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AT YALE UNIVERSITY

Box 2125, Yale University
New Haven, Connecticut 06520

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The "Dice" Model: Background and Structure Of a Dynamic Integrated Climate-Economy Model of the Economics of Global Warming

by

William D. Nordhaus

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Preface

This study is designed to present the methodological and technical assumptions and the results behind the Dynamic Integrated model of Climate and the Economy (the DICE model). It is a model that attempts to use the tools of modern economics to determine an efficient strategy for coping with the threat of global warming. The fundamental premise behind this study is that societies should undertake environmental policies only when their benefits, broadly construed, exceed their costs and that the level of environmental control should be at that point where the incremental benefits of additional controls no longer exceed the incremental costs.

In the area of global warming, this general strategy is easy to articulate and difficult to execute. The work embodied in this study lays out one approach -- the use of dynamic economic optimization -- to the construction of an efficient control strategy. The present study differs from earlier ones by the author and other researchers in two respects. First, it attempts to integrate both the economic costs and benefits of greenhouse-gas controls with a simple aggregate model linking economic growth with climate change. The primary new development here, however, is the inclusion of both economic and climate dynamics. Because of the long residence times of greenhouse gases and the long time lags in both economic activity and in climate change, dynamics are of the essence. Any study which overlooks the dynamics may produce misleading conclusions for the steps that we should take at the dawn of the age of greenhouse warming.

The present study represents the technical background and documentation for a model of economic growth in the presence of global warming. The model itself is relatively small by the standards of both economics and the related natural sciences, but many of the components will be unfamiliar to those outside the disciplines from which the individual components are derived. The purpose of this study, therefore, is to lay out in detail both the assumptions and the underlying derivations of the relationships underpinning the DICE model.

This study consists primarily of the background equations of the DICE model rather than analyzing the policy implications or analyzing the impacts of alternative specifications of the model. These further refinements remain on the agenda of future research.

The present study was supported by the National Science Foundation. It has benefitted from an invisible college of scholars who have contributed to the development of the field of the economics of global environmental issues. Early co-workers in the economics of the greenhouse effect who imparted their wisdom included Jesse Ausubel, William Clark, William Cline, Jae Edmonds, Howard Gruenspecht, Dale Jorgenson, Geoffrey Heal, William Hogan, Lester Lave, Alan Manne, Richard Morgenstern, David Pearce, John Reilly, Richard Richels, Thomas Schelling, John Weyant, the late David Wood, and Gary Yohe. The author is grateful for helpful suggestions from colleagues at Yale, including William Brainard, Richard Levin, and Herbert Scarf, as well as for comments on earlier versions of this work by Edward Barbier, Richard Cooper, Peter Diamond, Herbert Giersch, and Leo Schrattenholzer. Scientists in other disciplines have tutored me on innumerable occasions, and I am particularly indebted to Robert Chen, Thomas Lee, Thomas Malone, William Nierenberg, John Perry, Roger Ravelle, Stephen Schneider, Karl Turekian, and Paul Waggoner. Finally, I am enormously grateful to Zili Yang for dedicated and patient research assistance. All errors that have survived the interchanges with these colleagues are my sole responsibility.

THE "DICE" MODEL: BACKGROUND AND STRUCTURE OF A DYNAMIC INTEGRATED CLIMATE-ECONOMY MODEL OF GLOBAL WARMING

William D. Nordhaus
Yale University
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PART ONE. MODEL DESCRIPTION

I. INTRODUCTION

The possibility of greenhouse warming has received growing attention in recent years. Climatologists and other scientists have warned that the accumulation of carbon dioxide (CO_2) and other greenhouse gases (GHGs) is likely to lead to global warming and other significant climatic changes over the next century. Many scientific bodies, along with a growing chorus of environmental bodies and government, are calling for severe curbs on the emissions of greenhouse gases, as for example the reports of the Intergovernmental Panel on Climate Change (IPCC [1990]) and the Second World Climate Conference (October 1990). Governments are in the process of negotiating a "framework treaty," which will establish a process through which national efforts can be coordinated and will culminate in the Rio meeting in June 1992.

To date, the calls to arms and treaty negotiations have progressed more or less independently of economic studies of the costs and benefits of measures to slow greenhouse warming. Over the last few years, however, a growing body of economic evidence has pointed to the likelihood that greenhouse warming will have at most modest costs in industrial countries, while programs to impose deep cuts in GHG emissions will have substantial costs. Like two ships passing in the night, the economic studies and the treaty negotiations seems to be proceeding independently under their own steam.

Notwithstanding the difficulties of marrying the economic analysis with the policy process, the need to address the potential issues raised by future climate change is daunting for those who take policy analysis seriously. It raises formidable issues of data, modeling, uncertainty, international coordination, and institutional design. In addition, the economic stakes are enormous, involving investments on the order of hundreds of billions of dollars a year to slow or prevent climate change.

In earlier studies, I developed a simple cost-benefit framework for determining the optimal "steady-state" control of CO_2 and other greenhouse gases.[1] This earlier study came to a middle-of-the-road conclusion that the threat of greenhouse warming was sufficient to justify modest steps to slow the pace of climate change, but I found that the calls for draconian cuts in GHG emissions by 50 percent or more were not warranted by the current scientific and economic evidence on costs and impacts.

The earlier studies had a number of shortcomings, but one of the most significant from an analytical point of view was the inadequate treatment of the dynamics of the economy and the climate. The earlier work examined a "resource steady state," one in which all physical flows are constant (e. g., in which population, emissions, concentrations, and climate change have all stabilized in

[1] The latest version appears in abbreviated form in Nordhaus [1991a] and in greater detail in Nordhaus [1991b].

their steady state) although there might be improvements in real incomes because of technological change. It then went on to examine the optimal control strategy in the resource steady state.

The previous approach is unsatisfactory primarily because of the extraordinarily long time lags involved in the reaction of the climate and economy to greenhouse gas emissions. Current scientific estimates indicate that the major GHGs have an atmospheric residence time over 100 years; moveover, because of the vast thermal inertia of the oceans, the climate appears to have a lag in the order of 50 years behind the changes in GHG concentrations; and there are long lags in introduction of new technologies in human economies to changing economic conditions. It would appear, therefore, that the dynamics are of the essence and that an examination of the steady state may provide misleading conclusions for the steps that we should take at the dawn of the age of greenhouse warming.

The plan of the present study is to develop a dynamic, global model of both the impacts of and policies to slow global warming. It is an integrated model that incorporates both the dynamics of emissions and impacts and the economic costs of policies to curb emissions. I call it the <u>DICE model</u> as an acronym for a "Dynamic Integrated Climate-Economy model."[2] This new model is an advance over earlier studies in that it allows for different policies in the transition path from those in the ultimate steady state. It does this through the extension of the standard tools of modern optimal economic growth theory and adding to this analysis both a climate sector and a closed-loop interaction between the climate and the economy. The model is sufficiently small as to be transparent (or at least

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[2] In a background paper, Nordhaus [1992], I developed an optimal growth model in which to analyze the issue of the optimal response to the threat of climate change under conditions of certainty. That analysis showed that the optimal policy will depend upon a number of different parameters of both the climate system and the economy.

translucent), to allow a range of sensitivity analyses, and to be available for a number of further extensions.

The basic approach is to use a Ramsey model of optimal economic growth with certain adjustments and to calculate the optimal path for both capital accumulation and GHG-emission reductions. The resulting trajectory can be interpreted as either the most efficient path for slowing climate change given initial endowments or as the competitive equilibrium among market economies where the externalities are internalized using the appropriate social shadow prices for GHGs.

The purpose of this paper is to lay out in detail the structure of the model and the nature of the assumptions. The first section lays out the algebra of the model in simplified form. The following sections derive the parameters of the model in detail. The final sections then show some empirical runs of the model and provide estimates of the optimal policy along with some sensitivity runs of alternative assumptions.

II. AN OVERVIEW OF THE DICE MODEL (Version 1.2.3)

The economic model of climate change used here must take into account, above all, the long time lags between actions or policies and responses. Nations must decide whether they will take steps now (in effect, invest in emissions reductions) in order to slow climate change over the coming centuries. Few societal decisions, and no personal ones except those involving Pascal's wager or the afterlife, have comparable time horizons.

A useful way to view decisions involving such long time horizons is with the apparatus of optimal growth theory. This approach was developed by Frank Ramsey in the 1920s (see Ramsey [1928]), made rigorous by Tjalling Koopmans and others in the 1960s, and is summarized nicely by Robert Solow in his masterful exposition of economic-growth theory [1970]. In the neoclassical growth model, society invests in tangible capital goods, thereby abstaining from consumption today, in order to increase consumption in the future.

The issue of climate change is the exact analog with respect to emissions reductions: society must take steps today, reducing consumption by devoting resources to reducing greenhouse gas emissions, in order to prevent economically harmful climate change and therefore to increase consumption possibilities in the future.

A. A Verbal Overview

Begin with a verbal description of the approach. The model is an optimal-growth model of the world economy. It is designed to maximize the discounted "utility" or satisfaction from consumption subject to a number of economic and climatic constraints. The decision variables that are available to the world economy are the rate of investment in tangible capital and the rate of emissions reductions of greenhouse gases. The time path of decisions is designed to maximize the discounted utility.

The model contains both a traditional economic sector and a novel climate sector. First begin with the "traditional" sector of the economy--the economy without any incorporation of considerations of climate change. The global economy is assumed to produce a composite commodity. It is not necessary that the countries actually be identical. Rather, the goods produced must be perfect substitutes and the production functions must be identical except for multiplicative differences in productivity. In plain language, this means that countries can differ in their quantitative attributes, but that there cannot be large differences in the composition or relative proportions. While this is a restrictive assumption, preliminary work with a more complete multi-country model suggests that this assumption has little effect upon the major conclusions.

Our composite economy is endowed with an initial stock of capital and labor, and an initial level of technology, and all industries behave competitively. Each country maximizes an intertemporal objective function, identical in each region, which is the sum of discounted utilities of per capita consumption times population; the utility function is logarithmic in per capita consumption. Output is produced by a Cobb-Douglas production function in capital, labor, and technology. Population growth and technological change are exogenous, while capital accumulation is determined by optimizing the flow of consumption over time. There is no need for international trade since the outputs of the different countries are perfect substitutes.

Turning to the "non-traditional" part of the model, begin with a number of relationships that attempt to capture the major features of the forces affecting climate change. This part includes an emissions equation, a concentrations equation, a climate equation, and a damage relationship. Emissions represent all GHG emissions, although they are most easily viewed as CO_2. Uncontrolled emissions are a slowly declining constant times gross output--a relationship which is consistent with a complex set of assumptions about the underlying production functions. GHG emissions can be controlled by increasing the prices of factors or outputs that are GHG-intensive, and we represent the abatement cost schedule parametrically by drawing upon a number of studies of the cost of GHG reduction.

Atmospheric concentrations are increased with emissions, with concentrations reduced with an atmospheric residence time of 120 years. Climate change is represented by global mean surface temperature, and the relationship uses the consensus of climate modelers and a lag suggested by recent coupled ocean-atmospheric models. The economic impacts of climate change are assumed to be increasing in the realized temperature increase.

Note that the model can be interpreted either in an optimizing framework or as the outcome of idealized competitive markets. Under the latter interpretation, there is one major leap of faith: It assumes that the public goods nature of climate change is somehow overcome in an efficient manner. That is, it assumes that, through some mechanism, countries internalize in their decision making the global costs of their emissions decisions. This seems unlikely, but the current approach has the virtue of calculating the equilibrium that would emerge were each country to behave in such a farsighted and altruistic fashion.

B. Detailed List of equations

Turn next to a specific list of the equations of the DICE model. The relationships are divided into three groups: the objective function, the economic relationships, and the climate-emissions relationships. This section outlines the equations of the model while the next section discusses the empirical derivation of the parameters.

1. Objective function

The model maximizes a social welfare function that is the discounted sum of the utility of per capita consumption. In technical language, we desire to maximize the objective function:

$$\text{(II-0)} \qquad \max_{\{c(t)\}} \quad \sum_{t} U[c(t),L(t)](1+\rho)^{-t}$$

which is the discounted sum of the utilities of consumption, U[c(t),L(t)], summed over the relevant time horizon. Here U is the flow of utility or social well-being, c(t) is the flow of consumption per capita at time t, L(t) is the level of population at time t, and ρ is the pure rate of social time preference. The exact form of the utility function will be described below.

2. Economic Constraints

The maximization is subject to a number of constraints. The first set is the economic constraints, which are described in this subsection, while the second set is the novel climate-emissions relationships.

The first equation is the definition of utility, which is equal to the size of population [L(t)] times the utility of per capita consumption u[c(t)]. We take a power function to represent the form of the utility function:

(II-1) $$U[c(t)] = L(t)\{[c(t)]^{1-\alpha}\}/(1-\alpha)$$

In this equation, the parameter α is a measure of the social valuation of different levels of consumption, which is called the "rate of inequality aversion." In the experiments reported here, we take (the limit of) $\alpha = 1$, which is the logarithmic or Bernoullian utility function:

(II-1') $$U[c(t)] = L(t)\{\log[c(t)]\}$$

Output [Q(t)] is given by a standard Cobb-Douglas production function in technology [A(t)], capital [K(t)], and labor [L(t)]. Labor inputs are proportional to population.

(II-2) $$Q(t) = \Omega(t)A(t)K(t)^{\gamma}L(t)^{1-\gamma}$$

where γ is the elasticity of output with respect to capital. The term $\Omega(t)$ relates to climatic impacts and will be described in equation (II-11).The formulation assumes that the production function takes the log-linear or Cobb-Douglas form and that there are constant returns to scale in capital and labor.

The next equation shows the disposition of output between consumption [C(t)] and gross investment [I(t)]:

(II-3) $$Q(t) = C(t) + I(t)$$

This represents the accounting identity that output can be devoted either to investment in new capital goods or to consumption.

The next equation is the definition of per capita consumption:

(II-4) $$c(t) = C(t)/L(t)$$

Finally, we have the capital balance equation for the capital stock:

(II-5) $$K(t) = (1-\delta_K)K(t-1) + I(t)$$

where δ_K is the rate of depreciation of the capital stock.

3. Climate-Emissions-Damage Equations

The next set of constraints is more of a challenge because there are no well-established relationships or set of historical data that can be drawn upon to represent the linkage between economic activity and climate change. This group consists of a set of equations representing the relationship among economic activity, emissions, concentrations, and climate change, and then links up the climate change with the rest of the economy. As with the economic relationships, it is necessary to keep these equations very simple so that the theoretical model will be transparent and so that the optimization model is empirically tractable.

The first equation links greenhouse-gas emissions to economic activity. In the analysis that follows, we translate each of the GHGs into its CO_2 equivalent. To aggregate the different GHGs, we use a measure of the total warming potential, which is the contribution of a GHG to global warming summed over the indefinite future. Approximately 80 percent of the total warming potential of the major GHGs is due to CO_2, and we therefore devote most of our attention to that gas.

In modeling GHG emissions, assume that the ratio of GHG emissions to gross output declines slowing. The uncontrolled ratio of GHG emissions to output is represented by the parameter $\sigma(t)$. In what follows, it is assumed that the exogenous decline in σ is 1.25 percent per annum.

GHG emissions can be reduced through a wide range of policies. We represent the rate of emissions reduction by an "emissions control rate," $\mu(t)$. The control rate μ represents the fractional reduction of emissions relative to uncontrolled emissions. This variable is a central one in the present study; indeed, one of the key question investigated here is the optimal trajectory of emissions control. The emissions equation is given as:

$$(II-6) \qquad E(t) = [1-\mu(t)]\sigma(t)Q(t)$$

In this equation, $\sigma(t)$ is a technological parameter which represents the trend in the GHG-output ratio. In the empirical work, it is assumed that this is an exogenous trend. The variable $\mu(t)$ is determined by the optimization.

The next relationship in the economy-climate nexus represents the accumulation of GHGs in the atmosphere. Let E(t) be the emissions of GHGs (in CO_2-equivalent terms). Then the accumulation and transportation of emissions is assumed to follow the following equation:

$$(II-7) \qquad M(t) = \beta E(t) + (1-\delta_M) M(t-1)$$

where β is the marginal atmospheric retention ratio and δ_M is the rate of transfer from the rapidly mixing reservoirs to the deep ocean; the latter is assumed to be so large as to provide an infinite sink for carbon.

The next and the most difficult step concerns the relationship between the accumulation of greenhouse gases and climate change. Climate modelers have developed a wide variety of approaches for estimating the impact of rising GHGs on climatic variables. On the whole, existing models are much too complex to be included in economic models. Instead, we employ a small structural model that captures the basic relationship between GHG concentrations and the dynamics of climate change. In what follows, the climate system is represented by a multi-layer system; more precisely, there are three layers--the atmosphere, the mixed or upper layer of the oceans, and the deep oceans. Each of the layers is assumed to be internally well mixed.

The accumulation of GHGs leads to an increase in radiative forcing, F(t). Higher levels of forcing warm the atmospheric layer, which then warms the mixed ocean, which in turn diffuses into the deep oceans. The lags in the system are primarily due to the thermal inertia of the three layers.

We can write the model as follows:

$$T(t) = T(t\text{-}1) + (1/R_1)\{F(t) - \lambda T(t\text{-}1) - (R_2/\tau_{12})[T(t\text{-}1) - T^*(t\text{-}1)]\}$$

(II-8)

$$T^*(t) = T^*(t\text{-}1) + (1/R_2)\{(R_2/\tau_{12})[T(t\text{-}1) - T^*(t\text{-}1)]\}$$

where T(t) is the temperature in the atmosphere and the upper level of the ocean; $T^*(t)$ is the temperature in the deep oceans; F(t) is the rate of radiative forcing in the atmosphere; R_1 is the thermal capacity of the upper stratum while R_2 is the thermal capacity of the deep oceans. τ_{12} is the transfer rate from the upper layer to the lower layer, and λ is a feedback parameter. A fuller explanation of the model is given below.

The next link in the chain is the impact of climate change on human and natural systems. Estimating the damages from greenhouse warming has proven extremely elusive, and I have reviewed some of the issues in Nordhaus [1991b]. For the purpose of this study, it is assumed that there is a relationship between the damage from greenhouse warming and the realized amount of warming. More specifically, the relationship between global-temperature increase and income loss is given by:

(II-9) $$d(t) = Q(t)\theta_1 T(t)^{\theta_2}$$

where d(t) is the loss of global output, θ_1 is a parameter representing the scale of damage, and θ_2 is an exponent that represents the nonlinearity in the damage function.

The last major link in the chain is the costs of reduction of greenhouse gases. This is the one area that has been extensively studied and, while not without controversy, the general shape and extent of the cost function has been sketched on a number of occasions. The final form of the equation used in the model is:

(II-10) $$TC(t) = Q(t)\, b_1\, \mu(t)^{b2}$$

where $\mu(t)$ is the fractional reduction in GHG emissions and TC(t) is the total cost of the reduction. The coefficients b_1 and b_2 represent the scale and the nonlinearity of the cost function.

Combining the cost and damage relationships, we have the Ω relationship in the production function as follows (omitting the time subscripts):

(II-11) $$\Omega(t) = (1 - b_1\, \mu(t)^{b2})/[1 + \theta_1 T\, (t)^{\theta 2}]$$

Equations (II-0) through (II-11) form the mathematical model that is optimized and whose results are described in the text. The major variables are summarized in Table II.B-1.

III. <u>DERIVATION OF THE EMPIRICAL EQUATIONS IN THE "DICE" MODEL</u>

This section presents the derivation of the equations and parameters in detail. The plan of this section is to go through each equation and explain both the rationale for the functional form as well as the source of the parameters.

Table II.B-1. Major Variables in the DICE Model

Exogenous Variables

$A(t)$ = level of technology
$L(t)$ = labor inputs
$L(t)$ = population at time t, also equal to labor inputs
t = time

Parameters

α = rate of inequality aversion
b_1, b_2 = parameters of emissions-reduction cost function
β = marginal atmospheric retention ratio of GHGs
γ = elasticity of output with respect to capital
δ_K = rate of depreciation of the capital stock
δ_M = rate of transfer of GHGs from upper to lower reservoir
λ = feedback parameter in climate model
ρ = pure rate of social time preference
R_1 = thermal capacity of the upper layer
R_2 = thermal parameters of deep oceans
$\sigma(t)$ = emissions/output ratio
τ_{12} = transfer rate from lower to upper reservoir
θ_1, θ_2 = parameters of damage function

Endogenous Variables

$C(t)$ = consumption
$c(t)$ = per capita consumption
$d(t)$ = damage from greenhouse warming
$E(t)$ = emissions of greenhouse gases
$F(t)$ = radiative forcing from GHGs
$\Omega(t)$ = output reduction due to emissions controls and to damages from climate change
$K(t)$ = capital stock
$M(t)$ = mass of greenhouse gases in atmosphere
$Q(t)$ = gross world product
$T(t)$ = atmospheric temperature relative to base period
$T^*(t)$ = deep-ocean temperature relative to base period
$TC(t)$ = total cost of reducing GHG emissions
$u(t) = u[c(t)]$ = utility of per capita consumption

Policy Variables

$I(t)$ = gross investment
$\mu(t)$ = rate of emissions reduction

Section A goes through each equation and presents the parameters along with a brief discussion of their derivation. Then sections B through F present the derivation of the parameters or equations which pose greater difficulties. This part of the study is necessarily quite extensive, so it may be useful to note where the major novel approaches have been taken at the outset. For the most part, the economic sectors are conventional in their approach. The climate sector, however, required significant attention as no existing models were appropriate for inclusion in a dynamic macroeconomic model of the kind used here. Therefore, the climate module, the carbon cycle, the cost of emissions reduction, the damages from climate change, and the trend in energy-GHG ratios are presented in greater detail in sections III.B through III.F below.

A. EMPIRICAL EQUATIONS FOR THE DICE MODEL

A.0. The objective function

The objective function of the model is to maximize the discounted value of utility:

$$\text{(III.0)} \qquad \max_{\{c(t)\}} \quad \sum_t U[c(t),L(t)](1+\rho)^{-t}, \ \rho = .03 \text{ per year}$$

The only parameter in this equation is the pure rate of social time preference, ρ. This parameter is a social choice variable that is implicit in many fiscal and monetary variables. In conjunction with other variables, it is closely connected with the market rate of interest (or marginal productivity of capital) and with the savings rate. A value of ρ of .03 percent per year (or higher) is consistent with historical savings data and interest rates. By contrast, a value for ρ of .01 and .00 per year has sometimes been used in studies of greenhouse warming, but these will imply both too-high a savings rate and too-low a rate of return on capital.

This preference function, combined with the production function below, leads to rates of return on capital that are in the neighborhood of the observed levels in industrial countries (e. g., 6 to 7 percent per year in the 1970-89 period).

A.1. Utility function

(III.A.1) $$U[c(t)] = L(t)\{[c(t)]^{1-\alpha}\}/(1-\alpha)$$

This utility function contains exogenous data on population and an assumption concerning "inequality aversion," which is represented by the parameter α, the elasticity of the marginal utility of consumption. We use the logarithmic utility function ($\alpha=0$), which is traditional in economic growth theory. By a limiting argument, (III.A.1) reduces to:

(III.A.1') $$U[c(t)] = L(t)\{\log[c(t)]\}.$$

Population data are derived from a number of sources. The basic projection method is as follows: Population growth in the initial period is assumed to be equal to the historical rate over the period 1965-87. We then assume that the growth rate declines over time at a geometrically declining rate. More precisely, let $g_{pop}(t)$ be the growth rate of population in period t and δ_{pop} be the rate of decline rate in the growth of population. We then have the growth of population in period t as:

(III.A.1") $$g_{pop}(t) = g_{pop}(t-1)\ (1-\delta_{pop}),\ \delta_{pop} = .195 \text{ per decade;}$$
$$g_{pop}(1965) = 2.03 \text{ percent per annum;}$$
$$L(1965) = 3.369 \text{ billion}$$

This technique is easily verified to lead to a gradual leveling off of population. Its advantage is that the population trajectory can be represented by two parameters and can easily be fit to different projections. For the base case, the initial growth of population in the decade 1960-69 is 2.03 percent per year and the decadal decline in the growth rate is δ_{pop} = .195. This assumption leads to a asymptotic maximum population of 10.6 billion people.

A.2. Aggregate Production Function

The next set of parameters concerns the aggregate production function:

(III.A.2) $$Q(t) = \Omega(t)A(t)K(t)^{\gamma}L(t)^{1-\gamma},\ \gamma=.25$$

The historical output and capital data are gathered from a number of sources for the years 1960, 1965, 1970, 1975, 1980, 1985, and 1990. The basic technique is to gather data for the major countries or regions and to aggregate them using purchasing-power parity weights. We use the standard Solow identification procedure which allows us to calculate $1-\gamma$ as the share of labor in national income, which is approximately 0.75 in industrial countries; little reliable data is available for poorer countries.

The variable Ω is assumed to be one during the historical period, and projections for future periods are discussed below. The only unobservable variable is A(t), which can be derived from (2) by solving for A(t) as a function of all the other observable variables.

A major uncertainty in the model involves projecting the growth of A(t), or total factor productivity, into the future. For the period 1965-87, we estimate the average annual growth of total factor productivity to be 1.52 percent per annum, where this is the average for global Output, capital, and population. It is interesting that historical studies find little slowdown in the average growth of global output over the last century of so.

Most modelers project a significant slowdown in the growth of per capita incomes in the future. For example, Table III.A-1 shows the result of the Nordhaus-Yohe survey [1983] of long-term projections of per capita income growth:

Table III.A-1
Projected rate of growth of output per worker

[Average annual growth rate, percent per annum]

	1975-2000	2000-2025	2025 and beyond
Mean	2.3	1.6	1.0
Standard deviation	0.7	0.5	0.3
Extremes	1.2;3.4	0.9;2.3	0.5;1.5

Source: Nordhaus and Yohe [1983]. These represent the average, standard deviation, and extreme values of projected increases from a number of published studies.

This study follows the shape of the decline projected in the earlier studies, although it projects a continuous rather than a step change in productivity growth. More precisely, we use the same technique developed above for population growth to project productivity growth. Let $g_A(t)$ be the growth rate of total factor productivity in period t and δ_A be the decline rate in the growth of productivity. Productivity growth in period t is then:

(III.A.2') $g_A(t) = g_A(t-1)(1-\delta_A)$;

$g_A(1965) = 1.41$ percent per annum;

$\delta_A = .11$ per decade.

Again, this assumes a leveling of productivity growth. As with the representation of population, its advantage is that the productivity growth curve can be represented by two parameters. For the base case, the estimate of the growth of total factor productivity for 1961-70 is 1.41 percent per annum. We then assume that productivity growth declines by half every six decades, which is roughly consistent with the projections shown above. This gives an estimate of the parameter $\delta_A = .11$.

A.3. Output identity

The next equation represents the division of gross output into gross investment and consumption.

(III.A.3) $$Q(t) = C(t) + I(t)$$

Data on consumption and investment are from the same sources as described for equation (III.A.2).

A.4. Per capita consumption

The next equation, for per capita consumption, is an identity derived from other variables.

(III.A.4) $$c(t) = C(t)/L(t)$$

A.5. Capital-stock balance equation

The capital balance equation for the capital stock is the following:

(III.A.5) $$K(t) = (1-\delta_K)K(t-1) + I(t), \delta_K = .10 \text{ per year}$$

In this equation, the depreciation rate of capital is taken to be .10 percent per annum, which reflects an average lifetime of capital of ten years on a declining balance method.

A.6. Emissions Equation

The next equation is the one showing the emissions of greenhouse gases.

(III.A.6) $E(t) = [1- \mu(t)]\sigma(t)Q(t);$
$\sigma(1965) = .519.$

The variable E(t) is the total emissions of greenhouse gases in CO_2 equivalent terms, carbon weight. For this variable, the emissions of the four major greenhouse gases (CO_2, CFCs, CH_4, and N_2O) are aggregated using as weights their total warming potential as determined by the Lashof and Ahuja [1990]. The emissions-control factor μ is zero during the historical period and is determined endogenously by the model for the calculation period.

The major difficult variable in equation (III.A.6) is the parameter $\sigma(t)$, which reflects the trend in CO_2-equivalent emissions per unit of gross output in the absence of controls. Relying on historical information, we estimate that σ has declined between 1 and 1.5 percent annually due to both energy-efficiency improvements and a transition away from coal. For the future, we assume that σ declines at 1.25 percent annually.

(III.A.6') g_σ = -1.25 percent per annum

The derivation of this parameter is examined in section III.E below.

A.7. The carbon cycle

The next equation reflects the accumulation of carbon concentrations in the atmosphere:

(III.A.7) $M(t) = \beta E(t) + (1-\delta_M)\, M(t-1);$

$\beta = .64;\ \delta_M = .0833$ per decade;

$M(1965) = 677$ billion tons carbon

This parameterization assumes a three-box model of the transportation of GHGs. Of emissions of GHGs, the fraction β stays in the atmosphere in the short run. In addition, the GHGs in the short-lived reservoir are transported into a very large reservoir at a rate of .0833 per decade. The derivation of this equation is presented in section III.A. Note that this equation has been derived for CO_2 and does not apply as well to the other GHGs.

A.8. Determinants of climate change

The next set of equations is the relationship between CO_2-equivalent concentrations and climate change. The first set of equations reflect the increase in globally and annually averaged surface temperature in the atmospheric and deep oceans:

$$T(t) = T(t-1) + (1/R_1)\ \{F(t) - \lambda T(t-1) - (R_2/\tau_{12})[T(t-1) - T^*(t-1)]\}$$

(III.A.8)

$$T^*(t) = T^*(t-1) + (1/R_2)\ \{(R_2/\tau_{12})[T(t-1) - T^*(t-1)]\}$$

This set of equations is discussed in section III.B below. The parameters that are derived there are:

(III.A.8') $T(1960) = .2; R_1 = 41.7;$

$$R_2/\tau_{12} = .44; T^*(1960) = .10$$

The final link in the chain is the radiative forcing from CO_2 accumulations. There is relatively little controversy about the forcings, and we use the standard formula:

(III.A.8") $F(t) = 4.1 \log[M(t)/590]/\log(2)$

where M(t) is the atmospheric loadings of CO_2 (or its equivalent) in billions of tons of carbon and F(t) is the increase of surface warming in watts per square meter (W/m_2), which is the standard measure of radiative forcing.

A.9. The Impacts of Climate Change

The next equation shows the economic impact of climate change. Equation III.A.9 shows the damage from rising global temperatures and is further analyzed in section III-C:

(III.A.9) $d(t)/Q(t) = .013\ [T(t)/3]^2 = .00144\ T(t)^2$

This states that the damages from a 3 degree C rise in average temperature is 1.3 percent of world output.

A.10. The Costs of Emissions Reductions

Finally, equation (III.A.10) is the estimated cost of reduction of GHG:

(III.A.10) $TC(t)/GNP(t) = b_1\ \mu(t)^{b2} = .0686\ \mu(t)^{2.887}$

This equation is derived in section III-D.

Next, (III.A.9) and (III.A.10) are combined into a single factor in equation (III.A.11) that enters the aggregate production function in (III.A.2).

$$\text{(III.A.11)} \qquad \Omega = (1 - b_1\mu(t)^{b2})/[1 + d(t)]$$
$$= (1 - .0686\, \mu(t)^{2.887})/[1 + .00144\, T(t)^2]$$

The detailed list of parameters is shown in Table III-A-2.

B. A Minimodel of Climate Change

B.1. Introduction

Climate modelers have developed a wide variety of approaches for estimating the impact of rising GHGs on climatic variables. On the whole, existing models are, unfortunately, much too complex to be included in economic models of the kind employed here. The models generally taken to be the most satisfactory are the large general circulation models (GCMs). These require several hundred hours of supercomputer time simply to perform a simulation, and inclusion of these in an optimization model of the kind developed here is infeasible.

Another difficulty of current GCMs is that they have generally been used to estimate the equilibrium impact of a change in CO_2 concentrations upon the level of temperature and other variables. For economic analyses, it is essential to understand the dynamics or transient properties of the response of climate to GHG concentrations. At present, only a handful of models have been used to estimate the transient path, and these have only begun to incorporate the deep oceans into their calculations.

Table III.A-2. **Initial Values of Parameters in the DICE Model**

$\alpha = 0$

$b_1 = .0686$

$b_2 = 2.887$

$\beta = .64$

$\gamma = .25$

$\delta_K = .10$ per year

$\delta_M = .0833$ per decade

$\delta_A = -1.1$ percent per year

$\delta_{pop} = -19.5$ percent per decade

$g_{pop}(1965) = 2.03$ percent per year

$g_A(1965) = 1.14$ percent per year

$g_\sigma = -1.25$ percent per year

$K(1965) = 16.0$ trillion U. S. dollars, 1989 prices

$\lambda = 1.41$ degrees C/W-m^2

$M(1960) = 677$ billion tons CO_2 equivalent, carbon weight

$L(1965) = 3.369$ billion persons

$\rho = .03$ per year

$1/R_1 = .226$ °C-m^2/watt-years

$R_2/\tau_{12} = .44$ watts/°C-m^2

$Q(1965) = 8.519$ trillion U. S. dollars, 1989 prices

$\sigma(t) = .5368\ (.9875^{t-1990})$

$T(1960) = .2$ degrees C

$T^*(1960) = .1$ degrees C

$\theta_1 = .00144$

$\theta_2 = 2$

For the present purposes, it is necessary to have a relatively small model that links together GHG concentrations and the major climatic variables. We have chosen to include only the impact of GHGs on global mean temperature in the analysis that follows. Although this analysis focuses primarily upon globally averaged surface temperature, it is recognized that this variable is not the most important for impacts. Variables like precipitation or water levels and extremes of droughts or freezes are likely to be more important. Mean temperature is chosen because it is a useful index of climate change that tends to be associated with most other important changes. In the language of statistics, temperature is likely to be a "sufficient statistic" for the other variables that have an important impact upon human and natural societies.[3] This point is shown in Figure III.B-1, which shows the estimated impact of CO_2 doubling upon both mean temperature and precipitation in a number of models. This diagram shows the high correlation between the predicted temperature and precipitation changes. Other variables are also likely to be highly correlated with the extent of global temperature change.

The approach taken here begins with a simplified model used to represent the basic dynamics of climate change. We then use larger models to calibrate the major parameters of the "minimodel." It must be emphasized, however, that this representation is highly oversimplified and that a major effort by climatologists and climate modelers will be necessary to develop an adequate and parsimonious representation of the linkage between climate and GHG concentrations.

B.2. The model

We are interested in developing a model that represents the broad aggregates of climate over decadal time scales; this approach differs from the standard GCMs, which

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[3] See Savage [1954] for a discussion of sufficient statistics.

operate on the scale of minutes and hours. For our purpose, it will be useful to develop a globally averaged and highly aggregative model of the climate system.

In what follows, we represent the climate system by a multi-stratum system which includes an atmospheric-land portion and a number of ocean levels. In the simplest approach (which is called the "one-equation model"), the system contains an atmosphere and a single oceanic stratum. In the more complete approach (called the "two-equation model"), the system contains an atmosphere, an upper-ocean stratum, and a lower-ocean stratum. In both cases, the system has an atmosphere that is warmed by solar radiation, contains GHGs, and is in short-run radiative equilibrium. The system is represented by a single variable, average surface temperature. The atmosphere mixes quickly with the upper oceans, which impose a certain amount of thermal inertia on the system because of their heat capacity. In the two-equation model, the upper stratum of the ocean exchanges water with a lower stratum, representing the deep oceans, and the rate of heat transfer is proportional to the rate of water exchange.

This model is a box-diffusion model, which is simpler to include in economic models than the mixed box-diffusion and upwelling-diffusion approach that is widely used in medium- and large-scale models today.

The approach here follows closely the model developed by Schneider and Thompson [1981]. For conciseness, we describe the two-equation model, and the one-equation model is easily described after the more complex model is derived. Assume a coupled atmosphere-ocean model in which there are three strata--the atmosphere, the mixed stratum of the oceans, and the deep oceans. Each of the three strata is assumed to be well mixed, and in addition the atmosphere and the upper stratum of the ocean is assumed to be well mixed. The accumulation of GHGs warms the atmosphere, which then mixes with and warms the upper ocean, which in turn mixes with and heats the deep oceans. The lags in the system are primarily due to the thermal inertia of the three strata.

More precisely, assume the following. Let

$T_i(t)$ = temperature of stratum i in period t (relative to the pre-industrial period, degrees C)

$\mathbf{T}(t)$ = a column vector of the $T_i(t)$

τ_{ij} = the annual heat transfer from stratum i to stratum j

Γ = n x n matrix of τ's in which the ith element in row j is τ_{ij}

$F(t)$ = radiative forcing in atmosphere

$\mathbf{F}(t)$ = column vector of radiative forcing such that $\mathbf{F}' = [F\ 0 \ldots 0]$.

The system can then be written as follows:

(III.B.1) $$\mathbf{T}(t) = \Gamma\mathbf{T}(t-1) + \mathbf{F}(t-1)$$

In the two-equation approach, there are two "boxes" or layers, where T (light-faced) = temperature of the atmosphere, land, and upper stratum of the ocean and T* = temperature of the deep oceans. Schneider and Thompson suggest the following simplification of (III.B.1), which gives us the <u>two-equation model</u>:

(III.B.2)

$$T(t) = T(t-1) + (1/R_1)\ \{F(t) - \lambda T(t-1) - (R_2/\tau_{12})[T(t-1) - T^*(t-1)]\}$$

$$T^*(t) = T^*(t-1) + (1/R_2)\ [(R_2/\tau_{12})(T(t-1) - T^*(t-1))]$$

In this approach, the R_1 is the thermal capacity of the atmospheric layer and the upper oceans, R_2 is the thermal capacity of the deep oceans, and τ_{12} is the transfer rate from the upper level of the ocean to the deep oceans. A key parameter in all models is λ, or the "feedback parameter." This parameter is a way of representing the equilibrium impact of CO_2 doubling on climate, and I comment on its significance shortly.

It will be useful to compare the results of the two-equation model with a simpler version (the simpler version is often used to describe the equilibrium behavior of climate systems and was used in the static cost-benefit analysis in Nordhaus [1991b]). To obtain a one-equation model, we treat the ocean as a single well-mixed box (although it might be shallower than both strata of the ocean in the two-equation model). This then gives:

(III.B.3) $$T(t) = T(t-1) + (1/R_1)\,[F(t) - \lambda T(t-1)]$$

This "one-dimensional" model has been widely used in discussions of the impact of greenhouse forcing. By solving either (III.B.2) or (III.B.3) for a constant T, it is easily seen that the long-run or equilibrium impact of a change in radiative forcing is $\Delta T/\Delta F = 1/\lambda$. We use the parameter T_{2xco2} to represent the equilibrium impact of doubled CO_2 concentrations on global mean surface temperature. Solving (III.B.2) or (III.B.3), we see that $T_{2xco2} = 1/\lambda$. The derivation of λ is given in numerous sources.

A more complex variant of this system has been developed for purposes of describing the behavior of more complex GCMs. For example, a study of Schlesinger and Jiang [1990] developed a seven-layer model and parameterized it using GCM calculations of the Oregon State University model. In principle, that model could be incorporated into the economic model here, although it would probably not be as flexible or as transparent as the models used in this study. The major advantage of the approach used here is that the two major issues in climate change--the equilibrium sensitivity and the speed of adjustment--can be represented in terms of the parameters λ and $(1/R_1)$.

B.3. Calibration to General Circulation Models

The next step is to find the appropriate numerical representation of the simplified climate models in (III.B.2) and (III.B.3). We approach this by calibrating the models to transient runs of three climate models and then by estimating the models using historical data. This section describes the calibration to GCMs.

a. Models Used for Transient Runs

For calibration purposes, we have examined three different models.

ST: The first is the Schneider-Thompson runs [1981]. These have the advantage of using the explicit representation of equation set (III.B.2) and the disadvantage of being highly oversimplified relative to larger models. For this run, we can construct the model explicitly using the parameters developed in the original study.

SMB: The most completely developed model is a coupled atmospheric-ocean model developed by Stouffer, Manabe, and Bryan [1989]. This model is a complete representation of both the atmosphere and the oceans and provides a transient calculation of the impact of slowly rising CO_2 concentrations.

SJ: A third model, somewhat in the spirit of the approach here, is a parametric representation of the Oregon State University model in a small model of the coupled atmospheric and six-layer ocean model by Schlesinger and Jiang [1990]. This model uses the larger model to determine the parameters of the smaller model and then simulates the smaller model for transient runs over longer periods.

b. Parameter calibration

The models in (III.B.2) and (III.B.3) can be represented in terms of four parameters: R_1, R_2, τ_{12}, and λ. The first two parameters represent the heat capacity of the atmosphere, land, and upper stratum of the ocean (R_1) and of the deep ocean (R_2). The third parameter represents the transfer coefficient between the two strata. The parameter λ is the feedback coefficient which gives the crucial CO_2-temperature equilibrium relationship.

For calibration purposes, we take the feedback parameter (λ) from equilibrium runs of the three models described above (which implicitly assumes that there is a unique and stable long-run equilibrium for each radiative forcing). It is easily seen that $1/\lambda$ = equilibrium impact of CO_2 doubling on global mean surface temperatures, defined as T_{2xCO2}. Figure III.B-1 shows the value of T_{2xCO2} for a number of models reviewed by the IPCC. The recent report of the U. S. National Academy of Sciences [1991] concluded that value of T_{2xCO2} lay between 1° C and 5° C. The values of the parameter λ for the different models are shown in Table III.B-1.

The parameters for the transfer between the upper and lower levels are taken from both empirical studies and empirical estimates of the heat capacities of the different reservoirs, which are easily calculated once the sizes of the reservoirs are determined. ST takes the first reservoir to be the atmosphere, land, and the first 133.5 meters of the oceans. The second reservoir is the next 1500 meters of the deep oceans. This gives parameter values of R_1 = 13.2 watt-years/°C-m^2 and R_2 = 223.7 watt-years/°C-m^2. The major problematical assumption here is the depth of the boundary between the two ocean strata.

The other two parameters are highly uncertain and cannot be determined with precision from fundamental physics. The transfer coefficient (τ_{12}) refers to the speed at which water transfers between the deep and upper oceans. Schneider and Thompson propose a value of τ_{12} = 550 years based on observations of W. Broecker. Other studies use a shorter period. For calibration purposes, we take a slight downward adjustment of the ST value to τ_{12} = 500. By combining this value with the value for R_2 we obtain the last coefficient in the first equation of (III.B.2), which is R_2/τ_{12} = 223.7 (watt-years/°C-m²)/500 years = .44 watts/°C-m².

The final parameter is $1/R_1$, which is the effective heat capacity of the atmosphere and the upper level of the oceans. This is calculated to be $1/R_1$ = 1/(13.2 watt-years/°C-m²) = .076 by Schneider and Thompson. For the other two models, we estimate the parameter from the equation system in (III.B.2) and (III.B.3). The estimated number is likely to differ from ST's calculated value because the presence of the intermediate layers will give an apparent larger size to the upper stratum than will a two-stratum model.

The balance of the coefficients were determined as follows. For ST, we programmed the model and calculated the exact path of the temperatures for the different layers. We then estimated the parameters of both equations (III.B.2) and (III.B.3) using iterative least squares.[4] Because the temperatures were calculated using the equations in (III.B.2), the annual equations fit perfectly for the two-equation model. The equation fit less well for the one-equation model because the two layers were forced to behave as one. Table III.B-1 shows the estimated parameters of the models for both the one-equation and the two-equation versions. The last column (SEE) shows the standard error of estimate of the equation (in degrees C). This is not an error relative to actual observation; rather, it represents the error of the approximation in the "minimodel" relative to the larger model. On the whole, the errors are in the hundredths of a degree C and therefore represent the larger model calculations quite accurately.

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[4] By "iterative least squares" we mean the following: The equation set in (III.B.2) is a recursive set of nonlinear equations and cannot easily be estimated directly, and the lower temperature cannot be directly observed. We therefore estimated each equation using the values from the other equation, and iterated between the equations until the values of the endogenous variables converged. This procedure will underestimate the standard errors but is sufficient for the purposes of this paper.

For the SMB model, we began with the calculated temperature path for the model (kindly provided by Dr. Stouffer). This path was for years 5, 15,...,95 of a simulation in which CO_2 concentrations grew at 1 percent per annum compounded. We then fit a polynomial function to the path to obtain interpolated values. Next, we used the program for the ST model to calculate the values of temperature for the lower layer of the oceans. Finally we estimated the one-equation and two-equation models for the interpolated values for the atmosphere and the estimated values for the lower layer. Again the values are shown in Table III.B-1.

In addition, we show in Figure III.B-2 the original values from the SMB model (shown as asterisks), the interpolated values (TRFOR), and the estimated values for both the one-equation and two-equation models (TRFOREQ1 and TRFOREQ2, respectively). In addition, Table III.B-4 shows the correlation matrix of the four series. It is apparent that the equations of the "minimodel" fit the calculations quite well, with the average absolute error being less than 0.1 degrees C.

For the SJ model, we took follow the same approach as in the SMB model. We took from that study the calculated temperature increases for the atmosphere, upper layer of the ocean, and lower layer of the ocean at ten year intervals. We then interpolated the annual figures using polynomial interpolation. Finally, we estimated the free parameter using ordinary least squares. The estimated values of the parameters are shown in Table III.B-1.

B.4. Estimated Values from Historical Data

One of the major tasks of researchers in the area of global climate change is to use historical data to test or validate model calculations. This topic is of independent interest but in this study we compare model forecasts with the instrumental record to determine whether the models described in the last section fit tolerably well the actual historical record of temperature changes.

a. Methods

In this section, we estimate the equation sets (III.B.2) and (III.B.3) using historical data on atmospheric forcing and observed mean atmospheric surface temperatures. The temperature series covered the period 1862 to 1989 and relied on estimates of Jones et al. [1990]. These data are intended to cover both terrestrial and oceanic stations, although the coverage is sparse for several regions.

In addition, we calculated the radiative forcings from the four major greenhouse gases using a variety of data on atmospheric concentrations and combining those into a series on radiative forcing using conversions suggested by T. Wigley and used in the IPCC report [1990].

The estimation technique is the following. For concreteness, we describe the technique for the two equation model and then explain how it would apply to the one-equation model. Recall that the two-equation model can be written as follows:

$$T(t) = T(t-1) + \alpha_1 \{[F(t) - \alpha_2 T(t-1) - \alpha_3 [T(t-1) - T^*(t-1)]\}$$

(III.B.2)

$$T^*(t) = T^*(t-1) + \alpha_4 [T(t-1) - T^*(t-1)]$$

where $\alpha_1 = (1/R_1)$, $\alpha_2 = \lambda$, $\alpha_3 = R_2/\tau_{12}$, and $\alpha_4 = (1/\tau_{12})$. The historical data is insufficient to estimate more than two of the parameters, so we used physical data from the models to calibrate the two least important parameters, α_3 and α_4. On the basis of the last section, these are taken to be $\alpha_3 = .44$ and $\alpha_4 = 1/500$. Note that data on T^* are currently unavailable.

In addition, we assume that there are random terms reflecting either misspecifications of the structure (such as the oscillations due to El Nino) or random external forcings (such as volcanic influences or changes in solar luminosity). We specify these as a random term with a first-order autoregressive error. On the basis of estimating a linear approximation to (III.B.2), we obtain an estimate of the autocorrelation coefficient of 0.8, and this is used in the estimation:

(III.B.4) $u(t) = 0.8\ u(t-1) + \epsilon(t)$

where $\epsilon(t)$ is assumed to be independently and identically distributed.

Combining the assumptions, we then obtain the final equation to be estimated:

$$T(t) = T(t-1) + \alpha_1 \{F(t) - \alpha_2 T(t-1) - 0.44\ [T(t-1) - T^*(t-1)]\} + u(t)$$

(III.B.5)

$$T^*(t) = T^*(t-1) + .002\ [T(t-1) - T^*(t-1)]$$

It is difficult to estimate this equation set directly, so we use a number of procedures to estimate the parameters. The first procedure is to ignore the autoregressive term and estimate the equation by nonlinear least squares. In this procedure, we use two particular values for the temperature sensitivity coefficient and estimate the inertia term (α_1).

Table III.B-2 shows the results of these estimates. The inertial variables (α_1) do not differ greatly across the different specifications, ranging from .014 to 0.02. These yield highly different e-fold times (exponential half-lives) primarily because of the differing feedback coefficients.

The inertial parameter (α_1) is not well-determined even when the other parameters are specified, particularly when the T_{2xco2} parameter is at the high end of the range. The data indicate that the time lag of adjustment is longer than that in the GCM models in Table III.B-1 if T_{2xco2} is at the high end of the range of values (say 3 or more). For example, using a value of T_{2xco2} of 4 requires a time-lag coefficient of about two-thirds of the GCM models for the one-equation model and one-third to one-fifth of the GCM models for the two-equation studies. Put differently, for a T_{2xco2} of 4, the e-fold value (which gives the time to reach .63 times the equilibrium value) is around 63 years in the one-equation model for high T_{2xco2} as compared to values of 29 to 34 years in the GCMs.

The standard errors of the coefficients are given in brackets under the estimates in Table III.B-2. In addition, we have shown the standard errors of the equations as SEE. The SEEs are virtually equal for the four estimated equations; it is clear that the likelihood function is extremely flat over four specifications.

To capture the full specification in (III.B.5), we have provided a likelihood-function estimate of the equation for different combination of parameters. In this set of estimates, we have choose five different values of the temperature-sensitivity coefficient, T_{2xco2}, values of 1, 2, 3, 4, and 5° C per CO_2 doubling. In addition, we have estimated the likelihood of the equations for four different values of α_1 = .01, .02, .05, .10. These represent thermal inertias of from one-half to five times the figures from the theoretical models shown in Table III.B-2.

Table III.B-3 and Figure III.B-3 show the estimated likelihood of each of the 20 combinations of parameters. These estimates indicate that either the inertia coefficient (α_1) is very low (in the order of .01) or the temperature-sensitivity coefficient is at the lower end of the range. A low value of the inertia coefficient indicates that the observed temperature increase is quite low because the thermal inertia of the system is much higher than any of models indicates. This possibility is shown by the high value of the likelihood for temperature-sensitivity coefficients of 3.1 through 5.0 with α_1 = .01. The other set of coefficients with high likelihood is for values of T_{2xco2} of 1 and 2, where almost any of the inertia coefficients are plausible. Conditional on estimates of the inertial parameter generated by most models (in the order of α_1 = .02), the best estimate of T_{2xco2} is 2, although values of 1 and 3 are between one-half and two-thirds as likely. Integrating across the different values of α_1, values of T_{2xco2} of 1 and 2 have the highest and almost equal likelihood.

Figure III.B-4 shows the estimates contribution of historical concentrations of GHGs to temperature changes over the last 100 years according to four of the two-equation models shown in Figure III.B-6 and Table III.B-3. The middle two curves are the two combinations of parameters with the highest likelihood (T_{2xco2} = 2.0 and α_1 =.02; T_{2xco2} = 4.0 and α_1 =.01). These clearly track the historical data tolerably well. The upper path (T_{2xco2} = 4.2 and α_1 =.02) is close to the SMB model and is quite unlikely given the historical data. The lower path (T_{2xco2} = 1.0 and α_1 =.01) is one that has high inertia and low temperature sensitivity and is also moderately unlikely.

Figure III.B-5 shows another way of examining the differences among the alternative models. This shows, for the two-equation model, the results of an increase in CO_2 concentrations of 1 percent per year compounded (doubling in 70 years). The asterisks show the calculations of the SMB model. The three top curves show the transient runs generated by the three models analyzed here using the parameters shown in Table III.B-1. The lower two curves show the paths generated by the parameters estimated from the historical data (with the high assuming a T_{2xco2} = 4.0 and the low assuming a $T_{2xco2} = 1.0$). This figure shows the predicament that the GCM models appear to give transient temperature increases well above those consistent with the historical record.

B.5. Results and Discussion

On the whole, the historical data have great difficulty distinguishing among the different models. The reason for this is that the strength of the "greenhouse signal" is relatively weak over this period. There is not enough signal independently to estimate both the feedback factor and the time-lag coefficient.

In addition, the statistical estimates for the historical data can be biased if there are slowly moving trend variables that are correlated with the greenhouse gas signal. Two such variables have been recently identified: aerosols from combustion and ozone depletion. Both of these are likely to be negatively correlated with the greenhouse forcings, so the estimates may be biased downwards.

We can use the likelihood estimates to evaluate the likelihood of the three models evaluated here in terms of the historical data. The maximum likelihood of the twenty cases shown in Figure III.B-4 is $p = .111$. Using the parameters for the one-equation model in Table III.B-1, the likelihood for the ST model is $p = .0312$, for the SMB model $p = .0026$, and for the SJ model $p = .0470$. By conventional likelihood-ratio tests, these are not particularly attractive, but they cannot be definitively ruled out either.

The historical data pose a major issue for economic modeling of policies for slowing climate change. If the historical data provide the appropriate parameters, then climate will change much more slowly than the standard models estimate. This is a major uncertainty that will continue to plague policy studies.

Figure III.B-6 shows the estimates of the key parameters of the different models and the estimates for the historical data. This figure shows for both the one-equation model and the two-equation model the values of the temperature-doubling coefficient (T_{2xco2}) and the period required for reaching 63 percent of the equilibrium value (the e-fold time).

In the study here, for the base case we use the results of the Schlesinger-Jiang study. This study is in the middle of the pack of most of the GCM results and its value of T_{2xco2} is at the center of the judgmental figures of recent National Academy Panels as well as the views of the IPCC.[5]

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[5] One final transformation is required for the economic model. The time step in the model is ten years. We therefore took each of the models and simulated it using the one-year parameters shown in Table III.B-1. We then estimated using ordinary least squares the coefficients for the ten-year differences instead of the 1-year differences.

C. Estimates of the Impact of Climate Change

Perhaps the most controversial aspect of an economic analysis of climate change involves the estimates of the impact of climate change. At present, there are few estimates of the impacts, and these are generally concentrated in advanced industrial countries. The estimates used here are presented as extremely rough, order-of-magnitude figures; it is hoped that they can be improved considerably in the coming years as researchers focus more attention on the impacts of climate change.

Current analyses generally estimate the impact of climate change on a country or region at a particular time; most often, studies focus on the impacts of a CO_2 doubling or on a 3 degree C rise in temperature. This section draws upon impact estimates, but additionally presents a technique for estimating the impact of climate changes over time as a function of the composition of output and of the impacts of climate change by sector. The estimates rely upon studies of impacts by sector for the United States and then apply these to different countries as a function of the projected future industrial compositions.

C.1. Impact by Sector: Methods

The basic framework is that the damages in each country are a function of the sectoral composition of national output and the extent of climate change. More specifically, assume that output in sector i of country j at time t is given by $x_{ij}(t)$. For simplicity, we assume that there is no impact of different policies or of climate change on relative prices, so that the impacts can simply be summed across the different sectors.

To construct the impact of climate change, we assume that the impact of climate change is a function of the change in global mean temperature from pre-industrial times, this being T(t). The damage in a particular industry is assumed to take the form:

(III.C.1) $d_{ij}(t) = \eta_i \, T(t)^2 \, Q_{ij}(t)$

where $d_{ij}(t)$ is the loss of output in the sector, η_i is a damage coefficient that is specific to a particular industry, and the quadratic term on temperature reflects the assumption, consistent with a few studies, that the damage is quadratic in the temperature increase. For example, if the equation for agriculture is $d_{farm,US}(t) = \eta_{farm} \, T(t)^2 \, Q_{farm,US}(t) = .01 \, T(t)^2 \, Q_{farm,US}(t)$, this indicates that a rise of global mean temperature of 3 degrees C would lower farm output in the United States by 9 percent.

We have surveyed the estimates of climate damage elsewhere for the United States (see Nordhaus [1991b]). From that study, we derive the estimates of the η coefficients shown in Table III.C-1. The last column in that table shows the estimated damage from a 3 degree C warming as a percentage of the GNP originating in four sectors. The adjustments for each sector are as follows:

1. Agriculture. To apply this methodology for future growth we do the following. We first project the share of agriculture in each country over the next 100 years. This projection is made assuming that the share of agriculture is a function of the logarithm of per capita GNP. We assume that the coefficient relating the rate of decline of the share of agriculture and the growth of per capita income is two-thirds of the coefficient estimated for the 1965-87 period.

2. Energy. For energy losses, we assume that the energy sector is two percent of total GNP.

3. Sea level rise. The most difficult calculation involves the losses due to sea-level rise. We begin with studies for the United States, which are shown in Table III.C-1. To apply this technique to other countries requires a detailed investigation of the geography and state of development and infrastructure for each country, a massive task that has not yet been undertaken.

Instead, we use a very simple correction of the coastal activity taking into account the coastline of different countries. We begin with a measure of "coastal vulnerability," f_i, which is the ratio of the coastal area to total land area. We define the coastal area as that area within ten kilometers of the coastline. Figure III.C-1 shows the relationship between the index of coastal vulnerability and the logarithm of per capita GNP.[6] On the whole, there appears to be a positive association between coastal vulnerability and per capita GNP.

In addition, we have examined some crude regressions between per capita GNP and a number of variables. A representative relationship is:

$$\log(\text{per capita GNP}) = \text{constant} + \underset{[.15]}{.62}\ \log(\text{coastal vulnerability}) + \text{other variables}$$

where the standard error of the coefficient is in brackets. This relationship confirms the impression that coastal countries tend to have higher incomes, a relationship that probably goes back at least to Periclean Athens.

For the United States, we calculate that the cost to coastal activities of an equilibrium CO_2 doubling is $5.3 billion per year at 1981 prices and national income (see Nordhaus [1991b]), this representing about 0.2 percent of total output. For other countries, we assume that a country with the same coastal vulnerability index has the same cost (as a fraction of its GNP) as does the United States.[7] On the basis of the regression equation above, we assume that the impact is a function of the square root of

[6] We have excluded Hong Kong from the plot and have not adjusted for the irregularity of coastlines.

[7] Another approach would be to assume that the absolute losses are the same per unit of land threatened. This makes no sense as the contribution of land to economic activity varies immensely across different countries.

the vulnerability index, so that a country with four times the fraction of its land in coastal zones as the United States will have a GNP loss twice as much as the U. S.

4. Other sectors. For the balance of GNP, we assume that the total loss is three-quarters of one percent of GNP, which gives an impact coefficient of .71 percent.[8]

[8] This estimate is a residual one and is derived from the judgment that the total loss for the U. S. is around 1 percent of national output for an equilibrium CO_2 doubling. In the study of Nordhaus [1991b], measured losses were actually much smaller than 1 percent of GNP. Rather, the 1 percent estimate is a precautionary guess as to the magnitude of "surprises" from climate change that are likely to occur in the non-farm sectors. Studies by William Cline have identified losses of this order of magnitude for the non-farm sectors.

2. Data and Results for Individual Countries

Data on the various series were obtained primarily from the World Bank, World Development Report, 1989 and from the U. S. Central Intelligence Agency, World Factbook, 1990. Some adjustments were made to the per capita GNPs of Eastern Europe and the USSR to reflect recent estimates of their output levels.

This methodology gives an estimate of the impact of global warming on total output. The lower bound of the impact is .71 percent of output for landlocked states with no agriculture. For a state with a great deal of coastal activity and a large part of the economy in agriculture, the loss can exceed 4 percent of GNP.

To obtain a rough estimate of the distribution of different countries to climate change, we have calculated the estimated vulnerability of countries as measured by the estimated loss due to a 3 degree C warming. We then plot in Figure III.C-2 the estimated cumulative vulnerability to climate change for 1987, where the index of vulnerability is plotted against the percentage of world population with vulnerabilities at or above that level. Although the United States is estimated to have a vulnerability near 1 percent of national output, the majority of the world's population has output vulnerability around twice that (primarily because of the large size of farming in countries will large populations).

Figure III.C-3 shows for 2050 the same estimated distribution function as does Figure III.C-2 for 1987. As can be seen, there is a modest shift downward in the curve reflecting the projected decline in the farming sector in most countries. Finally, Figure III.C-4 shows the same graph for cumulative output in 1987 rather than cumulative population. The fraction of the output at risk is less than the fraction of the population at risk because of the smaller share of farming in high income countries.

3. Overall losses to world output

Finally, we estimate the vulnerability of global output to climate change. Total world output losses are derived by taking the weighted average of the losses of the different countries, weighted by GNP and by the share of each sector in each year. The total estimates are shown in Table III.C-2.

Overall, the estimates suggest that the global losses from an equilibrium CO_2 doubling is in the order of 1⅓ percent of national output. This estimate depends heavily on the estimates for the United States and the method of extending these estimates to other countries. Both of these are subject to large margins of errors.

The result of this calculation is to suggest that, using the United States impacts by sectors, the global losses are likely to be larger than those for the United States. The global impact of a CO_2 doubling is in the order of 1.33 percent of national output at 1987 output weights for all countries. The global impact is slightly larger than that for the United States because of the larger share of agriculture outside the U. S. The surprising finding is the small size of the differential.

The other major surprise is that the projected shifts in the sectoral and national distributions of output over the next century do not appear to have any substantial effect upon the estimated impact of climate change on total output. According to the calculations, the total impact of a CO_2 doubling is within .03 percentage points of 1.33 percent of output for the entire period from 1987 to 2100. The reason for the lack of change in global vulnerability is the following: On the one hand, agriculture is shrinking in most countries, which leads to a decline in vulnerability. On the other hand, those countries with a large part of their economies devoted to agriculture are growing more rapidly than the industrialized economies, and growth is strongest in countries which appear to have high coastal vulnerability. These two trends appear to offset each other over the next 100 years. It should be noted, again, that, because of infirmities in the underlying estimates of damage, this result does not inspire high levels of confidence.

On the basis of these calculations, we will use a baseline estimate of the impact of a CO_2 doubling on real income of 1.3 percent of output. In addition, there is evidence that the impact increases sharply as the temperature increase, and we assume that the relationship is quadratic.[9] Therefore, the final relationship between global temperature increase and income loss is:

$$d(t) = (1.3/9)\, T(t)^2\, Q(t) \qquad \text{(III.C.2)}$$

where d(t) is the loss of global output.

[9] The major piece of evidence about the nonlinear nature of the impact of temperature rise is from studies of coastline impacts. See EPA [1989] and studies by Gary Yohe. The nonlinear nature of the costs is also likely to hold because of the adaptive nature of the response of human activities to the existing climate.

D. The Costs of Reduction Emissions of Greenhouse Gases

One of the key components in devising sensible policies to slow global warming is the estimation of the costs of reducing emissions of greenhouse gases (GHGs). To motivate the approach used here, we concentrate on CO_2. We assume that there is a standard supply and demand functions for the carbon embodied in goods and services. For simplicity, assume that there are two goods, carbon-based and non-carbon-based output. Further assume that the schedules can be represented by standard iso-elastic demand and supply curves. Then we can write the demand and supply of carbon-based output (measured in terms of the CO_2 emissions) as follows:[10]

(III.D.1) $$E(t) = A(t)\ P(t)^{-\epsilon}, \quad \epsilon > 0.$$

(III.D.2) $$E(t) = B(t)\ P(t)^{\delta}, \quad \delta > 0.$$

In these equations, E(t) is the quantity supplied or demanded of the carbon-based output, which will lead to emissions of CO_2, while A(t) and B(t) are inessential functions of time, income etc. P(t) is the price of carbon-based output, ϵ is the price elasticity of demand, and δ is the price elasticity of supply.[11]

We are concerned with estimating the impact on GHG emissions of constraints in the form of taxes or regulations. In the model, we impose carbon constraints; this is economically equivalent to levying a carbon tax on the production or consumption of

[10] Note that this section uses some inessential notation differently from other sections.

[11] Note that the notation in this section is slightly different from that in the balance of this study.

GHGs. This is most easily represented by an ad valorem tax, $\theta(t)$, which is measured as a fraction of the post-tax price. P(t). This yields the transformed equations:

(III.D.3) $$E(t) = A(t)\, P(t)^{-\epsilon}$$

(III.D.4) $$E(t) = B(t)\, [P(t)/(1+\theta(t))]^{\delta}$$

where $P(t)/(1+\theta(t))$ is the "net" price, and P(t) is the actual market price. The market equilibrium is obtained by solving equations (III.D.3) and (III.D.4) for price and quantity. We transform the variables by assuming that lower-case Roman letters represent the natural logarithms of upper-case Roman letters. This yields:

(III.D.5) $$e(t) = a(t) - \epsilon\, p(t)$$

(III.D.6) $$e(t) = b(t) + \delta[p(t) - \ln(1+\theta(t))]$$

which yields

(III.D.7) $$p(t) = [a(t) - b(t) + \delta \ln(1+\theta(t))]/(\delta+\epsilon)$$

(III.D.8) $$e(t) = [\delta a(t) + \epsilon b(t) - \epsilon\delta \ln(1+\theta(t))]/(\delta+\epsilon)$$

Equation (III.D.8) shows the impact of changing carbon taxes on carbon consumption. For small taxes, $\ln(1+\theta) = \theta$. Taking the derivative of (III.D.8) for small taxes we obtain:

$$\text{(III.D.9)} \qquad \frac{\partial e(t)}{\partial \theta(t)} = - \frac{e\delta}{\delta + e}$$

To determine the welfare effects of taxation, we use the standard formula for local deadweight loss, which gives the deadweight loss for a given tax, $D(\theta)$. This yields:

$$D(\theta) = ½ [E(t)(\theta)-E(t)(0)] [P(t)(\theta)-P(t)(0)]$$

For small taxes, this reduces to:

$$D(\theta) = ½ [-\epsilon\delta/(\delta+\epsilon)] [\delta/(\delta+\epsilon)]\theta^2 P(t) Q(t)$$

or

$$\text{(III.D.10)} \qquad D(\theta) = - ½ \, e \, [\delta/(\delta+\epsilon)]^2 \, \theta^2 \, P(t) \, Q(t)$$

This equation shows that the dead-weight loss is proportional to the size of the industry and to the square of the tax rate, and is a complicated function of the supply and demand elasticities.

Finally, we can derive the relationship between the deadweight loss, the carbon tax, and the reduction in emissions as follows. The dollar value of the carbon tax for small taxes is given by $\theta(t)P(t)$. The percentage reduction of carbon usage for small taxes is equal to $-e(t)$. Therefore, we have from (III.D.9):

$$\text{(III.D.11)} \qquad \frac{\partial e(t)}{\partial [P(t)\theta(t)]} = - \frac{e\delta}{\delta + e} = - \gamma$$

where $\gamma = e\delta/(e+\delta)$. We can write this in terms of a formula to be estimated as follows:

(III.D.12) $E(\theta) = E(0)(1+\theta)^{-\gamma}$

or

(III.D.13) $\ln[1 - \mu(\theta)] = - \gamma \ln(1+\theta)$

or

(III.D.14) $\mu(\theta) \approx \gamma\theta$

where $\mu(\theta)$ is the fractional reduction in carbon production or emissions [= 1 - $E(\theta)/E(0)$]. Equation (III.D.14) states that (for small taxes) the fractional reduction of carbon emissions is proportional to the product of γ (which is a parameter determined by the elasticities of supply and demand) and the proportional tax rate on carbon.

Although this derivation was not known at the time, equation (III.D.13) was used in my survey (see Nordhaus [1991a]) and represents a convenient way to parameterize the marginal cost function for different models. It suggests that the appropriate way to compare models is in terms of (a) the percentage difference of the carbon emissions from a baseline (i. e., uncontrolled or untaxed) path and (b) the percentage carbon tax (or ratio of controlled to uncontrolled marginal cost of carbon fuels), or the costs relative to either total energy expenditures or GNP. Comparisons that look at the absolute levels of costs across different models are likely to provide differences simply due to different baseline runs.

In representing the costs of reducing GHGs in optimization models, it is convenient to work with the total cost function rather than the marginal cost function. To a first approximation, the equation represented by (III.D.10) shows the economic costs from reducing carbon emissions. We can then substitute (III.D.14) into (III.D.10) to obtain the total cost of reducing carbon emissions as a function of the reduction:

$$\text{(III.D.15)} \qquad D(\theta) = -\tfrac{1}{2}\, e\, \gamma^2 \theta^2 P(t)\, Q(t)$$

$$= -\tfrac{1}{2}\, e\, \gamma^2 (\mu/\gamma)^2 P(t)\, Q(t)$$

$$= -\tfrac{1}{2}\, e\, \mu^2 P(t)\, Q(t)$$

This equation states that the cost of reducing carbon emissions or output is linear in the demand elasticity, quadratic in the reduction, and proportional in the total expenditures on carbon output. For flexibility, we estimate (III.D.15) with free coefficients. For data, we use the total cost estimates from the survey in Nordhaus [1991]. This yields the following equation:

$$\text{(III.D.16)} \quad \ln[D(\mu)/GNP] = \underset{(.14)}{-3.54} + \underset{(.038)}{2.89} \ln[\mu(\theta)] \quad R^2 = .984$$

where the standard errors are in parentheses. The curve fits reasonably well. The curve has an exponent greater than 2 because of composition effects. The initial reduction of GHGs is extremely inexpensive because of the low cost of reducing CFC production; after that point, however, the costs become closer to the quadratic form in (III.D.15). One interpretation would be to determine the cost of reducing emissions to zero ($\mu = 1$). According to this equation, this yields an estimate of
$D = -\tfrac{1}{2}\epsilon\ P(t)\, Q(t) = e^{-3.54}\ GNP = .0290\ GNP$. Expenditures on total carbon-based fuels are in the order of $1400 billion out of world output of $20,000 billion, which gives an estimate of ϵ of .83 (for final demand). This implicit demand price elasticity is comfortably in the range of estimates of many studies.

Figure III.D-1 shows the estimated total cost curve for the reduction of GHGs from the survey in Nordhaus [1991] as the solid line; this shows the total cost as a fraction of world output. The dashed line in Figure III.D-1 shows the parameterization used in the optimal growth model.

III.E. Greenhouse Gas Emissions and Energy-Efficiency Improvements

A messy set of empirical issues surrounds the appropriate treatment of alternative greenhouse gases and the trends in the energy-GNP and GHG-GNP ratio. There are no major analytical issues involved here, but the treatment will have a significant impact upon the analysis.

1. Trends in Energy-GNP and CO_2-GNP Ratios

In the aggregative model used here, as well as in other energy models, it is necessary to make a judgment as to the trends in energy use and GHG emissions per unit output. In the actual model, this is represented by the parameter $\sigma(t)$, which is the ratio of the uncontrolled CO_2-equivalent emissions to total output.

We begin with an analysis of CO_2 trends and then discuss the role of other greenhouse gases. The current model does not explicitly model the energy sector. For this reason, the trends in energy use per unit output as well as CO_2 emissions per unit output must be taken from other studies as well as from historical data. We begin with a historical review and then look at model projections.

a. Historical trends

Tables III.E.1 and III.E.2 show trends in energy-GNP ratios and CO_2 emissions from the use of fossil fuels per unit of GNP for various regions over the period 1929-89. These data have been collated from a wide variety of sources and are of varying comparability and quality. The energy data consist primarily of commercial energy use and exclude firewood and biomass. The data are probably more reliable for the U. S. and for the OECD region, while the data for the USSR, China, and the Rest of the World are dubious for most of the period. In addition, the ratios are plotted in Figures III.E.1 and III.E.2.

We have constructed estimates of the rates of growth of these two ratios at the bottom of the tables. The rates of growth are for the entire period as well as for two sub-periods. Table III.E.1 shows that energy-GNP ratios have been declining steadily in most high-income countries during the last six decades. By contrast, the energy-GNP ratios appear to have increased in the USSR, China, and the rest of the developing world. Overall, energy-GNP ratios appear to have fallen around 1.1 percent per annum since 1929.

This study is primarily concerned with the trends in the CO_2-GNP ratio, shown in Table III.E.2. This ratio shows the same general pattern as for the energy-GNP ratio, with declines in the high-income regions and an increase over time in the poorer countries. Overall, the trend shows a decline around 1 percent per annum in the CO_2-GNP ratio since 1929.

The data for the United States are probably more reliable than for the other regions. The data suggest a more rapid decline in the CO_2-GNP ratio than for the global total--a trend consistent with the increase in CO_2 emissions per unit of output in many socialist and low-income countries. These data again show a more rapid decline in the

CO_2-GNP ratio for the period after the steep increase of fossil-fuel prices in the mid-1970s.

The appropriate concept for the analysis is the change in the CO_2-GNP ratio that would occur with the market-induced rise in fossil-fuel prices as they are gradually exhausted. This would appear to be consistent with a decline of between 1 and 1½ percent per annum in the CO_2-GNP ratio, with the lower number consistent with the data for all countries and the higher number consistent with the longer-term data for the United States.

b. Results from energy models

Energy models either build in or generate changes in the CO_2-GNP ratio. Table III.E.3 shows a sample of estimates from different models. Forecasts for the near term suggest decreases in the CO_2-GNP ratio for global models in the range of 1 to 1½ percent per annum, although longer term models find somewhat lower figures.

c. Conclusion

Clearly there is no right answer as to the future trend in the CO_2-GNP ratio even in the absence of steps to control greenhouse gases. Much depends upon price trends and the development of new technologies. The figure that seems most consistent with both historical trends and with energy models is for a long-term reduction in the CO_2-GNP ratio of 1¼ percent per annum. This might be as high as 1½ percent per year or as low as ½ percent per year. Clearly, as is shown in accompanying figures and tables, there is great uncertainty about the trends.

2. Modeling of Other Greenhouse Gases

The role of non-energy CO_2 emissions and non-CO_2 greenhouse gases poses formidable problems for a number of reasons. The major complication is that reliable models for projecting these other greenhouse gases do not today exist.

Fortunately (from a modeling perspective), the role of other greenhouse gases is likely to be relatively minor over the next century. Table III-E-4 shows alternative estimates of CO_2-equivalent emissions, using total warming potential as the weights for converting other GHGs into CO_2. The last column shows data from the IPCC and a conversion of the IPCC data to put it on the same basis as the present study. The estimates of CO_2 emissions used here are slightly higher than those used in the IPCC, whereas the non-CO_2 GHGs use the IPCC figures. Clearly, CO_2 is the most significant contributor. It is likely to be even more important in the coming decades as CFCs are phased out or, as recent evidence indicates, if there are negative radiative offsets to CFC accumulation. Moreover, there appears to have been a decline in the growth of concentrations in methane in the last few years, although the reasons are mysterious.

The treatment of non-energy-CO_2 greenhouse gases is as follows: The costs of reduction of both CO_2 and CFCs were explicitly included in the cost estimates for the reduction of GHGs in section III.D. Therefore this component of GHG accumulation is explicitly included in the calculations. In the future, it is assumed that the growth of the non-energy-CO_2 and the CFCs are proportional to that of the energy CO_2. This implies that non-energy CO_2 and CFCs will grow at the same rate as CO_2 in an uncontrolled economy. The controllable part of GHG emissions is confined to CO_2 and CFCs.

The other GHGs are assumed to be exogenous because too little is known today about their sources, sinks, and costs to include them into the economic estimates put forth year. N_2O is of little importance, and we simply add a trend from the IPCC estimates representing the incremental contribution of N_2O to radiative forcing. Methane is a serious problem because we know little about its sources and have no good way of controlling it. Therefore, we simply add to radiative forcing the low projection of the IPCC to obtain the direct and indirect contributions of methane.

Tables III.E.5 and III.E.6 show the assumptions and estimated forcings of the different greenhouse gases in the IPCC analysis and those used in the DICE model. The exogenous GHGs in this projection are assumed to grow at a rate approximately 1.3 percent per annum slower than real output in the base case.

III.F. Derivation of Equation for Carbon Cycle

This section lays out a simple model of the carbon cycle for use in the optimal growth model developed in this study. The first section lays out the model, while the second section provides the statistical estimation and the parameter estimates.

1. The Model

We assume that the GHG accumulation and transportation can be represented as a n-box system, in which each of the boxes is well mixed. In what follows, we will use the parameters of CO_2 to estimate the model because, once the CFCs have been phased out, CO_2 is the main area where policies are likely to be concentrated.

We assume that the carbon cycle can be represented by a linear (or more precisely, linearized) box-diffusion model. It should also be noted that because of uncertainty about the first-order fluxes, there are major questions about the accuracy of the flows in the model.

The essence is as follows. Let

$M_i(t)$ = the amount of carbon in box i at time t
$\mathbf{M}(t)$ = a column vector of the $M_i(t)$
α_{ij} = the annual transport from box i to box j
$\mathbf{A}$ = n x n matrix of α's, with the ith element of row j being α_{ij}
$\mathbf{E}(t)$ = column vector of carbon emissions.

The boxes are 1 = atmosphere, 2 = deep oceans, 3 = mixed layer of the ocean, 4 = long-term biosphere, 5 = short-term biosphere. Note that in describing the model, for notational convenience, we use the notation "M," or light-face M without a subscript, to represent M_1 or atmospheric carbon. In what follows, we assume that all emissions are into the atmosphere, box 1. Hence:

$$\mathbf{E}'(t) = [E(t) \;\; 0 \;\; 0 \ldots \;\; 0].$$

(where $\mathbf{E}'$ = transpose of $\mathbf{E}$). We can then write our system as:

(III.F.1) $$\mathbf{M}(t) = \mathbf{AM}(t-1) + \mathbf{E}(t-1)$$

In this treatment, we assume that the short-lived biosphere, the mixed level of the ocean (say 50 to 100 meters), and the atmosphere are well mixed. (A more complete treatment will change the results only slightly.) The uptake of the long-term biosphere will be omitted from this calculation.

Finally, we assume that the atmosphere contains the fraction β of the mass of carbon in the well mixed reservoirs (boxes 1, 3, and 5). It is then easily seen that β is the marginal atmospheric retention ratio, which is the fraction of an additional unit of emission that is retained and observed in the atmosphere in the first period. This should be distinguished from the usual definition of the atmospheric retention ratio, the observed ratio of total change in atmospheric M to total emissions of M (which is the average atmospheric retention ratio).

Recall that $M_1 = \beta(M_1 + M_3 + M_5)$, which given us

(III.F.2) $$M_1(t) = \alpha_{11} M_1(t-1) + [\beta\alpha_{21}] M_2(t-1) + \beta E(t-1)$$

(III.F.3) $$M_2(t) = (1-\alpha_{11}) [M_1(t-1)/\beta] + (1-\alpha_{21}) M2(t-1)$$

Equations (III.F.2) and (III.F.3) are the basic equations that we will estimate below.

2. Statistical Estimation

For the purposes of the optimal growth model in the study, it is desirable to find a simple representation of the carbon cycle. This is accomplished by applying the model in (III.F.2) and (III.F.3) to observed data on the carbon cycle.

For purposes of statistical estimation, we define all variables as deviations from a pre-industrial baseline, which is taken to be 590 billion metric tons of atmospheric carbon. we further assume that on the time scale that we are analyzing, the concentration of carbon in the deep oceans is unchanged, because it is such a huge reservoir, so that we can assume $M_{2,t}$ is constant . Then we can rewrite (III.F.2) as:

(III.F.4) $$M_1(t) = \alpha_{11} M_1(t-1)+ \beta E(t-1)$$

From carbon-cycle studies, we take α_{11} to be $1-1/\tau^M$, where τ^M is the turnover time in years for the deep oceans.[12] There is some controversy about the turnover time, but most estimates range between 100 and 500 years.[13] A simple one-equation representation such as (III.F.4) can be parameterized by examining the behavior of more complete models. Models developed by Siegenthaler and Oeschger [1987] and Maier-Reimer and Masselman [1987] suggest a $\tau^M = 120$ years, with a range of 50 to 200 years (IPCC [1990]). This then gives us the equation:

$$M_1(t) = [1-(1/\tau^M)]\, M_1(t-1) + \beta E(t-1) \qquad \text{(III.F.5)}$$

We applied this equation to data on estimated global carbon emissions and concentrations from Boden et al. [1990] for energy and cement. We augmented this by adding estimates from EPA for deforestation from EPA [1989]. Total emissions from the EPA report were 145 billion tons of carbon over the period 1860-1985, whereas the IPCC estimated the total emissions as 115 billions of tons of C. We therefore adjusted downward the EPA figure for each year by the ratio of 115/145. Figure III.F.1 shows the constructed series for CO_2 emissions along with the calculated increase in concentrations. It is likely that the emissions data are subject to greater error than the concentrations data, particularly since direct measurement of the latter began in 1958.

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[12] The major shortcoming of this approach would arise either because the quickly mixing boxes are not in fact in equilibrium in a short time period or if the slowly mixing boxes other than the deep ocean are quantitatively significant. It is possible, in particular, that carbon uptake in forests is on a shorter time scale than the deep oceans, and this could bias the statistical estimates.

[13] The IPCC states that "on average it takes hundreds to about one thousand years for water at the surface to penetrate to well below the mixed layer of the oceans" (IPCC [1990], p. 12).

Equation (III.F.5) is estimated with and without a constant (to allow for the possibility of a missing emission) and with and without first-order autocorrelation of the residuals (to allow for omitted variables). The time period was for 1860 to 1989. The results are shown in Table III.F-1.

Table III.F-1
Estimates of Carbon-Cycle Equation

	No constant			Constant		
	$\tau^M=200$	$\tau^M=120$	$\tau^M=50$	$\tau^M=200$	$\tau^M=120$**	$\tau^M=50$
β	.56	.64	.89	.61	.67	.87
se(β)+	(.015)	(.015)	(.015)	(.027)	(.027)	(.027)
SEE*	.527	.519	.514	.519	.517	.514

Sample period: 1880-1989
*SEE = standard error of equation
**For τ^M = 120, constant is -.127 (.09).
+Standard errors of coefficients are in parenthesis.

The first-order error correction did relatively little because the equation is virtually a first-different specification, and these results are not included.

As discussed above, there are alternative estimates about the effective turnover time of the oceans, and we have taken a preferred estimate of 120 years with alternative values of 50 and 200 years. If a constant term is included (representing errors of atmospheric retention ratio is 0.64($\pm$0.015). The equation with the constant says that there is a missing source of emissions of -.13 ($\pm$.09) gigatons of emissions and that the β is slightly higher at β = .67 ($\pm$.027).

Figure III.F.2 shows the actual and predicted from the regression for a lifetime of τ^M = 120 years. The prediction is a "dynamic" one; that is, it uses the initial conditions for the concentrations of CO_2 and simulates the equation without correcting for errors along the way. (Equations that use the actual values of the concentrations do much better.) As can be seen, the estimated equation without the constant term tracks the concentrations reasonably well, although the equation tends to overpredict in the period 1940-1960. From about 1960 on, the predicted growth is slightly less than the actual, so by the end of the period the dynamic prediction is close to the mark. In any case, these results are probably well within the errors of measurement of the underlying data.

Finally, Figure III.F.3 shows an estimate of the β parameter using "recursive least squares." This uses the lifetime estimate of τ^M = 120 years and suppresses the constant term. This equation shows estimates of the parameter for a sample period from 1880 through the date shown on the horizontal axis; in addition, an error band of plus and minus two standard errors of the coefficient is shown. The major impression from this figure is that the estimate of the marginal atmospheric retention ratio (β) is rising as more data come in over the last three decades. Estimates through 1960 give an estimate of 0.55, while those through 1989 give the estimate shown in Table III.F.1 of .64. The path of the standard errors suggests that the increase is unlikely to be due to sampling error. Other suspects are specification error and measurement error. One hypothesis is that there is growing saturation in the carbon sinks, so that the uptake in non-atmospheric sinks is decreasing over time.[14]

[14] Recall that for convenience we use the notation "M" or light-face M without a subscript to represent M_1 or atmospheric carbon.

PART TWO. MODEL RESULTS

Part One outlined the assumptions and model structure for the DICE model of the economics of global warming. In this part, we outline the results of the model. Section IV discusses the computational approach taken in estimating the model and then presents the results. The last section then discusses the results.

IV. PROJECTIONS OF THE "DICE" MODEL

A. Computational Procedures

1. Calibration to Historical Data

One of the difficulties of using mathematical programming (MP) approaches to energy models has been the complexity of calibrating the model to historical data. Unlike econometric models, MP models are not easily estimated. Moreover, the mapping from the parameters to the likelihood function is highly nonlinear, so even indirect techniques for calculating the "goodness of fit" are not currently available.

In the face of these difficulties, most practitioners calibrate their models to data in an intuitive fashion, either by selecting parameters on the basis of historical data, stylized facts, or statistical estimates. Generally, the "fitting" consists of a single observation (say one historical time period or the initial conditions). There are few attempts in the literature of MP models of attempting to match up the results of the models with actual data.

The present study addresses these issues in a crude way by using an extensive historical period as the first few time periods of the calculation. In the standard runs, we calculate both economic and climatic variables for the three historical periods 1965, 1975, and 1985; for the model, these dates are interpreted as the center point of the decades in which they fall. We can therefore make a rough judgment as to the validity of the model by examining the extent to which the model conforms to the historical data for this three-decade period. If should be emphasized, however, that there is a large number of combinations of parameter values that could explain the historical data, so conformity to history is but a partial validation of the model.

2. Algorithmics and Computational Record

The model is solved using a language and nonlinear programming algorithm known as the GAMS system. The technique used is a nested two-level algorithm. The inner algorithm is a refinement of the standard primal simplex method for solving linear programming problems first developed by G. Dantzig in the 1940s. The nonlinear constraints and objective function are solved with a reduced gradient and quasi-Newton method along with a projected Lagrangean algorithm due to S. M. Robinson. An introductory explanation to this is contained in Brooke et al. [1988].

The model has been run using the 386 version of the GAMS algorithm on various 386 compatible machines. The canonical runs presented below use a 40-period (400 year) calculation with terminal valuations (marginal values) on GHG concentrations, capital, and atmospheric temperature. These terminal valuations were obtained from a 60-period (600 year) run and are sufficient to stabilize the solution for the first 20 periods (200 years). The canonical 40-period run can be run from scratch in 1.67 minutes on an Intel 486/33 processor.

One of the difficulties of modeling dynamic growth processes is the need to truncate the estimation period, as discussed in the last paragraph. To ensure that the runs are not truncated too soon, we have undertaken two steps. First, we calculate in a 60-period run the shadow prices for carbon, capital, and the atmospheric temperature at period 40. We then impose these in the 40th period of the 40-period run to ensure that future decisions are not underweighted in the calculations. Second, we then make a number of runs of different lengths to test whether the values of the important variables are sensitive to the length of the period of calculation. By comparing the time-intensive 60-period run with the 40-period run used in the estimates below, we see that there is no change in any of the variables for the first 21 periods (that is, through the calculation period corresponding to 2165). A summary of the comparisons of the runs of different lengths and the impacts upon the important variables is shown in Appendix Table A-1.

B. Comparative Runs

In the analysis that follows, we analyze two specific sets of runs. The first is the "laissez-faire" or uncontrolled run in which there are no controls on greenhouse gases. This serves as a base case for comparison with other studies. The second is the optimal policy, the one in which greenhouse-gas controls are set so as to maximize the discounted value of the utility of consumption. A more precise description is the following:

o Uncontrolled (laissez-faire) case. The baseline run is one in which there are no controls or policies on greenhouse gases; i. e., no steps are taken to slow or reverse greenhouse warming. Individuals would, of course, adapt to the changing climate, but there would be no attempt to take policies that would curb greenhouse gas emissions or to "internalize" the greenhouse externality. This policy is one which has been followed for the most part by nations through 1989.

o Optimal policy. The second run is a calculation of the optimal policy. This run maximizes the discounted value of utility subject to the economic and climatic constraints described above. This policy can be thought of as one in which the nations of the world get together and set the efficient policy for "internalizing" the greenhouse externality. It is assumed that the policy is efficiently implemented, say through uniform carbon taxation or auctionable quotas with a perfect enforcement system. The policy is assumed to be implemented in the decade beginning 1990.

C. Results of the Two Runs

We now present the results of the model.

1. Economic Variables

We begin with a presentation of the major economic variables in the model. The model outputs in the mathematical programming analysis consist of both physical output (such as energy consumption or greenhouse-gas emissions) as well as economic values (such as the value of output and consumption). The economic values are generated as "present values," which look somewhat strange.[15] In the presentation that follows, however, we have converted all the present-value prices to constant prices which correspond to 1989 U. S. dollars.

Table IV-1 shows the levels of output and consumption in the uncontrolled and optimal runs. In addition, we show in the tables that follow the estimated historical values of the variables when those are available. The projected and actual data show rapid growth in the historical periods and assume continued rapid growth in the future in both output and consumption.

[15] A word of explanation of the difference between the variables is useful. The constant-dollar prices are the familiar indexes used to correct for inflation. They simply take the volumes of activity and multiply the values by prices that hold in a base year. The present-value prices use instead the prices discounted back to the present (or first period). For example, assume that the price of a ton of coal in the year 2015 is $45 and that the appropriate discount rate is 9.5 percent per year. To calculate the present-value price in 1990, we divide the $45 by the present-value factor of 9.5 percent per year compounded over 25 years, which is a factor of ten. This gives the present-value price of $4.50 for the coal. In the presentation here, we will translate the present-value prices into constant-dollar prices.

Table IV-2 shows the values of the gross savings rate (gross investment divided by gross world product) and the net rate of return on capital (measured as the current-period rate of return on capital net of depreciation).

In addition, we show in Figures IV-1 through IV-4 plots of the major economic variables. The first figure shows output in the uncontrolled and the optimal path. Figure IV-2 shows the trajectory of consumption per capita in the two cases. This shows the key assumption of continued rapid growth in per capital consumption over the coming decades. As can be seen in these figures, it is very difficult to see the impact of the optimal control program on world output or consumption in the coming decades.

Figures IV-3 and IV-4 show the difference between the optimal controls and the uncontrolled run on global output, both in absolute dollars and in percentage terms. There is very little difference between the optimal and the uncontrolled runs for about a half-century. This is because of the modest level of warming for the coming decades as well as the low levels of GHG controls in the near term. By the end of the next century, however, the optimal strategy enjoys annual output levels almost $200 billion higher than the uncontrolled path; this is the difference between the damage averted and the costs of controls.

Figures IV-5 and IV-6 show the trajectory of both the gross savings rate and the net rate of return in the two cases. As can be seen in these figures, there is no substantial difference between the rates of return and the return on capital between the optimal and the uncontrolled cases.

The decline in the net return on capital is important for evaluating different control strategies. Most analyses of environmental issues take the discount rate as constant and exogenous. In fact, the assumption of exogeneity seems satisfied in the case examined here, although the selection of the discount rate is a difficult issue in itself. On the other hand, given the assumptions that are built into the model, the discount rate should decline over time as economic and population growth slows.

2. Environmental Variables

The next set of runs shows the major environmental-climate variables. Table IV-3 and Figure IV-7 show the all-important control rate of greenhouse gases in the optimal run. The optimal control rate begins around 9 percent in the early decades and rises slowly over time, reaching 14 percent by the end of the next century. The increase in the control rate is the result of the growing size of the global economy as well as the decline in the discount rate on capital over this period.

Governments may choose to implement a greenhouse-gas control strategy by imposing carbon emission taxes; carbon taxes are defined as the tax that would impose an equivalent tax per unit of global warming potential on all emissions of greenhouse gases. These can be calculated in the optimal program as the level of the carbon tax that would lead to the desired control rate. Technically, this is calculated as the derivative of the objective function with respect to an additional unit of carbon emissions. Table IV-3 and Figure IV-8 show the optimal carbon tax over time in the optimal run. The tax begins around $5 per ton carbon in the 1991-2000 decade and then gradually increases over time to around $20 per ton carbon at the end of the 21st century (all in 1989 prices).[16]

The implications of the calculated optimal control of GHGs for emissions and global temperatures are shows in the next tables and figures. Table IV-4 and Figures IV-9 shows the rate of emissions of GHGs for the optimal and baseline runs. The two paths coincide for the period up through 1990 because no greenhouse controls were in place.

[16] Often, the carbon tax is expressed in terms of CO_2 weight rather than carbon weight. To obtain the relevant CO_2 weights, multiply by 3.67. To translate the carbon taxes into CO_2 taxes, divide by 3.67.

Beginning in 1991, the optimal controls lower GHG emissions modestly below the uncontrolled path. Recall that our estimates of GHG emissions include only CO_2 and CFCs, and the other GHGs are taken as exogenous for control purposes. Note that our assumptions find that the uncontrolled emissions of CO_2 equivalent are found to rise by a factor of three between now and the end of the next century.

The result of the lower emissions on GHG concentrations is shown in Table IV-4 and Figure IV-10. We calculate that the concentrations of GHGs (CO_2 and CFCs only) will have risen from 745 billion tons of carbon equivalent in 1990 to 1500 billion tons by the end of the 21st century in the uncontrolled case. As a result of imposing controls, GHG concentrations at the end of the next century have been reduced by slightly more than 100 billion tons of carbon equivalent, to around 1400 billion tons of carbon equivalent, in 2100.

The net effect of the policies on the climate is shown in Figure IV-11 and Table IV-5, where the calculated change in global mean surface temperature (relative to the base period of 1860) is shown. According to the model used here, global mean temperature has risen 0.6 degrees C from the base period to 1985. In the uncontrolled case, global mean temperature is estimated to rise an additional 2.8 degrees C by the end of the next century, for an average decadal increase of 0.24 degrees C per decade. In the optimal case, the global mean temperature is estimated to rise 2.6 degrees C by the end of the next century, for an average decadal increase of 0.22 degrees C.

Many readers of these results have been struck by the very modest impact that the optimal policy makes upon the concentrations and temperature trajectories. The reason for the modest impact is straightforward. According to our estimated, the impact of warming upon the global economy is relatively small, amounting to around 1.3 percent of global output for a 3 degree C average warming. The costs of slowing the warming are very small for the first increment of policy and then rise sharply as the degree of emissions restraint increases. The resultant of these two factors is that the optimal degree of slowing global warming is but a small part of the future warming.

Two other factors lead to the small decrease in the extent of warming in the optimal path. First, there is a great deal of momentum in the current climate given the existing degree of buildup of GHGs and the lags in the response of the climate to GHG increases. According to the model used here, if GHG concentrations were stabilized at the rate expected in the year 2000--an extremely ambitious target requiring a drop in emissions of about 70 percent over the next two decades--global mean temperature would still rise by around 1.6 degrees C above the base level.

The second reason why the reduction in the rate of warming is so slight is the non-linear relationship between GHG concentrations and warming. According to scientific studies, the relationship between equilibrium warming and CO_2 concentrations is logarithmic. This implies that moving from 300 to 315 ppm of CO_2 increases equilibrium temperature by .215 degrees C while moving from 585 to 600 ppm of CO_2 increases equilibrium temperature by only .111 degree C. The implication of this nonlinear relationship is that the first units of reduction of CO_2 concentrations have a relatively small impact upon the path of temperature.

The final figure on the runs, Figure IV-12, shows the net true savings rates in the two calculations. The net true savings rate attempts to measure both investments in physical capital and reductions of output that come from mitigation measures to slow global warming. Technically, this measure shows the difference between output in the uncontrolled case and consumption in either cases. As can be seen, there is relatively little difference between the savings rates in the two cases, although in the optimal policy the savings rate is slightly lower toward the end of the period.

3. Costs and Benefits of GHG Controls

We next address the issue of the overall impact of GHG controls on the economy. We can measure the economic impact by examining the attained maximum value of the objective function in the maximization. This difference between the attained maximum value in the uncontrolled and the optimal case provides an estimate of the cost of greenhouse-gas controls.

Table IV-6 shows the impact of controls on global output when optimal controls are imposed for the decade centered on 1995. For the "best-guess" parameter values, there is relatively little difference in output or consumption for the first few decades. Until the middle of the next century, the optimal GHG controls are at modest levels. For the second half of the next century, however, the value of controlling GHGs makes a substantial difference to global output, with net benefits (that is reduced damages less costs of controls) in the order of $200 billion in 1989 prices.

The discounted value of the net benefits from GHG controls is $205 billion, discounted back to the first period of control (the 1990s) in 1989 prices. This amounts to 0.029 percent of the total discounted value of consumption over the indefinite period from 1991 on (where consumption flows are discounted at the real market rate of interest). The flow impact on consumption is around one-quarter percent of consumption at its maximum impact, where this is a net benefit. In terms of gross damages and gross costs of abatement, the figures are somewhat larger than this in absolute value.

V. CONCLUSIONS

This paper is designed to present the methodological and technical assumptions and the results behind the Dynamic Integrated model of Climate and the Economy (the DICE model). This study differs from earlier ones by the author and other researchers in two respects. First, it attempts to integrate both the economic costs and benefits of greenhouse-gas controls with a simple aggregate model linking the economy with climatic change. In addition, it improves upon earlier models in examining the optimal dynamics of an optimal control strategy. In this section we summarize the principal conclusions.

First, an efficient strategy for coping with greenhouse warming must weigh the costs and benefits of different policies at different points of time. This study relies on earlier studies made by the present author and others as to the costs of abating greenhouse gases as well as the damages from greenhouse warming. Estimates of both costs and damages are highly uncertain and incomplete, and our estimates are therefore highly tentative. In terms of damages from the equivalent of a doubling of CO_2 in equilibrium, we estimate that the impact of climate change coming from a 3° C rise in global mean surface temperature, along with the associated changes in climate, is estimated to be about 1.3 percent of output for the global economy.

Second, we have examined a number of different studies of the costs of reducing GHG emissions. In this study, we examine controls on CO_2 and CFCs and assume that the other GHGs are exogenous from the point of view of control programs. The estimate used here is that a halving of controllable GHG emissions, relative to the path that would otherwise occur, would reduce total output by about 1 percent. On the base of today's output, the long-run marginal cost of reducing GHG emissions is $120 per ton CO_2 equivalent (carbon weight) for a 50 percent reduction. The total global costs of this reductions is about $200 billion for a 50 percent reduction at today's level of economic activity.

Third, we have included a number of linkages from the economy to climate change. These include an assumption of an exogenous improvement in the GHG-output ratio at a rate of 1¼ percent per annum, which reflects rising fossil fuel prices as well as improvements in underlying technical efficiency. In addition, we have included a simple emissions-concentrations equation that is consistent with both historical studies and with carbon-cycle models.

The most difficult link is that between GHG concentrations and climate. We have employed a simple first-order dynamic model linking radiative inputs and global mean surface temperature. We then calibrated this model to both existing general circulation models and to historical data. Using the central estimate from this calibration, we have included in the dynamic model a two-equation system relating radiative inputs and global mean temperature.

Fourth, we have put the different components together in a dynamic optimization model. This model calculates the trajectory of global output, consumption, GHG emissions and concentrations, and climate change, as well as climate damages over the indefinite future. By assuming there are no controls in place, we can calculate a baseline, uncontrolled run. We then compare this run with the "optimal" run in which the level of GHG controls is calculated to maximize the discounted value of the utility of consumption.

The results of these calculations were presented in the last section and will not be extensively reviewed here. The major results concern the optimal controls on GHGs. For the "best-guess" parameters used here, the optimal reduction in GHGs is calculated to be 8.8 percent for CO_2 and the CFCs taken together. This corresponds to a marginal cost of reduction of around \$5.3 per ton carbon in the decade of the 1990s. This "carbon tax" is determined to rise rapidly over time, reaching around \$20 per ton carbon by the end of the next century.

Fifth, the appropriate level of control depends critically upon a number of parameters of the climate-economic system. We have not performed sensitivity analysis for the present study, reserving that for the next round of experiments. But from earlier work, it is clear that the major uncertainties involve the real interest rate, the cost of control of GHGs, the damage to the human societies from greenhouse warming, the parameters of the climate system, and the time lags in the reaction of the climate to emissions.

Finally, it should be emphasized that this analysis remains incomplete and has a number important uncertainties. There are many important parameters of the system whose values are only beginning to be studied and which are imprecisely known. Virtually every relationship described above has a significant margin of error. Particularly uncertain is the damage function -- the impact of global warming upon human and natural societies. Moreover, this analysis excludes considerations of uncertainty, in which risk aversion and the possibility of learning may modify the optimal control strategy.

Whatever the infirmities of the present approach, it is hoped that integrated economy-climate models can become a useful tool for analyzing the societal issues involved in global warming. They are easily modified to include new information; in addition, they can be employed to investigate the value of information and the "risk premium" for uncertainty. They are at present the only tool for weighing both costs of control against damages from warming. Such balancing is necessary if nations are to design efficient strategies to cope with global warming in the years to come.

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TABLE III.B-1

Parameters for Three GCMs for Climate Minimodel

A. One equation model (equation III.B-3)

Model	Feedback parameter α_2 [= λ]	Temperature sensitivity coefficient T_{2xco2} [°C]	α_1 [$=1/R_1$]	α_3 [$=R_2/\tau_{12}$]	α_4 [$=1/\tau_{12}$]	e-fold [yrs]	SEE [°C]
Schneider-Thompson	1.33	3.08	0.026	0	0	29	0.034
Stouffer-Manabe-Bryan	0.98	4.20	0.030	0	0	34	0.011
Schlesinger-Jiang	1.41	2.91	0.024	0	0	30	0.064

B. Two-equation model (equation III.B-2)

Model	Feedback parameter α_2 [= λ]	Temperature sensitivity coefficient T_{2xco2} [°C]	α_1 [$=1/R_1$]	α_3 [$=R_2/\tau_{12}$]	α_4 [$=1/\tau_{12}$]	e-fold [yrs]	SEE [°C]
Schneider-Thompson	1.33	3.08	0.075	0.44	0.002	13	0.000
Stouffer-Manabe-Bryan	0.98	4.20	0.065	0.44	0.002	25	0.014
Schlesinger-Jiang	1.41	2.91	0.048	0.44	0.002	19	0.050

Source: Parameters calibrated to models as described in text.

Table III.B-2
Estimated Values for Models from Historical Data

A. One equation model (equation III.B-3)

Model	Feedback parameter α_2 [= λ]	Temperature sensitivity coefficient T_{2xco2} [°C]	α_1 [=1/R_1]	α_3 [=R_2/τ_{12}]	α_4 [=1/τ_{12}]	e-fold [yrs]	SEE [°C]
Low temperature sensitivity	4.00	1.03	0.020 [.0072]	0	0	13	0.121
High temperature sensitivity	1.00	4.20	0.014 [.0105]	0	0	63	0.124

B. Two-equation model (equation III.B-2)

Model	Feedback parameter α_2 [= λ]	Temperature sensitivity coefficient T_{2xco2} [°C]	α_1 [=1/R_1]	α_3 [=R_2/τ_{12}]	α_4 [=1/τ_{12}]	e-fold [yrs]	SEE [°C]
Low temperature sensitivity	4.00	1.03	0.020 [.0072]	0.44	0.002	14	0.121
High temperature sensitivity	1.00	4.10	0.016 [.0099]	0.44	0.002	96	0.123

Source: Data and estimation technique describe in text. Equations were fitted using ordinary or iterative least squares over period 1862 to 1989.

Table III.B-3

Likelihood of Different Parameters:
Two-Equation Model of Climate System

Inertia Parameter (α_1)	Temperature-Sensitivity Coefficient (T_{2xco2})					
	5.0	4.0	3.1	2.0	1.0	Sum
0.01	0.090	0.102	0.111	0.103	0.050	0.455
0.02	0.010	0.024	0.054	0.106	0.072	0.264
0.05	0.000	0.001	0.006	0.059	0.087	0.152
0.10	0.000	0.000	0.001	0.035	0.092	0.128
Sum	0.100	0.126	0.172	0.303	0.300	1.000

Table III.B-4

Correlations between Alternative Specifications in Stouffer-Manabe-Bryan Model

	Variable			
Variable	TR	TRFRO	TRFOREQ1	TRFOREQ2
TR	1.0000	0.9994	0.9963	0.9947
TRFOR		1.0000	0.9961	0.9951
TRFOREQ1			1.0000	0.9956
TRFOREQ2				1.0000

Notes to Table B-4:

TR = Actual model calculation from Stouffer, Manabe, Bryan [1989].

TRFOR = Polynomial interpolation that best fits the TR series.

TRFOREQ1 = Ordinary least squares estimate of one-equation model in (III.B-3) to TRFOR.

TRFOREQ2 = Ordinary least squares estimate of two-equation model in (III.B-2) to TRFOR.

Table III.C-1

Impact Coefficients for Different Sectors

Sector	GNP originating [billions of 1981$]	Impact [billions ot 1981$]	Impact [percent]
1. Farming	61.0	3.0	4.92
2. Energy	45.9	1.0	2.18
3. Coastal activities	76.6	5.3	6.91
4. Other	2231.5	15.9	0.71
TOTAL	2415.0	24.2	1.00

Notes on application to other countries:

Row 1: This is taken to be proportional to the GNP originating in agriculture.
Row 2: This is taken to be proportional to total energy consumption in value terms.
Row 3: The coastline vulnerability is taken to be the ratio of the coastline to the land area. The vulnerability is assumed to rise with the square root of the coastline vulnerability. The measure of GNP is recreation and real estate.
Row 4: This is proportional to the non-agricultural GNP.

Table III.C-2

Impact in Different Sectors for Global Economy: Effect of Growth and Changing Industrial Composition

	Impact by sector and total [percent of global output]					Memo: Share of Farming
Year	Total	Farming	Coastal	Energy	Other	[percent of output]
1987	1.326	0.306	0.272	0.044	0.704	6.1
2010	1.331	0.292	0.289	0.044	0.706	5.8
2050	1.336	0.271	0.312	0.044	0.709	5.4
2100	1.331	0.259	0.317	0.044	0.711	5.2

Notes: The estimate shows the impact on global output of climate change due to equilibrium CO_2 doubling (with 3 degrees C of warming). The estimates reflect the changing composition of national outputs and the differing vulnerabilities of different sectors and countries as shown in Tables III-C-1 and III-C-2.

Table III.E-1

Trends in Energy-GNP Ratios for Different Regions and Subperiods, 1929-1989

[Energy/GNP in tons of coal equivalent per thousands of 1989 US $]

	[1] USA	[2] Japan	[3] Rest of OECD	[4] USSR	[5] China	[6] Rest of World	[7] Total
1900	na	na	na	na	na	na	na
1913	na	na	na	na	na	na	na
1929	0.938	0.288	0.626	0.205	0.567	0.359	0.620
1938	0.816	0.342	0.556	0.491	0.619	0.350	0.567
1950	0.762	0.253	0.438	0.539	0.814	0.408	0.552
1960	0.555	0.243	0.283	0.389	0.452	0.209	0.578
1970	0.487	0.312	0.255	0.598	3.182	0.290	0.224
1980	0.566	0.255	0.318	0.754	1.518	0.336	0.260
1989	0.456	0.164	0.310	0.831	1.316	0.460	0.317

[Average annual growth rate of Energy-GNP ratio, subperiods, percent per year]

	[1] USA	[2] Japan	[3] Rest of OECD	[4] USSR	[5] China	[6] Rest of World	[7] Total
1929-89	-1.195	-0.935	-1.163	2.361	1.414	0.414	-1.113
1929-50	-0.986	-0.618	-1.686	4.718	1.742	0.613	-0.548
1950-89	-1.307	-1.106	-0.880	1.114	1.237	0.308	-1.416

Table III.E-2

Trends in CO2-GNP Ratios for Different Regions and Subperiods, 1929-1989

[CO2 Emissions/GNP in tons of C per thousands of 1989 US $]

	[1] USA	[2] Japan	[3] Rest of OECD	[4] USSR	[5] China	[6] Rest of World	[7] TOTAL
1900	na	na	na	na	na	na	na
1913	na	na	na	na	na	na	na
1929	0.600	0.189	0.428	0.135	0.391	0.237	0.409
1938	0.501	0.223	0.375	0.323	0.429	0.225	0.366
1950	0.446	0.156	0.287	0.358	0.568	0.264	0.343
1960	0.384	0.144	0.251	0.543	3.139	0.262	0.219
1970	0.393	0.180	0.245	0.584	2.100	0.317	0.219
1980	0.323	0.139	0.207	0.617	1.939	0.325	0.241
1989	0.256	0.088	0.164	0.586	1.270	0.333	0.232

[Average annual growth rate of CO2-GNP ratio, subperiods, % / y

	[1] USA	[2] Japan	[3] Rest of OECD	[4] USSR	[5] China	[6] Rest of World	[7] TOTAL
1929-89	-1.410	-1.272	-1.581	2.481	1.982	0.566	-0.941
1929-50	-1.408	-0.915	-1.892	4.764	1.791	0.506	-0.832
1950-89	-1.412	-1.464	-1.413	1.272	2.085	0.598	-0.999

Table III.E-3

Projections of Changes in Energy Efficiency and CO2-GNP Ratios in Different Studies

Study	Coverage	Rate of Change in Item per unit GNP [percent per annum] Energy	CO2
Nordhaus-Yohe [1]	Global		
1975-2025		-1.3	-1.7
2025-2100		-0.3	-0.6
Manne-Richels [2]	US	-0.5	
IEW-1991 [3]	World		
1990-2020			
Median forecast		-0.8	-1.4
Low [4]		-1.2	-1.2
High [5]		-1.8	-1.8

[1] Nordhaus and Yohe [1983], Table 2.15.

[2] This is change in "autonomous energy efficiency improvement. For a backcast, see A. Manne and R. Richels, "Estimating the Energy Conservation Parameter," November 1990.

[3] Presentation by Leo Schrattenholzer at International Energy Workshop, June 1991. This represents the median and range of forecasts of energy modelers.

[4] This represents the 16th percentile of forecasts.

[5] This represents the 84th percentile of forecasts.

Table III.E-4

Worksheet for Different Assumptions of Greenhouse Gas Emissions

	1985	1990
Earlier Study [1]		
CO2	6.50	
CFCs	0.26	
Others	1.16	
TOTAL	7.92	
This study		
CO2 [2]	6.64	7.36
CFC [3]	0.89	1.09
Others [4]	2.35	2.59
TOTAL	9.89	11.04
IPCC (100 year integration) [5].		
CO2		7.10
CFCs		1.30
Others		3.24
TOTAL		11.65
IPCC (200 year interpolated)		
CO2 [6]		7.10
CFCs [7]		1.09
Other [8]		2.59
TOTAL		10.78

Notes:
[1] from Nordhaus [1991].
[2] as explained in Appendix A.
[3] 1990 figure from IPCC 200 year integration as explained below; 1985 figure reduced by 4 percent per annum.
[4] Same as [3], but growth rate is 2 percent per annum.
[5] Use 200 year integration period to conform to the assumption of low (1.0 percent per annum) discounting on GHGs. This reduced CO2 equivalent by 12 percent for CFCs and 20 percent for other gases.

Table III.E-5

Assumptions for Endogenous Greenhouse Gases: Contribution to Warming [W/m2]

A. IPCC Assumptions

	[1]	[2]	[3]
Year	CO_2	CFCs	TOTAL
1900	0.37	0.00	0.37
1960	0.79	0.02	0.81
1970	0.96	0.07	1.03
1980	1.20	0.16	1.36
1990	1.50	0.29	1.79
2000	1.85	0.37	2.22
2025	2.88	0.52	3.40
2050	4.15	0.67	4.82
2075	5.49	0.76	6.25
2100	6.84	0.76	7.60

Source: IPCC, p. 54, 57.
[1] is "Business as Usual." [2] is "accelerated policies." Other CFCs than CFC-11, -12, and -22 are assumed to be .08 from 2000 on.

B. Data by Period Used in Economic Model

1965	0.88	0.04	0.92
1975	1.08	0.11	1.19
1985	1.35	0.22	1.57
1995	1.68	0.33	2.00
2005	2.06	0.40	2.46
2015	2.47	0.46	2.93
2025	2.88	0.52	3.40
2035	3.39	0.58	3.97
2045	3.90	0.64	4.54
2055	4.42	0.69	5.11
2065	4.95	0.72	5.68
2075	5.49	0.76	6.25
2085	6.03	0.76	6.79
2095	6.57	0.76	7.33
2100	6.84	0.76	7.60

Source: Interpolation from part A.

Table III.E-6

Assumptions for Exogenous Greenhouse Gases: Contribution to Warming [W/m2]

A. IPCC Assumptions

Year	[1] Methane: Direct	[2] Methane: H2O	[3] N2O	[4] TOTAL
1900	0.10	0.03	0.027	0.16
1960	0.24	0.08	0.045	0.37
1970	0.30	0.10	0.054	0.45
1980	0.36	0.12	0.068	0.55
1990	0.42	0.14	0.100	0.66
2000	0.45	0.16	0.120	0.73
2025	0.56	0.19	0.210	0.96
2050	0.65	0.22	0.310	1.18
2075	0.66	0.23	0.400	1.29
2100	0.66	0.23	0.470	1.36

Source: IPCC, p. 54, 57.
[1] and [2] are "Low Emissions."
[3] is business as usual.

B. Data Used in Economic Model

1965	0.27	0.09	0.050	0.41
1975	0.33	0.11	0.061	0.50
1985	0.39	0.13	0.084	0.60
1995	0.44	0.15	0.110	0.70
2005	0.47	0.17	0.138	0.78
2015	0.52	0.18	0.174	0.87
2025	0.56	0.19	0.210	0.96
2035	0.60	0.20	0.250	1.05
2045	0.63	0.21	0.290	1.14
2055	0.65	0.22	0.328	1.20
2065	0.66	0.23	0.364	1.25
2075	0.66	0.23	0.400	1.29
2085	0.66	0.23	0.428	1.32
2095	0.66	0.23	0.456	1.35
2105 and beyond	0.66	0.23	0.470	1.36

Source: Interpolation from part A.

Table IV-1. **Values of Output and Consumption in Uncontrolled and Optimal Runs of DICE Model**

Decade centered on year:		World Output		World Consumption	
		Uncont	Optimal	Uncont	Optimal
1965	proj	8,520	8,520	6,652	6,652
	act	8,519		na	
1975	proj	12,680	12,680	10,017	10,017
	act	12,708		na	
1985	proj	17,890	17,890	14,273	14,273
	act	17,819		na	
1995	proj	24,073	24,073	19,364	19,363
2005	proj	31,095	31,094	25,182	25,179
2025	proj	46,928	46,931	38,390	38,389
2075	proj	88,213	88,311	73,145	73,217

"Proj" = projected levels from model; "act" = actual levels.

"Uncont" = values when no controls are placed on greenhouse gases.

"Optimal" = values when GHG controls are set to maximize utility of consumption.

na = not available.

Table IV-2. **Savings Rate and Rate of Return in Uncontrolled and Optimal Policy Runs**

Decade centered on year:	Savings Rate [Percent of gross output] Uncon	Optimal	Diff	Rate of Return [percent per year] Uncon	Optimal	Diff
1965:proj	21.9	21.9	0.0%	6.8	6.8	0.0%
1975 proj	21.0	21.0	0.0	6.5	6.5	0.0
1985 proj	20.2	20.2	0.0	6.2	6.2	0.0
1995 proj	19.6	19.6	0.0	5.9	5.9	0.0
2015 proj	19.0	19.0	0.0	5.6	5.6	0.0
2025 proj	18.2	18.2	0.0	5.2	5.2	0.0
2075 proj	17.1	17.1	0.0	4.4	4.4	0.0

"Uncont" = values when no controls are placed on greenhouse gases.

"Optimal" = values when GHG controls are set to maximize utility of consumption.

Note: The "savings rate" is equal to the ratio of gross investment to gross output. The rate of return is calculated as the marginal product of capital on an annualized basis.

Table IV-3. **Level of Control and Carbon Tax Equivalent on Greenhouse Gases in Optimal Run of DICE Model**
[As percent of uncontrolled emissions]

Decade centered on year:	Rate of Control of GHGs [Percent]	Carbon Tax Equivalent [1989 $ per t C]
1965	0.0	$ 0.00
1975	0.0	0.00
1985	0.0	0.00
1995	8.8	5.29
2005	9.6	6.77
2025	11.1	10.03
2075	13.4	17.75

Note: Carbon tax is per ton of CO_2 equivalent, carbon weight.

Table IV-4. **Values of GHG Emissions and Concentrations in Base and Optimal Runs of "ICE" Model**
[billions of tons C equivalent]

Decade centered on year:		GHG Emissions [per year] Uncontr	GHG Emissions [per year] Optimal	GHG Concentrations [end of period] Uncontr.	GHG Concentrations [end of period] Optimal
1965	proj	4.42	4.42	677	677
	act	na	na	677	677
1975	proj	5.89	5.89	698	698
1985	proj	7.53	7.53	727	727
	act	7.53	7.53	na	na
1995	proj	9.28	8.46	764	764
2005	proj	11.07	10.07	809	803
2025	proj	14.62	13.00	921	902
2075	proj	21.96	19.01	1293	1221

"Proj" = projected levels from model; "act" = actual levels.
"Uncontr" = results of uncontrolled run
"Optimal" = results of run with emissions optimized.

Note: Total greenhouse gas emissions and concentrations are the CO_2 equivalent of both CO_2 and CFCs converted into the equivalent mass of CO_2 that would yield the equivalent radiative warming.

Table IV-5. **Values of Global Mean Surface Temperature in Uncontrolled and Optimal Runs of DICE Model**
[Difference from 1900 levels, degrees C]

Decade centered on year:	Temperature Increase: Optimal	Uncont
1965 proj	0.20	0.20
act		
1975 proj	0.40	0.40
act		
1985 proj	0.58	0.58
act		
1995 proj	0.76	0.76
2015 proj	0.96	0.96
2025 proj	1.38	1.40
2075 proj	2.55	2.68
2105 proj	3.20	3.40

"Proj" = projected levels from model; "act" = actual levels.

"Uncontr" = results of uncontrolled run.

"Optimal" = results of run with emissions optimized.

Appendix Table A-1

Comparison of Different Run Lengths for
DICE Model

VARIABLE	Length of Run	PERIOD NUMBER 1	4	11	21	31	41
Consumption (trillions of 1989 US $)							
	20	6.65	19.36	66.58			
	30	6.65	19.36	66.58	115.49		
	40	6.65	19.36	66.58	115.49	133.50	
	60	6.65	19.36	66.58	115.49	133.50	138.68
Savings Rate (gross investment as fraction of gross output)							
	20	0.219	0.196	0.210			
	30	0.219	0.196	0.172	0.165		
	40	0.219	0.196	0.172	0.165	0.163	
	60	0.219	0.196	0.172	0.165	0.163	0.163
GHG Control Rate (fraction)							
	20	0.000	0.088	0.135			
	30	0.000	0.088	0.131	0.151		
	40	0.000	0.088	0.131	0.148	0.153	
	60	0.000	0.088	0.131	0.148	0.150	0.150
Atmospheric Carbon Equivalent (billions of tons)							
	20	677	763	1151			
	30	677	763	1152	1804		
	40	677	763	1152	1806	2242	
	60	677	763	1152	1805	2243	2473
Interest rate (per annum)							
	20	0.068	0.059	0.045			
	30	0.068	0.059	0.045	0.038		
	40	0.068	0.059	0.045	0.038	0.036	
	60	0.068	0.059	0.045	0.038	0.036	0.035
Shadow price on GHG emissions (present value prices)							
	20	1.779	1.134	0.225			
	30	1.774	1.134	0.212	0.012		
	40	1.774	1.134	0.212	0.011		
	60	1.774	1.134	0.212	0.011	na	
Shadow price on GHG emissions (current prices, 1989 $)							
	20	9209	2162.8				
	30	9209	2162.8	127.66	4.403		
	40	9209	2162.8	127.66	4.403	0.202	
	60	9209	2162.8	127.66	4.403	0.202	
Carbon tax (1989 $ per ton C equivalent)							
	20	1.93	5.24				
	30	1.93	5.24	16.61	27.25	na	
	40	1.93	5.24	16.61	24.98	na	
	60	1.93	5.24	16.61	24.98	na	

Notes: The table shows the calculated values of each of the variables for runs of different lengths (given in the first column) and for different time periods (read across the top of the table).

na = not available because value too small to be calculated.

Table IV-6. Impact of Optimal Program on Consumption and on Value of Objective Function in DICE Model

Decade centered on year:	Difference betweeen Uncontrolled and Optimal Paths for: Output	Consumption
1995	0	-1
2005	-1	-3
2015	1	0
2025	3	-1
2035	10	3
2045	21	12
2055	39	26
2065	65	45
2075	98	72
2085	139	105
2095	187	145
2105	242	191
Discounted Value to 1989 prices:		
In billions:	(a)	205 (b)
As percent of total discounted global consumption, 1990 on		.029 percent

"Uncont" = values when no controls are placed on greenhouse gases.

"Optimal" = values when GHG controls are set to maximize utility of consumption.

(a) Because output double counts returns, the discounted value of output is not calculated.

(b) This value is the difference in the attained maximum values of the objective function converted to 1989 prices by using the marginal values of consumption for 1985 and 1995 and interpolating to 1989.

Figure III-B-1

Relationship Between Equilibrium Temperature Rise and Equilibrium Precipitation Increase in Models Surveyed by IPCC

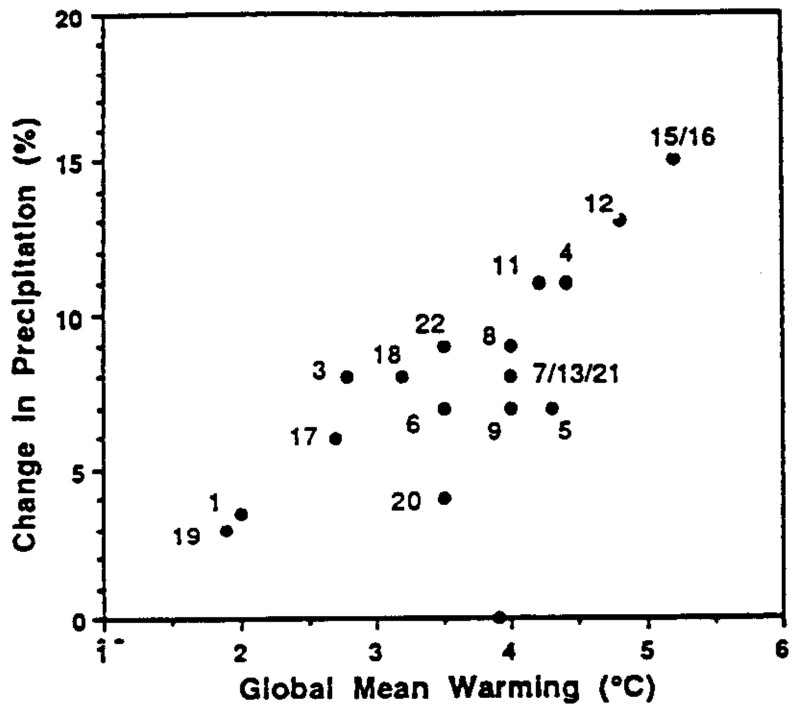
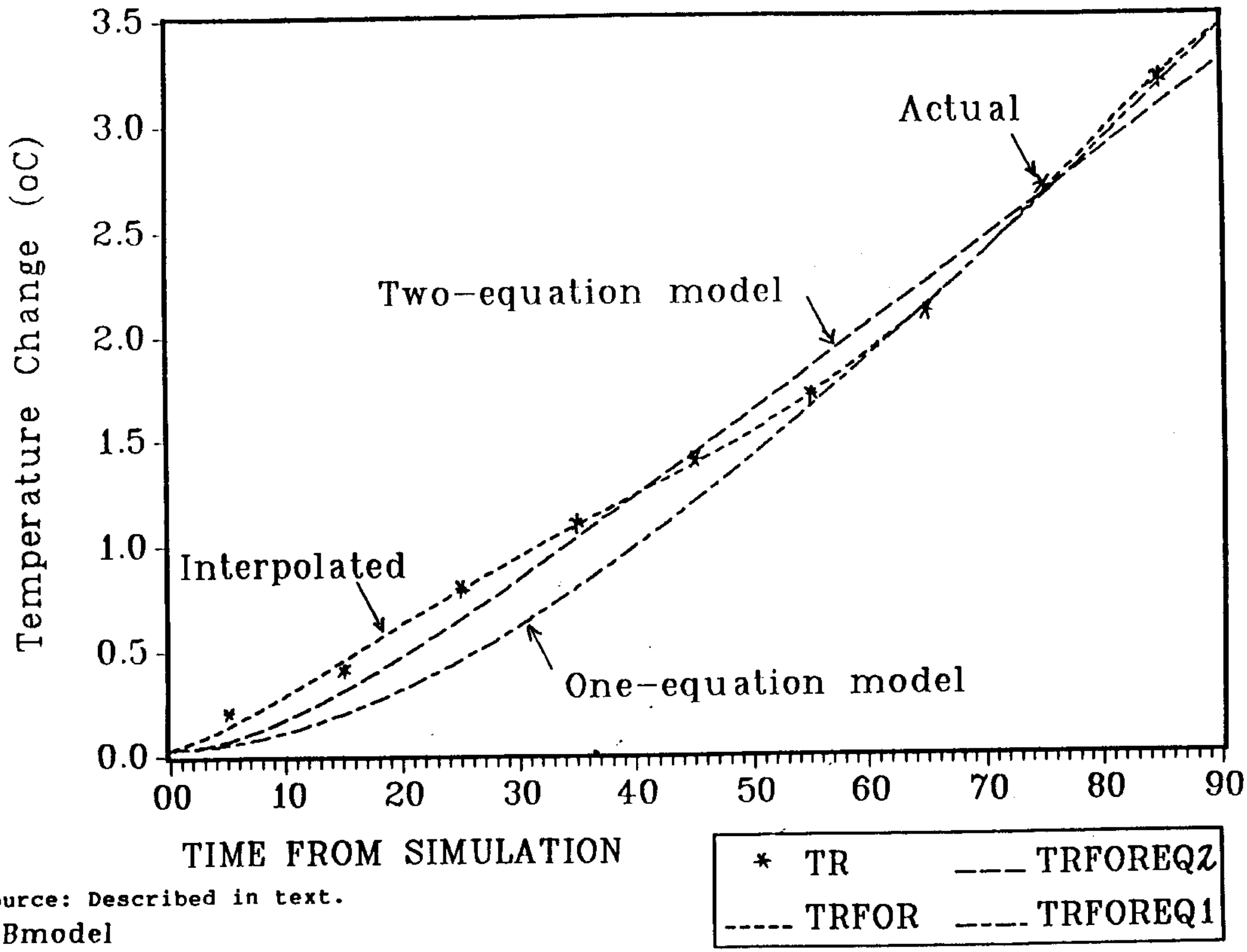


Figure 5.1: Percentage change in globally and annually averaged precipitation as a function of global mean warming from 17 models. The numbers refer to the entries describing the models in Table 3.2a.

Source: IPCC [1990], p. 138.

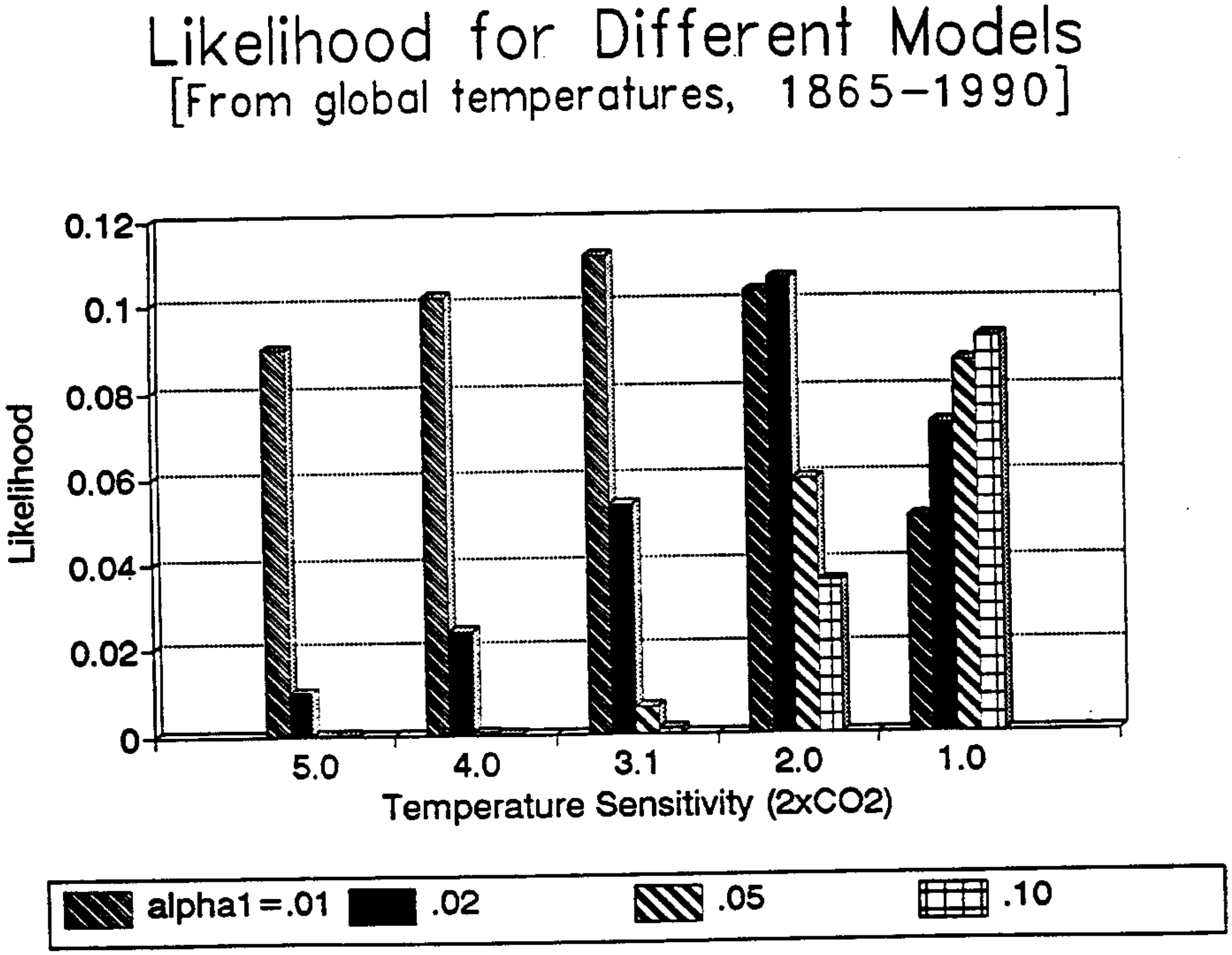
Figure III.B-2

ACTUAL, INTERPOLATED, AND MINIMODEL, S-M-B MODEL



Source: Described in text.
SMBmodel

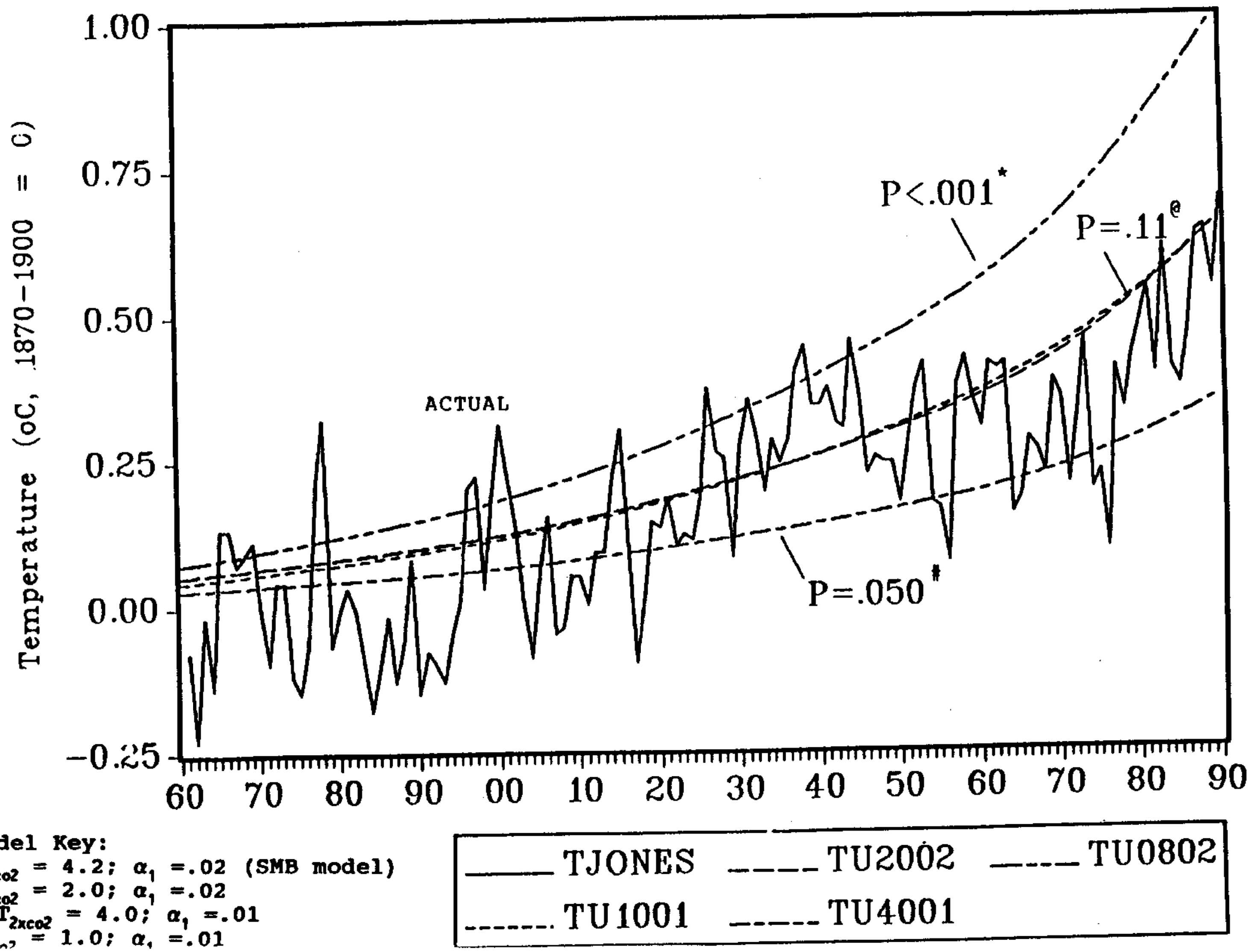
Table III.B-3



Source: Equation set (B-2) was estimated using data on radiative forcing and observed changes in globally averaged surface temperatures. Likelihood was formed for each of twenty sets of parameters (five temperature-sensitivity times four inertial coefficients) and these were normalized to sum to unity. Each bar shows the normalized likelihood for the particular parameter combination.

Figure III.B-4

ACTUAL AND PROJECTED TEMPERATURE INCREASE, 1860–1990



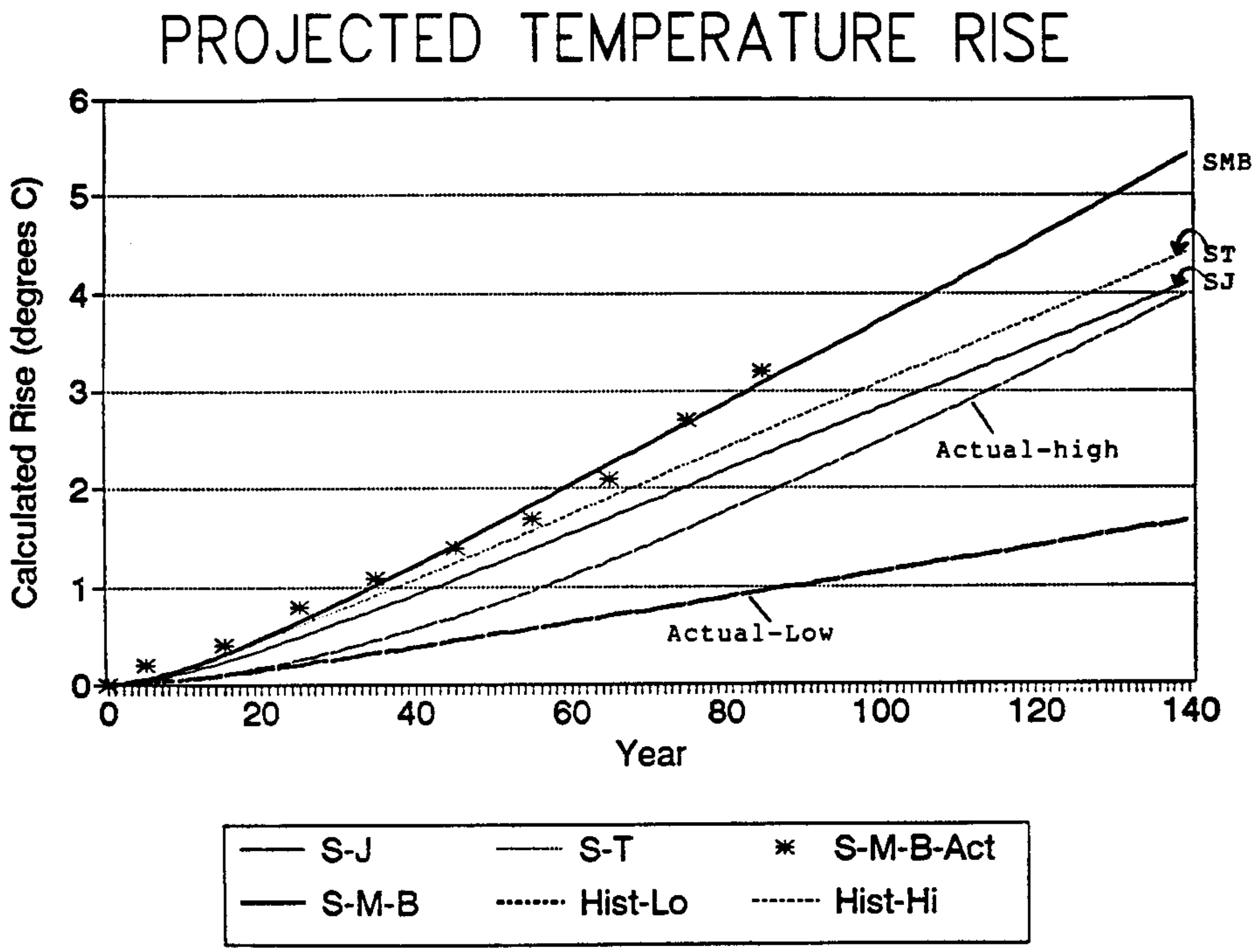
Model Key:

* T_{2xco2} = 4.2; α_1 =.02 (SMB model)

@ T_{2xco2} = 2.0; α_1 =.02 and T_{2xco2} = 4.0; α_1 =.01

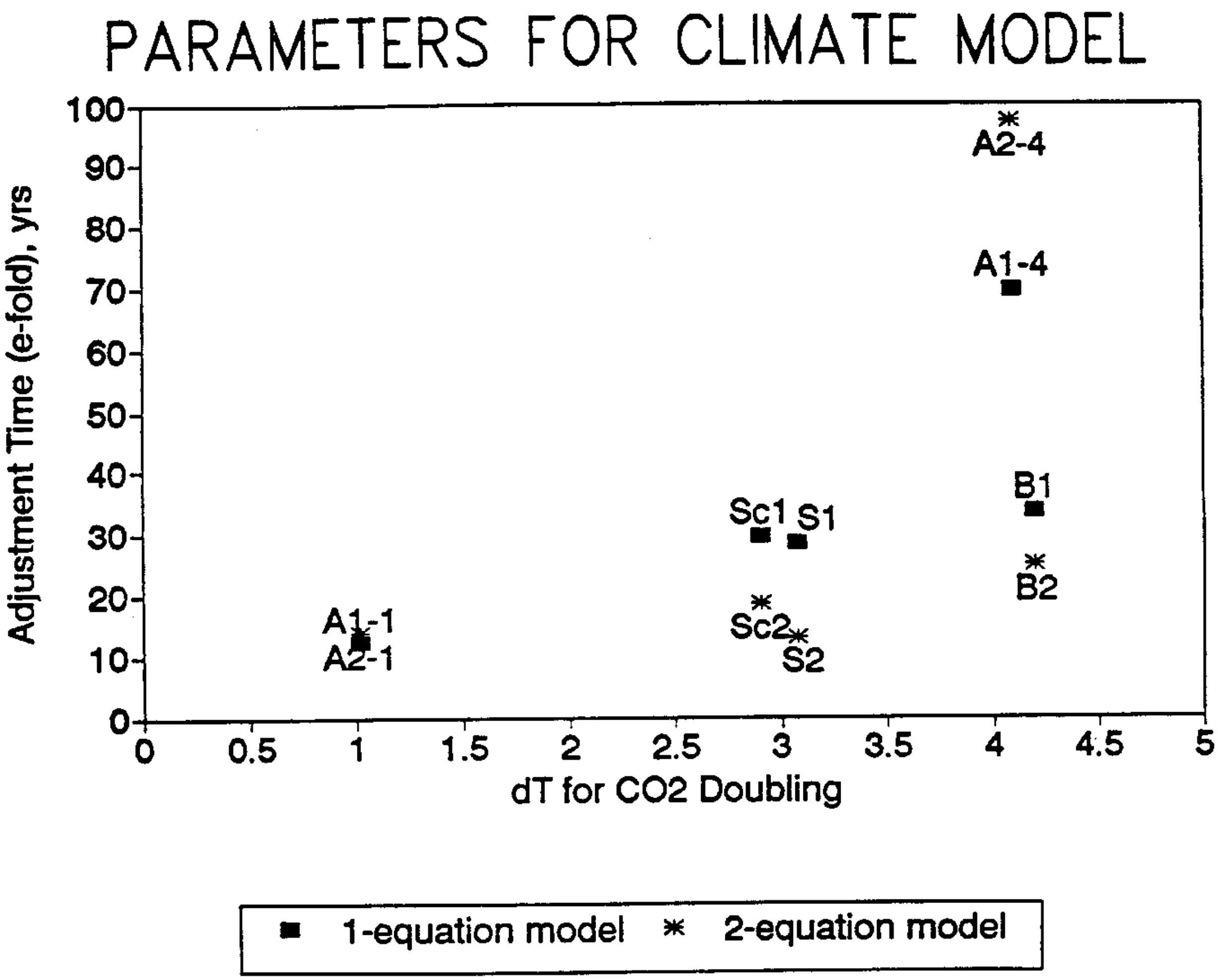
\# T_{2xco2} = 1.0; α_1 =.01

Figure III.B-5



Notes: Figure shows the calculated trajectory of climate change for a 1 percent per year compounded increase in CO_2 concentrations. The * show the calculation from SMB model. All others are for different parameters of the two-equation model in (B-2). Lower two curves show the trajectories consistent with actual historical temperature data.

Figure III.B-6



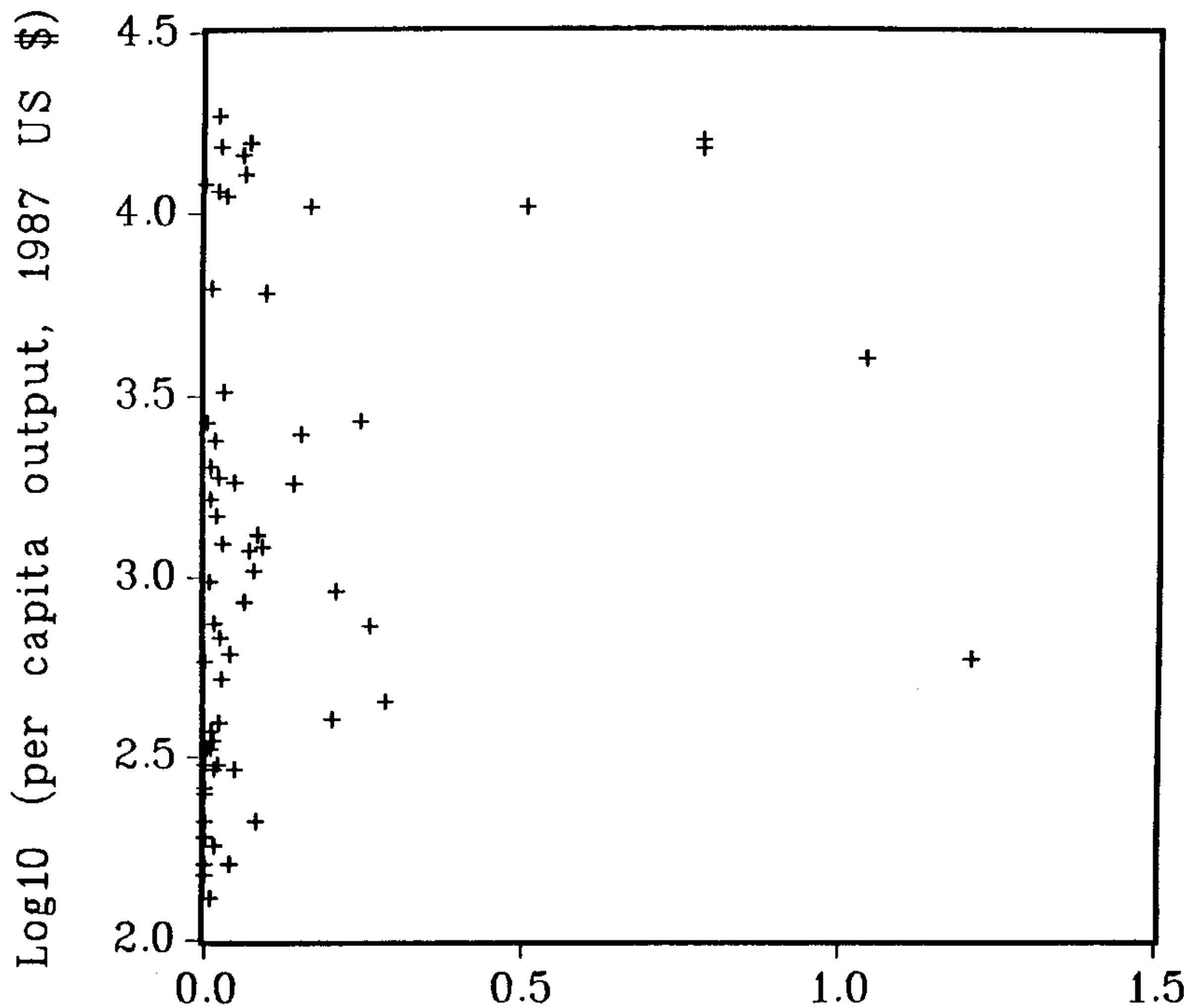
Each point shows the parameter pair for a particular model or estimate. Key is:

A1-1: History (T=1, 1 equation)
A1-4: History (T=4, 1 equation)
Sc1: S-J model (1 equation)
S1: S-T model (1 equation)
B1: S-M-B model (1 equation)

A2-1: Same (two equation model)
A2-4: Same (two-equation model)
Sc2: Same (two-equation model)
S2: Same (Two-equation model)
B2: Same (two-equation model)

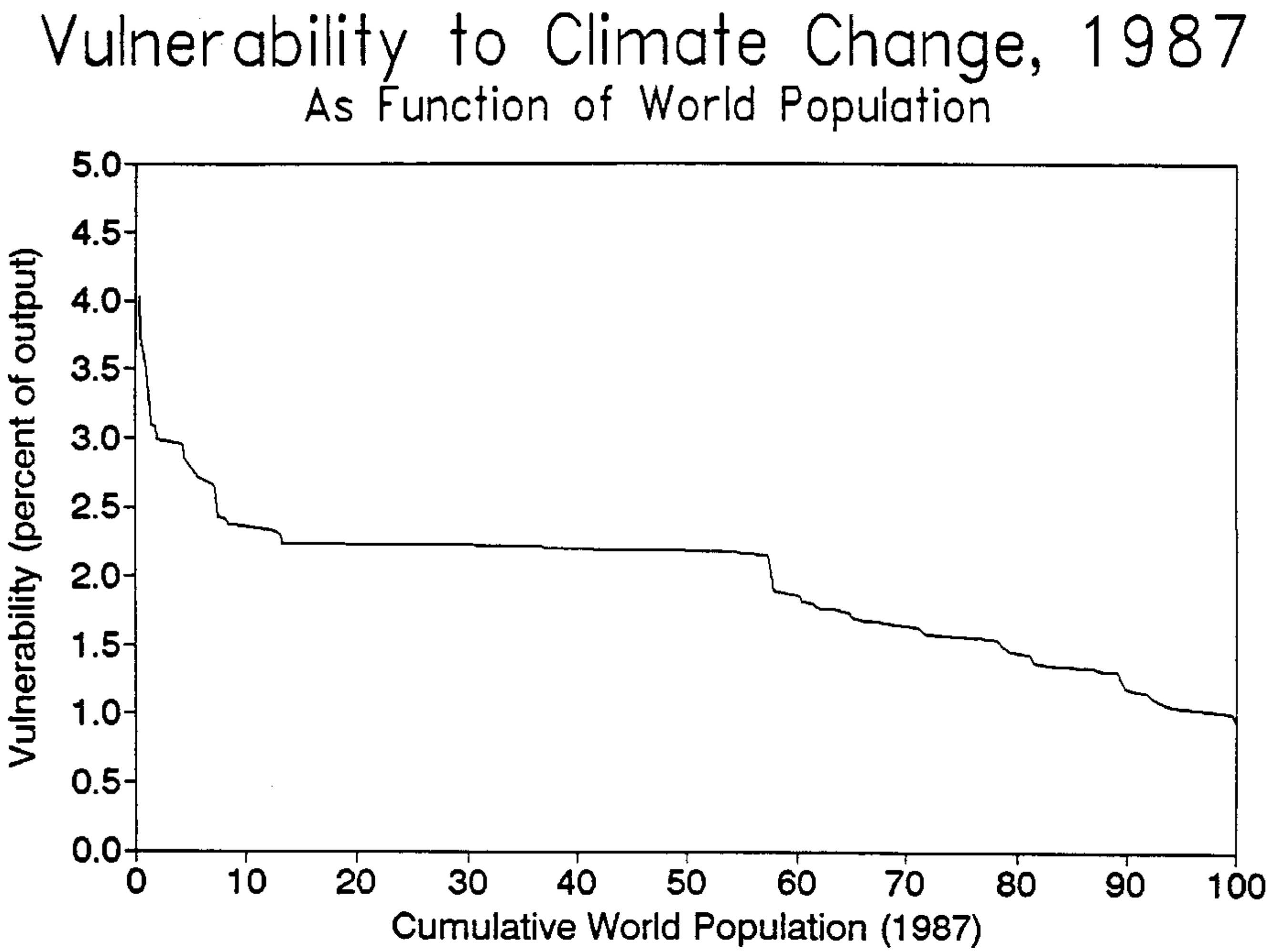
Figure III.C-1

Per capita GNP and coastal vulnerability



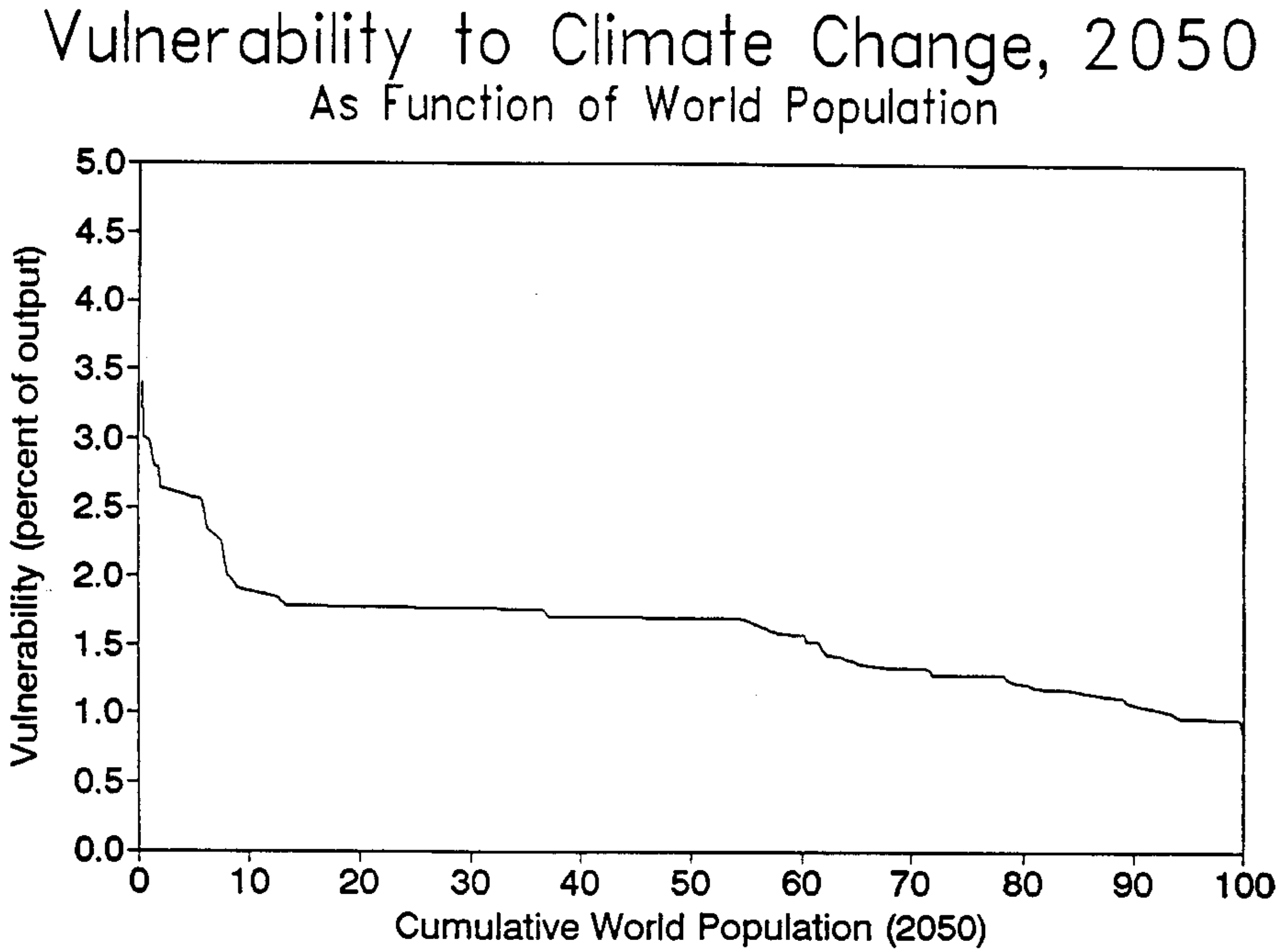
Note: Coastal vulnerability is fraction of land area within 10 km of coastline.

Figure III.C-2



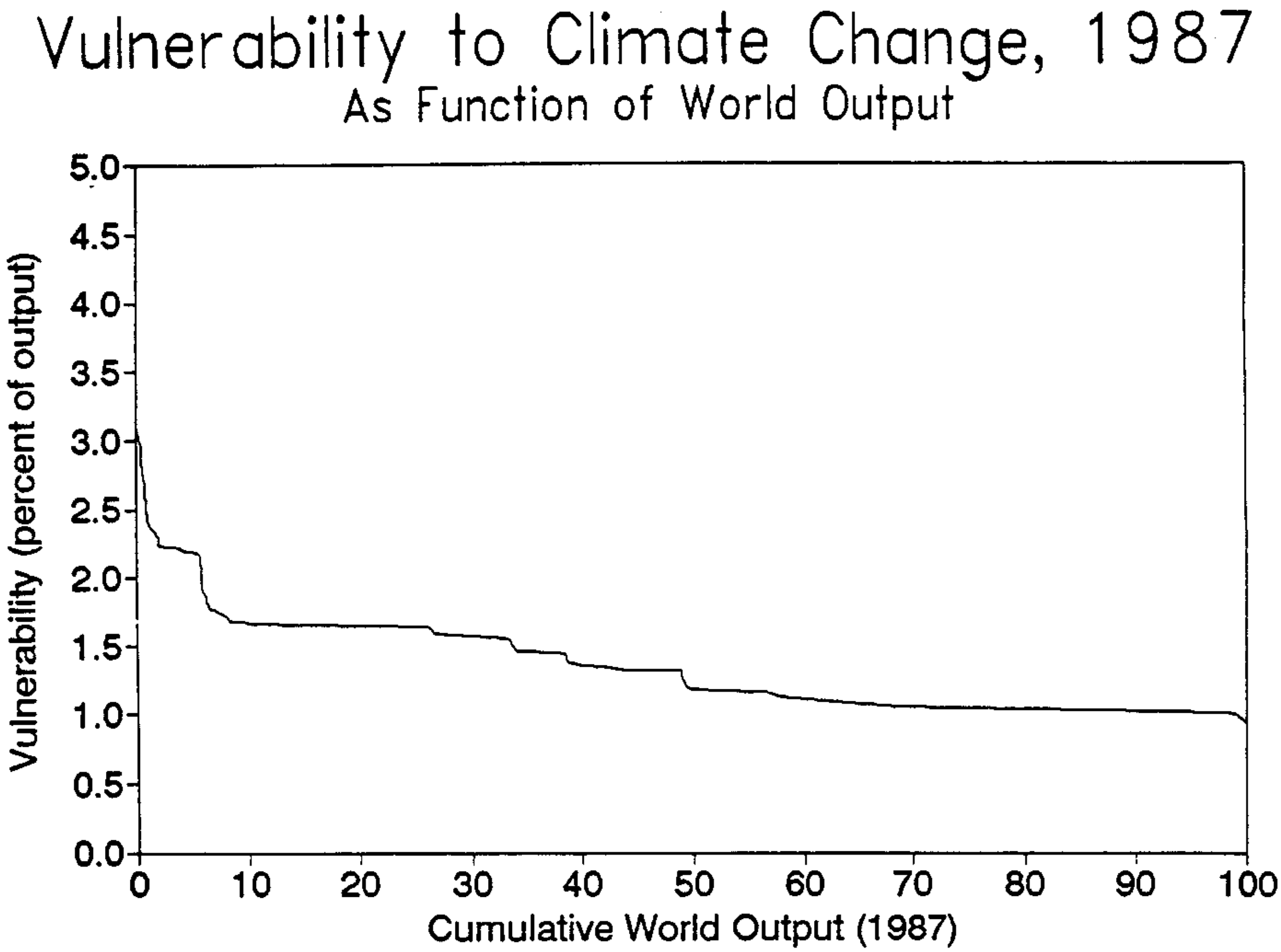
Note: Countries are ranked by their vulnerability to climate change as shown in Table C-2. Countries are then rank ordered and cumulated by 1987 population. Each point therefore shows the fraction of the world's population with vulnerability at or above the given level.

Figure III.C-3



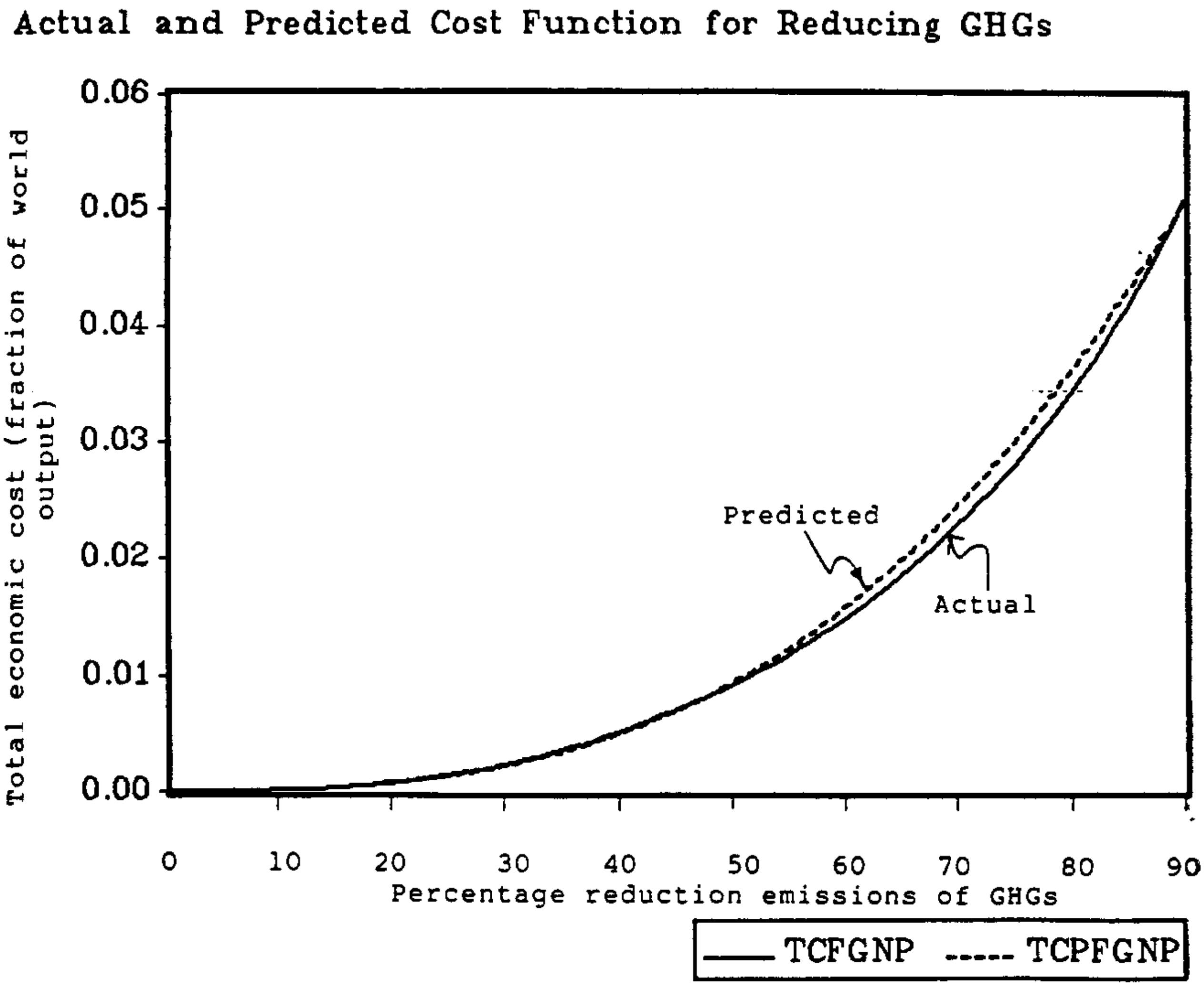
Note: See Figure C-2.

Figure III.C-4



Note: The technique for generating this curve is the same as for Figure C-2. However, countries are cumulated by their share of world output rather than world population.

Figure III.D-1



Note: "Actual" is estimate of total costs of GHG reduction from Nordhaus [1991], measured as a percent of world output. "Predicted" is from estimated parameterized equation in text.

Figure III.E-1

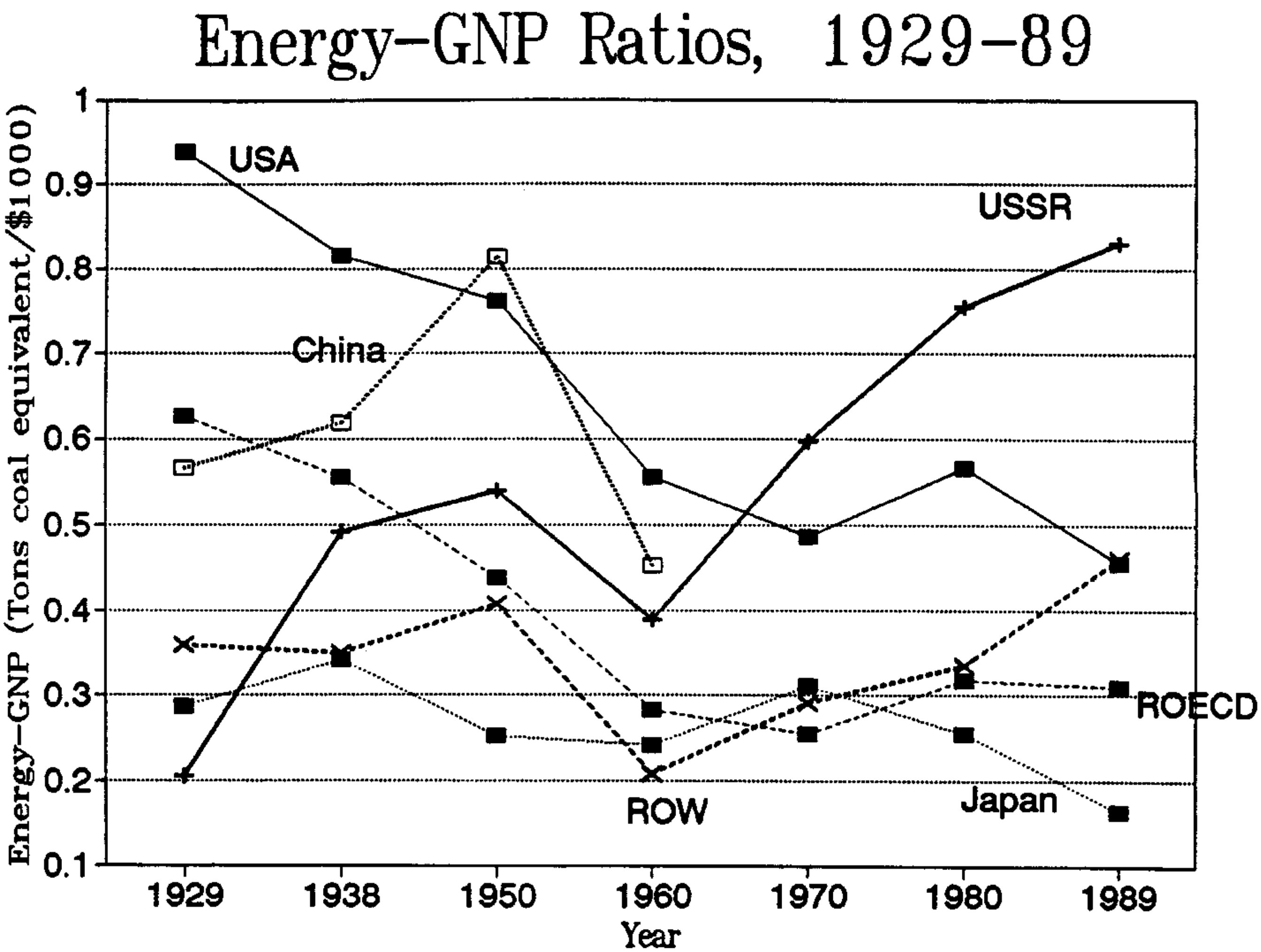


Figure III.E-2

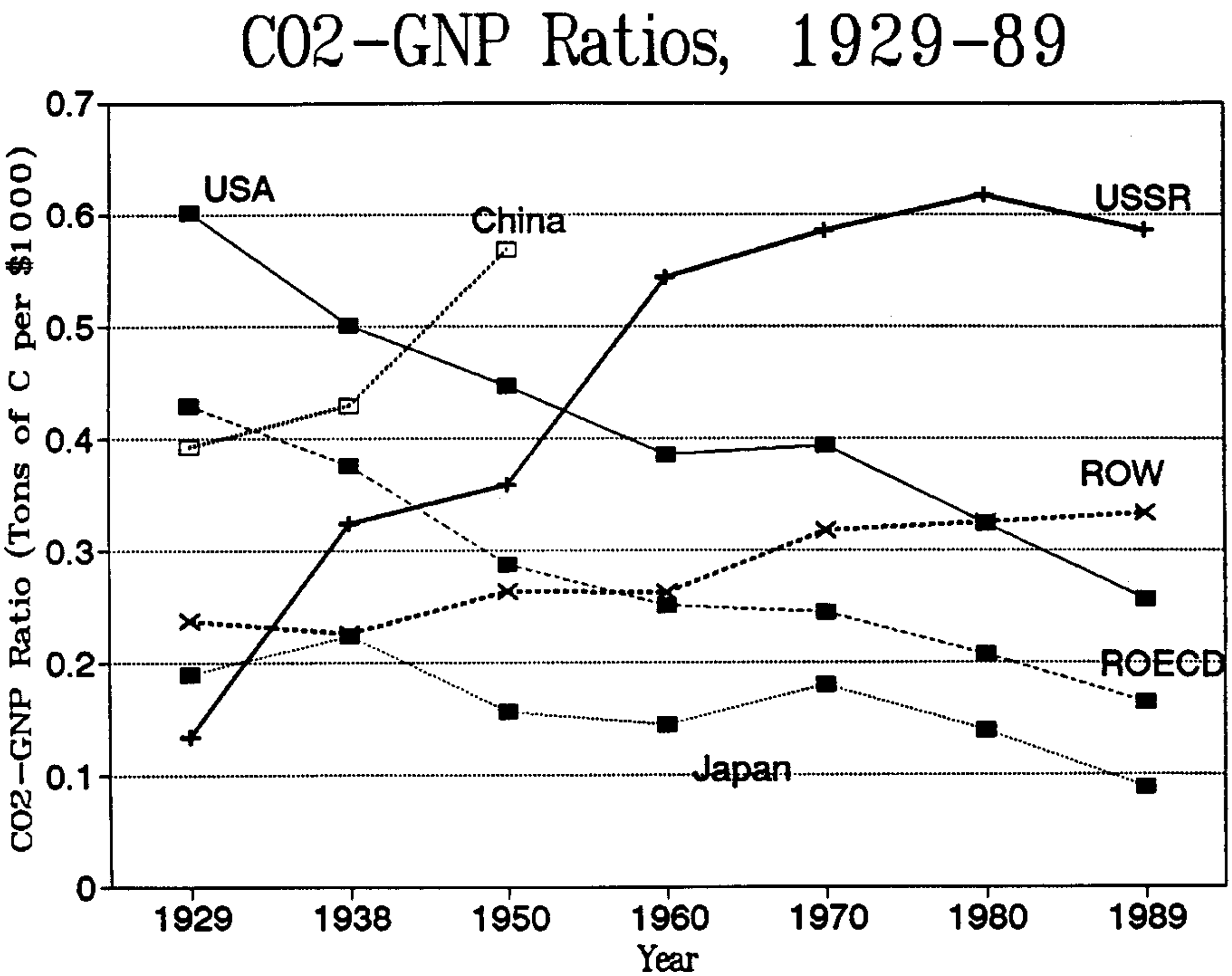
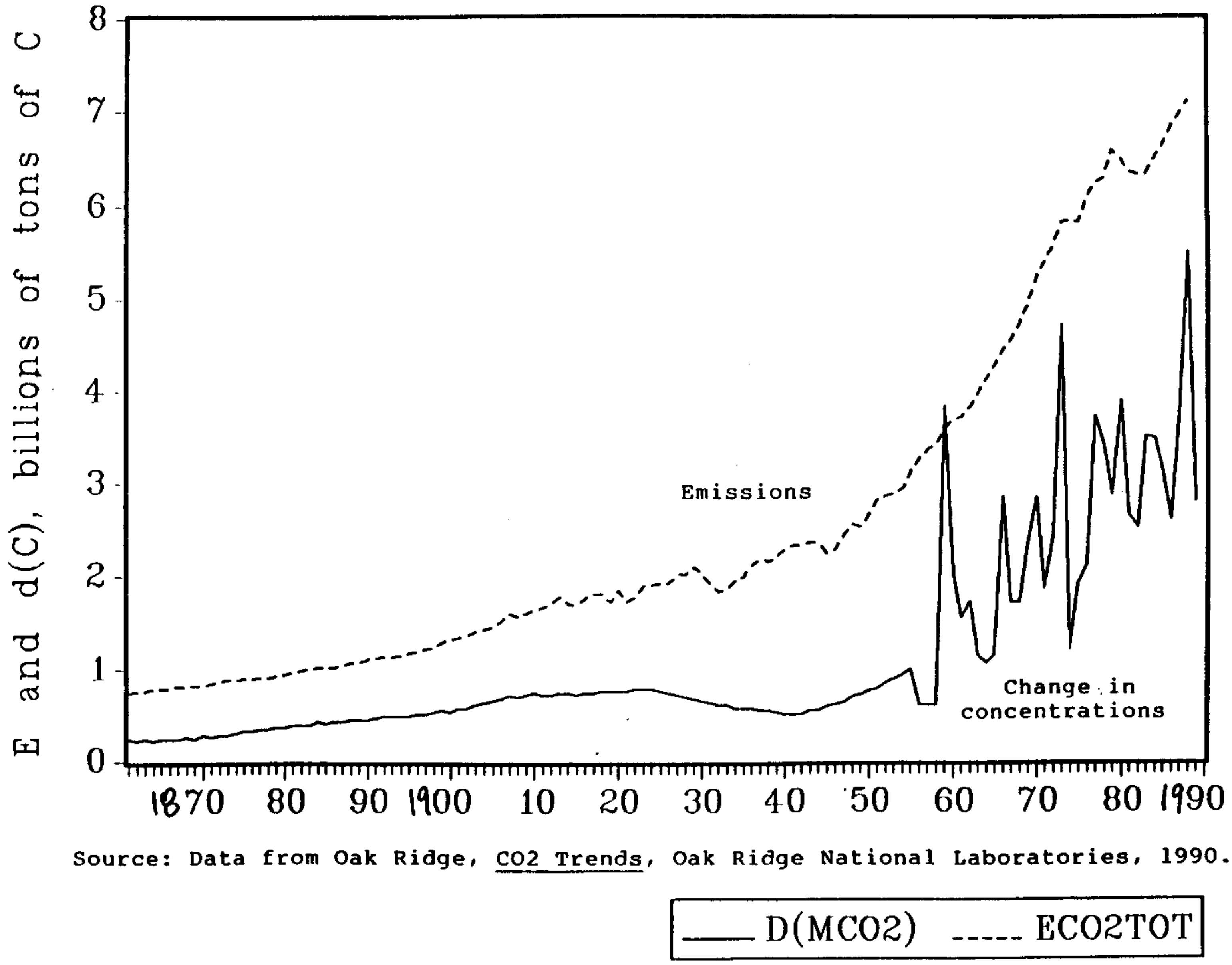


FIGURE III.F-1

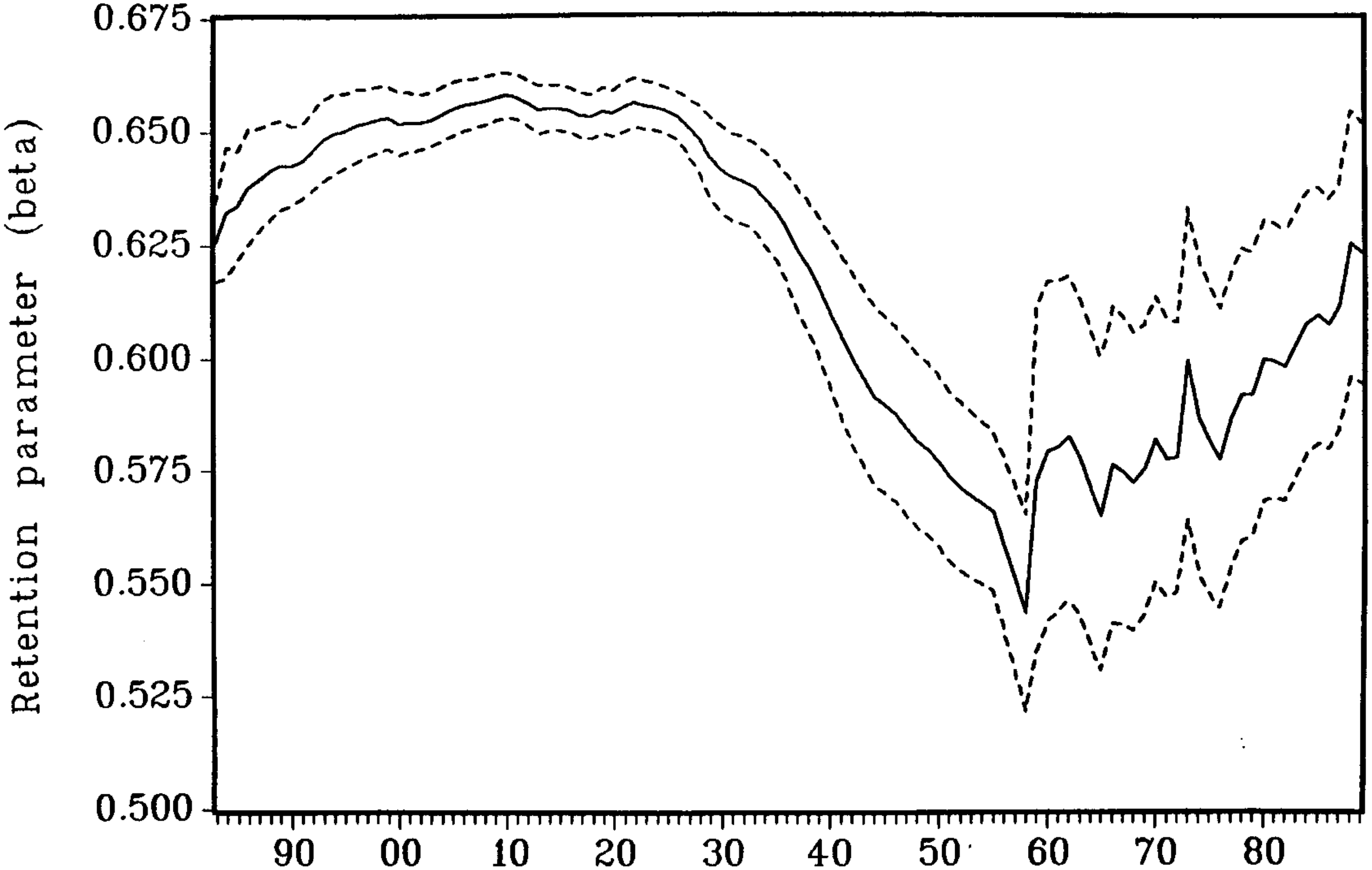
CO2 EMISSIONS AND CONCENTRATION CHANGES, 1860–1988



Source: Data from Oak Ridge, CO2 Trends, Oak Ridge National Laboratories, 1990.

Table III.F 3

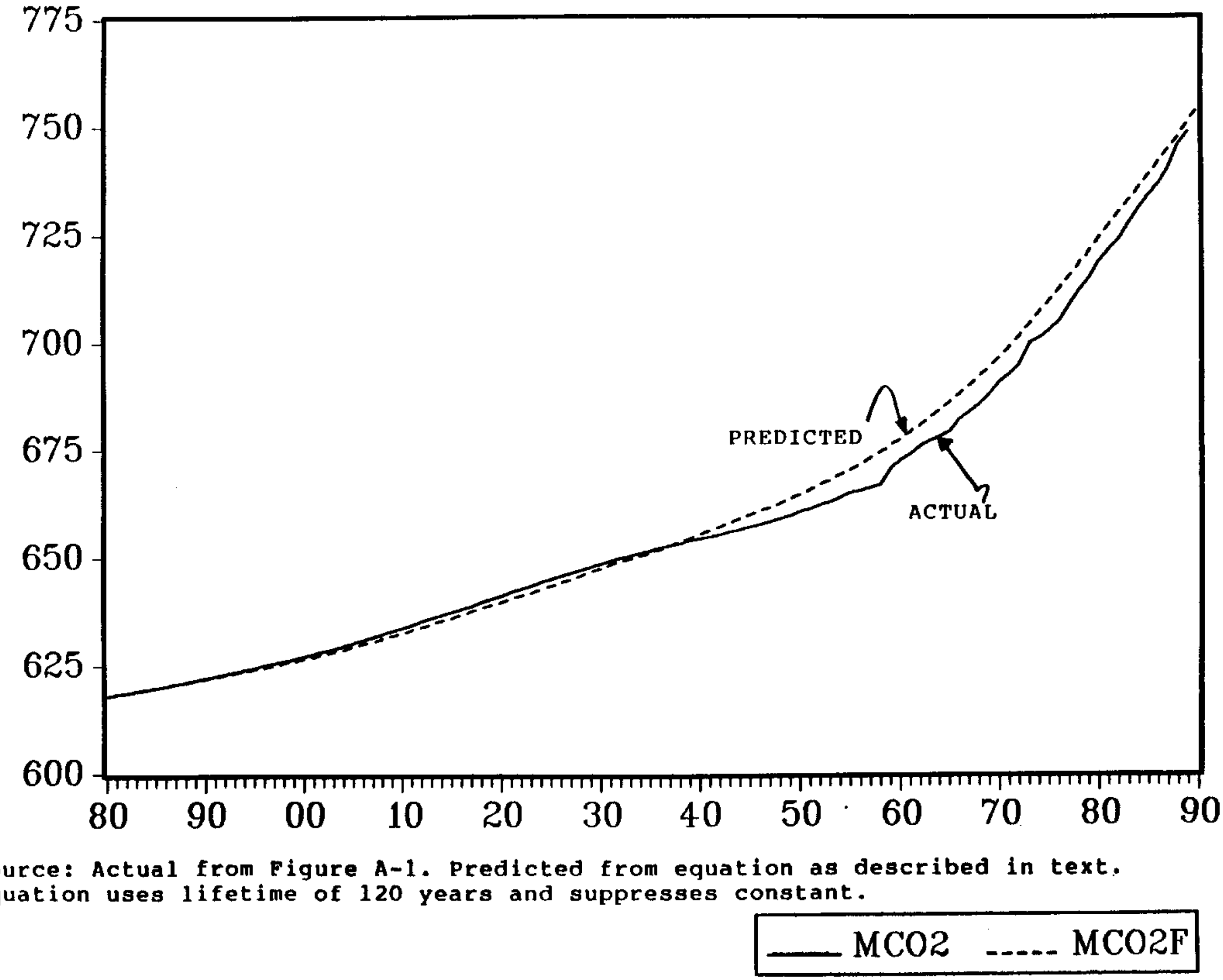
RECURSIVE ESTIMATE OF ATMOSPHERIC RETENTION PARAMETER



Source: Equation uses lifetime of 120 years and suppresses constant term. Observation for a given year shows the coefficient as solid line and coefficient plus and minus two standard errors of coefficient as dashed lines for 1880 through date shown.

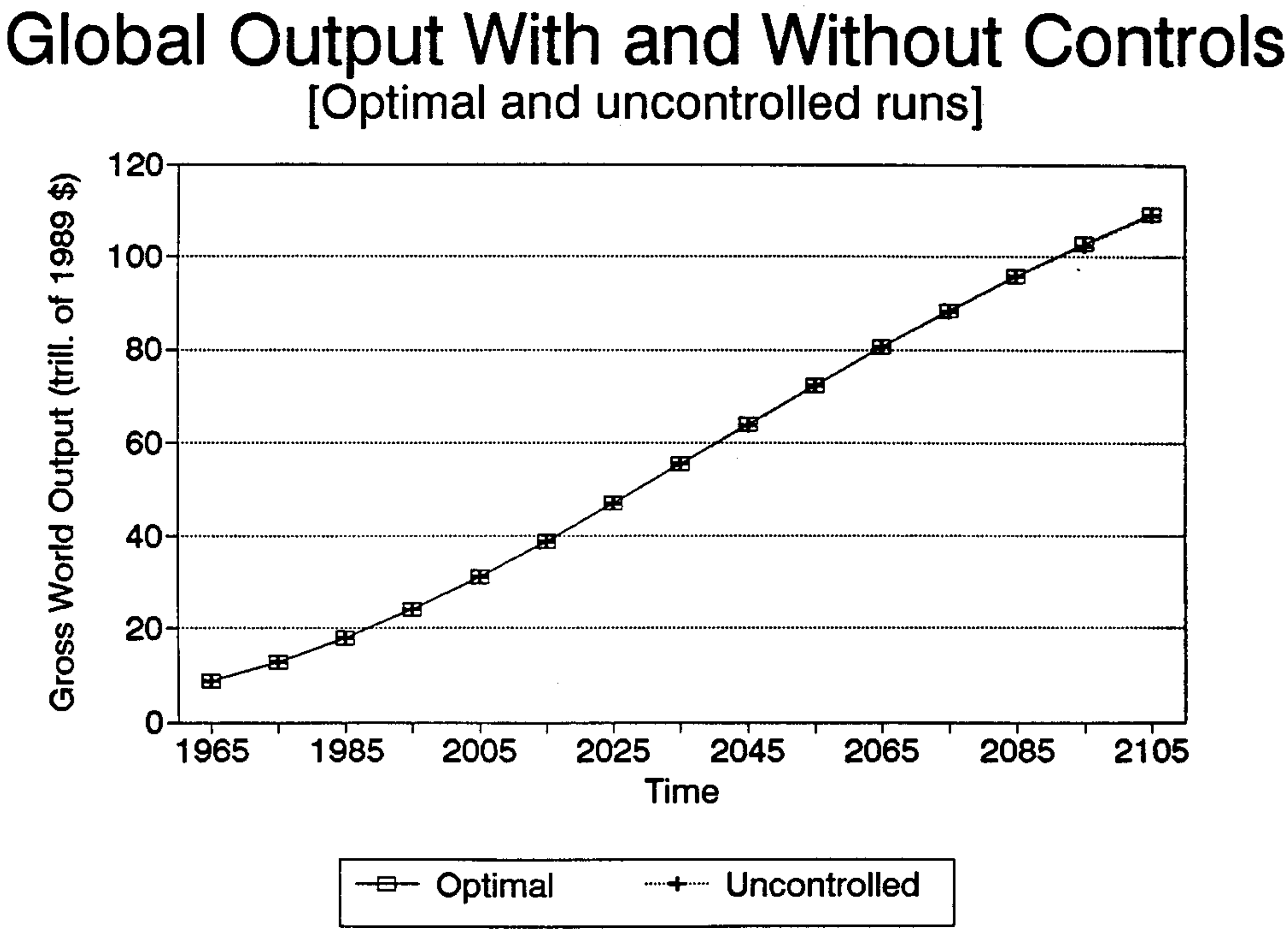
—— Recursive C(1) Estimates ----- +- 2 S.E.

Figure III.F-2
ACTUAL AND PREDICTED CO2 CONCENTRATIONS



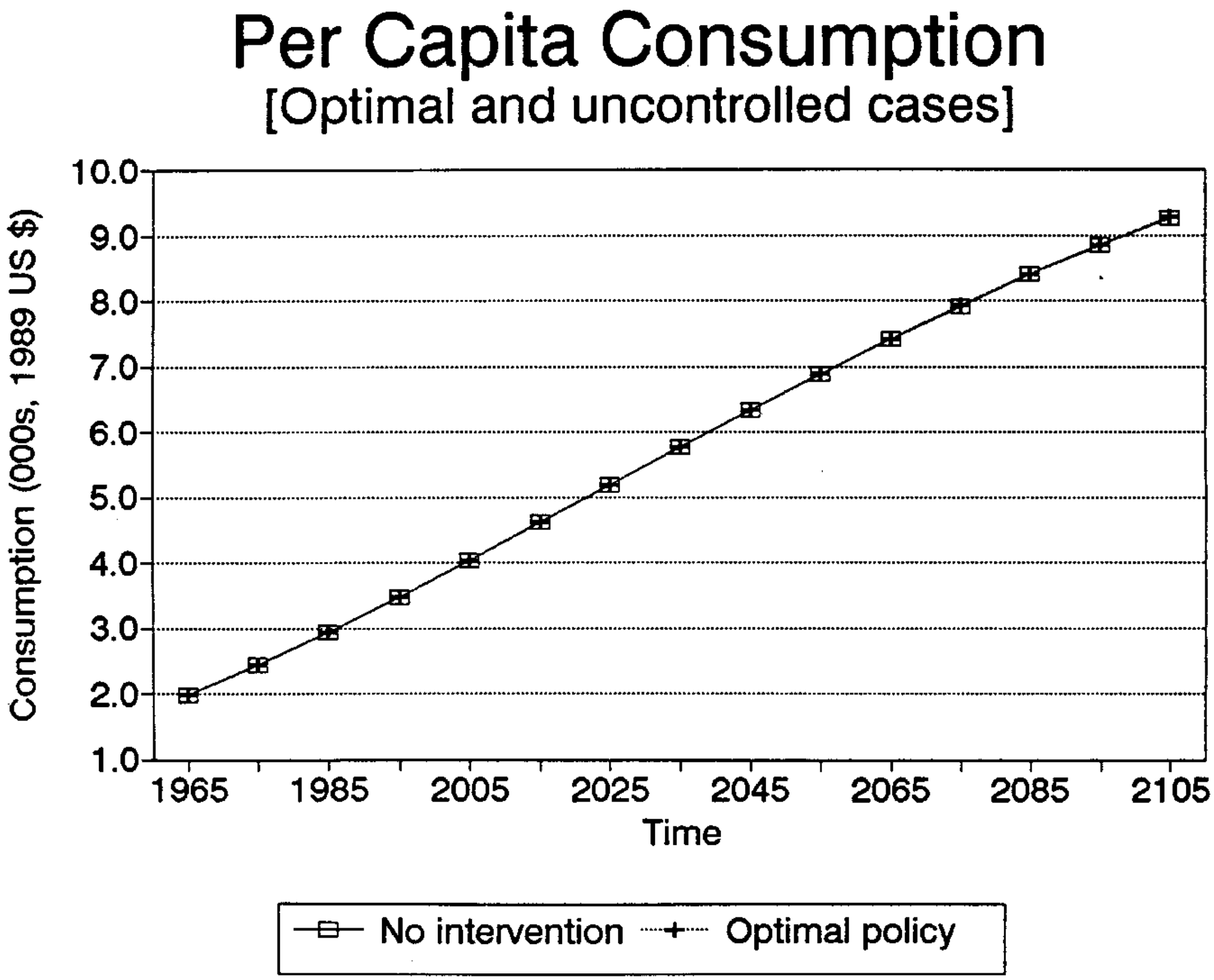
Source: Actual from Figure A-1. Predicted from equation as described in text. Equation uses lifetime of 120 years and suppresses constant.

Figure IV-1



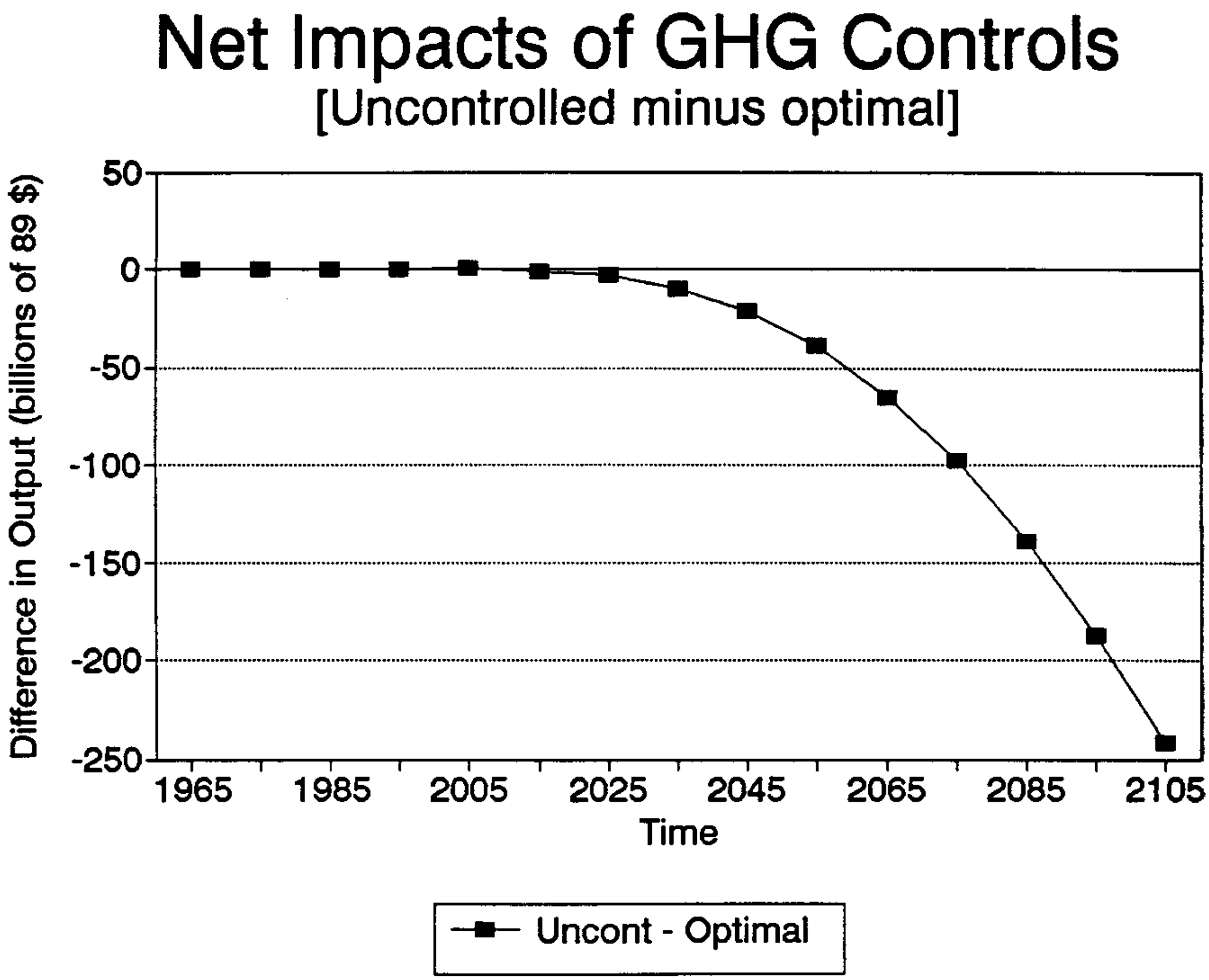
Global Output With and Without Controls
[Optimal and uncontrolled runs]
Gross World Output (trill. of 1989 $)
120
100
80
60
40
20
0
1965
1985
2005
2025
2045
2065
2085
2105
Time
Optimal
Uncontrolled

Figure IV-2



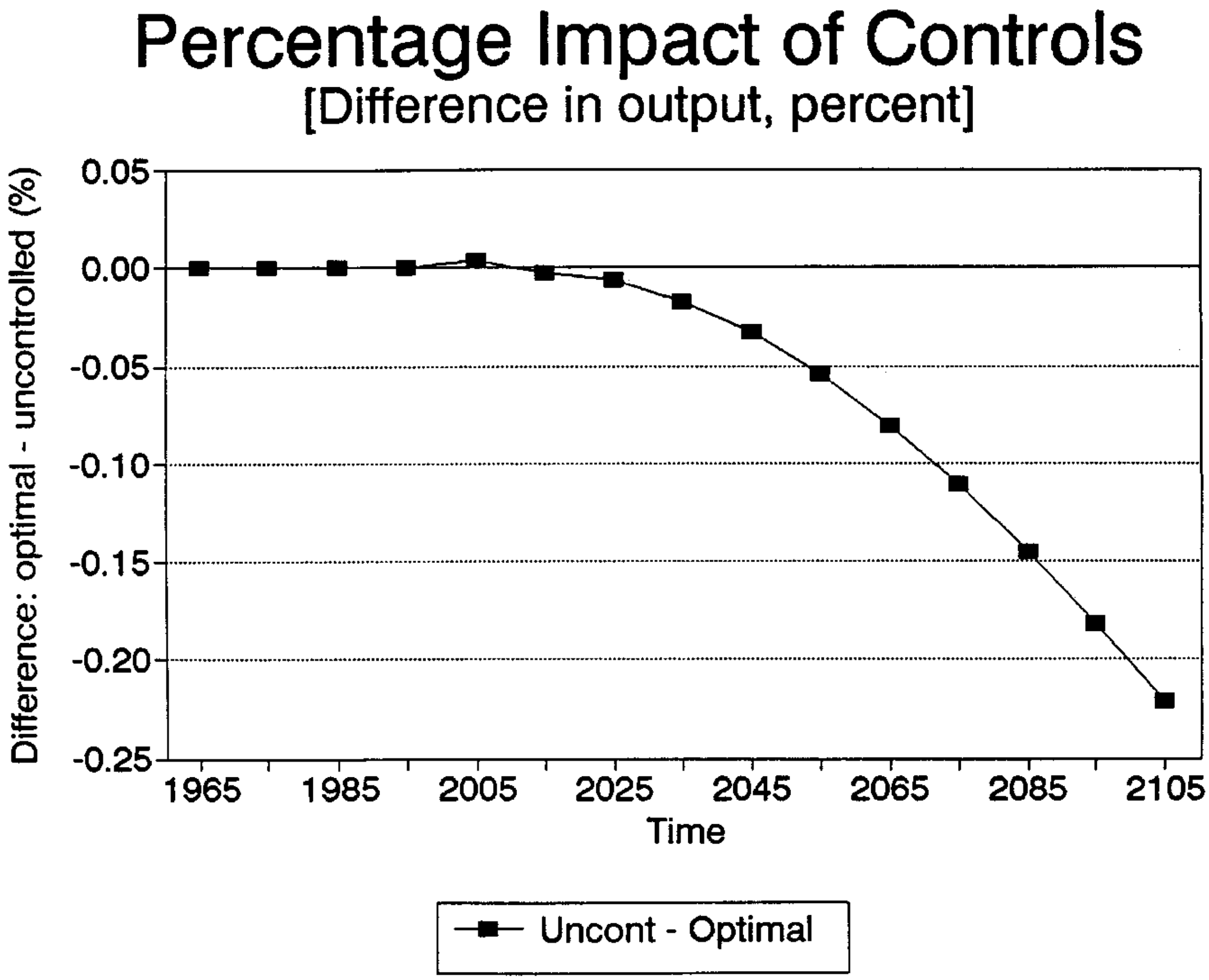
Per Capita Consumption
[Optimal and uncontrolled cases]
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
Consumption (000s, 1989 US $)
1965
1985
2005
2025
2045
2065
2085
2105
Time
No intervention
Optimal policy

Figure IV-3



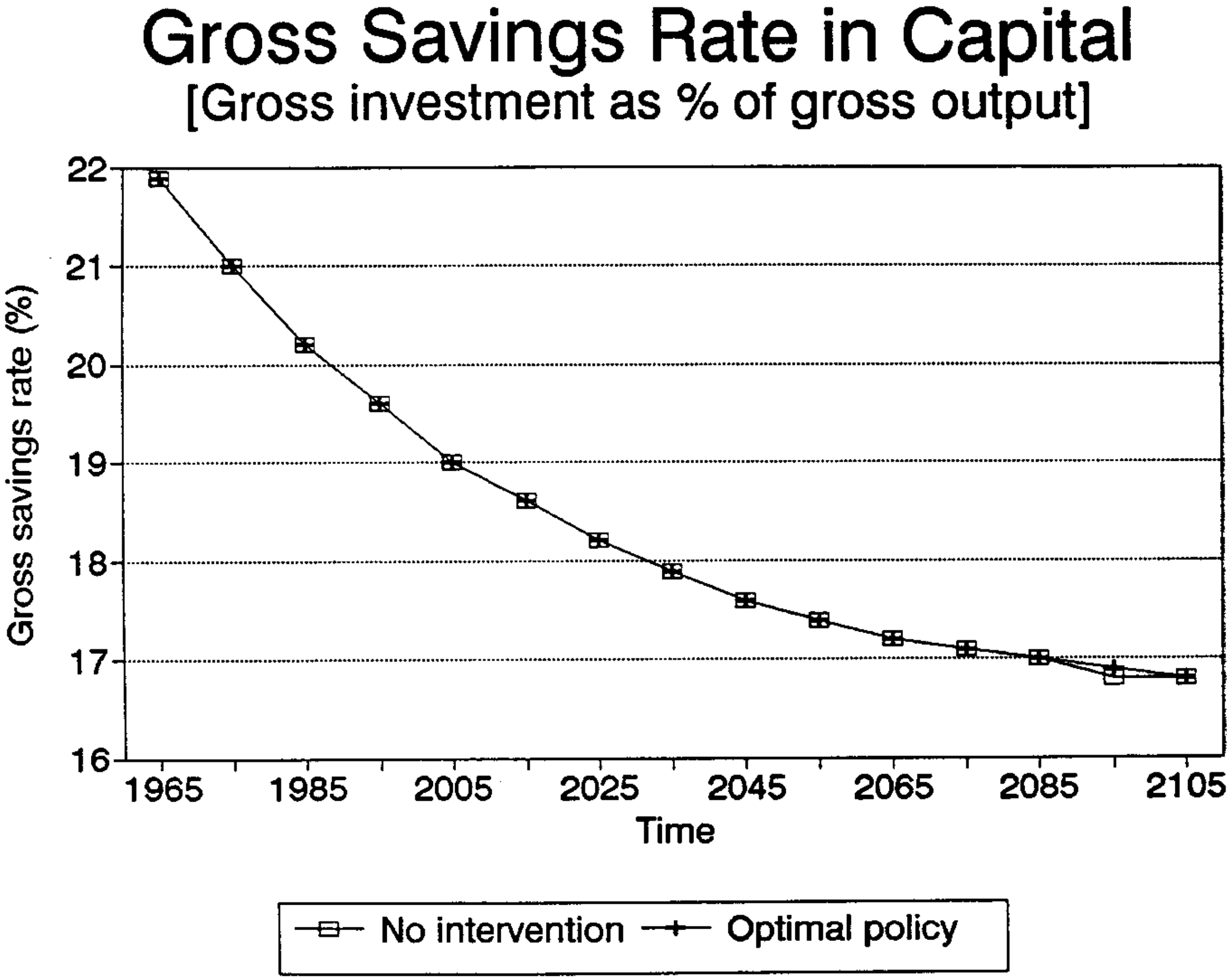
Net Impacts of GHG Controls
[Uncontrolled minus optimal]
Difference in Output (billions of 89 $)
50
0
-50
-100
-150
-200
-250
1965
1985
2005
2025
2045
2065
2085
2105
Time
Uncont - Optimal

Figure IV-4



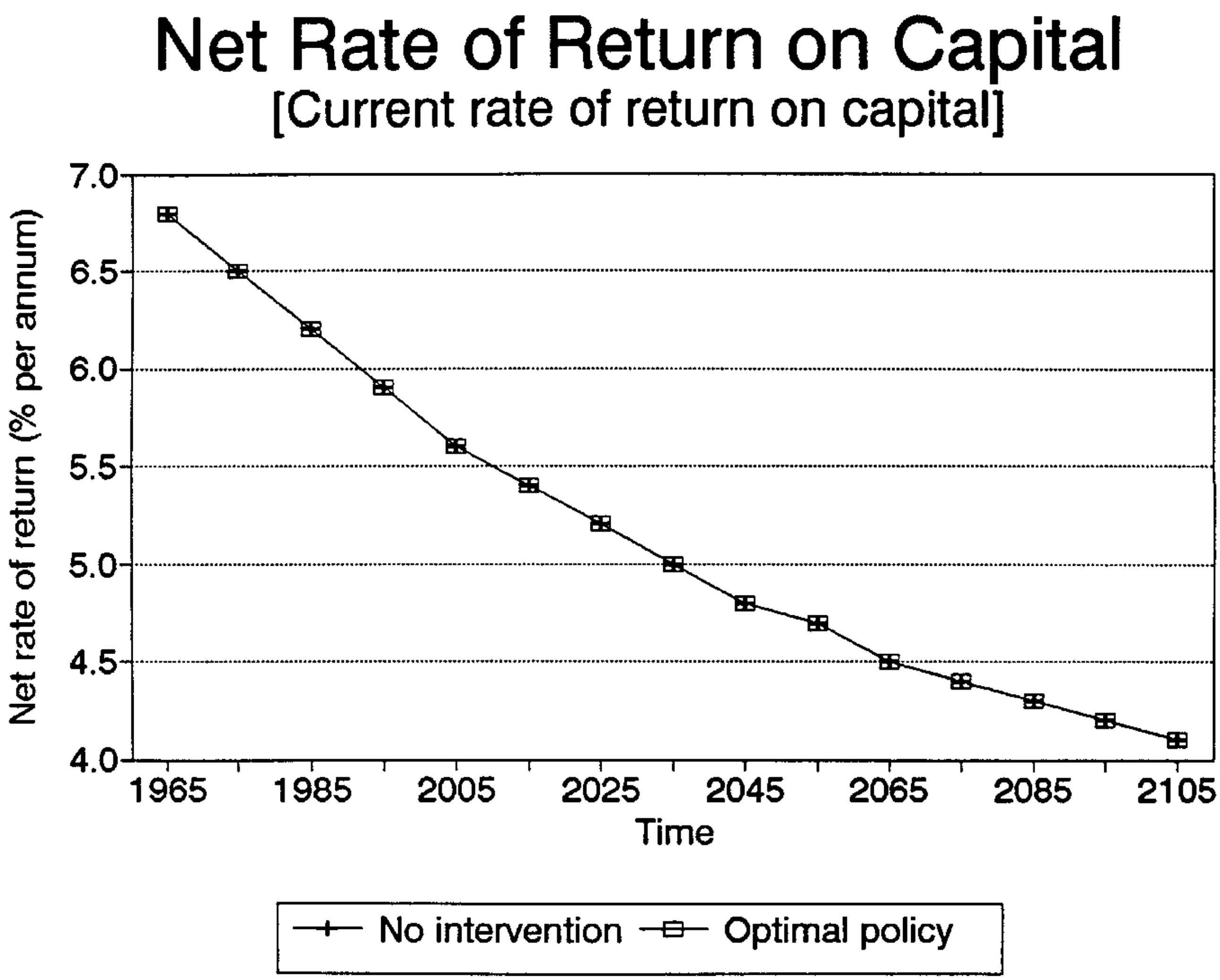
Percentage Impact of Controls
[Difference in output, percent]
Difference: optimal - uncontrolled (%)
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
1965
1985
2005
2025
2045
2065
2085
2105
Time
Uncont - Optimal

Figure IV-5



Gross Savings Rate in Capital
[Gross investment as % of gross output]
Gross savings rate (%)
22
21
20
19
18
17
16
1965
1985
2005
2025
2045
2065
2085
2105
Time
No intervention
Optimal policy

Figure IV-6



Net Rate of Return on Capital
[Current rate of return on capital]
Net rate of return (% per annum)
7.0
6.5
6.0
5.5
5.0
4.5
4.0
1965
1985
2005
2025
2045
2065
2085
2105
Time
No intervention
Optimal policy

Figure IV-7

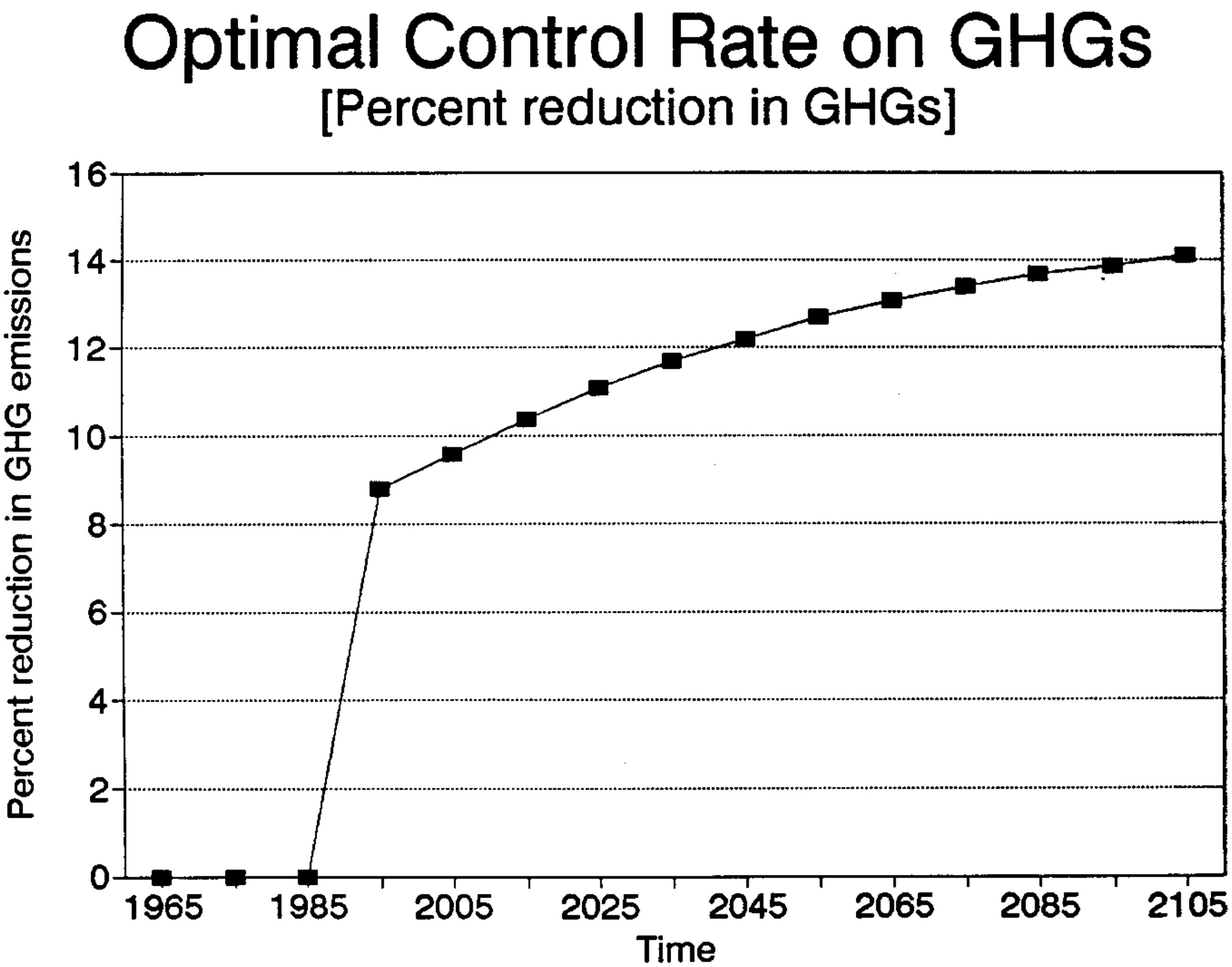

Optimal Control Rate on GHGs
[Percent reduction in GHGs]
Percent reduction in GHG emissions
16
14
12
10
8
6
4
2
0
1965
1985
2005
2025
2045
2065
2085
2105
Time

Figure IV-8

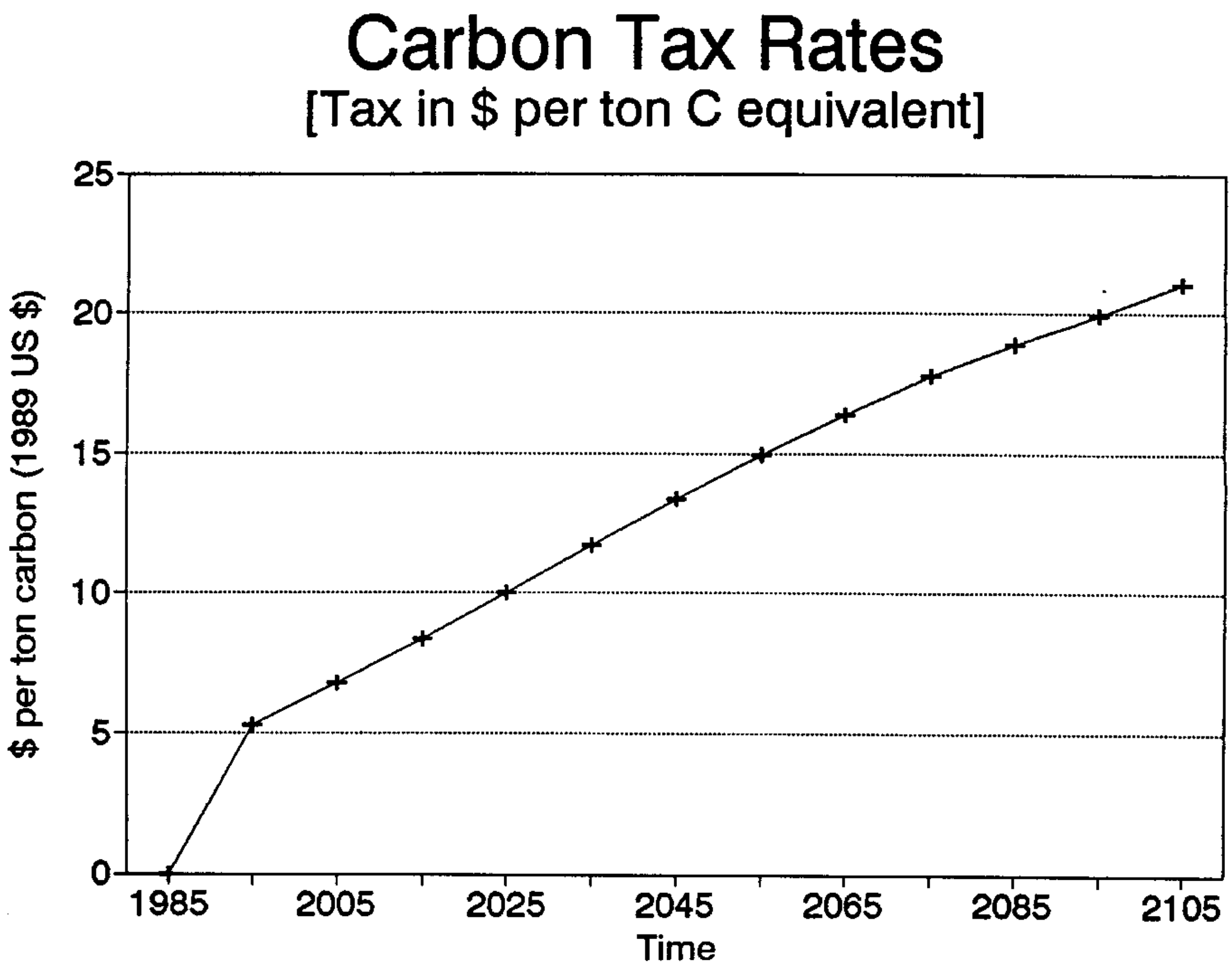
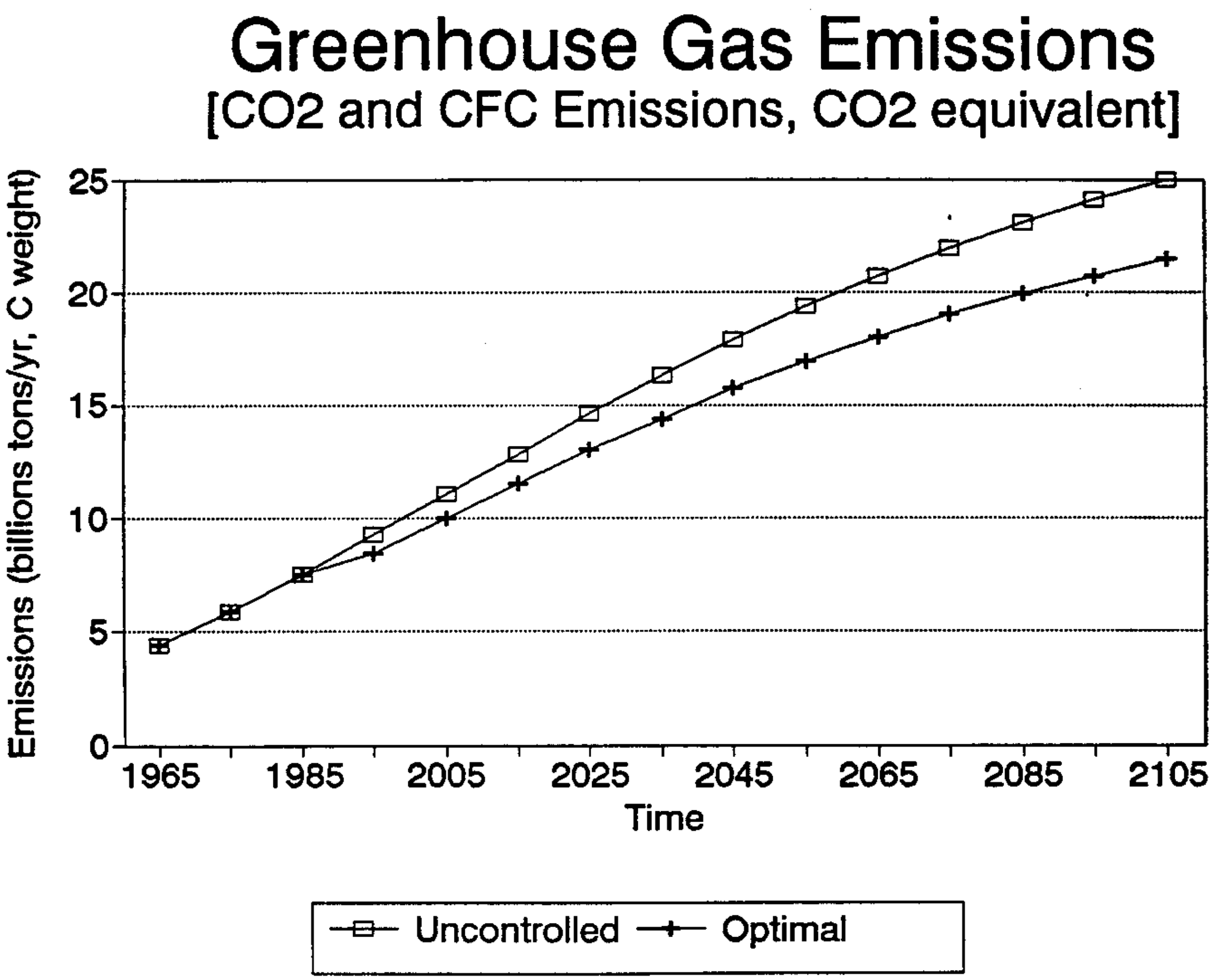

Carbon Tax Rates
[Tax in $ per ton C equivalent]
$ per ton carbon (1989 US $)
25
20
15
10
5
0
1985
2005
2025
2045
2065
2085
2105
Time

Figure IV-9



Greenhouse Gas Emissions
[CO2 and CFC Emissions, CO2 equivalent]
Emissions (billions tons/yr, C weight)
25
20
15
10
5
0
1965
1985
2005
2025
2045
2065
2085
2105
Time
Uncontrolled
Optimal

Figure IV-10

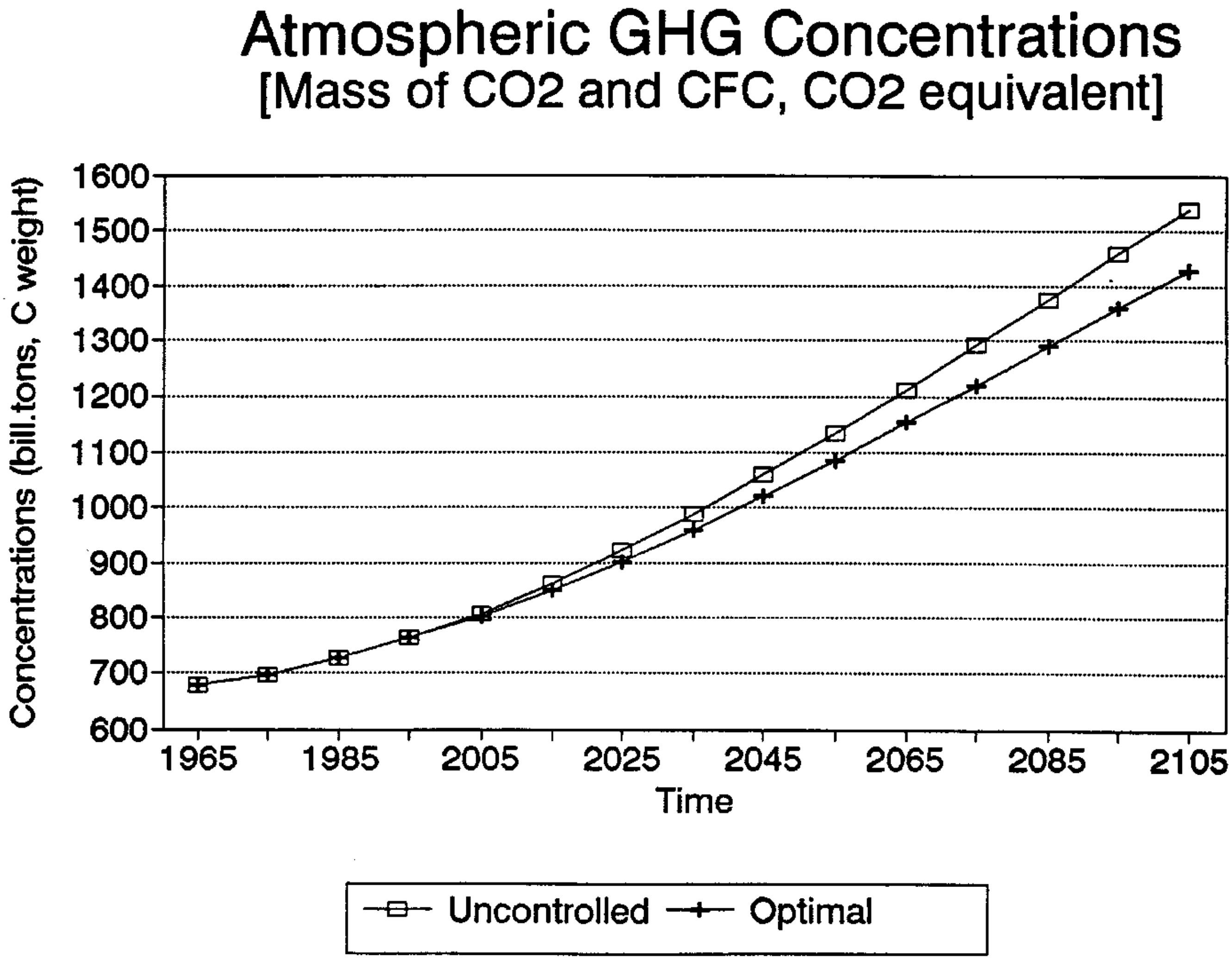
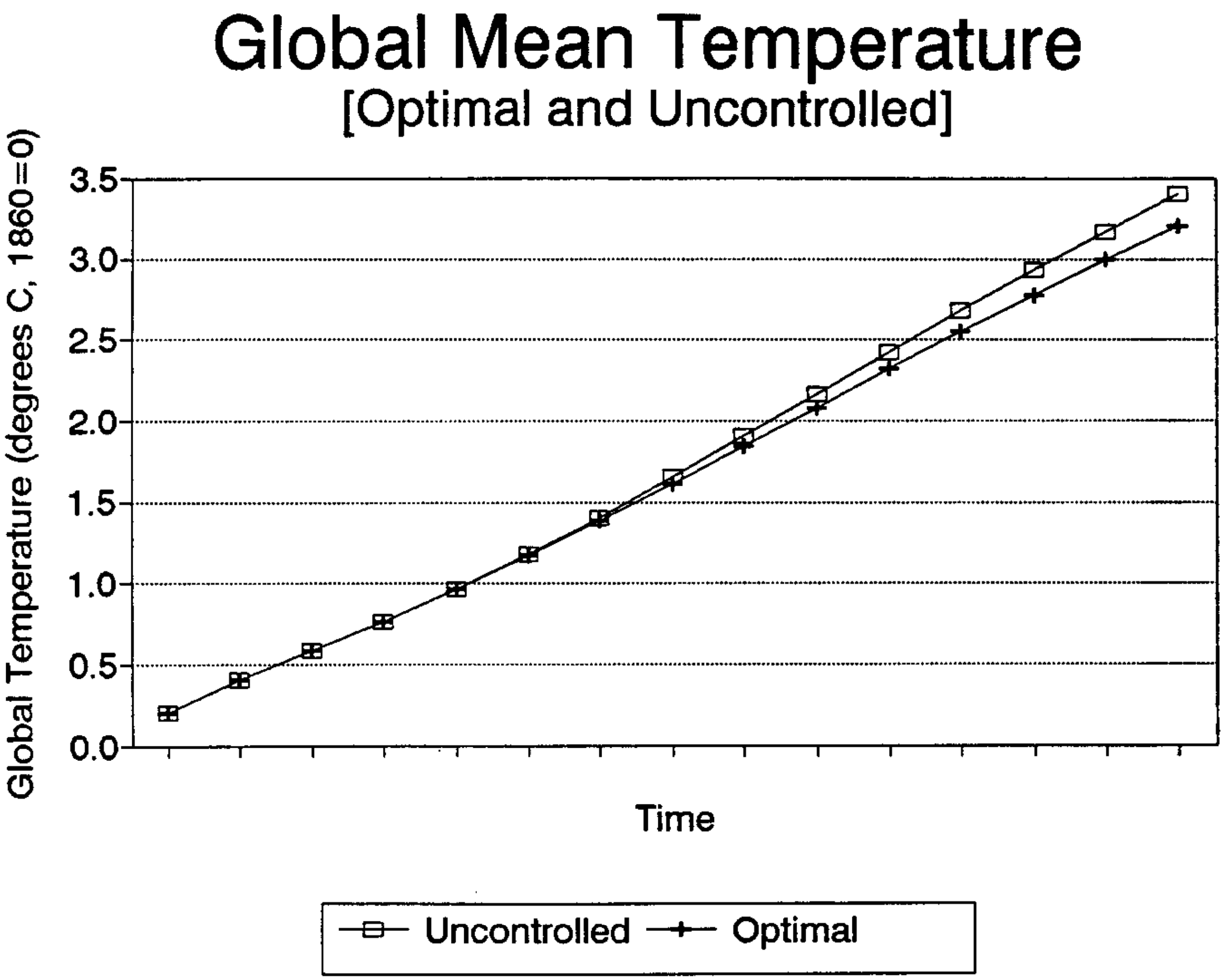
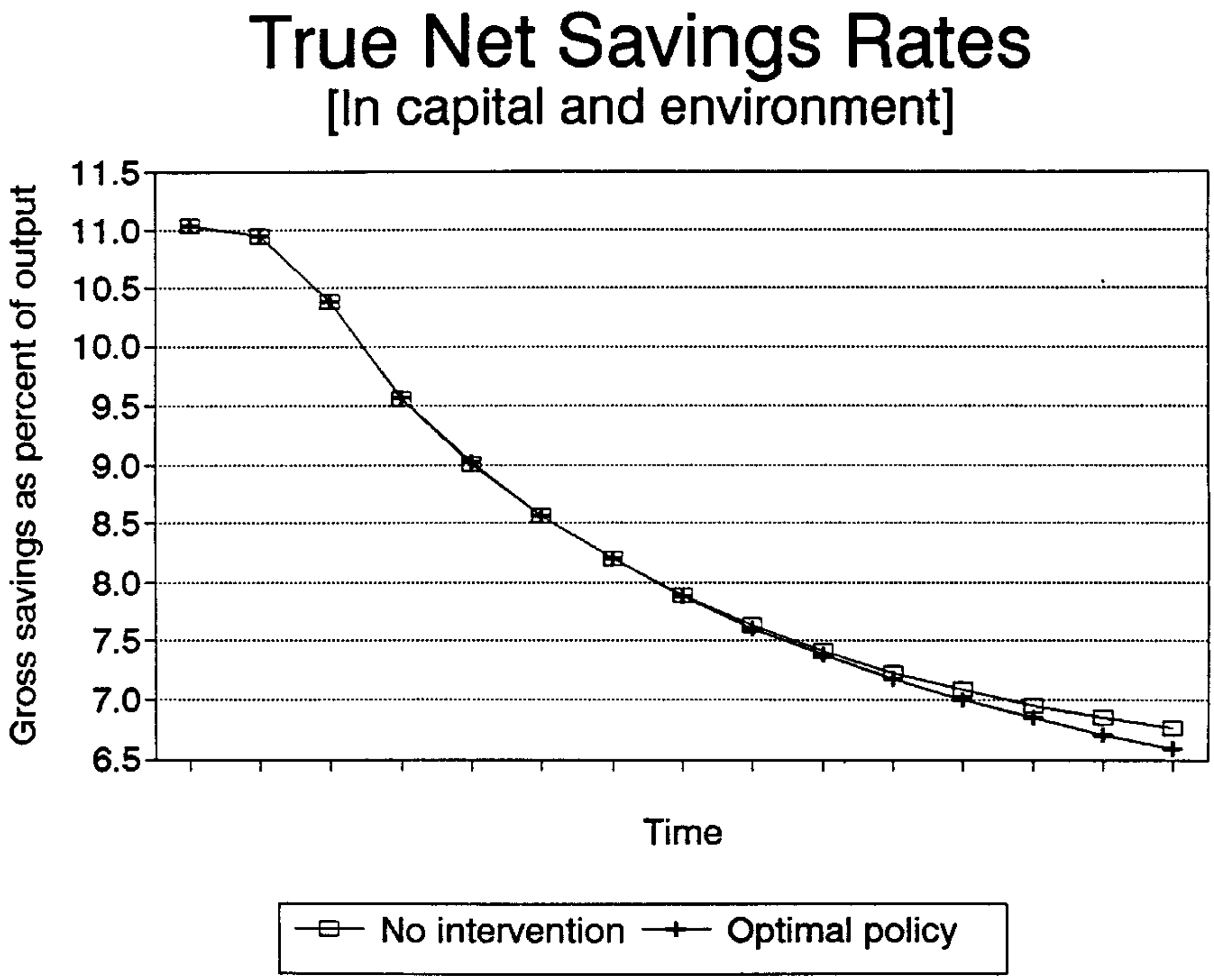

Atmospheric GHG Concentrations
[Mass of CO2 and CFC, CO2 equivalent]
Concentrations (bill.tons, C weight)
1600
1500
1400
1300
1200
1100
1000
900
800
700
600
1965
1985
2005
2025
2045
2065
2085
2105
Time
Uncontrolled
Optimal

Figure IV-11



Global Mean Temperature
[Optimal and Uncontrolled]
Global Temperature (degrees C, 1860=0)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Time
Uncontrolled
Optimal

Table IV-12


True Net Savings Rates
[In capital and environment]
Gross savings as percent of output
11.5
11.0
10.5
10.0
9.5
9.0
8.5
8.0
7.5
7.0
6.5
Time
No intervention
Optimal policy

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