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NONSTATIONARY TIME SERIES AND COINTEGRATION: RECENT BOOKS AND THEMES FOR THE FUTURE

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1. INTRODUCTION

We have recently witnessed the biggest and certainly the most exciting change to occur for several decades in the way we approach economic time series. This change has led to a new field of econometrics that concerns the way we deal with nonstationarity in the construction and estimation of time series models. Research on nonstationary time series exploded into the profession in the mid 1980's. Now some 10 years later we can begin to appreciate how time series econometrics has been changed by the developments that have occurred. Some measure of the extent and rapidity of these changes can be gauged by noting that when the Handbook of Econometrics was published over the period 1982–1985 it contained 35 chapters covering three volumes written by many of the world's leading econometricians. Yet in spite of its enormous coverage there was not a single article on nonstationary time series in the Handbook and scant mention of the subject in any of its chapters, even though it was of course widely recognized at the time of its publication that the predominant characteristic of most economic time series is their nonstationarity. The neglect of the subject was not confined to scientific handbooks. At the World Congress of the Econometric Society held in Boston in 1985 there were two or three papers that considered this topic out of hundreds of papers on econometrics overall.

The transformation in academic interest that has taken place since then is astonishing. Nowadays there is hardly an issue of any leading economics journal without some article dealing with nonstationary time series. Several major journals have run special issues on the subject, and one journal has considered developments to be so important that it has run special issues that cover ongoing work in the field twice within the last four years. Since 1985 there have been a score of small conferences, specialized workshops and research seminars on the subject. This background of subsequent activity is a remarkable testimony to the pace of change in econometrics and shows that scientific handbooks and big international conferences can be rapidly and completely overtaken by the march of events. One cannot help but wonder whether a similar fate waits in the wings for future handbooks and congresses.

2. THE SCOPE OF CURRENT INTEREST

The field of nonstationary time series is an unusually rich one that has engaged the attention of a full spread of participants from economics, econometrics, statistics and probability. Economists are interested in the subject because of their long standing fascination with the phenomenon of growth and efficient markets, their recent interest in the question of whether autonomous shocks to economic activity have persistent effects, and most recently the important international issue of whether economic growth paths of different countries converge in some sense over time. All of these substantive issues are affected by how we measure and treat nonstationarity in the data.

Econometricians have become excited by the subject because of its inherent empirical possibilities. We have always been interested in relationships between variables in levels or log-levels form. But since 1985 we have had the tools to analyze regressions in levels for integrated (I(1)) processes and this has made it possible to develop a theory of inference for such regressions. The arithmetic of I(1) analysis, as I call it, has opened the door to the careful econometric study of a whole host of interesting models, not the least of which are the error correction models that have been used in empirical research for many decades (the early empirical work on trade cycle and cyclical growth models being the leading examples) and which have now become popular in empirical studies of cointegration. We can also study and compare the effects of traditional detrending by polynomial regression against that of data differencing, and test for the one against the other. These possibilities have had an enormous impact on empirical time series research in economics and it seems likely that they will soon have an impact in other subject areas where nonstationary data is widespread, such as political science and communications.

Statisticians and probabilists have also become engrossed in the subject, partly because of its potential importance in applications but also because of fascinating features of the statistical theory. At a technical level, nonstationary regression theory is interesting because it provides a major statistical application of central limit theory on function spaces, rather than Euclidean spaces. This occurs because when shocks are persistent in the data, as they are when there is a unit root in the process, all of a time series trajectory becomes important in the behavior of regression statistics. We therefore need a way of carrying the whole time path of a variable with us as we study the sampling properties of statistics that depend on summary measures like sample moments. Functional limit theory enables us to do this in a simple way because it maps the observed trajectory into realizations of random elements that live in a much bigger space than the Euclidean space to which the finite observation vector belongs or even the infinite dimensional Euclidean space to which the entire (unobserved) trajectory belongs. Norbert Wiener was motivated to develop the probability space underpinnings for function-valued random variables after he observed the path of a fly crossing the room from one wall to another in a doctor's office where he was waiting for his appointment. Such a path, mused Wiener, is a realization of a random process and the random element which gives rise to it has values in a function space. Now, when we want to test hypotheses about the fly's flight path we need to take the whole realization into account. In a similar way when we analyze economic data in which the shocks that drive the system are persistent, we need to use all of the data and find a way of preserving its relational features (like that of the flight path of the fly) even as the sample gets infinitely large. Functional limit theory enables us to do just this.

3. SOME RECENT TEXTBOOK AND READINGS THAT SEEK TO COVER THE FIELD

With all the ongoing development in the field it would seem something of a perilous venture to review the literature. The field is moving so quickly that most reviews are dated as they are in the process of being composed. The authors of three recent book length reviews of the field are therefore to be congratulated for their energy and enterprise in putting some of the material together for students and researchers with interests in the area. The books are:

Hamilton, James, *Time Series Analysis*, Princeton University Press, 1994.

Banerjee, A., J. Dolado, J. Gailbraith and D. F. Hendry, *Cointegration, Error Correction and the Econometric Analysis of Nonstationary Data*, Oxford University Press, 1993.

Engle, R. and C. W. J. Granger, *Long-Run Economic Relationships: Readings in Econometrics*, Oxford University Press, 1993.

Hamilton's book is an ambitious undertaking that seeks to give an introduction to modern time series methods that covers recent developments in nonstationary time series. It is extensive in coverage, well researched, generally well written and certainly well presented with plenty of examples and exercises that will help guide beginning students. It is destined to have a strong market in North American graduate economics programs and final year undergraduate and graduate programs in the United Kingdom, Europe and Australasia. From a purely pedagogical perspective it is the soundest of the three offerings.

The book by Banerjee, Dolado, Gailbraith and Hendry (BDGH) has a similar market and audience but it is narrower in scope, a lot shorter in length and has a greater focus on cointegration and unit roots. It is well motivated and succeeds in integrating the three themes of cointegration, error correction and nonstationarity. The book has worked examples that are taken from the existing literature and tends to follow the original research articles from which it draws its content quite closely, with some amplification of the algebra to make it more accessible to students. In this respect it is very similar to Hamilton's book and there is little value added in terms of new material in either book. The exercises and wider scope of Hamilton's book do make it more valuable to an instructor for a full year's course on time series. But BDGH will be useful in half-year, semester and seminar courses.

The book by Engle and Granger is a collection of readings. It is the weakest of the three offerings and is of little pedagogical use to an instructor in any course on time series or even a more focused course on cointegration. This is because some of the articles that are included have now been overtaken by the literature, and because the collection itself is deficient in several important respects. First, the authors chose not to include any articles on the analytic methods of the new field and the development of the asymptotic theory of regression for nonstationary time series.

In consequence, some of the important articles that they do include which are heavily dependent on this machinery are at best incomplete and, for the uninitiated at least, virtually unreadable. Second, the volume has no articles at all on spurious regression. This is an egregious omission because spurious regression is the alternative to cointegration and serves as the null in the most frequently used single equation tests for cointegration. Understanding spurious regressions and the differences in the large sample behavior of such regression statistics and those of a cointegrating regression is very important from both a theoretical and practical empirical standpoint. Third, the volume does not include articles on the asymptotic theory and appropriate construction of residual based tests for cointegration. Since these tests are the mainstay of empirical work this omission is especially curious.

4. SOME THEMES FOR THE FUTURE

This section briefly outlines some new themes in the subject that are untouched in the books discussed in the previous section. They are themes that I find particularly interesting. Some of them are of general technical interest, others are more important for practical applications. Not all of them are likely to be of equal importance in the future, but hopefully they will give some idea of directions in which the subject is presently moving.

(a) Characterizing the Likelihood and an Optimal Theory of Estimation

One feature of nonstationary time series analysis that makes the subject especially interesting to probabilists and more technically minded econometricians is that the usual asymptotic theory breaks down at the unit circle. The failure of the usual central limit theory and the accelerated rate of convergence at the unit root in a simple autoregression have been known for many decades. Even the original Cowles Commission researchers had an interest in the statistical large sample properties of nonstationary autoregression estimators. John White (1958) was the first to dig a little deeper and he anticipated the role of functional limit theory in obtaining a neat solution to the problem. The approach was systematically explored in the 1980's in work by Solo (1984), Phillips (1987, 1989), Chan and Wei (1988), and Jeganathan (1991). The power of the apparatus in providing an accessible basis for the asymptotic theory of multiple regression for integrated time series was shown in Phillips (1986), Phillips and Durlauf (1986) and Park and Phillips (1988, 1989). A technical review of the field was given in Phillips (1988). Scores of papers on the subject have subsequently been published in econometrics and statistics journals.

In spite of all the work that has now been done on the asymptotic theory of regression for integrated processes there are still some fascinating issues that remain to be studied. In fact, the technical issues go quite a lot deeper than the accelerated convergence rate of unit root and cointegration estimators and their nonstandard limit theory, which are the issues that have so far attracted the attention of most econometricians, as is amply reflected in the books reviewed above. LeCam (1986) showed that in problems where the likelihood is locally asymptotically well approximated by a quadratic the limit theory for the maximum likelihood estimator is normal or mixed normal for almost all (in the Lebesgue measure sense) points in the parameter space — LeCam and Yang (1990) provide a readable account of this interesting property. Thus the exception to the limit theory that arises on the unit circle in an autoregression is a rather important exception. Moreover, the quadratic behavior of the likelihood itself varies as we move away from the unit circle in a local sense, as shown in Phillips (1989). In effect, the Fisher information is variable (i.e. varies with the extent of the parametric departure) as we move away from unity. The information is also random in the limit rather than fixed, as it is in the stationary case.

These aspects of the problem of nonstationarity have certainly been enough to make the statistical theory interesting even in the scalar case. But the problems in the multivariate case, where there are some unit roots and some cointegration in the data, are even more interesting and not yet fully resolved. In this case the matter of whether the rank of the cointegrating space is known turns out to be of major importance in determining the form of the likelihood asymptotically. When the rank is known the likelihood is locally asymptotically mixed normal. But when the rank of the cointegration space is not known the likelihood is a combination of a mixed normal and a Gaussian functional that is related to a matrix unit root distribution. The combination is parameter dependent and it turns out that this makes it very difficult to establish that the maximum likelihood estimator has good properties. I have been able to show recently that in some directions and for some non negligible (i.e. of positive Lebesgue measure) part of the neighboring parameter space one can indeed improve on the maximum likelihood estimator. Thus, the asymptotic theory of cointegrated vector autoregression is fascinating and seems to involve some interesting exceptions to the usual theory of the optimality of the maximum likelihood estimator.

(b) GLS Detrending and Efficient Tests for Unit Roots and Cointegration

Attention has recently shifted to principles that lead to unit root tests with good properties. Of course, the likelihood ratio principle leads to the Dickey–Fuller t–test and its various extensions such as the $Z(t)$ test in semiparametric modelling situations. Note, however, that t–ratio tests are usually employed in statistical problems where nuisance parameters need to be estimated and scaled out. The estimation of a unit root is not such a situation. Indeed the least squares coefficient estimator (the Gaussian estimator) is scale invariant. Thus, the use of a t–ratio is not strictly necessary in unit root testing and, just as in the comparison of a t–ratio regression test with the Normal test that can be used when the error variance is known, we expect the coefficient based unit root test and its semiparametric variants like the $Z(a)$ test to be the more powerful. This is indeed the case, as Monte-Carlo evidence confirms. We also know that the rate of divergence of the coefficient test under the alternative is greater than that of the t–ratio test. The main reason that t–ratio unit root tests are still used in practice is that these tests have been shown in Monte-Carlo simulations to have more stable size properties for a wider range of nuisance parameter constellations than coefficient based tests. But there are costs in terms of test power that have to be paid for this size stability. Other possibilities are to use coefficient based tests that have improved size stability. One way of achieving this is to employ data-based modelling methods to assist in the estimation of the nuisance parameters that cause the size instability. We have achieved some recent improvements along these lines in the $Z(a)$ unit root test by using such procedures in the estimation of the long-run variance (nuisance) parameter — see Lee and Phillips (1993) for details.

Point optimal unit root tests are another possibility and these can be constructed under Gaussian or other distributional assumptions (e.g. the exponential family) for alternatives that are local to unity. Recent work on this topic shows that there are virtually no power gains from using this approach over the coefficient test, at least for Gaussian data and for models where there is no trend or drift — see the simulations in Elliot, Rothenberg and Stock (1992). However, power gains do materialize in cases where there is a trend and detrending is performed under the alternative hypothesis in constructing the test. This type of procedure is used in the Elliot, Rothenberg and Stock paper and in Schmidt and Phillips (1992). But the reason for the power gains is not clear in those papers. It turns out that there is a simple explanation. A famous theorem of Grenander and Rosenblatt (1957) showed that ordinary least squares (OLS) regression is asymptotically efficient in the removal of a deterministic trend (i.e. is asymptotically equivalent to generalized least squares (GLS)). That theorem was one of the main reasons why trend removal by regression became so popular in empirical time series research: one could efficiently remove a trend by OLS regression irrespective of the process driving the stationary component of the model. However this theorem on the asymptotic equivalence of OLS and GLS regression does not apply when the model has a unit root or a root local to unity. In fact, when there is a unit root or a root local to unity, the trend can be more efficiently removed by GLS regression. The efficiency gain can be of the order of 25% or so. This gain in estimation efficiency translates into a local power gain in the test efficiency of unit root tests that utilize the more efficient GLS detrending method. Against stationary alternatives there are no asymptotic power gains, just as there is no efficiency gain in GLS regression for detrending in this case. An analysis of the efficiency gains of GLS regression in trend removal and its potential effects on the test efficiency of unit root tests when there are trended data has recently been given in Lee and Phillips (1994). Obviously it is of considerable interest to extend this analysis of efficiency gains in unit root tests to tests for cointegration in multivariable systems.

(c) "Reasonable" Spurious Regression

The term spurious regression carries a decidedly pejorative connotation. The thinking goes back at least as far as Yule (1926), who used the term nonsense correlation to describe correlations that are induced by attendant effects like trends that happen to be present in series that are otherwise uncorrelated. In econometrics the phenomenon has been extensively studied in cases where the trends are stochastic — see Granger and Newbold (1974) and Phillips (1986) for details. The books by Hamilton and BDGH both discuss the phenomenon and adopt the conventional view that such regressions are bad econometrics and should be avoided.

Here I want to put forward a contrarian perspective and argue that spurious regressions are often more benign than their name implies. The essential idea is well illustrated in the following examples.

Suppose the true data generating process is a model with a unit root

$$y(t) = y(t-1) + u(t) \tag{1}$$

where $u(t)$ is a stationary process. In place of (1) suppose we fit a linear trend to $y(t)$ giving the regression

$$y(t) = \widehat{b}t + \widehat{u}(t). \tag{2}$$

In conventional parlance (2) is a spurious regression. The analytics of this regression were studied in Durlauf and Phillips (1988). We have the following asymptotic behavior:

$$\widehat{b} \xrightarrow{p} 0, \tag{3}$$

$$t(b) = O(n^{0.5}). \tag{4}$$

Thus, $\widehat{b}$ is consistent for its true value 0 but the t–ratio statistic $t(b)$ diverges, so that inferences about the trend are wrong with probability that goes to one as $n \to \infty$. Hence the conventional view that the regression (2) is spurious.

But is (2) really spurious and are (3) and (4) really misleading? One can easily argue that the conventional view is strongly overstated. First, b does tend to 0, its true value in (1) — there is certainly nothing misleading about this. Second, the t–ratio diverges and this tells us that the linear trend is ultimately always significant even though the regression coefficient is zero in the limit. A reasonable view of the latter is that the statistical analysis is pointing to the presence of some type of trend in the data even though the regression coefficient itself may be arbitrarily small. The conclusion is that the linear trend in (2) doesn't measure much of a trend in the data in absolute terms but is nevertheless strongly significant. Within the context of the chosen fitted model (2), the statistical analysis actually does quite a reasonable job for us — it says b is zero or close to zero but there is a significant trend in the data anyway. Thus, although the regression is misspecified the general conclusion is surely quite reasonable. We cannot expect to do much better than this when a model is so seriously misspecified.

Next consider the converse case of the generating process

$$y(t) = bt + u(t) \tag{5}$$

where $u(t)$ is again stationary, and the fitted model

$$y(t) = \widehat{a}y(t-1) + \widehat{u}(t). \tag{6}$$

In this case, $\widehat{a}$ converges in probability to one and $n(\widehat{a}-1)$ converges to a constant. The fitted model (6) is considered a spurious regression and tests for a unit root are inconsistent, i.e. do not have unit power against stationary alternatives as $n \to \infty$. A good deal has been made of this in the literature and concern over the inconsistency of the unit root test here has led to insistence on the additional inclusion of deterministic trends in regressions like (6). But is the regression (6) that bad? Within the context of the fitted model, the statistical analysis simply confirms the presence of a trend in the data, which is surely a reasonable conclusion. In effect, the stochastic trend proxies for the determininistic trend and the statistical analysis is doing the best that it can within the context of the given model.

At a more general level, in empirical work we are always caught up in the difficulties of misspecified models. Within sample models may appear to do well in terms of regression diagnostics, but this is often because the modelling efforts are just sophisticated curve fitting exercises. It is fatuous to expect that the complications of trending mechanisms in economics are properly captured by the simple models of stochastic trends, determininistic trends or trend breaks that we presently use. Outside of sample, models and their fitted trending mechanisms inevitably break down. We can hope that the trends we have fitted continue to proxy the true evolutionary mechanism in a satisfactory way outside of sample but we should not expect it. Simply put, trends in econometric models are approximations like everything else and should be treated as such. When a statistical analysis confirms that there is some trending mechanism in the sample data, that conclusion will often be a very reasonable one, even if the model is wrong. The regression that delivers the result may be spurious in conventional terminology, but it is reasonable in terms of the inference made about the presence of trends in the data.

(d) Bayesian Approaches and Model Selection

The Bayesian approach to inference offers some special advantages in models where there are stochastic trends. The most important of these is that posterior distributions are asymptotically normally distributed even when the true process has a unit root, in contrast to classical asymptotic unit root distributions. This result was established in recent papers by Sims (1990), Kim (1994) and Phillips and Ploberger (1992). It was foreshadowed in earlier statistical literature by Heyde and Johnstone (1978) and Sweeting and Adekola (1987).

Another advantage of Bayesian analysis is that it is possible to design coherent procedures that integrate model selection and hypothesis testing. As the examples of the previous section illustrate, the choice of model can have an important effect on the large sample properties of tests for unit roots. It therefore seems sensible when conducting such tests to integrate modelling choices about the form of the trends in a model with other features of model choice like the lag length in an autoregression. This can be done in a Bayesian analysis and the choices can even be made simultaneously — see Phillips and Ploberger (1994). This integration of model choice and hypothesis testing is not possible in the classical approach to inference, where to deal with these issues it is necessary to employ sequential testing procedures which have their own attendant problems.

The application of these Bayesian model selection methods to multivariate models with some cointegration and some unit roots is especially useful in practice. In the Bayesian approach it is possible to treat cointegrating rank as an order selection problem. As such it can be combined with order selection of the lag length in a VAR to produce a joint order selection problem that can be treated coherently by the same methodology. The outcomes can be graphically displayed as an order selection surface and this highlights the extent to which lag length and cointegrating rank interact and shows how well determined the final estimated orders are. Phillips (1994) and Chao and Phillips (1994) provide the theory behind this approach and give some simulations and empirical applications to illustrate the approach.

(e) Non-Gaussian Data and Adaptive Methods

Many empirical applications of cointegration involve financial data like exchange rates that are heavy tailed and leptokurtotic. It is therefore useful to have available estimation and test procedures that are fairly robust to departures from Gaussianity in the data. Regression methods can in fact be easily modified to have more robust characteristics. It is easiest to work from traditional robust estimators like the LAD and M estimators and modify them to accommodate the nonstationary nature of the data. Regression estimates based on these principles have been constructed and analyzed in Phillips (1993).

A more ambitious undertaking is the development of a fully fledged adaptive estimation procedure, which adapts for the unknown form of the joint probability density of the equation errors and the shocks that drive the I(1) variables. Jeganathan (1995) offers some suggestions about how to proceed in this case. Even more complex is the construction of an adaptive reduced rank regression estimator which estimates the cointegration space in a VAR, while adapting for possibly non-Gaussian errors. Fortunately, many of the financial data sets for which such procedures seem most suited are long series, which run to many hundreds of weekly observations and thousands of daily observations. Adaptive procedures are likely to be quite feasible with such data sets.

5. CONCLUSION

Given the potential for empirical economic applications, the wide ranging problems of modelling methodology, and the host of technical issues and fascinating asymptotics that nonstationary time series has spawned, it is understandable why so much activity has been going on in the subject at so many different levels. It is probably fair to say that the subject of nonstationary time series has brought together a wider group of participants and excited more interest than any subject in econometrics since the development of simultaneous equations theory in the early days of the Cowles Commission. And just like simultaneous equations models, the subject has not been without its critics and controversies. Of late we have heard refrains of the following type: Do unit roots matter? Is cointegration really relevant and why aren't more empirical time series cointegrated? Trend stationarity is still alive and well! Unit root tests have low discriminatory power, so should we use them and, if we do, should we take the outcome seriously? Aren't Bayesian methods superior to classical unit root tests because the presence of unit roots does not affect Bayesian asymptotics? And so on. Reevaluations of this type are an inevitable and important part of scientific progress. If anything they have confirmed the importance of the field. And in spite of the critics and controversies some things are certain: we now know how to analyze nonstationary regressions in levels; that knowledge empowers us in our search for empirical answers to certain questions about the interconnections between economic variables over time; and applied econometric practice with macroeconomic time series has now absorbed much of the new toolkit and will, as a result, never be quite the same again.

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