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PONZI FINANCE, GOVERNMENT SOLVENCY AND
THE REDUNDANCY OR USEFULNESS OF PUBLIC DEBT

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Abstract.

We study how the government's ability to borrow depends on its capacity to tax. Using a two-period OLG growth model, we establish the following.

When lump-sum taxes are unrestricted, Ponzi finance is possible, regardless of whether the economy is dynamically inefficient and regardless of the relationship between the interest rate and the growth rate. Ponzi finance, and government debt generally, is *unessential* or redundant: it does not enlarge the set of allocations that can be supported as competitive equilibria.

When lump-sum taxes are restricted, Ponzi finance (public debt) may be *essential*.

Central to the paper is our characterization of feasible government fiscal-financial plans for an *infinite-lived* government facing a sequence of *finite-lived* overlapping private generations. The central idea is that the government does not bankrupt private agents. We contrast our criterion with the conventional government solvency constraint, . The conventional solvency constraint (the present value of future government debt is non-positive in the infinitely distant future) is neither necessary nor sufficient for our feasibility criterion. When the government must use distortionary taxes and the long-run interest rate exceeds the long-run growth rate, our feasibility criterion implies the conventional solvency constraint.

(I) INTRODUCTION.

When is Ponzi finance[1] feasible for an infinite-lived government? When does the opportunity to engage in Ponzi finance enhance the government's ability to influence private resource allocation? The traditional answer to the first of these two questions is that, in deterministic competitive perfect foresight OLG models, Ponzi finance is feasible only if the economy is dynamically inefficient (see e.g. Gale [1983], Tirole [1985], O'Connell and Zeldes [1988], Blanchard and Weil [1992] and Azariades [1993]). Under uncertainty, Blanchard and Weil [1992] argue that Ponzi schemes are feasible only if the competitive equilibrium allocation is not Pareto-efficient (because it fails to provide full intergenerational insurance), even if it is dynamically efficient. In answer to the second question, the existing literature points out that when the competitive equilibrium is Pareto-inefficient, Ponzi finance can be Pareto improving.[2]

We show that Ponzi finance may be feasible whether or not the competitive equilibrium is dynamically efficient or Pareto-efficient, and regardless of the relationship between the interest rate and the growth rate. The two key building blocks of our argument are the government's "capacity to tax", that is, the richness of the set of available lump-sum tax-transfer instruments and the specification of the feasibility constraints on the government's fiscal-financial plans. We prove that Ponzi finance is *always* feasible if the set of lump-sum taxes and transfers the government can choose from is unrestricted in how they vary across generations, time and states of nature. In this case Ponzi finance is, however, *unessential*; it does not enhance the set of allocations that can be supported as competitive equilibria.

Ponzi finance may also be feasible when lump-sum taxes and transfers are

restricted. In the presence of such restrictions, Ponzi finance is *essential*; the ability to engage in Ponzi finance allows the government to support competitive equilibrium allocations that cannot be attained without it. This includes the ability to achieve Pareto improvements in the special cases already noted in the literature.

Our proof of the feasibility of Ponzi finance depends on the characterization of feasible government fiscal–financial plans for an infinite–lived government[3] in an infinite–lived economy with overlapping generations of finite–lived households without private intergenerational gift motives. In our model, feasibility of the government's fiscal–financial plan is expressed as a set of three inequality constraints on admissible sequences of taxes, transfers, public debt and exhaustive public (consumption) spending. These are derived from the requirement that the private capital stock, private consumption by each generation and government consumptions be non–negative in each period. In other words, feasibility for the plans of the infinite–lived government is derived from the (well–understood) requirement of solvency (or non–bankruptcy) for each of an infinite sequence of finite–lived households.

Our feasibility constraints on government fiscal–financial plans are generally less restrictive than the ubiquitous conventional solvency constraint of an infinite–lived government, that the discounted value of government debt should be non–positive in the limit[4]. A central motivation for our paper was to *derive* the conventional solvency constraint from acceptable primitives. We find reasonable sufficient conditions for our feasibility constraints to imply that the conventional solvency constraint holds. Consider equilibria in which the long–run real interest rate is above the long–run growth rate of efficiency labor. The first sufficient condition is that the maximal long–run growth rate of transfers to the young and of taxes on the old is

less than the long–run interest rate. If, for instance, the long–run growth rate of taxes and transfers cannot exceed that of efficiency labor, or equivalently, if the ratio of taxes paid (transfers received) during a period by a generation to the value of the physical resources owned by it, is bounded, this condition would be met. Another example of a sufficient condition is that the net transfer payments by the government to a generation cannot change sign over the life cycle of that generation.

The essential–unessential distinction we make for Ponzi finance is extended to public debt as such. Whether or not public debt (or Ponzi finance) is essential depends on the restrictions imposed on the government's ability to use lump–sum taxes and transfers.

We discuss three benchmark cases of restrictions on the set of fiscal instruments available to the government and state three equivalence results. The first is the well–known proposition (see e.g. Wallace [1981], Chamley and Polemarchakis [1984] and Sargent [1987, Chapter 8]) that with unrestricted time–, age– and state–specific lump–sum taxes and transfers, the ability to depart from budget balance does not permit additional equilibria to be supported. Specifically, any intergenerational redistribution and insurance that can be provided with government borrowing or lending can also be provided with a balanced government budget.

While lump–sum taxes and transfers can and do vary to some extent both across the different generations alive at any given moment and over the life–cycle of each individual household, it is reasonable to recognize that they are constrained in the extent to which they can vary. To highlight the importance of limits on the government's capacity to tax, we choose to study two extreme cases of restrictions. One restricts differential taxation of generations alive at the same time. The other restricts variation in the tax

burden over the life cycle of each individual generation.

We prove a second equivalence result stating that, if the government is constrained to treat all overlapping generations the same during any given period, the ability to unbalance the budget permits it to support all equilibria that can be supported with completely unrestricted lumps-sum taxes and transfers. This second proposition will not hold if the conventional government solvency constraint is imposed rather than our less restrictive feasibility constraint. Essential Ponzi finance therefore characterizes these "generation-independent" tax-transfer policies.

We also establish a third equivalence proposition stating that even rather severe restrictions on the ability to vary taxes and transfers over the lifetime of a generation do not restrict the set of equilibria that can be supported, provided unbalanced budgets are permitted. This result holds even when the conventional solvency constraint is imposed and does therefore not require Ponzi finance.

Alternative government financing policies not only effect redistribution among generations (giving rise to familiar "financial crowding out" issues), in a stochastic environment they will also permit trades across states of nature or intergenerational insurance. There is by now quite a rich literature on this subject. Such intergenerational redistribution schemes as social security taxes and retirement benefits can provide insurance that either cannot be provided by the market or is provided inefficiently[5]. OLG models have incomplete market participation. Because individual households cannot enter into insurance contracts before they are born, there may be incomplete risk-sharing (Blanchard and Weil [1992]). Even in a dynamically efficient economy, the public provision of this insurance can have implications for Pareto-efficiency (see Zilcha [1990] and Blanchard and Weil [1992]).[6] Our model allows lump-sum taxes and

transfers and public sector debt to be used to make improvements in allocative efficiency by providing missing intergenerational insurance opportunities. Rather than investigating the many interesting positive and welfare aspects of intergenerational redistribution and of the provision of intergenerational insurance through the government budget (see e.g. Fischer [1983], Enders and Lapan [1982], Stiglitz [1983], Merton [1984], Gordon and Varian [1988], Pagano [1988] and Gale [1990]), we focus on the scope for and role of Ponzi finance, on the appropriateness of the conventional government solvency constraint and on the equivalence results.

The outline of the rest of this paper is as follows. Section II develops the model. Section III concerns the feasibility of Ponzi finance. Conditions under which the feasibility constraints on government fiscal-financial plans imply the conventional solvency constraint are stated in Section IV. Section V concerns the usefulness of Ponzi finance and presents the equivalence results. Section VI concludes.

(II) THE MODEL.

Consider the closed economy, one-good two-period OLG growth model of Diamond [1965], with government borrowing or lending and lump-sum taxes or transfers.

The private sector.

Individuals of the same generation are identical. Successive generations have the same utility functions and maximize expected utility. People live for two periods, work in the first period of life and retire in the second. There is no intergenerational gift or bequest motive. Labor supply is inelastic and scaled to unity for each young worker. The young have access to two stores of

value, claims on risky real capital and potentially risky public debt.[7]

Population and labor force size in natural units are both denoted by L. It grows at the constant proportional rate $n > -1$, and L_0 is set equal to 1. There is labor–augmenting productivity growth, and the level of productivity is denoted by θ. Labor in efficiency units is therefore equal to θL. The growth rate of efficiency labor will also be referred to as the natural rate of growth in what follows.

The optimization problem of a competitive representative consumer born in period t is given in equations (II.2) and (II.3a,b and c). c_t^i and τ_t^i, $i = 1, 2$, are consumption, respectively taxes paid, by a member of generation t in the ith period of her life. w_t is the wage rate in period t. k_{t+1}^d and b_{t+1}^d are the amounts of capital, respectively bonds or securities (measured for notational convenience in units of efficiency labor for generation $t+1$), demanded by a member of generation t at the end of period t. For simplicity, all securities are assumed to have a one–period maturity. p_t is the price, in terms of period t output, of a security that entitles one to a gross payment (coupon) of γ_{t+1} units of output in period $t+1$, with $\infty > \gamma_{t+1} > 0$. This payment may be stochastic in one of two ways. It can be state–contingent, where the states of nature are 'fundamental' or it can represent randomization of coupon payments by the issuer of the bond. The one–period interest rate on debt carried into period $t+1$, r_{t+1} is defined as by

(II.1) $$1 + r_{t+1} \equiv \gamma_{t+1}/p_t$$

In what follows we use the interest rate notation, because of its familiarity. ρ_{t+1} is the rental rate of a unit of capital in period $t+1$. E_t is the expectation operator conditional on information held at the beginning of period t.

(II.2) $$\max_{c_t^1, k_{t+1}^d, b_{t+1}^d, c_t^2} v(c_t^1) + \beta E_t v(c_t^2)$$ [8]

subject to the sequence of budget constraints given in *(II.3a,b)*.

(II.3a) $w_t - \tau_t^1 - c_t^1 \geq (k_{t+1}^d + p_t b_{t+1}^d)(1 + n)\theta_{t+1}$

(II.3b) $c_t^2 + \tau_t^2 \leq [(1 + \rho_{t+1})k_{t+1}^d + (1 + r_{t+1})p_t b_{t+1}^d](1+n)\theta_{t+1}$ [9] [10]

(II.3c) $c_t^1, \ c_t^2, \ k_{t+1}^d \geq 0$

Since utility is strictly increasing in c^1 and c^2, *(II.3a,b)* will hold with equality.

The interior first–order conditions[11] for a member of generation t are

(II.4a) $v'(c_t^1) = \beta E_t[(1 + \rho_{t+1})v'(c_t^2)]$

(II.4b) $v'(c_t^1) = \dfrac{\beta}{p_t} E_t[(1 + r_{t+1})p_t v'(c_t^2)]$

Output Y is produced by a twice continuously differentiable production function with constant returns to capital K and labor in efficiency units θL and positive and diminishing marginal products: $\dfrac{Y_t}{\theta_t L_t} \equiv y_t = f(\dfrac{K_t}{\theta_t L_t}) = f(k_t)$, $f(0) = 0; \ f' > 0; \ f'' < 0$. It also satisfies the Inada conditions.

Labor augmenting productivity can be random and is assumed to have positive support; for finite t, θ_t is also assumed to be bounded from above. Our equivalence results do not depend on this particular parameterization of uncertainty. The growth rate of labor–augmenting productivity ω_t is defined by $1 + \omega_t \equiv \theta_t/\theta_{t-1}$.

The public sector.

The government imposes lump–sum taxes (transfers when negative) on the young and/or the old, spends a non–negative amount on public consumption[12] and satisfies its single-period budget identity by borrowing or lending. B_t is the stock of government bonds at the beginning of period t and G_t the amount of exhaustive public spending. The single-period government budget identity

is
$$p_t B_{t+1} \equiv (1 + r_t)p_{t-1}B_t + G_t - \tau_t^1 L_t - \tau_{t-1}^2 L_{t-1} \; .$$

With $b_t \equiv B_t/(\theta_t L_t)$, and $g_t \equiv G_t/(\theta_t L_t)$, the single–period government budget identity can be rewritten as

$$(II.5) \qquad p_t b_{t+1}(1+n)(1+w_{t+1}) \equiv (1+r_t)p_{t-1}b_t + g_t - \theta_t^{-1}[\tau_t^1 + \tau_{t-1}^2(1+n)^{-1}]$$

The lump–sum taxes levied (transfers paid) by the government and the coupon payment on the public debt can be stochastic. We assume for simplicity that the government does not introduce additional noise into the system (G_t is non–stochastic), but that taxes and debt coupon payments can be made contingent on current and past realizations of the random variable θ. Let Θ_t be the sequence of current and past realizations of θ, that is $\Theta_t \equiv \{\theta_{t-i}; \; i \geq 0\}$. For simplicity we define the following notation:

$$(II.6a) \qquad \tau_t^1 = \tau_t^1(\Theta_t)$$

$$(II.6b) \qquad \tau_t^2 = \tau_t^2(\Theta_{t+1})$$

$$(II.6c) \qquad \gamma_t = \gamma_t(\Theta_t)$$

Factor and asset market equilibrium.

The labor market and capital rental market are competitive and clear, so

$$(II.7) \qquad w_t = \theta_t[f(k_t) - k_t f'(k_t)]$$

$$(II.8) \qquad \rho_{t+1} = f'(k_{t+1}) \; .$$

The economy–wide asset market equilibrium conditions are given by

$$B_{t+1} = L_{t+1}b_{t+1}^d$$

$$K_{t+1} = L_{t+1}k_{t+1}^d$$

Substituting the asset market equilibrium conditions into equation (II.3a) (assumed to hold with equality) yields:

$$(II.9) \qquad (p_t b_{t+1} + k_{t+1})(1 + n)(1+w_{t+1}) = (w_t - \tau_t^1 - c_t^1)\theta_t^{-1} \; .$$

Feasible fiscal–financial plans and government solvency.

Solving the government single–period budget identity forward in time to $T > t$, we get for all $t \geq 0$

$$(II.10) \qquad \delta_{t-1} p_{t-1} b_t \equiv \sum_{s=t}^{T-1} (\theta_{s+1})^{-1}\left[\frac{\tau_s^1}{1+n} + \left[\frac{1}{1+n}\right]^2 \tau_{s-1}^2 - \frac{\theta_s g_s}{1+n}\right]\delta_s$$

$$+ \delta_{T-1} p_{T-1} b_T$$

$$(II.11) \qquad \delta_s \equiv \prod_{j=0}^{s}\left[\frac{(1+n)(1+w_{j+1})}{1 + r_j}\right] \quad for \ s \geq 0 \ .$$

$$\equiv 1 \qquad\qquad for \ s = -1$$

We also define the market discount factor Δ as follows:

$$\Delta_s \equiv \prod_{j=0}^{s}\left[\frac{1}{1 + r_j}\right] = \delta_s/(\theta_{s+1} L_{s+1}).$$

Note that δ is the "labor–force–growth–and–productivity–growth–adjusted" discount factor. Both Δ_s and δ_s are assumed to be positive for finite values of s and non–negative in the limit as $s \to \infty$.

The conventional government solvency constraint, given in $(II.12)$ requires the discounted public debt to vanish in the long run for any realization of the discounted debt sequence.

$$(II.12) \qquad \lim_{T\to\infty} \delta_{T-1} p_{T-1} b_T \equiv \lim_{T\to\infty} \Delta_{T-1} p_{T-1} B_T = 0. \qquad [13]$$

Equations (II.12) implies (II.13)

$$(II.13) \qquad \delta_{t-1} p_{t-1} b_t = \lim_{T\to\infty} \sum_{s=t}^{T-1} (\theta_{s+1})^{-1}\left[\frac{\tau_s^1}{1+n} + \left[\frac{1}{1+n}\right]^2 \tau_{s-1}^2 - \frac{\theta_s g_s}{1+n}\right]\delta_s \qquad [14]$$

The solvency condition $(II.12)$ has the prima facie attractive property of implying the same kind of present value or intertemporal budget constraint $(II.13)$ for the infinite horizon case as for the finite horizon case. If $T-1$ is the finite terminal period, then the standard (and uncontroversial)

government solvency constraint is $p_{T-1}b_T \leq 0$ (the government does not owe anything at the end of the last period). A rational household sector ensures that $p_{T-1}b_T \geq 0$. From (II.10) these two weak inequalities imply, that the value of the current stock of debt is equal to the (expectation of) the present discounted value of future primary (non-interest) surpluses. This is the same as (II.13), with $\lim_{T\to\infty}$ dropped. The imposition of (II.12) has been virtually automatic in modern macroeconomic analysis. For a small sample see Barro [1979], Buiter [1985], Pagano [1988], Blanchard et. al. [1990], Auerbach and Kotlikoff [1987] and Auerbach, Gokhale and Kotlikoff [1991]. It has been the subject of extensive empirical testing (see e.g. Hamilton and Flavin [1986], Wilcox [1989], Corsetti [1990], Grilli [1990], Trehan and Walsh [1991] and Buiter and Patel [1992 and 1994]), with mixed results.

We believe that the analogy with the finite-horizon case is potentially misleading. It is by no means obvious what are, in an economy without a terminal date, the feasible debt strategies of an infinite-lived government facing an infinite sequence of finite-lived overlapping generations (see e.g. Shell [1971] and Wilson [1981]). As we shall see, without a-priori restrictions on taxes and transfers, our model has a surprising range of feasible debt strategies, many of which allow for Ponzi finance. This contrasts with the case of an economy with a finite number of infinite-lived consumers, where the conventional government solvency constraint is implied by household rationality (see McCallum [1984], Cass [1972], O'Connell and Zeldes [1988] and Bohn [1991]).

Formally, Ponzi finance is defined as follows for our model:

Definition 1: Ponzi finance.

The government engages in Ponzi finance if

(II.14) $p_t B_{t+1} \geq (1 + r_t) p_{t-1} B_t$ *for all* $t \geq 0$. [15]

The government engages in Ponzi finance if, in each period, t, the value of the debt carried into the next period, $t+1$, is at least as large as the cost of servicing the debt carried into period t. From the government's single-period budget identity it follows, that a government engages in Ponzi finance if $G_t - (\tau_t^1 L_t + \tau_{t-1}^2 L_{t-1}) \geq 0$ for all t, that is, if it never runs a primary (non-interest) budget surplus.

In Section V we are also interested in sequences of new debt minus old debt service, $\{p_t B_{t+1} - (1 + r_t) p_{t-1} B_t\}_{t=0}^{\infty}$ that, while not themselves Ponzi schemes, possess infinite subsequences $\{p_{t_j} B_{t_j+1} - (1+r_{t_j}) p_{t_j-1} B_{t_j}\}_{t_j=0}^{\infty}$ that are Ponzi schemes.

We proceed by investigating what kind of constraints the model of Section II imposes on the government's ability to issue debt. Equation *(II.9)*, stating that the savings of the young in period t equal the sum of the capital stock and the value of the stock of government debt carried into period $t+1$, can be rearranged as equation *(II.15)*

(II.15) $p_t B_{t+1} + \tau_t^1 L_t = - K_{t+1} + [w_t - c_t^1] L_t$

Equation *(II.3b)* (holding with equality), stating that the old consume all their after-tax resources, can be arranged as equation *(II.16)*

(II.16) $(1 + r_{t+1}) p_t B_{t+1} - \tau_t^2 L_{t-1} = -(1+\rho_{t+1}) K_{t+1} + c_t^2 L_t$

It is immediately obvious from *(II.15)* that, *for given* K_{t+1}, w_t, c_t^1 *and* L_t, the value of the public debt issued in period t, $p_t B_{t+1}$ can be made arbitrarily large (positive or negative) by making matching large (positive or

negative) period t transfers to the young, $-\tau_t^1 L_t$. Such an arbitrarily large (positive or negative) value of $p_t B_{t+1}$ is consistent with equation *(II.16) for given* τ_{t+1}, K_{t+1}, ρ_{t+1}, c_t^2 *and* L_t, as long as period $t+1$ taxes on the old, $\tau_t^2 L_t$ are assigned a matching large (positive or negative) value.

Since c_t^1 and K_{t+1} are non–negative, the constraint on public debt implied by *(II.15)* is $p_t B_{t+1} + \tau_t^1 L_t \leq w_t L_t$. There also is a lower bound on the amount of public debt that can be issued (or an upper bound on the stock of public credit to the private sector). It follows from non–negativity of consumption by the old in period t. From the resource constraint $K_{t+1} - K_t = w_t L_t + \rho_t K_t - c_t^1 L_t - c_{t-1}^2 L_{t-1} - G_t$ and $c_{t-1}^2 \geq 0$ it follows that $(w_t - c_t^1)L_t - G_t + (1 + \rho_t)K_t - K_{t+1} \geq 0$. From *(II.15)* this implies $p_t B_{t+1} + \tau_t^1 L_t \geq G_t - (1+\rho_t)K_t$.

These upper and lower bounds on the public debt in each period, together with the requirement that exhaustive public spending cannot be negative and cannot exceed the total physical resources available in any period, constitute our characterization of feasible fiscal–financial plans, that is a program of taxes, transfers, borrowing and exhaustive public spending that is feasible in a world with rational private agents[16].

Definition 2: Feasible fiscal–financial plans.

A government fiscal–financial plan is feasible if and only if its debt, taxes, transfer payments and exhaustive spending satisfy, for all $t \geq 0$, the single–period government budget identity given in (II.7) and

(II.17a) $\qquad\qquad p_t B_{t+1} + \tau_t^1 L_t \leq w_t L_t$

(II.17b) $\qquad\qquad p_t B_{t+1} + \tau_t^1 L_t \geq G_t - (1+\rho_t)K_t$

Equations (III.17a and b) plus non–negativity of G_t imply:

(II.17c) $\qquad\qquad 0 \leq G_t \leq w_t L_t + (1+\rho_t)K_t$

Note that this definition of feasibility of fiscal-financial plans can be generalized easily to all OLG models with finite household horizons. It relies only on the reasonable postulate that in the last period of its life, each household disposes of all real and financial assets (including public debt), pays off any debts it has carried into that period and does not purchase any new assets or incur any new debt.

Equations *(II.17a,b)* can be rewritten as

(II.17a') $\qquad\qquad (1 + r_t)p_{t-1}B_t - \tau_{t-1}^2 L_{t-1} \le w_t L_t - G_t$

and

(II.17b') $\qquad\qquad (1 + r_t)p_{t-1}B_t - \tau_{t-1}^2 L_{t-1} \ge -(1+\rho_t)K_t$

Since the government feasibility constraints *(II.17a,b)* are derived from the requirement that c_t^1, c_{t-1}^2 and $K_t \ge 0$ for all $t \ge 0$, another way of interpreting them is that the government refrains from policies that will bankrupt the private sector: it does not select sequences for taxes, transfer payments, debt and exhaustive spending that will cause the non-negativity constraints on consumption by both generations and on the private capital stock to become binding [17] [18].

The feasibility conditions *(II.17a and b')* can be rewritten as

(II.18a) $\qquad p_t B_{t+1} + \tau_t^1 L_t \le [f(k_t) - k_t f'(k_t)]\theta_t(1+n)^t$

(II.18b') $\qquad \tau_{t-1}^2 L_{t-1} - (1 + r_t)p_{t-1}B_t \le [1+f'(k_t)]k_t\theta_t(1+n)^t$

Equation *(III.18a)* implies that the long-run growth rate of the total resource transfer from the young generation to the government (whether through (voluntary) purchases by the young of government debt or through (involuntary) taxes on the young) cannot exceed the long-run growth rate of efficiency labor. Note that there is no constraint on $p_t B_{t+1}$ or $\tau_t^1 L_t$ separately, only on their sum.

Equation *(II.18b')* implies that the long-run growth rate of the total resource transfer from the old generation to the government (whether through explicit taxes on the old or through the servicing of debt to the government incurred by the old) cannot exceed the long-run growth rate of efficiency labor. Note that there is no constraint on $\tau^2_{t-1}L_{t-1}$ or $-(1 + r_t)p_{t-1}B_t$ separately, only on their sum.

If the long-run interest rate exceeds the long-run growth rate of efficiency labor $(\lim_{t\to\infty} \Delta_t\theta_t(1+n)^t = 0)$ then the feasibility constraints *(II.17a and b)* imply

$$\lim_{\substack{t' \to \infty \\ t' \leq t \leq \infty}} \inf \{\Delta_t[p_tB_{t+1} + \tau^1_tL_t]\} \leq 0$$

$$\lim_{\substack{t' \to \infty \\ t' \leq t \leq \infty}} \sup \{\Delta_t[p_tB_{t+1} + \tau^1_tL_t]\} \geq 0,$$

If the limit inferior and the limit superior are the same, we get

(II.19)
$$\lim_{t\to\infty} \Delta_t[p_tB_{t+1} + \tau^1_tL_t] = \lim_{t\to\infty} [\tau^2_{t-1}L_{t-1} - (1 + r_t)p_{t-1}B_t] = 0$$

Note how this differs from the conventional solvency constraint $\lim_{t\to\infty} \Delta_t p_tB_{t+1} = 0$. Equation *(II.19)* states that the present discounted value of the total resource transfer from the young to the government and the present discounted value of the total resource transfer from the old to the government should converge to zero. Without further restrictions on τ^1_t and τ^2_{t-1}, equation *(II.19)* does not constrain the behavior of the public debt or the public credit in the long run.

(III) THE FEASIBILITY OF PONZI FINANCE WITH UNRESTRICTED TAXES AND TRANSFERS.

In this Section we show that with unrestricted lump-sum taxes and transfers, Ponzi finance is both always feasible and *unessential*, that is it does not enlarge the set of allocations that can be supported as competitive

equilibria.

As a simple illustration of the kind of borrowing policies that are feasible with unrestricted taxes and transfers, consider the deterministic version of our model with a logarithmic utility function, $v(c_t^1) + \beta v(c_t^2) = (1-\eta)\ln c_t^1 + \eta \ln c_t^2$, $0 \leq \eta \leq 1$. The consumer's equilibrium in this case is given by

$$(III.1) \qquad c_t^1 = (1-\eta)(w_t - \tau_t^1 - \frac{\tau_t^2}{1+r_{t+1}})$$

$$(III.2) \qquad c_t^2 = (1+r_{t+1})\eta(w_t - \tau_t^1 - \frac{\tau_t^2}{1+r_{t+1}})$$

$$(III.3) \qquad p_t B_{t+1} + K_{t+1} = (w_t - c_t^1 - \tau_t^1)L_t$$

$$= [\eta(w_t - \tau_t^1) + (1-\eta)\frac{\tau_t^2}{1+r_{t+1}}]L_t$$

Consumption when young and old, c_t^1 and c_t^2, are functions only of the present discounted value of life–time resources, $w_t - \tau_t^1 - \frac{\tau_t^2}{1+r_{t+1}}$. Saving by the young (their aggregate demand for government bonds and real capital), however, is, for well–known life–cycle reasons, not a function of the present discounted value of life–time resources alone. We can rewrite *(III.3)* as follows:

$$p_t B_{t+1} + K_{t+1} = [\eta(w_t - \tau_t^1 - \frac{\tau_t^2}{1+r_{t+1}}) + \frac{\tau_t^2}{1+r_{t+1}}]L_t$$

The young of period t will demand more financial assets, cet. par., if they expect to have to pay a larger tax τ_t^2 when they are old, regardless of the present discounted value of their life–time resources. The demand for saving by the young depends on the actual distribution of disposable (after–tax) resources over the lifetime. They will save more while young if the distribution of lifetime disposable resources is skewed towards youth. If

the government has the ability to tax the members of any given generation differently when they are young than when they are old, it can influence the savings behavior of the young and with it the demand for its debt. By raising $[-\eta\tau_t^1 + (1-\eta)\frac{\tau_t^2}{1+r_{t+1}}]L_t$ while keeping $\tau_t^1 + \frac{\tau_t^2}{1+r_{t+1}}$ and G_t constant, the government can raise saving by the young by any amount without affecting consumption by the young or the old (or the demand for capital as a productive input). The government can therefore increase its debt without bound. Consider two equilibria, the single-star equilibrium and the double-star equilibrium. Assume that *(III.4)* holds for all $t \geq 0$, and that the initial capital stock K_0 and the sequence of exhaustive public spending in the two equilibria are identical.

$$(III.4) \qquad \tau_t^{1**} + \frac{\tau_t^{2**}}{1+r_{t+1}^{**}} = \tau_t^{1*} + \frac{\tau_t^{2*}}{1+r_{t+1}^{*}}$$

For concreteness, let the single-star equilibrium have a balanced budget in each period and zero public debt, that is

$$\tau_t^{1*}L_t + \tau_{t-1}^{2*}L_{t-1} = G_t \text{ and } B_t^* = 0 \text{ for all } t \geq 0$$

We define τ_t^{1**} and τ_{t-1}^{2**} as follows:

$$\tau_t^{1**} = \tau_t^{1*} - \epsilon_t \text{ and } \tau_{t-1}^{2**} = \tau_{t-1}^{2*} + (1+r_t)\epsilon_{t-1}$$

It follows that

$$(III.5) \qquad p_t B_{t+1}^{**} - (1+r_t)p_{t-1}B_t^{**} \equiv G_t - [\tau_t^{1**}L_t + \tau_{t-1}^{2**}L_{t-1}]$$

$$= G_t - [(\tau_t^{1*} - \epsilon_t)L_t + (\tau_{t-1}^{2*} + (1+r_t)\epsilon_{t-1})L_{t-1}]$$

$$= [(1+n)\epsilon_t - (1+r_t)\epsilon_{t-1}]L_{t-1} .[19]$$

Thus, by choosing appropriately growing values for ϵ_t, $t \geq 0$, (that is values such that $\epsilon_t/\epsilon_{t-1} > (1+n)^{-1}(1+r_t)$), we can raise the growth rate of public debt in any period to any level. Since the present discounted value of lifetime taxes is the same in the single star and the double star equilibrium,

the equilibrium private consumption sequences are the same, and so will be the wage rate, capital stock, interest rate and debt price sequences. By making a larger transfer to the young of generation t, the government provides the young with the means for increasing their saving. By levying a larger tax on that same generation when old, the government provides the young with an incentive to save in order to pay these higher taxes.

Since $\tau_t^{1**}L_t = \tau_t^{1*}L_t - \epsilon_t L_t$ and $\tau_{t-1}^{2**}L_{t-1} = -\tau_t^{*1}L_t + (1+r_t)\epsilon_{t-1}L_{t-1} + G_t$, we note that when a Ponzi game is played $(\epsilon_t/\epsilon_{t-1} > (1+n)^{-1}(1+r_t))$, the total tax on the young, $\tau_t^{1**}L_t$, will ultimately become negative and increasingly large in absolute value, while the tax on the old, $\tau_{t-1}^{2**}L_{t-1}$, will become an increasingly large positive number. We will therefore see the lifetime pattern of taxes becoming one of ever increasing receipts of transfer payments when young and ever increasing tax payments when old. The lifetime pattern of taxes therefore has to change sign or zig-zag. It is obvious that this property generalizes to any finite household horizon OLG model.

Another way of interpreting this is that the debt can grow without bound (and at a rate higher than the interest rate) without affecting the equilibrium allocations for consumption and the capital stock, because the government can, effectively, tax the debt held by the old to pay for the servicing of the debt held by the old. Government debt held by the old increases the "base" on which lump-sum taxes on the old can be levied. [20]

McCallum [1984], made this point in the context of an infinite-lived representative agent model [21](see also Bohn [1991]). Spaventa [1987, 1988] also emphasizes the distinction between models in which only endowments can be taxed (such as Pagano [1988]) and models in which interest income too is taxable. He, however, does not make the distinction between taxes on the young, taxes on the old and aggregate taxes. As we shall see below, aggregate

taxes can be zero, and therefore less than the endowment (wage income) and less that the sum of the endowment and interest income, while the debt grows at a rate at least equal to the rate of interest forever.

The foregoing discussion suggests the following Proposition for the general case when there are no restrictions, other than those implied by the fiscal-financial feasibility constraint *(II.17a and b)*, on lump-sum taxes and transfers.

Proposition 1.

With unrestricted age-, time- and state-contingent taxes and transfers, any equilibrium for consumption by the young and the old and for the capital stock, can be supported with an infinity of Ponzi schemes.

Proof: We assert that, if there exists an equilibrium (the single star equilibrium, say) c_t^{1*}, c_{t-1}^{2*}, w_t^*, k_t^*, r_t^*, p_t^*, τ_t^{1*}, τ_t^{2*}, b_t^* for $t \geq 0$, for a given feasible sequence of exhaustive public spending, g_t, $t \geq 0$, then there also exist, for the same sequence of exhaustive public spending, (infinitely many) equilibria (the double star equilibria) c_t^{1**}, c_{t-1}^{2**}, w_t^{**}, k_t^{**}, r_t^{**}, p_t^{**}, τ_t^{1**}, τ_t^{2**}, b_t^{**} for $t \geq 0$ such that $c_t^{1**} = c_t^{1*}$, $c_{t-1}^{2**} = c_{t-1}^{2*}$, $w_t^{**} = w_t^*$, $k_t^{**} = k_t^*$, $r_t^{**} = r_t^*$, $p_t^{**} = p_t^*$ and $p_t^{**}b_{t+1}^* - p_{t-1}^{**}b_t^*(1+n)^{-1}(1+\omega_{t+1})^{-1} \geq r_t^{**}p_{t-1}^{**}b_t^*(1+n)^{-1}(1+\omega_{t+1})^{-1}$ for all $t \geq 0$

Given k_0 and $(1+r_0)p_{-1}b_0$, a competitive equilibrium of our two-period OLG model satisfies equations *(III.6)* to *(III.10)* for all $t \geq 0$.

(III.6) $\qquad v'(c_t^1) = \beta E_t[(1 + f'(k_{t+1}))v'(c_t^2)]$

(III.7) $\qquad v'(c_t^1) = \beta E_t[(1 + r_{t+1})v'(c_t^2)]$

(III.8) $\qquad c_t^2 + \tau_t^2 = \theta_{t+1}(1+n)[(1+f'(k_{t+1}))k_{t+1} + (1+r_{t+1})p_t b_{t+1}]$

$(III.9) \qquad (\theta_t[f(k_t) - k_t f'(k_t)] - \tau_t^1 - c_t^1)\theta_t^{-1}$

$$= (k_{t+1} + p_t b_{t+1})(1+n)(1+\omega_{t+1})$$

$(III.10) \quad p_t b_{t+1}(1+n)(1+\omega_{t+1}) \equiv (1+r_t)p_{t-1}b_t + g_t - \theta_t^{-1}[\tau_t^1 + (1+n)^{-1}\tau_{t-1}^2]$

For the double star taxes and debt to support the same consumption and capital stock equilibrium, it is necessary and sufficient that

$(III.11a) \quad (1+n)(1+\omega_{t+1})^{-1}p_t^* b_{t+1}^* + \theta_t^{-1}\tau_t^{1*} = (1+n)(1+\omega_{t+1})^{-1}p_t^* b_{t+1}^{**} + \theta_t^{-1}\tau_t^{1**}$

or

$(III.11b) \quad (1+r_t)p_{t-1}b_t^* - (1+n)^{-1}\theta_t^{-1}\tau_{t-1}^{2*} = (1+r_t)p_{t-1}b_t^{**} - (1+n)^{-1}\theta_t^{-1}\tau_{t-1}^{2**}$

for all $t \geq 0$.

For the debt to grow at least as fast as the rate of interest forever, it must be true that for all $t \geq 0$

$(III.12) \quad (1+n)(1+\omega_{t+1})p_t^* b_{t+1}^{**} - (1+r_{t+1})p_{t-1}^* b_t^{**} = g_t - \theta_t^{-1}[\tau_t^{1**} + \tau_{t-1}^{2**}(1+n)^{-1}] \geq 0$

The two choice variables during period t in equation $(III.12)$ are τ_t^{1**} and τ_{t-1}^{2**}. No matter what value is assigned to τ_{t-1}^{2**}, τ_t^{1**} can always be assigned a large enough negative value to ensure that $(III.12)$ is satisfied: the debt grows at least as fast as the rate of interest.

From the single–period government budget identity, it follows that $(III.11a)$ and $(III.11b)$ are the same constraint. No matter what value is assigned to τ_t^{1**}, a value can be assigned to τ_{t-1}^{2**} that ensures that $(III.11a,b)$ are satisfied for any values of τ_t^{1*}, τ_{t-1}^{2*}, b_t^*, b_{t+1}^*, b_t^{**} and b_{t+1}^{**}.□

Another way of putting this is that, by increasing $-\tau_t^{1**}L_t$ for any given values of $\tau_{t-1}^{2**}L_{t-1}$ and G_t and for any inherited value of $(1+r_t)p_{t-1}B_t^{**}$, it is possible to raise the growth rate of the public debt to any positive level without affecting $p_t B_{t+1}^{**} + \tau_t^{1**}L_t$, the term on the left–hand side of the fiscal–financial feasibility constraints $(II.17a$ and $b)$. $\tau_{t-1}^{2**}L_t$ can then be chosen to ensure that $(1+r_t)p_{t-1}B_t^{**} - \tau_{t-1}^{2**}L_{t-1}$, the term on the left–hand side of $(II.17a'$ and $b')$ satisfies these inequalities. The government simply

reshuffles a constant total resource transfer away from the young in period t, $p_t B_{t+1}^{**} + \tau_t^{1**} L_t$, between borrowed resources, $p_t B_{t+1}^{**}$, and explicit taxes, $\tau_t^{1**} L_t$. Appropriating for its own use an amount of resources equal to the value of exhaustive public spending, G_t, it pays out the remainder to the old, either as debt service, $(1+r_t)p_{t-1}B_t^{**}$ or as transfers $-\tau_{t-1}^{2**} L_t$. Again it is only the total, $(1+r_t)p_{t-1}B_t^{**} - \tau_{t-1}^{2**} L_t$ that matters for the consumption of the old.

The Corollaries given below follow immediately from Proposition 1.

Corollary 1.

With unrestricted taxes and transfers, the competitive equilibrium model with the finite–lived OLG household sector does not require any bounds on the level or rate of growth of public debt. Ponzi finance is therefore always possible, regardless of the relationship between the interest rate and the growth rate, regardless of whether the economy is dynamically efficient and regardless of whether the economy is Pareto–efficient.

Corollary 2.

With unrestricted taxes and transfers, Ponzi finance is "unessential", that is, it does not permit additional equilibria to be supported.

(IV) RESTRICTIONS ON TAXES AND TRANSFERS AND THE CONVENTIONAL GOVERNMENT SOLVENCY CONSTRAINT.

Without restrictions on the government's ability to freely choose lump–sum taxes and transfers, the fiscal–financial feasibility constraints *(II.17a,b)* do not imply the conventional government solvency constraint given by *(II.12)* or *(II.13)* and allow Ponzi finance. In this section we consider

restrictions on the government's set of available lump–sum taxes and transfers that imply that the conventional government solvency constraint applies. These restrictions rule out Ponzi finance. Other restrictions, including restrictions on the extent to which taxes and transfers can vary across generations alive at the same time, are shown in the next section to allow (essential) Ponzi finance.

The first restriction we consider is that not all taxes and transfers are assumed to be lump–sum.

Distortionary taxes.

If the taxes paid or transfers received by a generation are distortionary, it is unlikely that the long–run growth rate of per capita taxes or transfers can exceed the long–run growth rate of productivity. There will be some finite upper bound on the ratio of taxes and transfers per generation per period to the before–tax resources owned by that generation that period. Tax administration and collection costs that are strictly convex increasing functions of the amount of taxes raised will also put a finite upper bound on the ratio of taxes paid to real resources owned (see Barro [1976], McCallum [1984], Kremers [1989] and Bohn [1991]).

In our model, the simplest interesting example involving distortionary taxes is the following. Assume that the young continue to be taxed in lump–sum fashion (τ_t^1 remains lump–sum for all t). The old in period $t+1$ are taxed through a proportional tax at rate a_{t+1} on the gross return to their savings (their gross resources in old age), that is,

$$(IV.1) \qquad \tau_t^2 L_t = a_{t+1}[(1 + \rho_{t+1})K_{t+1} + (1 + r_{t+1})p_t B_{t+1}]$$

This proportional tax (at a rate $a_{t+1}/(1-a_{t+1})$) on consumption by the old is interesting because, by including (gross) debt income, $(1+r_{t+1})p_t B_{t+1}$,

in the tax base, we do not prejudge the issue of whether taxes can grow faster than GDP forever, as we would if only the gross income from physical capital, $(1 + \rho_{t+1})K_{t+1}$, were taxed. We can safely restrict the analysis to the case where before-tax resources of the old in period $t+1$ are non-negative and $0 \leq a_t \leq 1$. When $a_t = 1$, $c_t^2 = 0$ and members of generation t don't save anything, so $K_{t+1} = 0$ and the economy collapses.

For notational simplicity we ignore uncertainty in what follows, so, with $\rho_{t+1} = r_{t+1} = f'(k_{t+1})$, the first-order condition for the household becomes

$$(IV.2) \qquad v'(c_t^1) = (1-a_{t+1})(1+f'(k_{t+1}))\beta v'(c_t^2)$$

Substituting (IV.1) and (IV.2) into the life-time budget constraint of a member of generation t (assumed to hold with strict equality) yields

$$(IV.3) \qquad c_t^1 + \frac{c_t^2 \beta v'(c_t^2)}{v'(c_t^1)} = c_t^1 + c_t^2 D_{t+1} = w_t - \tau_t^1$$

where $D_{t+1} \equiv [(1-a_{t+1})(1+r_{t+1})]^{-1}$ is the (after-tax) market discount factor.

Note that (IV.3) implies that, if τ_t^1 is positive (positive taxes are levied on the young), it cannot grow faster, in the long run, than w_t and therefore no faster than the natural rate of growth of the economy. With c_t^1, c_t^2 and v' non-negative, (IV.3) implies $\tau_t^1 \leq w_t$. Ponzi finance with growing public credit and long-run subsidies to saving ($a_{t+1} < 0$) is infeasible. Ponzi finance with ever-growing transfers to the young (negative values of τ_t^1) and the tax rate a approaching 1 is also infeasible, if the long-run interest rate exceeds the long-run growth rate of efficiency labor.

Consider the case where τ_t^1 is negative and growing in absolute value. We rewrite (IV.3) in growth rates as follows:

$$(IV.4) \qquad a_t^1 \left[\frac{c_{t+1}^1 - c_t^1}{c_t^1}\right] + (1-a_t^1)\left[\frac{c_{t+1}^2 - c_t^2}{c_t^2}\right] + (1-a_t^1)\frac{c_{t+1}^2}{c_t^2}\left[\frac{R_{t+1}-R_t}{R_{t+1}}\right]$$

$$= \left[\frac{w_t}{w_t-\tau_t^1}\right]\left[\frac{w_{t+1}-w_t}{w_t}\right] + \left[\frac{-\tau_t^1}{w_t-\tau_t^1}\right]\left[\frac{-\tau_{t+1}^1-(-\tau_t^1)}{-\tau_t^1}\right]$$

where $a_t^1 \equiv \dfrac{c_t^1}{w_t-\tau_t^1}$ is the share of expenditure on c_t^1. It is clear from $(IV.3'')$

that if the growth rate of $-\tau_t^1$, per capita transfers to the young, exceeds that for wages and for D_t in the long run, then consumption by either the young or the old at each date grows faster than output, violating the feasibility constraints. On the other hand, if D_t grows at least as fast as $-\tau_t^1$ (which in turn grows faster than the wage rate and output per capita), then consumption by the young must be at least asymptotically approaching the growth rate of $-\tau_t^1$ given that $v'(c) > 0$ for all $c \geq 0$. This is true because the relative price of c_t^1 in terms of c_t^2 approaches zero while the present value of resources grows with $-\tau_t^1$. It holds regardless of whether a_t^1 converges to zero. Therefore, Ponzi finance is infeasible when the long-run interest rate exceeds the long-run natural rate of growth in this deterministic example.[22][23].

No "sign reversal" in the net tax burden over the life cycle.

The second restriction we consider is that net taxes in each of the two periods of a household's life must have the same sign:

$$(IV.5) \qquad \tau_t^2 = \lambda_t \tau_t^1 \qquad \lambda_t \geq 0 \text{ for all } t \geq 0.$$

While this restriction may seem somewhat artificial, it covers many of the tax-transfer patterns actually found in the literature. These include (a) taxes on the young only (that is, $\lambda = 0$) and (b) common taxes on young and old (that is, $\lambda = 1$). We need just one of the weak inequalities of the

fiscal-financial feasibility constraint in order to show that, under restriction *(IV.5)*, Ponzi finance is possible only if the long-run interest rate is below the long-run growth rate of efficiency labor. Consider equations *(II.17a)* and *(II.17a')*, rewritten for this case as

(IV.6a) $\quad p_t B_{t+1} \leq [f(k_t) - k_t f'(k_t)]\theta_t L_t - \tau_t^1 L_t$

(IV.6b) $\quad (1+r_{t+1})p_t B_{t+1} \leq [f(k_{t+1}) - k_{t+1}f'(k_{t+1})]\theta_{t+1}L_{t+1} + \lambda_t \tau_t^1 L_t - G_{t+1}$

From *(IV.6a)*, the only way for the debt to grow faster than the growth rate of efficiency labor forever, is for τ_t^1 to be negative and for $-\tau_t^1$ to grow at a rate higher than the growth rate of labor productivity. If the debt grows faster than the growth rate of efficiency labor forever, *(IV.6b)* can only be satisfied if $\lambda_t \tau_t^1 L_t$ is positive and has a growth rate higher than the growth rate of efficiency labor. That is impossible since $\lambda_t \geq 0$. We conclude that $-\tau_t^1$ can grow no faster than the growth rate of labor productivity and that the growth rate of the debt can therefore be no higher than the growth rate of efficiency labor. The debt can therefore grow faster than the interest rate forever only if the long-run interest rate is below the long-run growth rate of efficiency labor.

Note that *(IV.5)* covers quite a variety of fiscal rules, including per capita taxes (or transfers) that are constant across generations at a point in time, and growing over time at a constant proportional rate ν $(\tau_t^1 = \tau_{t-1}^2 = \tau_t = \tau_0(1+\nu)^t$ for all $t \geq 0)$. Note that our argument implies that ν cannot be permanently higher that the growth rate of labor productivity.

Blanchard and Weil [1992] considered the special case of the model with *(IV.5)* where the labor force is constant $(n = 0)$, there is no productivity growth $(\theta_t = \theta$ for all $t \geq 0)$, there is no exhaustive public spending $(G_t = 0$ for all $t \geq 0)$ and there are no taxes or transfers $(\tau_t = 0$ for all $t \geq 0)$. They consider whether, starting from a zero initial public debt, a (small)

increase in the stock of public debt can be rolled over forever. In their model, debt obviously cannot grow faster than wage income in the long run. In the deterministic version of their model, this means that only in a dynamically inefficient equilibrium can there be viable Ponzi schemes, with the public debt growing forever at least as fast as the interest rate but no faster than the growth rate of labor income. [24]

Even if net taxes can change sign over the life cycle, if the long-run rate of interest exceeds the long-run growth rate of the *disposable* income of the young, that is, if $\lim_{t \to \infty} \{\Delta_t[\theta_t(f(k_t)-k_tf'(k_t))-\tau_t^1](1+n)^t\} = 0$, equations

(II.17a, or a') imply

(IV.7) $$\lim_{\substack{t' \to \infty \\ t' \leq t < \infty}} \inf \{\Delta_t p_t B_{t+1}\} \leq 0$$

From the public credit constraint *(II.17b)* or *(II.17b')* it follows that, if the long-run rate of interest exceeds the rate of growth of the *disposable* income of the old (capital income minus taxes on the old), that is, if $\lim_{t \to \infty} \{\Delta_t[(1+f'(k_t))k_t\theta_t-(1+n)^{-1}\tau_{t-1}^2](1+n)^t\} = 0$, we have

(IV.8) $$\lim_{\substack{t' \to \infty \\ t' \leq t < \infty}} \sup \{\Delta_t p_t B_{t+1}\} \geq 0.$$

If $\lim_{\substack{t' \to \infty \\ t' \leq t < \infty}} \inf \{\Delta_t p_t B_{t+1}\} = \lim_{\substack{t' \to \infty \\ t' \leq t < \infty}} \sup \{\Delta_t p_t B_{t+1}\} = 0$, then we also have $\lim_{t \to \infty} \Delta_t p_t B_{t+1} = 0$. This means that when the long-run rate of interest exceeds the long-run growth rate of the *disposable* income of the young and of the old, the conventional solvency constraint applies.

We summarize this discussion in the following Proposition:

Proposition 2.

The conventional government solvency constraint $(\lim\limits_{t \to \infty} \Delta_t p_t B_{t+1} = 0)$[25] is implied

by the fiscal–financial feasibility constraint (II.17a,b and c) if

(a) The long–run interest rate exceeds the long–run growth rate of efficiency labor $(\lim\limits_{t \to \infty} \Delta_t \theta_t (1+n)^t = 0)$

and

(b) Either, the net tax paid by any generation at a given age cannot change sign over the lifetime of that generation, or the long–run growth rate of taxes paid or transfers received at a given age by a generation is less than the long–run natural rate of growth. Distortionary taxes are sufficient to prevent the long–run growth rate of taxes and transfers from exceeding the long–run natural rate of growth.

In order for the public debt to grow at least as fast as the rate of interest forever, when the long-run rate of interest is above the long-run growth rate of efficiency labor, it must be possible to make transfer payments to a generation when it is young and to tax it when it is old; in addition, the growth rate of these taxes and transfers must be at least as high as the interest rate[26]. Note that it is only the taxes on or transfers to each generation separately that must have a growth rate at least as high as the interest rate. Aggregate taxes net of transfers, $\tau_t^1 L_t + \tau_{t-1}^2 L_t$, need not grow at all and can indeed be equal to zero.

(V) UNESSENTIAL AND ESSENTIAL PONZI FINANCE AND PUBLIC DEBT.

We first restate an equivalence or irrelevance result, due to Wallace [1981], Chamley and Polemarchakis [1984] and Sargent [1987]. It generalizes the familiar proposition that an equilibrium with positive public debt financed with taxes on the young is equivalent to a balanced budget, pay-as-you-go (or unfunded) social security retirement scheme in which lump-sum taxes on the young are paid out as lump-sum transfers to the old (see also Calvo and Obstfeld [1988a,b]).

This Proposition shows that, in a deterministic model, any intergenerational redistribution that can be supported by debt, taxes and transfers can also be supported just with taxes and transfers and without debt. In the stochastic case it shows that any intergenerational insurance scheme supported with public debt, unbalanced budgets and unrestricted lump-sum taxes and transfers can also be provided with a balanced budget and without public debt.

Proposition 3.

> Given initial values k_0 and b_0 and a feasible sequence g_t, $t \geq 0$, any equilibrium for p_t, k_t, c_t^1 and c_t^2 with arbitrary paths of debts b_t and of lump-sum taxes and transfers τ_t^1 and τ_t^2 for all $t \geq 1$ can be replicated without debt and deficits, that is, by using balanced-budget lump-sum taxes and transfers only.

Proof: An equilibrium is characterized for all $t \geq 0$ by equations (III.6) to (III.10), the government solvency constraint given in (II.17a, b, and c) and initial conditions k_0 and b_0. We rewrite the first two inequalities of the solvency constraint as follows:

$$(V.1) \qquad (1+w_{t+1})(1+n)p_t b_{t+1} + \theta_t^{-1}\tau_t^1 \leq f(k_t) - k_t f'(k_t)$$

$(V.2) \qquad (1+w_{t+1})(1+n)p_t b_{t+1} + \theta_t^{-1}\tau_t^1 \geq g_t - (1+\rho_t)k_t$

The proof is direct and constructive. First consider a "reference equilibrium" supported by given sequences of taxes and debt, denoted by $*$ (single star). Next consider an alternative set of sequences of taxes and debt (denoted by $**$ (double star)), which include the balanced-budget, zero public debt sequences. Finally, check by direct computation that the double star sequences support the same equilibrium sequences of consumption and capital accumulation as the single star sequences (note that exhaustive public spending is the same under both policies). Formally, consider paths b_t^*, τ_t^{1*} and τ_t^{2*} for all $t \geq 0$ that support equilibrium paths p_t, k_t, c_t^1 and c_t^2 for all $t \geq 1$ for given k_0 and b_0. We show that for *any* other set of debt paths b_t^*, $t \geq 1$, there exists associated paths for lump-sum taxes and transfers τ_t^{1**} and τ_t^{2**}, $t \geq 0$ that support the same equilibrium paths p_t, k_t, c_t^1 and c_t^2 for $t \geq 1$. Let the double star debt and tax sequences satisfy $(V.3)$ and $(V.4)$

$(V.3) \qquad p_t(b_{t+1}^{**} - b_{t+1}^*) = \left[\frac{1}{1+n}\right]\theta_{t+1}^{-1}\left[\tau_t^{1*} - \tau_t^{1**}\right]$ for all $t \geq 0$

$(V.4) \qquad \left[\tau_t^{2**} - \tau_t^{2*}\right] = (1 + r_{t+1})p_t\left[b_{t+1}^{**} - b_{t+1}^*\right]\theta_{t+1}(1 + n)$

Equations $(V.3)$ and $(V.4)$ imply $(V.5)$

$(V.5) \qquad -\left[\tau_t^{1**} - \tau_t^{1*}\right] = \frac{1}{1 + r_{t+1}}\left[\tau_t^{2**} - \tau_t^{2*}\right]$ for all $t \geq 0$. [27]

Equation $(V.3)$ ensures that the economy-wide capital market equilibrium condition $(III.9)$ will be satisfied for the same values of p_t, c_t^1, c_t^2 and k_t (and therefore also the same values of w_t). Equation $(V.4)$ ensures that the budget constraint of the old in period t given in $(III.8)$ will be satisfied for the same values of c_t^2, k_{t+1} and r_{t+1}. It is easily verified that the government budget identity in $(III.10)$ will also be satisfied under the double star policies. Finally, it is obvious that if the fiscal-financial feasibility constraint is satisfied for the single star policy it will also be satisfied for the

double star policy.

The remaining equilibrium conditions *(III.6)*, and *(III.7)* also hold under the double star policy. To get Proposition 3 we set $b_t^{**} = 0$ for all $t \geq 1$. It can be checked easily that Proposition 3 remains valid if the conventional government solvency constraint *(II.12)* or *(II.13)* were imposed.□

Proposition 3 implies that with unrestricted lump–sum taxes and transfers, public debt is redundant or "unessential".

We next consider restrictions on how taxes and transfer can differ in any period across overlapping generations. Three simple restrictions fitting this category are:

(a) Equal taxes or transfers per generation for all generations alive during any given period, that is $\tau_t^1(1+n) = \tau_{t-1}^2$ for all $t \geq 0$.

(b) Equal per capita taxes or transfers for all generations alive during any given period, that is $\tau_t^1 = \tau_{t-1}^2$ for all $t \geq 0$.

(c) Equal taxes per unit of efficiency labor for all generations alive during any given period, that is $\tau_t^1 = (1+\omega_t)\tau_{t-1}^2$ for all $t \geq 0$.

The next Proposition states that any intergenerational redistribution and intergenerational insurance supported with a balanced–budget and unrestricted lump–sum taxes and transfers, can also be supported with lump–sum taxes and transfers constrained to fall equally in per capita terms on all overlapping generations (case (b)), provided the public sector budget can be unbalanced. Note that, from Proposition 3, there is no loss of generality in taking the benchmark equilibrium of Proposition 4 to have a balanced budget and zero public debt. The proof of case (b) can be extended easily to the other two cases.

What drives these results is that even though the two generations (the young and the old) alive in any given period are treated in the same way

during that period, we can still vary the present discounted value of lifetime taxes and transfers freely for each generation, making transfer payments to them while young and taxing them when old. In our model, everyone in any given generation is identical. With intra-generational heterogeneity, our equivalence propositions do not, of course, apply when tax-transfer schemes without debt are unrestricted as to how they can vary within each generation (in addition to across generations, time and states of nature). If we restrict our attention to competitive equilibria that can be supported using only tax-transfer schemes that do not vary within generations, then our results go through.

Proposition 4.

Given an initial value k_0 and a sequence of exhaustive public spending g_t, $t \geq 0$, any equilibrium for p_t, k_t, c_t^1 and c_{t-1}^2 for all $t \geq 0$ supported by unrestricted lump-sum taxes and transfers but without public debt and with balanced public sector budgets, can also be supported with equal per capita lump-sum taxes or transfers for both generations alive in any given period, provided unbalanced public sector budgets are allowed.

Proof: Variables with single stars represent the benchmark balanced-budget policy with age-dependent taxes and transfers. Variables with double stars represent the age-independent tax/transfer case with an unbalanced budget. Note that $b_t^* = 0$, $\tau_t^{1*} = -(1 + n)^{-1} \tau_{t-1}^{2*} + \theta_t g_t$ and $\tau_{t-1}^{2**} = \tau_t^{1**} = \tau_t^{**}$ for all $t \geq 0$.

From equation *(III.8)* it follows that, if equivalence holds, it must be true that

$(V.6) \qquad p_t b_{t+1}^{**} = (1 + r_{t+1})^{-1} \theta_{t+1}^{-1} [\tau_{t+1}^{1*} + \tau_{t+1}^{**}(1+n)^{-1} - \theta_{t+1} g_{t+1}]$

From *(V.6)* and the government's single-period budget identity, (or equivalently from the economy-wide capital market equilibrium condition *(III.9))*, it follows that, if the double star regime supports the same equilibria as the single star regime, it must be true that *(V.7)* holds.

(V.7)
$$p_t b^{**}_{t+1} = (1 + n)^{-1} \theta^{-1}_{t+1} [\tau^{1*}_t - \tau^{**}_t]$$

For any τ^{1*}_t and τ^{**}_t it is clear that values of τ^{**}_{t+1} and $p_t b^{**}_{t+1}$ can be found to satisfy *(V.6)* and *(V.7)*. The other equilibrium conditions *(III.6)* and *(III.7)* are also satisfied under the double star regime. The solvency constraint under the single star regime is

(V.8)
$$\theta^{-1}_t \tau^{1*}_t \leq f(k_t) - k_t f'(k_t)$$

and

(V.9)
$$\theta^{-1}_t \tau^{1*}_t \geq g_t - (1+\rho_t)k_t$$

Under the double star regime the solvency constraint is

(V.10)
$$p_t b^{**}_{t+1} + (1 + n)^{-1} \theta^{-1}_{t+1} \tau^{1**}_t \leq f(k_t) - k_t f'(k_t)$$

and

(V.11)
$$p_t b^{**}_{t+1} + (1 + n)^{-1} \theta^{-1}_{t+1} \tau^{1**}_t \geq g_t - (1+\rho_t)k_t$$

It is clear from *(V.7)* that if the solvency constraint is satisfied under the single star regime *((V.8)* and *(V.9)* hold*)*, then it will also be satisfied under the double star regime *((V.10)* and *(V.11)* hold*)*. □

While this completes the proof of Proposition 4, it is instructive to investigate the behavior of taxes and of the public debt under the double star regime. It turns out that Ponzi finance of a special kind (the sequence of government debt will have infinite subsequences that grow faster than the rate of interest forever if the population growth rate is positive), will in general be necessary for the age-independent tax-transfer regime to support the same equilibria as the unrestricted tax-transfer regime. We therefore have the following Corollary to Proposition 4:

Corollary 1.

Ponzi finance is essential when lump–sum per capita taxes and transfers are restricted be equal for overlapping generations.

Proof:

Equations *(V.6)* and *(V.7)* imply

$$(V.12) \qquad \tau^{**}_{t+1} = -(1+r_{t+1})\tau^{**}_t + (1+r_{t+1})\tau^{1*}_t - (1+n)\tau^{1*}_{t+1} + (1+n)\theta_{t+1}g_{t+1}$$

Note that the homogeneous part of equation *(V.12)* changes sign each period (imparting a saw–tooth pattern) and grows at a proportional rate $1 + r$ in absolute value. The saw–tooth pattern of tax receipts is passed on to the value of the per capita debt through the government budget identity under the double star policy, given in *(V.13)* below.

$$(V.13) \qquad p_t b^{**}_{t+1} = (1+\omega_{t+1})^{-1}(1+n)^{-1}(1+r_t)p_{t-1}b^{**}_t + (1+\omega_{t+1})^{-1}(1+n)^{-1}g_t$$
$$- (1+\omega_{t+1})^{-1}(1+n)^{-2}\theta_t^{-1}(2+n)\tau^{**}_t$$

Equations *(V.7)* and *(V.13)* imply that

$$(V.14) \quad p_t b^{**}_{t+1} = -(1+\omega_{t+1})^{-1}(1+r_t)p_{t-1}b^{**}_t - (1+\omega_{t+1})^{-1}g_t + (1+n)^{-1}\theta_{t+1}^{-1}(2+n)\tau^{1*}_t$$

Equation *(V.14)* can be rewritten as

$$(V.15) \qquad p_t B^{**}_{t+1} = -(1+n)(1+r_t)p_{t-1}B^{**}_t - (1+n)G_t + (2+n)\tau^{1*}_t L_t$$

The value of the public debt under the age–independent tax, unbalanced budget policy, $p_t B^{**}_{t+1}$, is likely to zig–zag from a positive value in one period to a negative value in the next. If, for instance, τ^{1*}_t and L_t were constant over time, the saw–tooth pattern of the public debt, with debt in the homogeneous equation of *(V.15)* having a growth factor of $-(1+r_t)$ each period, is immediately apparent. Calvo and Obstfeld [1988b] noted such a pattern in an economy without population growth or productivity growth.

Over a two–period horizon, the public debt evolves according to

$$(V.16) \qquad p_t B_{t+2} = (1+n)^2(1+r_{t+1})(1+r_t)p_{t-1}B_t - (1+n)[G_{t+1} - (1+n)(1+r_{t+1})G_t]$$
$$+ (2+n)[\tau^{1*}_{t+1} - (1+r_{t+1})\tau^{1*}_t]L_{t+1}$$

Consider the simple example where $G_{t+1} = G_t = 0$ and $\tau^{1*}_{t+1} = \tau^{1*}_t = \tau^{1*}$ for all $t \geq 0$. Equation $(V.16)$ simplifies to

$$p_t B^{**}_{t+2} = (1+n)^2(1+r_{t+1})(1+r_t)p_{t-1}B^{**}_t - (2+n)r_{t+1}\tau^* L_{t+1}$$

When τ^* is negative (the balanced-budget scheme redistributes from the old to the young) and r_{t+1} is non-negative, the public debt will, over a two-period horizon grow at a proportional rate at least equal to the sum of the real interest rate and the growth rate of population. If n is non-negative, the sequence of the public debt will therefore have infinite subsequences that are characterized by Ponzi finance. Public credit too will, over a two-period interval, grow at a rate asymptotically equal to the sum of the interest rate and the growth rate of population. This proves that "subsequence Ponzi finance" is "essential" in this case. □

Analogous results to Proposition 4 hold under the restriction that taxes per generation alive in any given period are equal and under the restriction that taxes per efficiency unit of labor in any given period are equal. Ponzi finance is also essential in these cases.

The next proposition presents an example where public debt is essential, but Ponzi finance is infeasible, unless the long-run rate of interest is below the long-run growth rate of efficiency labor.

Proposition 5.

Given an initial value k_0 and a sequence of exhaustive public spending g_t, $t \geq 0$, any equilibrium for p_t, k_t, c_t^1 and c_{t-1}^2 for all $t \geq 0$ that can be supported with a balanced budget and unrestricted lump-sum taxes and transfers, can also be supported with taxes net of transfers that are required to have the same sign during the lifetime of each generation, provided unbalanced budgets are allowed.

Proof: Under the balanced budget (single star) reference policy, $B_t^* = 0$ and $\tau_t^{1*}L_t + \tau_{t-1}^{2*}L_{t-1} = G_t$ for all $t \geq 0$. Under the double star policy, we have $p_t B_{t+1}^{**} \equiv (1+r_t)p_{t-1}B_t^{**} + G_t - \tau_t^{1**}L_t - \tau_{t-1}^{2**}L_{t-1}$ and $\tau_t^{2**} = \lambda_t \tau_t^{1**}$, $\lambda_t > 0$ for all $t \geq 0$. For the two policies to support the same equilibrium, it must be the case that $p_t B_{t+1}^{**} + \tau_t^{1**}L_t = \tau_t^{1*}$.

The behavior of the public debt under the double star policy is governed by

$$(V.17) \qquad p_t B_{t+1}^{**} = (1+r_{t+1}+\lambda_t)^{-1}[\tau_{t+1}^{1*}L_{t+1} + \lambda_t \tau_t^{1*}L_t - G_{t+1}]$$

Noting that

$$(V.18) \qquad p_t B_{t+1}^{**} + \tau_t^{1**}L_t = \tau_t^{1*}L_t \, ,$$

we see that equations *(V.17)* and *(V.18)* can be satisfied through the appropriate choice of (positive or negative) values for $p_t B_{t+1}^{**}$ and τ_t^{1**}, for any given positive value of λ_t and given feasible values of τ_{t+1}^{1*}, τ_t^{1*} and G_{t+1}. $\square$

Proposition 3 states that public debt and deficits (and by implication Ponzi finance) are redundant policy instruments as long as the fiscal authority has unrestricted lump-sum taxes and transfers. Proposition 4 and Proposition 5 emphasize that a fiscal authority with a restricted tax-transfer instrumentarium may be able to use public debt and deficits as perfect substitutes for the missing age-specific taxes and transfers, provided the

government's fiscal-financial feasibility constraint is specified as in *(II.17a,b and c)*. Essential (subsequence) Ponzi finance may be a feature of these government borrowing and lending strategies.

(VI) CONCLUSION.

Our results fall into two categories. The first contains a number of propositions that bring out how the government's ability to issue debt is constrained by its "capacity to tax". The second contains a number of propositions about how the government's ability to issue debt may expand the set of equilibria that can be supported when taxes are restricted.

The ability of the government to engage in Ponzi finance depends on its "capacity to tax," that is, the set of tax and transfer instruments available to it. We show that an infinity of Ponzi finance schemes are feasible if the government can make unrestricted lump-sum transfer payments to each generation when young and impose unrestricted lump-sum taxes on the resources of that generation (including the gross interest earned on its holdings of public debt) when old. The feasibility of Ponzi finance with unrestricted lump-sum taxes and transfers does not depend on whether a competitive equilibrium is dynamically efficient or Pareto efficient. It also does not depend on the relationship between the interest rate and the natural growth rate.

When the long-run rate of interest exceeds the long-run growth rate of efficiency labor, Ponzi finance is feasible as long as (1) the tax burden can vary freely over the life cycle of each generation and (2) gross transfers and taxes (per generation per period) can grow at least as fast as the rate of interest.

When taxes and transfers are unrestricted, Ponzi finance is *unessential*, in the sense that it does not enlarge the set of allocations that can be

supported as competitive equilibria. Ponzi finance may also be feasible when lump-sum taxes and transfers are constrained, again regardless of Pareto efficiency of the equilibrium. Ponzi finance is essential in such cases. More generally, when taxes and transfers are restricted, the ability to unbalance the budget increases the range of intergenerational distribution and insurance schemes the government can implement. Note that the ability to engage in Ponzi finance when the long-run interest rate exceeds the long-run natural rate of growth is restricted to the government. While private agents can make transfer payments (unrequited non-negative payments to others) only the government can impose taxes (unrequited positive transfers to itself).

An important role is played in the proofs of these results by our specification of the government's fiscal-financial feasibility constraints. These require the government not to pursue policies that force the private sector into bankruptcy. The private sector is bankrupt when the non-negativity constraints on consumption by the young, consumption by the old or the private capital stock become binding. The stock of public debt is therefore limited by the condition that the total amount of resources taken by the government from the young, whether through borrowing or through taxes, cannot exceed the wage income of the young. The stock of public credit is likewise limited by the condition that the total amount of resources taken by the government from the old, whether through the old servicing their debt to the government or through taxes, cannot exceed the capital income of the old.

Our approach to the feasibility of fiscal-financial plans has implications for the empirical approaches to testing for government solvency (see e.g. Hamilton and Flavin [1986], Wilcox [1989], Corsetti [1990], Grilli [1990], Trehan and Walsh [1991] and Buiter and Patel [1992 and 1994]). All these papers use variants of the conventional solvency criterion and study the

long-run behavior of the discounted public debt; typically, they test whether the (expectation of the) present discounted value at time t_0 of the future stock of public debt at $t_0 + T$ goes to zero in the limit as T goes to infinity. The conventional solvency criterion is neither necessary nor sufficient for our fiscal-financial feasibility conditions to be satisfied. Intuitively, the conventional solvency constraint concerns a (scalar) present value relationship using equilibrium prices. Our feasibility constraint concerns the equilibrium (physical) allocation vector. The news for (tests of) the conventional solvency constraint is not all bad, however. Our fiscal-financial feasibility constraints imply the conventional government solvency constraint when the long-run interest rate exceeds the long-run growth rate of efficiency labor and when the long-run growth rate of taxes and transfers cannot exceed the long-run growth rate of efficiency labor. This second condition will be satisfied when there are distortionary taxes and/or transfers, or when there are (strictly convex) tax collection and transfer administration costs. This is a further illustration of how the government's ability to borrow is restricted by its capacity to tax.

Finally, the tax-smoothing proposition (Barro [1979]) demonstrates how public debt can be useful for conventional efficiency reasons when there are no non-distortionary taxes and transfers. Our paper complements this by showing how public debt can be useful in the pursuit of distributional objectives and efficient intergenerational insurance schemes, if there is a restricted menu of lump-sum taxes and transfers.

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NOTES

[1]A government engages in Ponzi finance if, after some date, it never runs a primary (non-interest) budget surplus despite having a positive stock of debt outstanding. Equivalently, the value of the additional debt issued each period is at least as large as the interest payments made on the debt outstanding at the beginning of that period.

[2]With distortionary (non-lump-sum) taxes and transfers, real equilibrium allocations will almost always be affected by the ability, offered by unbalanced budgets, to vary the pattern over time of the excess burdens associated with the use of distortionary instruments. The same is true when there are tax collection or transfer administration costs that are increasing and strictly convex functions of marginal or average tax rates. See e.g. Barro [1979].

[3]It is the *institution* of government that must be infinite-lived, not any particular set of incumbent politicians. Specifically, it is required that a government does not repudiate the debt incurred by its predecessors.

[4]Equivalently, the sum of the present discounted values of future primary surpluses is at least as large as the current government debt.

[5]Private intergenerational risk sharing motivated through altruism was analyzed by Hayashi, Altonji and Kotlikoff [1991].

[6]Apart from the incomplete market participation that is intrinsic to OLG models without operative intergenerational gift motives or the institution of hereditary slavery, more standard types of insurance market failures can also create a potential welfare-improving role for taxes, transfer payments and public debt. For instance, in the presence of uncertain lifetimes (a feature that is absent from our model), a compulsory social security retirement scheme can provide an annuity that is actuarially fairer than those provided by the voluntary private annuities market which is impaired by adverse selection (see Abel [1988] and Feldstein (1989)). Eaton and Rosen [1980], Varian [1980], Feldstein [1988], Kimball and Mankiw [1989] and Kaplow [1991] discuss how income taxation can serve as social insurance against uncertainties in labor income. The positive and welfare consequences of such social insurance will of course depend on the availability and nature of private insurance arrangements and the reasons for the absence of a set of complete insurance markets. Typically, adverse selection problems can be mitigated by compulsory social insurance through the tax-transfer mechanism, while moral hazard problems affect efficient public provision of insurance as much as private provision.

[7]Allowing for longer maturity debt would add notation but would not affect the equivalence results.

[8]The single-period utility function v is twice continuously differentiable, strictl concave, increasing in c^1 and c^2 and satisfies the Inada conditions. Note that this util: function has the property that both c^1 and c^2 are normal goods.

[9]Since taxes, coupon payments and the marginal product of capital can be stochastic, it may not be possible to satisfy *(II.3a,b)* for non-negative values of c_t^1 and/or c_t^2. While labor productivity and the marginal product of capital are assumed to be positive, it may not be possible to satisfy the constraints

$c_t^1 \geq 0$, $c_{t-1}^2 \geq 0$, $K_{t+1} \geq 0$ and $G_t \geq 0$ for arbitrary public debt, tax and transfer sequences. Our government solvency constraint is in fact exactly the constraint that households are not forced into bankruptcy by government policy.

Note that for arbitrary government policies, private bankruptcy might occur even if individuals cannot borrow from the government $(b_{t+1}^d \geq 0)$, because even without private debt to the government, the old might not have enough resources to pay the stochastic taxes. If the constraints c_t^1, $c_t^2 \geq 0$ were imposed, even non-contingent debt issued by private individuals $(b_{t+1}^d < 0)$ would in general be risky debt, with gross rate of return $1 + r_{t+1}$ if there is no bankruptcy $(c_t^2 > 0)$ and gross rate of return

$max\{0, [(1 + \rho_{t+1})k_{t+1}^d - \tau_t^2]/(-b_{t+1}^d)\}$ otherwise. With or without private debt to the government, the old might not have enough resources to pay taxes. Assuming that taxes owed to the government have the same seniority as interest owed to the government, actual taxes plus interest paid by the old would be given by $min\{\tau_t^2 - (1 + r_{t+1})p_t b_{t+1}^d, (1 + \rho_{t+1})k_{t+1}^d\}$. It is even possible that the young would not be able to pay their taxes. This would be the case if w_t plus the maximal amount the young could borrow were less than τ_t^1. Allowing for this would greatly complicate the exposition but would not affect our equivalence results, as long as taxes and debt service owed would be subject to the same treatment.

[10]If in the household decision problem k^d is interpreted as the demand for equity, that is, ownership claims to the stock of physical capital, there is no reason the household cannot go short in it $(k^d < 0)$. The constraint $k^d \geq 0$ should in that case be omitted. An alternative interpretation of k^d is that it represents the demand for physical capital by the household (which owns the production technology) in period one of its life, to be carried over into the second period of its life, in order to be used (together with hired labor) in household production. In this case $k^d \geq 0$ would be a sensible constraint. The paper assumes it does not become binding. We do not impose the constraint $b^d \geq 0$. Government credit is allowed. We also allow households to issue state-contingent debt. What we are implicitly assuming is that the debt they issue is identical to government debt. The introduction of private debt does therefore not increase the asset menu.

[11]The consumer's optimum will be turn out to be interior because (1) the utility function satisfies the Inada conditions, (2) the wage rate, the gross return on debt and the gross return on capital are positive and (3) government policy does not drive consumers to bankruptcy. Equations *(II.4a,b)* anticipate points (2) and (3), which are introduced later in this Section.

[12]Public consumption can be an argument in the private utility function. As long as it enters in an additively separable way, it will not affect the first-order conditions for private consumption. Since we are interested in

characterizing feasible fiscal strategies rather than optimal ones, we model public consumption as a pure waste of resources. Public sector capital formation could be added to our list of fiscal instruments in a straightforward manner and is omitted only because of space limitations.

[13]Often the weaker solvency criterion that *(II.12)* hold *in expectation*, is imposed for stochastic models, that is, $E_t \lim_{T\to\infty} \delta_{T-1} p_{T-1} b_T \equiv E_t \lim_{T\to\infty} \Delta_{T-1} p_{T-1} B_T$ $= 0$. Bohn [1990] argues quite convincingly, however, that the solvency criterion should apply to each realization of the discounted debt process, and not just to its mathematical expectation. See also Blanchard and Weil [1992].

[14]When only the expectation of the discounted debt is required to go to zero in the limit, equation (II.13) is replaced by

$$\delta_{t-1} p_{t-1} b_t = E_t \lim_{T\to\infty} \left[\sum_{s=t}^{T-1} (\theta_{s+1})^{-1} \left[\frac{\tau_s^1}{1+n} + \left[\frac{1}{1+n}\right]^2 \tau_{s-1}^2 - \frac{\theta_s g_s}{1+n} \right] \delta_s \right]$$

[15]If we extended the government financial liability menu, say by allowing longer maturity debt, the definition would have to be adapted to the specific set of government debt instruments allowed. The statement that the government never runs a primary surplus, despite having a positive stock of debt outstanding, always defines Ponzi finance.

[16]This is in the spirit of O'Connell and Zeldes [1988], who point out that in order for the government to run a "rational" Ponzi scheme, a rational private sector must be willing to be at the receiving end of such a scheme.

[17]Note that in our model both the wage rate and the marginal product of capital are positive, because we restrict the level of labor–augmenting productivity to be postive. Without a government sector, private bankruptcy would therefore not occur. If the technology were to permit private bankruptcy even without a government, our feasibility constraints would be modified as follows. The government does not select sequences for taxes, transfer payments, debt and exhaustive spending that will cause the non–negativity constraints on consumption by both generations and on the capital stock to become binding *if there exist alternative sequences of the government instruments that would avoid this.*

[18]Kotlikoff [1989] points out that the ultimate constraint on the government is that it cannot take, in present value, more from each household than the present value of its (pre–tax) resources. Our criterion is more general than this, since, involving only period–by–period physical (non–negativity) constraints, it does not rely on equilibrium prices and interest rates. It therefore works even if markets are incomplete. Where present values can be defined properly, our criterion implies that suggested by Kotlikoff. Note, however, that Kotlikoff [1989] continues to impose the conventional government solvency constraint.

[19]Note that the total transfer to the government by the young during period t evolves according to

$$p_t B_{t+1}^{**} + \tau_t^{1**} L_t = (1+r_t)(p_{t-1} B_{t-1}^{**} + \tau_{t-1}^{1*} L_{t-1}) + [(1+n)\tau_t^{1*} - (1+r_t)\tau_{t-1}^{1*}] L_{t-1}$$

[20]Note that such a tax is not perceived by those investing in government debt as a "tax on debt" affecting expected returns from holding debt, even if the tax is fully anticipated. The individual perceives it as lump–sum, that

is, the amount paid is perceived to be independent of the actions of the individual taxpayer, including her portfolio choice. The aggregate tax collections, however, can move systematically with the total amount of debt outstanding.

[21]McCallum's specification of the private sector solvency constraint still implies that the long-run growth rate of the public debt must the less than the private rate of time preference. If the time preference rate exceeds the growth rate of efficiency labor, the stock of debt per unit of efficiency labor can grow without bound in McCallum's model.

[22]With logarithmic single-period utility, $v(c) = \ln (c)$, the proof that transfer payments to the young cannot grow faster than the natural rate of growth forever is immediate, since in that case (IV.3) becomes $\tau_t^1 = w_t - (1+\beta)c_t^1$ Since $w_t \geq 0$, this implies $-\tau_t^1 \leq (1+\beta)c_t^1$.

[23]Homothetic preferences would also rule out Ponzi finance. With homothetic preferences, the increase in life-time resources $w-\tau^1$ associated with an increase in τ^1 would, at a given intertemporal relative price, cause demand for c^1 to increase at the same proportional rate as $-\tau^1$. Any decline in the relative price of period 1 consumption would raise the growth rate of c^1 above that of $-\tau^1$. This would eventually violate the feasibility constraints.

[24]In Tirole's extension of Diamond's OLG model, a speculative bubble has many of the features of public debt in our version of the Diamond model. However, his bubble is a private asset whose behavior is restricted by private agents' budget constraints and voluntary exchange. Our potentially explosive debt bubbles rely on the government's unique capacity to tax. In Tirole's case, "*As bubbles crowd out productive savings and cannot grow faster than the economy, their existence is naturally shown to rely on the comparison between the asymptotic rates of growth and interest in the bubbleless economy.*" (Tirole [1985], p. 1500).

[25]Strictly speaking this should be $\lim_{\substack{t' \to \infty \\ t' \leq t < \infty}} \inf \{\Delta_t p_t B_{t+1}\} \leq 0$ and $\lim_{\substack{t' \to \infty \\ t' \leq \tau < \infty}} \sup \{\Delta_t p_t B_{t+1}\} \geq 0$. If the *lim inf* and the *lim sup* are both equal to zero then $\lim_{t \to \infty} \Delta_t p_t B_{t+1} = 0$.

[26]What we require, strictly speaking, is that the sequences of taxes and transfers per generation, $\{\tau_t^1 L_t\}_{t=0}^{\infty}$ and $\{\tau_{t-1}^2 L_{t-1}\}_{t=0}^{\infty}$ have infinite subsequences $\{\tau_{t_j}^1 L_{t_j}\}_{t_j=0}^{\infty}$ and $\{\tau_{t_j-1}^2\}_{t_j=0}^{\infty}$ whose elements have a growth rate at least as high as the interest rate.

[27]Note that, although τ_t^{2**} can depend on Θ_{t+1} and therefore on θ_{t+1}, τ_t^{2**} —

τ_t^{2*} can depend only on Θ_t and therefore not on θ_{t+1}. If you tax the young more in period t under the double star policy than under the single star policy, you will borrow less (equation *(III.9)*). In period $t+1$ the taxes on the old generation can be lower under the double star policy by $(1+r_{t+1})$ times the amount by which the taxes they paid in period t (when they were young) were higher. This leaves the life–time budget constraint unaffected.