COWLES FOUNDATION FOR RESEARCH IN ECONOMICS
AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut 06520

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ECONOMIC EQUILIBRIUM AND SOVIET ECONOMIC REFORM

Herbert E. Scarf

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by

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I. Introduction

One of the major goals of the proposed economic reform in the Soviet Union is the introduction of competitive markets to replace the procedures of centralized economic decision making established over the last seven decades. Prices for all of the goods and services in the competitive sectors of the economy, rather than being set by government planning agencies, are to be determined by the interplay between supply and demand, and to reflect underlying economic values. Factories, machinery and other forms of capital, as well as land, raw materials, stocks of finished goods and other factors of production are to be owned individually or through the joint ownership of some collective entity. The role of private incentives in the choice of productive techniques is to be greatly expanded, and disparities in the distribution of income are to be tolerated--even encouraged--if they result from specialized skills, entrepreneurial abilities and socially useful forms of economic activity.

I can think of no precedent for this extraordinary economic experiment. There have been many instances in the course of this century, in which the major factors of production in a particular country have been abruptly

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nationalized and the techniques of economic planning have replaced an earlier form of economic organization based on private ownership. But there is no example of such a dramatic attempt to discard the instruments of centralized political and economic control in favor of market forms, in a country of the size and diversity of the Soviet Union. There is no trustworthy body of practical experience to which Soviet economic reformers can turn, and it may be necessary to look for guidance in some branches of economic theory. I take it as my assignment to provide an outline of those aspects of price theory--in particular the theory of general economic equilibrium--that may offer some theoretical insights about the economic problems to be encountered during the transition from socialism to private markets.

The general equilibrium model is a conceptual framework for analyzing an economic system whose agents are assumed to respond in a decentralized fashion to competitive prices. The basic units in the economy are typically divided into two broad classes. One class consists of consumers, who command their supplies of labor and own all of the assets of the economy either directly or indirectly--by means of financial claims or shares in manufacturing entities. The second class of economic agents are producers, whose business is to transform productive inputs into those goods and services that are valued by consumers themselves, or used as intermediary goods by other producers. Of course, the same individual may engage in acts of consumption and production simultaneously.

There may be many ways for a given producer to take the factors of production at his disposal--labor of varying skills, capital, a great diversity of raw materials, energy and other inputs--and transform them into outputs. Consumer goods can be manufactured by labor intensive methods, by using an automated assembly line or acquired directly by importing from abroad. Clothing can be restricted in its variety and quality or the most elegant designer wardrobes can be made available for those with sufficient incomes. Electrical energy can be produced by furnaces using coal or oil, by nuclear power plants or by hydroelectric installations. Goods can be transported by rail, water or along a network of superhighways. Food can be produced on small plots with primitive implements, or on large farms making use of the most advanced forms of agricultural machinery. Steel can be produced in plants of varying sizes, independently, or integrated with enterprises using steel as inputs. During the Great Leap Forward in China, it was even proposed that steel be manufactured in individual back yards. How are these choices to be made?

General equilibrium theory makes the assumption that the individual producers in the economy are faced with competitive prices for all of the factors of production, and with competitive prices for the outputs of production. If all of the input and output prices are known by the firm, then any particular production plan will have a profit associated with it: the value of output at these prices minus the cost of those factors--including a charge for the services of capital--used in production. It is then customary to assume that the goal of the manufacturing entity is to select, from the list of all possible plans that are available to it, that particular production plan which maximizes its profit. This decision results in a net supply for the individual firm, which is a function of the price of its output and of its factors of production. The net supply function for the entire economy is obtained by aggregation over all firms.

Prices also enter into the consumer side. Each consuming unit owns its own labor and its share of the assets of the economy, and is able to evaluate its income or wealth once the prices of these assets are known. Given the income of each consumer, and the prices of goods to be purchased, the individual consumer's demand for the outputs of production can be specified as well defined functions of price. If we add up the individual demand functions we obtain the market demand functions, which tell us the quantity demanded of each of the goods and services in the economy as a function of the entire set of prices faced by consumers.

Market demands arise from the consumer side of the economy, market supplies from the producer side. At an arbitrary selection of prices the demand for each commodity need not equal its supply. If the price of apples is too high, consumers may wish to spend their income on oranges; if the price of clothing is too low in comparison with wages and the cost of materials, manufacturers may not be able to cover their costs of production and may be unwilling to supply those quantities which are adequate to meet demand. Only certain prices--equilibrium prices--will equilibrate the demand and supply for all commodities. It is these prices--and these prices alone--which permit the economy to achieve an optimal allocation of resources in the decentralized fashion celebrated by economic theory.

Section II of the paper contains a mathematical formulation of the general equilibrium model, a sketch of a proof of the existence of equilibrium prices and a brief discussion of the problems created by costly disposal and public goods. Section III contains some remarks about the relevance of the general equilibrium model for economic reform in the Soviet Union, including some reference to the literature on socialist economic planning from the early decades of this century. This is followed by an outline of the welfare properties of a competitive equilibrium and some comments on the ability to detect Pareto improvements by a calculation of profitability. A numerical example of a general equilibrium model is presented in Section IV, and Section

V contains a discussion of the problems of uniqueness and stability of the competitive equilibrium. Section V includes some comments on the relevance of the classical price adjustment mechanism to a description of the dynamic path that might be followed by the Soviet economy in its transition to a market economy.

Section VI is an account of my personal view of the problems--for equilibrium theory and welfare economics--caused by the presence of economies of scale in production. In this section, I dwell on the absence of a coherent analytical formulation, with the generality of the Walrasian model, which is capable of incorporating increasing returns to scale in production. The implications of this discussion may be quite different for the Soviet Union and for market economies such as the United States. Central planning may have been the appropriate mechanism for the rapid exploitation of economies of scale in heavy industry and military defense in the 1920s and 30s, but, by all accounts, manufacturing entities in the Soviet Union are far too large at present and should--in the interest of efficiency--be broken down into a number of smaller entities exposed to the forces of competition. Finally, Section VII contains a suggestion for the construction of an applied general equilibrium model of the Soviet economy to obtain some gross estimates of the circumstances in which the economy might find itself after the introduction of competitive markets.

II. The Mathematical Formulation of the General Equilibrium Model

The basic themes of the competitive model have been in the process of continued intellectual refinement and elaboration for perhaps two centuries, since their original appearance in Adam Smith's Wealth of Nations. One of the major achievements of economic theory has been the gradual translation of the verbal concepts of equilibrium into a formal mathematical model whose properties can then be examined by the varied techniques of mathematical analysis. For example, the important issue of the internal consistency of the equilibrium model is resolved by providing a mathematical proof of the existence of prices that equilibrate supply and demand in all markets; the welfare properties of an equilibrium can be explored using the theory of convex sets; the question of stability of the competitive equilibrium can be studied in terms of a formal system of differential equations relating the movement of prices to the difference between supply and demand when the system is not in equilibrium; and the possibility of using the equilibrium model as a practical tool for the evaluation of changes in economic policy is much enhanced by a numerical formulation which permits the utilization of modern computing capabilities.

It may be useful to provide a brief mathematical treatment of the general equilibrium model. We begin by making a list of all of the commodities to be included in the model: labor of varying skills, machinery and other durable inputs into production, energy, raw materials, intermediate goods and the entire menu of outputs of production--at whatever level of disaggregation seems appropriate to the discussion. Each of the commodities may also be distinguished by its location and the time of its availability. If uncertainty is to play a major role in the analysis, the commodities may even be differentiated according to various states of the world which are not fully predictable in advance. If the number of commodities is n, then a specification of the quantity of all of the goods and services in the economy is given by a vector $x = (x_1, \ldots, x_n)$. It is, of course, a questionable assumption of general equilibrium analysis that the list of all potential goods and services is known in advance.

We let the components of the vector $p = (p_1, \ldots, p_n)$ represent the prices at which the various commodities can be purchased and sold. Assuming that the disposal of unwanted commodities is costless, it is traditional to restrict our attention to prices that are greater than or equal to zero. We also normalize prices by an arbitrary convention such as $\Sigma p_i = 1$, thereby avoiding--as is customary in general equilibrium analysis--the vital issue of the absolute price level. The ith consumer is presumed to have a utility function $u_i(x)$ which describes his preferences for the consumption of commodity bundles x, and a vector of initially owned commodities w^i. At prices p, the income or wealth of the ith consumer is given by pw^i, and the assumption is typically made that the consumer allocates his income among potential items of consumption by maximizing utility subject to his budget constraint $px \leq pw^i$. Under conventional assumptions on the individual preferences, this results in a continuous function $x^i(p)$, satisfying the identity $px^i(p) = pw^i$, and describing the ith consumer's demand for all of the goods and services in the economy as a function of their as yet unknown prices. The market demand functions, $x(p)$, obtained by summing the individual demand functions, are then continuous and satisfy what is known as the Walras law: $px(p) = pw$, with w the sum of the commodities initially owned by all of the consumers in the economy. If the market <u>excess demand</u> functions are defined by $f(p) = x(p) - w$, they will satisfy the Walras law in the form $pf(p) = 0$; an identity which holds for <u>all</u> price vectors on the unit simplex $\{p | \; p \geq 0, \; \Sigma p_i = 1\}$.

There is a well developed theory of consumer behavior which describes the properties of individual demand functions obtained by the process of utility maximization. In addition to continuity and satisfaction of the individual budget constraints, individual demand functions satisfy the Slutsky equations--or its alternative: the strong axiom of revealed preference. These conditions are necessary and sufficient for a potential demand function to arise from utility maximization; individual demand functions are far from arbitrary. The situation is, however, quite different for market demand functions obtained by the aggregation of individual demand functions. The Slutsky equations and the strong axiom do not survive aggregation; and one of the more significant mathematical conclusions about market excess demand functions is that they are arbitrary aside from the conditions of continuity and the Walras law--unless special assumptions are imposed on the preferences of individual consumers. More specifically, if $f_1(p),\ldots,f_n(p)$ is _any_ set of continuous functions defined on some closed subset of the interior of the unit simplex, and satisfying the Walras Law at all such price vectors, then a collection of consumers can be constructed, with well defined preferences and a distribution of initially owned assets, whose market excess demand functions are identical with f on the given subset. It is precisely this arbitrariness which makes the problems of existence, stability and computability of the competitive equilibrium so difficult.

Before describing the production side of the economy, some words of caution may be in order. No one has ever quite seen a utility function and economists may, because of habitual use, be attributing more realism to this construction than is warranted. It may not be plausible to assume the existence of stable preferences involving commodities which have never previously been imagined, at dates far in the future and at exotic locations. Moreover, consumers do not spring to life fully equipped with a set of preferences for all of the potential goods and services in the economy; preferences depend on social customs and educational experiences, on the reports of travelers about exotic spices, on advertising and the influence of the media. It is unlikely that preferences for television sets and personal computers existed before these items became commercially available in large quantities. In much of equilibrium analysis the major use of consumer preferences or utility functions is simply to assert the existence of a stable relationship between prices and market demands--a relationship which could perhaps be found by a statistical analysis, rather than by an explicit aggregation of individual demand functions. A specification of individual utility functions is required by welfare theory, where we are concerned with the effect on the consuming units of the economy of alternative production and distribution plans.

The basic construction on the production side of the general equilibrium model is a complete description of the current state of technical knowledge about the alternative ways in which inputs into production can be transformed into outputs. A specific production plan can be represented by a vector $y = (y_1,\ldots,y_n)$ in n-dimensional space with the convention that the negative entries in y represent quantities of the corresponding goods required as factors of production and the positive entries are levels of output. The totality of all technically feasible production plans is then described by the <u>production possibility set</u> Y, which may be presented either by an activity analysis model, by a series of specific production functions or in some other fashion.

In neoclassical economic theory, it is crucial for the demonstration of the existence of a competitive equilibrium to assume that the production possibility set is convex. If it is technically possible not to produce at all, i.e. if the zero vector is contained in Y, then convexity implies that the production possibility set exhibits constant or decreasing returns to scale. Moreover, if all of the factors of production--including land of differing qualities and specialized managerial skills--are explicitly accounted for, it is also plausible to make the additional assumption of <u>free</u>

entry. From a mathematical point of view, free entry implies additivity of the production set Y: if y^1 and y^2 are both technically feasible then they can be employed simultaneously so that $y = y^1 + y^2$ is also in Y. In conjunction with convexity, additivity implies that the production possibility set is a convex cone.

A production possibility set which is a convex cone will exhibit constant returns to scale: if y is a feasible production plan then λy will also be feasible for all non-negative values of λ. This is an extremely important--and to my way of thinking extremely restrictive--assumption about the production side of the economy. The property of constant returns to scale implies that the mix of inputs needed to produce a particular assortment of outputs be unchanged as the scale of production varies; that it is just as efficient to manufacture steel in our own back yards, as it is using a fully integrated assembly line. This is a terribly unrealistic assumption which excludes the possibility of economies of scale and forces us to ignore one of the central features of economic life in the last two centuries: the large industrial firm whose size is based on the economic advantages of large-scale production. This is a point to which I shall return.

The general equilibrium model is fully specified by a list of consumers with utility functions u_i and vectors of initial holdings w^i and by the production possibility set Y, which we take to be a convex cone. A competitive equilibrium consists of a price vector p* and a feasible production plan y*, such that the following conditions are satisfied:

1. $f(p^*) = y^*$,
2. $p^*y^* = 0$, and $p^*y \le 0$ for all other feasible production plans y.

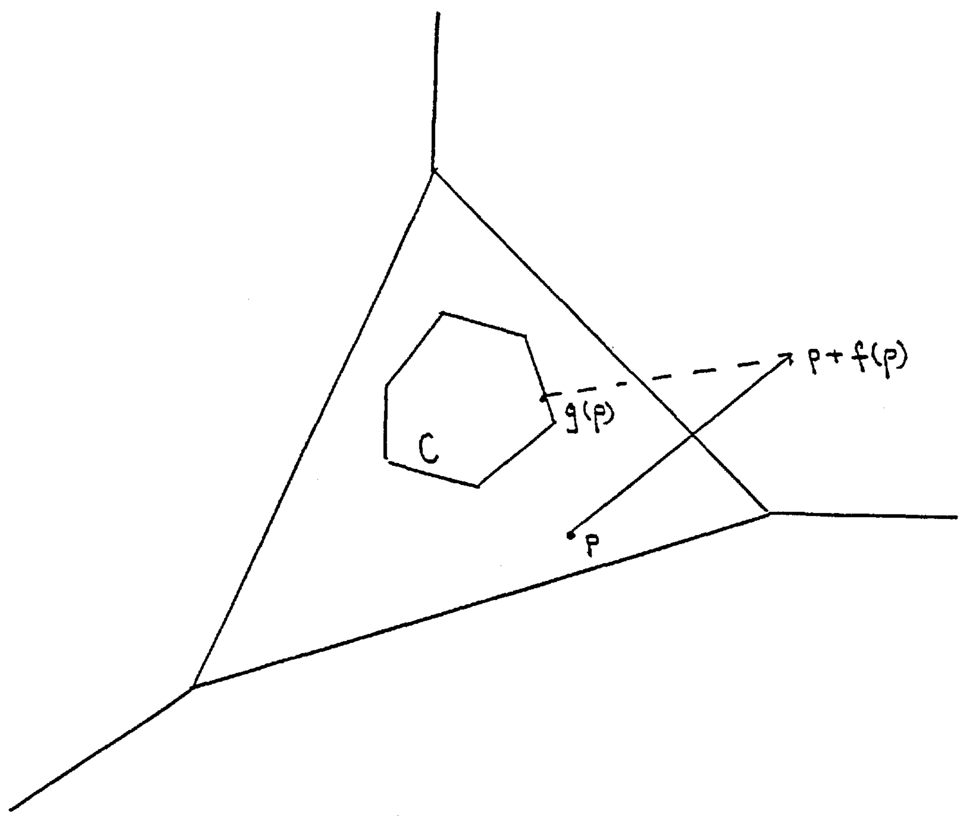
The first of these conditions states that the market excess demand f(p*) is equal to the net supply y*. The second condition says that the equilibrium production plan maximizes profit at the proposed equilibrium prices p^*, and that this maximum profit, which includes charges for the services of capital, is equal to zero. If the profit at equilibrium were positive, then from the assumption of constant returns to scale in production, it could be increased without limit and the system would not be in equilibrium; if the profit were negative, profits could be increased by selecting the zero production plan. On the other hand, if we had chosen a formulation with decreasing returns to scale at the level of the firm, positive profits--to be distributed to consumers--would have been generated at equilibrium. These profits could, however, be considered as a return to the specialized capital embodied in the firm's production set.

Consistency of the general equilibrium model is demonstrated by a formal mathematical proof of the existence of an equilibrium price vector and production plan satisfying the two conditions defining an equilibrium. The customary proofs make use of fixed point theorems, the simplest of which is Brouwer's theorem:

A continuous mapping $p' = g(p)$ of the price simplex $\{p: p \geq 0, \Sigma p_i = 1\}$ into itself necessarily has a fixed point: $p^* = g(p^*)$.

For completeness I include the following compact argument for the existence of a competitive equilibrium in the case in which the production possibility set is given by an activity analysis matrix A. The non-technical reader may wish to skip the formal proof.

‖Let C be the set of price vectors on the unit simplex which satisfy $pA \leq 0$, i.e. the set of prices yielding a non-positive profit for all conceivable production plans. For each p on the simplex let g(p) be that vector in C which is closest, in the ordinary Euclidean distance, to $p + f(p)$, with f(p) the market excess demand at prices p. (The reader should not be disturbed by the addition of prices and quantities in the definition of the mapping; presumably appropriate units of measurement have been selected.) For each p, g(p) will be the solution to the quadratic programming problem: $\min\{(g - p - f(p))^2 : g \geq 0, \Sigma g_i = 1, gA \leq 0\}$. It is a straightforward exercise to verify that a fixed point of this mapping, p*, will be an equilibrium price vector, and that the Lagrange multipliers associated with the constraints $gA \leq 0$ will, in fact, be the equilibrium activity levels.‖



THE CONTINUOUS MAPPING

In this presentation of the general equilibrium model, I have implicitly assumed that all consumer goods have positive marginal utilities and that disposal is costless. If these assumptions were not made, then some of the equilibrium prices might indeed be negative; economic agents might be tempted to avoid making the required payments for the disposal of unwanted goods, if their activities were not easily detected. It is customary in economic theory to refer to this possibility as a negative externality, the classic example being the dumping of soot by a steel mill on a neighboring laundry. My own personal preference is to consider the use of resources required for disposal--without proper payment--as an act of theft, similar in spirit to non-payment for the use of other factors of production. This raises certain legal questions about the precise definition of the rights associated with the ownership of particular goods.

I have also assumed in the definition of a competitive equilibrium that all of the commodities are private goods, whose total supply must be allocated among the various consumers in order to be enjoyed. In contrast, the supply of a public good such as a park or a television network may enter into each utility function at the same level without being divided among the consumers. The Lindahl solution is an extension of the concept of economic equilibrium which allows for the possibility of public goods in consumption. In this solution each consumer has his own private price for the public good--selected in such a way that each private act of utility maximization will lead to the same level of the public good, and such that the sum of the private payments is adequate to cover costs. Unfortunately, individuals may be motivated to misrepresent their own marginal utilities for public goods in the hope that costs will be borne by other members of society. An even more demanding problem arises if the public good is, in fact, a public bad: a commodity like atmospheric pollution which is distressing merely by virtue of its existence. In this case the private prices required by the Lindahl solution would be negative and extremely difficult both to ascertain and to collect. But the Lindahl solution does provide an analytical model, which, in theory at least, would be capable of yielding the appropriate prices to be charged.

III. Applications to Socialist Planning

The general equilibrium model formed the basis for the fascinating discussion of economic planning under socialism during the early decades of this century. One of the earliest participants in the debate was the Italian economist Enrique Barone who, in 1908, presented a formal mathematical model for the selection of productive techniques and the distribution of outputs among the consuming units of a socialist economy. Barone's analysis was virtually identical with the model of general economic equilibrium previously introduced by Walras, aside from allowing the distribution of income to be partially under state control rather than arising solely from the private ownership of factors of production.

Barone was skeptical about using the equilibrium model as an instrument for central planning on the grounds that the computational difficulties were insurmountable. He described the production side of the economy by an activity analysis model, and argued that if we knew precisely which activities were to be used at equilibrium, then relative prices--for all of the goods and services in the economy--could be determined by solving the system of linear equations stating that the profit associated with each of these activities was zero. For Barone, the major difficulty in applying the equilibrium model to a socialist economy was that no computational procedure existed for solving the vast number of nonlinear equations--and inequalities--required to select precisely the correct set of activities to be used at equilibrium.

von Mises, writing in 1920, and Hayek--more than a decade later--both argued most convincingly that a rational allocation of resources was impossible without the use of competitive prices. For von Mises, market-clearing prices could not be determined without the incentives conferred by private ownership of productive factors. He does not refer explicitly to Barone's earlier contribution, but his bias against the possibility of a mathematical solution is revealed in several curious pages of diffuse annoyance at Cassel and other writers on the subject. Hayek was aware of the possibility that equilibrium prices could conceivably be calculated by a central planning agency, rather than arising from the decentralized responses to market prices of individual economic agents, but he was extremely dubious about the possibility of assembling all the relevant information about individual preferences and the current state of technical knowledge in the offices of the central planners. Hayek stressed the importance of private rewards as an incentive for amassing information about productive possibilities, for exploring alternative technical choices and for selecting production plans congruent with the ultimate desires of consumers.

Lange, a committed socialist, was also doubtful about the possibility of centralized computation. He proposed, in his celebrated paper of 1936, "On the Economic Theory of Socialism," that conventional free markets be used to determine competitive prices in a socialist economy; non-equilibrium prices would be continually revised by market forces in response to the discrepancy between supply and demand. For Lange, a major virtue of socialism was its ability to correct the gross inequalities of income resulting from the private ownership of factors of production in a capitalist economy. One of Lange's suggestions was that the returns to collectively owned factors of production could be allocated directly to the consuming units of the economy by means of a "social dividend"--based on an equal ownership by each consumer of stock in a single national corporation.

Why was Lange's suggestion of using market clearing prices never adopted in the Soviet Union? During the 1920s and 1930s, there was an overwhelming concern about the rapid buildup of heavy industry, followed by massive investments in defense to prepare for the Nazi threat. But it is conceivable that the inadequate supplies of foodstuffs and consumer goods--an important motivation for the current move towards private enterprise--could have been remedied if market forces had been permitted in the post World War II period, or if centralized calculations had been used to provide rough approximations to competitive prices. It may be useful to quote Lange, himself, from an address given in 1957.

> Methods which are necessary and useful in the period of social revolution and of intensive industrialization become an obstacle to further economic progress when they are perpetuated beyond their historic justification. They become obstacles because they are characterized by a lack of flexibility. They are rigid, and they lead therefore to waste of resources, resulting from this inflexibility; they require a wasteful bureaucratic apparatus and make it difficult to adjust production to the needs of the population. However, it seems that the greatest obstacle to further progress results from the lack of proper economic incentives in this bureaucratic centralistic type of management. This hampers proper economic utilization of resources, encourages waste and also hinders technical progress.

At first glance, Barone's earlier concern about computation seems no longer to be valid, given the emergence of the modern computer and the development of efficient computer codes for calculating equilibrium prices. Models with, say, several hundred variables can be solved on a personal computer. Given the value of this calculation, it would have seemed appropriate to use a dozen supercomputers full time in order to provide Soviet planners with those prices and production decisions which would allocate resources in an optimal fashion. Like all administrators, Soviet politicians are reluctant to surrender political power; a Cray supercomputer would be a trivial investment if queues at food stores and clothing shops could be avoided without sacrificing centralized political control of economic decisions, and without allowing the vast disparities in income that are an inevitable consequence of private economic initiative.

But, of course, a general equilibrium model of the Soviet economy involving even several hundred carefully selected sectors is not suitable as an instrument for detailed control of an economy consisting of millions of goods and services. In what was surely a temporary lapse, Lange seriously overestimated the computational power of the modern computer when he returned to the debate with Hayek and wrote, in an article which appeared posthumously in 1967:

> Were I to rewrite my essay today, my task would be much simpler. My answer to Hayek and Robbins would be: so what's the trouble? Let us put the simultaneous equations on an electronic computer, and we shall obtain the solution in less than a second. The market process with its cumbersome tâtonnements appears old-fashioned. Indeed it may be considered as a computing device of the pre-electronic age.

A general equilibrium model with millions of equations and unknowns could conceivably be solved in months--or possibly years--of constant computing on a supercomputer. But the disaggregated data on consumer preferences and the technical possibilities of production would be virtually impossible to assemble, and even if some heroic approximation were achieved, the numerical inputs would certainly be out of date by the time the computation were completed.

A general equilibrium model with several hundred sectors may be quite useful in an entirely different way: as an analytical instrument for investigating the consequences of changes in economic policy--such as the imposition of a quota on imported oil, a modification of the tax code, or the drastic reductions in tariffs envisaged in the European Economic Community.

But it is well understood that in this type of exercise the results depend on the taste and skill of the analyst, and on the choice of variables and modes of aggregation; they are suggestive at best, and certainly not sufficiently disaggregated for economic decision making in a large, modern economy. Nevertheless, I am astonished that some stylized version of a general equilibrium model was not used to suggest revisions in Soviet prices from their rigid levels of the last several decades.

Clearly, the major attraction of markets over centralized calculation for President Gorbachev and his economic reformers is to be found in Hayek's description of the merits of private incentives in assembling technical information about production, in evaluating the profitability of new activities in terms of competitive prices and in establishing some degree of concordance between productive decisions and the preferences of consumers. If an economy is to be responsive to the novel conditions of daily life--and to engage the energies and skills of millions of self-interested economic actors--it seems necessary to use the market, rather than the computer, as the algorithm for solving the equations of economic equilibrium.

The production and distribution plan arising from a competitive equilibrium is Pareto optimal in the sense that there is no alternative plan which improves the utility of all consumers. Moreover, any Pareto optimal production and distribution plan is, itself, a competitive equilibrium if sufficiently flexible lump sum redistributions of income are allowed. A competitive equilibrium is also in the <u>core</u> of the economy: no proper coalition of consumers can improve their own utilities by an alternative use of the coalition's initial assets and the productive knowledge available to the economy as a whole. And if the number of consumers is sufficiently large, a production and distribution plan which cannot be improved upon by any coalition of consumers will, in fact, be a competitive equilibrium, without requiring any redistribution of income at all.

Suppose that the economy is in equilibrium and that a discovery is made of a new way to make sausages out of sawdust, or a new way to transport electrical energy using superconducting wires. Shall this new activity be used? Committed central planners could recalculate the equilibrium on the supercomputer. Or they could make use of a powerful result of economic theory--perhaps the most important theorem of microeconomic analysis--which provides an immediate necessary and sufficient condition for an affirmative answer to the question: can all consumers be made better off if the new activity is used? The condition is remarkably simple: all consumers can be made better off if and only if the new activity makes a positive profit at the current equilibrium prices. If a profitable project is undertaken, the equilibrium prices will change. But it is not necessary to calculate the new vector of prices to address the issue of a Pareto improvement; the current system of prices is fully adequate for this task.

The relationship between the market test of profitability and the test for a Pareto improvement is one of the major intellectual justifications for allowing private incentives to operate in the presence of decentralized markets. Of course, substantial redistributions of income may be required to improve the welfare of all agents in the economy; the specialized skills of the workers in a firm may become worthless if the discovery of a profitable technology elsewhere in the economy condemns the firm to bankruptcy; the owners of shares in a manufacturing entity whose outputs are challenged on the market may face a sudden capital loss. Redistributions required for a Pareto improvement are rarely carried out in practice--a technical innovation introduced in Japan may call for the unlikely transfer of income from the

Japanese government to inhabitants of the United States. Without explicit compensation, Soviet citizens will be forced to tolerate substantial inequalities in the distribution of income, if these market tests are to be undertaken by economically motivated agents, whose rewards depend on precise computations of economic profitability.

IV. A Numerical Example

The general equilibrium model can be used to discuss the consequences of a change in some significant parameter of the economy, such as the abrupt increase in the cost of imported oil experienced by the United States twice in the last fifteen years (in 1973-74 and 1979-80), changes with extraordinary consequences for the future development of the U.S. economy. To analyze this experience it is necessary to agree on an acceptable model of the economy before and after the shock. To model the consumer side of the economy we specify the assets--for example, labor, capital and durable goods--owned by each class of consumers, and the consumers' preferences for the goods and services potentially available to them. To model the production side, we need an explicit mathematical description of the techniques available to producers--given perhaps by an input/output table for the economy as a whole, or by a series of activity analysis matrices for each of the firms in the economy. A formal mathematical presentation of the model of equilibrium is required for this exercise in comparative statics to be carried out on the computer.

It may be useful, at this point, to provide an explicit numerical example of a general equilibrium model--an extremely simple example which I designed a number of years ago to illustrate the possible effects on the economy of the United States of the substantial decrease in the real price of imported oil experienced during the mid 1980s. The example consists of five sectors:

1. services
2. manufactured goods
3. labor
4. capital
5. energy.

The production side of the economy is described by the following activity analysis matrix:

services	100	100	0	0	0	0
manufactured goods	-10	-10	100	100	100	-100
labor	-60	-62	-45	-52	-55	0
capital	-27	-20	-50	-40	-30	0
energy	-3	-10	-5	-8	-18	100

in which columns 1 and 2 represent alternative plans for the production of services; columns 3,4 and 5 alternative activities for the production of manufactured goods; and the final column the possibility of importing energy from abroad by exporting manufactured goods on a one for one basis. The activities, which differ substantially in their labor, capital and energy requirements, can be run simultaneously at arbitrary non-negative levels.

The consumption side of the economy is described by two aggregated consumers, the first of whom owns 16000 units of labor and nothing of the other goods. The second consumer--the capitalist in the example--owns no labor, 10000 units of capital and 900 units of energy. Each consumer has a Cobb-Douglas utility function with the following percentages of income spent on the first three commodities--services, manufactured goods and leisure:

$$\begin{array}{l} \text{Workers} \\ \text{Capitalists} \end{array} \begin{bmatrix} .3 & .6 & .1 \\ .6 & .4 & .0 \end{bmatrix}$$

The model is unrealistic in many ways. There is a stock-flow problem in the treatment of capital: capital should be permitted to depreciate over time, and its services are the correct input into production. Energy is most appropriately treated as an intermediary good, rather than as a primary factor of production. In the model, neither consumer saves, in the sense of committing income to the purchase of future capital goods or stocks of energy, nor is there an explicit treatment of investment opportunities and the possibility of production in subsequent periods of time. Presumably features of this sort would be included in a more realistic model.

The units of measurement have been selected so that the competitive equilibrium in this example is given by the price vector

$$p^* = (.2,.2,.2,.2,.2),$$

in conjunction with the production plan obtained by running the six activities at the following levels:

Activity	Level	Profit
1.	113.4	.0
2.	0.0	-.4
3.	71.18	.0
4.	84.47	.0
5.	0.0	-.6
6.	4.72	.0.

All capital and the stocks of domestic and imported energy are fully used in production, as are the 16000 units of available labor minus the 1600 units reserved for the leisure of the working class. The net production plan y* is given by (11340, 13960, 1600, 0, 0) and the final allocation of goods to the two consumers is

	Services	Manufactured goods	Leisure
Workers	4800	9600	1600
Capitalists	6540	4360	0.

To illustrate the consequences of a substantial decrease in the price of imported energy the final column of the activity analysis model was replaced by one in which 100 units of imported energy could be purchased by exporting 40 units of manufactured goods, rather than at the previous cost of 100 units. No other changes were made in the production possibility set or in the parameters describing the preferences or endowments of the two consumers. After this change, the new equilibrium price vector is

$$p^* = (.234, .220, .267, .191, .088),$$

and the activities in use and their corresponding levels become

Activity	Level	Profit
1.	.0	-.193
2.	105.5	.0
3.	141.6	.0
4.	.0	-.227
5.	27.1	.0
6.	13.49	.0.

As can be seen, there is a substantial increase in the quantity of imported energy and a substitution of production in the direction of energy-intensive activities. The increase in the ratio of the wage rate to the price of capital causes a substantial shift from activity 4 to activity 3. The net production plan becomes y* = (10552, 15270, 1600, 0, 0) which is allocated to the two consumers in the following fashion:

	Services	Manufactured goods	Leisure
Workers	5468	11657	1600
Capitalists	5084	3613	0.

The large increase in wages has made the class of workers considerably better off in real terms than they were before the decrease in the price of imported energy. The capitalists, largely because of the decreased value of their energy stocks, experience a fall in their levels of consumption. A Pareto improvement over the previous equilibrium would, in this example, require a donation of income from workers to capitalists.

V. Uniqueness and Stability

Each of the two examples presented above has a unique equilibrium price vector and associated production plan. It is possible, however, for a general equilibrium model to have more than one solution; there is a well known example in the theory of international trade with three distinct equilibria. In such a case, there is no a priori argument for selecting one of the several equilibria; explicit consideration must be given to the path taken from the current state of disequilibrium.

There are several mathematical conditions on the specification of the model, each of which is sufficient to guarantee uniqueness of the equilibrium. For example, in a model of exchange in which production is not explicitly considered, there will be a unique equilibrium if the market excess demand functions satisfy the assumption of gross substitutability, i.e., the demand for any particular commodity will increase if the price of any other commodity rises. The property of gross substitutability aggregates over consumers; if the condition is satisfied for each individual consumer then it will be satisfied for the market demand functions as well. The property is certainly satisfied if each consumer has preferences described by a Cobb-Douglas utility function or more generally by a utility function with constant elasticity of substitution greater than one.

Unfortunately, gross substitutability of the market excess demand functions is no longer sufficient for the uniqueness of equilibrium in the more realistic case in which production is included in the model. If the production possibility set is a convex cone, it is easy to demonstrate uniqueness if the market excess demand functions f(p) satisfy the weak axiom of revealed preference: for no two distinct price vectors p^1 and p^2 can the two inequalities $p^1 f(p^2) \leq 0$ and $p^2 f(p^1) \leq 0$ simultaneously hold. Moreover, this is the only property of the market excess demand functions which guarantees uniqueness. If the market excess demand functions do not satisfy the weak axiom, then a production possibility set can be constructed, exhibiting convexity and constant returns to scale, which yields multiple equilibria in combination with the given demand functions. But again we have a problem: the weak axiom does not aggregate over consumers. It is satisfied for individual consumers with any well behaved preferences, but if there is any diversity of preferences among the consumers, it may not be satisfied for the market. To illustrate this point, an example of an economy has been constructed--with four commodities, three consumers whose utility functions are of the Cobb-Douglas form, and with production described by an activity analysis model--which possesses three distinct equilibria.

But the issue is perhaps more theoretical than practical. Since the development of efficient numerical algorithms for the solution of general equilibrium models more than two decades ago, thousands of large, disaggregated models have been constructed, not one of which has exhibited multiple equilibria. In practice, the equilibrium seems to be unique, unless there are clear and compelling economic reasons for multiplicity.

There is, however, a related question of stability of the competitive equilibrium which, in my opinion, cannot be dismissed as easily. The issue concerns the dynamic path of adjustment followed by a system which is not in equilibrium, for example, the Soviet economy in transition from the disequilibrium induced by central planning to the presumed equilibrium to be achieved by the introduction of competitive markets. If the system is not at equilibrium at current prices, there will be a discrepancy between supply and demand, leading to queues or inventories of unwanted goods, or opportunities for profitable productive activities. In the first case, we would expect the prices of goods in excess demand to rise, and those in excess supply to fall; in the second case we would expect a change in the supplies offered to consumers by the productive side of the economy. Do these adjustments - motivated by a search for welfare improvements - lead to a path of prices and production plans which ultimately tend to a state of equilibrium?

It is my impression that neo-classical economists of the last century clearly expected there to be a tendency towards equilibrium although they were not specific about the precise nature of the path to be followed. Lange, in 1936, made the explicit suggestion that the Walrasian tâtonnement be institutionalized as the mechanism for arriving at market-clearing prices in a socialist economy. The tâtonnement, or price adjustment mechanism introduced by Walras for the model of exchange, is a mathematical formalization of the tendency to equilibrium: given a price vector p, the price of the ith good will rise if the excess demand for the ith good, $f_i(p)$, is positive and will fall if the excess demand is negative - unless the price is already zero, in which case it will remain at this level. If the adjustment is assumed to take place continuously in time, the tâtonnement process can be converted into a system of differential equations, $dp_i/dt = f_i(p)$, whose solution will yield a dynamic path of prices. The significant question is whether the path, starting in disequilibrium, will converge to a price vector at which demand and supply are equal.

As has been remarked, the market excess demand functions, $f(p)$, are completely general aside from continuity and the Walras law $pf(p) = 0$. This latter property has the peculiar consequence - dependent on the choice of units and certainly not significant from an economic point of view - that $d\Sigma(p_i)^2/dt = 2\Sigma p_i(dp_i/dt) = 2pf(p) = 0$, so that $\Sigma(p_i)^2$ is constant along the solution to the system of differential equations. But aside from this property--that the price vector stays at a constant distance from the origin --there are no other restrictions on the path of solutions, which can be made to trace out an arbitrary curve on the sphere in n-dimensional space by an appropriate selection of the excess demand functions.

As in the study of uniqueness of equilibrium, some additional assumptions on the market excess functions are required to guarantee stability in this sense. It comes as no surprise that precisely the same conditions are sufficient: the tâtonnement process, in the case of pure trade, will be stable if the market excess demand functions satisfy the condition of gross substitutability or the weak axiom of revealed preference.

It is interesting to compare the Walrasian tâtonnement with the economic interpretation of the simplex method for the solution of linear programming problems. From an economic point of view, the typical linear program can be regarded as a problem of maximizing the value of output subject to constraints on the availability of factors of production, when the underlying production possibility set is given by an activity analysis model. At each step of the simplex method, a trial solution to the linear program is proposed. To test the optimality of this solution, we find those prices which yield a profit of zero for the activities in use and use these prices to calculate the profitability of the remaining activities. The trial solution is optimal if none of the remaining activities makes a positive profit; if one of them is profitable we simply increase the level of its use from zero, making compensating changes in the previous activity levels until one of them falls to zero. The algorithm continues until a trial solution is found which passes the pricing test for optimality.

The simplex method mimics the search for decentralized prices which equilibrate the supply and demand for factors of production. In much the same fashion, the Walrasian price adjustment mechanism can be shown to converge for an arbitrary convex programming problem: max $g_0(x)$, subject to the constraints $g_i(x) \le b_i$ on the factors of production. A vector of factor prices, p, is proposed, and the economic agent is instructed to maximize profit: $g_0(x) - \Sigma p_i g_i(x)$, without explicit consideration of the availability of these factors. The maximization problem will lead to demands for the factors of production as a function of the current price vector, and the system will be globally stable if prices are revised according to the discrepancy between demand and supply. In summary, the price adjustment mechanism, in one of its forms, converges for neo-classical problems which are purely on the production side of the economy. It is the presence of income effects associated with consumer demand functions which impedes the corresponding process of adjustment in a general equilibrium model of the economy.

There are efficient numerical algorithms for calculating the equilibrium prices and production plans for a disaggregated general equilibrium model of the economy. These algorithms, which do yield a time dependent path of economic variables on the computer, have been designed by human beings, and it is unlikely that they are more sophisticated than the combined calculations of highly motivated agents responding to market forces. The major difference between these computational methods and our specification of the price adjustment mechanism is that fixed point methods are based not simply on the value of excess demand at the current disequilibrium prices, but on the derivatives of the market demand functions as well. The path of prices in these globally stable numerical procedures depends on estimates of the local behavior of demand with respect to small changes in price, rather than on the value of the demand function alone.

But even if some variant of the Walrasian price adjustment mechanism were always globally stable, there would be serious doubts about its economic realism. The mechanism executes in virtual time; the excess demand functions are unchanged over time and no meaningful economic transactions occur during the transition from an arbitrary initial state to the final state of equilibrium. New and profitable production plans are suggested, but they are never set in motion - machinery and manufacturing plants are not constructed, no purchases are made of factors of production and output is not distributed to consumers. The revision of prices called for by the discrepancy between supply and demand has no consequences for the income of consumers and no real penalties are exacted by exchanges at disequilibrium prices. The real costs of unemployment of labor and other inputs into production are not experienced by any of the agents in the economy. There is no unintended saving or investment. Everything takes place on the drawing board.

Of course, dynamic elements can be consciously introduced in the general equilibrium model. If the goods and services in the economy are differentiated according to the time of their availability, if the utility functions of consumers involve the levels of consumption today and at various times in the future, and if the production possibility set allows for intertemporal flows of inputs and outputs, then the Walrasian model will account for prices, production plans and consumption levels which depend explicitly on time. But this formulation involves either the introduction of perfect futures markets, at a degree of detail and specificity far from what occurs in practice, or the explicit incorporation of expectations for future prices, which may be thoroughly inconsistent with those prices as they are subsequently realized.

In either of these formulations, the one-period model of equilibrium is supplanted by an even more ambitious construction in which all future markets equilibrate at the very beginning of time. The path constructed by the Walrasian price adjustment mechanism would be replaced by a path in which all economic variables are constantly in equilibrium and evolve in a perfectly predictable fashion. But what is the dynamic process for arriving at this even more complex equilibrium?

VI. Increasing Returns to Scale in Production

The assumption that the production possibility set is a convex cone - required for the existence of equilibrium in the general equilibrium model - is incompatible with the possibility of increasing returns to scale in production. If increasing returns to scale are present, the production possibility set will be non-convex, and there may not be an equilibrium with producers selecting their production plans so as to maximize profit with respect to fixed market prices. Moreover, the core of the economy - an ideal solution to the problem of production and distribution - may be empty in the sense that any proposed production and distribution plan consistent with endowments and the current state of technological knowledge can be improved upon by some coalition of consumers.

In his recent book, Scale and Scope, the eminent business historian Alfred D. Chandler has provided a comprehensive account of the rise of the large industrial corporation during the last century - in the United States, England and Germany. For Chandler, and possibly for his predecessor Schumpeter as well, the ultimate source of the extraordinary benefits of modern industrial capitalism is to be found in the ability of this new organizational form to exploit the economies of scale in production made available by continued scientific and technical innovation.

In the classical case of constant returns to scale the firm can be disaggregated into small units, which then interact with each other by means of market prices; there is no requirement for a sophisticated theory of the firm. If increasing returns to scale prevail, however, the enterprise cannot be disaggregated into competitive units and alternative forms of industrial organization are necessary to provide a hospitable environment in which scale economies can be realized.

How shall the Soviet economic reform accommodate itself to the possibilities of economies of scale in production? Who shall own the large firm and how shall profits be distributed among the consumers in the economy? Shall some version of marginal or average cost pricing be used to decide on the price of output? How shall the firm be managed, and what are the criteria for the selection of projects? Shall the large firm be regulated so as to protect consumers from monopolistic behavior or shall legislation be used to break up the firm into smaller competitive units, with the possible sacrifice of economies of scale? What enterprises shall be centrally owned and administered: shall agriculture and small manufacturing be in private hands but not telephone networks, railroads and the postal service? And what is the appropriate size of the large manufacturing enterprise if a balance is to be struck between economies of scale and the difficulties of managerial coordination? Perhaps, as many analysts have suggested, the scale of Soviet firms is already far too large and they should be broken up into a number of competing entities.

Economists have been concerned for many years about the need to incorporate the possibility of increasing returns to scale in their analytic formulations. An older school of economists held the opinion that efficiencies of large-scale production were caused by indivisibilities, i.e., large, lumpy aggregates of capital - assembly lines, railroad and telephone networks, oil pipelines, bridges - whose economic advantages could not be realized at low levels of production. Abba Lerner, for instance, devoted two chapters of his famous book <u>The Economics of Control</u> to the study of indivisibilities. I quote from Chapter 15, to illustrate his position on this subject:

> We see then that indivisibility leads to an expansion in the output of the firm, and this either makes the output big enough to render the indivisibility insignificant, or it destroys the perfection of competition. Significant indivisibility destroys perfect competition.

In the case of constant returns to scale, institutional arrangements such as competitive markets are directly suggested by numerical methods for the solution of linear programming problems. If the analogy were to be maintained, we might expect corresponding insights about the internal organization of large firms from the study of decision methods for the solution of maximization problems involving indivisibilities.

Indivisibilities are introduced into a linear programming problem by requiring that some, or all, of the activity levels take on integral values, rather than arbitrary real values; linear programming problems with integer activity levels are known as integer programs. The first algorithm for solving the general integer programming problem was introduced in the late 1950s by Ralph Gomory. The early methods were not robust: a slight change in one of the parameters of the problem could transform an easy problem into an intractable one. In contrast to the simplex method, which performs remarkably well on most linear programming problems, integer programming algorithms are time consuming and capricious. And perhaps even more significantly for economic theory, none of these early algorithms was capable of being interpreted -- by even the most sympathetic student -- in meaningful economic terms.

Let me try to illustrate the basic difficulty introduced by indivisibilities by returning to the earlier discussion of the role played by prices in the solution of linear programs. For such problems, prices have their customary economic interpretation as "marginal value products" -- the marginal change in the optimal value of output if a particular factor of production is increased by a small amount. But, as we have seen, prices are also used to determine whether a specific feasible solution, one which satisfies the constraints of the problem, is actually the optimal solution. Given a feasible solution to a linear program, we find those prices that yield a zero profit -- net of all costs, including the rental of capital -- for the activities being used. Then a necessary and sufficient condition that the proposed feasible solution be optimal is that all of the remaining activities be unprofitable when their profitability is evaluated at these same prices.

This test for optimality is not available for integer programs; there simply need not be a set of competitive prices that yields a zero profit for those activities in use at the optimal solution and a non-positive profit for the remaining activities. Let us look at the following example of an integer program with a single constraint and two non-negative integer variables:

$$\text{maximize } x + 3y, \text{ subject to}$$
$$2x + 3y \leq 5,$$
$$x,y \geq 0, \text{ and integral.}$$

The solution to the corresponding linear program - with no requirement of integrality for the activity levels - is $(x,y) = (0,5/3)$, and the price of the constraint is equal to one. At this price the second activity makes a profit of zero and the first activity - which is not used - has a negative profit. But the optimal solution for the integer program is $(x,y) = (1,1)$; both activities are used and there is no price at all which yields a zero profit for the two activities simultaneously.

This is, of course, not an accident of this extremely elementary example. Except for very special integer programs, there will not be a vector of prices which provides a profit of zero for those activities used in the optimal solution and a non-positive profit for the remaining activities. Moreover, if we have solved a specific integer program by one device or another, and a new activity is discovered, there is no conclusive pricing test which tells us whether the new activity can be used so as to improve the objective value.

A similar difficulty arises in the model of general economic equilibrium. Suppose that the economic system is in equilibrium at certain prices and that a new activity is discovered which can only be used at an integral level. Is its profitability at the equilibrium prices a necessary and sufficient condition for a Pareto improvement - for the possibility that everyone can be made better off using this new activity? The answer, unfortunately, is no: a new technical process may be profitable and yet it cannot be used to improve the utility of all consumers. And if several activities are simultaneously discovered, all of which must be used at integral levels, improvements may require the use of a complex mixture of activities that are profitable at current prices and those which are not. The correct mixture of activities can be found by solving a non-linear integer programming problem, but the simple market test, relating profitability at current prices to the possibility of a Pareto improvement, is not available in the presence of indivisibilities in production. This important observation qualifies, to some extent, the argument for classical market forms of economic organization when economies of scale are present.

There has been a considerable revival of interest during the last decade in noncompetitive pricing rules for the firm whose production possibility set exhibits increasing returns to scale. Existence theorems have been demonstrated for marginal and average cost pricing equilibria in models of substantial generality, and algorithms have been introduced for their calculation. The historical justification for marginal cost pricing is its presumed relation to efficiency and Pareto optimality; the justification is somewhat weakened by a recent series of examples in which none of the marginal cost pricing equilibria is optimal.

VII. A Suggestion

Economic theory has not been able to provide a generally convincing account of the dynamic behavior of an economic system tending towards equilibrium in real rather than virtual time. No version of the general equilibrium model is adequate to describe the painfully slow recovery of market economies to the Depression of the 1930s. In this sense, theory has relatively little to say about the transition of the Soviet economy from its central planning mode to a market economy. We cannot be precise about the duration of the transition and the magnitude of the costs, both social and economic, that will be incurred prior to the achievement of competitive markets. These costs may very well depend on aspects of Soviet cultural, legal, and institutional traditions which are impossible to describe in purely economic terms alone.

My own personal suggestion is that the costs of the transition could be estimated in a crude way by constructing an explicit disaggregated general equilibrium model of the Soviet economy designed to assess the magnitude of the changes to be expected along the way to a market economy. The model could be constructed so as to extend over several years and could plausibly be based on the assumption of perfect foresight. The results of the computation would be suggestive about the deviation of currently controlled prices from their equilibrium levels and from international relative prices; about the magnitude of the excess demand for various goods and services; about the dates of availability and prices of agricultural products; about the ultimate mix of imports and exports under a market system with flexible exchange rates; about requisite investments in productive activities which take advantage of Soviet natural endowments and technical skills and about the distribution of income to be achieved by the introduction of competitive markets. We would obtain some indication of those sectors of the Soviet economy in which substantial changes from the current state might be expected to occur and which might, therefore, require centralized intervention and some form of "indicative planning." And, finally, the discipline imposed by constructing a detailed general equilibrium model might be of considerable benefit in the organization of Soviet economic statistics and the establishment of sound accounting procedures. I have no doubt about the value of the exercise for the proposed economic reform in the Soviet Union, if the results were properly seen as being indicative of significant tendencies rather than being accurate to several decimal places.

The numerical computations should not be prohibitive. Several hundred sectors could be accommodated if the computation were carried out in the space of commodities and a larger degree of disaggregation might be achievable if the model were based on a small number of consuming units in each major region of the economy. In addition, there are many excellent scholars in the field of applied general equilibrium analysis whose judgment and considerable experience in this type of enterprise could be effectively utilized. In my opinion, the major difficulty would be in the collection of data and in decisions about the basic structural elements to be included in the model. Utility functions for classes of Soviet consumers might not be easy to obtain and would be founded more upon hunches than refined econometric analysis. There would also be questions about precisely how to incorporate the public ownership of land, factories and other forms of capital so that the returns to these factors are reflected in market demands. A decision would be required, furthermore, as to whether some form of increasing returns to scale should be included in the production possibility set, and if so what notion of equilibrium should be used. And finally, there might be some apprehension about the reliability of the production data provided by central planning agencies.

In a way, these concerns are reminiscent of those expressed by Hayek in the 1930s, and they would be as relevant today if our goal were the construction of a general equilibrium model as an instrument of detailed economic surveillance and control. But the purpose of this exercise would be quite different: to provide valuable insight about the magnitude of the changes to be experienced in the transition to a market economy, quantitative estimates of the ultimate gains to be achieved, and an overall sense of where the Soviet economy might be heading.

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