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A REEXAMINATION OF THE CONSUMPTION FUNCTION USING FREQUENCY DOMAIN REGRESSIONS

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0. ABSTRACT

This paper reexamines the permanent income hypothesis (PIH) in the frequency domain. Using a simple model, we demonstrate that the PIH implies the marginal propensity to consume (MPC) out of zero frequency income is unity. The PIH also implies that the MPC out of transitory (or high frequency) income is smaller than the long-run MPC. The paper employs a systems spectral regression procedure to test the PIH that accommodates stochastic trends in the consumption and income series as well as the joint dependence in these series. Monte Carlo simulations suggest that single equation techniques can produce inefficient tests of the PIH and that system spectral regression methods provide substantially better tests. New empirical estimates of the consumption function and tests of the PIH based on systems spectral regression methods are reported for U.S. aggregate consumption and income data over the period 1948-1990. The empirical results provide partial support for the theoretical implications of the PIH in the frequency domain.

I. INTRODUCTION

The aim of this paper is to reexamine the permanent income hypothesis (PIH) in the frequency domain. In particular, we build on the work of Engle (1974), who first applied band spectral regression techniques to test the PIH in the frequency domain. As Engle (1974, p.9) states:

> Noticing that transitory components of a series are primarily high frequency components, the Friedman hypothesis suggests that regressions (between consumption and income) using only high frequency components would behave differently from those with only low frequencies. In particular, the marginal propensity to consume would be substantially lower.

Engle then conducts a simple test of the hypothesis that permanent and transitory components have the same marginal propensity to consume. The statistical analysis was based on single equation band spectral regression.

This paper extends Engle's (1974) paper in two ways. First, in order to formalize Engle's proposition, we show that a simple representative agent model implies that the aggregate consumption function exhibits a marginal propensity to consume (MPC) that is unity with respect to long-run (or zero frequency) income and declines as we move to higher frequency (or transitory) income components. The model highlights the importance of using a systems approach and is similar in spirit to that developed in Campbell (1987), where the PIH is tested using a VAR which includes savings and the change in labor income. In this paper, we study a variant of Campbell's two equation model in the frequency domain and derive its implications for the MPC over different frequencies.

Second, we take into account the presence of stochastic trends in the consumption and income series, as well as their joint dependence, in testing the PIH.[1] We base our empirical analysis on the Engle and Granger (1987) theory of co-integration and Phillips' (1991) spectral regression procedure for co-integrated regressions. Unlike single equation spectral regression, which does not properly deal with stochastic trends, the system procedure

[1]There are numerous other papers that account for nonstationarity when studying the PIH in the time domain, rather than the frequency domain as studied here. Among them, see Cochrane (1991) and Stock (1988).

employed in this paper yields estimates of the long-run MPC which are efficient and asymptotically median unbiased. Hypothesis testing is conducted using standard asymptotic chi-squared tests.

The paper is organized as follows. Section II provides a simple version of the representative agent's decision problem where the MPC is a decreasing function of the frequency. Section III outlines the econometric methodology we employ to estimate the MPC. Section IV describes Monte Carlo experiments designed to evaluate the systems spectral estimator relative to the single equation band spectral regression procedure. Section V examines the time series and frequency domain properties of the consumption and income data used in our empirical study. We provide estimates of the MPC from full band and partial band systems spectral techniques. Tests of the constancy of the MPC across frequencies are also performed. Section VI gives concluding remarks.

II. CONSUMPTION THEORY

We adopt the simplified version of the PIH outlined in Sargent (1987). A representative agent chooses a plan for consumption that maximizes the present discounted expected value of a quadratic utility function

$$U(C_t) = -\frac{1}{2}(b-C_t)^2 + C_t\varepsilon_t^c$$

where b is the bliss level of consumption and ε^c_t denotes stochastic preference shocks capturing transitory consumption. We assume that ε^c_t is a serially uncorrelated random process with mean zero which is uncorrelated with the labor income process w_t (that is, $E[\varepsilon^c_t\, w_s] = 0\ \forall\ t,s$). The agent knows present and past values of ε^c_t, but not future values at the time of choice.

The problem is to maximize

$$E_t\left[\sum_{j=0}^{\infty} \rho^j U(C_{t+j})\right] \tag{1}$$

subject to the sequence of budget constraints

$$A_{t+1} = R[A_t + w_t - C_t] \tag{2}$$

and the transversality condition

$$\lim_{j\to\infty} R^{-j} A_{t+j} = 0 \quad a.s.$$

where ρ is the discount factor ($0<\rho<1$), $R=(1+r)$ is the gross return on wealth between t and t+1, A_t is assets or indebtedness, and w_t is an exogenously given process for disposable labor income. Taking conditional expectations, the sequence of resource constraints may be written as the lifetime budget constraint

$$\sum_{j=0}^{\infty} R^{-j} E_t C_{t+j} = A_t + \sum_{j=0}^{\infty} R^{-j} E_t w_{t+j} \tag{3}$$

Under the assumption $\rho = R^{-1}$, the first order conditions imply

$$E_t \rho R U'(C_{t+1}) = U'(C_t) \quad or \quad E_t C_{t+1} = C_t - \varepsilon_t^c \tag{4}$$

Performing recursions on (4) and substituting into (3) yields the following version of the PIH

$$C_t = \frac{r}{R}\left[A_t + \sum_{j=0}^{\infty} R^{-j} E_t w_{t+j}\right] + \varepsilon_t^c \tag{5}$$

which states that consumption is equal to permanent income plus a transitory shock. Lastly, we note that total disposable income can be expressed $Y_t = (r/R)A_t + w_t$.

With little loss of generality, we assume that disposable labor income is governed by the simple stochastic process $w_t = \mu + \varepsilon^w_t$, where ε^w_t is serially uncorrelated. This process obviously implies that consumption and income are co-integrated since[2]

$$C_t = Y_t - R^{-1}\varepsilon_t^w + \varepsilon_t^c \tag{6}$$

The disposable labor income process and the asset transition equation implies that total disposable income can be

[2] Cochrane and Sbordone (1988) point out that C_t and Y_t are cointegrated if w_t is a stationary ARMA process, stationary about a linear trend, or stationary about a geometric trend of order less than R^{-1}, so that the sum in (5) converges.

represented by the following integrated stochastic process

$$(1-L)Y_t = (1-R^{-1}L)\varepsilon_t^w - rL\varepsilon_t^c \tag{7}$$

Equation (7) can be solved for ε^w_t and substituted back into (6) to yield the PIH consumption function:

$$C_t = \left(\frac{1-R^{-1}}{1-R^{-1}L}\right)Y_t + \left(1 - \frac{rL}{R(1-R^{-1}L)}\right)\varepsilon_t^c \tag{8}$$

We now discuss the frequency domain version of (8). Let (C_t, Y_t) be discrete time series with spectral density representations

$$C_t = \int_{-\pi}^{\pi} e^{it\lambda} dZ_C(\lambda) \quad and \quad Y_t = \int_{-\pi}^{\pi} e^{it\lambda} dZ_Y(\lambda) \tag{9}$$

where (Z_C , Z_Y) are random processes with orthogonal increments (and possibly infinite variances at $\lambda=0$ to accommodate the I(1) nature of C_t and Y_t). Let $Z_u(\lambda)$ be a random process with orthogonal increments and finite variance $E[Z_u(\lambda)Z_u(\lambda)^*] = F_u(\lambda)$. We can represent (8) in the frequency domain as

$$dZ_C(\lambda) = \beta(\lambda)dZ_Y(\lambda) + dZ_u(\lambda) \tag{10}$$

in which the true MPC parameter varies in the frequency domain since it is determined by the filter $(1-R^{-1})/(1-R^{-1}L)$ in (8). The coherence between consumption and income at high frequencies falls, since the transfer function of the filter is given by

$$\left[\frac{1-R^{-1}}{1-R^{-1}e^{-i\lambda}} \cdot \frac{1-R^{-1}}{1-R^{-1}e^{i\lambda}}\right]^{\frac{1}{2}} = \left[\frac{(1-R^{-1})^2}{1+R^{-2}-2R^{-1}\cos\lambda}\right]^{\frac{1}{2}}$$

which is decreasing in λ over the interval $[0,\pi]$. This result may be considered a frequency domain version of Friedman's PIH with an MPC out of permanent income (zero frequency) equal to 1.[3]

[3]Corbae and Whiteman (1991) derive a similar, but more general, result using Weiner-Hopf techniques.

III. ECONOMETRIC METHODOLOGY

The co-integrated system embodied in equations (6) and (7) is analogous to the general set-up in Phillips (1991, equations (1) and (2)). Phillips develops a spectral regression estimator for co-integrated systems based upon a triangular ECM representation of the co-integrated system. The estimator, appropriately modified, may also be applied to the levels of the co-integrating regression itself. We use the levels approach in this paper.

In order to use Phillips' estimator, we first transform (8) to frequency domain format by taking discrete Fourier transforms

$$\omega_c(\lambda) = \beta(\lambda)\omega_y(\lambda) + \omega_u(\lambda) \qquad (11)$$

where

$$\beta(\lambda) = \beta$$

$$\omega_c(\lambda) = (2\pi T)^{-1/2} \sum_{t=1}^{T} c_t \, e^{it\lambda}$$

$$\omega_y(\lambda) = (2\pi T)^{-1/2} \sum_{t=1}^{T} y_t \, e^{it\lambda}$$

$$\omega_u(\lambda) = (2\pi T)^{-1/2} \sum_{t=1}^{T} u_t \, e^{it\lambda}$$

for $\lambda \in [-\pi,\pi]$. Note that we have assumed $\beta(\lambda) = \beta$ for $\lambda \in [-\pi,\pi]$; this assumption is not necessary and is relaxed below. Phillips' estimator relies on an estimate of the spectrum of $v = [u,\Delta y]'$. We may use the residuals from an initial Hannan (1963) efficient single equation regression on (11) to estimate u. The spectrum of v may then be estimated by a variety of methods, e.g. by smoothing the periodogram, viz:

$$\hat{f}_{vv}(\theta_j) = \frac{2M}{T} \sum_{B_j} \omega_v(\lambda_s)\, \omega_v(\lambda_s)^*$$

where the summation is over the frequency band B_j given by

$$\lambda_s \in B_j = \left(\theta_j - \frac{\pi}{2M} < \lambda \le \theta_j + \frac{\pi}{2M} \right)$$

Minimization of the Hermitian form

$$\sum_{B} tr\left([\omega_v(\lambda_s)\ \omega_v(\lambda_s)^*]\ \Phi(\lambda_s) \right)$$

with respect to β, where $\Phi(\lambda_s) = \hat{f}_{vv}(\omega_j)^{-1}\ \forall\ \lambda_s \epsilon B_j$, leads to the following estimator:

$$\hat{\beta} = \left[\frac{1}{2M} \sum_{j=-M+1}^{M} e' \hat{f}_{vv}^{-1}(\theta_j)\ e\ \hat{f}_{yy}'(\theta_j) \right]^{-1} \left[\frac{1}{2M} \sum_{j=-M+1}^{M} \hat{f}_{y*}(\theta_j)\ \hat{f}_{vv}^{-1}(\theta_j)\ e \right] \tag{12}$$

where

$$\hat{f}_{yy}(\theta_j) = \frac{2M}{T} \sum_{B_j} \omega_y(\lambda_s)\omega_y(\lambda_s)^*$$

$$\hat{f}_{y*}(\theta_j) = \frac{2M}{T} \sum_{B_j} \omega_y(\lambda_s)\omega_{[c,\Delta y]}(\lambda_s)^*$$

and $e'=(1,-\gamma)$. Note that if there is no coherence between u and Δy at frequency zero, the spectrum of v is diagonal for $\lambda \in [-\pi,\pi]$, and Phillips' estimator would be equivalent to the Hannan efficient single equation estimator if the true error spectrum were employed in (12). We also use estimates of the marginal propensity to consume which rely only on spectral estimates at the origin. This estimator is given by

$$\hat{\beta}(0) = \frac{\hat{f}_{yy}(0)^{-1}\hat{f}_{y*}(0)\hat{f}_{vv}^{-1}e}{e'\hat{f}_{vv}^{-1}(0)e} \tag{13}$$

Phillips (Theorem 3.1) proves that the limit distributions of $\hat{\beta}$ and $\hat{\beta}(0)$ are the same normal mixture distributions, and that the estimates are asymptotically median unbiased, and fully efficient under Gaussian errors. Furthermore, we can conduct tests of the hypothesis of constancy of the mpc across frequencies, or more general hypotheses about the co-integration space such as:

$$H_o: h(\beta) = 0, \quad H_1: h(\beta) \neq 0$$

where h( ) is a twice continuously differentiable function of restrictions on β. To test H_o against H_1 we may employ the Wald statistic in its usual form. Thus, for the estimator $\hat{\beta}$ we would form

$$X_1 = h(\hat{\beta})' [\ \hat{H}\ V_T\ \hat{H}'\ ]^{-1}\ h(\hat{\beta})$$

where

$$\hat{H} = H(\hat{\beta}) \quad and \quad V_T = \frac{1}{T}\left[\frac{1}{2M}\sum_{j=-M+1}^{M} \gamma'\hat{f}_{ww}^{-1}(\theta_j)\gamma\hat{f}_{yy}'(\theta_j)\right]^{-1}$$

Here V_T is the conventional estimate of the asymptotic variance matrix of β from spectral regression theory (see Hannan (1970),p.442). Phillips proves that the test statistic X_1 converges asymptotically to the chi square distribution with degrees of freedom = 1.

When β changes with λ, an approximation to the frequency domain consumption function given in the time domain may be obtained as follows. By inversion of (10) we have:

$$C_t = \int_{-\pi}^{\pi} e^{it\lambda}\beta(\lambda)dZ_Y(\lambda) + u_t, \quad where \quad u_t = \int_{-\pi}^{\pi} e^{it\lambda}dZ_u(\lambda) \tag{14}$$

Assuming that the frequency dependent mpc $\beta(\lambda)$ obeys the step function:

$$\beta(\lambda) = \begin{cases} \beta_1, & \lambda \in B_1 = [-\pi/3,\pi/3] \\ \beta_2, & \lambda \in B_2 = [-\pi,-\pi/3] \cup [\pi/3,\pi] \end{cases} \tag{15}$$

then (14) has the simple time domain form:

$$C_t = \beta_1 Y_t + u_t + \int_{-\pi}^{\pi} e^{it\lambda}(\beta(\lambda)-\beta_1)dZ_C(\lambda)$$

or

$$C_t = \beta_1 Y_t + \eta_t, \quad where \quad \eta_t = u_t + \int_{B_2} e^{it\lambda}(\beta_2-\beta_1)dZ_y(\lambda) \tag{16}$$

Equation (16) is a co-integrated system between C_t and Y_t since η_t is stationary. It follows that Phillips' spectral regression approach may still be used to estimate β_1. Moreover, restricting the regression to the lower frequencies will affect the asymptotic properties of the estimate of β_1 since η_t is stationary.

Estimation of the high frequency parameter, β_2, is more involved. One approach is to estimate β_2 by restricting the spectral regression to the high frequencies. However, this is unlikely to yield satisfactory estimates for β_2 since η_t and Y_t, though stationary over the high frequencies, will be coherent and this results in simultaneous equations bias. Since the limit variates for $\lambda\epsilon B_2$ all have finite variance and $\omega_Y(\lambda)$ and $\omega_u(\lambda)$ are correlated in the limit, we need to instrument $\omega_Y(\lambda)$ to get consistent estimates of β_2. For example, if the generating mechanism for ΔY is $\omega_{\Delta Y}(\lambda)=\delta\omega_Z(\lambda)+\omega_\zeta(\lambda)$ and Z_t is independent of ζ_t and η_t, we can use $\omega_Z(\lambda)$ as an instrument in a spectral regression on (16). For the purpose of the simulations we shall use a frequency domain Generalized Instrumental Variable Estimator (GIVE). This estimator is based on the following formula, which may be interpreted as a spectral version of Sargan's (1989,p.63) GIVE estimator:

$$\beta^{GIVE} = \frac{\left[[\sum_{\theta_j\epsilon B_2} f_{yz}(\theta_j)f_{uu}^{-1}(\theta_j)] \; [\sum_{\theta_j\epsilon B_2} f_{zz}(\theta_j)f_{uu}^{-1}(\theta_j)]^{-1} \; [\sum_{\theta_j\epsilon B_2} f_{zc}(\theta_j)f_{uu}^{-1}(\theta_j)] \right]}{\left[[\sum_{\theta_j\epsilon B_2} f_{yz}(\theta_j)f_{uu}^{-1}(\theta_j)] \; [\sum_{\theta_j\epsilon B_2} f_{zz}(\theta_j)f_{uu}^{-1}(\theta_j)]^{-1} \; [\sum_{\theta_j\epsilon B_2} f_{zy}(\theta_j)f_{uu}^{-1}(\theta_j)] \right]} \qquad (17)$$

with asymptotic variance matrix:

$$V(\beta^{GIVE}) = \frac{2M}{T}\left[[\sum_{\theta_j\epsilon B_2} f_{yz}(\theta_j)f_{uu}^{-1}(\theta_j)] \; [\sum_{\theta_j\epsilon B_2} f_{zz}(\theta_j)f_{uu}^{-1}(\theta_j)]^{-1} \; [\sum_{\theta_j\epsilon B_2} f_{zy}(\theta_j)f_{uu}^{-1}(\theta_j)] \right]^{-1}$$

In both cases, feasible estimates are obtained by employing estimated spectra in these formulae. Note that (17) is based on frequencies in the band B_2 and is therefore a band spectral GIVE estimator for the parameter β_2 in (10) and (15).

IV. SIMULATIONS

We now report the results of simulations designed to assess the merit of Phillips' systems procedure (henceforth denoted by SYS) relative to single equation band spectral regression (denoted by SNG). Our evaluation will be based on two criteria: (a) whether the estimator yields t-statistics with the correct size (given a nominal significance level); and (b) power.

Let $\omega_Y(\lambda)=(2\pi T)^{-1/2}\Sigma e^{it\lambda}Y_t$ denote the discrete fourier transform of Y_t. The data generation process we employ may be represented as:

$$\omega_C(\lambda) = \beta(\lambda)\omega_Y(\lambda) + \omega_{u1}(\lambda)$$

where

$$\omega_{\Delta Y}(\lambda) = \omega_{u2}(\lambda)$$

$$\beta(\lambda) = \begin{cases} 1.00, & \lambda = [0,\pi/3] \\ 0.25, & \lambda = (\pi/3,\pi] \end{cases}$$

and

$$Y_t = Y_{t-1} + u_{2t}, \qquad t=1,2,3,....T$$

$$\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} = u_t = \xi_t + \psi\xi_{t-1}, \quad \xi \sim N(0,\Sigma)$$

$$\psi = \begin{bmatrix} 0.3 & 0.4 \\ \psi_{21} & 0.6 \end{bmatrix}, \qquad \Sigma = \begin{bmatrix} 1 & \sigma_{12} \\ \sigma_{21} & 1 \end{bmatrix}$$

Following Phillips and Loretan (1991), we consider values of {0.8, 0.4, 0.0, -0.8} for ψ_{21} and {-0.85, -0.5, 0.5} for σ_{21}. Note that the error term, u_1, is coherent with u_2 at frequency zero for every combination of σ_{21} and ψ_{21}. Thus the distribution of the SNG estimator should suffer from second-order bias (see Phillips (1991)).

Let $\hat{\beta}_i(0)$ (i = SYS, SNG) denote the parameter estimate for the true β over the low frequencies (i.e. $\lambda = 0$) and $t(\hat{\beta}_i=\beta)$ the t-statistic for the null hypothesis that $\hat{\beta}_i(0) = \beta$. Column 2 of Table 1 (Part A) contains the variance ratio of SYS/SNG for $\hat{\beta}(0)$. Column 3 reports the bias ratio of SYS to SNG. Columns 4 and 5 report the mean bias in the two estimators. Size computations are reported in columns 6 and 7, and power computations are reported in columns 8 and 9. The size computations are based on the t-statistic for the null hypothesis that the low frequency $\beta = 1$ (using a 5 per cent level of significance). Power is computed by changing the true value to β=0.95. All the simulations were conducted for M = 2, a sample size of 256, and 10,000 replications[4].

The results in Table 1 (Part A) suggest that the SYS estimator is superior to the SNG estimator since the SYS estimator consistently yields a smaller variance and bias than the SNG estimator. The simulations demonstrate that the variance ratio falls as the coherence between u_1 and u_2 increases. This is to be expected since the asymptotic variance of the SYS estimator depends on the conditional variance of u_1 given u_2. The efficiency gains can be large (see model C, F, and, in particular, J). The SYS estimator typically dominates the SNG estimator in terms of power across the 12 simulations. Interestingly, both estimators suffer from little size distortion. This suggests that the second-order bias effect due to the coherence of u_1 and u_2 does not have a great impact on the location of the finite sample distribution (at least for the parameter combinations chosen in these simulations).

Part B of Table 1 reports the results of applying the spectral regression estimator over the frequency band $[\pi/3, \pi]$. Column 2 contains the bias in the estimator, while column 3 presents empirical size. The results clearly demonstrate the simultaneity bias arising from the coherence of u_1 and u_2 over the high frequency. Except for model F, where the coherence over the high frequency is negligible, the level of bias is substantial. These simulations underscore the importance of using GIVE over the high frequencies.

The results of applying GIVE to the same model are presented in columns 4 and 5 of Table 1 (Part B), using an instrument which has zero coherence with u_1 (the true error term) for all frequencies. The instrument was generated by (recursively) accumulating the residuals from a regression of u_2 on u_1, which is obviously orthogonal to u_1. The size of the correlation between u_2 and the instrument is inversely proportional to the covariance between u_2 and u_1. GIVE clearly overcomes the bias problem. However, the size of the t-statistic is consistently conservative; that is, less than the nominal setting of the test, which is 5% in this case. The power of the statistic is satisfactory.

[4] The simulations are representative of the findings which were obtained using a sample size of 128 and 512.

V. EMPIRICAL RESULTS

This section provides a statistical analysis of U.S. aggregate consumption and income data. All data are seasonally adjusted quarterly series for the period 1948:1 to 1990:3, and are taken from the National Income and Product Accounts. The series examined are per capita non-durable consumption and per capita disposable income.

Table 2 provides tests of the unit root hypothesis for consumption and disposable income using Phillips' (1987) Z_α and Z_t statistics and the Park and Choi (1988) G(p,q) statistics. For the Z_α and Z_t statistics, the maintained hypothesis is a unit root. For the G(p,q) statistic, the null hypothesis is stationarity around a p'th order time polynomial; the statistic possesses (asymptotically) a χ^2_{q-p} distribution. The statistical results imply that detrended consumption and income possess a unit root. First differences of the detrended data are stationary according to the same set of tests.

Table 3 presents OLS estimates of the co-integrating regression between consumption and income. This table also reports the results of applying the Z_α and Z_t statistic to the estimated residuals of the co-integrating regression. Both the Z_α and the Z_t statistics are significant at the 5% level of significance, thus providing support for the alternative hypothesis of co-integration.[5]. This conclusion is supported by Park's (1988) H(0,5) statistic, which uses a null hypothesis of co-integration; the p-value of the statistic is 0.091.

Table 4 presents band spectral estimates of the marginal propensity to consume using Phillips' systems procedure (SYS), comparing it to the single equation technique (SNG) and, when appropriate, the GIVE estimator. The first panel provides estimates for the true marginal propensity to consume using equation (13) [$\hat{\beta}(0)$], with 64 periodogram ordinates to estimate the spectrum at frequency zero. The second and third panels provide estimates of the marginal propensity to consume over the restricted high frequency interval $[\pi/3, \pi]$, partitioning it into bands based on M=2. They are obtained from the GIVE estimator using real per capita government expenditure and the monetary base as instruments for real per capita income. Estimates are provided for the levels of the data (panel 2), as well as for first differences (see panel 3).

[5]We note that the results do not depend on the choice of q. We were not able to reject the null hypothesis of cointegration for q=3,4, and 5 using a 5 percent level of significance.

Using the SYS estimator, the point estimate for the long-run mpc is approximately 0.73. We can easily reject the null hypothesis that the long-run mpc is unity at the 5% level of significance. Using GIVE and levels of non-durable consumption and disposable income, we obtain a point estimate of 0.792 for the mpc over the high frequencies ($[\pi/3, \pi]$), which is larger than the long-run estimate. At face value, this finding would cast doubt on the validity of the PIH. However, the short-run mpc estimate is distorted because of leakage in the periodogram from the low frequencies to the high frequencies.[6] Given the unit-root behavior of the data, leakage from the low frequency ordinates to the high ordinates can be large, resulting in poor estimates of the short-run mpc. One solution is to filter the data by taking first differences and re-estimate the model. Note that the spectral GIVE estimator is still consistent in this filtered model provided the instruments are valid. When first differences of the consumption and income data are used (see Table 4, panel 3), we find that the high frequency mpc is small, negative, and insignificantly different from zero at the 5% level of significance. The point estimate for the short-run mpc is -0.0658. Furthermore, the point estimate for the low frequency mpc using first differences is 0.7117, so that filtering the data does not appear to have induced the lower mpc over the high frequency band.

The statistical results for the test of constancy of the MPC over the two frequency domain intervals is also provided in Table 4. We may perform a formal statistical test as outlined in Section III. The computed χ^2_1 statistic for the null hypothesis of parameter constancy is 9.2937, which exceeds the critical value of 3.84 at the 5% level. Thus we reject the null of constancy of the MPC across frequencies. Given our theoretical finding that the MPC should decrease over higher frequencies (see equation (13)), these results provide some support for this feature of the PIH.

Lastly, we remark that the SYS and SNG estimators have produced very similar estimates for the long-run mpc. This is due to the fact that the residuals of the co-integrating regression and the first difference of disposable income have negligible coherence over $[-\pi, \pi]$. The efficiency gains in moving from SYS to SNG in this particular application are small.

[6]The leakage problem does not occur in our Monte Carlos since we generated the simulated data such that the actual frequencies are mapped one-for-one to the frequencies used by the discrete fourier transform. The discretization of the actual (continuous) consumption data results in leakage from one frequency to another, and impairs the performance of the estimator, which is acute at the high frequencies where super consistency does not apply.

VI. CONCLUSIONS

This paper has investigated the properties of the consumption function in the frequency domain. The PIH implies a system of two equations, one of which is a co-integrating regression between consumption and income and the other is an equation of motion for income. We examined this model in the frequency domain and its implications for the marginal propensity to consume at both low and high frequencies. The model suggests that the MPC is unity at the zero frequency and that the MPC falls as we move to higher frequencies. New frequency domain estimates of the long run mpc which account for the stochastic trends in aggregate consumption and disposable income data, as well as their joint dependence, are provided. The hypothesis that the long-run (or zero frequency) MPC is unity can be rejected in this model and dataset. However, the null hypothesis of constancy of the MPC across frequency bands can also be rejected. Thus, our empirical results provide partial support for the PIH in the frequency domain.

TABLE 1

PART A: MONTE CARLO RESULTS FOR MODEL 1 (LOW FREQUENCY), T=256 (10,000 replications)

			Bias Level		Size		Power at β=0.95	
Model	Var Ratio (SYS/SNG)	Bias Ratio (SYS/SNG)	SYS	SNG	SYS	SNG	SYS	SNG
(A)	0.8745	0.1719	-0.3350E-04	-0.1948E-02	5.43	4.43	94.90	92.37
(B)	0.8951	-0.061	-0.7911E-06	0.1277E-02	5.12	4.69	97.97	98.15
(C)	0.2292	0.0044	0.1429E-06	0.3227E-02	5.23	1.57	100.00	99.91
(D)	0.6938	0.1209	-0.2939E-03	-0.2430E-02	4.26	2.92	99.10	97.01
(E)	0.9827	0.2385	0.7158E-06	0.3001E-03	5.81	5.47	97.47	97.51
(F)	0.3565	-0.022	-0.7381E-06	0.3325E-02	6.00	2.25	99.89	99.72
(G)	0.5735	0.1218	-0.2770E-03	-0.2773E-02	3.74	2.35	99.90	92.27
(H)	0.9472	0.1354	-0.9990E-06	-0.7377E-03	5.22	4.52	98.70	98.32
(I)	0.5361	0.0025	0.8250E-07	0.3323E-02	5.66	2.80	98.90	98.18
(J)	0.4839	0.1182	-0.2095E-03	-0.1771E-02	1.90	1.20	99.99	99.95
(K)	0.7699	0.1949	-0.3024E-03	-0.1551E-02	4.80	3.50	99.92	99.76
(L)	0.9331	0.0495	0.6894E-06	0.13901E-02	5.75	5.04	92.19	92.79

LEGEND

model code	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)
σ_{21}	-0.85	-0.50	0.50	-0.85	-0.50	0.50	-0.85	-0.50	0.50	-0.85	-0.50	0.50
ϕ_{21}	0.80	0.80	0.80	0.40	0.40	0.40	0.00	0.00	0.00	-0.80	-0.80	-0.80
Coherence of u_1, u_2 at zero frequency	-0.36	0.30	0.88	-0.59	0.02	0.81	-0.72	-0.21	0.68	-0.83	-0.52	0.22

PART B: MONTE CARLO RESULTS FOR MODEL 1 ($\pi, \pi/3$), T=256 (10,000 replications)

	SNG		SPECTRAL GIVE		
Model	Bias	Size	Bias	Size	Power
(A)	-0.2997	76.1	0.3654E-02	2.62	90.8
(B)	0.2197	66.9	0.5830E-02	2.50	95.0
(C)	0.6504	99.0	0.4392E-02	3.26	100.0
(D)	-0.4131	95.6	0.4419E-02	2.96	99.1
(E)	0.0219	5.30	0.9396E-02	2.78	94.6
(F)	0.6729	99.5	0.6641E-02	3.18	99.8
(G)	-0.4234	97.8	0.4526E-02	3.40	100.0
(H)	-0.1306	37.8	0.5291E-06	2.76	97.6
(I)	0.6102	98.5	0.4173E-02	3.06	97.4
(J)	-0.3689	99.3	0.3752E-02	3.72	100.0
(K)	-0.2497	90.0	0.5404E-02	3.06	99.9
(L)	0.2223	46.3	0.6887E-02	2.18	82.5

LEGEND

model code	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)
a_{21}	-0.85	-0.50	0.50	-0.85	-0.50	0.50	-0.85	-0.50	0.50	-0.85	-0.50	0.50
θ_{21}	0.80	0.80	0.80	0.40	0.40	0.40	0.00	0.00	0.00	-0.80	-0.80	-0.80
Coherence of u_1, u_2, at zero frequency	-0.36	0.30	0.88	-0.59	0.02	0.81	-0.72	-0.21	0.68	-0.83	-0.52	0.22

TABLE 2

UNIT ROOT TESTS FOR NON-DURABLE CONSUMPTION AND DISPOSABLE INCOME

	Levels			First Differences		
	Z_t	Z_α	G(1,3)	Z_t	Z_α	G(0,3)
C p-value	-2.6087	-7.665	30.914 (0.0000)	-10.246	-130.460	5.3514 (0.1478)
Y p-value	-2.3741	-9.3731	22.026 (0.0000)	-13.086	-171.183	1.3650 (0.7137)

Notes: The computed Zt and Za and G(p,q) statistics are based on 5 lags. T-statistics are given in parentheses. All statistics are based on detrended data. Critical values (5%): Z_t = -2.8837; Z_α = -14.0751.

TABLE 3

CO-INTEGRATING REGRESSION: $C_t = \kappa + \beta Y_t$

Non-Durable Consumption and Disposable Income			
κ	β	Z_t	Z_α
527.58 (15.437)	0.7326 (176.34)	-5.1156	-43.549
H(0,5) = 9.4831; p-value = 0.0913			

Notes: The Z statistics are based on 5 lags. T-statistics are reported in the parentheses. Critical values (5%) for Z_t = -3.4660, Z_α = -19.614.

TABLE 4

SPECTRAL ESTIMATES OF THE MARGINAL PROPENSITY TO CONSUME CO-INTEGRATING REGRESSION: $c_t = \kappa + \beta y_t$

Non-Durable Consumption and Disposable Income			
Zero Frequency Estimator: SYS		Zero Frequency Estimator SNG	
κ	$\beta(0)$	κ	$\beta(0)$
525.04	0.7328	523.40	0.7329
(15.437)	(176.34)	(15.437)	(176.22)
GIVE Freq $(\pi/3,\pi)$ Estimator (using levels)			
β			
0.7927			
(138.20)			
GIVE Freq Estimator (using first differences)			
Zero Frequency		Frequency $(\pi/3,\pi)$	
β		β	
0.7117		-0.0658	
(5.5539)		(0.2498)	
H_o: $\beta(0)=\beta(\pi/3,\pi)$, H_1: $\beta(0)\neq\beta(\pi/3,\pi)$			
$\chi^2(1) = 9.2937$			

Notes: SYS denotes Phillips' (1991) systems spectral estimator while SNG denotes Hannan's (1963) single equation estimator as employed in Engle (1974). t-statistics are reported in different from unity at the 5% level of significance. GIVE uses real per-capita government expenditure and the monetary base as instruments for real per-capita income.

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