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DEFAULT AND BANKRUPTCY IN A MULTISTAGE EXCHANGE ECONOMY

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1. INTRODUCTION

In several previous publications Shubik and Wilson (1977), Dubey and Shubik (1979,1988) and Dubey, Geanakoplos and Shubik (1988) considered the need for and varieties of a default or bankruptcy penalty in a one period game with borrowing.

From the viewpoint of the law there are important distinctions among the various levels of default and bankruptcy which we do not make here. Specifically we are concerned with the situation where an individual debtor does not honor his debt regardless of his possessions at that point. The penalty involved may be extraeconomic (such as prison) or it may be strictly economic such as the garnishing of salary or the recapture of assets.

In the first investigations one period trade without exogenous uncertainty was considered. It was shown that all that is really needed in order to guarantee optimal trade is a "harsh enough" penalty to prevent strategic bankruptcy, i.e. a willful default of debt. There are many ways this can be achieved. Without going into detail all that is required is that an economically endogenous penalty like exile, prison, slavery is nasty enough that it does not pay to elect strategic bankruptcy.

In spite of the fact that history has many examples where bankruptcy involved penalties of the variety noted above, some economists might claim that these penalties are "too arbitrary" and instead would be happier with a more economically endogenous penalty which would make as much use as possible of actions with respect to goods and markets. Dubey and Shubik(1988) showed that it was possible to do this *if* one made the assumption that after trade there is an interval before consumption where goods can be confiscated from individuals who have defaulted.

There are two simple rules which might be applied. The forgiving rule and the no information rule.

Suppose an individual goes bankrupt for an amount A and obtains a final endowment of $\bar{x}$. The referee removes a vector of resources b such that

$$\max \varphi_i(\bar{x}_1^i - b_1^i, \bar{x}_2^i - b_2^i, \ldots, \bar{x}_m^i - b_m^i)$$

$$\text{subject to } \bar{p} \cdot b = A$$

where $\bar{p}$ are the equilibrium prices. This is the forgiving rule and is exactly the same as though the individual himself were required to adjust to a loss of income of A. Thus he is indifferent between not defaulting or defaulting and being subject to this penalty.[1] With such penalties the Nash Equilibria of continuum strategic market games will be Walrasian.

The second rule has the important advantage that it requires no knowledge of the individual preferences by the referee. b is selected arbitrarily such that

$$\bar{p} \cdot b = A \quad \text{otherwise} \quad b = \bar{x}.$$

If the referee is able to garnish assets before they have been dissipated this is at least as harsh as the first rule and is an easy economic rule which will discourage strategic bankruptcy. The resources obtained from a bankrupt go to the referee and could be used for redistribution. However in the discussion of the one period model without exogenous uncertainty there is no need to be concerned with creditors as we know that we obtain Pareto optimal trade with no bankruptcy.

Matters change considerably when we consider incomplete markets. Dubey, Geanakoplos and Shubik (1988) were able to show that an equilibrium could exist with active bankruptcy, and furthermore one could show that some finite penalty would dominate either a zero or an infinite penalty. In particular it was argued that in varying the intensity of the penalty one is able to perform a sensitivity analysis over a class of games where the only difference between any two is the level of the penalty. For an economy with only two types of trader and a unique pure strategy NE one can illustrate a range for the bankruptcy penalty which gives rise to NEs which Pareto dominate the NEs arising from any other penalties. Unfortunately although this observation is suggestive, when one considers a general economy with many types of traders and goods, although one can show the existence of an optimal range of bankruptcy settings, the characterization of the optimal set poses considerable difficulties.

The general problem and insight, however can be stated relatively simply. When there are incomplete markets and there is a nonzero chance that any individual may obtain no income in some state then an infinite

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[1]But this rule has the considerable inconvenience that it requires, for its implementation, the knowledge of agents' utility functions, unless the referee gives the agent the opportunity to name the assets to be sold--a strategic freedom he may not wish to grant because of features such as adverse selection.

default penalty will completely wipe out borrowing. A zero penalty will wipe out lending. Somewhere in between the society can Pareto improve. Thus the selection of the penalty is in essence a public good.

Although the Dubey, Geanakoplos and Shubik (1988) work showed equilibrium with bankruptcy, as it was constrained to one play of the game no indication was given as to what happens to the bankrupts in the further evolution of the economy. As soon as we consider more than one play the treatment of reorganization and the position of the creditors must be spelled out.

In this paper the new problems which arise in considering bankruptcy in a multistage game are considered. They are:

The class of bankruptcy rules in a multistage game without exogenous uncertainty considered for:

(a) optimality

(b) exogenous or endogenous penalty

(c) reorganization of debtors and payments to creditors.

The important case of sequential bankruptcy with exogenous uncertainty or incomplete markets will be dealt with in a subsequent paper.

At first glance the production of a mechanism which covers period by period bankruptcy appears to be extremely *ad hoc* and institutional. However we suggest that a fruitful approach is to attempt to consider the properties called for and then to construct as simple a mechanism as possible, analyze the resulting game and then consider the possible generalizations of the mechanism.

2. THE RELEVANCE OF THE PROPERTIES OF ECONOMIC MATTER

Possession is nine tenths of the law.

2.1. Secured Lending and Assets

Trustless trade is simple trade and it permits more anonymity than trade involving trust. Trustless trade can be achieved by exchanges where value is always given for value. In a multistage economy with assets there are items of value which will not be consumed for many periods. They present an economic opportunity and pose an economic problem. The opportunity is that they can serve as hostages, or by trading them back and forth they enlarge the domain of trustless or value for value trade. The problem they pose is in evaluation. If a durable lasts for 50 years (such as a house) how do we attach a present value to the asset. If we have already made the enormous simplifying assumption of no exogenous uncertainty, we can suspend common sense even further and (with a continuum of agents) argue for a mystical rational expectations which enables individuals to sum accurately expected discounted income streams.

If someone wishes to consume more than his immediate money and income and he has assets, he can increase his purchasing ability by borrowing or by selling assets.

What determines the choice between borrowing or selling an asset? We suggest that in actuality much of this choice is dependant on the microeconomic details of transactions costs and the physical properties of many immoveable goods such as houses and most personal durable possessions. For much economic analysis it is handy as a first order approximation to treat items such as houses or automobiles as fungible, but for other purposes this approximation is not adequate. Renters behave differently from owners. Many tastes are idiosyncratic. One might far prefer to borrow against one's land, than sell it with a buy back clause.

At the level of abstraction used here, without these extra transactions costs and considerations, a secured loan and a related sale, lease-back, buy-back may be regarded as equivalent in a world without exogenous uncertainty and with perfect foresight.

What assets serve as security for loans? What are the properties which make some more desirable than others? We suggest that there are several important physical properties involving the ability of the debtor to abscond, hide, cheat, steal, destroy or otherwise devalue or dissipate his assets. Immovability, low or no physical depreciation make land attractive. Up to a point owner occupied housing is attractive. Automobiles are far less attractive due to their mobility and high depreciation. At a slightly higher level of sophistication ownership paper such as 1,000 shares of AT&T serves as an excellent security.

Ownership paper such as a deed to land or better still part ownership of a major corporation symbolized by fungible and marketable stock is highly desirable as a hostage; as long as the legal system holds. The physical depreciation and inventory carrying costs of paper are minimal.

In actuality the problem with a security such as a stock is the ability of the individuals to assess present value accurately relating it to its discounted income stream. But in the simplified world postulated here that poses no problem.

In actual practice no one pretends to have perfect foresight. But experience teaches that the price of some assets is far more volatile than the price of other assets. The secured lending rule of thumb therefore involves a "haircut." Thus one may be willing to lend a certain percentage of the assessed value of a house, but a different percentage against a new sports car. If we believe that the lenders are conservative and that prices might be volatile a cheap and reasonably accurate way of modelling this conservatism is to introduce a "haircut" parameter for the secured loan rather than pretend to believe in the rational expectations story.

There has not yet been any experimental gaming with a multistage strategic market game without exogenous uncertainty, but it is straightforward to do so and I conjecture that even in a game with a unique equilibrium

point it is highly unlikely that play will be at the equilibrium. However I suspect that with an appropriate "haircut" secured lending can and will take place.

2.2. A Theorem on Secured Loans

There is a paradoxical result in the style of Modigliani-Miller which covers secured lending. In the theory based upon highly simplistic assumptions the result may be correct and useful in illustrating the implications of these assumptions. But in practice the result may be false and as such challenges the economist to observe more and ask which assumptions must be given up in exchange for the construction of models closer to economic reality.

THEOREM: *In a finite production and exchange economy without exogenous uncertainty and no inputs beyond the initial resource endowment of capital goods and consumption goods at the start all lending can be secured and hence all lending is unnecessary as it can be replaced by sale, lease and buy back.*

In the usual discussion of a T time period exchange economy it is assumed that individuals may obtain new endowments at any period. In contrast with this assumption, when we study growth theory, in general, the initial endowments of land, labor and capital are given together with the production transformation set and the dynamics of growth are studied.

The interpretation of new resources entering the economy from the outside after the initial conditions have been given, can be made in terms of new discoveries or political, military or other windfalls into an open economy. For purposes of our current analysis, after the initial endowments have been specified we rule out further inputs.

Two case must be considered. In the first intratemporal trades are netted out; but in the second money must be used to pay even for intratemporal trading.

Model 1: The following assumptions are made. We consider a modified general equilibrium economy which has n types of traders, trading in m commodities where the economy lasts for T time periods. There are also k firms which are treated as price takers maximizing profits to be paid out to the traders at the end of the economy. The traders all begin with initial endowments of the goods and shares in the firms. The goods can be divided into perishables, storable consumables, consumer durables; producer storables and producer durables. The production transformation sets are specified and are owned by the firms.

As it stands this model is nothing more than a specialization of the Arrow-Debreu model with production and stockholders, the existence of CEs follows immediately, but we require the economy to be run without debt,

hence there must be a balance of the individual budgets each period. But this follows immediately from the initial conditions and the existence of an equilibrium. At equilibrium the value of all current assets must equal the appropriately discounted value of their income streams. But this is the value of all of the final consumption in every period. Thus taking into account the initial value of his goods and shares held at the start each individual has immediate assets valued at his lifetime budget. He can thus follow an optimal policy by consuming *a la* Arrow-Debreu and by trading physical assets and shares each period so that his budget is balanced each period.

One institutional feature which must be covered to make the above argument complete involves the treatment of labor. If, for example, the only initial asset held by an individual is himself (and hence his labor), then selling this asset amounts to slavery or some form of indentured servitude. Historically, however, in Rome for example, it was possible for a slave to own property and to buy himself out of slavery. The role of human capital in economics does not lend itself to being treated in the same manner as a steel mill or die press.

With the *caveat* concerning how we treat human capital as an asset we not only arrive at the observation that there is no need for borrowing in this economy, but two further observations follow. (1) if transactions costs are zero, secured lending using the assets as the security, instead of trading them will yield an equivalent result. (2) If trade is netted out each period there is no need whatsoever for a commodity money. A unit of account would serve equally well as all the need for trust is in the clearing house which balances the books every period. The price level would be determined by the default penalty.

Model 2: Suppose that all trades have to be made in cash. In a world without trust a commodity money will be required to cover the intraperiod transactions. There will be enough of a commodity money if it is in sufficient supply to satisfy the period with the maximum amount of trade. The full meaning of "enough money" in a multistage economy has been covered in Dubey, Geanakoplos and Shubik (1990). The key element to note here is that two of the different uses of money are clearly operationally distinguished here. The transactions need is for the covering of intraperiod or short term transactions' trust and the store of value need is for balancing intertemporal trade.

2.3. Why Have Lending if Asset Trading Is Sufficient?

We have gone to some lengths to suggest that in a fully closed economy without changes in technology from the outside, there is no need for lending as asset trading will suffice. Yet the economies of the world all have highly developed loan markets with many of them dealing in secured loans. This suggests that at least five

factors go against the asset trading idealization as a close model of actual trade. They are: (1) the incompleteness of markets; (2) differences in transactions costs in various forms of trade; (3) exogenous changes in resources and production methods; (4) the treatment of humans as assets and (5) the accuracy in the evaluation of the present worth of assets.

If we merely consider the Arrow-Debreu model with exogenous assets after the first period and we rule out the selling of human capital, optimal exchange will call for loans.

2.4. A Few Crude Facts

The percentage of US homeowners with mortgages in 1982 was 57 (SAUS 1987 T823) with a mean debt of $27,147. In the age group 25-34 the percentage was 84 and the mean debt $32,266. The total mortgage debt in 1985 on 1-4 family homes was $1,469 billion; and the value of this housing stock was estimated at $4,338 billion with an average age of 24.8 for the housing(SAUS 1987, T1282). In 1983 the total number of units occupied was 84,638,000. In general the rate of interest charged is a function of the percentage of the house value the individual borrows. The usual borrowing levels for a first mortgage are several points above prime. Furthermore insurance against destruction of the house is in general required.

The number of automobiles in the United States in 1985 was 137.3 million, in that year there was $206.5 billion of automobile paper outstanding. Other consumer credit involves appliances bought on time.

The amount of corporate bonds and other long term debt in 1983 was $1,323 billion. Corporations also had $760 billion of debt due within a year and $671 of accounts payable, for a total of $2,754. The total value of retained earnings, paid in capital surplus and capital stock was ($1,274+874+787) $2,935 billion. (SAUS, 1987, T872).

At first glance it appears that only a modest fraction of lending is secured. In some instances furthermore there may be a considerable misestimate between the ascribed worth of the asset offered as security and its actual market value. The aspects of loan security do not appear to have been subject to much empirical study in the economic theory literature. Even the few crude figures given above to sweeten the intuition are deceptive in the sense that there are considerable differences in different countries. These differences merit separate study.

3. DEFAULT RULES WITHOUT EXOGENOUS UNCERTAINTY

3.1. The Exogenous Penalty

In order to model default we may modify the utility function.

Formally, without arguing the finer points we can merely modify the T period utility function replacing

$$\varphi_i(x^i_{11}, ..., x^i_{mT}) \quad \text{by} \quad \varphi_i(x^i_{11}, ..., x^i_{mT}) + \mu_i \min\left[\sum_{i=1}^{n}\sum_{j=1}^{m} p_{ji}(x^i_{ji} - a^i_{ji}),\ 0\right]$$

where for a harsh, or high enough default penalty no one will elect strategic bankruptcy. Because there will be no bankruptcy at equilibrium there is a temptation to argue that we do not need to bother with describing settlement rules and rehabilitation rules if a bankruptcy were to occur. Unfortunately the failure to do so would leave us without a well defined game. As soon as there is more than one period, it becomes necessary to fully specify how a possible bankruptcy in period 1 will influence the game in subsequent periods. We could handwave an argument for a tough enough exogenous penalty which makes strategic bankruptcy almost always undesirable. Even so when many periods must be considered the specifics of the penalties and losses of each agent must be given.

When we try to construct a specific model covering the treatment of creditors and debtors extra physical facts are obviously relevant. Common business comments such as "10 cents in the dollar" serve to indicate that assets can be wasted and the administrative aspects of settlement can be expensive. How fast and how expensive it is to settle become critical questions. Here we opt for as high a level of stylized simplicity as possible.

Before we can be specific, however two items must be discussed. The first is the general desirable properties of default settlement rules. The second is more specific. The structure of the loan market must be given and rules concerning the possibility of rollover and Ponzi games must be fully defined.

3.2. General Desirable Properties of Settlement Rules

The agents: The principal parties involved in a default settlement are the direct debtors and the creditors. However the public as a whole forms a class of (often diverse) third parties with a stake in the nature of the settlement. The administrators who make a living of running the mechanism form a party of considerable size that is often overlooked in utopian discussion.

Major factors: There are four main causal factors in default which are often difficult to sort out, but call for different considerations. They are:

(1) purposeful strategic default;

(2) default through incompetence;

(3) default through the incompetence of others, and

(4) default through bad luck.

When there is no exogenous uncertainty present only the first three are relevant.

In a world where *homo oeconomicus* makes no errors the prime goal of the default penalty is a simple Benthamite utilitarian rule that the expected disutility of the settlement or "punishment" is greater than or equal to the added utility of the default. Thus as we have noted elsewhere (Shubik and Wilson, 1977; Dubey and Shubik, 1979) a sufficiently harsh exogenous penalty is to modify the utility of the individual in a manner indicated in equation (1) where the μ is selected so that

$$\mu = \max[\lambda_1^k, \ldots, \lambda_n^k]$$

where the index k stands for the k^{th} CE of the associated exchange economy. If we select μ such that it is the largest for all k then the penalty will be sufficiently harsh that all CEs can be attained as NEs.

In a world where error occurs a new consideration enters. If an individual defaults through his or others' incompetence we may wish to avoid a punitive penalty. A way in which the penalty suggested can be cut down is to have it "personalized" so that instead of a single number μ selected as indicated above, we select a vector where each μ_i is set precisely to be the marginal value of income to each individual i. There are clear administrative reasons why this is impractical, as are noted below.

Administrative considerations: The costs of data gathering, fact finding and administration can be overwhelming. Dicken's savage parody of Jarndyce vs. Jarndyce[2] had the legal fees of a contested estate eat away the whole estate. There are at least five administrative considerations in the design of a settlement mechanism. They are:

(1) The less the administrators need to know about personal details, the better.

(2) The more available and cheap to obtain, information concerning the assets of the debtor and their location, the better.

(3) The cheaper the costs for the recapture of assets, the better.

(4) The more fungible and marketable the recaptured assets, the better.

(5) The lower the expected price variation of the assets the better.

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[2]Bleak House.

The desirability of low personal knowledge tells us that in a mass society the idea of handtailoring a bankruptcy penalty to each individual though logically possible is hardly practical. In history and currently there are only a few distinctions made in differentiating bankruptcy penalties for individuals. Basically the two distinguished individuals are natural persons and corporations.

The ease with which assets can be located depends on the disclosure required at the time of the loan and on the physical properties of the assets. It is harder to hide or abscond with a block of midtown Manhattan than a bag of uncut diamonds.

The administrative costs of taking over the possessions of a debtor can be considerable. A reason for some settlements which leave the debtor in possession is the recognition that there is more to be regained from an ongoing enterprise managed by someone who knows the business, than its corpse managed by a trustee.

The more assets there are which are fungible and easily marketable the easier it becomes to obtain an economic resolution of default. But clearly in a bankruptcy with liquidation value below the size of the debt some creditors must lose.

Assets sold off will be sold at tomorrow's prices, not yesterday's. Thus the less the period by period price changes are expected to be the easier it becomes to calculate expected liquidation values.

From the above considerations we may conclude:

PROPOSITION 1: *The poorer the society, the more economically exogenous must be the efficient default penalties.*

PROPOSITION 2: *The higher the proportion of durables and storables to consumables the more economical feasible endogenous penalties become.*

PROPOSITION 3: *The higher the administrative costs the less societally satisfactory the process.*

With these general considerations we turn to the possibility of designing specific mechanisms.

3.3. The Loan Markets and Refinancing Rules

Before the default settlement mechanism is considered the structure of markets and loan markets must be specified.

We limit the formal model to one involving only exchange. There are n trader types, m goods plus gold as money[3]. Three periods are sufficient to illustrate all new problems. Further simplification can be made by assuming that intraperiod trade may be netted out.

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[3]The model with fiat as the money follows immediately with some fairly straightforward modifications.

We rule out all futures markets. In the three period model there are only spot markets for goods and two one period loan markets and one two period loan market for money.

Figure 1 shows the full structure of the markets. It must be noted that a one period loan must include within it at least two market sessions because presumably an individual borrows to buy in today's market because he is running a deficit. He will be only able to repay *after* tomorrow's market when presumably he will run a sufficient surplus.

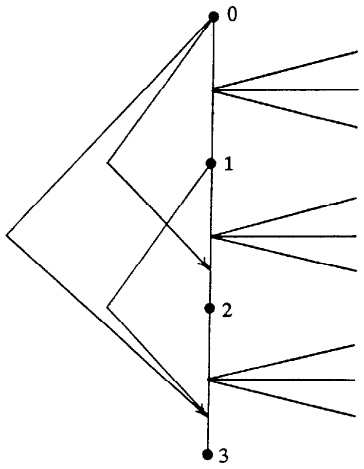

Figure 1

Three period exchange with loan markets

We first consider the economy where gold is used as money and where all other goods are assumed to be perishables. The extensive form is given in Figure 2. In the description of the game we must specify the possibility for the capturing of assets from a debtor. After he has traded does he have the opportunity to consume before or after he has settled his outstanding debt due at that time. Selecting the most extremely pessimistic possibility the extensive form shows consumption before repayment.

Suppose that we restrict ourselves to the set of games where there is enough gold for efficient trade (see Shubik and Yao, 1990; Dubey, Geanakoplos and Shubik, 1990) but that it is badly distributed. Under these circumstances it can be shown that optimal trade can be achieved with an active loan market with the rate of

interest reflecting the forgone marginal value of the utilization of gold between any two periods. Clearly an exogenous noneconomic penalty that is harsh enough will discourage strategic default.

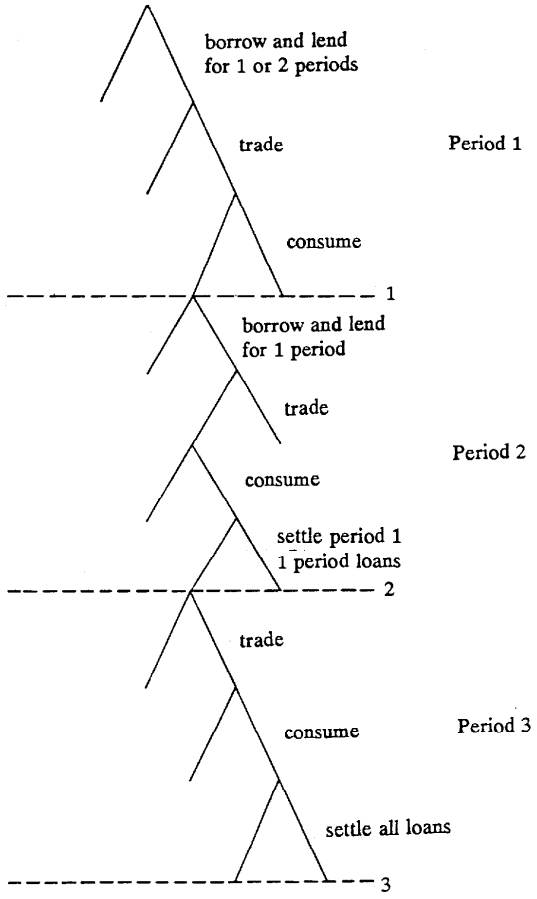

Figure 2

Extensive form of three period exchange with loans

The question to be addressed is can the penalty be made more economic in the sense that it depends only in some form on quantities of goods and prices.

The immediate answer for the game described above and related games where the number of periods of trade is an arbitrary T and all loan markets exist is as follows:

For $T = 1$ or 2 there is no economic form of default rule as there is no way creditors can reclaim consumed goods and not enough time to garnish future goods hence there will be no active loan markets and efficient trade cannot be achieved.

For $T > 2$ depending on the distribution of future endowments of consumable goods there will be robust regions of games where loan markets are active and the penalty is completely economic and of the form of the garnishing of future income and the freezing of further borrowing until loans have been repaid.

In essence the possibility for an economic penalty depends upon the ability to divert resources from the debtor when he has defaulted. In games of two periods or less as he does not default until the end of the second period it is too late to claim resources as he has none. Thus loan markets will close down for any finite game in the last two periods, but there is no backward induction unravelling. Up until the last two periods the question is, is there enough to garnish that it pays the debtor to repay rather than to opt for strategic default.

The following simple settlement rule is sufficient. If individual i is in default at time t he is excluded from the loan markets until his debt (with interest) is paid. His resources at time $t+1$ are taken and sold. Any residuum above his debt (with interest) is given to him at the end of $t+1$. If he still has debt outstanding the process is repeated at time $t+2$. The proceeds from the sale of the debtor's assets are paid out *pro rata* to the creditors in order of seniority. Thus if several loans of different length are all due simultaneously, if they all have the same seniority the debt is aggregated and the pro rata is on aggregate debt. If seniority is say, in order of length of loan with the longest loan the most senior, then the prorating is only among the senior debt holders to start with and then goes down according to seniority.

It is straightforward to construct exchange economies where optimality can be achieved without any need to borrow for the last two periods. As long as the debtors have enough assets to lose in the surviving periods of the game optimal exchange can be achieved and bankruptcy is avoided

4. FURTHER PROBLEMS

We looked only at the worst case, the introduction of assets gives the game more hostages hence increases the potential for optimal lending.

The process outlined certainly is sufficient for a game without exogenous uncertainty and errors by the players. If errors are to be expected the process although extremely simple may be unduly harsh. If the individual owes \$1 his whole array of assets for the next period is put up for sale. A variant would be to use last

period's price multiplied by a safety factor reflecting concern for the expected movement in price and ability to garnish assets.

Although the penalty is adequate for exchange without exogenous uncertainty, new difficulties arise if exogenous uncertainty is present. In particular although the settlement process is equally well defined for exchange with exogenous uncertainty, proof of existence of equilibria appears to be more difficult and we lose the clear optimality properties of the games without exogenous uncertainty. It is conjectured however that an extension of the Dubey, Geanakoplos and Shubik (1988) results is possible with the same type of optimality results obtained by introducing a parameter g whose value will be less than one which measures the "forgiveness" of the system in the sense of a willingness to settle for "g cents in the dollar."

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