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ROBUST TESTS OF FORWARD EXCHANGE MARKET EFFICIENCY  
WITH EMPIRICAL EVIDENCE FROM THE 1920's

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FORWARD EXCHANGE MARKET EFFICIENCY  
WITH EMPIRICAL EVIDENCE FROM THE 1920'S\***

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## 0. ABSTRACT

This paper provides a robust statistical approach to testing the unbiasedness hypothesis in forward exchange market efficiency studies. The methods we use allow us to work explicitly with levels rather than differenced data. They are statistically robust to data distributions with heavy tails, and they can be applied to data sets where the frequency of observation and the futures maturity do not coincide. In addition, our methods allow for stochastic trend nonstationarity and general forms of serial dependence. The methods are applied to daily data of spot exchange rates and forward exchange rates during the 1920's, which marked the first episode of a broadly general floating exchange rate system. The tail behavior of the data is analyzed using an adaptive data-based method for estimating the tail slope of the density. The results confirm the need for the use of robust regression methods. We find cointegration between the forward rate and spot rate for the four currencies we consider (the Belgian and French francs, the Italian lira and the US dollar), we find support for a stationary risk premium in the case of the Belgian franc, the Italian lira and the US dollar, and we find support for the simple market efficiency hypothesis (where the forward rate is an unbiased predictor of the future spot rate and there is a zero mean risk premium) in the case of the US dollar.

## 1. INTRODUCTION

The relationship between forward exchange rates and future spot rates has been the focus of many studies of market efficiency in the foreign exchange market. Recent work on this topic has used levels (or, more precisely, log-levels) data to test whether forward exchange rates are unbiased forecasts of future spot rates and has relied on the modern theory of regression for nonstationary time series to justify the statistical methods employed. In particular, Hakkio and Rush (1989a) and Baillie and Bollerslev (1989a) used residual based cointegration tests (see Engle and Granger, 1987; Phillips and Ouliaris, 1990) to test the joint hypothesis of market efficiency and risk neutrality, and to assess the evidence in support of the existence of a stationary risk premium. More recently, several studies (Corbae *et al.*, 1993; Moore, 1992; McFarland *et al.*, 1992) have employed direct nonstationary regression procedures that are due to Phillips and Hansen (1990) and Park (1992) to estimate the relationship between forward exchange rates and future spot rates.

Direct regression procedures have many natural advantages over other methods, including vector autoregression, error correction model and reduced rank regression approaches. As mentioned recently by Moore (1992), the latter all encounter some difficulty in the case of overlapping observations where the duration of the futures maturity contract exceeds the time interval between observations. This is because overlapping observations induce moving average effects, as shown by Hansen and Hodrick (1980), and these are only approximately handled by autoregressive formulations. Both Moore (1992) and McFarland *et al.* (1992) use the fully modified least squares (FM-OLS) regression method of Phillips and Hansen (1990) as an alternative way of dealing with this difficulty. Since FM-regression treats equation errors in a semiparametric way moving average effects are automatically accommodated by this method. Also, FM-regression allows us to test the forward efficiency hypothesis directly in the context of the usual formulation of the model, *i.e.*

$$s_{t+k} = a + bf_{t,k} + u_{t+k} \quad (1)$$

where  $f_{t,k}$  is the (log) forward exchange rate for a given currency contracted at time  $t$  for delivery at time  $t+k$ ,  $s_{t+k}$  is the (log) spot exchange rate for the currency at  $t+k$ , and  $u_{t+k}$  is a stationary error. Thus, this regression method has some advantages in the context of exchange rate models

like (1).

Market rationality and a zero mean risk premium lead to the “simple efficiency hypothesis”

$$\mathcal{H}_0 : a = 0, b = 1$$

in (1).  $\mathcal{H}_0$  can be tested using a straightforward Wald statistic based on the FM–OLS regression coefficients of  $(a, b)$  in (1) and their estimated variance covariance matrix (using formulae from Phillips and Hansen, 1990). The method allows for stochastic nonstationarity in the data  $\{s_{t+k}, f_{t,k}\}$  and serial dependence in the error  $\{u_{t+k}\}$  of (1). Corbae *et al.* (1993) use the canonical cointegrating regression (CCR) procedure of Park (1992), which is very closely related to FM–OLS, and they fit models like (1) that allow for a linear time trend in the regressors, *viz.*

$$s_{t+k} = a + ct + bf_{t,k} + u_{t+k}, \quad (1')$$

and multi-currency effects, *viz.*

$$s_{t+k}^i = a_i + c_i t + \sum_{j=1}^m b_j f_{i,k}^j + u_{t+k}, \quad (1'')$$

where the additional affixes “*i*” and “*j*” signify the (currencies of) country *i* and *j*, respectively. For model (1') the simple efficiency hypothesis has the form

$$\mathcal{H}'_0 : a = 0, c = 0, b = 1$$

and for model (1'') the multi-market efficiency hypothesis takes the form

$$\mathcal{H}''_0 : a_i = 0, c_i = 0, b_i = 1, b_{j \neq i} = 0.$$

One aim of the present paper is to test the unbiasedness hypothesis using a methodology that is more robust than the FM–OLS and CCR regression procedures. The use of robust statistical methods seems important in this context because it is well known that the densities of financial asset returns typically exhibit heavy tails. This is especially true of frequently sampled data like daily exchange rate returns and is a theme in recent studies that use ARCH approaches to model conditional return data (*e.g.* Baillie and Bollerslev, 1989b; and Hsieh, 1989), as well as in empirical work on the unconditional distributions of returns. In four recent studies of exchange rate returns, Koedijk *et al.* (1990) (with weekly data on European currencies–US dollar exchange rates), Hols and DeVries (1991) (with weekly data on the Canadian–US dollar exchange rate),

Koedijk and Kool (1992) (with monthly exchange rate returns for seven East European currencies) and Loretan and Phillips (1993) (with daily exchange rate returns for several OECD countries) all found strong evidence that the maximal moment exponent of the densities of these series was less than four, *i.e.* fourth moments of the distributions appear to be not finite and in some cases variances appear to be not finite also. Each of these studies uses procedures that are based on order statistics to estimate the tail slope of the density and are therefore not reliant on specific distributional forms for exchange rate returns for their validity. Our own empirical analysis of the tail behavior of exchange rate returns uses similar order statistic methods but employs an adaptive data-based procedure to determine the number of order statistics to be used in estimating the tail slope. Our results on tail shape, like those of the studies cited above, call into question the appropriateness of regression procedures like least squares, which give prominence to outlier observations, in the context of models such as (1)–(1'') for heavy tailed exchange rate data. It therefore seems desirable to fit these models and test the unbiasedness hypothesis using methods which work well with such data, while at the same time maintaining the desirable features of the FM–OLS procedure especially its capacity to accommodate nonstationary log-levels data and serial dependence in the equation errors.

The present paper employs a new procedure due to Phillips (1993) that is designed to achieve these objectives. The procedure is called fully modified least absolute deviations (FM–LAD) and it is based on a “fully modified” extension of the least absolute deviations (LAD) regression estimator. The LAD estimator is well known to be a robust regression procedure whose asymptotic properties in conventional regression models have been known for some time (since Bassett and Koenker, 1978). The method is also applicable in time series regressions models (*e.g.* Bloomfield and Steiger, 1983) and has very recently been shown to have good properties in models with an autoregressive unit root (Knight, 1989, 1991; and Phillips, 1991). The FM–LAD estimator that we use in the empirical work of this paper has all of the robust features of the LAD estimator but is also applicable in models for nonstationary time series like (1) where there is endogeneity in the regressors and serial dependence in the errors. Moreover, the estimator and its associated tests apply irrespective of the tail thickness of the data. In fact, the procedures we employ here are applicable (and valid) even when the data have no finite variances, which seems to be the case for at least some exchange rate return series (*e.g.* those studied in Koedijk *et al.*, 1990).

Like FM-OLS, the FM-LAD estimator is a semiparametric procedure that treats nuisance parameters (like serial dependence effects) in a nonparametric way but regression coefficients (like those in (1)) parametrically. In this respect the approach is quite different from the quasi-maximum likelihood technique used recently by Baillie, Bollerslev and Redfearn (1993) to cope with non-Gaussian data in a univariate GARCH analysis of weekly foreign exchange returns.

The paper proceeds as follows. Section 2 discusses the data and seeks to characterize their tail slope characteristics using adaptive order statistic methods. The cointegration properties of the forward rate and spot rate series for each country are considered in Section 3. Recursive residual based cointegration tests are used in this section to highlight the relationship between the forward and spot rates over the full sample period. Section 4 describes the FM-LAD procedure and our empirical results and tests of the unbiasedness hypothesis are given in Section 5. Some concluding observations are made in Section 6.

## 2. THE DATA AND ITS TAIL SLOPE CHARACTERISTICS

The data set employed in this study was gathered by one of the authors (PCM) from back issues of *The Manchester Guardian* and consists of daily observations of spot and one-month forward exchange rates over the period beginning 1 May 1922 and ending 30 May 1925. The currencies are the Belgian franc, French franc, Italian lira and US dollar all measured in terms of the UK pound sterling. Observations include Saturdays and therefore cover six days a week, giving 966 observations of each series in total.

Our data set does not provide exact matching of the (one-month) forward rate series with the settlement date (see Levine, 1989, for a description of the mechanics of this exact matching process). Instead, we use a fixed period of  $k = 26$  (*i.e.* a typical month of 31 days less 5 Sundays) for the number of days to settlement of the forward contract. However, in view of the nonparametric treatment by our FM-LAD procedure of the regression errors in (1) and the shocks to the regressors themselves, the fact that we do not employ an “exact” matching of the forward rate settlement and the spot rate data is not important, at least asymptotically, for the good properties of our procedure.

This period in the 1920's marks the first generalized episode of financial experimentation

with floating exchange rates during which many countries abandoned the gold standard in an effort to reconcile their external balances. Financial reconstruction following World War I was a destabilizing experience for many countries and the period was characterized by speculative attacks against currencies and financial turbulence. For these reasons the data offer an especially interesting opportunity to explore the empirical support for foreign exchange market efficiency theories.

Figures 1-4 graph the levels of the series with each figure showing the spot rate (SR) at  $t + k$  and the forward rate (FR) at  $t$  for  $k = 26$ . The turbulent behavior of these series over certain subperiods is apparent from the figures, notably the winter of 1924 for the Belgium and French francs and the year of 1922 for the Italian lira.

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 Figures 1-4 about here  
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The tail shapes of the data are characterized in Figures 5-8, which plot the tails of the empirical distribution functions of the exchange rate return data in double-logarithmic coordinates. More precisely, these figures plot  $\log_{10}\{P(X < -x)\}$  against  $\log_{10} x$  for  $x > 0$ , *i.e.* we give the results here for the left tails of the distributions. (Similar graphs were obtained for the right tails, although the tail slopes were a little different in each case.) Note that in these coordinates the Pareto distribution, for which  $P(X < -x) = Cx^{-\alpha}$  for constants  $C$  and  $\alpha$ , appear as a straight line with slope =  $-\alpha$ . The case  $\alpha = 2$  is critical since it divides off finite variance distributions from infinite variance distribution. We therefore show a straight line of slope =  $-2$  in each of the figures and graph it against the tail of the empirical distribution for ease of comparison. As is apparent from these graphs the tail of the empirical distributions is reasonably well described by a Pareto distribution for the Belgium, French, and Italian data, but is less well characterized by a straight line in these coordinates for the US data. For all currencies the tail slope seems to be at least as steep as  $-2$  but is very close to  $-2$  for the Belgian data.

Direct estimates of the tail slope can be obtained as follows. Suppose  $X_t$  is an iid sequence whose distribution has tail behavior of the Pareto-Lévy form, *viz.*

$$P(X > x) = pCx^{-\alpha}(1 + \alpha_1(x)), \quad x > 0 \quad (2)$$



$$P(X < -x) = qCx^{-\alpha}(1 + \alpha_2(x)), \quad x > 0, \quad (3)$$

where  $\alpha_i(x) \rightarrow 0$  ( $i = 1, 2$ ) as  $x \rightarrow \infty$  and  $p \geq 0, q \geq 0$ . The parameter  $C$  in these tail formulae is a scale dispersion parameter and  $\alpha$ , which determines the tail slope, is the maximal moment exponent in the sense that  $\alpha = \sup\{r > 0 : E|X|^r < \infty\}$ . The slope parameter  $\alpha$  in (2) and (3) can be estimated by means of order statistics. If  $X_{n1} \leq X_{n2} \leq \dots \leq X_{nn}$  are the order statistics of  $\{X_t\}_1^n$  in ascending order then Hill's (1975) estimators of  $\alpha$  and  $C$  are

$$\hat{\alpha}_s = (s^{-1} \sum_{j=1}^s \ln X_{n,n-j+1} - \ln X_{n,n-s})^{-1}, \quad (4)$$

$$\hat{C}_s = (s/n) X_{n,n-s}^{\hat{\alpha}_s}. \quad (5)$$

These were developed as conditional maximum likelihood estimators (conditional on the  $s$  largest order statistics of the sample under precisely Pareto tails, *i.e.*  $\alpha_1(x) = 0$  in (2)). Hall (1982) derived an asymptotic theory for these estimators in the case of distributions whose tails are of the form (2) & (3), *i.e.* only asymptotically Pareto, and showed that it is optimal when  $\alpha_i(x) = O(x^{-\gamma})$ ,  $\gamma > 0$ , in (2) and (3) to choose the order statistic truncation number  $s = s(n)$  so that it tends to infinity with  $n$  and is of order  $n^{2\gamma/(2\gamma+\alpha)}$ . When  $s = o(n^{2\gamma/(2\gamma+\alpha)})$  we have the limit theory,

$$s^{1/2}(\hat{\alpha}_s - \alpha) \xrightarrow{d} N(0, \alpha^2) \quad (6)$$

(Theorem 2 of Hall, 1982), which facilitates inference about  $\alpha$ . This methodology has been used in some earlier work on foreign exchange rate and stock returns by Koedijk *et al.* (1991), Jansen and DeVries (1991), Hols and DeVries (1991), and Loretan and Phillips (1993). Kearns and Pagan (1992) recently conducted a Monte Carlo study comparing this procedure with other methods of estimating the tail slope, concluding that the Hill estimator was the preferable procedure for practical applications but that the tail slope is imprecisely determined even in large samples especially when there is dependence in the series.

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Figures 5–8 about here

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None of the aforementioned studies use data-based methods for selecting the order statistic truncation number  $s = s(n)$ . An “optimal” choice of  $s(n)$  can be deduced from the asymptotic

theory in Hall (1982) in terms of the minimum asymptotic mean squared error. For distributions in which the tail behavior has the form given in (2) & (3) with  $\alpha_i(x) = Dx^{-\alpha} + o(x^{-\alpha})$  and  $p = q = 1$ , the mean squared error of the limit distribution of  $\hat{\alpha}_s$  is minimized by choosing

$$s = s(n) = [\lambda n^{2/3}], \text{ with } \lambda = (2C^2/D^2)^{1/3},$$

and where  $[ \ ]$  signifies the integer part of its argument. Hall and Welsh (1985) show that the parameter  $\lambda$  may be estimated adaptively by

$$\hat{\lambda} = |\hat{\alpha}_s/2^{1/2}(n/t_1)(\hat{\alpha}_{t_1} - \hat{\alpha}_s)|^{2/3}$$

Here  $\hat{\alpha}_{t_1}$  and  $\hat{\alpha}_s$  are preliminary estimates of  $\alpha$  obtained by using formula (4) with data truncations  $s(n) = [n^\sigma]$  and  $s(n) = [n^\tau]$ , respectively, where  $0 < \sigma < 2/3$  and  $2/3 < \tau < 1$ . We employed this adaptive approach in our empirical work, setting

$$\hat{s} = [\hat{\lambda} n^{2/3}], \text{ with } \sigma = 0.60 \text{ and } \tau = 0.90.$$

The estimates given below in Table 1 for  $\hat{s}$  and  $\hat{\alpha}_{\hat{s}}$  are not very sensitive to alternative choices of  $\sigma$  and  $\tau$ , especially over the intervals  $\sigma \in [0.5, 0.65]$  and  $\tau \in [0.8, 0.95]$ . In our experience, choices of  $\sigma$  and  $\tau$  outside these intervals do lead to a wider range of estimates of  $s$ , particularly when  $\tau$  is close to its lower limit of  $2/3$ . But even in such cases the estimates of the tail slope parameter seem fairly stable. Table 2 reports a sensitivity analysis of this feature of the adaptive estimators  $\hat{s}$  and  $\hat{\alpha}_{\hat{s}}$  for our data sets and the results given there support these general observations.

Table 1 presents estimates of the tail slope using formula (4) above for a range of values of the order statistic truncation number  $s$  up to around 10% of the sample size (following the suggestions of Dumouchel, 1983). Standard errors of the slope estimate are given in parentheses. These are based on (6) and therefore (like the asymptotic theory) assume homoskedasticity. There is some recent evidence from simulations (Kearns and Pagan, 1992) that this formula may poorly estimate the variation when ARCH effects are present, although simulations that use the adaptive procedure employed here have not yet (to our knowledge at least) been performed. The final row for each country gives the adaptive estimate of the tail slope, using the data-determined truncation number  $\hat{s}$ . Right tails, left tails and the combined (two) tails of the exchange return data were analyzed in this way. Table 1 also reports results for the spread variable  $s_{t+k} - f_{t,k}$ . Asymptotic standard errors were computed using (6). The results given in the table in the ‘‘Forward Rate’’

TABLE 1: Point Estimates of Tail Slope Parameters

Country	s	Forward Rate			Spot Rate			Spread			Estimated AR Order PIC/BIG
		Left Tail	Right Tail	Two Tail	Left Tail	Right Tail	Two Tail	$s_{t,t} - f_{t,t}$			
Belgium	15	2.803 (0.723)	3.095 (0.799)	3.196 (0.825)	2.814 (0.729)	3.121 (0.805)	3.198 (0.825)	3.383 (0.873)			
	25	2.277 (0.455)	2.171 (0.543)	3.090 (0.618)	2.294 (0.458)	2.716 (0.543)	3.012 (0.602)	3.511 (0.702)			
	50	1.857 (0.262)	2.186 (0.308)	2.570 (0.363)	1.879 (0.265)	2.179 (0.308)	2.568 (0.363)	2.927 (0.413)		4	
	75	1.707 (0.197)	1.889 (0.218)	2.429 (0.281)	1.713 (0.197)	1.898 (0.219)	2.451 (0.283)	2.769 (0.319)			
	$\hat{s}$	2.092[41] (0.326)	2.150[49] (0.307)	2.473[71] (0.293)	2.176[43] (0.332)	2.316[48] (0.334)	2.442[72] (0.287)	3.048[66] (0.375)			
France	15	2.420 (0.625)	4.057 (1.047)	5.050 (1.304)	2.625 (0.677)	4.309 (1.113)	3.852 (0.994)	4.721 (1.219)			
	25	2.859 (0.571)	2.926 (0.585)	3.298 (0.659)	2.877 (0.575)	3.089 (0.617)	3.460 (0.691)	4.628 (0.925)			
	50	1.985 (0.280)	2.306 (0.326)	2.935 (0.415)	2.222 (0.314)	2.362 (0.334)	2.948 (0.417)	3.633 (0.513)		4	
	75	1.766 (0.203)	2.002 (0.231)	2.464 (0.284)	1.878 (0.216)	2.011 (0.232)	2.500 (0.288)	3.231 (0.373)			
	$\hat{s}$	2.445[42] (0.377)	2.392[58] (0.314)	2.629[71] (0.312)	2.238[44] (0.337)	2.269[58] (0.297)	2.493[72] (0.293)	3.232[66] (0.397)			
Italy	15	3.358 (0.867)	3.576 (0.923)	3.390 (0.875)	3.374 (0.871)	3.641 (0.940)	3.265 (0.843)	3.059 (0.789)			
	25	2.822 (0.564)	3.393 (0.678)	3.308 (0.661)	2.856 (0.571)	3.271 (0.654)	3.132 (0.626)	2.969 (0.593)			
	50	2.118 (0.299)	3.364 (0.475)	3.274 (0.463)	2.077 (0.293)	3.420 (0.483)	3.208 (0.453)	3.329 (0.470)		3	
	75	2.151 (0.248)	2.660 (0.307)	2.900 (0.334)	2.099 (0.242)	2.613 (0.301)	2.969 (0.342)	3.216 (0.371)			
	$\hat{s}$	2.701[39] (0.433)	3.346[44] (0.504)	2.890[68] (0.350)	2.747[40] (0.434)	3.400[44] (0.512)	2.836[66] (0.349)	3.546[66] (0.436)			
USA	15	3.442 (0.887)	2.917 (0.753)	2.853 (0.736)	3.849 (0.993)	2.883 (0.744)	2.844 (0.734)	5.504 (1.421)			
	25	3.279 (0.655)	2.544 (0.508)	3.153 (0.630)	3.091 (0.618)	2.582 (0.516)	3.289 (0.657)	4.183 (0.836)			
	50	3.173 (0.448)	2.349 (0.332)	2.791 (0.394)	3.017 (0.426)	2.322 (0.328)	2.767 (0.391)	3.016 (0.300)		1	
	75	2.331 (0.269)	2.101 (0.242)	2.664 (0.307)	2.370 (0.273)	2.023 (0.233)	2.550 (0.294)	2.602 (0.300)			
	$\hat{s}$	3.115[49] (0.445)	2.207[55] (0.297)	2.649[78] (0.299)	2.942[46] (0.434)	2.144[53] (0.294)	2.634[79] (0.296)	2.634[78] (0.298)			

[ ] =  $\hat{s}$  = adaptive estimate of order statistic truncation number; ( ) = standard error of  $\hat{\alpha}$ ,

and “Spot Rate” panels are for the return series. Very similar results were obtained for prefiltered data using the residuals from autoregressions of the return series with various lag lengths in the range  $p = 1 - 26$ . The spread variables,  $s_{t+k} - f_{t,k}$ , are strongly autocorrelated, as is clear from Figures 1–4. In consequence, we prefiltered these data by taking the residuals from autoregressions whose lag orders were selected by BIC (see Schwarz, 1978) and by PIC (see Phillips and Ploberger, 1994), the latter allowing for possible nonstationarity in the spread series. In fact, BIC and PIC determined the same lag order for each series, as shown in the final panel of Table 1 and the order chosen ranged from one lag to four lags in these autoregressions.

**TABLE 2: Sensitivity of Truncation Estimates and Tail Slope to  $\sigma/\tau$  Settings**

Country	$\sigma$	$\tau$	Forward Rate (Two Tail)			Spot Rate (Two Tail)		
			$\hat{s}$	$\hat{\alpha}$	st.error	$\hat{s}$	$\hat{\alpha}$	st.error
Belgium	0.35	0.75	39	2.822	0.451	39	2.866	0.459
	0.50	0.80	53	2.506	0.344	52	2.477	0.343
	0.60	0.090	71	2.473	0.293	72	2.442	0.287
	0.65	0.95	79	2.391	0.269	77	2.423	0.276
	0.65	0.99	82	2.356	0.260	81	2.457	0.273
France	0.35	0.75	78	2.380	0.269	78	2.439	0.276
	0.35	0.75	34	3.109	0.533	36	3.229	0.538
	0.50	0.80	50	2.935	0.415	50	2.948	0.417
	0.60	0.90	71	2.629	0.312	72	2.493	0.293
	0.65	0.95	82	2.428	0.268	81	2.393	0.265
Italy	0.65	0.99	84	2.332	0.254	82	2.336	0.256
	0.35	0.75	77	2.508	0.285	78	2.507	0.283
	0.35	0.75	62	2.887	0.3667	63	2.851	0.359
	0.50	0.80	58	2.838	0.3726	58	2.905	0.381
	0.60	0.90	68	2.890	0.3504	66	2.836	0.349
USA	0.65	0.95	74	2.916	0.339	72	2.969	0.349
	0.65	0.99	80	2.917	0.326	81	2.879	0.319
	0.35	0.75	79	2.887	0.324	80	2.856	0.319
	0.35	0.75	39	2.941	0.470	39	2.866	0.459
	0.50	0.80	49	2.870	0.410	48	2.861	0.413
USA	0.60	0.90	78	2.649	0.299	79	2.634	0.296
	0.65	0.95	79	2.673	0.300	79	2.634	0.296
	0.65	0.99	82	2.580	0.284	82	2.578	0.284
	0.35	0.75	78	2.649	0.299	78	2.617	0.296

The adaptive point estimates given in Table 1 are all less than 3.0 when the two tails are combined, and all the estimates but three are less than 3.0 when the tails are considered separately. The truncation numbers  $\hat{s}$  in the adaptive estimates seem quite well determined and are in the range 40–58 for the individual tails and 66–79 for the combined tails. The Belgian series seem to have the thickest tails and this is borne out by the graphs of the tails shown earlier in Figures 5–8. There is some doubt in the case of the Belgian series whether a finite variance distribution

is an appropriate model for the data. In all cases the tails seem to be much heavier than those of a Gaussian distribution. There would therefore seem to be a definite advantage to using robust regression procedures with these data.

The tails of the spread series  $s_{t+k} - f_{t,k}$  seem to be somewhat thinner for the Belgian franc, French franc and Italian lira and each have adaptively estimated tail slopes that are greater than 3.0 but less than 4.0. For the US dollar spread, the adaptive tail slope estimate is very close to those of the individual return series. In fact, the estimate is identical to that of the US dollar spot rate return series, but has a slightly different truncation statistic estimator ( $\hat{s} = 78$  for the spread, whereas  $\hat{s} = 79$  for the spot rate return). Thus, the tail behavior of the US dollar exchange rate spread is very similar to that of the exchange rate returns themselves.

### 3. COINTEGRATION OF THE SPOT RATE AND FORWARD RATE

Cointegration tests were conducted to assess the adequacy of the model (1) over the sample period. Residual based tests of the Phillips and Ouliaris (1990) type were carried out for each of the currencies and the results are reported in Table 3. The  $Z_a$  and  $Z_t$  tests were computed using (i) a fixed lag (set at  $\ell = 10$ ) long-run variance estimator and (ii) the data-based long-run variance estimator of Lee and Phillips (1993). Similarly, the ADF tests used a fixed AR lag length ( $p = 10$ ) and a data-based (BIC) AR order selector  $\hat{p}$ . All of the tests confirm that the relationship (1) is cointegrating for each currency. The 5% critical values given in Table 3 are from the Phillips and Ouliaris (1990) tables (as updated in the GAUSS software package COINT 2.0 — see Ouliaris and Phillips, 1994) and rely on the Brownian motion limit theory derived in that paper for these tests. As in the case of unit root tests, these residual based cointegration tests are still useable where the data is heavy tailed (with possibly infinite variance) much as in the analysis of Phillips (1990), although this has not been formally demonstrated and the tests are likely to be conservative just like the unit roots test in this case (see Chan and Tran, 1989). Table 3 also includes the results of the Phillips and Ploberger (1991, 1994) posterior information criterion (PIC) unit root test applied to the residuals from the regression (1) for each currency. This test involves a data-based model selection procedure prior to the construction of the odds ratio. The odds in favor of cointegration range from 4.5:1 in the case of the US dollar to 309:1

in the case of Belgian francs. These outcomes corroborate the conclusions of the residual based tests.

**TABLE 3: Residual Based Tests of Cointegration**

Currency	$\ell = 10$ $Z(a)$	Data-based $\hat{\ell}$ $Z(a)$	$\ell = 10$ $Z(t)$	Data-based $\hat{\ell}$ $Z(t)$	$p = 10$ ADF	Data-based $\hat{p}$ ADF	Phillips-Ploberger PIC odds in favor of cointegration
Belgium	-37.236	-44.771	-4.377	-4.786	-4.213	-4.786	309.39
France	-35.499	-30.739	-4.233	-3.940	-4.908	-4.563	232.86
Italy	-28.691	-36.095	-3.767	-4.228	-4.240	-3.752	8.65
USA	-30.468	-28.207	-3.925	-3.776	-4.377	-3.834	4.58
5% cv's: $Z(a)$ cv = -20.4517; $Z(t)$ cv = -3.358							

Since the period 1922–1925 involves interesting subperiods of financial turbulence, it is informative to plot the outcome of the cointegration tests as we move progressively through the sample data. Figures 9–12 show such recursive calculations for the residual based  $Z_a$  test for each of the currencies starting with the 100'th observation and running through to the final observation (966). The recursive  $Z_a$  statistic was calculated using a data-based long-run variance estimator so that the bandwidth parameter was automatically adjusted on a period by period basis as the recursive calculations proceeded. We used a Parzen kernel and the corresponding “optimal” bandwidth computed by the “plug in” method (*e.g.*, see Andrews, 1991) preceded by AR(1) data-prefiltering and subsequent recoloring (as suggested in Andrews and Monahan, 1992). This method is much faster than the data-based prefiltering method of Lee and Phillips (1993) that is computationally more demanding especially in extensive recursive calculations of this type.

Figures 9–12 give the 5% critical values of the  $Z_a$  statistic for sample sizes over the relevant range and the line shown is a step function for sample sizes over the intervals  $100 \leq n < 200$ ,  $200 \leq n < 300$ ,  $300 \leq n < 400$ , and  $n \geq 500$ . Within each interval these critical values are based on simulations involving 10,000 replications and they were obtained from the COINT 2.0 cointegration regression library of Ouliaris and Phillips (1994). The results shown in Figures 9–12 are interesting and reveal some important differences between the series. In the case of the Belgian and French francs the financial turbulence of the currencies during the winter and spring of 1924 is apparent in the volatile behavior of the  $Z_a$  test. For the French franc the cointegration test statistic swings from acceptance to rejection and back to acceptance again, and similar swings in the statistic are evident for the Belgian franc although the inference about cointegration is not

reversed by the swings in this case.

The volatility of the  $Z_a$  statistic for the French franc during the spring of 1924 coincides with a famous interlude known as the “Poincaré bear squeeze,” which amounted to a major government intervention in the foreign exchange market. Owing to domestic economic difficulties and growing inflation, the French franc depreciated substantially against the British pound during the fall of 1923 and had lost nearly 50% of its value by March 1924. To counteract this devaluation, the French Premier Poincaré raised secret loans with US and British banks to purchase large quantities of francs and thereby arrest (and reverse) the bear market. The move was highly successful. In the week following 11 March 1924 the franc appreciated from 117F/£ to 90F/£ and within three weeks rose to 64.92F/£. During this period the forward and future spot rates were severely out of alignment (as is apparent from Figure 2), and this disparity in the rates leads directly to the huge swing in value of the  $Z_a$  test during March 1924 that is evident in Figure 10. In the period following the spring of 1924 the forward rate and future spot rate moved into a closer pattern of alignment and the cointegrating test statistic  $Z_a$  declines in value in a corresponding way. Indeed, after the summer of 1924 the test statistic has a monotonically declining value (*i.e.*, is increasingly negative) leading to increasingly strong acceptance of cointegration in the spot and forward rates for both currencies. If equation (1) holds for these currencies, this is exactly the behavior in the statistic that we would expect as the sample size grows and more information on the currencies accumulates, since  $Z(a)$  diverges to  $-\infty$  as the sample size  $n \rightarrow \infty$  when (1) holds.

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 Figures 9–12 about here  
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We also notice this behavior as the sample size  $n$  increases in the case of the Italian lira (Figure 11) and the US dollar (Figure 12). For the lira the evidence in favor of (1) holding becomes very strong from 1924 onwards. For the US dollar the evidence for cointegration is mixed during 1924 but becomes stronger (and monotonic) during 1925. Prior to 1924 there is no evidence in support of cointegration for the US dollar exchange rates and little evidence for cointegration in the forward and spot exchange rates of the other three countries.

#### 4. THE FM-LAD ESTIMATOR AND TESTS

This section briefly describes the fully modified least absolute deviations (FM-LAD) regression procedure developed recently in Phillips (1993). Like the fully modified least squares (FM-OLS) method of Phillips and Hansen (1990), the FM-LAD estimator is designed for levels regressions of nonstationary time series and makes modifications to the usual regression procedure (here least absolute deviations or LAD) to deal with (i) the endogeneity of the regressors and (ii) serial correlation in the errors and in the first differences of the regressors. The LAD estimator in conventional regression models (see Bassett and Koenker, 1978) is particularly suited to situations where the equation errors are heavy tailed and in such contexts is consistent even when least squares regression is inconsistent. In a similar way, the FM-LAD estimator is suited to the estimation of cointegrating regression equations when the data and the equation errors are heavy tailed.

To fix ideas we consider the cointegrated system

$$y_t = \beta' x_t + u_{0t} \quad (7a)$$

$$\Delta x_t = u_{xt} \quad (7b)$$

where  $u'_t = (u_{0t}, u'_{xt})$  is a stationary  $m$ -vector time series ( $m = 1 + m_x$ ) with spectral density matrix  $f_{uu}(\lambda)$ . The long-run covariance matrix of  $u_t$  is

$$\Omega_{uu} = 2\pi f_{uu}(0) = \begin{bmatrix} \Omega_{00} & \Omega_{0x} \\ \Omega_{x0} & \Omega_{xx} \end{bmatrix}, \quad (8)$$

where the partition is conformable with that of the vector  $u_t$  and we assume  $\Omega_{xx} > 0$  (*i.e.*,  $\Omega_{xx}$  is positive definite), so that  $x_t$  in (7a) is a full rank I(1) process in the sense that the number of unit roots in the stochastic process  $x_t$  is equal to  $m_x$ , the dimension of  $x_t$ . Cases where  $\Omega_{xx}$  is positive semi-definite and some of the  $x_t$  regressors are cointegrated are also of potential interest (*e.g.* in models like (1'') with several currencies). It can be shown (although we do not attempt it here) that the FM-LAD procedure is applicable in such situations also — see Phillips (1994) for an analysis of this type of situation with regard to FM-OLS estimation. In cases where  $u_t$  does not possess finite second moments the matrix  $\Omega$  in (8) is not well-defined. However, it is still possible in such cases to construct a pseudo long-run variance of  $u_t$ , as discussed in Phillips (1990, p. 51), and we will proceed as if this has been done.



Since we work below with the transformed error  $v_t = \text{sign}(u_{0t})$ , it is also helpful to define the long-run covariance matrix of  $w_t' = (v_t, u_{xt}')$  as

$$\Omega_{ww} = 2\pi f_{ww}(0) = \begin{bmatrix} \Omega_{vv} & \Omega_{vx} \\ \Omega_{xv} & \Omega_{xx} \end{bmatrix},$$

partitioned conformably with  $w_t$ . Note that  $v_t$  is bounded and has finite moments of all orders. But this is not true of  $u_{xt}$  and in cases where the second moments of  $u_{xt}$  do not exist we may again employ a pseudo variance interpretation of  $\Omega_{ww}$ .

The LAD estimator of  $\beta$  in (7a) is the extremum estimator

$$\beta_{\text{LAD}} = \text{argmin}\{n^{-1}\sum_1^n |y_t - x_t'\beta|\}. \quad (9)$$

When the regressors  $x_t$  are fixed this estimator has an asymptotic normal distribution and is  $\sqrt{n}$  consistent for  $\beta$  in (7a). When  $x_t$  is an I(1) process and (7b) holds, this asymptotic theory no longer applies in general. Instead, the LAD estimator, just like OLS, suffers from bias and nonscale nuisance parameter problems even in the limit as  $n \rightarrow \infty$ .

The FM-LAD estimator is designed to address these difficulties that are encountered by the LAD estimator while at the same time retaining its robustness features with regard to heavy tailed errors. As with the FM-OLS estimator, we modify LAD to account for possible endogeneities in the  $x_t$  regressor variables and serial dependence in the errors. The FM-LAD estimator is defined by

$$\beta_{\text{LAD}}^+ = \beta_{\text{LAD}} - (1/2\hat{f}(0))(X'X)^{-1}(X'\Delta X - n\hat{\Delta}_{xv}^+), \quad (10)$$

where  $X'X = \sum_1^n x_t x_t'$ ,  $X'\Delta X = \sum_1^n x_t \Delta x_t'$  and  $\hat{f}(0)$  is a consistent estimator of the probability density of  $u_{0t}$  at the origin.

The matrix  $\hat{\Delta}_{xv}^+$  in (10) is a consistent estimator of the one-sided long-run covariance matrix

$$\Delta_{xv}^+ = \sum_{k=0}^{\infty} E(u_{x0} v_k^+), \quad (11)$$

where

$$v_t^+ = v_t - \Omega_{vx} \Omega_{xx}^{-1} \Delta x_t, \quad (12)$$

and

$$v_t = \text{sign}(u_{0t}). \quad (13)$$

In order to estimate  $\Delta_{xv}^+$ , we need first to estimate the modified error  $v_t^+$ , which in turn involves the estimation of  $v_t$ . This is achieved by a first stage LAD regression which produces the error estimate  $\widehat{u}_{0t} = y_t - \beta_{\text{LAD}}^t x_t$  and  $\widehat{v}_t = \text{sign}(\widehat{u}_{0t})$ . We then construct

$$\widehat{v}_t^+ = \widehat{v}_t - \widehat{\Omega}_{vx} \widehat{\Omega}_{xx}^{-1} \Delta x_t \quad (14)$$

using conventional kernel estimates of the long-run covariance matrices  $\Omega_{vx}$  and  $\Omega_{xx}$ , whereupon we can estimate  $\Delta_{xv}^+$  in (11) directly by using a kernel estimate of the one-sided long-run covariance of  $u_{xt}$  and  $\widehat{v}_t^+$  (see Park and Phillips, 1988 and Andrews, 1991). Note that we can write

$$\Delta_{xv}^+ = \Delta_{xv} - \Delta_{xx} \Omega_{xx}^{-1} \Omega_{xv}, \text{ where } \Delta_{xv} = \sum_{k=0}^{\infty} E(u_{x0} v_k), \Delta_{xx} = \sum_{k=0}^{\infty} E(u_{x0} u'_{xk}), \quad (15)$$

so that the estimation of  $\Delta_{xv}^+$  effectively involves the estimation of the four submatrices  $\Delta_{xv}$ ,  $\Delta_{xx}$ ,  $\Omega_{xx}$  and  $\Omega_{xv}$ . In our empirical work reported below we used the Parzen kernel and the associated ‘‘optimal’’ data-based bandwidth in the estimation of these long-run variances and one-sided covariances. Once again, if variances were infinite we could employ pseudo-variance interpretations of these quantities and the pseudo-variances could all be estimated in the same way as we have done with finite samples of data.

An asymptotic theory for the estimator  $\beta_{\text{LAD}}^+$  given in (10) is developed in Phillips (1993). It is shown there (Theorem 4.4, *op. cit.*) that, when the system (7) has finite variance errors,  $\beta_{\text{LAD}}^+$  is asymptotically mixed normal, *i.e.*

$$(\beta_{\text{LAD}}^+ - \beta) \sim N(0, (1/2f(0))^2 [\omega_{vv \cdot x} \otimes (X'X)^{-1}]) \quad (16)$$

where  $\omega_{vv \cdot x} = \Omega_{vv} - \Omega_{vx} \Omega_{xx}^{-1} \Omega_{xv}$ . Moreover, because  $\beta_{\text{LAD}}^+$  is conditionally asymptotically normal, Wald statistics can be constructed in the usual way from this mixture normal approximation to test restrictions on the parameter vector  $\beta$  and such statistics have limiting chi-squared distributions with degrees of freedom equal to the number of restrictions — see Phillips (1993, Remark 4.5(ii)) for details). The same limit theory applies when the regressor variable  $x_t$  in (7a) has a constant or deterministic trend, but in this case the conditioning that appears in the covariance matrix  $\omega_{vv \cdot x}$  applies only to the I(1) components of  $x_t$ .

When the system (7) has infinite variance errors it is shown in Phillips (1993) that (15) still holds but with  $\omega_{vv \cdot x} = \Omega_{vv}$ , and the convergence rate is faster than  $O(n)$ . Thus, for the FM-LAD estimator the asymptotic mixed normal approximation (16) applies whether or not the error

variances in the system (7) are finite. The FM-LAD estimator is therefore not only a robust estimator, but it also has the attractive feature that its limit theory in the case of cointegrated systems like (7) is robust and may be used for statistical inference irrespective of the tail thickness of the errors. This feature makes the estimator and its associated tests very useful in the context of foreign exchange market data as in the present case, where the empirical distributions clearly have heavy tails.

## 5. ROBUST TESTS OF MARKET EFFICIENCY

Equation (1) was estimated for each of the four currencies by OLS, FM-OLS, LAD and FM-LAD. The results are shown in Table 4. Standard errors,  $t$ -ratios and Wald statistics for testing the joint hypothesis  $\mathcal{H}_0 : a = 0, b = 1$  are given in the table for the FM-OLS and FM-LAD procedures only, since conventional inferential formulae are inappropriate in the case of the OLS and LAD estimators in view of the nonstationarity and temporal dependence of the data. (Note, however, that both OLS and LAD coefficient estimates are consistent, even though they suffer from second order bias).

TABLE 4: Empirical Estimates of Equation (1)

Currency	Estimation Method	Parameters, standard errors and $t$ -ratios						Joint Test of Efficiency
		$\hat{a}$	$s_a$	$t_a = \hat{a}/s_a$	$\hat{b}$	$s_b$	$t_b = (\hat{b}-1)/s_b$	
Belgium	OLS	0.745			0.835			
	FM-OLS	0.546	0.179	3.045 <sup>††</sup>	0.880	0.040	-2.957 <sup>***</sup>	12.942 <sup>***</sup>
	LAD	0.344			0.926			
	FM-LAD	0.232	0.139	1.667	0.952	0.031	-1.526	12.601 <sup>***</sup>
France	OLS	0.779			0.823			
	FM-OLS	0.604	0.199	3.030 <sup>††</sup>	0.863	0.046	-2.955 <sup>***</sup>	12.066 <sup>***</sup>
	LAD	0.357			0.922			
	FM-LAD	0.269	0.136	1.972 <sup>†</sup>	0.942	0.032	-1.806 <sup>*</sup>	18.835 <sup>***</sup>
Italy	OLS	0.835			0.821			
	FM-OLS	0.641	0.256	2.506 <sup>†</sup>	0.863	0.055	-2.464 <sup>**</sup>	13.368 <sup>***</sup>
	LAD	0.468			0.901			
	FM-LAD	0.241	0.199	1.207	0.950	0.043	-1.144	17.742 <sup>***</sup>
USA	OLS	0.061			0.960			
	FM-OLS	0.063	0.067	0.942	0.959	0.044	-0.917	2.131
	LAD	0.091			0.940			
	FM-LAD	0.029	0.073	0.395	0.982	0.048	-0.375	0.913
(i) One-tail significance: * = 5%    ** = 1%    *** = 5%								
(ii) Two-tail significance: † = 5%    †† = 1%    ††† = 0.5%								

The main results to emerge from these regressions are as follows.

(i) There are substantial differences in the coefficient estimates (of both the slope and the intercept parameter in (1)) between FM-OLS and FM-LAD. For example, in the case of the Belgian franc we have  $b_{\text{LAD}}^+ = 0.952$  and  $b_{\text{OLS}}^+ = 0.880$ , which differ by more than two FM-LAD asymptotic standard errors. Similarly for the French franc we have  $b_{\text{LAD}}^+ = 0.942$  compared with  $b_{\text{OLS}}^+ = 0.863$ , again more than two standard errors apart. A similar comment holds for the Italian lira.

In all these cases the FM-LAD estimates are much closer to the value  $b = 1$ . Deviations of  $b$  from unity have an important impact on the statistical properties of the risk premium

$$p_{t,k} = f_{t,k} - E_t(s_{t+k}) = (1-b)f_{t,k} - a - E_t(u_{t+k}). \quad (17)$$

In particular, the risk premium  $p_{t,k}$  is nonstationary if  $b \neq 1$ . For the Belgian and French francs, the hypothesis

$$\mathcal{H}_b : b = 1$$

is rejected at the 0.1% level using the FM-OLS estimates, and the hypothesis is rejected at the 1% level for the Italian lira based on FM-OLS. However,  $\mathcal{H}_b$  is accepted for the Belgian franc and the Italian lira, and only marginally rejected for the French franc when the robust FM-LAD estimates are used. Thus, the FM-LAD estimates support the hypothesis that the risk premium is stationary for the Belgian franc and Italian lira, and give only marginal evidence in support of a nonstationary risk premium for the French franc.

(ii) For the US dollar, both FM-OLS and FM-LAD procedures fail to reject  $\mathcal{H}_a$  and  $\mathcal{H}_b$ . The FM-LAD estimates give stronger support to the “acceptance” of these hypotheses than FM-OLS. Thus, the FM-LAD empirical results for the US dollar appear to support strongly the forward rate unbiasedness hypothesis.

(iii) Tests of the joint hypothesis of market efficiency, viz.  $\mathcal{H}_0 : a = 0, b = 1$ , are unambiguous. The Wald tests constructed from both the FM-OLS and FM-LAD estimates concur for each currency: efficiency is clearly rejected (at the 0.1% level) for the Belgian, French and Italian currencies; and efficiency is “accepted” in the case of the US dollar.

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 Figures 13a & b about here  
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Figures 13a & b graph kernel estimates of the probability densities of the equation error  $u_{0t}$  and the exchange rate return  $u_{xt} = \Delta x_t$  for the Belgian franc. In each case a normal kernel was used with a data-based (“plug in”) optimal bandwidth (see Silverman, 1986). (The same method was used to estimate the density of  $u_{0t}$  at the origin,  $\hat{f}(0)$ , as required for the FM-LAD estimator and its associated Wald statistics — see (10) and (15) above.) On each figure we also graph a normal density with variance  $s^2$  equal to the sample variation of the data — here either the FM-LAD residual  $\hat{u}_{0t}$  or  $u_{xt} = \Delta x_t$ .

**TABLE 5: Empirical Estimates of the Multi-Currency Model:**

$$s_{i+k}^i = a_i + \sum_{j=1}^m b_j f_{i,k}^j + u_{i+k}$$

Currency	Estimation Method	$a_i$	Belgium $b_1$	France $b_2$	Italy $b_3$	USA $b_4$	Joint Test of Multi-market Efficiency
Belgium	FM-OLS	0.293 (0.478)	0.919 (-0.462)	-0.046 (-0.226)	0.004 (0.028)	0.168 (0.632)	13.623*
	FM-LAD	-0.423 (-0.893)	1.259 (1.934)	-0.376 (-2.396) <sup>†</sup>	0.254 (1.926)	-0.164 (-0.796)	19.029**
France	FM-OLS	-0.071 (-0.112)	0.066 (0.365)	0.761 (-1.129)	0.138 (0.781)	0.124 (0.449)	13.781*
	FM-LAD	-0.172 (-0.394)	0.185 (1.503)	0.725 (-1.896)	0.152 (1.259)	-0.096 (-0.506)	21.227***
Italy	FM-OLS	0.811 (2.901) <sup>††</sup>	-0.195 (-2.469) <sup>†</sup>	0.231 (2.496) <sup>†</sup>	0.709 (-3.732) <sup>†††</sup>	0.266 (2.192) <sup>†</sup>	28.789***
	FM-LAD	0.026 (0.123)	-0.059 (-0.991)	0.016 (0.232)	0.963 (-0.625)	0.231 (2.512) <sup>†</sup>	44.107***
USA	FM-OLS	0.065 (0.637)	-0.098 (-3.374) <sup>†††</sup>	0.111 (3.263) <sup>††</sup>	0.004 (0.141)	0.913 (-1.916)	17.828**
	FM-LAD	-0.023 (-0.237)	-0.070 (-2.498) <sup>†</sup>	0.078 (2.365) <sup>†</sup>	0.021 (0.747)	0.936 (-1.469)	13.704*
<i>t</i> -ratios in parentheses (centered on unity for own currency. (i) one-tail significance: * = 5%    ** = 1%    *** = .1% (ii) two-tail significance: † = 5%    †† = 1%    ††† = .1%							

The leptokurtosis and heavy tailed features of the data are evident from the estimated densities. To save space we report the graphs only for the Belgian franc, but similar patterns were evident for the other currencies and these were shown in the original version of the paper, which is available upon request. The heavy tails of the exchange returns corroborate our earlier findings for the tail slopes of the distribution of these data that were reported in Section 2. The form of these estimated densities, especially when taken in contrast to the approximating Gaussian  $N(0, s^2)$  distributions, confirms the appropriateness of robust estimation procedures in the present context.

To examine multi-country effects we estimated equation (1'') with  $\gamma_i = 0$  for each of the

currencies. The results are shown in Table 5, which gives both FM-OLS and FM-LAD regression statistics. The Wald statistic for testing the null hypothesis  $\mathcal{H}'_0 : a_i = 0, b_j = 0 (j \neq i), b_i = 1$  of multi-market efficiency is rejected at the 5% level for each currency. Some of the individual currency effects are particularly interesting. For instance, looking at the US dollar we see that the forward rates for the French and Belgian francs both have a significant predictive effect on the dollar's future spot rate. These effects hold for both FM-OLS and FM-LAD regressions.

In each of these multi-country regressions the own currency forward rate is the strongest predictor of the future spot rate, and the FM-LAD estimates are not significantly different from unity. There are some notable differences between the FM-LAD and FM-OLS coefficient estimates in these regressions — the Belgian franc and Italian lira, in particular. This confirms our earlier findings in the own currency regressions. In sum, the overall evidence does not support multi-currency efficiency and there is some evidence (strong in the case of the US dollar) that other currency forward rates do influence future spot rate movements in a currency.

## 6. CONCLUSION

The early 1920's was a period of financial experimentation in the foreign exchange markets when many countries abandoned the gold standard and adopted flexible exchange rates. Our daily data on one-month forward rates and spot rates for four currencies (the Belgian franc, French franc, Italian lira and the US dollar all measured against the UK pound sterling) provide an opportunity to examine empirical support for the forward unbiasedness hypothesis and simple market efficiency during this interesting financial period. Since the data are, like most high frequency exchange rate series, heavy tailed and leptokurtotic, conventional regression methods of analysis seem inappropriate. Like other recent work in this field we also need to address the important issues of data nonstationarity and temporal dependence so that we can analyze the data directly in log levels form.

The regression methodology of this paper relies on a fully modified least absolute deviations (FM-LAD) estimator developed recently in other work (see Phillips, 1993). The FM-LAD estimator is statistically robust to data distributions with heavy tails and is designed explicitly for nonstationary cointegrating regressions of the form that is involved in equation (1). Our empirical

results show substantial differences between robust cointegrating regression estimates and those of least squares based procedures like the fully modified least squares estimator of Phillips and Hansen (1990). Our findings confirm that the forward rate and spot rate for the four currencies are cointegrated, especially over the latter part of the sample period from mid 1924 onwards. We find support for the presence of a stationary risk premium in the case of the Belgian franc and the US dollar. The market efficiency hypothesis  $\mathcal{H}_0$  which requires that the forward rate be an unbiased predictor of the future spot rate and the risk premium have mean (or median) zero is empirically supported for the US dollar–UK pound sterling exchange rate but not for the other currencies. Multi-currency regressions reveal that other currency forward rates do influence future spot rate movements in a currency and that multi-market efficiency is not supported for any of the currencies, including the US dollar.

## 7. REFERENCES

- Andrews, D. W. K. (1991), "Heteroskedasticity and autocorrelation consistent covariance matrix estimation," *Econometrica*, 59, 817–858.
- Baillie, R. T. and T. Bollerslev (1989a). "Common stochastic trends in a system of real exchange rates," *Journal of Finance*, 44, 167–181.
- Baillie, R. T. and T. Bollerslev (1989b). "The message in daily exchange rates: A conditional variance tale," *Journal of Business and Economic Statistics*, 7, 297–305.
- Baillie, R. T., T. Bollerslev and M. R. Redfearn (1993). "Bear squeezes, volatility spillovers and speculative attacks in the hyperinflation 1920's foreign exchange," *Journal of International Money and Finance*, October.
- Bassett, G. and R. Koenker (1978). "Asymptotic theory for least absolute error regression," *Journal of the American Statistical Association*, 73, 618–622.
- Bhansali, R. J. (1988). "Consistent order determination for processes with infinite variance," *Journal of the Royal Statistical Society, B*, 50, 46–60.
- Bloomfield, P. and W. L. Steiger (1983). *Least Absolute Deviations: Theory Applications and Algorithms*. Boston: Birkhauser.
- Chan, N. H. and L. T. Tran (1989). "On the first order autoregressive process with infinite variance," *Econometric Theory*, 5, 354–362.
- Corbae, D., K-G Lim and S. Ouliaris (1993). "On cointegration and tests of forward market unbiasedness," *Review of Economics and Statistics*, forthcoming.
- Davis, R. A. and S. I. Resnick (1985). "Limit theory for moving averages of random variables with regularly varying tail probabilities," *Annals of Probability*, 13, 179–195.

- Davis, R. A. and S. I. Resnick (1986). "Limit theory for the sample covariance and correlation functions of moving averages," *Annals of Statistics*, 14, 533–558.
- Dumouchel, W. H. (1983). "Estimating the stable index  $\alpha$  in order to measure the tail thickness: A critique," *Annals of Statistics*, 11, 1019–1031.
- Engle, R. F. and C. W. J. Granger (1987). "Cointegration and error-correction: Representation, estimation and testing," *Econometrica*, 55, 251–276.
- Hakkio, C. S. and M. Rush (1989). "Market efficiency and co-integration: An application to the Sterling and Deutschmark exchange markets," *Journal of International Money and Finance*, 8, 75–88.
- Hall, P. (1982). "On some simple estimates of an exponent of regular variation," *Journal of the Royal Statistical Society, Series B*, 44, 37–42.
- Hall, P. and A. H. Welsh (1985). "Adaptive estimates of parameters of regular variation," *Annals of Statistics*, 13, 331–341.
- Hannan, E. J. and J. Rissanen (1982). "Recursive estimation of ARMA order," *Biometrika*, 69, 273–280 [Corrigenda, *Biometrika*, 1983, 70].
- Hansen, L. P. and H. J. Hodrick (1980). "Forward exchange rates as optimal predictors of future spot rates: An econometric analysis," *Journal of Political Economy*, 88, 829–853.
- Hill, B. M. (1975). "A simple general approach to inference about the tail of a distribution," *Annals of Statistics*, 3, 1163–1174.
- Hols, M. C. A. B. and C. G. DeVries (1991). "The limiting distribution of extremal exchange rate returns," *Journal of Applied Econometrics*, 6, 287–302.
- Hsieh, D. (1989). "Modeling heteroscedasticity in daily foreign exchange rates," *Journal of Business and Economic Statistics*, 7, 307–317.
- Jansen, D. W. and C. G. DeVries (1991). "On the frequency of large stock returns: Putting booms and busts into perspective," *Review of Economics and Statistics*, 73, 18–24.
- Kanter, M. and W. L. Steiger (1974). "Regression and autoregression with infinite variance," *Advances in Applied Probability*, 6, 768–783.
- Kavalieris, L. (1991). "A note on estimating autoregressive-moving average order," *Biometrika*, 78, 920–922.
- Kearns, P. and A. R. Pagan (1992). "Estimating the density tail index for financial time series," Australian National University, mimeographed.
- Kearns, P. and A. Pagan (1993). "Estimating the density tail index for financial time series," Australian National University, mimeographed.
- Knight, K. (1989). "Limit theory for autoregressive parameters in an infinite variance random walk," *Canadian Journal of Statistics*, 17, 261–278.
- Knight, K. (1991). "Limit theory for  $M$ -estimates in an integrated infinite variance process," *Econometric Theory*, 7, 200–212.



- Koedijk, K. G. and C. J. M. Kool (1992). "Tail estimates of East European exchange rates," *Journal of Business and Economic Statistics*, 10, 83-96.
- Koedijk, K. G., M. M. A. Schafgans and C. G. deVries (1990). "The tail index of exchange rate returns," *Journal of International Economics*, 29, 93-108.
- Lee, Chin and P. C. B. Phillips (1993). "An improved estimator of the long-run variance with an application to unit root tests," in preparation, Yale University.
- Levine, R. (1989). "The pricing of forward exchange rates," *Journal of International Money and Finance*, 8, 163-179.
- Loretan, M. and P. C. B. Phillips (1993). "Testing covariance stationarity of heavy tailed time series: An overview of the theory with applications to several financial datasets," *Journal of Empirical Finance* (forthcoming).
- McFarland, J. W., P. C. McMahon and Y. Ngama (1992). "Forward exchange rates and expectations during the 1920's: A reexamination of the evidence," mimeographed, Tulane University.
- Moore, M. J. (1992). "Some new tests of efficiency in the forward exchange market," mimeographed, Queens University of Belfast.
- Ouliaris, S. and P. C. B. Phillips (1994). "COINT 2.0: GAUSS procedures for cointegrated systems," *Predicta Software Inc.*, Madison, CT.
- Park, J. Y. (1992). "Canonical cointegrating regressions," *Econometrica*, 60, 119-143.
- Phillips, P. C. B. (1990). "Time series regression with a unit root and infinite variance errors," *Econometric Theory*, 6, 44-62.
- Phillips, P. C. B. (1991). "A shortcut to LAD estimator asymptotics," *Econometric Theory*, 7, 450-463.
- Phillips, P. C. B. (1993). "Robust nonstationary regressions" Cowles Foundation Discussion Paper No. 1064, Yale University.
- Phillips, P. C. B. (1994). "Fully modified least squares and vector autoregression," Cowles Foundation Discussion Paper No. 1047, Yale University.
- Phillips, P. C. B. and B. E. Hansen (1990). "Statistical inference in instrumental variables regression with I(1) processes," *Review of Economic Studies*, 57, 99-125.
- Phillips, P. C. B. and S. Ouliaris (1990), "Asymptotic properties of residual based tests for cointegration," *Econometrica*, 58, 165-194.
- Phillips, P. C. B. and J. Y. Park (1988). "Asymptotic equivalence of ordinary least squares and generalized least squares in regressions with integrated regressors," *Journal of the American Economic Association*, 83, 111-115.
- Phillips, P. C. B. and W. Ploberger (1991). "Time series modeling with a Bayesian frame of reference: Concepts, illustrations and asymptotics," Cowles Foundation Discussion Paper No. 980.

- Phillips, P. C. B. and W. Ploberger (1994). "Posterior odds testing for a unit root with data-based model selection," *Econometric Theory* (forthcoming).
- Schwarz, G. (1978). "Estimating the dimension of a model," *Annals of Statistics*, 6, 461–464.
- Silverman, B. W. (1986). *Density Estimation for Statistics and Data Analysis*. London: Chapman and Hall.

Figure 1: Belgium FR & SR: 1922–1925

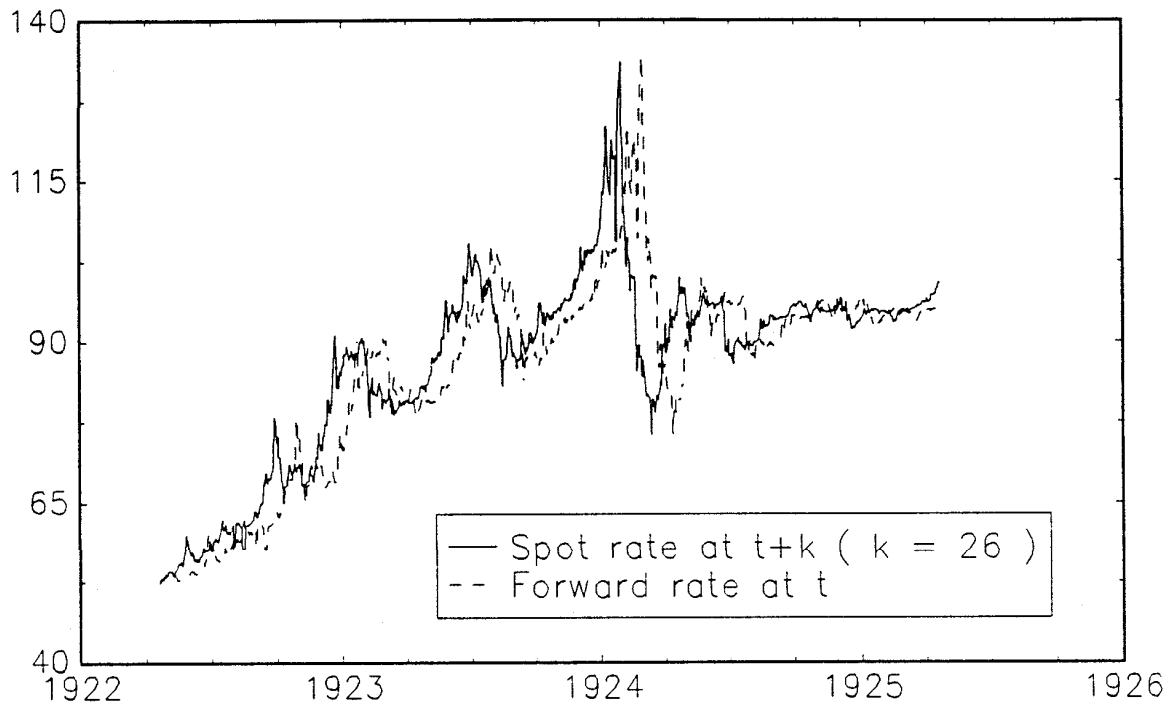


Figure 2: France FR & SR: 1922–1925

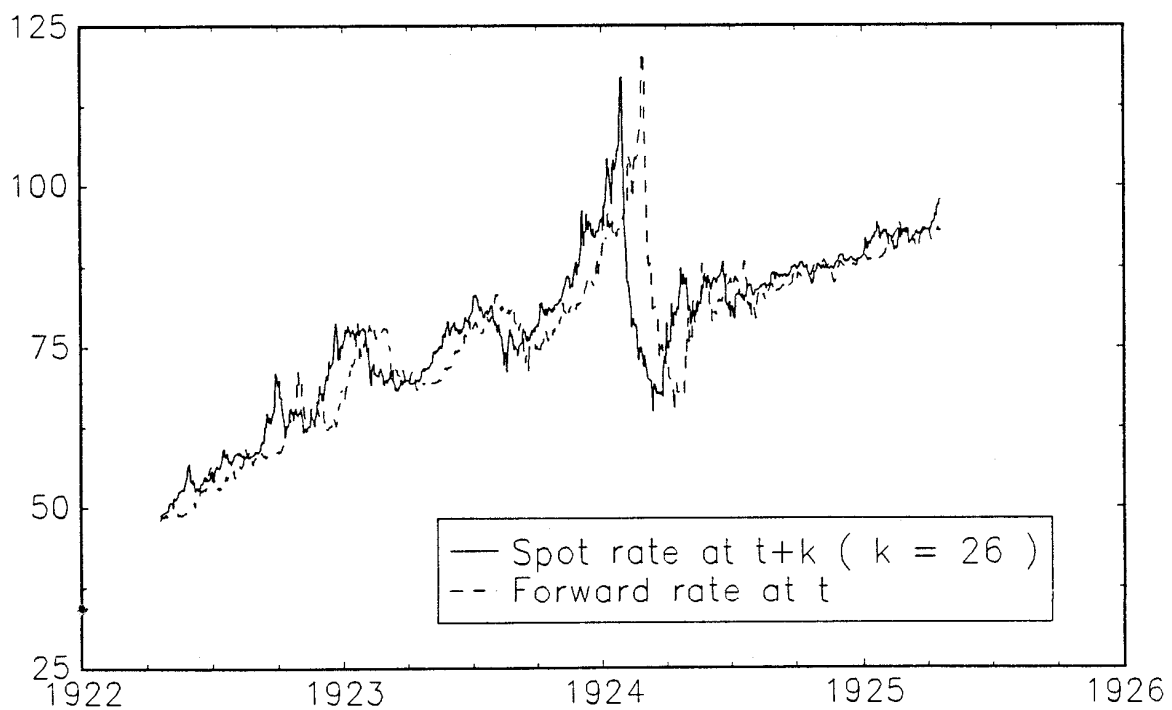


Figure 3: Italy FR & SR: 1922–1925

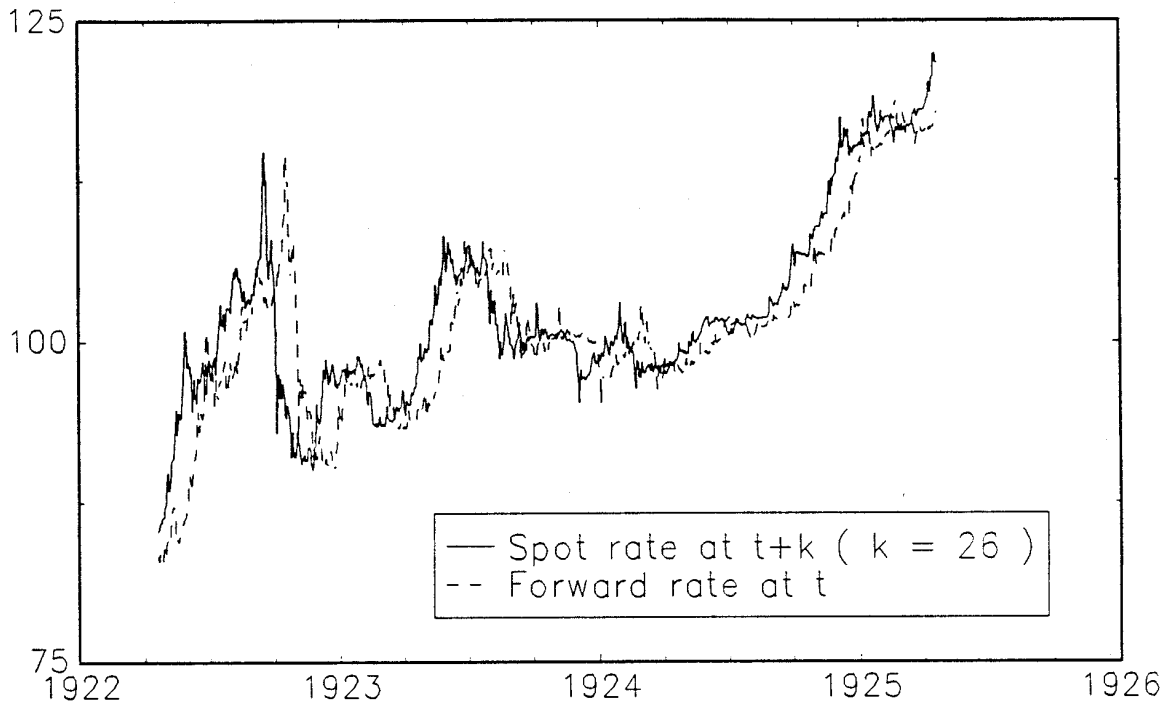


Figure 4: USA FR & SR: 1922–1925

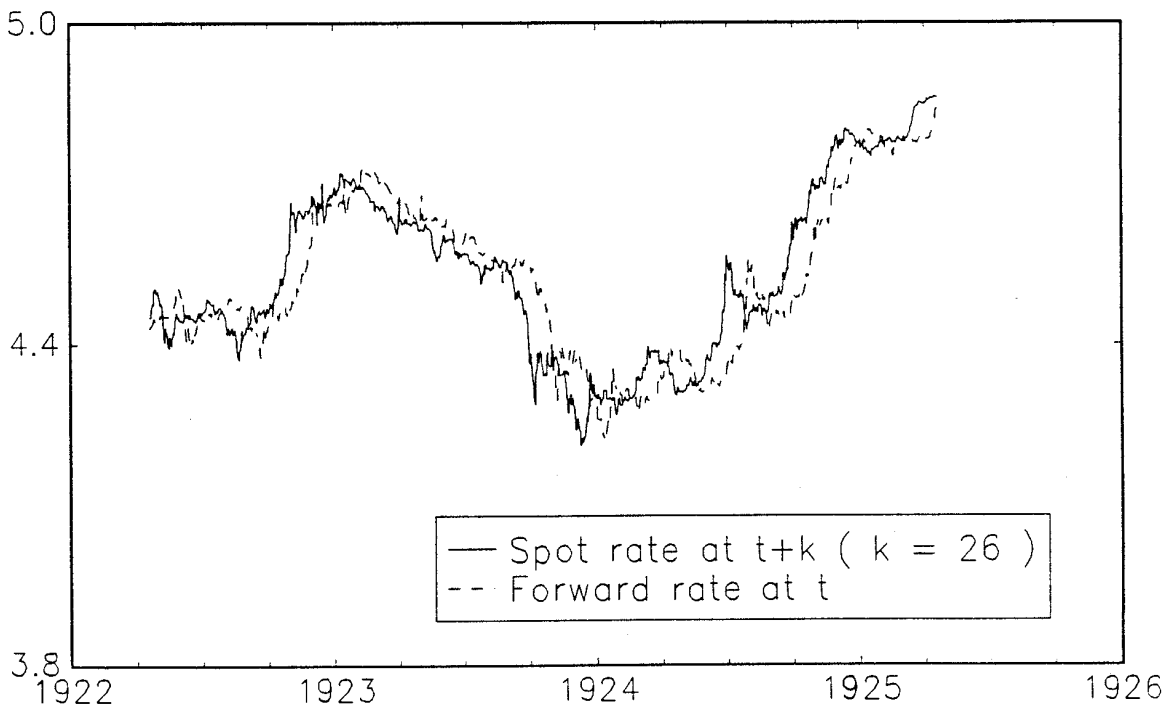


Figure 5: Empirical cdf  
Belgium Forward Exchange rate

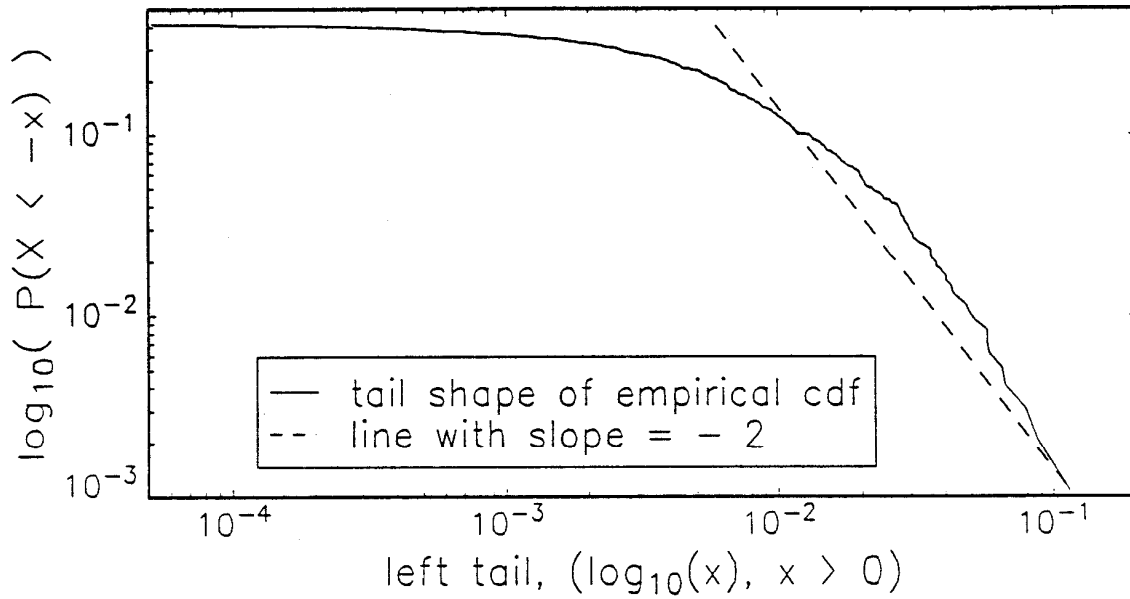


Figure 6: Empirical cdf  
France Forward Exchange rate

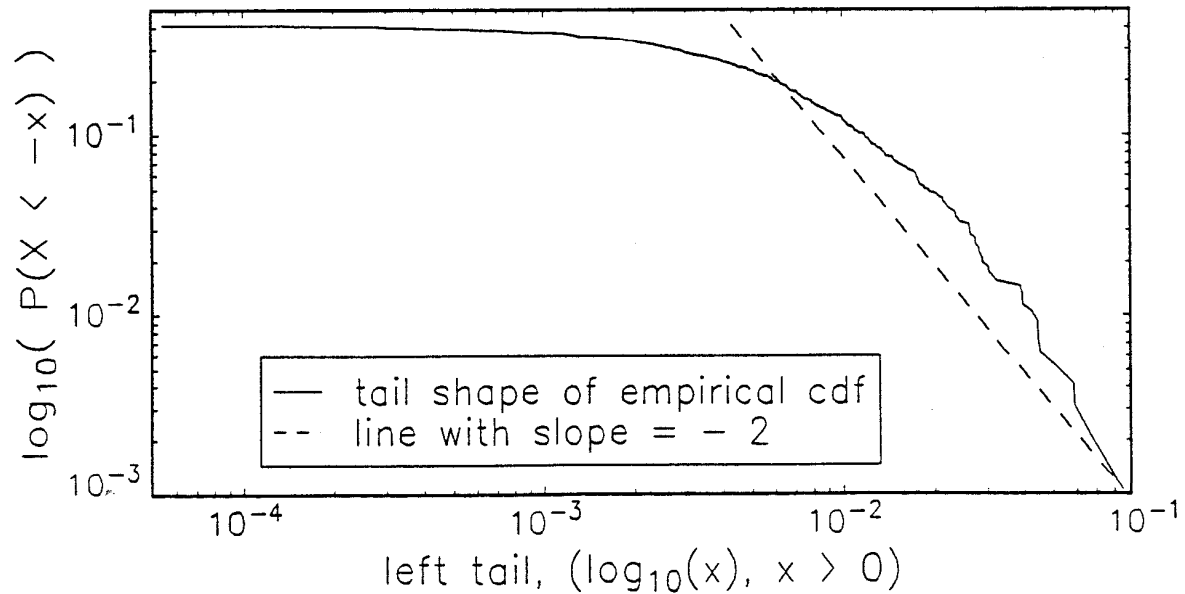


Figure 7: Empirical cdf  
Italy Forward Exchange rate

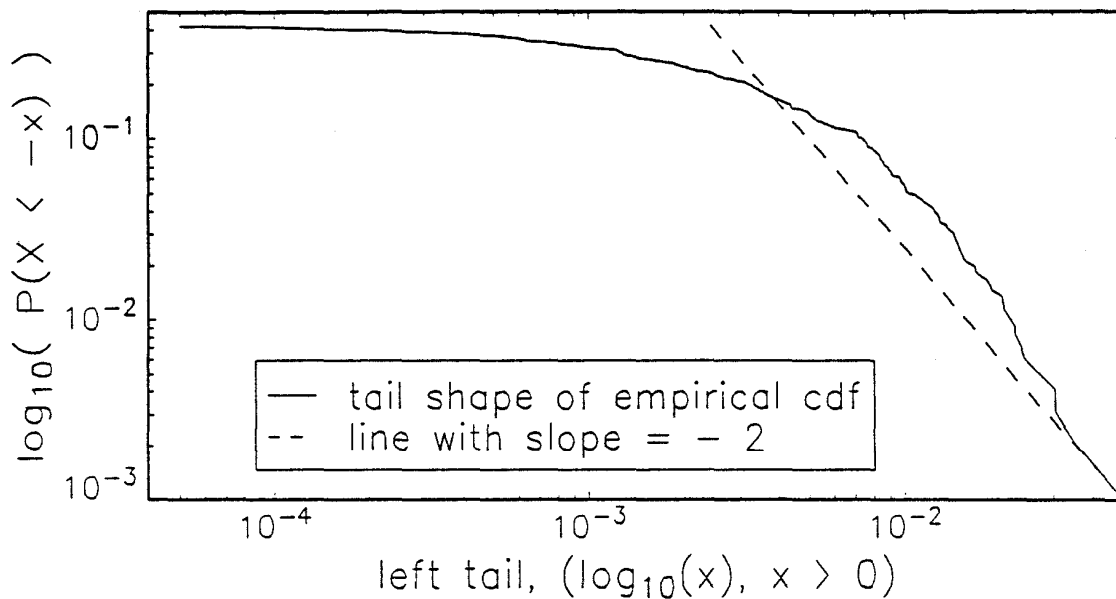


Figure 8: Empirical cdf  
USA Forward Exchange rate

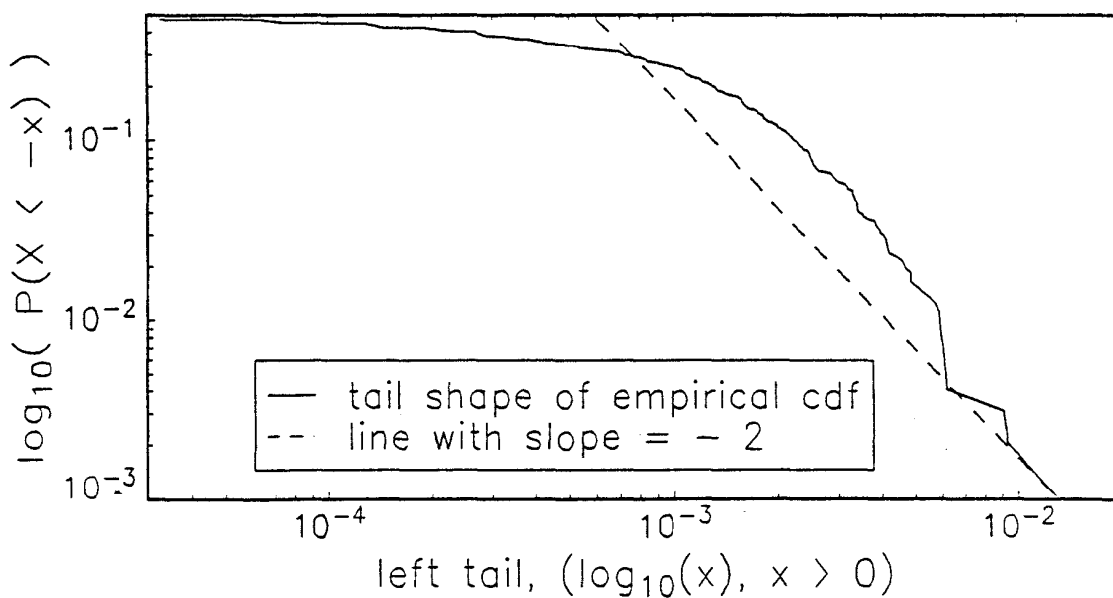


Figure 9: Belgium – Recursive plot of residual based  $Z_a$  Cointegration Test

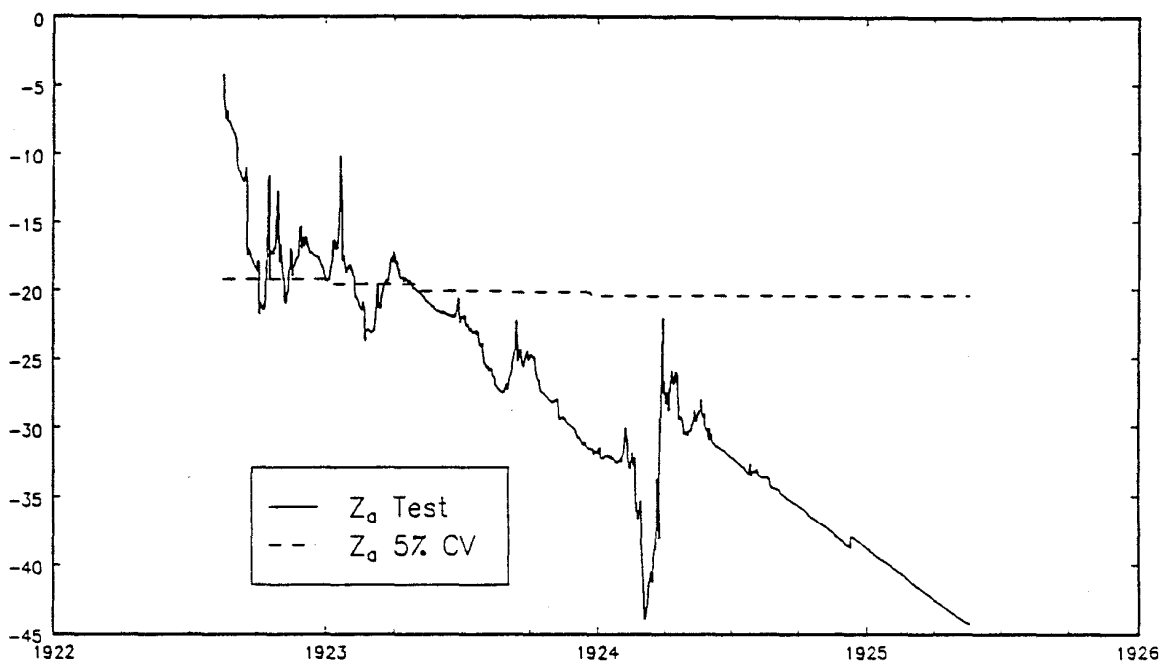


Figure 10: France – Recursive plot of residual based  $Z_a$  Cointegration Test

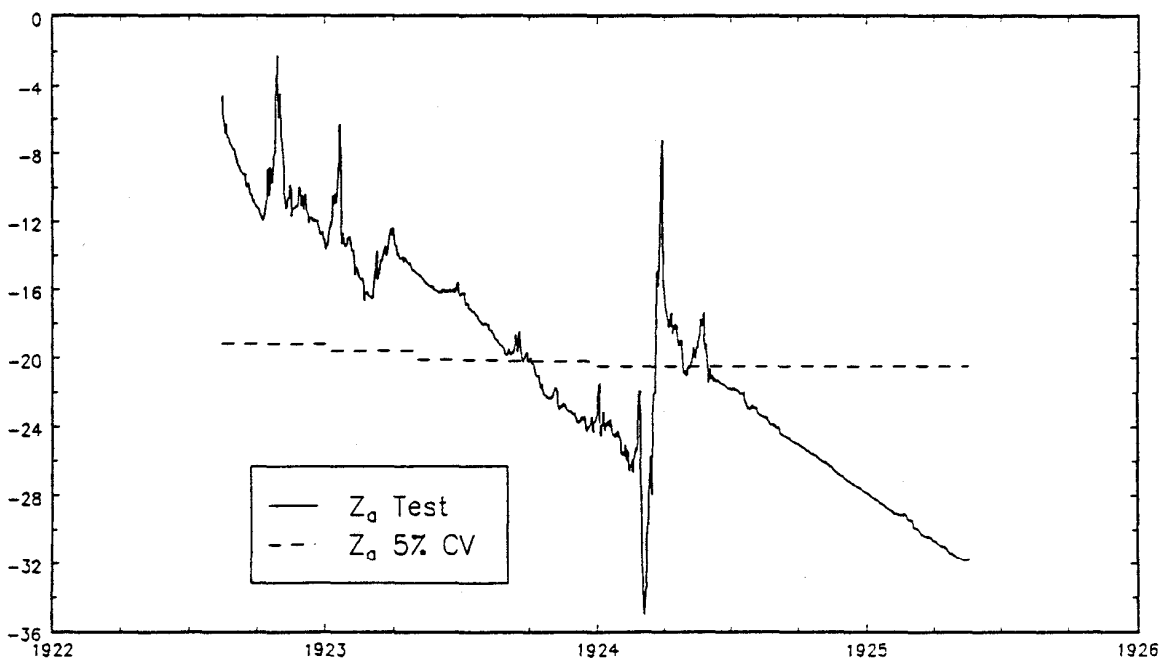


Figure 11: Italy – Recursive plot of residual based  $Z_a$  Cointegration Test

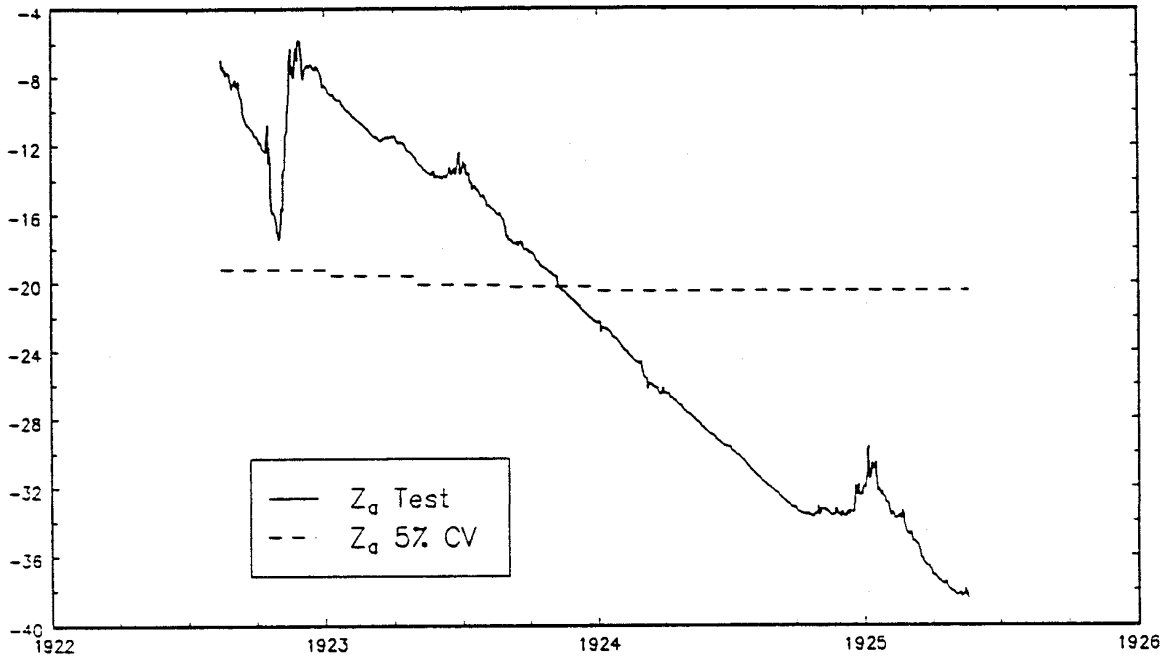


Figure 12: USA – Recursive plot of residual based  $Z_a$  Cointegration Test

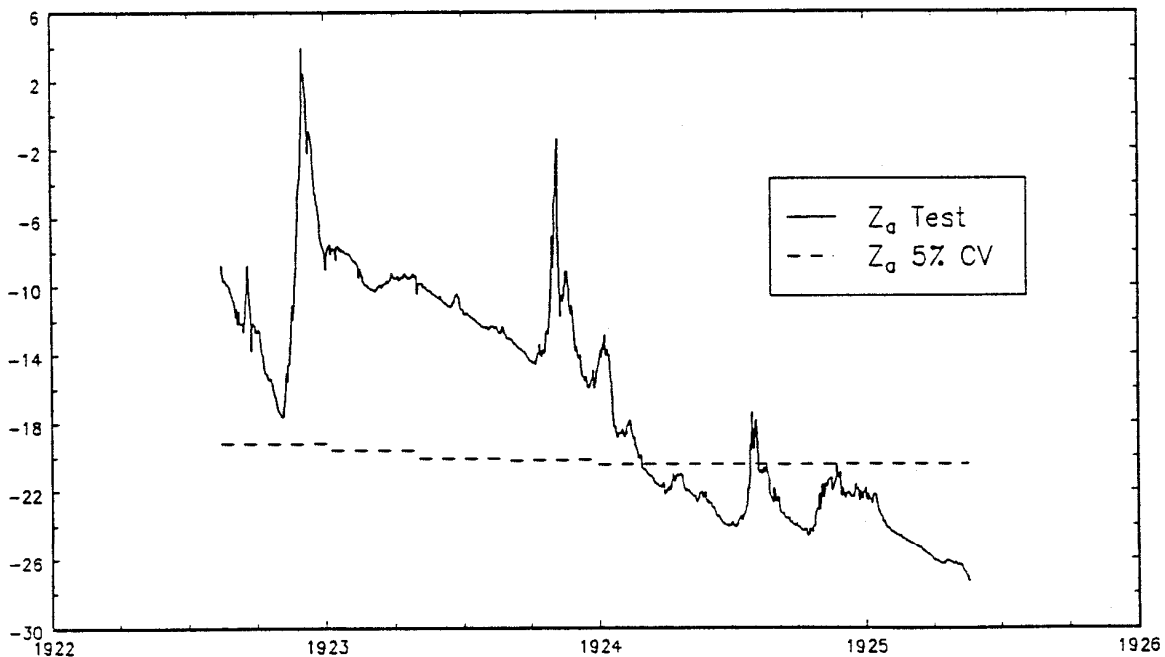




Figure 13a: Belgium – density of  $u_0(t)$

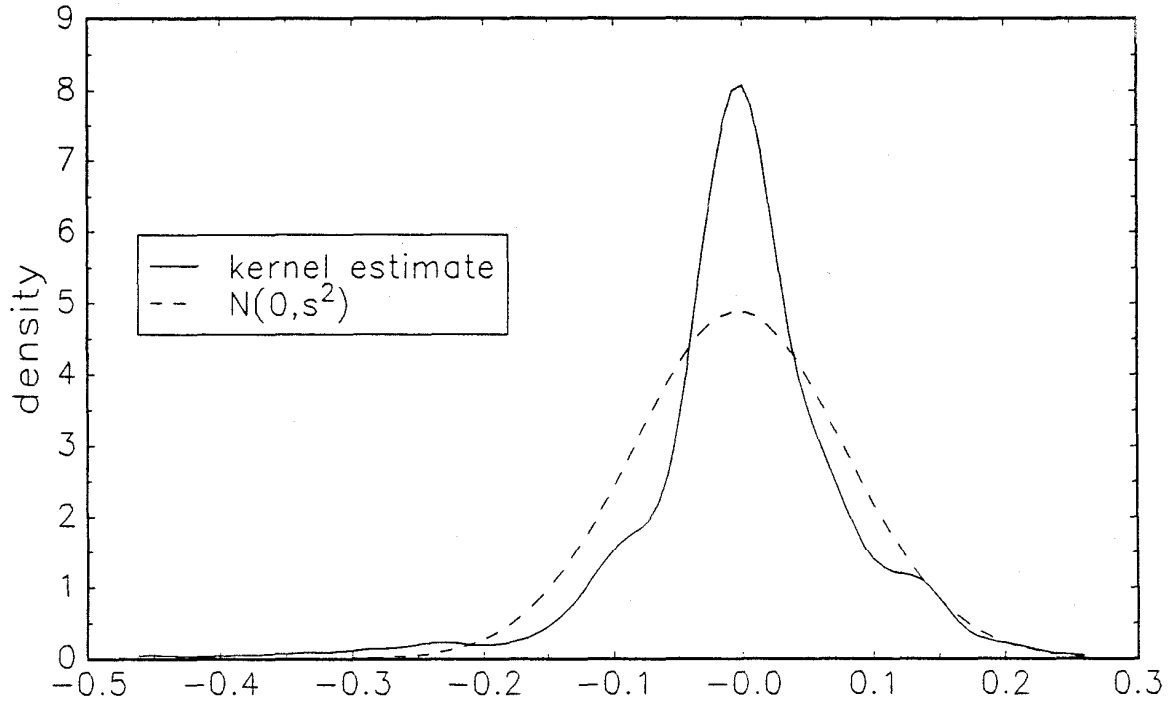


Figure 13b: Belgium – density of  $u_x(t)$

