notes are also required. It must be stressed that although there may be considerable latitude in selecting a specific institutional form, there is little latitude in covering function. For example, there are many variants of bankruptcy rules, but no matter which is chosen some method must be given for specifying what happens if an individual cannot pay back a debt.

Beyond the primary basic functions of exchange, borrowing and repayment there are secondary functions which modify the basics. For example bank and clearinghouse money provide more flexibility than a system with fiat alone. But they are not needed in the simple models noted here; however in any attempt at modeling an actual economy they are called for.

4.2 Behavior, strategy, inference and expectations

The approach adopted here has been to begin by staying close to the equilibrium assumptions of basic general equilibrium theory in order to be able to contrast the results obtained. But because full process models are constructed we may then consider differences as well as similarities. In particular in models regarded as open to time we may consider solutions which are not based on backwards induction, but on extrapolating from the past forward, inferring future behavior from past performance. An open question concerns what class of expectations lead to convergence, if an equilibrium exists. The mere existence of equilibrium tells us little about the dynamics.

4.3 Institutions: The self-correcting system?

In Section 3 I have suggested the special role for institutions and laws in serving not only as carriers of process, but in providing the appropriate bounds to guarantee the existence of equilibrium. In Section 3.3 the possibility was conjectured that with imperfect institutions panics, best illustrated by power laws, could be present. But, as observed above, unlike natural power laws such as that which may govern earthquakes, individuals do have some control over financial power laws. When the game becomes too disastrous for all, they change the game. This is tantamount to postulating an elemental structure to the financial games-within-the-game. All participants take the rules of the game as given and act as context-constrained local optimizers, where the context is the set of laws and institutions of their society. When a disaster of a sufficiently large magnitude strikes enough of the economic agents are hurt that legal, political and social change modifies the economic environment.

I have entitled this section "The self-correcting system?" with a stress on the "?". The evidence from 1700 (when fiat money, stockmarkets and modern financial institutions started to proliferate) unto this present day is that there has been a series of panics and crashes of various magnitudes, but that after each the rules have changed and (at least with Candide bifocals) it appears that the laws and institutions, by some measures may have improved.

REFERENCES

Arrow, K. J. and G. Debreu, 1954. "Existence of an Equilibrium for a Competitive Economy," *Econometrica* 22: 265-290.

Arthur, B., 1994. "Inductive Reasoning, Bounded Rationality and the Bar Problem," *American Economic Review (Papers and Proceedings)* 84.

Dubey, P. and J. Geanakoplos, 1990. "The Value of Money in a Finite Horizon Economy: A Role for Banks," mimeographed, Cowles Foundation.

Ijiri, Y. and H. A. Simon, 1977. *Skew Distributions and the Sizes of Business Firms*. Amsterdam: North-Holland Publishing.

Karatzas, Y., M. Shubik and W. D. Sudderth, 1994. "Construction of Stationary Markov Equilibria in a Strategic Market Game," *Journal of Mathematical Operations Research* 19,4: 975-1006

Karatzas, Y., M. Shubik and W. D. Sudderth, 1995. "A Strategic Market Game with Secured Lending," CFDP 1099.

Mandelbrot, B., 1963. "The Variation of Certain Speculative Prices," *Journal of Business* 36: 307-332.

Mandelbrot, B., 1967. "The Variation of Some Other Speculative Prices," *Journal of Business of the University of Chicago* 40: 393-413.

Minsky, H., 1986. *Stabilizing an Unstable Economy*. New Haven: Yale University Press.

Quint, T. and M. Shubik, 1995a. "Varying the Money Supply, Part I: A General Equilibrium Model with Perishables, Storables and Durables," CFPP 950511.

Quint, T. and M. Shubik, 1995b. "Varying the Money Supply, Part IIa: Trade in Spot Markets Using Money — The Three Period Models," mimeographed, Cowles Foundation.

Quint, T. and M. Shubik, 1995c. Varying the Money Supply, Part IIb: Trade in Spot Markets Using Money: The Infinite Horizon," mimeographed, Cowles Foundation.

Shubik, M., 1996. *The Theory of Money and Financial Institutions*. MIT Press, forthcoming.

Shubik, M. and W. Whitt, 1973. "Fiat Money in an Economy with One Nondurable Good and No Credit (A Noncooperative Sequential Game)," in A. Blaquiere, ed., *Topics in Differential Games*. Amsterdam: North Holland, 1973, pp. 401-449.

Woodford, M. and W. J. Muller III, 1988. "Determinacy of Equilibrium in Stationary Economies with Both Finite and Infinite Lived Consumers," *Journal Economic Theory* 46: 255-290.