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COWLES FOUNDATION DISCUSSION PAPER NO. 905

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Market Innovation and Entrepreneurship:

A Knightian View

by

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April 1989

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ABSTRACT

Stimulated by Frank Knight's work, *Risk, Uncertainty and Profit*, I present a theory of innovation based on what I term Knightian decision theory. This theory includes a concept of uncertainty aversion, a behavioral property that makes people reluctant to undertake new unevaluatable risks. This aversion is compounded when individuals are obliged to cooperate in undertaking risks. The theory leads directly to the conclusion that innovation in business is the natural domain of individual investors with unusually low levels of uncertainty aversion. Also, it should be difficult to innovate new markets for insurance of unevaluatable risks, for the success of a new market requires that many people overcome their aversion to uncertainty and enter the market.

This work has been supported by National Science Foundation Grant SES-8806730.

1. INTRODUCTION

In this paper, I use what I call Knightian decision theory to give formal expression to Frank Knight's ideas on uncertainty, insurance and entrepreneurship. This formulation of Knight's ideas has interesting implications. I also touch on related ideas of Keynes about the vagueness and instability of investor expectations.

Knight (1921) distinguished between risk and uncertainty. A gamble is risky if the probabilities of outcomes are known. If the probabilities are unknown, the gamble is uncertain. (This distinction will be maintained throughout this paper.) Knight claimed that uncertain gambles cannot be insured and that the role of the entrepreneur is to initiate uncertain investments. Investments involving only risk would be readily marketed and so would not require a special person to undertake them. These ideas are hard to interpret in terms of Bayesian decision theory, but they do make sense in the framework of Knightian decision theory.

In giving formal expression to Knight's ideas, one gains theoretical economic insights, which may have some empirical validity. For instance, in explaining why it is difficult to insure uncertain gambles, one obtains a new explanation of why markets are incomplete, which complements the usual reasons of moral hazard, adverse selection, and transactions costs. The usual explanations are convincing in many cases but seem inadequate in others where the Knightian explanation could apply. Examples are the markets for distant future delivery of standardized commodities and markets for unemployment insurance. In the theory of implicit labor contracts, there has been no good explanation of why firms do not provide at least partial unemployment insurance if they are willing to insure workers' wages.

Another insight has to do with betting. Everyday experience leads one to believe that probability assessments of some events vary widely among individuals. Such differences of opinion help explain some economic phenomena. But if decision makers are Bayesian, any two with different opinions about an observable event could gain *ex ante* by

exchanging a bet on the event. One rarely observes such bets, though, so that we are led to assume that the differences of opinion are due to differences of information, for no bets would be made if differences of opinion were due solely to differences of information (see Geanakoplos and Sebenius (1983)). But it is not always reasonable to assume that people with different opinions have different information. Knightian decision theory explains how people could have different opinions based on the same information and yet not want to exchange bets. The explanation of the absence of betting also gives a resolution of the Ellsberg paradox.

Differences of opinion, which seem natural in a Knightian framework, provide easy explanations of two puzzles in finance, the lack of diversification of individual investment portfolios and the fact that closed end mutual funds often sell at a discount.

Under the Knightian characterization, the entrepreneur is described as someone with unusual opinions or an unusually low level of uncertainty aversion. If this description were valid, the pooling and diversification of entrepreneurial risks would tend to discourage entrepreneurship. This conclusion is the opposite of that one would draw if risk aversion rather than uncertainty aversion were the principal inhibitor of entrepreneurship. The Knightian description also makes it possible to visualize a world in which waves of innovation occur in a natural way. Entrepreneurship, through innovation, creates knowledge. This knowledge in turn reduces the uncertainty about the prospects of other possible innovations. Since uncertainty inhibits innovation, reducing uncertainty tends to stimulate new innovations, which, whether successful or not, create new knowledge, and the process can feed on itself indefinitely.

Keynes (1935, 1937) had ideas about uncertainty similar to those of Knight. But unlike Knight, Keynes associated uncertainty with instability of investor expectations. In Section 16, his ideas are related to Knightian decision theory.

Knightian preferences and rational expectations make sense in mutually exclusive settings. If people could estimate probabilities precisely and unambiguously, then these

probabilities would be the only subjective probabilities of a Knightian decision maker, just as they would be for a Bayesian decision maker. If one admits that in economic life people often cannot estimate probabilities precisely, then it is natural to explore the consequences for intertemporal general equilibrium theory of replacing rational by Knightian expectations. This subject is only touched on here. An example is given which shows that if expectations are not rational, then traders may not be able to learn future spot markets prices through forward trading, even if markets are complete. Since people can trade on spot markets in the future, their expectations of future spot prices can so modify their demands and supplies on forward markets that forward prices have no value as predictors of future spot prices. This example is given to justify the assumption that people do not know future spot prices as a function of exogenous states of nature.

This paper is part of a program of research exploring the economic implications of Knightian decision theory. This program is undertaken in the spirit of experimentation. It is not claimed that Knightian decision theory represents a good approximation to reality. However, it would be encouraging if Knightian decision theory explained a number of otherwise mysterious economic phenomena. That this decision theory is certainly a fallible guide to human behavior is clear, because it does not differ from the expected utility hypothesis in the case of choice among risky alternatives, and experimental work finds many divergences from the predictions of that hypothesis (see Schoemaker (1982)). But it is too much to expect any theory of rationality to give correct predictions in all cases for all individuals, since human behavior is so clearly erratic. All that is needed for economic theory is a theory of behavior that is roughly correct most of the time. The expected utility hypothesis is certainly not the only reasonable theory of rational choice under uncertainty, and many economic phenomena are difficult to explain using that hypothesis. It therefore seems worthwhile to experiment with other definitions of rationality in the hope of finding one that yields simple economic models giving a better fit to reality than the ones we now have.

The experiment presented here should not be seen as an attempt to interpret Knight's work, but rather as an attempt to develop ideas suggested by his work in order to gain economic insights.. LeRoy and Singell (1987) have argued that by uncertainty Knight meant "situations in which insurance markets collapse because of moral hazard or adverse selection" (quoted from their abstract). They may be right. Knight's work is too informal to permit a precise interpretation. His work is nevertheless very stimulating.

The paper is organized as follows. Section 2 contains a review of Knightian decision theory. Betting and the Ellsberg paradox are discussed in Sections 3 and 4, respectively. The discussion of insurance is contained in Sections 6–8. Futures markets are discussed in Sections 9–11. Section 11 contains an example showing that forward prices may be bad predictors of future spot prices. Portfolio diversification and closed end mutual funds are discussed in Section 12. Sections 13–15 contain the discussion of entrepreneurship. Keynes' ideas on the instability of investors' expectations are discussed in Section 16.

2. OUTLINE OF KNIGHTIAN DECISION THEORY

I here describe briefly the main ideas of the Knightian decision theory proposed in a previous paper (1986). One modifies the usual axiomatic basis of Bayesian decision theory by eliminating the assumption that preferences over lotteries are complete and by adding a hypothesis called the inertia assumption.

Dropping the completeness assumption replaces the single subjective distribution of Bayesian theory with a convex family of distributions, denoted by Π . One lottery is preferred to another if and only if it has higher expected utility according to all distributions in Π . That is, if S is a state space with measurable subsets \mathcal{S} and if $x : S \rightarrow (-\infty, \infty)$ and $y : S \rightarrow (-\infty, \infty)$ are lotteries, measurable with respect to \mathcal{S} , then x is preferred to y if and only if $\int u(x(s), s)\pi(ds) > \int u(y(s), s)\pi(ds)$, for $\pi \in \Pi$, where $u(z, s)$ is the utility of payoff z in state s and each $\pi \in \Pi$ is a probability measure on \mathcal{S} . The multiplicity of the subjective distributions, π , expresses ignorance of the true distribution. It is assumed

that if the probability of an event is known, then its subjective probability equals the known probability, for all $\pi \in \Pi$. Thus, the theory does not contradict that of von Neumann and Morgenstern. New information is incorporated by conditioning or applying Bayes' rule to each of the distributions $\pi \in \Pi$, so that learning is treated in the Knightian theory just as it is treated in the Bayesian theory. If a sequence of observations would identify asymptotically the true stochastic mechanism governing the environment, then as observations accumulated the set of subjective distributions, Π , would shrink down to the true distribution.

In the Knightian theory, one cannot say that lottery x is revealed to be preferred to lottery y if x is chosen when y is available. When preferences are incomplete, one can say only that y is revealed not to be preferred to x .

The inertia assumption says, roughly, that x is revealed to be preferred to y if x is chosen when y is available and y is the *status quo*. In decision theoretic language, the *status quo* is defined to be planned behavior or the full contingent program defined for the whole decision tree. The inertia assumption is that the *status quo* is abandoned only when new alternatives become available that are preferred to it. Also, the new alternative adopted must be preferred to the *status quo*. An alternative is defined to be new if the decision maker had never thought that it might become available and so had not incorporated it in his plans. A new alternative is said to be acceptable if it is preferred to the *status quo*. The *status quo* is in the decision maker's head and so has the same logical standing as a preference ordering.

It is not obvious how to relate the inertia assumption to observed behavior. Inertia is defined in terms of a decision tree and a program for that tree, and yet people clearly do not formulate such programs unless they have been trained to do so when solving certain specific problems. Nevertheless, for mathematical convenience and because in economic matters people do seem often to act fairly rationally, we model people as adopting such programs. But even if we assume that a person acts as if he followed a full contingent

program for a decision tree, we do not know precisely how the tree and program are formulated. It is this one needs to know when applying the inertia assumption. This ambiguity is the chief theoretical weakness of Knightian decision theory, I believe. It arises because the inertia assumption applies to those circumstances where the decision tree model makes the least sense as a model of behavior. In applications, the *status quo* is probably best thought of as routines or rules used for guiding behavior in typical situations.

The inertia assumption and the multiplicity of subjective distributions lead to a notion of uncertainty aversion. A decision maker with a fat set of subjective distributions would be reluctant to abandon the *status quo* and so can be said to be averse to uncertainty. Uncertainty aversion is distinct from risk aversion. Also, uncertainty aversion does not involve aversion to uncertainty itself but to movement from the *status quo*. The *status quo* might involve the possibility of great loss. But if the probability of the loss were not known, an uncertainty-averse decision maker would be reluctant to move from the *status quo* to an alternative involving the possibilities of both smaller losses and smaller gains.

Except when choosing between the *status quo* and new alternatives, a Knightian decision maker acts just like a Bayesian one. That is, he chooses a program that is optimal with respect to one of his subjective distributions. This assertion follows from the natural assumption that a Knightian decision maker adopts a program that is undominated in the sense that no other program is preferred to it. If one allows randomization over programs, then it follows from Minkowski's separation theorem that an undominated program is optimal with respect to some $\bar{\pi} \in \Pi$. This $\bar{\pi}$ will be referred to as the decision maker's opinion. The opinion is revealed by the decision maker's choice of program. It is, however, not a part of his preference ordering. Knightian decision theory has nothing to say about the selection of the opinion, which as an unconditional probability distribution remains unaltered as long as the decision maker sticks with his *status quo* program. However, new information does affect his opinion through conditioning. At any time, the decision maker's opinion may be viewed as his original opinion, conditioned on the information then

available. In fact, all the probability distributions $\pi \in \Pi$ are updated through conditioning as new information becomes available.

If the decision maker abandons the *status quo* in favor of a new opportunity, then he may change his opinion, as the following example illustrates. There are two states, *a* and *b*. The variables x_a and x_b are the utility payoffs in states *a* and *b*, respectively. The original choice set is *OAB*. The *status quo* is *C*. The cone bounded by *cCc'* is assumed to be the set of points preferred to *C*. The original opinion is $\pi = (\pi_a, \pi_b)$. The new alternative is *D* and the new choice set is *OADB*. The point *D* is the optimal point in *OADB* according to π , but *D* is not preferred to *C*. By the inertia assumption, a point such as *C'*, preferred to *C*, will be chosen. The new opinion is, therefore, the probability vector, π' , perpendicular to the line *DB*.

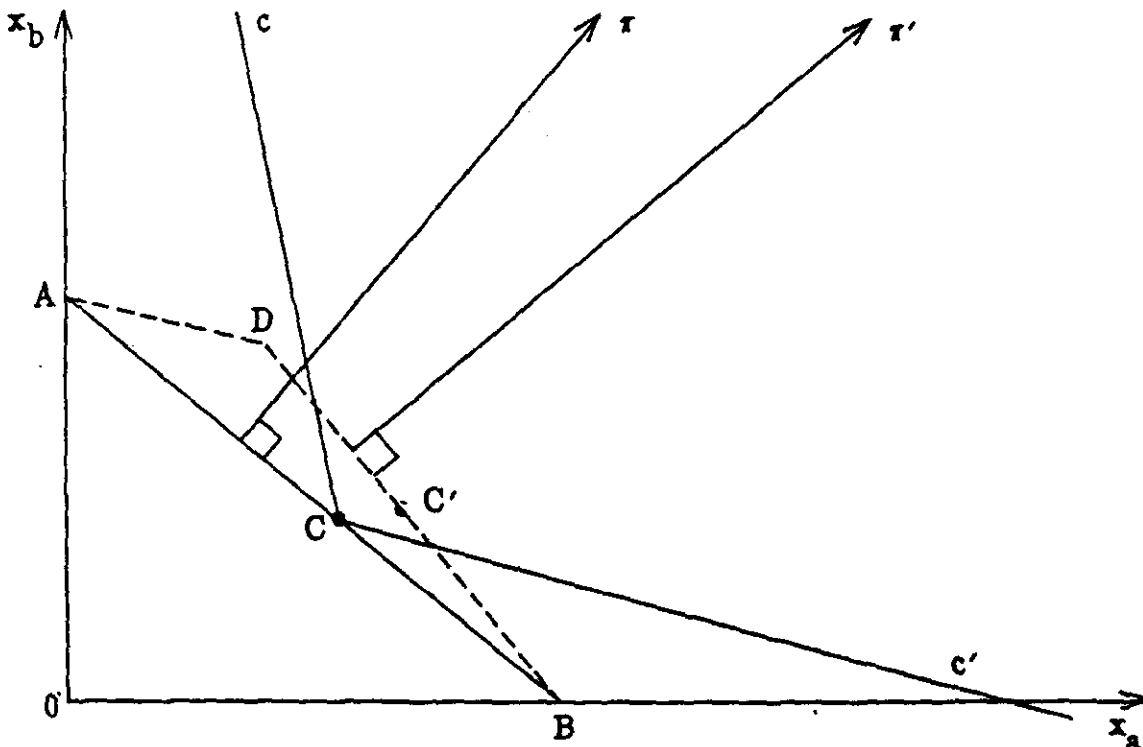


Figure 1

New alternatives to the *status quo* may be presented as a surprise by the external environment or may be things the decision maker thinks of himself. Knightian decision theory is not intended to model the entire thinking apparatus of the decision maker. If one were to view the decision maker as boundedly rational, then it would be possible for him to perceive new opportunities that had previously been present but had gone unnoticed. Knightian decision theory itself does not require that the decision maker be boundedly rational. Preference incompleteness does not imply irrationality nor any bound on computational capacity.

3. THE ABSENCE OF BETTING

Knightian decision theory yields an easy explanation of the infrequency of betting on events of unknown probability. The explanation may be grasped easily by imagining two individuals, 1 and 2, who may exchange a bet on event A of unknown probability. Suppose the individuals each attach an interval of probabilities to the event, this interval being $[\underline{\pi}_n, \bar{\pi}_n]$, for individual n , for $n = 1, 2$. Consider a bet according to which individual 1 receives b dollars if A occurs and pays one dollar if A does not occur. Think of the bet as small, so that risk aversion may be ignored, and suppose that the event does not affect the personal fortunes of the individuals, so that the value of money to them does not depend on the occurrence of A . Next assume that for both individuals the possibility of making a bet on A comes as a surprise or is not part of their cultural habits. More formally, assume that their *status quo* programs do not cover the possibility of making such a bet. Then, individual 1 will make the bet if and only if $b\pi - (1-\pi) > 0$ or $\pi > (1+b)^{-1}$, for all $\pi \in [\underline{\pi}_1, \bar{\pi}_1]$. Similarly, individual 2 will make the bet if and only if $-b\pi + (1-\pi) > 0$ or $\pi < (1+b)^{-1}$, for all $\pi \in [\underline{\pi}_2, \bar{\pi}_2]$. Thus, the bet would be acceptable to both individuals only if $\bar{\pi}_2 < (1+b)^{-1} < \underline{\pi}_1$. One sees that a bet at some odds would be acceptable to both if and only if the intervals $[\underline{\pi}_1, \bar{\pi}_1]$ and $[\underline{\pi}_2, \bar{\pi}_2]$ do not overlap. If the intervals overlapped, no betting would occur.

Even if the intervals did overlap, the opinions of the individuals could differ, the opinion of each being an arbitrary point in his interval. The opinions would rationalize those decisions covered by the *status quo* program. Thus, the individuals might express widely differing views on the likely outcome of some political situation and yet not be willing to exchange a bet on the outcome.

The geometry of the argument may be seen in the Edgeworth box diagram below. The horizontal axes are the payoffs in the event A . The vertical axes are the payoffs in the complementary event, B , and the cone K_n , for $n = 1, 2$, is $\{(x_A, x_B) \mid \pi x_A + (1-\pi)x_B \geq w_n, \text{ for all } \pi \in [\underline{\pi}_n, \bar{\pi}_n]\}$, where w_n is the initial wealth of individual n . If $[\underline{\pi}_1, \bar{\pi}_1]$ and $[\underline{\pi}_2, \bar{\pi}_2]$ overlap, the cones K_1 and K_2 do not intersect and no betting occurs.

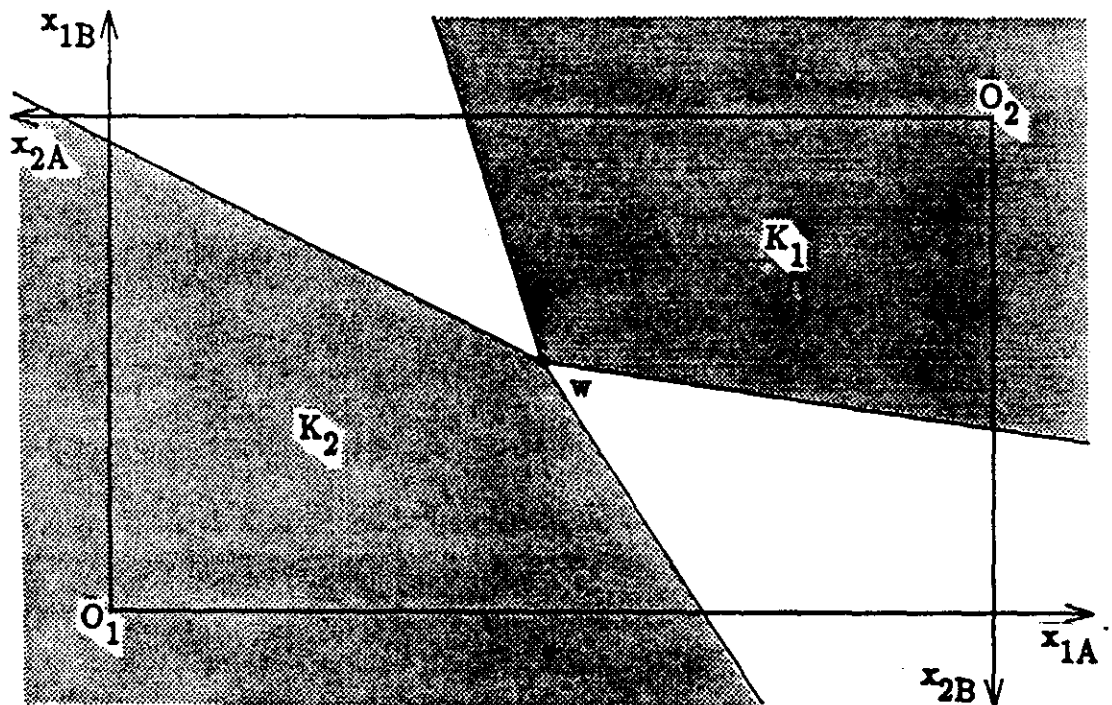


Figure 2

The argument is easily generalized to the case of many events and individuals. Let \mathcal{A} be a finite collection of events in a state space, S . The events in \mathcal{A} need not be disjoint and need not cover S . Let the set of subjective probability distributions of individual n over S be Π_n and let $\Pi_n^{\mathcal{A}} = \{(\sum_{A' \in \mathcal{A}} \pi(A'))^{-1}(\pi(A)) \mid \pi \in \Pi_n\}$. A sys-

tem of bets or events in \mathcal{A} is a set of functions $x_n : \mathcal{A} \rightarrow (-\infty, \infty)$, for $n = 1, 2, \dots, N$, such that $\sum_{n=1}^N x_n = 0$. The system is acceptable if for all n , $\sum_{A \in \mathcal{A}} \pi(A) x_n(A) > 0$, for all $\pi \in \Pi_n^{\mathcal{A}}$.

3.1) Proposition. There is an acceptable system of bets if and only if $\bigcap_{n=1}^N \Pi_n^{\mathcal{A}} = \phi$.

Proof. Let $R^{\mathcal{A}}$ be the set of functions $x : \mathcal{A} \rightarrow (-\infty, \infty)$. For each individual n , let $K_n = \{x \in R^{\mathcal{A}} \mid \sum_{A \in \mathcal{A}} \pi(A) x(A) > 0, \text{ for all } \pi \in \Pi_n^{\mathcal{A}}\}$. Let $K = \{x \in \bigcap_{n=1}^N (K_n \cup \{0\}) \mid \sum_{n=1}^N x_n = 0\}$. There is an acceptable system if and only if $K \neq \{0\}$. Clearly, if $\bigcap_{n=1}^N \Pi_n^{\mathcal{A}} \neq \phi$, then $K = \{0\}$. Let $X = \sum_{n=0}^N (K_n \cup \{0\})$. If $K = \{0\}$, then $X \cap R_-^{\mathcal{A}} = \{0\}$, where $R_-^{\mathcal{A}} = \{x \in R^{\mathcal{A}} \mid x(A) \leq 0, \text{ for all } A\}$. Let $\pi \in R^{\mathcal{A}}$ separate X from $R_-^{\mathcal{A}}$. Normalize π so that $\sum_{A \in \mathcal{A}} \pi(A) = 1$. Then $\pi \in \bigcap_{n=1}^N \Pi_n^{\mathcal{A}}$, so that $\bigcap_{n=1}^N \Pi_n^{\mathcal{A}} \neq \phi$. \square

If the events in \mathcal{A} are uncertain and disjoint and if individuals are uncertainty averse, then the condition $\bigcap_{n=1}^N \Pi_n^{\mathcal{A}} \neq \phi$ is plausible, for each set $\Pi_n^{\mathcal{A}}$ is of full dimension in the convex set $\{(\sum_{A' \in \mathcal{A}} \pi(A'))^{-1} (\pi(A))_{A \in \mathcal{A}} \mid \pi \text{ is a probability distribution over } S\}$.

One might well ask why one needs an explanation for the absence of betting when gambling is so common. Gambling at known odds, as in casinos, seems bizarre from the standpoint of both the Bayesian and Knightian decision theories. Because the house takes a share, the expected value of such gambling is negative. One must conclude that rational gamblers either are risk lovers or love the excitement of gambling. In some gambling, such as horse racing, the probability of winning is unknown, so that gamblers can legitimately imagine that their bets have positive expected value, even though the house or track gets a share of the bets. From the Knightian viewpoint, such betting is simply part of the *status quo*, and individuals bet because their opinions differ. From the Bayesian point of view,

one must assume that the bettors have different prior distributions over the horses' prospects. If they had the same priors and different information, then no betting should occur. Thus, betting on horse races by risk averse people would seem to contradict the common assumption that different individuals have the same priors.

Proposition 3.1 should be thought of as expressing resistance to innovation in betting. It is meant to answer the common argument that individuals in contact who have the same information should have the same subjective distributions over observed events, for if they did not, they would exchange bets, and such betting is not common. Proposition 3.1 is not contradicted by the existence of institutionalized betting, though one does wonder how it could have gotten started in a world of rational people. One can only hope that in the context of serious economic decision making people behave more sensibly.

4. THE ELLSBERG PARADOX

The argument of the previous section gives an explanation of the Ellsberg paradox (Ellsberg (1961, 1963)). Ellsberg observed experimentally that people prefer a lottery giving a dollar with probability one half to a lottery giving a dollar with completely unknown odds. According to both the Knightian and Bayesian points of view, the first lottery is worth fifty cents (or slightly less if one allows for risk aversion). If a Bayesian applies the principle of insufficient reason to the second lottery, then it is also worth fifty cents to him—hence the paradox.

In the Knightian framework, the paradox disappears if one assume that betting at known odds is covered by the *status quo* program, whereas the bet at unknown odds is not. It is easy to imagine that betting at known odds would be covered by the *status quo*, for a simple rule determines the optimal choice—maximize expected utility. With these assumptions about the *status quo*, the choice of the lottery with known odds does not reveal a preference for that lottery. It simply indicates that the individual probably had difficulty comparing the two lotteries and found the lottery with known odds familiar.

One might try to modify the Ellsberg experiment so as to make the lottery with unknown odds the *status quo*, but it is hard to imagine how one could do so. One could give the lottery with unknown odds to the subject or make him acquire it by offering him the choice between it and, say, five cents. According to the formal definition of the *status quo*, possession of the lottery could then become part of the *status quo*. But in fact one would not know how the lottery fitted into the rules of behavior which constitute the loose interpretation of the *status quo*. These rules might not deal with such a lottery at all, even after one was acquired. If one succeeded in incorporating the lottery with unknown odds into the *status quo*, one could ask the subject to exchange this lottery for the one with known odds. If this offer were unanticipated, the subject should retain the lottery with unknown odds. But it would be hard to make this offer be totally unanticipated, especially if the *status quo* contained rules of behavior which covered the offer.

5. A SIMPLISTIC TREATMENT OF INSURANCE

An easy generalization of the results of Section 3 provides a simplistic explanation of the absence of markets for the insurance of uncertain events. Let the set of states S , be finite. For each $s \in S$, let $u_n(x, s)$ be the utility of individual n for money held in state s , where $n = 1, \dots, N$. Assume that for all n and s , $u_n(\cdot, s)$ is increasing, concave and differentiable. Let ω_{ns} be the monetary endowment of individual n in state s and assume that $\omega_{ns} > 0$, for all n and s . Let Π_n be the set of subjective distributions over S of trader n and let \mathcal{A} be a collection of subsets of S of unknown probability. An \mathcal{A} -trade for individual n is a function $x : \mathcal{A} \rightarrow (-\infty, \infty)$ such that $\omega_n(s) + x(A) \geq 0$, for all $A \in \mathcal{A}$ and $s \in A$. An \mathcal{A} -reallocation is a vector $x = (x_1, \dots, x_N)$, where x_n is an \mathcal{A} -trade for individual n . The \mathcal{A} -reallocation is feasible if $\sum_{n=1}^N x_n(A) = 0$, for all $A \in \mathcal{A}$. If x_n is an \mathcal{A} -trade for individual n , let $(\omega_n + x) : S \rightarrow [0, \infty)$ be defined by $(\omega_n + x)(s) = \omega_n(s) + \sum \{x(A) : A \in \mathcal{A} \text{ and } s \in A\}$. Assume that the *status quo* of each individual does

not cover trade in contracts contingent on events in \mathcal{A} . Then, trade in such contracts will occur only if there exists an acceptable \mathcal{A} -reallocation, x , where $x = (x_1, \dots, x_N)$ is acceptable if, for all n , $\sum_{s \in S} \pi(s)[u_n(x + \omega_n)(s), s) - u_n(\omega_n(s), s)] > 0$, for all $\pi \in \Pi_n$.

Let $\lambda_{ns} = \frac{d}{dx} u_n(\omega_n(s), s)$, for all s and n and define $\Pi_n^{\mathcal{A}, \lambda} = \left\{ \left(\sum_{A' \in \mathcal{A}} \sum_{s \in A'} \pi(s) \lambda_{ns} \right)^{-1} \times \left(\sum_{s \in A} \pi(s) \lambda_{ns} \right) \mid \pi \in \Pi_n \right\}$. Proposition 3.1 generalizes easily to the following.

5.1) Proposition. There exists an acceptable feasible \mathcal{A} -allocation if and only if

$$\bigcap_{n=1}^N \Pi_n^{\mathcal{A}, \lambda} = \phi.$$

The geometric intuition behind this proposition may be seen in the Edgeworth box diagram below, for the case with two states, a and b , and with $\mathcal{A} = \{a\} \cup \{b\}$. The sets K_n are now defined to be $\{(y_a, y_b) \mid \pi(a)u_n(y_a, a) + \pi(b)u_n(y_b, b) > 0, \text{ for all } \pi \in \Pi_n\}$, for $n = 1, 2$.

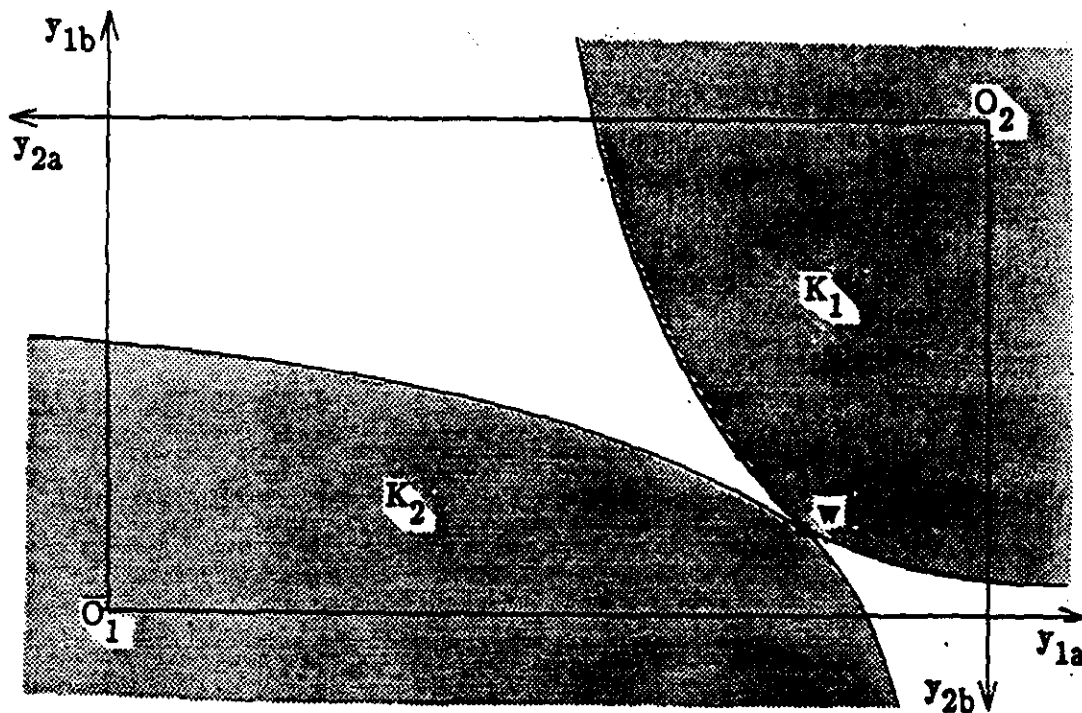


Figure 3

If the conditions of the proposition hold, then there would be no trade when markets for insurance of events in \mathcal{A} were introduced, so that the market innovation would fail.

6. INSURANCE WITH GILBOA-SCHMEIDLER PREFERENCES

Gilboa and Schmeidler (1986) have given an axiomatic characterization of a class of preferences which yield a form of uncertainty aversion related to but distinct from that of Knightian decision theory. Their class of preferences explains the rarity of betting on uncertain events and explains the Ellsberg paradox, but does not explain the difficulty of insuring uncertain events. In fact, their class of preferences makes insurance even more likely than does the usual Bayesian or Savage class of preferences.

Using the notation of the previous section, a Gilboa-Schmeidler utility function, defined on vectors $x:S \rightarrow [0, \infty)$ of money holdings, is of the form $U(x) = \min_{s \in S} \{ \sum \pi(s)u(x(s), s) \mid \pi \in \Pi \}$. The utility function shows aversion to uncertainty if Π contains more than one point. Because the preference ordering represented by U is complete, an inertia assumption is not needed in order to define uncertainty aversion. Roughly speaking, an uncertainty averse individual with Gilboa-Schmeidler preferences wants to equate the level of utility in different states. Individuals with Bayesian or Savage preferences wish simply to equate the marginal utilities of money in different states.

If individuals have Gilboa-Schmeidler preferences, then the more uncertainty averse they are, the more they will want to trade in insurance. This fact is illustrated in the following Edgeworth box diagram, with the notation the same as in Figure 3. The shaded area represents the area of mutually advantageous trade.

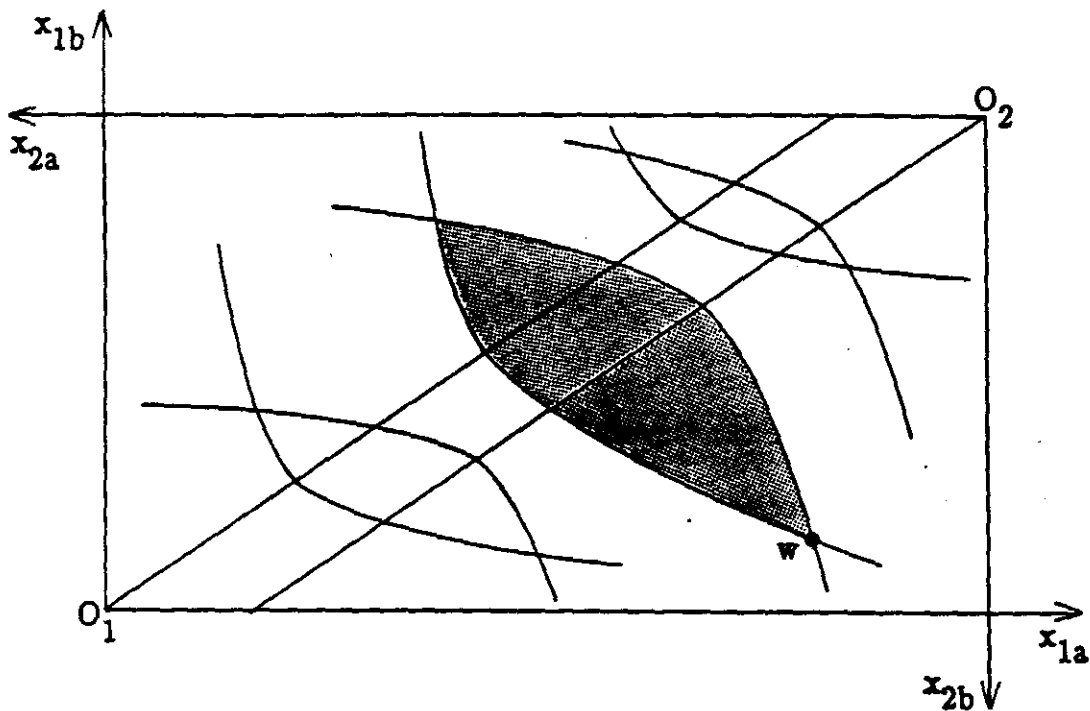


Figure 4

7. COUNTEREXAMPLES

According to proposition 5.1, there is no trade on new markets for contracts on events in \mathcal{A} if

$$7.1) \quad \bigcap_{n=1}^N \Pi_n^{\mathcal{A}, \lambda} = \phi.$$

This condition is not plausible if there is an event of known probability which is a union of finitely many events in \mathcal{A} and is not the union of all the events in \mathcal{A} . Condition 7.1 is not even a sufficient condition for no trade if it is possible to trade in insurance on events not in \mathcal{A} .

If one tries to generalize proposition 5.1 to an intertemporal model, one finds that there are obvious dated events of known measure, namely, the dated event that a particular period, t , occurs. Also, if one allows borrowing and lending, one must allow trade in

contracts conditional on particular dates. Thus, in an intertemporal model condition 7.1 is neither plausible nor sufficient for no trade.

In this section, an intertemporal model is described and an example is given showing that condition 7.1 does not exclude trade in insurance if borrowing and lending are possible. There then follows an example showing that condition 7.1 need not hold if there is an event of known probability which is a finite union of events in \mathcal{A} .

An intertemporal model may be described as follows. There are T periods. The set of states of the world is finite and is still denoted by S . The partition of S into events occurring at time t is \mathcal{S}_t . For each t , \mathcal{S}_{t+1} is a refinement of \mathcal{S}_t . Also, $\mathcal{S}_T = \{\{s\} | s \in S\}$, and $\mathcal{S}_1 = \{S\}$. Hence, the partitions $\mathcal{S}_1, \dots, \mathcal{S}_T$ form a dated event tree with vertices $\Gamma = \{(t, A) | 1 \leq t \leq T \text{ and } A \in \mathcal{S}_t\}$. There are N individuals. Individual n , for $n = 1, \dots, N$, has a set, Π_n , of subjective probabilities over S . The opinion of individual n is denoted by $\bar{\pi}_n$. The utility of individual n in period t and event $A \in \mathcal{S}_t$ is $u_{nt}(\cdot, A)$, assumed to be differentiable, increasing and concave. An allocation (of money) to an individual is a function $y : \Gamma \rightarrow [0, \infty)$. The utility of an assignment y to individual n , evaluated according to $\pi \in \Pi_n$ is
$$\sum_{t=1}^T \sum_{A \in \mathcal{S}_t} \pi(A) u_{nt}(y_{(t,A)}, A).$$

Individual n 's initial allocation is denoted ω_n and is assumed to be positive in every component. The initial marginal utility of money of individual n at $(t, A) \in \Gamma$ is
$$\lambda_{n,(t,A)} = \frac{d}{dx} u_{nt}(\omega_{n,(t,A)}, A).$$

Any probability distribution π on S induces a probability measure π_Γ on subsets G of Γ , defined by
$$\pi_\Gamma(G) = T^{-1} \sum_{(t,A) \in G} \pi(A).$$
 The notation π_Γ will have this interpretation throughout the rest of the paper. A subset G of Γ is called uncertain if $\pi_\Gamma(G)$ varies as π varies over Π_n , for some n . In what follows, \mathcal{A} denotes a collection of disjoint subsets of Γ of unknown measure and \mathcal{B} denotes a partition of Γ into sets of known measure. The field generated by \mathcal{B} includes the sets $G_t = \{(t, A) | A \in \mathcal{S}_t\}$, for $t = 1, \dots, T$, for these each have measure T^{-1} . It is assumed that there exist markets

for contingent claims on each of the sets in B and that markets are proposed for the sets in A . Also, the *status quo* of each individual does not cover trade in contracts contingent on sets in A . If C is a collection of disjoint subsets of Γ , a C -trade, a C -reallocation, and an acceptable C -reallocation are all defined as in Section 5. Let

$$\Pi_n^{A,\lambda} = \left\{ \left(\sum_{G' \in A} \sum_{(t,A) \in G'} \pi(A) \lambda_{n,(t,A)} \right)^{-1} \left(\sum_{(t,A) \in G} \pi(A) \lambda_{n,(t,A)} \right)_{G \in A} \mid \pi \in \Pi \right\}.$$

Proposition 5.1 generalizes without difficulty to the above intertemporal model.

That is, there is an acceptable A -allocation if and only if $\bigcap_{n=1}^N \Pi_n^{A,\lambda} = \phi$. However, one

cannot conclude from this statement that no trade in contracts on sets in A would occur if

$\bigcap_{n=1}^N \Pi_n^{A,\lambda} \neq \phi$, for trade in contracts on sets in A could induce changes in positions on the

markets for contracts contingent on sets in B . The interesting question is whether there

exists an acceptable $(A \cup B)$ -reallocation when all individuals start in equilibrium on the

markets for contracts contingent on events in B . The next example shows that there may

exist such an allocation even if $\bigcap_{n=1}^N \Pi_n^{A,\lambda} \neq \phi$.

7.2) Example. There are two periods and two states, labelled unemployed (U) and employed (E). Let $S = \{U, E\}$. There are two traders, a worker, W , and a firm, F .

The subjective probability interval for state U is $\left[\frac{1}{8}, \frac{1}{2} \right]$ for both traders. The opinion of each is that the probability of U is $\frac{1}{3}$. The marginal utilities of money of the firm are

$\lambda_{F,(1,S)} = \lambda_{F,(2,E)} = \lambda_{F,(2,U)} = 1$. The marginal utilities of money for the worker are

$\lambda_{W,(1,S)} = 1$, $\lambda_{W,(2,E)} = \frac{1}{2}$, and $\lambda_{W,(2,U)} = 2$. The sets of A are $\{(2,E)\}$ and

$\{(2,U)\}$. That is, a market for unemployment insurance is proposed. The sets in B are

$\{(1,S)\}$ and $\{(2,E), (2,U)\}$. That is, there exist perfect markets for loans.

It is easy to see that both traders are in equilibrium in the market for loans before trade in contracts on events in A occurs. This fact is seen by verifying the equation

$\lambda_{n,(1,S)} = \bar{\pi}_U \lambda_{n,(2,U)} + (1 - \bar{\pi}_U) \lambda_{n,(2,E)}$, for $n = F, W$, where $\bar{\pi}(U) = \frac{1}{3}$ is the traders' opinion about the probability of U .

In this example, $\Pi_F^{\mathcal{A},\lambda} = \left\{ (\pi_U, 1-\pi_U) \mid \frac{1}{8} \leq \pi_U \leq \frac{1}{2} \right\}$ and $\Pi_W^{\mathcal{A},\lambda} = \left\{ (\pi_U, 1-\pi_U) \mid \frac{4}{11} \leq \pi_U \leq \frac{4}{5} \right\}$, so that $\Pi_F^{\mathcal{A},\lambda} \cap \Pi_W^{\mathcal{A},\lambda} \neq \emptyset$ and there does not exist an acceptable \mathcal{A} -reallocation. However, there does exist an acceptable $(\mathcal{A} \cup \mathcal{B})$ -reallocation, if the traders' utility functions for money are concave and differentiable and if their endowments of money are positive in every dated event. Suppose that the worker borrows $\epsilon > 0$ dollars from the firm in period 1 for repayment in period 2 and that the worker makes a contract with the firm giving the firm ϵ dollars in event E in exchange for $\frac{3}{4}\epsilon$ dollars in event U . The net change in position for the worker is $x_W = (x_{W,(1,S)}, x_{W,(2,E)}, x_{W,(2,U)}) = \epsilon \left[1, -2, -\frac{1}{4} \right]$, and that of the firm is $x_F = -x_W$. Up to an infinitesimal of order $o(\epsilon)$, the change in utility for the firm is $x_{W,(1,S)}\lambda_{W,(1,S)} + x_{W,(2,E)}\lambda_{W,(2,E)}(1-\pi_U) + x_{W,(2,U)}\lambda_{W,(2,U)}\pi_U = \frac{1}{2}\epsilon\pi_U > 0$, when evaluated at any $\pi_U \in \left[\frac{1}{8}, \frac{1}{2} \right]$. Similarly, the change in the firm's utility is approximately $x_{F,(1,S)}\lambda_{F,(1,S)} + x_{W,(2,E)}\lambda_{W,(2,E)}(1-\pi_U) + x_{W,(2,U)}\lambda_{W,(2,U)}\pi_U = \epsilon \left[1 - \frac{7}{4}\pi_U \right] > 0$, for all $\pi_U \in \left[\frac{1}{8}, \frac{1}{2} \right]$. It follows that for ϵ sufficiently small, the trades x_W and x_F lead to an acceptable $(\mathcal{A} \cup \mathcal{B})$ -reallocation. The worker's ability to insure against unemployment makes it desirable for him to borrow.

One could have made the previous example a one period example and have replaced the first period by an event of known probability. Then, it would prove possible to introduce unemployment insurance because it would induce changes in positions on preexisting markets for insurance.

The next example shows how condition 7.1 may fail if unions of events in \mathcal{A} are of known probability.

7.3) Example. There is one period and there are two traders, labelled a and b, and four states, labelled 1 through 4. The set of subjective probabilities of both traders is $\Pi = \left\{ (\pi_1, \pi_2, \pi_3, \pi_4) \mid \pi_2 = \frac{1}{2} - \pi_1, \pi_4 = \frac{1}{2} - \pi_3, \frac{1}{14} \leq \pi_1 \leq \frac{1}{4} \text{ and } \frac{1}{14} \leq \pi_3 \leq \frac{1}{4} \right\}$. The marginal utilities of money for trader a are $(\lambda_{a1}, \lambda_{a2}, \lambda_{a3}, \lambda_{a4}) = \left[1, 1, 4, \frac{1}{2} \right]$ and those for trader b are $(\lambda_{b1}, \lambda_{b2}, \lambda_{b3}, \lambda_{b4}) = \left[4, \frac{1}{2}, 1, 1 \right]$. Let $\mathcal{A} = \{\{1\}, \{2\}, \{3\}, \{4\}\}$.

Condition 7.1 fails, that is, $\Pi_a^{\mathcal{A}, \lambda} \cap \Pi_b^{\mathcal{A}, \lambda} = \phi$, because if $\alpha = (\alpha_1, \alpha_2, \alpha_3, \alpha_4) \in \Pi_a^{\mathcal{A}, \lambda}$, then $\alpha_1 \alpha_2^{-1} \leq 1$, whereas if $\alpha \in \Pi_b^{\mathcal{A}, \lambda}$, then $\alpha_1 \alpha_2^{-1} \geq \frac{4}{3}$. Hence, by proposition 5.1, it could be possible to introduce insurance on the events in \mathcal{A} .

This example shows that it could be easier to introduce insurance on uncertain events if insurance were offered on all intersections of events of unknown probability with events which were already insurable. Let \mathcal{A}' be the partition $\{\{1,3\}, \{2,4\}\}$ and let B' be the partition $\{\{1,2\}, \{3,4\}\}$. The events in \mathcal{A}' are uncertain and those in B' are of known probability. Suppose that events in B' are insurable and that insurance on events in \mathcal{A}' is introduced. Let the opinion of person a be $(\pi_1, \pi_2, \pi_3, \pi_4) = \left[\frac{1}{4}, \frac{1}{4}, \frac{1}{14}, \frac{6}{14} \right]$ and let the opinion of person b be $\left[\frac{1}{14}, \frac{6}{14}, \frac{1}{4}, \frac{1}{4} \right]$. With these opinions, both people are in equilibrium in the markets for insurance of events in B' . Also these opinions give rise to the vector $(v_{\{1,3\}}, v_{\{2,4\}}, v_{\{1,2\}}, v_{\{3,4\}}) = \frac{1}{62}(15, 19, 14, 14) \in \Pi_a^{\mathcal{A}' \cup B', \lambda} \cap \Pi_b^{\mathcal{A}' \cup B', \lambda}$, so that by proposition 5.1 no trade would occur if there were markets for event only in $\mathcal{A}' \cup B'$. However, if insurance were introduced on events in $\mathcal{A} = \mathcal{A}' \vee B'$, then we know that trade would occur. ($\mathcal{A}' \vee B'$ is the partition made up of events which are intersections of events in \mathcal{A}' and B' .)

8. EXPLANATIONS OF THE ABSENCE OF INSURANCE

In this section, I describe two approaches to a Knightian explanation of the uninsurability of uncertainty. The explanations are made in terms of the model of the previous section, with markets existing for contracts on the events of known measure in the partition B and with markets proposed for contracts on the sets of unknown measure in A . One approach is to assume that, for practical reasons, the new contracts are contingent on sets only in $A \cup B$ and not on those of $A \vee B$. This assumption avoids example 7.3, though not example 7.2. One can show that trade in the new markets will be proportional to the changes in position in old markets induced by the opening of the new markets. The other approach is to assume that the new markets are for contracts contingent on all the events in $A \vee B$, but that the events in A and B are economically and statistically independent. Once again, one finds that trade on the new markets would be proportional to the induced changes in trading on the old markets.

If one believes that the induced changes in trade on the old markets would be small, then one concludes that there probably would be little interest in trade on the new markets for insurance, at least initially after they were introduced. Therefore, if there are costs associated with running the markets, they might fail to survive. Thus, the Knightian explanation for the uninsurability of uncertainty requires the existence of some transaction costs.

The first approach mentioned above is expressed formally by the following proposition. In this proposition, $\text{int } \Pi_n^{A,\lambda}$ denotes the interior of $\Pi_n^{A,\lambda}$ relative to the set of all probabilities on A . Also, an $(A \cup B)$ -trade will be written as (x^A, x^B) , where $x^A: A \rightarrow (-\infty, \infty)$ and $x^B: B \rightarrow (-\infty, \infty)$. Similarly, an $(A \cup B)$ -reallocation is written as $(x^A, x^B) = (x_1^A, \dots, x_N^A; x_1^B, \dots, x_N^B)$, where (x_n^A, x_n^B) is an $(A \cup B)$ -trade for individual n , for each n . Finally, $\|x\|$ denotes the Euclidean length of the vector x .

8.1) Proposition. If $\bigcap_{n=1}^N \text{int } \Pi_n^{A,\lambda} \neq \emptyset$, then there is a positive number γ such that if a feasible $(A \cup B)$ -reallocation (x^A, x^B) is acceptable, then $\|x^A\| \leq \gamma \|x^B\|$.

Proof. Suppose the proposition is false. Then, for each $k = 1, 2, \dots$, there exists a feasible and acceptable $(A \cup B)$ -reallocation (x^{Ak}, x^{Bk}) such that $\|x^{Ak}\| > k \|x^{Bk}\|$. Since (x^{Ak}, x^{Bk}) is acceptable, $\sum_{A \in \mathcal{A}} x_{nA}^{Ak} \sum_{g \in A} \pi_{\Gamma(g)} \lambda_{ng} + \sum_{B \in \mathcal{B}} x_{nB}^{Bk} \sum_{g \in B} \pi_{\Gamma(g)} \lambda_{ng} > 0$, for all $\pi \in \Pi_n$ and for all n . By passing to a subsequence, one may assume that $\|x^{Ak}\|^{-1} x^{Ak}$ converges, say, to \bar{x}^A . Clearly, \bar{x}^A is an A -reallocation. Also, $\|\bar{x}^A\|^{-1} x^{Bk}$ converges to zero. Hence, by dividing both sides of the above inequality by $\|x^{Ak}\|$ and passing to the limit, one obtains $\sum_{A \in \mathcal{A}} \bar{x}_{nA}^A \sum_{g \in A} \pi_{\Gamma(g)} \lambda_{ng} \geq 0$, for all $\pi \in \Pi_n$ and for all n . That is, for all n , $\sum_{A \in \mathcal{A}} \pi(A) \bar{x}_{nA}^A \geq 0$, for all $\pi \in \Pi_n^{A,\lambda}$. Since \bar{x}^A is an A -reallocation, the condition $\bigcap_{n=1}^N \text{int } \Pi_n^{A,\lambda} \neq \emptyset$ implies that $\bar{x}^A = 0$. But this contradicts the fact that $\|\bar{x}^A\| = 1$. \square

I now turn to the definition of the independence of events in \mathcal{A} and \mathcal{B} . For each $B \in \mathcal{B}$, let $\nu(B)$ be the known measure of B . That is, for all n and all $\pi \in \Pi_n$, $\pi_{\Gamma}(B) = \nu(B)$.

Definition. \mathcal{A} and \mathcal{B} are *statistically independent* if for all n and all $\pi \in \Pi_n$, $\pi_{\Gamma}(A \cap B) = \pi(A)\nu(B)$, for all $A \in \mathcal{A}$ and $B \in \mathcal{B}$.

In the example of unemployment insurance, \mathcal{A} consists of two events, those of being employed and being unemployed. The field generated by \mathcal{B} includes all insurable events, such as accidents, together with the sets $G_t = \{(t, A) | A \in \mathcal{S}_t\}$, for $t = 1, 2, \dots, T$. The sets G_t can be insured via borrowing and lending. Since one is either employed or unemployed in the first period, G_1 cannot intersect both events in \mathcal{A} . Thus, if \mathcal{A} and \mathcal{B} are to be statistically independent, the events in \mathcal{A} must be "employed in any of periods 2, ..., T" and "unemployed in any of periods 2, ..., T."

Motivated by this example, I assume that B is partitioned into B_1 and B_2 . In the unemployment insurance example, B_1 would correspond to $\{G_1\}$. Let $\Gamma_i = \cup\{B \mid B \in B_i\}$, for $i = 1, 2$. Γ_1 and Γ_2 form a partition of Γ . Assume that \mathcal{A} is a partition of Γ_2 .

It is also assumed that before any new markets are introduced, events in B are perfectly insurable. Let p_B be the price of one dollar contingent on event $B \in B$, the price being in terms of dollars of period 1. If $G \subset \Gamma$ and $\pi \in \Pi_n$, let $\lambda_{n,G}(\pi) = \pi_{\Gamma(G)}^{-1} \sum_{g \in G} \pi_{\Gamma(g)} \lambda_{ng}$. Then, for each n , there exists $\bar{\pi}_n \in \Pi_n$ and $\alpha_n > 0$ such that $\nu(B) \lambda_{n,B}(\bar{\pi}_n) = \alpha_n p_B$, for all $B \in B$.

Definition. \mathcal{A} and B_2 are *economically independent* with respect to $\pi \in \Pi_n$ if for all $A \in \mathcal{A}$, the vector $(\lambda_{n,A \cap B}(\pi))_{B \in B_2}$ is collinear with the vector $(\nu(B)^{-1} p_B)_{B \in B_2}$.

In the example of unemployment insurance, \mathcal{A} and B_2 are economically independent if the relative impact of unemployment on the marginal utility of money is the same for all periods and no matter whether sickness or accident occurs at the same time.

Now let $\hat{\Pi}_n^{\mathcal{A}, \lambda} = \{[\sum_{A' \in \mathcal{A}} \pi_{\Gamma(A')} \lambda_{n,A'}(\pi)]^{-1} | (\pi_{\Gamma(A)} \lambda_{n,A}(\pi))_{A \in \mathcal{A}} \mid \pi \in \Pi_n \text{ and } \mathcal{A} \text{ and } B_2 \text{ are economically independent with respect to } \pi\}$. Also, let $\text{int } \hat{\Pi}_n^{\mathcal{A}, \lambda}$ be the interior of $\hat{\Pi}_n^{\mathcal{A}, \lambda}$ in the $(|\mathcal{A}| - 1)$ -dimensional simplex containing $\hat{\Pi}_n^{\mathcal{A}, \lambda}$, $|\mathcal{A}|$ being the cardinality of \mathcal{A} .

Suppose that new markets are opened for all events in $\mathcal{A} \vee B_2$. Then, the events on which contingent contracts may be traded are those of B_1 and $\mathcal{A} \vee B_2$, that is, the partition $B_1 \cup (\mathcal{A} \vee B_2)$ of Γ .

8.2) Proposition. If \mathcal{A} and B_2 are statistically independent and if $\bigcap_{n=1}^N \text{int } \hat{\Pi}_n^{\mathcal{A}, \lambda} \neq \emptyset$, then there is a positive number γ such that if a feasible $(B_1 \cup (\mathcal{A} \vee B_2))$ -reallocation, $(x^{B_1}, x^{\mathcal{A} \vee B_2})$, is acceptable, then $\|x^{\mathcal{A} \vee B_2}\| \leq \gamma \|x^{B_1}\|$.

Proof. First of all, let $x = (x_n)$ be an acceptable $(\mathcal{A} \vee \mathcal{B}_2)$ -reallocation. Let $v \in \prod_{n=1}^N \hat{\Pi}_n^{\mathcal{A}, \lambda}$. Then, for each n , there is $\pi_n \in \Pi_n$ such that $v = [\sum_{A' \in \mathcal{A}} \pi_{n\Gamma}(A') \lambda_{n,A'}(\pi_n)]^{-1} (\pi_{n\Gamma}(A) \lambda_{n,A}(\pi_n))_{A \in \mathcal{A}}$ and \mathcal{A} and \mathcal{B}_2 are economically independent with respect to π_n . Since \mathcal{A} and \mathcal{B}_2 are economically and statistically independent with respect to π_n , the vector $(\pi_{n\Gamma}(A \cap B) \lambda_{n,A \cap B}(\pi_n))_{B \in \mathcal{B}_2}$ is collinear with the vector $(p_B)_{B \in \mathcal{B}_2}$, for all $A \in \mathcal{A}$ and for all n . It follows that the vector $\delta_n (\pi_{n\Gamma}(A \cap B) \lambda_{n,A \cap B}(\pi_n))_{A \in \mathcal{A}} = \beta_B v$, for all $B \in \mathcal{B}_2$ and for all n , where $\beta_B > 0$ and $\delta_n = [\sum_{A \in \mathcal{A}} \pi_{n\Gamma}(A) \lambda_{n,A}(\pi_n)]^{-1}$.

Since x is acceptable, $0 < \sum_{A \in \mathcal{A}} \sum_{B \in \mathcal{B}} \delta_n \pi_{n\Gamma}(A \cap B) \lambda_{n,A \cap B}(\pi_n) x_n(A \cap B) = \sum_{A \in \mathcal{A}} \sum_{B \in \mathcal{B}} \beta_B x_n(A \cap B)$, for all n , which contradicts the fact that $\sum_{n=1}^n x_n(A \cap B) = 0$.

The rest of the proof is similar to that of proposition 8.1. \square

It is hard to know whether the conditions of the previous proposition are plausible. They become somewhat more believable when one realizes that the numbers $\lambda_{n,A \cap B}(\pi)$ could vary widely as π varied. Suppose that $A \cap B$ had many states and that the probabilities of these states conditional on $A \cap B$ were unknown. Suppose also that λ_{ng} varied widely as g varied over $A \cap B$. Then, as π varied over Π_n , $\lambda_{n,A \cap B}(\pi)$ would vary over a wide interval.

One might argue that propositions 8.1 and 8.2 cannot apply to reality because the existence of Lloyd's of London proves that uncertainty is insurable. However, this objection is not valid, for Lloyd's of London is not really a market for insurance nor an insurer, but a brokerage house which arranges special insurance contracts between individuals. It is completely in the spirit of Knightian decision theory that there should exist unusual individuals of low uncertainty aversion who would be willing to engage in such insurance.

The propositions of this section apply to the case where one or a few large firms are able to provide insurance for many potential insurees. Let the insurer be indexed by $n = 1$ and let $n = 2, \dots, N$ index the insurees. Considering the conditions of proposition 8.1, one would expect that $\bigcap_{n=2}^N \Pi_n^{\mathcal{A}, \lambda} \neq \phi$. The condition $\bigcap_{n=1}^N \Pi_n^{\mathcal{A}, \lambda} \neq \phi$ implies that no potential buyer would be interested in buying much insurance at a price of interest to the potential seller. However, realistically one might expect that there would be a small group, $I \subset \{1, \dots, N\}$, of unusual potential buyers for whom $\Pi_1^{\mathcal{A}, \lambda} \cap \left(\bigcap_{n \in I} \Pi_n^{\mathcal{A}, \lambda} \right) \neq \phi$, so that meaningful trade could occur between 1 and the members of I . However, if I were small, the trade might not be enough to keep the market alive.

This sort of argument makes some sense of the often heard assertion that insurance companies are extremely reluctant to undertake unevaluable risk.

It is not hard to generalize the results of this section to the case where individuals may trade in financial assets as well as in insurance. Similar results apply provided the returns on assets are not closely correlated with events in \mathcal{A} . If a close correlation did exist, there would be little need for insurance.

9. AN EXPLANATION OF THE ABSENCE OF FUTURE MARKETS

A Knightian argument can be used to explain the absence of some forward markets, especially those for distant futures. By a forward market, I mean a contract to deliver a certain quantity of a specified good or service at a stated price and date, with payment made at the time of delivery. Consider the following simple two-period model of forward trade in a single commodity. Let there be N_S sellers, indexed by $n = 1, \dots, N_S$, and N_B buyers, indexed by $n = N_S + 1, \dots, N_S + N_B$. The forward trade could be contingent on a particular event, but for simplicity think of it as uncontingent. Let p be the spot price of the commodity in period two. From the point of view of period one, p is a random variable with unknown distribution. Let θ_n be a random variable observed by

trader n in period two. The demand of trader n in period two is $x_n(p, \theta_n)$. If trader n is a supplier, $x_n(p, \theta_n)$ is negative. Trader n 's utility function for money in period two is $V_n(\cdot, \theta_n)$. Also, Π_n denotes trader n 's set of subjective probability distributions at time one over the random variables p and θ_n . Forward transactions are made at price p_F . Letting x_{F_n} be the amount trader n buys forward, the part of his utility function relevant for forward trading is $V_n(p(x_{F_n} - x_n(p, \theta_n)) - p_F x_{F_n}, \theta_n)$. For simplicity, it is assumed that $x(p, \theta_n)$ does not depend on money holdings in period two.

Now suppose that the forward market is opened for the first time and that the *status quo* programs of each trader do not include trade on this market. Then, trade will occur if and only if for at least one buyer, n , $\frac{d}{dx_{F_n}} E_\pi V_n(p(x_{F_n} - x_n(p, \theta_n)) - p_F x_{F_n}, \theta_n) \Big|_{x_{F_n}=0} > 0$, for all $\pi \in \Pi_n$, and for at least one seller, n , $\frac{d}{dx_{F_n}} E_\pi V_n(p(x_{F_n} - x_n(p, \theta_n)) - p_F x_{F_n}, \theta_n) \Big|_{x_{F_n}=0} < 0$, for all $\pi \in \Pi_n$, where E_π denotes expectation with respect to π . The derivative is $E_\pi[(p - p_F)V'_n(-px_n(p, \theta_n), \theta_n)] = \text{cov}_\pi(V'_n(-px_n(p, \theta_n), \theta_n), p) + (E_\pi p - p_F) \cdot E_\pi V'_n(-px_n(p, \theta_n), \theta_n)$, which is positive or negative according as p_F is less than or exceeds $E_\pi p + [E_\pi V'_n(-px_n(p, \theta_n), \theta_n)]^{-1} \text{cov}_\pi(V'_n(-px_n(p, \theta_n), \theta_n), p)$. In these expressions, cov_π denotes covariance with respect to π . One would normally expect that $\text{cov}_\pi(V'_n(-px_n(p, \theta_n), \theta_n), p)$ would have the same sign as x_n . The covariance term expresses the insurance value of forward trade.

The above discussion implies that there will be no forward trade if

$$\max_{n=N_S+1, \dots, N_B+N_B} \min_{\pi \in \Pi_n} \{E_\pi p + [E_\pi V'_n(-px_n(p, \theta_n), \theta_n)]^{-1} \text{cov}_\pi(V'_n(-px_n(p, \theta_n), \theta_n), p)\} > \min_{n=1, \dots, N_S} \max_{\pi \in \Pi_n} \{E_\pi p + [E_\pi V'_n(-px_n(p, \theta_n), \theta_n)]^{-1} \text{cov}_\pi(V'_n(-px_n(p, \theta_n), \theta_n), p)\}.$$

The fact that the covariance term is negative for sellers and positive for buyers tends to make forward trade more likely. But if for all traders, $E_\pi p$ varies widely as π varies over Π_n , there may be little or no forward trade. Thus,

Knightian uncertainty would discourage the innovation of new forward markets, in spite of the insurance value of forward trade.

10. HEDGING

It appears that in existing forward markets, a great deal of selling is done by hedgers. In fact, the primary function of such markets seems to be to provide an outlet for hedging. In this section, it is argued that those who produce or store a commodity would probably be willing to trade on either side of a new forward market with a due date corresponding to the period of production or storage. Thus, one imagines that it would be easy to establish such markets, but perhaps difficult to establish markets with due dates more distant than the period of production or storage. Once the markets were established, risk aversion would make most producers or storers of the good hedge.

Hedging will be analyzed in terms of a simple two period model of a firm which uses K inputs in period one to produce one output in period two. The firm's production function is $f(y, \theta)$, where y is a K -vector of inputs in period one and θ is a random variable observed in period two. Assume that f is differentiable, strictly concave and increasing with respect to y . The spot selling price in period two is a random variable p . The firm's owner is assumed to have a family, Π , of probability distributions over θ and p . It is assumed that the distribution of θ is known and is statistically independent of p , for all $\pi \in \Pi$. Let q be the L -vector of input prices in period one. For simplicity, it is assumed that the firm's owner is risk neutral and that the interest rate is zero.

Suppose the firm is in equilibrium in a world without forward trade and that suddenly forward trade is proposed. Before forward trade is proposed, the firm acts so as to solve $\max_y E_{\bar{\pi}}[pf(y, \theta) - q \cdot y]$, where $\bar{\pi} \in \Pi$ is the owner's opinion. Let \bar{y} be the solution of this problem and assume that $\bar{y}_k > 0$, for all k . The first order conditions are

$$q = (E_{\bar{\pi}} p) E(Df(\bar{y}, \theta)),$$

where Df is the derivative of f with respect to y .

Suppose that forward trade is proposed at price p_F and that the firm sells x_F forward and changes its inputs by Δy . Expected profits according to $\pi \in \Pi$ are $E_\pi[p(f(\bar{y} + \Delta y, \theta) - x_F) + p_F x_F - q \cdot (\bar{y} + \Delta y)]$. Since f is concave, this expression exceeds $E_\pi[pf(\bar{y}, \theta) - q \cdot \bar{y}]$, for small Δy , if and only if $0 < E_\pi[p(Df(\bar{y}, \theta) \cdot \Delta y - x_F)] + p_F x_F - q \cdot \Delta y$. Setting $x_F = E(Df(\bar{y}, \theta)) \cdot \Delta y = (E_{\bar{\pi}} p)^{-1} q \cdot \Delta y$, the above inequality becomes $(p_F - E_{\bar{\pi}} p) E(Df(\bar{y}, \theta)) \cdot \Delta y > 0$. Notice that this condition does not depend on $\pi \in \Pi$, so that even if inertia applies to the use of the new forward market, it will be used provided $p_F \neq E_{\bar{\pi}} p$. Recall that $\bar{\pi}$ is the owner's opinion. One imagines that if there were little knowledge of the distribution of p , then opinion would vary widely from firm to firm, so that many would be willing to trade forward at any value of p_F .

One sees that inertia does not discourage firms from using the forward market for their product, if the due date matches their period of production. In the above discussion, the firm would buy or sell forward according as p_F was less than or exceeded $E_{\bar{\pi}} p$. If the firm's owner were risk averse, he would be more likely to sell forward, that is, to hedge.

The results described above do not generalize to the case where θ and p are dependent, which would be the more plausible case for a farmer, but not for a storer of the good.

11. FORWARD MARKETS AS BAD PREDICTORS

The explanation given in Section 9 for the absence of forward markets depended on their being uncertainty about future spot prices. One might imagine that one could make this uncertainty nearly disappear by making the forward market be for delivery in a very narrowly defined dated event. Traders should then know fairly precisely their future demands and supplies in the dated event, so that the price on the forward market would be a good indicator of the future spot price in that event, especially if one also had a market for unit of account contingent on the event.

This argument meets two objections. First of all, there might be Knightian uncertainty about the dated event itself, which would make it hard to establish the contingent forward market. Secondly, even if there were no uncertainty about the dated event, the forward price would not necessarily be a good indicator of the future spot price in the dated event. Because traders could trade on the future spot market, they would use the forward market to speculate against what they imagined the future spot price would be. For this reason, their forward demands and supplies would not necessarily reflect their anticipated demands and supplies in the dated event, and so forward prices would not necessarily even be close to future spot prices. A simple example should make this point clear.

11.1) Example. There are two periods, two traders, and two goods. There is only one state of the world in the second period. Trader 1 is endowed with 2 units of good 1 and none of good 2 in period one. Trader 2 is endowed with $5/2$ units of good 2 and none of good 1 in period one. Neither trader has any endowment in period two, but, for $i = 1, 2$, trader i may store good i from period one to period two. Let a consumption allocation be denoted by $(x_{11}, x_{12}, x_{21}, x_{22})$, where x_{tk} denotes the consumption of good k in period t . The utility function of trader 1 is $u_1(x_{11}, x_{12}, x_{21}, x_{22}) = u(x_{11}) + u(x_{22})$, where $u(x) = x - \frac{x^2}{4}$. Similarly, the utility function of consumer 2 is $u_2(x_{11}, x_{12}, x_{21}, x_{22}) = u(x_{12}) + u(x_{21})$. In period one, the traders may trade forward with payment made in period two. Let good 1 in period 2 be the *numeraire* and let p be the spot price in period two for good 2. Let E_i be the expectations operator of trader i , for $i = 1, 2$. This operator represents the opinion of the trader. Assume that $E_1 p^{-1} = \frac{1}{2}$, $E_1 (1 - p^{-1})^2 = \frac{1}{3}$, $E_2 p \equiv \frac{1}{2}$ and $E_2 (1 - p)^2 = \frac{3}{8}$.

The utility maximization problem of trader 1 is $\max_{y_1, y_{F1}} [u(2 - y_1) + E_1 u(p^{-1}(y_1 - y_{F1}) + p_F^{-1} y_{F1})]$, where y_1 is the quantity of good 1 stored and y_{F1} is the quantity sold forward at the forward price p_F . The utility maximization problem

of trader 2 is $\max_{y_2, y_{F2}} [u(\frac{5}{2} - y_2) + E_2 u(p(y_2 - y_{F2}) + p_F y_{F2})]$, where y_2 is the quantity of good 2 stored and y_{F2} is the quantity sold forward. Using these objective functions, it is not hard to verify that an equilibrium is $p_F = 1$, $y_1 = \frac{1}{2}$, $y_2 = 1$, and $y_{F1} = y_{F2} = \frac{5}{2}$. The equilibrium consumption allocation of trader 1 is $[\frac{3}{2}, 0, 0, 1]$ and that of trader 2 is $[0, \frac{3}{2}, \frac{1}{2}, 0]$. The actual spot price in period 2 is $p = 4/3$. In this equilibrium, traders sell more forward than they store. They make up the difference by spot purchases in period two. The equilibrium allocation is not Pareto optimal. It is Pareto dominated by the allocation giving trader 1 the allocation $[\frac{5}{4}, 0, 0, \frac{5}{4}]$ and trader 2 the allocation $[0, \frac{5}{4}, \frac{3}{4}, 0]$. This allocation is obtained by storing $\frac{1}{4}$ more units of both goods than in the equilibrium. In the equilibrium, the traders' pessimistic expectations about the future spot price discourage storage.

If the traders' had rational expectations, then the phenomenon illustrated by the example could not occur. The phenomenon would not occur either if both traders believed the current forward price equaled the future spot price. But the example shows that they need not be equal. By playing with examples, one may see that there need be no systematic relation between forward prices and future spot prices. Such a relationship would appear only if circumstances repeated themselves so that rational expectations would be appropriate. The example would again be impossible if traders calculated future spot prices from knowledge of the future endowments and production and utility functions incorporated in the description of the dated event. From a theoretical standpoint, such a calculation is possible. However, to have individuals calculate equilibria in this manner is contrary to the spirit of general equilibrium theory. Part of the magic of general equilibrium is supposed to be that the market makes such calculations, it being understood that they are too difficult for any individuals.

Knightian decision theory is of interest only in a world completely different from one with rational expectations. In general equilibrium models with rational expectations,

it is usually assumed that people know not only the true probabilities of dated events, but the spot prices associated with these events. In a Knightian world, both of these things may not be known. From a Knightian viewpoint, the uncertainty about future prices not only makes it difficult to introduce new forward markets, but it adds to the ambiguity of future marginal utilities of money in the various dated events, and so adds to the difficulty of introducing new forms of insurance. That is, ambiguity about future marginal utilities of money makes the conditions of propositions 8.1 and 8.2 slightly more plausible.

The point made by example 11.1 reinforces a common criticism of rational expectations. The criticism is that it is not clear how people could know spot prices as a function of future dated events. (See, for instance, Drèze (1971).) The example simply shows that forward trading cannot be counted on to reveal future prices. The example does not depend on traders having Knightian preferences. It requires simply that they not have rational expectations. The point made by the example must have occurred to many who have thought about forward markets, though I have not found a reference.

12. ASSET TRADING AND DIVERSITY OF OPINION

It should be clear that the Knightian viewpoint has many implications for the theory of finance. Two obvious implications are mentioned here because they are closely related to the Knightian theory of entrepreneurship. Whether one believes preferences to be Knightian or Bayesian, it is natural to imagine that subjective probability assessments vary among individuals when the assessments have no sound objective basis. What follows depends only on there being a variation of opinion among investors.

One of the puzzles of finance is why individual investors hold very undiversified portfolios of securities. (See Blume and Friend (1978), Chapter 2.) If investors' expectations were rational and markets were in equilibrium, then diversification would reduce risk without leading to a loss of expected return. Common explanations of the

puzzle are transactions costs, investor ignorance of alternative investments, and ignorance of the advantages of diversification.

An obvious additional explanation is possible when opinions vary. Investors buy those few securities which they believe have the highest returns. Lack of diversification may be interpreted as indirect evidence of the existence of variation in opinion. Harrison and Kreps (1978) have developed a stock market model with diversity of investor opinion. In their model, it is clear that this diversity could cause individual portfolios to be concentrated in a few stocks.

One might object that differences of opinion should lead to short selling, and short sales are not very common. Investors should sell short the securities about which they are most pessimistic and use the proceeds to buy those about which they are most optimistic. Margin requirements discourage short sales. At present, half the value of a short position must be held in non-interest bearing cash. With this rule, an investor would not sell a stock short unless its expected return were less than half that of the security in which he intended to invest the proceeds. Perhaps the chief deterrent to short sales is that the potential loss from a short position is unbounded, whereas that from a long position is no more than the amount invested.

Another puzzle from finance is that closed end mutual funds often sell at a discount from the total value of their investments. Part of the puzzle, perhaps, is why arbitrage operations don't eliminate the discount, but this should be no mystery since margin requirements discourage such operations and a possible widening of the discount would make them dangerous. The puzzle addressed here is why the discount exists even if arbitrage were impossible.

The discount can be interpreted as evidence for variation in investor opinion, just as can the lack of diversification. If the assets of the fund were sold separately, each would be bought by the investor who valued it most. Selling the assets jointly obliges buyers to accept what they value little along with what they value highly.

In order to see this point more clearly, consider a simple two-period model with K assets and N investors. Let the price of each asset in period one be one and let R_k be the random value of asset k plus its dividend, in period two. Let $y_k > 0$ be the proportion of the fund's assets invested in the k^{th} asset and let $r > 0$ be the interest rate. Finally, let E_n be the n^{th} investor's expectation operator. The value of one "share" of the fund to investor n is $(1+r)^{-1} \sum_{k=1}^K y_k E_n R_k$, so that if the aggregate wealth of the investors is large relative to the size of the fund, the value of one share should be nearly $(1+r)^{-1} \max_n \sum_{k=1}^K y_k E_n R_k$. If the assets of the fund were sold separately, the value should be nearly $(1+r)^{-1} \sum_{k=1}^K y_k \max_n E_n R_k$, which clearly exceeds $\max_n (1+r)^{-1} \sum_{k=1}^K y_k E_n R_k$, if the $E_n R_k$ vary as n varies. In fact, the difference is a measure of the variability of investor opinion.

Dow and Werlang (1988) have investigated the implications of Gilboa-Schmeidler preferences for trading in securities. Such preferences imply a kind of inertia with respect to the position of holding none of an asset. They mention that Gilboa-Schmeidler preferences could explain lack of portfolio diversification. I note in passing that it would be inappropriate to apply my own inertia assumption to asset trading, unless one is considering the purchase or sale of a completely new form of asset.

13. THE ENTREPRENEUR

Frank Knight described entrepreneurs as those who undertake investments with unevaluable and therefore uninsurable risks. This intuition is at least logically sound when it is expressed formally in terms of Knightian decision theory. If the risks of a new enterprise were evaluable, one imagines that markets would be organized for contingent claims on those risks. The entrepreneur would then become the hired manager or agent of a large group of claims holders, investors or insurance companies. The risk would be

marketed and the entrepreneur would not need to bear it himself. If the risks of a new enterprise are difficult to evaluate, then by what has been said in Section 8, it might be difficult to organize insurance markets or groups of investors to spread the risk. The entrepreneur or initial investors would therefore have to be people who were unusual in their willingness to undertake the risk. If the enterprise were something new and unexpected, then the inertia assumption would imply that the initial investors would be people with low aversion to uncertainty and with subjective probabilities that favored the enterprise. If the enterprise were not a new idea, then the inertia assumption would not apply and one concludes that the opinions of the initial investors were unusually favorable to the enterprise.

The two characterizations just given of the entrepreneur may give some insight into the innovation process. For instance, they imply that more innovation will occur if the decision to innovate is dispersed among many individuals than if it is concentrated in the hands of a few. Also, an enterprise would be less likely to be undertaken if a group of investors all had to agree to undertake it than if any one of the group could do so. Thus, large organizations might be more conservative than individuals acting independently. These conclusions are similar in spirit to those of the previous section.

Turning now to the more formal description of entrepreneurship, I continue to use the notation of Section 7. Thus, Γ is the set of dated events and Π is a set of subjective probability distributions on the state space underlying Γ . If $\pi \in \Pi$, π_Γ is the probability measure on Γ defined by $\pi_\Gamma(G) = \sum_{(t,A) \in G} \pi(A)$. A potential enterprise is described by a subset Y of R^Γ , where R^Γ is the set of real-valued functions on Γ . If $y \in Y$, y_g is the monetary net return in dated event $g \in \Gamma$. Consider a potential entrepreneur with subjective probability distributions Π and Knightian preferences. For simplicity, assume that he is risk neutral and attaches the same utility to money in all dated events. Also, assume that the interest rate is zero. Then, the enterprise is profitable

according to $\pi \in \Pi$ if, for some $y \in Y$, $\pi_{\Gamma} \cdot y \equiv \sum_{g \in \Gamma} \pi_{\Gamma}(g) y_g > 0$.

Assume now that the potential entrepreneur had not previously been aware of the existence of Y , so that the inertia assumption applied to the decision to undertake Y . Assume also that the only alternative use of funds is to hold money. Then, the enterprise will be undertaken if and only if there is $y \in Y$ such that $\pi_{\Gamma} \cdot y > 0$, for all $\pi \in \Pi$. Therefore, the enterprise is, roughly speaking, the more likely to be undertaken the smaller is Π , that is, the less averse to uncertainty the individual is.

If the enterprise has long been under consideration by the individual, then he undertakes it if it is profitable according to his opinion $\bar{\pi} \in \Pi$. Since the individual did not undertake the enterprise before, one must imagine either that new information has just made his opinion more favorable or that he only just acquired the means to carry out the enterprise.

If one believes that risk aversion seriously inhibits the undertaking of new enterprises, then their creation should be encouraged by diversification and the spreading of risks. That is, the most daring innovators should be large organizations which pool the funds of many investors and invest in many new enterprises. However, according to the Knightian theory, both diversification and risk sharing discourage entrepreneurship.

Consider, first of all, the effects of diversification. Let there be K possible new enterprises, the k^{th} being represented by $y_k \in R^{\Gamma}$, for $k = 1, \dots, K$. Suppose that the investor has long been familiar with the possible enterprises, so that they are evaluated according to his opinion, $\bar{\pi} \in \Pi$. Then, as was indicated in the previous section, even if the investor were risk averse he would probably be most willing to invest if he could concentrate his investments in those few enterprises, k , for which $\bar{\pi}_{\Gamma} \cdot y_k$ was highest relative to the initial investment. Suppose next that the investor has just become aware of the possibility of investing in the K new enterprises. Suppose that the K possible enterprises are independent in the sense that the range of values of $\pi_{\Gamma} \cdot y_k$ is independent of the

values of $\pi_{\Gamma} \cdot y_k$, for all distinct k and k' . More precisely, let $R_k = \{(\pi_{\Gamma} \cdot y_1, \dots, \pi_{\Gamma} \cdot y_{k-1}, \pi_{\Gamma} \cdot y_{k+1}, \dots, \pi_{\Gamma} \cdot y_K) | \pi \in \Pi\}$ and assume that, for each k , the interval $\{\pi_{\Gamma} \cdot y_k | \pi \in \Pi \text{ and } \pi_{\Gamma} \cdot y_{k'} = r_{k'}, \text{ for all } k' \neq k\}$ is independent of $(r_{k'}) \in R_k$. If the investor considers the choice between investing in the K enterprises together or not at

all, he will participate only if $0 < \min\{\pi_{\Gamma} \cdot (\sum_{k=1}^K y_k) | \pi \in \Pi\} = \sum_{k=1}^K \min\{\pi_{\Gamma} \cdot y_k | \pi \in \Pi\}$,

where the equality holds because of the independence assumption. If the investor considers each enterprise separately, he will invest in at least one if $0 < \min\{\pi_{\Gamma} \cdot y_k | \pi \in \Pi\}$, for some k . Thus, the investor is more likely to participate in at least one enterprise than all together. To this extent, diversification over independent enterprises would tend to discourage investment in new businesses.

The diversity of opinion and preferences also makes it difficult to share unevaluatable risks. Suppose there are N investors who must decide whether to invest in one new enterprise. Let the enterprise be represented by $y \in R^{\Gamma}$ and let Π_n be the set of subjective probabilities of the n^{th} investor, for $n = 1, 2, \dots, N$. Suppose the investors must decide by majority vote. Suppose, first of all, that the inertia assumption does not apply and let $\bar{\pi}_n \in \Pi_n$ be the opinion of the n^{th} investor. Then, the investment will be undertaken only if $\bar{\pi}_n \cdot y > 0$, for a majority of n . If the investors acted independently and each could finance the investment himself, then the enterprise would be undertaken if $\max_n \bar{\pi}_n \cdot y > 0$. Suppose now that the inertia assumption applies. If the investors acted separately, the enterprise would be undertaken if $\min\{\pi_{\Gamma} \cdot y | \pi \in \Pi_n\} > 0$, for some n . If they acted together, it would be necessary that this inequality apply for a majority of n . Thus, cooperation lessens the likelihood of innovation.

Any kind of cooperation by a group can lead to a strong form of collective uncertainty aversion, much discussed in the literature on shareholder control. (See, for instance, Drèze (1985).) Suppose N individuals must act together and decide by unanimity and suppose that none of the individuals are uncertainty averse. Then, unless the group finds a

way to compromise, any action taken must have positive expected value according to the subjective distribution of each individual. This requirement makes the group uncertainty averse, unless all individuals have the same subjective distribution. This form of uncertainty aversion is stronger than that attributed to individuals by the inertia hypothesis, for it applies to all actions, not just departures from the *status quo*.

In conclusion, Knightian decision theory leads to the idea that innovation and the foundation of new enterprises are the natural domain of unusual individual investors acting alone or nearly alone. One imagines that the risk associated with new enterprises must be difficult to evaluate. As an enterprise becomes established and develops a history, its prospects should be easier to evaluate and so could attract the interest of a large number of investors.

The theory of entrepreneurship presented here may remind one of the work of Schumpeter. Schumpeter's ideas were quite different, however. He did not envision the entrepreneur as an investor but as a leader of men (Schumpeter (1939), pp. 102–104).

14. NON-OPTIMALITY OF DISPERSED INNOVATION

The lack of coordination among investors gives rise to inefficiency, even though innovation is encouraged by their independence. The inefficiency results from diversity of opinion and the inability of investors to sell their future profits forward. The following example illustrates the point.

14.1) Example. There are two periods and two types of people. There are ten people of type 1 and two of type 2. People of type 1 live only in period 1. People of type 2 live in both periods. Consumption occurs in period 2. Production and production decisions occur in period 1. There are four types of consumption goods. Each person of type 1 is endowed with one unit of good 1. People of type 2 can produce any of goods 2, 3 or 4 for period 2 using labor in period 1. Each person of type 2 has one unit of labor and can produce the

goods in any relative quantities at the rate of one unit of good per unit of labor. The utility function of a person of type 2 is $u(x_1, x_2, x_3, x_4) = x_1$ where x_k is the quantity of good k consumed. The utility functions of all people of type 1 are equal to $v(x_1, x_2, x_3, x_4) = x_1 + x_2 + \frac{3}{2}x_3$ in state s_v and are equal to $w(x_1, x_2, x_3, x_4) = x_1 + x_2 + \frac{3}{2}x_4$ in state s_w . In period 1, people of type 2 do not know the true state, but they learn it in period 2. The set of subjective probability distributions of people of type 2 is $\Pi = \left\{ (\pi(s_v), \pi(s_w)) \mid \pi(s_v) + \pi(s_w) = 1 \text{ and } \frac{1}{4} \leq \pi(s_v) \leq \frac{3}{4} \right\}$.

In period 2, commodities are traded in a competitive equilibrium. Let the unit of account in period 2 be good 1. If goods 2, 3 or 4 are produced, their prices are, respectively, 1, $\frac{3}{2}$, and 0 in state s_v and 1, 0, and $\frac{3}{2}$ in state s_w . People of type 2 realize in period 1 that these will be the period 2 prices as a function of the state.

Give the name V to one of the people of type 2. Call the other W . Let the opinion of V be $(\pi(s_v), \pi(s_w)) = \left[\frac{3}{4}, \frac{1}{4} \right]$ and that of W be $(\pi(s_v), \pi(s_w)) = \left[\frac{1}{4}, \frac{3}{4} \right]$. Then, V will produce good 3 and W will produce good 4. In equilibrium, people of type 1 have utility 1 in both states, V has utility $\frac{3}{2}$ in state s_v and 0 in state s_w , and W has utility 0 in state s_v and $\frac{3}{2}$ in state s_w .

This outcome is not Pareto optimal. Suppose that both V and W produced good 2, that in state s_v , W gives 1 unit of good 2 to V , and that in state s_w , V gives 1 unit of good 2 to W . Then, person V would have utility 2 in state s_v and 0 in state s_w , and person W would have utility 0 in state s_v and 2 in state s_w . People of type 1 would continue to have utility 1 in both states. The new outcome clearly Pareto dominates the equilibrium in both states. The new outcome is what would occur if one had forward trading with rational expectations.

The equilibrium would be Pareto optimal if both people of type 2 had the same opinion. In fact, if there were no diversity of opinion and, as in the example, no need for insurance, equilibrium would be Pareto optimal in a quite general model.

Diversity of opinion causes suboptimality for roughly the same reason that changes of opinion are suboptimal for an individual. The loss to an individual from opinion changes may be seen in the diagram below, where x_a and x_b are the utility payoffs in states a and b , respectively, and $OABC$ is the set of feasible choices. Suppose the decision maker chooses twice before the state is revealed, and that his total reward is the average of his rewards from the two choices. If the decision maker chooses A once and C once, using opinions π and π' , respectively, his average reward is at D , which is dominated by B . The analogy with example 14.1 may be seen by replacing utility by social welfare.

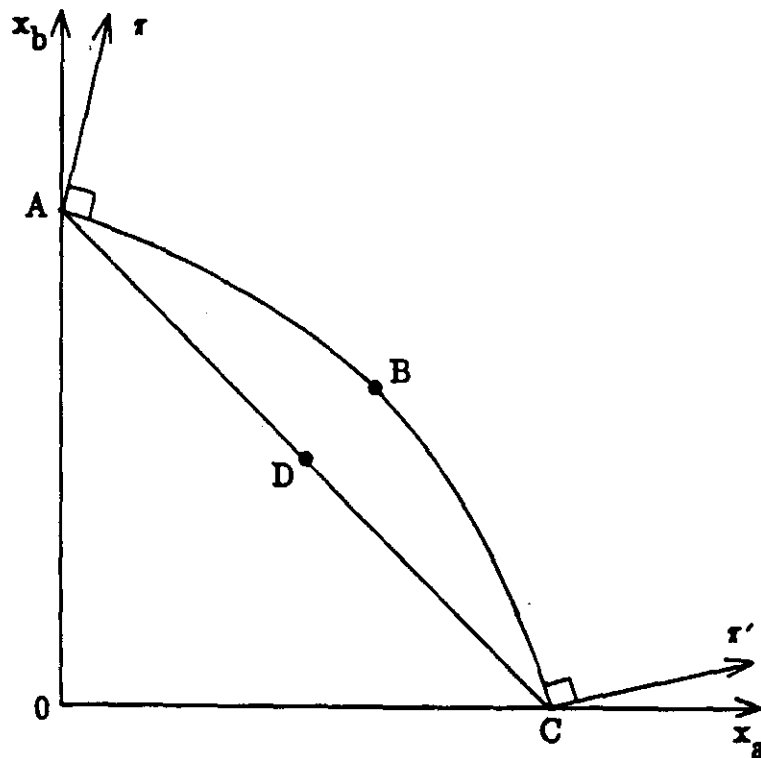


Figure 5

15. THE INFORMATIONAL EXTERNALITY OF INNOVATION

As is well-known from the literature on patent rights, innovation creates knowledge and so gives rise to an externality. In the Knightian model of entrepreneurship, one can see how this externality could cause one innovation to stimulate further innovation and so lead to the waves of innovation that Schumpeter claimed to observe (1939, p. 100). The knowledge created by one innovation reduces the uncertainty associated with other possible innovations and so makes them more attractive to investors. The knowledge also focuses subjective probabilities on those innovations which are more likely to succeed and so tends to stimulate them.

There are at least two sorts of knowledge created by innovation. One has to do with new concepts or ideas, and the other has to do with experimental verification. The conception of new ideas has to do with bounded rationality and so has no place in the Knightian framework. However, the process of experimental verification can be expressed in the Knightian theory in the same way that it can be in the Bayesian theory. As pointed out in Section 2, an individual uses information to update by conditioning all of his subjective distributions. Also, information tends to reduce uncertainty by confining the set of subjective distributions to a smaller region around the true distribution.

The following example illustrates how, in a Knightian model, the informational externality can cause innovations to stimulate each other in an endless chain. The example also demonstrates how a person with an odd opinion can initiate such a chain.

15.1) Example. Consider an overlapping generations model where people live two periods. Each person is endowed with one unit of labor in the first period of life, which may be used to produce a consumption good for the succeeding period. The production technology is linear, with production at the rate of one unit of consumption good per unit of labor. All consumption occurs in the second period of life.

There are countably many types of consumption goods. One is indexed simply by 1 and the others are indexed by all possible finite sequences of symbols a and b . Let C denote the set of all such finite sequences and let $X = \{x : \{1\} \cup C \rightarrow [0, \infty) \mid x_c \neq 0 \text{ for at most finitely many values of } c\}$. X is the consumption set.

There are two types of people, types 1 and 2. Type 1 people can produce only good 1. Those of type 2 can produce any good. There are 100 people of type 2 and $1000 \cdot 2^t$ of type 1 in period t .

The utility function for a person of type 2 is $v : X \rightarrow [0, \infty)$, defined by $v(x) = x_1$. People of type 2 do not know the utility of a person of type 1. The set of possible types of utility functions is $\{1\} \cup S$, where $S = \{(s_1, s_2, \dots) \mid s_n = a \text{ or } b, \text{ for all } n\}$. The utility function corresponding to 1 is v . The utility function corresponding to $s \in S$ is $v_s : X \rightarrow [0, \infty)$, defined by
$$v_s(x) = x_1 + \sum_{n=1}^{\infty} \left[\left(\frac{10}{9} \right)^n x_{s_1, \dots, s_n} + \left(\frac{10}{9} \right)^{n-1} x_{s_1, \dots, s_{n-1}, s_n^*} \right],$$
 where $s_n^* = a$, if $s_n = b$ and $s_n^* = b$ if $s_n = a$. Give S the usual Borel σ -field generated by the product of the discrete topologies on $\{a, b\}$. Each person of type 2 has as a set of subjective distributions $\Pi = \left\{ \pi \mid \pi \text{ is a probability distribution on } \{1\} \cup S \text{ such that } \frac{1}{14} \leq \pi(1) \leq \frac{3}{14}, \frac{1}{4} \leq \pi(s_{n+1} = b \mid S \text{ and } (s_1, \dots, s_n) = (\bar{s}_1, \dots, \bar{s}_n)) \leq \frac{3}{4}, \text{ for } n = 0, 1, 2, \dots \text{ and all } (\bar{s}_1, \dots, \bar{s}_n) \right\}$, where $\pi(\cdot \mid \cdot)$ is the conditional distribution. The true utility function of a person of type 1 is $u = v_{b^*}$, where $b^* = (b, b, \dots, b) \in S$.

In each period, exchange is determined by a competitive equilibrium.

Suppose that all people of type 2 have been producing the good of type 1 up until period 1, so that they have no information about the utility function of people of type 1. Suppose the inertia assumption does not apply to the production decisions of people of type 2. No good other than 1 has ever been produced because people's opinions have not favored event S sufficiently. Now suppose that in period one an odd young person of type 2 has the opinion that the true state lies in S with probability $\frac{13}{14}$ and that if the state is in S , it

satisfies $s_1 = a$ with probability $\frac{3}{4}$. Then, if he produces good a he can expect to gain $\frac{13}{14} \left[\frac{10}{9} \left(\frac{3}{4} \right) + \frac{1}{4} \right] = \frac{13}{14} \left(\frac{13}{12} \right) > 1$. If he produces good b , he gains a utility level of 1. Therefore, he produces good a . Once that good is produced, it will be discovered in period 2 that the true state is in S and that $s_1 = b$. It is then easy to calculate that in each period t , people of type 2 would know that $s_1 = \dots = s_{t-1} = b$ and each person of type 2, regardless of his opinion, would produce either $(c_1, \dots, c_t) = (b, \dots, b, a)$ or $(c_1, \dots, c_t) = (b, \dots, b)$, thus revealing in period $t+1$ that $s_t = b$.

In this example, one could equally well have allowed the inertia assumption to apply to all innovations and have had the initiator of the chain of innovations be a person with an unusually low aversion to uncertainty.

The existence of the informational externality is, of course, a well-known argument for subsidizing innovation. One sees that any inadequacy in such subsidization may be offset, to some extent, by the diversity of investor opinion. As was shown in the previous section, that diversity may lead to excessive experimentation. From the Knightian point of view, one could estimate only very imprecisely the appropriate size of any subsidy, because all the relevant quantities are imponderables. In fact, innovations must be the domain of economic life where rational expectations makes the least sense. How can one know the true probability of success of an experiment never before attempted?

16. KEYNES' IDEAS ON INVESTOR INSTABILITY

Keynes had ideas about uncertainty similar to those of Knight. Keynes claimed that it made little sense to associate specific probabilities with the outcomes of a business venture. He associated the vagueness of investor expectations with their instability, and claimed that such instability caused damaging fluctuations in security prices.

The question arises naturally whether it is possible to reconcile the instability of expectations with Knightian decision theory. At first glance, it would appear to be impossible to do so. An investor's expectations are his opinion, and this should change only in

response to a change of *status quo* or in response to new information. However, updating of opinion in response to new information could lead to apparently wide fluctuations of opinion if the opinion itself was based on little information. Knightian uncertainty exists, presumably, in environments where there is little information on which to base subjective probability judgments. Therefore, an environment which gives rise to uncertainty could also give rise to instability of expectations, but that instability would be rational, not frivolous.

It is easy to see that expectations based on little information can be very sensitive to new information. As an extreme example, consider a random variable which is normally distributed with known variance but completely unknown mean. If we express complete ignorance by a uniform improper prior on the mean, then after one observation, x , the posterior mean of the random variable is x .

It does not seem necessary to elaborate this point, for it is well-known that dispersed prior distributions lead to extreme sensitivity of posterior distributions to just a few observations.

In the context of investment, if one imagines that the value of information decays over time, then investors' opinions could forever be sensitive to new scraps of information.

Perhaps part of the entrepreneurial temperament is captured by low uncertainty aversion and unstable expectations. The first property would be a personality trait whereas the second may simply be imposed by circumstances.

17. CONCLUSION

Knightian decision theory may give insight into the process of innovation in business. Uncertainty and aversion to uncertainty make innovation difficult. A single entrepreneur with low aversion to uncertainty can initiate new products, production processes and forms of business. Entrepreneurs in a committee would tend to hinder each others' initiatives. Not only would they have to overcome each others' aversion to

uncertainty, but their opinions would have to be in near agreement. Diversity of opinion among them would lead to a kind of collective aversion to uncertainty. Innovation of a financial market is all the more difficult if the financial instruments traded involve uncertainty. Not only does the market innovator have to overcome his own aversion to uncertainty, but all the traders on the market must overcome their uncertainty aversion as well, since the financial instruments are new to them.

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