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ON THE PERIODIC STRUCTURE OF THE BUSINESS CYCLE

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1. INTRODUCTION

The influential work of Burns and Mitchell [Mitchell and Burns (1983) and Burns and Mitchell (1946)] has led the National Bureau of Economic Research (NBER) to record and publish a chronology of the U.S. business cycle and has initiated the construction of leading coincident and lagging indicators. The construction of a statistical series of peaks and troughs, dating back to the nineteenth century, has lead to several insightful research projects on economic fluctuations and has originated many foundational debates on business cycle analysis, not to mention the now famous "measurement without theory" debate [see Koopmans (1947)]. Several papers critically reviewed the chronology and suggested alternative dates [see, for instance, Ayres (1939), Cloos (1963a, b), Fels (1959), Moore (1961), Moore and Zarnowitz (1986), Persons (1931), Romer (1991), Trueblood (1961)]. The chronology was also used to investigate asymmetries in business cycle behavior [Weeker (1978), Neftçi (1979, 1984)] and the analysis of duration dependence in cycles [McCulloch (1975), Savin (1977), de Leeuw (1987), Diebold and Rudebusch (1990a)]. Several statistical procedures were also suggested which parallel the NBER dating procedure [Hamilton (1989) and Stock and Watson (1989)]. Moreover, the historical record of cycles has also been used recently to investigate the postwar stabilization hypothesis [Diebold and Rudebusch (1990b), Romer (1991), Watson (1992)].

In this paper, we look at a different set of issues which relate to the analysis of economic fluctuations via turning point chronologies. Taking the NBER dates or any alternative chronology, as given, we test whether a regime shift from expansion to recession, or vice versa, is ceteris paribus equally likely throughout the year. If not, then it may, for instance, be less likely to get out of a recession in the middle of the winter than it is, say, in the spring or summer. There are many good reasons why one might expect that business cycle turning points and durations do not appear to be evenly distributed throughout the year. One can easily find two major reasons for an uneven distribution. The first one can be called the false signal factor, or error of type II factor. The business cycle dating committees of the NBER face the difficult (impossible) task of disentangling regular seasonal variations and business cycle turning points. Many researchers involved in the business cycle chronology have cautioned that there is a fair amount of uncertainty and controversy about the precise month of the peak or trough. It may indeed be difficult to decide whether a recession started late during the year while the economy was still being fueled by the end-of-the-year holiday fever or whether it really turned around at the same time as the usual winter bust starting in January. This is even more the case for many of the series which make up the composite index of leading indicators, like claims for unemployment, building permits or changes in inventories. Besides this factor due to false signal extraction, it is clear that there may be more fundamental reasons why the turnaround of the economy is not evenly distributed and varies with the season. It is fairly plausible that entrepreneurs defer decisions such as lay-offs until after Christmas, for instance. Likewise, it is equally plausible that capital and equipment expansions are accelerated to be ready by the regular annual surge in demand and may therefore advance the beginning of an expansion. Another intrinsic factor may be due to institutionalized periodic regularities such as fiscal policies that are voted on and implemented every year.

The question we empirically investigate has to our knowledge not been considered before. There are good reasons why *seasonal patterns* in business cycle turning points and business cycle durations were never considered. It has been common to classify business cycle and seasonal comovements as mutually independent. Burns and Mitchell, and many others afterwards, advocated the separation of both types of cycles and advanced the idea of studying business cycles, and, more specifically, business cycle turning points, after having adjusted economic time series for seasonality [see, for example, Ghysels (1990) for a recent survey]. Independence of both types of cycles is, of course, a general concept. In practice, however, the removal of seasonal variations was essentially viewed as a signal extraction problem based on unobserved component *linear* time series models [for excellent literature reviews see Nerlove, Grether and Carvalho (1979), Bell and Hillmer (1984) and Hylleberg (1986)]. *Our paper investigates issues which go beyond linear dependence between the two types of cycles.*

There are several possible ways of investigating whether regime switching probabilities are invariant to the season, depending on whether (1) the NBER chronology is used or not and (2) parametric or nonparametric methods are exploited. The advantage of using the NBER chronology is that it makes "recessions" and expansions observable states of the world. The other advantage is that the chronology is implicitly a multivariate selection process for turning points, as the committee members take several series simultaneously into account to assess the dates of regime switches. The alternative to using the NBER chronology is to treat states of the world as unobservable and to estimate a stochastic switching regime model similar to that of Hamilton (1989). Such an approach, pursued in Ghysels (1992a), has the advantage of being detached from the NBER or any other chronology, but for all pratical purposes is unlike the chronology and only based on a single time series such as real GNP. In this paper, we take advantage of parametric Markov chain models estimated from the NBER chronology as well as an alternative set of dates recently suggested by Romer (1991) to test the hypothesis whether switching probabilities are uniform throughout the year. The use of parametric Markov chain models entails some restrictive auxiliary assumptions with regard to the duration dependence of business cycles [see Diebold and Rudebusch (1989), for instance, for a discussion). Moreover, the estimation theory is asymptotic. By taking advantage of the nonparametric statistical hypothesis testing theory applied to business cycle duration data, it is possible to invoke exact finite sample distribution theory and remove the need to rely on the auxiliary assumptions regarding duration dependence. We use nonparametric distribution-free tests in complementary work to this paper, reported in Ghysels (1991, 1992b). While extremely useful for testing purposes, this approach has some disadvantages in comparison to the one taken in this paper. Namely, in section 2, the parametric Markov chain models will enable us to make some explicit statements about differences in switching probabilities throughout the year. We introduce a Markov switching regime model that will be used to estimate and test the parametric and nonparametric tests. Section 3 discusses estimation and testing procedures. Empirical results appear in section 4, followed by conclusions in section 5.

2. A PERIODIC MARKOV SWITCHING REGIME MODEL

Following the work of Neftçi (1984), Hamilton (1989) and others, consider the following transition matrix of 2×2 homogeneous Markov chain :

(2.1)

	Expansion	Recession
Expansion	p	1 - p
Recession	1 - q	q

The economy moves in and out of two states of the world describing several stages of the business cycle. If in an expansion, the economy stays there with probability p and moves to a recession with probability 1 - p. The same interpretation applies to the second line in (2.1), when a recession is the current state. Consider now the case where the transition matrix, determined by p and q, depends on the time of the year; namely, p_i and q_i depend on $i = 1, \ldots, s$, where $s = 4$ or 12 depending on whether we sample quarterly or monthly. The transition matrix is now time varying as it depends on $i = 1, \ldots, s$ (assuming $p_i \neq p_j$ or $q_i \neq q_j$ for some $i \neq j$). Since the transition marix varies through time, we no longer have an homogeneous Markov chain. One way to write the Markov chain transition matrix would be as follows :

(2.2)

	Expansion	Recession
Expansion	$\sum_{i=1}^{s} d_{i1}\, p_i$	$1 - \sum_{i=1}^{s} d_{it}\, p_i$
Recession	$1 - \sum_{i=1}^{s} d_{it}\, q_i$	$\sum_{i=1}^{s} d_{i1}\, q_i$

where d_{it} represents a seasonal dummy process. Fortunately, there is an easy way to rewrite this process as a time-invariant homogeneous Markov chain, using a "stacked" state space. The idea is similar to that of expanding the state space to transform periodic models into aperiodic stationary ones where it is commonly used in linear time series models. See, e.g., Gladysev (1960), Tiao and Grupe (1980), Osborn and Smith (1989), Hansen and Sargent (1990) and Todd (1990). While most interest goes to a monthly seasonal, as the NBER chronology is based on a monthly classification. For convenience of presentation, however, we shall consider $s = 4$, i.e., a quarterly model. The monthly case is basically, mutatis mutandis, the same. A homogeneous (periodic) Markov chain representation is as follows :

(2.3)

		Q1		Q2		Q3		Q4	
		E	R	E	R	E	R	E	R
Q1	E	0	0	p_1	$1 - p_1$	0	0	0	0
	R	0	0	$1 - q_1$	q_1	0	0	0	0
Q2	E	0	0	0	0	p_2	$1 - p_2$	0	0
	R	0	0	0	0	$1 - q_2$	q_2	0	0
Q3	E	0	0	0	0	0	0	p_3	$1 - p_3$
	R	0	0	0	0	0	0	$1 - q_3$	q_3
Q4	E	p_4	$1 - p_4$	0	0	0	0	0	0
	R	$1 - q_4$	q_4	0	0	0	0	0	0

where E stands for expansion and R for recession, while Q1 through Q4 denote the four quarters. The 8×8 transition matrix contains, in fact, four 2×2 submatrices similar to the typical transition matrix appearing in (2.1). Note that if $p_i = p_j$ and $q_i = q_j$ $\forall$ i, j, we have redundancy as the 8×8 matrix is just a repetition of four 2×2 transition matrices, i.e. conditioning on the season does not make any difference. One should also observe the location of the four 2×2 submatrices in (2.3). We move from Q1 to Q2, from Q2 to Q3, etc. and as we do, we can stay or switch regime. This pattern determines the location of the four submatrices. Markov chains which are periodic, as in (2.3), have a particular property; namely, when the transition matrix is raised to the power of its periodicity, one obtains a block diagonal matrix. More specifically, when we raise the transition matrix appearing in (2.3) to the power four (since this is a quarterly version with s = 4), we obtain the following matrix :

Fourth Power of Transition Matrix

(2.4)

		Q1 E	Q1 R	Q2 E	Q2 R	Q3 E	Q3 R	Q4 E	Q4 R
Q1	E	α_1	$1 - \alpha_1$	0	0	0	0	0	0
	R	$1 - \beta_1$	β_1	0	0	0	0	0	0
Q2	E	0	0	α_2	$1 - \alpha_2$	0	0	0	0
	R	0	0	$1 - \beta_2$	β_2	0	0	0	0
Q3	E	0	0	0	0	α_3	$1 - \alpha_3$	0	0
	R	0	0	0	0	$1 - \beta_3$	β_3	0	0
Q4	E	0	0	0	0	0	0	α_4	$1 - \alpha_4$
	R	0	0	0	0	0	0	$1 - \beta_4$	β_4

This property results from the fact that starting from, say, Q1, we end up back in Q1 after four steps which may involve one or more (up to four) regime switches. The block diagonal structure in (2.4) will be convenient to motivate the discussion of the stochastic structure of the processes.

First, one should note that the (α_i, β_i) i = 1, ... , 4 in (2.4) are implicit functions of the (p_i, q_i) i = 1, ... , 4 which determine the matrix (2.3). After some tedious algebra, one obtains :

$$\alpha_i = 1 - \Big[q_{i-1}\Big[q_{i-2}\Big[q_{i-3}(1 - p_i) + p_i(1 - p_{i-3})\Big] + \Big[(1 - q_{i-3})\Big[(1 - p_i) + p_i\, p_{i-3}\Big](1 - p_{i-2})\Big] + \Big[(1 - q_{i-2})\Big[q_{i-3}(1 - p_i) + p_i(1 - p_{i-3})\Big] + \Big[(1 - q_{i-3})(1 - p_i) + p_i\, p_{i-3}\Big]p_{i-2}\Big](1 - p_{i-1})\Big]$$

$$\beta_i = 1 - \Big[(1 - q_{i-1})\Big[q_{i-2}\Big[q_i\, q_{i-3} + (1 - q_i)(1 - p_{i-3})\Big] + \Big[q_i(1 - q_{i-3}) + (1 - q_i)p_{i-3}\Big](1 - p_{i-2})\Big] + \Big[(1 - q_{i-2})(q_i\, q_{i-3} + (1 - q_i)(1 - p_{i-3})\Big] + \Big[q_i(1 - q_{i-3}) + (1 - q_i)p_{i-3}\Big]p_{i-2}\Big]p_{i-1}\Big]$$

where $i \equiv$ i mod 4. Obviously, when $p_i = p_j$ and $q_i = q_j\ \forall$ i, j, then $\alpha_i = \alpha_j$ and $\beta_i = \beta_j\ \forall$ i, j. The block diagonal structure allows us to derive easily the steady state properties of the stochastic switching regime process. Namely, since

(2.5) $$P^4 \equiv \text{diag}\begin{bmatrix}\alpha_i & 1 - \alpha_i \\ 1 - \beta_i & \beta_i\end{bmatrix}$$

it follows that

(2.6) $$P^{4n} = \text{diag}\left[\begin{bmatrix}\Pi_E^i & \Pi_R^i \\ \Pi_E^i & \Pi_R^i\end{bmatrix} + \frac{(\alpha_i + \beta_i - 1)}{2 - \alpha_i - \beta_i}\begin{bmatrix}1 - \alpha_i & \alpha_i - 1 \\ \beta_i - 1 & 1 - \beta_i\end{bmatrix}\right]$$

where $\Pi_E^i \equiv (1 - \beta_i) / (2 - \alpha_i - \beta_i)$ and $\Pi_R^i \equiv 1 - \Pi_E^i$ are the steady state probabilities of being in an expansion or recession in quarter i. Again, when there is no periodicity, we have that Π_E^i and Π_R^i are independent of i, i.e. each quarter has the same share of expansions and recessions. The block diagonal structure of P^4 also allows us to derive the "duration" distribution of an expansion or a recession conditional on the quarter it started and assuming an annual sampling frequency. Namely, the probability of a duration spell of k years is as follows :

(2.7) Probability of expansion lasting k years when started in quarter i =

$$(1 - \alpha_i)\alpha_i^k \quad , \quad k = 1, 2, \ldots$$

(2.8) Probability of recession lasting k years when started in quarter i =

$$(1 - \beta_i)\beta_i^k \quad , \quad k = 1, 2, \ldots$$

One has to be careful though with the interpretation of this duration distribution. Both α_i and β_i represent transition probabilities of "being back" in the same regime one year ahead. This does not exclude the possibility that within the year regime switches happen, as long as one returns to the same regime within the year. The point which is of interest here, however, is the fact that the duration distribution will differ when expansions or recessions are conditioned on the quarter they started (or ended). This feature is particularly useful when duration data are considered and exploited for the purpose of nonparametric hypothesis testing [see Ghysels (1991, 1992b)].

The periodic switching regime model described by transition matrix (2.3) has, of course, its limitations. We still assume an homogeneous transition matrix after allowing for the periodic time variations described in (2.2). In general, the transition probabilities p_i and q_i may still depend on other covariates, such as the duration of the cycle or possibly other seasonal variables. While it would be relatively straightforward to extend our periodic Markov chain model to nonhomogeneous cases, it is easy to see that there are two basic points to be retained from the periodic Markov chain switching regime model which would easily generalize. They are the following : (1) the model implies unequal but periodic transition probabilities for regime shifts and (2) the model also implies an unequal duration distribution of cycles which depend on the time of the year the cycle started as well as an unequal steady state distribution.

3. MAXIMUM LIKELIHOOD ESTIMATION OF MARKOV CHAINS AND TESTS OF PERIODICITY

We turn our attention now to parametric estimation of Markov switching regime models which exhibit possibly a periodic structure in the transition matrix as described in the previous section. We first briefly review the essential elements of the standard ML estimation theory for Markov chains in section 3.1. Section 3.2 is devoted to hypothesis testing.

3.1 A brief review of the estimation theory

A comprehensive treatment of the asymptotic distribution theory for MLE of Markov chains with general state space appears in several sources, most notably Billingsley (1960) and Basawa and Prakasa Rao (1980). For the purpose of presentation, we shall consider a generic example of an homogeneous Markov chain transition probability matrix (keeping in mind that it may exhibit a periodic structure) :

$$(3.1) \qquad P = [p_{ij}(\theta)]$$

where θ is the set of parameters determining the matrix P and $\sum_{j=1}^{\delta} p_{ij} = 1$, δ being the number of states. Typically, the set will consist of $\theta = (p_i, q_i)_{i=1}^{s}$ where p_i and q_i are the probabilities of no regime switch in expansion or recession in season i = 1, ... , s [cfr. the example appearing in (2.3)]. Furthermore, let the stochastic process $y_i(t)$ be defined as :

$$(3.2) \qquad y_i(t) = \begin{cases} 1 \text{ if in state i at time t} \\ 0 \text{ otherwise} \end{cases}$$

where i = 1, ... , 2s; namely, the number of states equals twice the seasonal sampling frequency, i.e. δ = 2s = 8 or 24. From (3.2) we define the vector $y(t) \equiv (y_i(t))_{i=1}^{2s}$. Finally, let $P_i(0)$ be the initial state condition at time t = 0, i.e. probability being in state i at time 0, where $\sum_{i=1}^{2s} P_i(0) = 1$ and $P(0) \equiv (P_i(0))_{i=1}^{2s}$, then the likelihood function over a sample $(y(t))_{k=0}^{T}$ is defined as follows :

$$(3.3) \qquad L((y(t)_{k=0}^{T}, \theta, P(0)) \equiv \left[\prod_{i=1}^{\delta} P_i(0)^{y_i(0)}\right] \left[\prod_{t=1}^{T} \prod_{i=1}^{\delta} \prod_{j=1}^{\delta} (P_{ij}(\theta))^{y_i(t-1)y_j(t)}\right]$$

where the first term on the right-hand side represents the inital state condition. To express the log likelihood function it will be convenient to define :

$$(3.4) \qquad n_{ij} = \sum_{t=1}^{T} y_i(t-1)y_j(t)$$

so that the log likelihood is :

(3.5) $$\ell((y(t))_{t=0}^{T}, \theta, P(0)) = \sum_{i=1}^{s} y_i(0) \log p_i(0) + \sum_{i=1}^{s} \sum_{j=1}^{s} n_{ij} \log p_{ij}(\theta).$$

Under suitable regularity conditions the initial state term, i.e. the first term on the right-hand side of (3.5), vanishes asymptotically [see Billingsley (1961, Condition 1.2)]. Moreover, the standard MLE asymptotic distribution theory goes through [again under suitable regularity conditions, see Billingsley (1961, Theorem 2.2)]. More specifically, the MLE $\hat{\theta}$ is $\sqrt{T}$ consistent and asymptotically normal with the covariance matrix equal to the inverse of Fisher's information matrix.

The log likelihood function, as formulated in (3.5), can be further simplified. It should first be noted that many of the transition probabilities $p_{ij}(\theta)$ are a priori zero because of the structure of (2.3). To simplify notation, let us define $(n_{EE}^{i}, n_{ER}^{i}, n_{RR}^{i}, n_{RE}^{i})_{i=1}^{s}$ as the observed transitions, similar to n_{ij} in (3.4), corresponding to $(p_i, 1 - p_i, q_i, 1 - q_i)_{i=1}^{s}$ in (2.3). Second, in the remainder of the paper, we will work under the assumption that P(0) has a negligible effect so that we operate on the likelihood function conditional on the first observation. Taking into account both modifications yields the following log-likelihood function :

(3.6) $$\ell((y(t))_{t=0}^{T}, \theta) = \sum_{i=1}^{s} [n_{EE}^{i} \log p_i + n_{ER}^{i} \log(1 - p_i) + n_{RR}^{i} \log q_i + n_{RE}^{i} \log(1 - q_i)]$$

where, as before, $\theta = (p_i, q_i)_{i=1}^{s}$.

As a reminder, it may also be useful to note that the information matrix is, again under the standard regularity conditions, a diagonal matrix. Consequently, the asymptotic covariance matrix of the MLE parameter estimates is diagonal and the $\hat{\theta} = (\hat{p}_i, \hat{q}_i)_{i=1}^{s}$ are asymptotically mutually independent.

3.2 Hypothesis testing

To test whether the transition matrix has a periodic structure, we will rely on LR as well as LM and Wald tests. Before we discuss the reason for using both tests, let us first formalize the hypothesis of interst :

$$(3.7) \qquad H_0 : p_i = \bar{p} \text{ and } q_i = \bar{q} \qquad \forall\, i = 1, \ldots, s$$

$$(3.8) \qquad H_0^E : p_i = \bar{p} \qquad \forall\, i = 1, \ldots, s$$

$$(3.9) \qquad H_0^R : q_i = \bar{q} \qquad \forall\, i = 1, \ldots, s.$$

The first hypothesis (3.7) restricts all transition probabilities across seasons to be equal in either of the two regimes. The second and third ones restrict the probabilities to be equal in either expansions; hence, H_0^E or recessions as in (3.9).

The LR test has the usual structure. That is to say, let $\hat{\theta}_u$ the unconstrained ML estimate of the parameter vector $\theta = (p_i, q_i)_{i=1}^s$ and $\hat{\theta}_2$ the restricted one, then

$$(3.10) \qquad LR = -2(\ell((y(t))_{t=0}^T, \hat{\theta}_2) - \ell((y(t)_{t=0}^T, \hat{\theta}_u)) \stackrel{d}{\to} \chi^2 \text{ df}$$

where df equals the number of restrictions imposed across p_i and q_i; namely, $df = 2(s - 1)$ in (3.7) and $df = s - 1$ in (3.8) and (3.9). The LM statistic is also relatively straightforward to derive from the score function obtained from (3.6) and the information matrix. The fact that the latter is diagonal, as noted before, greatly simplifies the formula of the LM statistic. In particular, let :

$$(3.11) \qquad LM^E \equiv \sum_{i=1}^{s} \left[\left[\frac{n_{EE}^i}{\hat{p}} - \frac{n_{ER}^i}{1 - \hat{p}} \right]^2 \left[\frac{n_{EE}^i}{\hat{p}^2} + \frac{n_{ER}^i}{(1 - \hat{p})^2} \right]^{-1} \right]$$

$$(3.12) \qquad LM^R \equiv \sum_{i=1}^{s} \left[\left[\frac{n_{RR}^i}{\hat{q}} - \frac{n_{RE}^i}{1 - \hat{q}} \right]^2 \left[\frac{n_{RR}^i}{\hat{q}^2} + \frac{n_{RE}^i}{(1 - \hat{q})^2} \right]^{-1} \right]$$

$$(3.13) \qquad LM \equiv LM^E + LM^R.$$

Where LM^E and LM^R correspond to hypotheses (3.8) and (3.9), respectively, while LM tests the restrictions imposed in (3.7). The LM statistic involves, of course, only parameter estimates under the null; namely, $\hat{p}$ and $\hat{q}$.

Anticipating some of the results reported in the next section, we need to discuss the issues that arise because of corner solutions, i.e., $\hat{p} = 1$ or $\hat{q} = 1$, of the ML estimates which may occur in small samples. Indeed, when any of the n^i_{RE} or n^i_{ER} equals zero, no regime switch has occurred over a particular sample of size T *during a particular season i*. Such corner solutions for parameter estimates invalidate the regularity conditions yielding the aforementioned standard distribution theory, and hence complicate the task of hypothesis testing. Two approaches will be pursued to address this. A first approach consists of using the LM test only involving parameter estimates $\hat{p}$ and/or $\hat{q}$ under the null. Such parameter estimates, as it turns out, will not be ill-behaved as $\sum_{i=1}^{S} n^i_{ER} > 0$ and $\sum_{i=1}^{S} n^i_{RE} > 0$ for all samples that were verified. Hence, the use of LM tests over LR tests has some advantage in situations where corner solutions occur. The presence of corner solutions is, of course, a small sample phenomenon, as we have not observed enough regime switches to have at least one occur every season. This raises the question about size distortion and power of the LM statistic, making use of its asymptotic χ^2 distribution critical values. It is beyond the scope of this paper to actually discuss this in detail [see, however, Ghysels (1992a) for further discussion].

A second approach of dealing with corner solutions consists of pooling seasons together, i.e., grouping months together, where one assumes that within each group of months the transition probabilities are identical (for each regime). One possible grouping consists of taking a quarterly classification, i.e., taking all months belonging to a quarter as having the same p or q. Another possibility, inspired by the results obtained in Ghysels (1992b), is to rely on a more economically based classification. The one we will be specifically interested in, given the discussion in Ghysels (1992b), consists of distinguishing what is loosely defined as Christmas and spring months from the other "off-season" months. For convenience, the classification consists of six months in each category; namely, March through July and December represent a combination of Christmas and spring effects, while the remaining six months are classified as off-season. This leads to two sets of transition probabilities (p_c, q_c) and (p_0, q_0) where index c refers to Christmas/spring and index 0 stands for other or off-season. The six months in either group are assumed to have the same transition probabilities. Given the results reported in Ghysels (1992b), the following hypothesis will be of particular interest :

(3.14) $$H_0^c : p_c = p_0 \text{ and } q_c = q_0$$

against

(3.15) $$H_A^c : p_c = q_0 \text{ and } q_c \neq q_0.$$

The hypothesis will be tested with a LR statistic analogous to (3.10). The hypothesis formulated in (3.14) and (3.15) tells us that economic recoveries are possibly more likely to occur during the six months categorized as Christmas/spring. Strictly speaking, we only test $q_c \neq q_0$ and not $q_c < q_0$, just for simplicity. Please note also that we keep $p_c = p_0$ as there is little evidence for different switching probabilities when in an expansion [based on results obtained from (3.7) through (3.9) as well as those reported in earlier work].

What is also important, from a practical point of view, is that as a result of the pooling, no boundary solutions appear in any of our empirical examples. While this is, of course, a useful consequence of pooling, it is important to stress that the prime motivation for pooling months into a Christmas/spring classification is not so much to avoid corner solutions. Instead, it is more inspired by the results and discussions appearing in Ghysels (1992b) where the economically plausible classification is suggested.

A final relatively easy and standard test used in the next section, will be a Wald test. Because of the diagonal structure of the information matrix, the test for comparison of two transition probabilities, say p_i and p_j is as follows :

$$W = (\hat{p}_i - \hat{p}_j)^2 (\text{var } \hat{p}_i + \text{var } \hat{p}_j)^{-1}.$$

Such a test will not be reported systematically or any more general Wald test. Instead, it will only be used in the discussion and comments to the tables with empirical results.

4. EMPIRICAL RESULTS

A business cycle chronology of turning points readily yields data on the frequency distribution of switching versus staying in a particular regime. Indeed, using the notation introduced in section 3, one can easilly calculate quantities like n^i_{EE}, n^i_{ER}, n^i_{RR} and n^i_{RE} for any season i corresponding to the number of times one either stayed in an expansion, moved to a recession, stayed in a recession or started a recovery respectively by the end of season i. The NBER chronology is a set of dates reaching back to the middle of last century yielding over 1,600 observations on transitions from one month to the next for expansions and recessions. Since the location of peaks and troughs in the earlier part of the chronology, in particular before 1914, has been the subject of much criticism, we shall also rely on an alternative chronology recently proposed by Romer (1991). The latter chronology is the outcome of a coherent effort to remove some of the apparent inconsistencies in the early part of the NBER chronology. Moreover, the alternative chronology relies on a more complete data set.

Besides drawing inference from two alternative chronologies, we will also investigate subsamples in each case, judiciously chosen to account for the possible effects of heterogeneity through time. Namely, one typically separates cycles before and after World War II to account for the "stabilization effect" of macroeconomic policy after WWII [see Diebold and Rudebusch (1991b), Romer (1991) and Watson (1992) for futher discussion], or the change in structure in the U.S. economy. War era expansions or contractions are often also excluded because of their possible different characters. We will not report, however, results where war eras were excluded to avoid overloading the tables with empirical results. In particular, as we found out, removing war eras did not effect in any substantial way any of our results. We will report results for the NBER chronology excluding observations before 1885, so that comparisons will be possible across the two chronologies of cycles. All configuratioins together yield a total of five sample configurations for the pre- and post-WWII samples as well as the 1885-WWII and 1885-present data sets (present meaning the end of 1990). For the alternative chronology, there are three sample configurations, as the entire sample coincides with 1885-present and pre-WWII coincides with 1885-WWII.

Table 4.1 contains the ML estimates of the monthly transition probabilities $\hat{p}_i$ and $\hat{q}_i$ for i = 1, ..., 12 for the eight samples as well as the constrained (aperiodic) ML estimates $\hat{p}$ and $\hat{q}$ in each of the samples. Let us first concentrate on the NBER chronology over the entire sample. We observe little variation in the monthly estimates $\hat{p}_i$ of staying in an expansion. Indeed, the two most extreme values, 0.946 for January, 0.989 for February and September are a mere 4 % apart. If we were to rely on a simple Wald test, we would not reject the null of $\hat{p}_i = p$ across all months. For the recession, the results are probably quite different. First, we observe a boundary parameter estimate for September, indicating that over the 1855–1990 period, the U.S. economy never recovered from a recession in that month (there is indeed no trough for that month over the entire NBER chronology, as shown in the Appendix). Such boundary parameter estimates yield a zero standard deviation. This is simply a consequence of the ML estimator based on the inverse of the information matrix. Apart from the one boundary parameter estimate, we also observe a wider dispersion of estimates $\hat{q}_i$ ranging from 0.872 and 0.889 for December and June to 0.977 and 0.979 for January, February and August. Based on a pairwise comparison of, say, December and January, one would conclude from a Wald test that both probability estimates are significantly different from each other at 5 %. The point estimates also differ significantly in economic terms, as there is more than 10 % probability difference between, say, December and February. In general, we should also note that all the relative low estimates for $\hat{q}_i$ occur in the months of March through July and December. This corresponds to the analysis already referred to earlier in Ghysels (1992) regarding the classification of Christmas/spring.

When we turn our attention to the subsamples drawn from the NBER chronology, we observe primarily two key features : namely, (1) more boundary parameter estimates appear as samples become smaller and (2) the relatively little variation of $\hat{p}_i$ as opposed to the larger differences among $\hat{q}_i$ persists throughout the various sample configurations. The months covering most of spring and the end of the year remain as those with relatively low $\hat{q}_i$. The alternative chronology proposed by Romer (1991) shows similar features. In fact, the variation across $\hat{q}_i$ over the full sample 1885–1990 is much larger, sparing more than 25 % difference between the highest and lowest transition probability estimates of $\hat{q}_i$. When we turn to the smallest sample reported in

TABLE 4.1 : Maximum Likelihood Estimates of Periodic and Aperiodic Markov Transition Probabilities

Month	$\hat{p}_i$	$1-\hat{p}_i$	St. Dev.	$\hat{q}_i$	$1-\hat{q}_i$	St. Dev.	$\hat{p}_i$	$1-\hat{p}_i$	St. Dev.	$\hat{q}_i$	$1-\hat{q}_i$	St. Dev.
	NBER Entire Sample						NBER Pre-WWII					
1.00	0.946	0.054	0.0235	0.977	0.023	0.0230	0.918	0.082	0.0391	0.971	0.029	0.0281
2.00	0.989	0.011	0.0110	0.979	0.021	0.0210	1.000	0.000	0.0000	1.000	0.000	0.0000
3.00	0.978	0.022	0.0158	0.915	0.085	0.0407	0.957	0.043	0.0300	0.921	0.079	0.0437
4.00	0.978	0.022	0.0155	0.956	0.044	0.0307	0.979	0.021	0.0210	0.973	0.027	0.0266
5.00	0.967	0.033	0.0187	0.933	0.067	0.0371	0.936	0.064	0.0356	0.946	0.054	0.0371
6.00	0.967	0.033	0.0190	0.889	0.111	0.0468	0.935	0.065	0.0365	0.868	0.132	0.0549
7.00	0.967	0.033	0.0184	0.930	0.070	0.0389	0.979	0.021	0.0205	0.944	0.056	0.0382
8.00	0.967	0.033	0.0184	0.977	0.023	0.0230	0.959	0.041	0.0283	0.971	0.029	0.0281
9.00	0.989	0.011	0.0110	1.000	0.000	0.0000	0.979	0.021	0.0205	1.000	0.000	0.0000
10.00	0.966	0.034	0.0192	0.957	0.043	0.0300	0.936	0.064	0.0356	1.000	0.000	0.0000
11.00	0.977	0.023	0.0158	0.936	0.064	0.0356	1.000	0.000	0.0000	0.975	0.025	0.0247
12.00	0.978	0.022	0.0158	0.872	0.128	0.0487	0.978	0.022	0.0219	0.872	0.128	0.0536
Aperiodic	0.972	0.028	0.0045	0.943	0.057	0.0100	0.963	0.037	0.0077	0.953	0.047	0.0100
	NBER 1885-1990						Alternative 1885-1990					
1.00	0.934	0.066	0.0285	0.967	0.033	0.0327	0.963	0.037	0.0207	0.917	0.083	0.0564
2.00	0.986	0.014	0.0138	0.971	0.029	0.0290	0.988	0.012	0.0122	0.920	0.080	0.0542
3.00	0.986	0.014	0.0138	0.912	0.088	0.0487	0.976	0.024	0.0170	0.875	0.125	0.0675
4.00	0.986	0.014	0.0134	0.938	0.062	0.0428	0.988	0.012	0.0118	0.913	0.087	0.0587
5.00	0.960	0.040	0.0226	0.903	0.097	0.0531	0.964	0.036	0.0202	0.955	0.045	0.0444
6.00	0.986	0.014	0.0134	0.871	0.129	0.0602	0.988	0.012.	0.0122	0.917	0.083	0.0564
7.00	0.961	0.039	0.0221	0.893	0.107	0.0585	0.951	0.049	0.0239	0.739	0.261	0.0915
8.00	0.961	0.039	0.0221	0.964	0.036	0.0351	0.964	0.036	0.0202	0.952	0.048	0.0465
9.00	0.987	0.013	0.0134	1.000	0.000	0.0000	0.988	0.012	0.0122	1.000	0.000	0.0000
10.00	0.986	0.014	0.0134	0.935	0.065	0.0442	0.975	0.025	0.0173	0.958	0.042	0.0407
11.00	0.973	0.027	0.0187	0.900	0.100	0.0548	0.988	0.012	0.0122	0.920	0.080	0.0542
12.00	0.974	0.026	0.0184	0.931	0.069	0.0470	0.975	0.025	0.0173	0.917	0.083	0.0564
Aperiodic	0.973	0.027	0.0055	0.932	0.068	0.0130	0.976	0.024	0.0045	0.915	0.085	0.0167

TABLE 4.1 (continued)

Month	$\hat{p}_i$	$1-\hat{p}_i$	St. Dev.	$\hat{q}_i$	$1-\hat{q}_i$	St. Dev.	$\hat{p}_i$	$1-\hat{p}_i$	St. Dev.	$\hat{q}_i$	$1-\hat{q}_i$	St. Dev.
			NBER 1885-WWII						Alternative 1885-WWII			
1.00	0.875	0.125	0.0585	0.955	0.045	0.0444	0.925	0.075	0.0416	0.857	0.143	0.0935
2.00	1.000	0.000	0.0000	1.000	0.000	0.0000	0.974	0.026	0.0253	0.933	0.067	0.0644
3.00	0.966	0.034	0.0339	0.920	0.080	0.0542	0.974	0.026	0.0253	0.867	0.133	0.0877
4.00	1.000	0.000	0.0000	0.958	0.042	0.0407	0.975	0.025	0.0247	1.000	0.000	0.0000
5.00	0.903	0.097	0.0531	0.913	0.087	0.0587	0.949	0.051	0.0354	0.933	0.067	0.0644
6.00	0.967	0.033	0.0327	0.833	0.167	0.0761	0.974	0.026	0.0259	0.875	0.125	0.0827
7.00	0.970	0.030	0.0298	0.905	0.095	0.0640	0.923	0.077	0.0427	0.733	0.267	0.1142
8.00	0.941	0.059	0.0404	0.950	0.050	0.0487	0.975	0.025	0.0247	1.000	0.000	0.0000
9.00	0.970	0.030	0.0298	1.000	0.000	0.0000	0.974	0.026	0.0253	1.000	0.000	0.0000
10.00	0.969	0.031	0.0308	1.000	0.000	0.0000	1.000	0.000	0.0000	1.000	0.000	0.0000
11.00	1.000	0.000	0.0000	0.957	0.043	0.0425	1.000	0.000	0.0000	0.938	0.062	0.0605
12.00	0.968	0.032	0.0318	0.909	0.091	0.0613	0.974	0.026	0.0259	0.867	0.133	0.0877
Aperiodic	0.960	0.040	0.0100	0.941	0.059	0.0141	0.968	0.032	0.0084	0.917	0.083	0.0205
			NBER Post-WWII						Alternative Post-WWII			
1.00	0.973	0.027	0.0266	1.000	0.000	0.0000	1.000	0.000	0.0000	1.000	0.000	0.0000
2.00	1.000	0.000	0.0000	0.889	0.111	0.1047	1.000	0.000	0.0000	0.889	0.111	0.1047
3.00	1.000	0.000	0.0000	0.875	0.125	0.1169	0.973	0.027	0.0266	1.000	0.000	0.0000
4.00	0.974	0.026	0.0259	0.857	0.143	0.1322	1.000	0.000	0.0000	0.778	0.222	0.1386
5.00	1.000	0.000	0.0000	0.857	0.143	0.1322	0.974	0.026	0.0259	1.000	0.000	0.0000
6.00	1.000	0.000	0.0000	1.000	0.000	0.0000	1.000	0.000	0.0000	1.000	0.000	0.0000
7.00	0.947	0.053	0.0362	0.833	0.167	0.1522	0.972	0.028	0.0274	0.750	0.250	0.1531
8.00	0.973	0.027	0.0266	1.000	0.000	0.0000	0.946	0.054	0.0371	0.857	0.143	0.1323
9.00	1.000	0.000	0.0000	1.000	0.000	0.0000	1.000	0.000	0.0000	1.000	0.000	0.0000
10.00	1.000	0.000	0.0000	0.875	0.125	0.1169	0.944	0.056	0.0382	0.875	0.125	0.1169
11.00	0.946	0.054	0.0371	0.714	0.286	0.1707	0.971	0.029	0.0281	0.889	0.111	0.1047
12.00	0.973	0.027	0.0266	1.000	0.000	0.0000	1.000	0.000	0.0000	1.000	0.000	0.0000
Aperiodic	0.982	0.018	0.0063	0.909	0.091	0.0307	0.982	0.018	0.0063	0.919	0.081	0.0274

Note : The chronologies from which the data and estimates are drawn appear in the Appendix.

Table 4.1, namely the post-WWII sample for both chronologies, we observe that the parameter estimates for $\hat{q}_i$ especially become very imprecise. The standard errors exceed 10 percentage points in many cases, reaching as high as 17 %. With such imprecision, we will refrain from testing hypotheses in this particular sample, as they will be mostly uninformative.

We turn our attention now to testing the hypotheses H_0, H_0^E and H_0^R defined in (3.7) through (3.9) via the LM statistics formulated in section 3.2. The LR statistic will not be reported, because the unconstrained Markov models involve boundary estimates in all samples. Table 4.2 contains the empirical results drawn from both chronologies. The pattern observed from the parameter estimates reported in Table 4.1 is largely confirmed by the LM statistics. There is clear evidence of heterogeneity in switching probabilities throughout the year, especially for recession switching probabilities. For all samples, we resoundingly reject H_0^R; namely, equal transition probabilities for starting a recovery when in a recession. In contrast, we also find that for most samples, except pre-WWII, there is little evidence against H_0^E; i.e., $p_i = p \; \forall i$. Since, we observed relatively low values for q during the loosely defined Christmas/spring months, March through July and December. We turn our attention to hypotheses (3.14) and (3.15) using the suggested pooling of months. In Table 4.3, the parameter estimates $\hat{q}_c$ and $\hat{q}_0$ are reported alongside the LR statistic for H_0^c against H_A^c. As the $p_i = p$ in each case, we do not report these parameter estimates, since they can be recovered from Table 4.1. First thing to note about Table 4.3, of course, is the absence of boundary parameter estimates. Indeed, the pooling has resulted in the disappearance of such estimates. The parameter estimates $\hat{q}_c$ are consistently smaller than the corresponding $\hat{q}_0$ in each sample reported in Table 4.3. Hence, the probability of recovering from a recession 1 - q is much higher during the Christmas/spring months. Applying a simple Wald test pairwise comparison yields strong rejections of the null that $\hat{q}_c = \hat{q}_0$. If we test the more complete hypothesis H_0^c against H_A^c (involving also $\hat{p}$), we find strong evidence against equal switching probabilities, except perhaps for the p-values below 10 % but above 5 % for the 1885-1990 sample with both chronologies.

TABLE 4.2 : Lagrangian Multiplier Tests for Periodic Structure

	NBER Chronology				Alternative Chronology	
	Entire Sample	1885-1990	Pre-WWII	1885-WWII	1885-1990	1885-WWII
H_0	61.41 (0.00)	41.20 (0.01)	213.30 (0.00)	164.90 (0.00)	35.88 (0.03)	141.60 (0.00)
H_0^E	6.41 (0.84)	8.41 (0.68)	95.30 (0.00)	93.70 (0.00)	6.88 (0.81)	78.90 (0.00)
H_0^R	55.00 (0.00)	32.80 (0.00)	118.00 (0.00)	71.20 (0.00)	29.00 (0.00)	62.70 (0.00)

Notes : The hypotheses H_0, H_0^E and H_0^R are defined in (3.7) through (3.9). The LM statistics are defined in (3.11) through (3.13). The entries to the table are the p-values of the test statistic. The statistics are distributed χ^2 with df = 11 for H_0^R and H_0^E and with df = 22 for H_0.

TABLE 4.3 : Likelihood Ratio Tests of Christmas/Spring Periodic Structure

	$\hat{q}_c$	St. Dev.	$\hat{q}_0$	St. Dev.	H_0^c against H_A^c
			NBER Chronology		
Entire sample	0.915	0.0169	0.969	0.0114	8.03 (0.00)
1990	0.908	0.0212	0.956	0.0151	3.45 (0.06)
Pre-WWII	0.920	0.0181	0.986	0.0078	12.14 (0.00)
1885-WWII	0.906	0.0247	0.977	0.0129	6.67 (0.01)
			Alternative Chronology		
1885-1990	0.886	0.0269	0.944	0.0193	3.09 (0.08)
1885-WWII	0.878	0.0345	0.956	0.0217	3.69 (0.05)

Note : The hypothesis and parameter definitions appear in (3.14) and (3.15).

5. CONCLUSIONS

One of the most robust results that transpires through the different tests and approaches used in this paper is that "jump starting" the economy out of a recession, does not seem to happen with a uniform probability throughout the year. It is also found that a classification of months along the lines of Christmas and spring versus the off-season reveals a statistically significant and strong difference in transition probabilities from recession to expansion. Besides this fairly robust result, there is some but comparatively little evidence of periodicity for expansions.

As it was pointed out in the beginning of the paper, there are two very different interpretations to the basic conclusion of this paper. It may well be simply an error of type II phenomenon that can only be attributed to the difficulties the various dating committees of the NBER faced. If that is the case, this paper does have nothing to contribute to the actual mechanism that generate an end to recessions. On the other hand, however, if the propensity for the economy to turn around is season-dependent, then we do have to think about what can really explain this and what should be learnt from it in terms of the nature of recessions and expansions. The fact that periodicity only appears in recession switching probabilities certainly hints at intrinsic causes as there are probably just as many reasons for type II errors in both regimes. Besides the periodicity, the analysis in this paper also uncovered an interesting asymmetry in switching probabilities; namely, a periodic structure in one regime versus no periodicity in the other. The patterns of periodicity discussed here will, of course, not be uncovered by simple linear seasonal adjustment filters or variants of it like X-11. This is discussed in more detail in Ghysels (1992a) which elaborates further on the stochastic process theory of periodic Markov chains and their relation to linear time series processes.

APPENDIX

Table A.1 : NBER Business Cycle Reference Dates and Durations

Trough	Peak	Contractions	Expansions
-	June 1857	18	-
December 1858	October 1860	8	22
June 1861	April 1865	32	46
December 1867	June 1869	18	18
December 1870	October 1873	65	34
March 1879	March 1882	38	36
May 1885	March 1887	13	22
April 1888	July 1890	10	27
May 1891	January 1893	17	20
June 1894	December 1895	18	18
June 1897	June 1899	18	24
December 1900	September 1902	23	21
August 1904	May 1907	13	33
June 1908	January 1910	24	19
January 1912	January 1913	23	12
December 1914	August 1918	7	44
March 1919	January 1920	18	10
July 1921	May 1923	14	22
July 1924	October 1926	13	27
November 1927	August 1929	43	21
March 1933	May 1937	13	50
June 1938	February 1945	8	80
October 1945	November 1948	11	37
October 1949	July 1953	10	45
May 1954	August 1957	8	39
April 1958	April 1960	10	24
February 1961	December 1969	11	106
November 1970	November 1973	16	36
March 1975	January 1980	6	58
July 1980	July 1981	16	12
November 1982	July 1990	-	92

Source : Diebold and Rudebusch (1990a).

Table A.2 : Alternative Business Cycle Reference Dates and Durations

Trough	Peak	Contractions	Expansions
–	February 1887	5	–
July 1887	January 1893	13	66
February 1894	December 1895	13	22
January 1897	April 1900	8	39
December 1900	July 1903	8	31
March 1904	July 1907	11	40
June 1908	January 1910	16	19
May 1911	June 1914	5	37
November 1914	May 1916	8	18
January 1917	July 1918	8	18
March 1919	January 1920	18	8
July 1921	May 1923	14	22
July 1924	March 1927	9	32
December 1927	September 1929	34	21
July 1932	August 1937	10	61
June 1938	December 1939	3	18
March 1940	October 1948	12	103
October 1949	August 1953	12	46
August 1954	August 1957	8	36
April 1958	May 1960	9	25
February 1961	October 1969	13	114
November 1970	November 1973	20	36
July 1975	March 1980	4	56
July 1980	July 1981	21	12
April 1983	July 1990	–	87

Source : See Romer (1991), Tables 2 and 3.

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