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THE INTERACTION OF IMPLICIT AND EXPLICIT CONTRACTS

IN

REPEATED AGENCY

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ABSTRACT

Traditional agency theory assumes that the principal has no more information about the agent's actions than the enforcement authorities have. This is unrealistic in many settings, and in repeated models, additional information possessed by the principal changes the nature of the problem. Such information can be used in implicit, self-enforcing contracts between principal and agent, that supplement the usual explicit contracts. This paper studies the way in which the two kinds of contract are combined in constrained efficient equilibria of the agency supergame. The agent's compensation is comprised of both guaranteed payments and voluntary bonuses from the principal. We give a simple characterization of the composition of remuneration in the optimal dynamic scheme.

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1. Introduction

In this paper, we investigate the relationship between two ways of sustaining cooperation in repeated principal-agent models. A principal can typically make commitments to an agent by offering him a legally enforceable contract that specifies payments contingent on information available to the courts. If the principal can observe more than the publicly verifiable information, implicit self-enforcing agreements between principal and agent supplementing the terms of the explicit contract may be mutually beneficial. We show how explicit contracts are designed to support constrained efficient equilibria of the agency game and emphasize the role of renegotiation of legal contracts in providing the appropriate incentives for both parties.

Although it is usually realistic to assume that the principal has information (related to the agent's action) beyond what is verifiable in a court of law, the distinction has played little role in agency theory.[1] The reason is that in a static setting, unverifiable observations are useless, except in conjunction with certain "revelation schemes" discussed later. In indefinitely repeated relationships, however, such information is vital to implicit agreements that exploit the multiplicity of equilibria in the supergame to make both players' payoffs depend on this information. For example, an agent whose actions are observed directly by the principal expects that by failing to exert the implicitly promised amount of effort, he would adversely affect his future payoffs, beyond the indirect effects his failure might have on the compensation guaranteed by the explicit contract he holds.

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[1] The recent literature on incomplete contracts is an exception that we discuss below.

Selecting an institutional setting in which to explore these ideas involves specifying the kinds of contracts that will be enforced, and the opportunities open to the agent. Should a court enforce a contract having lotteries as contingent payments, for example? Over what period are contracts valid, and are they binding on both parties? How much access does the agent have to capital markets or storage technologies? The modelling is fairly traditional in these respects. The explicit contracts considered are one-sided commitments requiring the principal to pay the agent deterministic amounts contingent on the realizations of publicly observable random variables (taken for simplicity to be gross profits). The agent cannot borrow, lend or save but can in any period elect to abandon the principal in favor of earning a fixed salary elsewhere. The agent's action is observable to the principal but not to the courts. Only single-period contracts are enforceable.

We find that optimal equilibria exhibit a form of renegotiation: on the equilibrium path, the compensation actually received by the agent usually differs from that which he is promised ex ante by his legal contract. At the end of each period the agent receives a "bonus" whose reciprocal varies directly with the compensation promised by the explicit contract and with the agent's expected payoff from the remainder of the equilibrium. This transfers some risk from the risk-averse agent to the risk-neutral principal without diminishing the agent's incentives to take the correct action.

There is a large body of work on infinitely repeated principal-agent problems beginning with Rubinstein (1979) and Radner (1985); to our knowledge, none of this studies the interplay of explicit and implicit contracts. Our paper is related to the growing literature on incomplete contracts (see, for example, Green and Laffont (1987), Grossman and Hart (1986), Hart and Moore (1988), Huberman and Kahn (1986) and Tirole (1986)) which emphasizes the distinction between observability and verifiability of information, and gives renegotiation a prominent role. The analysis is quite different, however; because these papers consider two or three period games, questions of optimal long-run relationships do not arise. The equilibria that we study are unusual in that all the renegotiation that eventually takes place is perfectly predictable at the beginning of the game as a function of verifiable information, and yet the ultimate terms of compensation cannot be specified in the original explicit contract.

In their paper on incomplete contracts, Hart and Moore (1988) consider revelation schemes in which the terms of the explicit contract depend upon messages sent simultaneously by the two contracting parties to the courts; this is a powerful device having its origins in the literature on the implementation of social choice rules (see, for example, Maskin (1977)). Even in a one-shot moral hazard problem in which the principal (but not the courts) could observe the agent's action, there are schemes of this kind having fully efficient equilibria. We disallow such contracts on the grounds that they are not likely to be legally enforceable. We suggest one theoretical justification for (but probably not an "explanation of") courts' unwillingness in practice to enforce these revelation contracts. Suppose that provisions such as the following were deemed enforceable: two parties independently send the court the message "conformed" or "did not conform", and both are assessed enormous penalties if the messages disagree. This amounts to writing the contracting parties a judicial blank check. While the provision could be used to ensure that an employee exert the appropriate amount of effort and that the principal fully insure him, it could equally well support an implicit agreement that the agent will be compensated for illegal activities such as committing perjury or murder.

Section 2 presents the underlying agency problem and characterizes the constrained efficient equilibria of the supergame. Brief concluding remarks are found in Section 3.

2. Constrained Efficiency in the Agency Supergame

This section studies a model in which a principal and a single agent interact repeatedly, but the explicit contract held by the agent in any particular period commits the principal to (output-contingent) payments in that period only. Such a restriction is relevant if long-term contracts are either unenforceable or prohibitively costly.

Information and Payoffs in the Component Game

The agent chooses from the finite set of available actions $A = \{a_0, a_1, \ldots, a_n\}$. The principal observes the choice, but the court sees only whether or not a_0 is chosen. Whereas $a_1, \ldots, a_n$ are alternative actions the agent may take while working for the principal (perhaps effort levels or degrees of care), a_0 represents "non-participation". If the agent selects a_0, he works for some outside employer at the reservation salary $r > 0$. Define

$\overline{\mathbb{R}} := \mathbb{R} \cup \{-\infty\}$. The agent's von Neumann-Morgenstern utility function $U : \mathbb{R}_+ \times A \to \overline{\mathbb{R}}$ takes the form (with an abuse of notation)

$$U(c,a_i) = U(c) - d_i ,$$

where c is his monetary compensation or consumption, and $d_i = d(a_i)$ is the <u>disutility</u> associated with taking action a_i.

Each action induces a probability distribution over verifiable outcomes, which are assumed for simplicity to be finite in number. Without essential loss of generality the outcomes are taken to be the gross profits of the principal. Let $\Pi = \{\pi_1,\ldots,\pi_n\}$ be the space of gross profits, and $p_{ik} = P[\pi_k|a_i]$, $i = 0,\ldots,n$; $k = 1,\ldots,m$, where $P[\pi_k|a_i]$ denotes the probability of π_k conditional on a_i.

<u>Assumptions</u>:

(A1) U is differentiable, strictly concave and strictly increasing in c.

(A2) $p_{ik} > 0$, $\quad i = 1,\ldots,n$; $k = 1,\ldots,m$.

(A3) $\sum_{k=1}^{m} p_{ik}\pi_k$ is minimized when $i = 0$.

<u>Normalizations</u>:

$$d_0 = 0 \quad \text{and} \quad \sum_{k=1}^{m} p_{0k}\,\pi_k = 0 .$$

The agent's sole sources of money are the principal and the reservation salary. The strict concavity of U reflects his risk-aversion. The principal is risk-neutral, caring only about the actuarial value of his profits, net of the payments to the agent; he faces no bankruptcy constraints.

The discounted (average) value to the agent of an infinite sequence $\{(c^t, a^t)\}$ of compensations and actions is

$$\frac{1-\delta}{\delta} \sum_{t=1}^{\infty} \delta^t [U(c^t) - d^t] ,$$

where d^t is the disutility corresponding to action a^t and $\delta \in (0,1)$ is the discount factor. Similarly, the principal's value of the stream $\{(\pi^t, c^t)\}$ of outcomes and compensations is

$$\frac{1-\delta}{\delta} \sum_{t=1}^{\infty} \delta^t [\pi^t - c^t] .$$

Explicit Contracts

At the beginning of each period t, $t = 1,2,\ldots$, the principal offers the agent a legally enforceable contract (called an explicit contract) making payment contingent on the outcome and/or the agent's participation. Thus an explicit contract is a vector $s \in \mathbb{R}_+^{m+1}$, where

s_0 = payment to the agent if he takes action a_0 (doesn't participate)

s_k = payment guaranteed to the agent when he participates and outcome k arises, $k = 1,\ldots,m$.

The guaranteed payments s_k are called salaries, and are received by the agent at the end of the period once output is realized. The principal may then choose to supplement the salary with some bonus $b_k \geq 0$, so that the agent's total compensation when π_k occurs is

$c_k := s_k + b_k$. Allowing for such gifts introduces the possibility that in equilibrium, the agent's consumption may depend not only on the verifiable outcome, but also on his choice of action. (One can show that no purpose is served by considering two-sided gift-giving: as we explain later, for any equilibrium in which the agent voluntarily returns some of his salary in some contingency, there exists another equilibrium with the same actions and patterns of consumption in which this does not occur. It is clearly redundant to allow for a bonus corresponding to nonparticipation; such a bonus could be incorporated into the salary.)

It is convenient to introduce a "public randomization device" as follows: at the beginning of each period, before the contract is offered, the players commonly observe the realization of a random variable uniformly distributed on the interval [0,1]. The random variables are independent (across time). A <u>strategy for the agent</u> in the supergame specifies for each t the action to be taken by the agent in period t as a measurable function of the entire history of the game until that time, including realizations of the randomization devices and gross profits, the contracts offered by the principal in periods 1 to t inclusive, the agent's previous choices of action, and the bonuses paid. A <u>strategy for the principal</u> specifies for each t the explicit contract to be offered as a measurable function of everything that has happened in the first $t - 1$ periods and the outcome of the period t randomization device, and specifies for each t the bonus to be paid as a measurable function of the history, including the period t values of the randomization device, the contract offered, the action taken, and the realization of gross profits. A pair E of strategies for the agent and the principal, respectively, induces a probability distribution over streams of actions, outcomes, and compensations, and hence yields a pair $v(E)$ of expected values for the respective players.[2]

We are interested in the pure strategy subgame perfect equilibria[3] of the repeated game, which will usually simply be called equilibria. There is no need to employ a more sophisticated definition of equilibrium because the game is one of perfect information. Denote by Γ the infinitely repeated game we have described. The equilibrium value set is

$$V := \{v(E) | E \text{ is a subgame perfect equilibrium of } \Gamma\} \subseteq \mathbb{R}^2.$$

We denote by $\hat{\Gamma}$ the (infinite-horizon) subgame of Γ that follows any realization of the public randomizing device in period 1, and let $\hat{V}$ be the equilibrium value set of $\hat{\Gamma}$. Notice that the equilibria of Γ are probability distributions over equilibria of $\hat{\Gamma}$, and hence $V = \text{co } \hat{V}$. It is convenient to abuse notation by letting $v(E)$ represent the payoff pair associated with E even when E is a profile of $\hat{\Gamma}$ rather than of Γ. Any particular value in $V \subseteq \mathbb{R}^2$ can be expressed as a convex combination of no more than three points in

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2 For some strategy profiles a player's expected value may not be finite, but such profiles cannot arise in equilibrium.

3 It can be shown that for these games, by introducing public randomization devices before each move of each player (rather than just at the beginning of periods) one renders redundant the consideration of mixed strategies. This would have been cumbersome but would not have changed the nature of our results.

$\hat{V}$; therefore, without loss of generality we henceforth restrict attention to equilibria which randomize over at most three equilibria of $\hat{\Gamma}$ at the beginning of any period.

Theorem 3 of the Appendix establishes the existence of an equilibrium of $\hat{\Gamma}$ (and hence of Γ, since the probability distribution over equilibria of $\hat{\Gamma}$ can be taken to be degenerate) and the compactness of the equilibrium value set V. The following notation is used extensively in the analysis below, and is illustrated in Figure 1. Let

$$\underline{u} := \min\{u | (u,v) \in V\}$$

$$\bar{u} := \max\{u | (u,v) \in V\}$$

$$\underline{v} := \min\{v | (u,v) \in V\}$$

$$\bar{v} := \max\{v | (u,v) \in V\}$$

$$\bar{u}^P := \max\{u | (u,\bar{v}) \in V\}$$

For each $u \in [\underline{u},\bar{u}]$ define $f(u) := \max\{v|(u,v) \in V\}$. The set $\text{upper}(V) := \{(u,f(u))|u \in [\underline{u},\bar{u}]\}$ is called the <u>upper frontier</u> of V. Since V is convex, $f : [\underline{u},\bar{u}] \to \mathbb{R}$ is <u>concave</u>. The pair $(u,v) \in V$ is <u>Pareto-efficient in V</u> if for all $(u',v') \in V$ distinct from (u,v), either $u' < u$ or $v' < v$. An equilibrium E of either Γ or $\hat{\Gamma}$ is <u>efficient relative to V</u> if $v(E)$ is Pareto-efficient in V.

(Insert Figure 1 here)

Our analysis is in the spirit of dynamic programming, and in addition owes much to Abreu (1988). It is useful to view the value to a player of an equilibrium of $\hat{\Gamma}$ as being the sum $(1-\delta)$(first period payoff) + δ (expected future value), where we mean by "expected future value" the player's payoff in equilibrium from the beginning of the second period onward. A similar decomposition is often a convenient way to check the equilibrium incentive constraints associated with a player's first-period choices. When the principal, for example, considers not paying (or "seizing") the bonus in period 1, he weighs the immediate savings against the resulting change in the expected future value. Similarly, in determining whether to deviate from the action prescribed by an implicit agreement, the agent takes into account the immediate gain, if any (including the change in the bonuses) and the change in the future payoff. The incentives to stay on the equilibrium path are strongest when a deviation is followed by the worst possible equilibrium for the deviating party in the resulting subgame. Thus we restrict attention without loss of generality to equilibria in which **the principal's expected future value if he seizes the bonus in any period is $\underline{v}$, his payoff from offering the wrong contract does not exceed $\underline{v}$ from the beginning of the current period onward,[4] and a deviating agent has expected future value $\underline{u}$ and receives bonus zero.** As noted earlier, allowing for two-sided gift-giving serves no purpose. In an equilibrium in which the principal and agent simultaneously exchange positive gifts after some history, no incentive constraint is tightened

[4] By the definition of $\underline{v}$, there cannot exist a deviation to some contract s^* such that the principal's payoff in the worst equilibrium of the ensuing subgame exceeds $\underline{v}$. But if the contract he offers uniformly promises extravagant salaries, for example, the principal's worst equilibrium continuation value could fall short of $\underline{v}$.

(and some are loosened) if the gifts are reduced by the same amount until one of them is zero. If it is the agent's gift that remains positive, it cannot exceed his salary (consumption is constrained to be non-negative); reduce the salary by the amount of the gift, and change the agent's gift to zero. Incentives for conforming to the equilibrium are preserved, and equilibrium payoffs are unchanged.

To demonstrate the existence of an equilibrium of $\hat{\Gamma}$ with some desired properties, it is often easiest to specify the equilibrium path in the first period (the contract s offered, the agent's action a_i, and the bonuses b_k, $k=1,\ldots,m$) and expected future values $(u_k,v_k) \in V$, $k = 0,\ldots,m$, where (u_0,v_0) is the expected value following nonparticipation.[5] If the players' incentive constraints are satisfied in period 1 (when they believe that conformity results in the future values (u_k,v_k) and deviations are met with the severest "punishments" described above), then it is easy to check that there exists an equilibrium of $\hat{\Gamma}$ with first-period path (s,a_i,b) and continuation values (u_k,v_k). For any profile E of $\hat{\Gamma}$, we denote the components of the first-period path induced by E, by

$\sigma(E)$:= the contract offered

$\alpha(E)$:= the action taken by the agent

$\beta(E)$:= the vector of bonuses $\beta_k(E)$, $k = 1,\ldots,m$.

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[5] Although the expected future value following nonparticipation could, in some equilibrium, depend on current realized profits, only the average (u_0,v_0) of these expected values is relevant.

Our goal is to characterize the (constrained) efficient equilibria of the agency supergame. For δ very near 1 or 0, this is quite simple. If players are extremely impatient, their future payoffs are of little concern to them, so self-enforcing agreements collapse: the solution coincides in every period with that of the static agency problem. If instead players are sufficiently patient, the folk-theorem of Fudenberg and Maskin (1986) implies that the first-best payoffs can be approximated in equilibria of $\hat{\Gamma}$, without recourse to explicit contracts. We are interested in the intermediate cases in which some implicit cooperation can be sustained, but incentives pose a substantial constraint. While the results presented below appear to characterize only the _first_ period of an efficient equilibrium, they hold at every point on the equilibrium path. This follows from the fact that _an efficient equilibrium induces, after any t-period equilibrium history, a continuation equilibrium that is itself efficient_: when an inefficient continuation equilibrium is replaced by a Pareto-superior equilibrium (without changing what is specified after any alternative history), incentives to conform to equilibrium play are improved (or at worst unchanged). The result is not immediate in other agency environments; this issue is explored in Fudenberg, Holmstrom and Milgrom (1987).

A central feature of optimal implicit agreements is the way in which the agent's rewards are divided among salaries, bonuses, and expected future values. Considerations of efficiency place some powerful restrictions on the pattern of rewards following any t-period history h, as long as players are not so patient that incentives compatibility is consistent with the agent's receiving a constant compensation (regardless of realized profits) in period $t + 1$ following h. Notice that if we could ignore the principal's temptation to seize the bonus, it would always be useful to decrease some salary and increase the corresponding bonus by the same amount, leaving players' payoffs unchanged but strengthening the agent's incentive to take the appropriate action (since one of the bonuses he might lose by cheating is now larger). This strict incentive compatibility would allow an adjustment of salaries that would decrease the variation in compensation received in the period in question, shifting risk from the agent to the principal. Taking the principal's incentives into account, one sees that a bonus can be increased only until it equals the wedge between the principal's expected future value after paying the bonus, and the worst punishment value $\underline{v}$ he can be given for seizing the bonus. Thus, an equilibrium of $\hat{\Gamma}$ is inefficient if the bonuses are not equal to the appropriate "wedges"; Theorem 4 makes this precise.

<u>Theorem 4</u>:

Let E be an equilibrium of $\hat{\Gamma}$. Let $s := \sigma(E)$, $a_i := \alpha(E)$, $b := \beta(E)$, and $(u_k, v_k) \in V$, $k = 0,\ldots,m$, be its expected future values. Suppose

i) $i \neq 0$

ii) there exists ℓ such that $b_\ell \neq [\delta/(1-\delta)](v_\ell - \underline{v})$ and $s_\ell > 0$

iii) consumption $c_k := s_k + b_k$ is not constant in k.

Then there exists an equilibrium E^* that Pareto dominates E:

$$\nu_A(E^*) > \nu_A(E) \quad \text{and} \quad \nu_P(E^*) = \nu_P(E) \ .$$

Proof:

Note that $b_k \leq [\delta/(1-\delta)](v_k - \underline{v})$ for each $k = 1,\ldots,m$. Otherwise, for some k, seizing the bonus b_k would be a profitable deviation for the principal: the adverse effect on his expected future equilibrium payoff is at most $[\delta/(1-\delta)](v_k - \underline{v})$. Hence, (ii) implies $b_\ell < [\delta/(1-\delta)](v_\ell - \underline{v})$.

We first construct an equilibrium E' as follows: $\sigma(E') := s'$, $\alpha(E') := a_i$, $\beta(E') := b'$, and the continuation profiles (from the second period onwards) are equilibria of Γ with values (u_k,v_k), $k = 0,\ldots,m$, where

$$s'_k := \begin{cases} s_k & \text{if } k \neq \ell \\ s_\ell - \varepsilon & \text{if } k = \ell \end{cases} \quad , \quad b'_k := \begin{cases} b_k & \text{if } k \neq \ell \\ b_\ell + \varepsilon & \text{if } k = \ell \end{cases} ,$$

and ε is any number in $(0,\min\{s_\ell, [\delta/(1-\delta)](v_\ell - \underline{v}) - b_\ell\})$. It is easily checked that E' is indeed an equilibrium, and that $\nu(E') = \nu(E)$. However, in E' the agent strictly prefers a_i to any other action a_j with $j \neq 0$: his payoff from choosing a_i is the same as in E, whereas deviating to a_j entails a greater loss in bonus when π_ℓ occurs, and $p_{j\ell} > 0$.

Note that $c_k = s'_k + b'_k$ for each $k = 1,\ldots,m$. Let

$$w := \sum_{k=1}^{m} p_{ik}U(c_k) \quad \text{and} \quad \bar{c} := \sum_{k=1}^{m} p_{ik}c_k$$

Define the contract s^* by

$$s_k^* + b_k' = (1-\lambda)c_k + \lambda\bar{c} \qquad k = 1,\ldots,m,$$

where $\lambda \in (0,1)$ is to be determined below. The consumption distribution $c_k^* := s_k^* + b_k'$, $k = 1,\ldots,m$, is "smoother" than c: the movement from c^* to c is a mean-preserving spread. Consider a profile E^* such that $\sigma(E^*) = s^*$, $\alpha(E^*) = a_i$, $\beta(E^*) = b'$, and with continuation profiles that are equilibria with values (u_k, v_k), $k = 0,\ldots,m$. We claim that E^* is an equilibrium. Since in the first period of E' the agent strictly prefers a_i to any other action a_j, $j \neq 0$, for sufficiently small $\lambda > 0$, this remains true in E^* (recall that the action set A is finite). By assumption, the first period compensation c of E is not constant, therefore $U(\bar{c}) > w$ and

$$w^* := \sum_{k=1}^{m} p_{ik}U(c_k^*) > (1-\lambda) \sum_{k=1}^{m} p_{ik}U(c_k) + \lambda U(\bar{c}) > w .$$

Hence $\nu_A(E^*) > \nu_A(E') = \nu_A(E)$. Incentives to pay the first-period bonuses are the same as in the equilibrium E', because for each k, the size of the bonus b_k' and the wedge $v_k - \underline{v}$ are the same in E^* and E'. Since $\bar{c}^* := \Sigma_{k=1}^{m} p_{ik}c_k^* = \bar{c}$, we have $\nu_P(E^*) = \nu_P(E') = \nu_P(E)$, and the principal's payoff from conforming and deviating, respectively, are not changed in the transition from E' to E^*. Thus E^* is an equilibrium with the required properties. Q.E.D.

In the notation of Theorem 4, if $v_\ell > \underline{v}$ and $c_\ell > 0$, <u>the bonus b_ℓ can be assumed (strictly) positive</u> without loss of generality even if condition (iii) does not hold. (If b_ℓ is zero, s_ℓ must be positive; s_ℓ can be decreased slightly and b_ℓ correspondingly increased without violating incentive constraints.) One way of guaranteeing that compensation is non-zero in equilibrium is to impose the following restriction on the agent's utility function, which will be in force for the remainder of this section.

<u>Assumption</u>: (A4) $\lim_{c \to 0} U(c) = -\infty$.

<u>Definition</u>: Let C be the inverse of the utility function U. (C can be viewed as a cost function.)

If any salary offered on the equilibrium path were zero, the agent's incentive constraints in the corresponding contingency would be slack: taking the wrong action results with positive probability in a consumption of zero, which has utility $-\infty$. Consequently, the salary could be increased slightly and the bonus reduced by an equal amount, while maintaining the correct incentives. Without loss of generality, then, we confine attention to equilibria in which <u>all salaries offered on the equilibrium path are positive</u>.

We simplify the statements of Proposition 2 and Theorem 5 below by assuming that the function f defined in Figure 1 is differentiable (at the relevant point); the discussion following the proof of Theorem 5 indicates the nature of the results in the absence of differentiability. Proposition 2 provides another link between current and future rewards, essentially stating that the marginal cost to the principal of increasing the agent's utility in some contingency should be the same whether he gives extra compensation today, or forgoes some profit tomorrow (moving clockwise along the efficient frontier of V). The result is similar to Proposition 1 of Rogerson (1985).

Proposition 2:

Let E be an equilibrium of $\hat{\Gamma}$, efficient relative to V. Let $s := \sigma(E)$, $a_i := \alpha(E)$, and $b := \beta(E)$, and suppose that $i \neq 0$ and E has continuation values $(u_k, v_k) \in V$, $k = 0,\dots,m$. Let $c_k := s_k + b_k$, $k = 1,\dots,m$. Then for each $k = 1,\dots,m$ for which $\underline{u} < u_k < \bar{u}$,

$$C'\big(U(c_k)\big) = -f'(u_k) \ .$$

Proof:

Let $(u,v) := \nu(E)$. Since E is efficient, $v = f(u)$ and $v_k = f(u_k)$ for each $k = 1,\dots,m$. Let ℓ be such that $\underline{u} < u_\ell < \bar{u}$. Then since $v_\ell = f(u_\ell)$ and f is concave, $v_\ell > \underline{v}$. Therefore it is unrestrictive to assume $b_\ell > 0$.

By contradiction, assume that $C'(U(c_\ell)) > -f'(u_\ell)$ (the case $C'(U(c_\ell)) < -f'(u_\ell)$ is similar). Let Δ be a small positive number. Define

$$b_k^* := \begin{cases} b_k & \text{if } k \neq \ell \\ b_\ell - \Delta & \text{if } k = \ell \end{cases}, \quad u_k^* := \begin{cases} u_k & \text{if } k \neq \ell \\ u_\ell^* & \text{if } k = \ell \end{cases},$$

$$v_k^* := f(u_k^*) \qquad k = 0,\dots,m,$$

where u_ℓ^* solves the equation

$$(1-\delta)U(s_\ell + b_\ell^*) + \delta u_\ell^* = (1-\delta)U(s_\ell + b_\ell) + \delta u_\ell \ . \tag{1}$$

Let E^* be a profile with $\sigma(E^*) = s$, $\alpha(E^*) = a_i$, $\beta(E^*) = b^*$, and equilibrium continuation profiles with values (u_k^*, v_k^*), $k = 0,\ldots,m$. Define $U_\ell := U(c_\ell)$ and $U_\ell^* := U(s_\ell + b_\ell^*)$. We have

$$v_\ell^* = f(u_\ell^*) \doteq f(u_\ell) + f'(u_\ell)(u_\ell^* - u_\ell)$$

$$= v_\ell + f'(u_\ell)[(1-\delta)/\delta](U_\ell - U_\ell^*) \qquad \text{(from (1))}$$

$$\doteq v_\ell + \frac{1-\delta}{\delta} f'(u_\ell)U'(c_\ell)\Delta$$

$$= v_\ell + \frac{1-\delta}{\delta} \frac{f'(u_\ell)}{C'(U_\ell)} \Delta \; > \; v_\ell - \frac{(1-\delta)}{\delta} \Delta \qquad \text{(by assumption)}$$

The expected values to the principal of deviating in the first period of E^* are the same as they were in E; thus he has no incentives to deviate from E^* if his payoff from conformity is no lower in E^* than in E. This is the case, because

$$\delta v_\ell^* - (1-\delta)b_\ell^* > \delta\left(v_\ell - \frac{(1-\delta)}{\delta} \Delta\right) - (1-\delta)(b_\ell - \Delta) = \delta v_\ell - (1-\delta)b_\ell ,$$

so $\nu_P(E^*) > \nu_P(E)$. The agent has the same incentives in E^* as in E to take action a_i: equation (1) implies that action a_i is equally lucrative in the profiles E^* and E. Therefore E^* is an equilibrium and $\nu_A(E^*) = \nu_A(E)$. This is a contradiction: $\nu(E)$ is not efficient in V. Q.E.D.

Theorem 5 describes precisely how the explicitly and implicitly promised rewards are used in combination to create the appropriate incentives for the agent. Suppose that it is desirable to give the agent greater total rewards following some history h (ending in some profit realization) than following another history h'. The current compensation, the salary, and the agent's expected future payoff are all higher after h than after h'. But the bonus moves in the opposite direction: the more an agent is being rewarded, the smaller is the bonus he receives. Although this initially sounds counterintuitive, note that when an agent is being rewarded, his (guaranteed) salary is relatively high, and this is only partially offset by a low bonus. The variation in the bonuses has a moderating effect on total compensation _when the agent conforms to the implicit agreement_; this enhances efficiency without threatening incentives. The differences between contractual guarantees and compensation actually paid can be viewed as _anticipated_ renegotiations of the terms of payment.

Theorem 5:

Adopt the assumptions and notation of Proposition 2, and suppose that compensation c_k is not constant. Then salaries s_k _must_ be positive, and the following statements are equivalent:

(i) $c_k < c_\ell$

(ii) $u_k < u_\ell$

(iii) $v_k > v_\ell$

(iv) $b_k > b_\ell$

(v) $s_k < s_\ell$.

Proof:

If s_k were zero for some k, the agent's payoff from deviating in period 1 would be $-\infty$, and hence none of the agent's incentive constraints (not including the participation constraint) would be binding. Then compensation could be smoothed slightly (as in Theorem 4) thereby improving the agent's payoff without violating incentive compatibility. This contradicts the fact that E is efficient relative to V.

From Proposition 2, $C'(U(c_q)) = -f'(u_q)$, $q = k,\ell$, and since U is strictly concave, C' is strictly increasing. Therefore (i) and (ii) are equivalent. Since E is efficient relative to V, (u_q,v_q) is an efficient point of V, $q = k,\ell$, so (ii) is equivalent to (iii). Theorem 4 implies that $b_q = v_q - \underline{v}$ for $q = k,\ell$, hence (iii) is equivalent to (iv). Also, $s_q = c_q - b_q$, so (i) (and (iv)) implies (v). Finally, suppose $c_k \geq c_\ell$. Then $b_k \leq b_\ell$ and $s_k = c_k - b_k \geq c_\ell - b_\ell = s_\ell$. Hence [not (i)] implies [not (v)]. Q.E.D.

Without invoking (A4) or differentiability of the efficient frontier of the equilibrium value set, one can obtain somewhat less tidy versions of the results above. The equality $-C'(U(c_k)) = f'(u_k)$ in Proposition 2 is replaced by the statement that $-C'(U(c_k))$ lies in the subdifferential of f at u_k. Consequently, it is possible for two different compensations to correspond to the same expected future payoff. Nonetheless, a slight modification of Theorem 5 still applies: if any one of the conditions (i) through (v) (in the statement of the theorem) holds (with strict inequality as before), the other four conditions must be satisfied as weak inequalities. Thus, a comparison of rewards in two equilibrium contingencies will never reveal bonuses moving in the same direction as salaries, compensations, or expected future values.

4. Conclusion

In a repeated agency model in which the principal has better information than the courts regarding the agent's actions, optimal cooperation between the players requires the use of both explicit and implicit agreements. This paper illustrates the interplay of external and self-sustaining enforcement mechanisms by studying the constrained efficient equilibria of supergames based on a particular, fairly standard agency model. The equilibria conform to a simple pattern. When an equilibrium history calls for the agent to be rewarded generously, his guaranteed salary is high, his total current compensation (salary plus bonus) is high, his expected future payoff is high, and the bonus (the voluntary component of the payment made by the principal) is low. The presence of the bonuses helps to discourage the agent from cheating, and their variation (across contingencies) partially smooths the risk-averse agent's consumption.

The discrepancy between realized and contractually guaranteed payments can be viewed as a kind of renegotiation. We are currently working on a model with long-term explicit contracts which we conjecture exhibits a more involved form of renegotiation: long-term contingent contracts are frequently replaced on the equilibrium path by new long-term contracts, despite the fact that the eventually realized terms of the relationship could have been specified in the first contract that was offered. While renegotiation of explicit contractual arrangements can be explained by appealing to various problems of complexity and incomplete information, we wish to emphasize that it also emerges naturally as a way of exploiting multiple equilibria to provide incentives as efficiently as possible.

APPENDIX

We show here that the equilibrium value set V of the supergame is nonempty and compact. This appendix draws extensively on results in Abreu, Pearce and Stacchetti (1986, 1987). The proofs of Theorem 1 and 2 below are omitted; they are straightforward modifications of the self-generation and factorization theorems found in those papers. The reader is directed to Abreu, Pearce and Stacchetti (1987) for a more formal and detailed account.

The definition of Admissibility below captures all incentive constraints that an equilibrium of the repeated game must satisfy in the first period. An equilibrium E of $\hat{\Gamma}$ is factorizable into its first period recommendation $(s,a_i,b) \in \mathbb{R}_+^{m+1} \times A \times \mathbb{R}_+^m$ to the players, and the values $(u_k,v_k) \in V$, $k = 0,\ldots,m$, of the strategies induced by E on the subgames beginning in period 2. Since each subgame beginning in period 2 is identical to Γ, the strategies induced by E on these subgames must be equilibria of Γ, and therefore their values must be in $V = \text{co}\ \hat{V}$. Initially we do not know the set $\hat{V}$, so we draw values from an arbitrary set $W \subseteq \mathbb{R}^2$, and use them as if they were equilibrium values of $\hat{\Gamma}$. Thus, the conditions (i) - (iv) of Admissibility have the following interpretation:

(i) Continuation values implicitly promised in the first period are equilibrium values.

(ii) The agent has no incentives to deviate in the first period: $F_A^*(s,a_i,b,u)$ is the supergame payoff he expects in equilibrium, and $F_A(s,a_j)$ is his expected payoff when he chooses action a_j instead.

(iii) The bonus b_k promised in equilibrium is no greater than the wedge between the principal's expected future value and the worst punishment value.

(iv) The principal has no incentive in the first period to offer a contract s' different from the contract s prescribed by the equilibrium: $F_P^*(s,a_i,b,v)$ is the supergame payoff he expects in equilibrium, and $F_P(s')$ is his expected payoff if he offers contract s' instead.

<u>Definition</u>: Admissibility

Assume $W \subseteq \mathbb{R}^2$ is bounded, and let

$$\underline{w}_A := \inf \{w_A | (w_A,w_P) \in W \text{ for some } w_P\},$$

$$\underline{w}_P := \inf \{w_P | (w_A,w_P) \in W \text{ for some } w_A\}.$$

The tuple $(s,a_i,b,u,v) \in \mathbb{R}_+^{m+1} \times A \times \mathbb{R}_+^m \times \mathbb{R}^{m+1} \times \mathbb{R}^{m+1}$ is <u>admissible w.r.t. W</u> if:

i) $(u_k,v_k) \in \text{co } W$ for each $k = 0,1,\ldots,m$.

ii) $F_A^*(s,a_i,b,u) \geq F_A(s,a_j)$ for each $j \neq i$, where

$$F_A^*(s,a_i,b,u) := \begin{cases} \sum_{k=1}^{m} p_{ik}[(1-\delta)U(s_k+b_k) + \delta u_k] - (1-\delta)d_i & \text{if } i \neq 0 \\ (1-\delta)U(r+s_0) + \delta u_0 - (1-\delta)d_0 & \text{if } i = 0 , \end{cases}$$

and

$$F_A(s,a_j) := \begin{cases} \sum_{k=1}^{m} p_{jk}[(1-\delta)U(s_k) + \delta \underline{w}_A] - (1-\delta)d_j & \text{if } j \neq 0 \\ (1-\delta)U(r+s_0) + \delta \underline{w}_A - (1-\delta)d_0 & \text{if } j = 0 . \end{cases}$$

iii) $(1-\delta)b_k \leq \delta(v_k - \underline{w}_p)$ for each $k = 1,\ldots,m$.

iv) $F_p^*(s,a_i,b,v) \geq F_p(s')$ for all $s' \in \mathbb{R}_+^{m+1}$, where

$$F_p^*(s,a_i,b,v) := \begin{cases} \sum_{k=1}^{m} p_{ik}[\delta v_k + (1-\delta)(\pi_k - s_k - b_k)] & \text{if } i \neq 0 \\ \delta v_0 + (1-\delta)\left(\sum_{k=1}^{m} p_{0k}\pi_k - s_0\right) & \text{if } i = 0 , \end{cases}$$

and $F_p(s')$ is the optimal value of the following optimization problem:

$$\inf F_p^*(s',a_j,b',v')$$

$$\text{s.t. } a_j \in A,\ b' \in \mathbb{R}_+^m,\ (u_k',v_k') \in \text{co } W \text{ for each } k=1,\ldots,m,$$

$$F_A^*(s',a_j,b',u') \geq F_A(s',a_\ell) \text{ for each } \ell \neq j$$

$$(1-\delta)b_k' \leq \delta(v_k' - \underline{w}_p) \text{ for each } k = 1,\ldots,m.$$

The value of a tuple $(s,a,b,u,v) \in \mathbb{R}_+^{m+1} \times A \times \mathbb{R}_+^m \times \mathbb{R}^{m+1} \times \mathbb{R}^{m+1}$ is $F^*(s,a,b,u,v) = (F_A^*(s,a,b,u),F_p^*(s,a,b,v))$.

Definition:

For each $W \subseteq \mathbb{R}^2$ bounded, let

$$B(W) := \{F^*(s,a,b,u,v) | (s,a,b,u,v) \text{ is admissible w.r.t. } W\}.$$

Proposition 1:

If $W \subseteq \mathbb{R}^2$ is compact, $B(W)$ is compact.

Proof:

The reader may check that if W is bounded, $B(W)$ is bounded. Hence, we show that $B(W)$ is closed. Let $\{(s^q,a^q,b^q,u^q,v^q)\}$ be a sequence of admissible tuples w.r.t. W such that $F^*(s^q,a^q,b^q,u^q,v^q) =: w^q \to w^*$.

We will show that $w^* \in B(W)$. Since W is bounded, $\{u^q\}$, $\{v^q\}$ and $\{b^q\}$ are bounded (for the latter use condition (iii) of admissibility), and therefore we can assume w.l.o.g. that $u^q \to u^*$, $v^q \to v^*$, and $b^q \to b^*$. Since A is finite, we can assume that $a^q = a_i$ for all q. Finally, if $i = 0$, condition (iv) of admissibility implies that $\{s_0^q\}$ is bounded, and w.l.o.g. we assume that $s_k^q = 0$ for each $k = 1,\dots,m$ and $q \geq 1$. Similarly, if $i \neq 0$, w.l.o.g. we assume that $s_0^q = 0$ for all $q \geq 1$, and condition (iv) implies that $\{s^q\}$ is bounded. Therefore, in either case we can assume that $\{s^q\}$ is bounded and that $s^q \to s^*$.

Because co W is compact, $(u_k^*, v_k^*) \in$ co W, and since $b_k^q \leq v_k^q - \underline{w}_P$ for all q, $b_k^* \leq v_k^* - \underline{w}_P$ for all $k = 1,\dots,m$. Clearly, $F_A^*(s,a,b,u)$ is continuous in (s,b,u) and $F_A(s,a)$ is continuous in s. Hence, for all $j \neq i$, $F_A^*(s^q,a_i,b^q,u^q) \geq F_A^*(s^q,a_j)$ for all q implies that $F_A^*(s^*,a_i,b^*,u^*) \geq F_A(s^*,a_j)$. Finally $F_P^*(s,a,b,v)$ is continuous in (s,b,v) so $F_P^*(s^*,a_i,b^*,v^*) \geq F_P(s')$ for all s'. Thus (s^*,a_i,b^*,u^*,v^*) is admissible w.r.t. W, and since by continuity $w^* = F^*(s^*,a_i,b^*,u^*,v^*)$, $w^* \in B(W)$. Q.E.D.

<u>Definition</u>:

$W \subseteq \mathbb{R}^2$ is self-generating if $W \subseteq$ co $B(W)$.

<u>Theorem 1</u>: Self-Generation

If $W \subseteq \mathbb{R}^2$ is <u>compact</u> and self-generating, then co $B(W) \subseteq \hat{V}$.

The following theorem corresponds to the factorization theorem in Abreu, Pearce and Stacchetti (1987). The converse inclusion is stated below in the Corollary of Theorem 3. However, to show the converse inclusion it is first necessary to establish, for example, that the worst continuation values for the agent and the principal are attained; this is implied by the fact that V is compact.

Theorem 2:

$V \subseteq \text{co } B(V)$.

Let (s^*,a^*,b^*) be the path of a Nash equilibrium of the principal-agent component game described in Section 2. (Clearly we can assume $b^* = 0$ w.l.o.g.) Let E^* be the profile of $\hat{\Gamma}$ specifying that in every period, after any history, the principal offers s^* and pays no bonuses, and the agent takes a myopic best response to the current contract in that period (if there is more than one, choose one that is best for the principal). Suppose $a^* = a_i$, and let

$$u^* := \begin{cases} \sum_{k=1}^{m} p_{ik}U(s_k^*) - d_i & \text{if } i \neq 0 \\ U(s_0^* + r) - d_0 & \text{if } i = 0 \end{cases}$$

$$v^* := \begin{cases} \sum_{k=1}^{m} p_{ik}[\pi_k - s_k^*] & \text{if } i \neq 0 \\ \sum_{k=1}^{m} p_{0k}[\pi_k - s_0^*] & \text{if } i = 0 . \end{cases}$$

It is easy to see that E^* is an equilibrium of $\hat{\Gamma}$ with value $\nu(E) = (u^*,v^*)$. Therefore $\hat{V} \neq \emptyset$.

Let $\bar{\pi} := \max_{1 \leq i \leq n} \sum_{k=1}^{m} p_{ik}\pi_k$, and recall that by A3, $0 = \min_{0 \leq i \leq n} \sum_{k=1}^{m} p_{ik}\pi_k = \sum_{k=1}^{m} p_{0k}\pi_k$. The principal can guarantee himself a supergame payoff of 0 by offering the contract $s \equiv 0$ in every period. On the other hand, the best outcome he can ever expect is that in every period the agent chooses the best action for the principal and receives no payment. This gives the principal an expected supergame payoff of $\bar{\pi}$. Let $\underline{p} := \min \{p_{ik} | 1 \leq i \leq n, 1 \leq k \leq m\}$ and $\underline{d} := \min \{d_i\}$. Since the principal knows that his continuation value, after any history, is in the interval $[0,\bar{\pi}]$, we can assume he will never offer a contract s having $s_0 > \delta\bar{\pi}/(1-\delta)$ or $s_k > \delta\bar{\pi}/[\underline{p}(1-\delta)]$ for some $k = 1,\ldots,m$. The agent can guarantee himself a salary r in every period by taking his alternative job in every period. Hence, for each $(u,v) \in \hat{V}$,

$$U(r) - d_0 \leq u \leq U\left(\frac{\delta\bar{\pi}}{\underline{p}(1-\delta)}\right) - \underline{d} \quad \text{and} \quad 0 \leq v \leq \bar{\pi} \, .$$

Since $V = \text{co}\,\hat{V}$, V is nonempty and bounded.

In the next theorem we use the fact that B is monotone in the sense that if $W \subseteq W' \subseteq \mathbb{R}^2$, then $B(W) \subseteq B(W')$.

<u>Theorem 3</u>:

V is a nonempty compact set.

<u>Proof</u>:

We need only show that V is closed. We have $V \subseteq \text{co}\,B(V) \subseteq \text{co}\,B(c\ell\,V)$, and since V is bounded, $c\ell\,V$ is compact and $\text{co}\,B(c\ell\,V)$ is compact. Hence, $c\ell\,V \subseteq \text{co}\,B(c\ell\,V)$, and by self-generation, $c\ell\,V \subseteq V$. Therefore, V is closed. Q.E.D.

<u>Corollary</u>:

$V = \text{co}\,B(V)$.

REFERENCES

Abreu, D. (1988), "On the Theory of Infinitely Repeated Games with Discounting", Econometrica 56, 383-396.

Abreu, D., D. Pearce and E. Stacchetti (1986), "Optimal Cartel Equilibria with Imperfect Monitoring", Journal of Economic Theory 39, 251-269.

Abreu, D., D. Pearce and E. Stacchetti (1987), "Toward a Theory of Discounted Repeated Games with Imperfect Monitoring", mimeo, Stanford University.

Fudenberg, D., B. Holmstrom and P. Milgrom (1987), "Short-Term Contracts and Long Term Agency Relationships", mimeo, Yale University.

Fudenberg, D. and E. Maskin (1986), "The Folk Theorem in Repeated Games with Discounting or with Incomplete Information", Econometrica 54, 533-554.

Green, J. and J.-J. Laffont (1987), "Renegotiation and the Form of Efficient Contracts", Discussion Paper No. 1338, Harvard Institute of Economic Research.

Grossman, S. and O. Hart (1986), "The Costs and Benefits of Ownership: A Theory of Vertical Integration", Journal of Political Economy 94, 691-719.

Hart, O and J. Moore (1988), "Incomplete Contracts and Renegotiation", Econometrica 56, 755-786.

Huberman, G. and C. Kahn (1986), "Strategic Renegotiation and Contractual Simplicity", mimeo.

Maskin, E. (1977), "Nash Equilibrium and Welfare Optimality", forthcoming in Mathematics of Operations Research.

Radner, R. (1985), "Repeated Principal-Agent Games with Discounting", Econometrica 53, 1173-1198.

Rogerson, W. (1985), "Repeated Moral Hazard", Econometrica 53, 69-76.

Rubinstein, A. (1979), "Offenses that May Have Been Committed by Accident--An Optimal Policy of Retribution", in Applied Game Theory, ed. by S.J. Brams, A. Schotter, and G. Schwödiauer. Würzburg: Physica-Verlag.

Tirole, J. (1986), "Procurement and Renegotiation", Journal of Political Economy 94, 235-259.

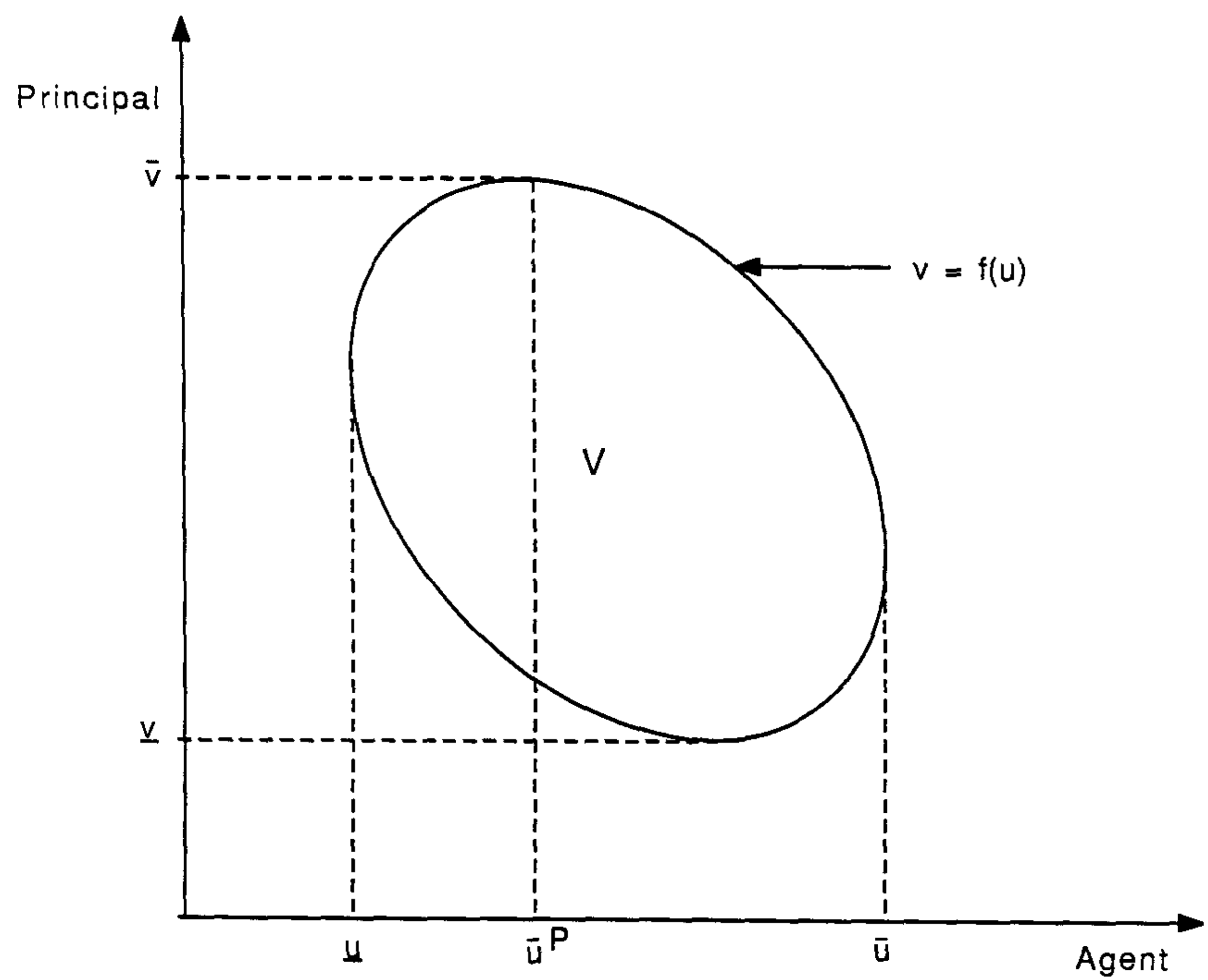

Figure 1