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GAME THEORY AND EXPERIMENTAL GAMING

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by

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1. SCOPE

This article deals with experimental games as they pertain to game theory. As such there is a natural distinction between experimentation with abstract games devoted to testing a specific hypothesis in game theory and games with a scenario from a discipline such as economics or political science where the game is presented in the context of some particular activity.

John Kennedy, professor of Psychology at Princeton and one of the earliest experimenters with games suggested that if you could tell him the results you wanted and give him control of the briefing he could get you the results.

Context appears to be critical in the study of behavior in games. R. Simon, in his Ph.D thesis (1967) controlled for context. He used the same two person zero sum game with three different scenarios. One was abstract, one described the game in a business context and the other in a military context. The players were selected from majors in business and in military science. As the game was a relatively simple two person zero sum game a reasonably strong case could be made out for the maxmin solution concept. The performance of all students was best in the abstract scenario. Some of

^{*}I would like to thank the Pew Charitable Trust for its generous support.

the military science students complained about the simplicity of the military scenario and the business students complained about the simplicity of the business scenario.

In teaching the basic concepts of game theory, for several years I have used several simple games. In the first lecture when most of the students have had no exposure to game theory I present them with three matrices, the Prisoner's Dilemma, Chicken and the Battle of the Sexes (see Figures 1a, b and c). The students are asked if they have had any experience with these

A	1	2	B	1	2	C	1	2
1	5,5	-9,10	1	-10,-10	5,-5	1	2,0	0,0
2	10,-9	0,0	2	-5,5	0,0	2	0,0	1,2

FIGURE 1

games. If not, they are asked to associate the name with the game. Table 1 shows the data for 1988.

	Battle of Chicken	Prisoner's the Sexes	Dilemma
Chicken	7	11	8
Battle of the Sexes	9	12	8
Prisoner's Dilemma	10	7	9

TABLE 1

There appears to be no significant ability to match the words with the

matrices.¹

There are several thousand articles on experimental matrix games and several hundred on experimental economics alone, most of which can be interpreted as pertaining to game theory as well as to economics. Many of the business games are multipurpose and contain a component which involves game theory. War gaming, especially at the tactical level and to some extent at the strategic level, has been influenced by game theory [see Brewer and Shubik (1979)]. No attempt is made here to provide an exhaustive review of the vast and differentiated body of literature pertaining to blends of operational, experimental and didactic gaming. Instead a selection is made concentrating on the modeling problems, formal structures and selection of solution concepts in game theory.

Roth (1986) in a provocative discussion of laboratory experimentation in economics has suggested three kinds of activities for those engaged in economic experimental gaming. They are "speaking to theorists"; "searching for facts" and "whispering in the ears of princes." The third of these activities, from the viewpoint of society is possibly the most immediately useful. In the United States the princes tend to be corporate executives, politicians and generals or admirals. However the topic of operational gaming merits a separate treatment, and is not treated here.

¹The rows and columns do not all add to the same number because of some omissions among 30 students.

2. GAME THEORY AND GAMING

Before discussing experimental games which might lead to the confirmation or rejection of theory, or to the discovery of "facts" or regularities in human behavior it is desirable to consider those aspects of game theory which might benefit from experimental gaming.

Perhaps the most important item to keep in mind is the fundamental distinction between the game theory and the approach of social psychology to the same problems. Underlying a considerable amount of game theory is the concept of external symmetry. Unless stated otherwise all players in game theory are treated as though they are intrinsically symmetric in all non-specified attributes. They have the same perceptions, abilities to calculate and they have the same personalities. Much of game theory is devoted to deriving normative or behavioral solution concepts to games played by personality free players in context free environments. In particular one of the impressive achievements of formal game theory has been to produce a menu of different solution concepts which suggest ingeniously reasoned sets of outcomes to nonsymmetric games played by externally symmetric players.

In bold contrast the approach of the social psychologists has for the most part been to take symmetric games and observe the broad differences in play which can be attributed to nonsymmetric players. Personality, passion and perception count and the social-psychologist is interested to see how they count.

A completely different view from either of the above underlies much of the approach of the economists interested in studying mass markets. The power of the mass market is that it turns the social science problem of competition or cooperation among the few into a nonsocial science problem. The

message of the mass market is not that personality differences do not exist, but that in the impersonal mass market they do not matter. The large market has the individual player pitted against an aggregate of other players. When the large market becomes still larger, all concern for the others is attenuated and the individual can regard himself as playing against a given inanimate object known as the market.

2.1. The testable elements of game theory

Before plunging into a description of who has been testing what, a question which needs to be asked is what is there to be tested concerning game theory. A brief listing is given and discussed in the subsequent sections.

Context: Does context matter? Or do we believe that strategic structure in vitro will be treated the same way by rational players regardless of context? There has been much abstract experimental gaming with no scenarios supplied to the players. But there also is a growing body of experimental literature where the context is explicitly economic, political, military or other.

External symmetry and limited rationality: The social psychologists are concerned with studying decisionmakers as they are. A central simplification of game theory has been to build upon the abstract simplification of rational man. Not only are the players in game theory devoid of personality they are endowed with unlimited rationality. What is learned from Maschler's school children, Smith's or Roth's undergraduates or Battaglio's rats [Maschler (1978), Smith (1986), Roth (1983), Battaglio et al. (1985)] requires considerable interpretation to relate the experimental results with the game abstraction.

Rules of the game and the role of time: In much game theory time plays little role. In the extensive form the sequencing of moves is frequently critical, but the cardinal aspects of the length of elapsed time between moves is rarely important (exceptions are in dueling and search and evasion models). In particular one of the paradoxes in attempting to experiment with games in coalitional form is that the key representation, the characteristic functions (and variants thereof) implicitly assumes that the bargaining, communication, deal making and tentative formation of coalitions is costless and timeless. Whereas in actual experiments the amount of time taken in bargaining and discussion is often a critical limiting factor in determining the outcome.

Utility and preference: Do we carry around utility functions? The utility function is an extremely handy fiction for the mathematical economist and the game theorist applying game theory to the study of mass markets, but in many of the experiments with matrix games or market games the payoffs have tended to be money (on a performance or hourly basis or both), points or grades [see Rapoport, Guyer and Gordon (1976) for example, Mosteller and Noguee (1951)] attempted to measure the utility function of individuals with little success. In most experimental work little attempt is made to obtain individual measures before the experiment. The design difficulties and the costs make this type of pretesting prohibitively difficult to perform. Depending on the specifics of the experiment it can be argued that in some experiments all one needs to know is that more money is preferred to less; in others the existence of a linear measure of utility is important; in still others there are questions concerning interpersonal comparisons. See however Roth and Malouf (1979) for an experimental design

to account for these difficulties.

Solution concepts: Given that we have an adequate characterization of the game and the individuals about to play, what do we want to test for? Over the course of the years various game theorists have proposed many solution concepts offering both normative and behavioral justifications for the solutions. Much of the activity in experimental gaming has been devoted to considering different solution concepts.

2.2. Experimentation and representation of the game

The two fundamental representations of games which have been used for experimentation have been the strategic and cooperative form. Rarely has the formal extensive form been used in experimentation. This appears to be due to several fundamental reasons. In the study of bargaining and negotiation in general we have no easy simple description of the game in extensive form. It is difficult to describe talk as moves. There is a lack of structure and a flexibility in sequencing which weakens the analogy with games such as Chess where the extensive form is clearly defined.

In experimenting with the game in matrix form, frequently the experimenter provides the same matrix to be played several times; but clearly this is different from giving the individual a new and considerably larger matrix representing the strategic form of a multistage game. Shubik, Wolf and Poon (1974) experimented with a game in matrix form played twice and a much larger matrix representing the strategic form of the two stage game to be played once. Strikingly different results were obtained with considerable emphasis given in the first version to the behavior of the opponent on his first trial.

How big a matrix can an experimental subject handle? The answer appears

to be a 2×2 unless there is some form of special structure on the entries, thus for example Fouraker, Shubik and Siegel (1961) used a 57×25 matrix in the study of triopoly, but this matrix had considerable regularities (being generated from continuous payoff functions from a Cournot triopoly model).

3. ABSTRACT MATRIX GAMES

3.1. Matrix games

The 2×2 matrix game has been a major source for the provision of didactic examples and experimental games for social scientists with interests in game theory. Limiting ourselves to strict orderings there are $4! \times 4! = 576$ ways in which two payoff entries can be placed in each of four cells of a 2×2 matrix. When symmetries are accounted for there are 78 strategically different games which remain [see Rapoport, Guyer and Gordon (1976)]. If ties are considered in the payoffs then the number of strategically different games becomes considerably larger. Guyer and Hamburger (1968) counted the strategically different weakly ordinal games and Powers (1986) made some corrections and provided a taxonomy for these games. There are 726 strategically different games. O'Neill in unpublished notes (1987) has estimated that the lower bound on the number of different $2 \times 2 \times 2$ games is $(8!)^3 / (8 \times 6) = 1,365,590,016,000$. For the 3×3 game we have $(9!)^2 / (3!^2) \times 2$. Thus we see that the reason why the $2 \times 2 \times 2$ and the 3×3 games have not been studied, classified and analyzed in any generality is that the number of different cases is astronomical.

Before considering the general nonconstant sum game, the first and most elementary question to ask concerns the playing of two person zero and con-

stant sum games. This splits into two questions, the first is how good a predictor is the saddlepoint in two person zero sum games with a pure strategy saddlepoint. The second is how good a predictor is the maxmin mixed strategy of a player's behavior. Possibly the two earliest published works on the zero sum game were Lieberman (1960) who considered a 3 x 3 game with a saddlepoint and Morin (1960). Lieberman (1962) also considered a zero sum 2 x 2 game played against a stooge using a minimax mixed strategy. Rather than approach an optimal strategy the live player appeared to follow or imitate the stooge.

Rapoport, Guyer and Gordon (1976, Chapter 21) note three ordinally different games of pure opposition. Using 4 to stand for the most desired outcome and 1 for the least desired Game 1 below has a saddlepoint with dominated strategies for each player, Game 2 has a saddlepoint but only one player has a dominated strategy; and Game 3 has no saddle point.

	1	2		1	2		1	2
1	2	4	1	3	4	1	2	4
2	1	3	2	2	1	2	3	1

FIGURE 2

They report on one shot experiments by Frenkel (1973) with a large population of players with reasonably good support for the saddlepoints but not for the mixed strategy. The saddlepoint for small games appears to provide a good prediction of behavior. Shubik has used a 2 x 2 zero sum game with a saddlepoint as a "rationality test" prior to having students play in nonzero sum games and one can bet with safety that over 90% of the students will

select their saddlepoint strategy. The mixed strategy results are by no means that clear.

More recently O'Neill (1987) and Brown and Rosenthal (1987) have considered a repeated zerosum game with a mixed strategy solution. The O'Neill experiment had 25 pairs of subjects play a card matching game 105 times. Figure 3 shows the matrix. The + is a win for Row and the - a loss. There is a unique equilibrium with both players randomizing using probabilities of .2, .2, .2, .4, each.

		Column Player			
		1	2	3	J
Row Player	1	-	+	+	-
	2	+	-	+	-
	3	+	+	-	-
	4	-	-	-	+

FIGURE 3

The students were given \$2.50 each and at each stage the winner received 5 cents from the loser. At best experimental subjects have trouble producing an i.i.d. distribution, yet the aggregate data involving 2625 joint moves showed high consistency with the minimax strategy hypothesis (no standard deviation for any of the 16 cells was larger than .01). Brown and Rosenthal (1987) reexamined the data reanalyzed the O'Neill data obtaining "extensive evidence of serial dependence" and no general evidence that winning percentages or play approached minimax predictions.

One might argue that minimax is a normative theory which is an extension of the concept of rational behavior and that there is little to actu-

ally test beyond observing that naive players do not necessarily do the right thing. In saddle point games there is reasonable evidence that they tend to do the right thing and as the results depend only on ordinal properties of the matrices this is doubly comforting. The mixed strategies however require the full philosophical mysteries of the measurement of utility and choice under uncertainty.

It should be noted, taking a cue from Miller's (1956) classical article that unless a matrix has special structure it may not be reasonable to experiment with any size larger than 2×2 .

One of the most ambitious of research projects devoted to studying matrix games has been that of Rapoport, Guyer and Gordon (1976). In their book entitled the 2×2 game, not only do they count all of the different games, but they develop a taxonomy of all the 78 games. They begin by observing that there are 12 ordinally symmetric games and 66 nonsymmetric games. They classify them into games of complete, partial and no opposition. Games of complete opposition are the ordinal variants of two person constant sum games. Games of no opposition are classified by them as those which have the best outcome to each in the same cell. Space limitations do not permit a full discussion of this taxonomy, however it is my belief that the problem of constructing an adequate taxonomy for matrix games is valuable both from the viewpoint of theory and experimentation. In particular although much stress has been laid on the noncooperative equilibrium for games in strategic form there appear to be a host of considerations which can all be present simultaneously which contribute towards driving a solution to a predictable outcome. These may be reflected in a taxonomy. A partial list is noted here. (1) uniqueness of equilibria; (2) symmetry of

the game; (3) safety levels of equilibria; (4) existence of dominant strategies; (5) Pareto optimality; (6) interpersonal comparison and sidepayment conditions; (7) information conditions.²

Of all of the 78 strategically different 2×2 games I suggest that the following one is structurally the most conducive to the selection of its equilibrium point solution.

	1	2
1	4,4	3,2
2	2,3	1,1

The cell (1,1) will be selected not merely because it is an equilibrium point but it is unique, Pareto optimal, results from the selection of dominant strategies, is symmetric and coincides with maxmin $(P_1 - P_2)$. Rapoport et al. (1976) have this tied for first place with a closely related no conflict game. with the off diagonal entries reversed. This new game does not have the coincidence of the maxmin difference solution with the equilibrium.

The work of Rapoport, Guyer and Gordon (1976) to this date is the most exhaustive attempt to ring the changes on conditions of play as well as structure of the 2×2 game.

²In an experiment with several 2×2 games played repeatedly with low information the ability to predict the outcome depended not merely on the noncooperative equilibrium but on the coincidence of an outcome having several other properties [see Shubik, 1962].

3.2. Games in coalitional form

If one wishes to be a purist a game in coalitional or characteristic function form cannot actually be played. It is an abstraction with playing time and all other transactions costs set at zero. Thus any attempt to experiment with games in characteristic function form must take into account the distinction between the representation of the game and the differences introduced by the control and the time utilized in play.

With allowances made for the difficulties in linking the actual play with the characteristic function there have been several experiments utilizing the characteristic function especially for the three person game and to some extent for larger games. In particular the invention of the bargaining set by Aumann and Maschler (1964) was motivated by Maschler's desire to explain experimental evidence [see Maschler (1963, 1978)].

The book of Kahan and Amnon Rapoport (1984) has possibly the most exhaustive discussion and report on games in coalitional form. They have eleven chapters on the theory development pertaining to games in coalitional form, a chapter on experimental results with three person games, a chapter³ on games for $n \geq 4$ followed by concluding remarks which support the general thesis that the experimental results are not independent of normalization and of other aspects of context; that no normative game theoretic solution emerges as generally verifiable (often this does not rule out their use in special context rich domains).

It is my belief that a fruitful way to utilize a game in characteristic

³These two chapters provide the most comprehensive summary of experimental results with games in characteristic form up until the early 1980s. These include Apex games, simple games, Selten's work on the equal division core and market games.

function form for experimental purposes is to explore beliefs or norms of students and others concerning how the proceeds from cooperation in a three person game with sidepayments should be split. Any play by three individuals of a three person game in characteristic function form is no longer represented by the form but has to be adjusted for the time and resource costs of the process of play as well as controlled for personality.

In a normative enquiry Shubik (1975, 1978) has used three characteristic functions for many years. They were selected so that the first game has a rather large core; the second a single point core that is highly nonsymmetric and the third has no core. For the most part the subjects were students attending lectures on game theory who gave their opinions on at least the first game without knowledge of any formal cooperative solution

$$\text{Game 1: } v(1) = v(2) = v(3) = 0; v(12) = 1, \quad v(13) = 2, \\ v(23) = 3; \quad v(123) = 4.$$

$$\text{Game 2: } v(1) = v(2) = v(3) = 0; v(12) = 0, \quad v(13) = 0, \\ v(23) = 4; \quad v(123) = 4.$$

$$\text{Game 3: } v(1) = v(2) = v(3) = 0; v(12) = 2,5, \quad v(13) = 3, \\ v(23) = 3,5; \quad v(123) = 4.$$

Table 2 shows the percentage of individuals selecting a point in the core for games 1 and 2. The smallest sample size was 17 and largest was 50. The last column shows the choice of an even split in the game without a core.

	Percentage in Core		Percentage even split
	Game 1	Game 2	Game 3
1980	86	28.3	36
1981	86.8	5	32.5
1983	87	21.8	7.5
1984	89.5	19	—
1985	93	20	—
1988	92	11	5

TABLE 2

There was no discernable pattern for the game without a core except for a selection of the even split, possibly as a focal point? There was little evidence supporting the value or nucleolus.

When the core was "fat" and not too nonsymmetric (game 1) it was a good predictor. When it is one point but highly skewed its attractiveness is highly diminished (game 2).

3.3. Other games

One important but somewhat overlooked aspect of experimental gaming is the construction and use of simple paradoxical games to illustrate problems in game theory and to find out what happens when these games are played. Three examples are given. Hausner, Nash, Shapley and Shubik (1964) in the early 1950s constructed a game called "so long sucker" in which the winner is the survivor; in order to win it is necessary, but not sufficient to form coalitions. At some point it will pay someone to "doublecross" his partner. However to a game theorist the definition of doublecross is difficult to make precise. When John McCarthy decided to get revenge for being double-

crossed by Nash, Nash pointed out that McCarthy was "being irrational" as he could have easily calculated that Nash would have to do what he did given the opportunity.

The dollar auction game [see Shubik (1971)] is another example of an extremely simple yet paradoxical game. The highest bidder pays to win a dollar, but the second highest bidder must also pay but receives nothing in return. Experiments with this game belong primarily to the domain of social psychology and a recent book by Tegler (1980) covers the experimental work. O'Neill (1987) however has suggested a game theoretic solution to the auction with limited resources.

The third game is a simple game suggested by Rosenthal. The subjects are confronted with a game in extensive form as indicated in the game tree in Figure 4. They are told that they are playing an unknown opponent and that they are player 2 and have been given the move. They are required to select their move and justify the choice.

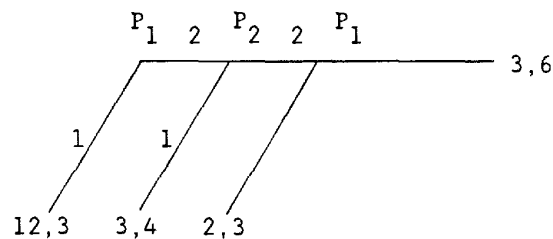


FIGURE 4

The paradox comes in the nature of the expectations that player 2 must form about player 1's behavior. If player 1 were "rational" he would have ended the game hence the move should have never come to player 2.

In a classroom exercise in 1985 17 out of 21 acting as player 2 chose to continue the game and 4 terminated giving reasons for the most part in-

volving the stupidity or the erratic behavior of the competition.

4. EXPERIMENTAL ECONOMICS

The first adequately documented running of an experimental game to study competition among firms was that of Chamberlin (1948). It was a report of a relatively informal oligopolistic market run in a class in economic theory. This was a relatively isolated event (occurring before the publication of the Nash equilibrium (1951) and the general idea of experimentation in the economics of competition did not spread for around another decade. Perhaps the main impetus to the idea of experimentation on economic competition came via the construction of the first computer based business game by Bellman et al. (1957). Within a few years business games were widely used both in business schools and in many corporations. Hoggatt (1959, 1969) was possibly the earliest researcher to recognize the value of the computer based game as an experimental tool.

In the past two decades there has been considerable work by Vernon Smith primarily using a computer based laboratory to study price formation in various market structures. Many of the results appear to offer comforting support for the virtues of a competitive price system. However it is important to consider that possibly a key feature in the functioning of an anonymous mass market is that it is designed implicitly or explicitly to minimize both the sociopsychological and the more complex game theoretic aspects of strategic human behavior. The crowning joy of economic theory as a social science is its theory of markets and competition. But paradoxically markets appear to work the best when the individuals are converted into nonsocial anonymous entities and competition is attenuated, leaving a set of

one person maximization problems.

This observation is not meant to detract from the value of the experimental work of Smith and many others who have been concerned with showing the efficiency of markets but to contrast the experimental program of Smith, Plott and others concerned with the design of economic mechanisms with the more general problems of game theory.

Are economic and game theoretic principles sufficiently basic that one might have cleaner experiments using rats and pigeons rather than humans? Hopefully animal passions and neuroses are simpler than those of humans. Kagel et al. (1975) began to investigate consumer demand for rats. The investigations have been primarily aimed at isolated individual economic behavior of animals, including an investigation of risky choice [Battaglio et al. (1985)]. Kagel (1987) offers a justification of the use of animals, noting problems of species extrapolation and cognition. He points out that (he believes that) animals unlike humans are not selected from a background of political, economic and cultural contexts, which at least to the game theorist concerned with external symmetry is helpful.

Especially given the growth of interest in game theory applications to animal behavior it would be possible and of interest to extend experimentation with animals to two-person zero and nonconstant sum games. In much of the experimental work in game theory anonymity is a key control factor. It should be reasonably straightforward to have animals (not even of the same species) in separate cages, each with two choices leading to four prizes from a 2 x 2 matrix. There is the problem as to how or if the animals are ever able to deduce that they are in a two-animal competitive situation. Further more at least if the analogy with economics is to be carried further

can animals be taught the meaning and value of money; as money figures so prominently in business games, much of experimental economics, experimental game theory and in the actual economy. I suspect that the answer is no. But how far can animal analogies be made seems to be a reasonable question.

Another approach to the study of competition has been via the use of robots in computerized oligopoly games. Hoggatt (1969) in an imaginative design had a live player play two artificial players separately in the same type of game. One artificial player was designed to be cooperative and the other competitive. The live players tended to cooperate with the former and to compete with the latter.

Shubik, Wolf and Eisenberg (1972), Shubik and Riese (1972), Liu (1973), and others have over the years run a series of experiments with an oligopolistic market game model with a real player versus either other real players or artificial players. In the SDC experiments markets with 1, 2, 3, 5, 7, and 10 live players were run to examine the prediction value of the noncooperative equilibrium and its relationship with the competitive equilibrium. The average of each of the last few periods tended to lie between the beat-the-average and noncooperative solution.

In a series of experiments run with students in game theory courses the students each first played in a monopoly game and then played in a duopoly game against an artificial player [Shubik and Sobel, manuscript in preparation]. The monopoly game posed a problem in simple maximization. There was no significant correlation between performance (measured in terms of profit ranking) between those who performed well in monopoly and those who performed well in duopoly.

Business games have been used for the most part in business school

education and corporate training courses with little if any experimental concern. Business game models have tended to be large complex and ad hoc, but games to study oligopoly have been kept relatively simple in order to maintain experimental control as can be seen by the work of Sauermann and Selten (1959); Hoggatt (1959); Fouraker, Shubik and Siegel, (1961); Fouraker and Siegel (1963); Friedman (1967, 1969); Friedman and Hoggatt (1980). A difficulty with the simplification is that it becomes so radical that the words (such as firm production cost, advertising, etc.) bear so little resemblance to the complex phenomena they are meant to evoke that experience without considerable ability to abstract may actually hamper the players.⁴

The first work in cooperative game experimental economics was the prize winning book of Siegel and Fouraker (1960) which contained a series of elegant simple experiments in Bilateral monopoly exploring the theories of Edgeworth (1881), Bowley (1928) and Zeuthen (1930). This was interdisciplinary work at its best with the senior author being a quantitatively oriented psychologist. The book begins with an explicit statement of the nature of the economic context, the forms of negotiation and the information conditions. In game theoretic terms the Edgeworth model leads to the no-sidepayment core, the Zeuthen model calls for a Nash-Harsanyi value and the Bowley solution is the outcome from a two stage sequential game with perfect information concerning the moves.

In the Siegel Fouraker experiments subjects were paid, instructed to maximize profits and were anonymous to each other. Their moves were relayed

⁴Two examples where the same abstract game can be given different scenarios are the Ph.D thesis of R. Simon, already noted and a student paper by Bulow where the same business game was run calling the same control variable advertising in one run and product development in another.

through an intermediary. In their first experiments they obtained evidence that the selection of a Pareto optimal outcome increased with increasing information about each others profits.

The simple economic context of the Siegel-Fouraker experiments combined with care to maintain anonymity gave interesting results consistent with economic theory. In contrast the imaginative and rich teaching games of Raiffa (1983) involving face to face bargaining and complex information conditions illustrate how far we are from a broadly acceptable theory of bargaining backed with experimental evidence. Roth (1983) recognizing especially the difficulties in observing bargainers preferences has devised several insightful experiments on bargaining. In particular he has concentrated on two-person bargaining where failure to agree gives the players nothing. This is a constant threat game and the various cooperative fair division theories such as those of Nash, Shapley, Harsanyi, Maschler-Perles, Raiffa, Zeuthen can all be considered.

Both in his book and in a perceptive article Roth (1979, 1983) has stressed the problems of the game theoretic approach to bargaining and the importance of and difficulties in linking theory and experimentation. The value and fair division theories noted are predicated on the two individuals knowing their own and each others' preferences. Abstractly a bargaining game is defined by a convex compact set of outcomes S and a point t in that set which represents the disagreement payoff. A solution to a bargaining game (S,t) is a rule applied to (S,t) which selects a point in S . The game theorist by assuming external symmetry concerning the personalities and other particular features of the players and by making the solution depend just upon the payoff set and the no agreement point has thrown away

most if not all of the social psychology.

Roth's experiments have been designed to allow the expected utility of the participants to be determined. Participants bargained over the probability that they would receive a monetary prize. If they did not agree within a specified time both received nothing. The experiment of Roth and Malouf (1979) was designed to see if the outcome of a binary lottery game⁵ would depend on whether the players are explicitly informed of their opponent's prize. They experimented with a partial and a complete information condition and found that under incomplete information there was a tendency towards equal division of the lottery tickets, whereas with complete information the tendency was towards equal expected payoffs.

The experiment of Roth and Murningham (1982) had one player with a \$20 prize and the other with a \$5 prize. A 4 x 2 factorial design of information conditions and common knowledge was used. (1) neither player knows his competitor's prize; (2) and (3) one does; (4) both do. In one set of instances the state of information is common knowledge, in the other it is not common knowledge.

The results suggested that the effect of information is primarily a function of whether the player with the smaller prize is informed about both. If this is common knowledge there is less disagreement than if it is not. Roth and Schoumaker (1983) considered the proposition that if the players were Bayesian utility maximizers then the previous results could indicate that the effect of information was to change player's subjective beliefs.

⁵If a player obtained 40% of the lottery tickets he would have a 40% chance of winning his personal prize and a 60% chance of obtaining nothing.

The efforts of Roth and colleagues complement the observations of Raiffa (1983) on the complexities faced in experimenting with bargaining situations. They clearly stress the importance of information. They avoid face to face complications and they bring new emphasis to the distinction that must be made between behavioral and normative approaches. Perhaps one of the key overlooked factors in the understanding of competition and bargaining is that they are almost always viewed by economists and game theorists as resolution mechanisms for individuals who know what things are worth to themselves and others. An alternative view is that the mechanisms are devices which help to clarify and to force individuals to value items where previously they had no clear scheme of evaluation. In many of the experiments money is used for payoffs possibly because it is just about the only item which is clearly man made and artificial that exists in the economy in such a form that it is normatively expected that most individuals will have a conscious value for it even though they hardly have worked out valuations for most other items.

Schelling (1961) suggested the possible importance of "natural" focal points in games. Stone (1958) performed a simple experiment in which two players were required to select a vertical and a horizontal line respectively on a payoff diagram as shown in Figure 5. If the two lines intersect

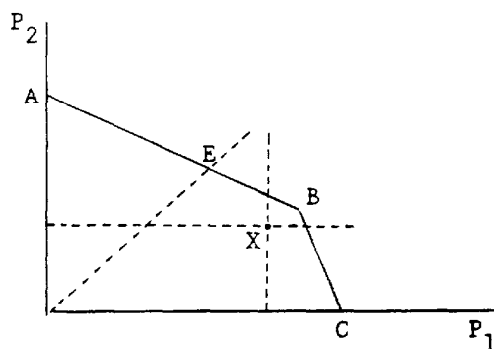


FIGURE 5

within or on the boundary (indicated by ABC) the players receive payoffs determined by the intersection of the lines (X provides an example). If the lines intersect outside of ABC then both players receive nothing. Stone's results shows a bimodal distribution with the two points selected being B, a "prominent point" and E, an equal split point. In class I have run a one-sided version of the Stone experiment six times. The results are shown in Table 3 and are bimodal.

	E	B	Other Offers*
1980	9	16	13
1981	10	11	11
1983	12	8	4
1984	8	6	5
1985	8	10	7
1988	24	15	7

*These are scattered on several other points.

TABLE 3

A more detailed discussion of some of the more recent work on experimental gaming in economics is to be found in the survey by Roth (1988), this

includes the considerable work on bidding and auction mechanisms [a good example of which is provided by Radner and Schotter (1988)].

5. EXPERIMENTATION AND OPERATIONAL ADVICE

Possibly the closest that experimental economics comes to military gaming is in the testing of specific mechanisms for economic allocation and control of items involving some aspects of public goods. Plott (1987) discusses policy applications of experimental methods. This is near to Roth's category of "whispering into the ears of princes," or at least bureaucrats. He notes agenda manipulation for a flying club [Levine and Plott (1977)]; rate filing policies for inland water transportation [Hong and Plott (1982)] and several other examples of institution design and experimentation. Alger (1986) considers the use of oligopoly games for policy purposes in the control of industry. Although this work is related to the concerns of the game theorist and indicates how much behavior may be guided by structure, it is more directly in the domain of economic application than aimed at answering the basic questions of game theory.

6. EXPERIMENTAL SOCIAL PSYCHOLOGY, POLITICAL SCIENCE AND LAW

6.1. Social psychology

It must be stressed that the same experimental game can be approached from highly different viewpoints. Although interdisciplinary work is often highly desirable much of experimental gaming has been carried out with emphasis on a single discipline. In particular the vast literature on the Prisoners' dilemma contains many experiments strictly in social psychology where the questions investigated include how play is influenced by sex dif-

ferences or differences in nationality.

6.2. Political science

We may divide much of gaming experiments in political science into two parts. One concerned with simple matrix experiments used for the most part for analogies to situations involving threats and international bargaining. The second devoted to experimental work on voting and committees.

The gaming section of Conflict Resolution contains a large selection of articles heavily oriented towards 2 x 2 matrix games with the emphasis on the social psychology and political science interpretations. Much of the political science gaming activities are not of prime concern to the game theorist.

The experiments dealing with voting and committees are more directly related to strictly game theoretic concerns. An up-to-date survey is provided by McKelvey and Ordeshook (1987). They note two different ways of viewing much of the work. One is a test of basic propositions and the other an attempt to gain insights into poorly understood aspects of political process. In the development of game theory at the different levels of theory, application and experimentation a recognition of the two different aspects of gaming is critical. The cooperative form is a convenient fiction. In the actual playing of games, mechanisms and institutions cannot be avoided. Frequently the act of constructing the playable game is a means of making relevant case distinctions and clarifying ill specified models used in theory.

6.3. Law

There is little of methodological interest to the game theorist in experimental law and economics. A recent lengthy survey article by Hoffman and Spitzer (1985) provides an extensive bibliography on auctions, sealed bids and on experiments with the provision of public goods a topic on which Plott and colleagues have done considerable work. The work on both auctions and public goods is of substantive interest to those interested in mechanism design. But essentially as yet there is no indication that the intersection among law, game theory and experimentation is more than some applied industrial organization where the legal content is negligible beyond a relatively simplistic misinterpretation of the contextual setting of threats.

The cultural and academic gap between game theorists and the laissez-faire school of law and economics is typified by virtually totemistic references to "The Coase Theorem." This theorem, to the best of this writer's understanding amounts to a casual comment by Coase (1960) to the effect that two parties who can harm each other, but who can negotiate will arrive at an efficient outcome. Hoffman and Spitzer (1982) offer some experimental tests of the Coase so-called theorem, but as there is no well defined context free theorem it is somewhat difficult to know what they are testing beyond the proposition that if people can threaten each other but also can make mutually beneficial deals they may do so.

7. GAME THEORY AND MILITARY GAMING

The earliest use of formal gaming appears to have been by the military. A history and discussion of war gaming has been given elsewhere [Brewer and Shubik (1979)]. The expenditures of the military on games and simulations for training and planning purposes are orders of magnitude larger than the expenditures of all of the social sciences on experimental gaming. Although the games devoted to doctrine, i.e. the actual testing of overall planning procedures can in some sense be regarded as research on the relevance and viability of overall strategies, at least in the open literature there is surprisingly little connection between the considerable activity in war gaming and the academic research community concerned with both game theory and gaming.

War games such as the Global War Game at the U.S. Naval War College may involve up to around 800 players and staff and take many weeks spread over several years to play. The compression of game time as compared to clock time is not far from one to one. In the course of play threats and counter-threats are made. Real fears are manifested as to whether the simulated war will go nuclear. Yet there is little if any connection between these activities and the scholarly game theoretic literature on threats.

8. WHERE TO WITH EXPERIMENTAL GAMING

Is there such a thing as context free experimental gaming? At the most basic level the answer must be no. The act of running an experimental game involves process. Games that are run take time and require the specification of process rules.

(1) In spite of the popularity of the 2 x 2 matrix game and the profu-

sion of cases for the $2 \times 2 \times 2$ and 3×3 several basic theoretical and experimental questions remain to be properly formulated and answered. In particular with the 2×2 game there are many special cases, games with names which have attracted considerable attention. What is the class of special games for the larger examples. Are there basic new phenomena which appear with larger matrix games. What, if any, are the special games for the $2 \times 2 \times 2$ and the 3×3 ?

(2) Setting aside game theory, our whole theory of risk is suspect. The critique of Kahneman and Tversky (1984), the various explanations of the Allais paradox indicate that our understanding of one person reaction to risk is still open to new observations, theory and experimentation. It is probable that different professions and societies adopt different ways to cope with individual risk. This suggests that studies of fighter pilots, scuba divers, high rise construction workers, demolition squads, mountaineers and other vocations or avocations with high risk characteristics is called for.

The social psychologists indicate that group pressures may influence corporate risk taking [Janis (1982)]. In the economics literature it seems to have been overlooked that probably over 80% of the economic decisions made in a society are fiduciary decisions with someone acting with someone else's money or life at stake. Yet no experimentation and little theory exists to account for fiduciary risk behavior.

A socialized individual has a far better perception of the value of money than the value of thousands of goods. Yet little direct consideration has been given to the important role of money as an intermediary between goods and their valuation.

(3) The experimental evidence is overwhelming that one point predictions are rarely if ever confirmed in few player games. Thus it appears that solutions such as the value are best considered as normative or as benchmarks for the behavior of abstract players. In mass markets or voting much (but not all) of the social psychology may be wiped out hence chances for prediction are improved. For few person games more explicitly interdisciplinary work is called for to consider how much of the observations are being explained by game theory, personal or social factors.

(4) Game theoretic solutions proposed as normative procedures can be viewed as what we should teach the public, or as reflecting what we believe public norms to be. Solutions proposed as reflecting actual behavior (such as the noncooperative theory) require a different treatment and concern for experimental validation. The statement that individuals should select the Pareto optimal noncooperative equilibrium if there is one is representative of a blend of behavioral and normative considerations. There is a need to clarify the relationship between behavioral and normative assumptions.

(5) Little attention appears to have been paid the effects of time. Roth, Murningham and Schoumaker (1987) have evidence concerning the importance of the end effect in bargaining. But both at the level of theory and experimentation clock time seems to have been of little concern, yet in military operational gaming questions concerning how time compression or expansion influences the game are of considerable concern.

(6) The explicit recognition that increasing numbers of players changes the nature of communication and information⁶ requires more exploration of

⁶ Around thirty years ago at M.I.T. there was considerable interest in the structure of communication networks. None of this material seems to have made any mark on game theoretic thought.

numbers between 3 and say 20. How many is many has game theoretical, psychological and socio-psychological dimensions. One can study how game theoretic solutions change with numbers, but at the same time psychological and socio-psychological possibilities change.

(7) Game theoretic investigations have cast light on phenomena such as bluff, threat and false revelation of preference. All of these items at one time might have been regarded as being almost completely in the domain of psychology or social-psychology, yet they were susceptible to game theoretic analysis. In human conflict and cooperation words such as faith, hope, charity, envy, rage, revenge, hate, fear, trust, honor and rectitude all appear to play a role. As yet they do not appear to have been susceptible to game theoretic analysis and perhaps they may remain so if we maintain the usual model of rational man. It might be different if a viable theory can be devised to consider context rational man who, although he may calculate and try to optimize is limited in terms of capacity constraints on perception, memory, calculation and communication. It is possible that the instincts and emotions are devices which enable our capacity constrained rational man to produce a sufficiently simple aggregated cognitive map of the context of his environment that he can act reasonably well with his limited facilities. Furthermore actions based on emotion such as rage may be powerful, aggregated signals.

The development of game theory represented an enormous step forward in the realization of the potential for rational calculation. Yet paradoxically it showed how enormous the size of calculation could become. Computation information and communication capacity considerations suggest that at best in situations of any complexity we have common context rather than common

knowledge.

There is constant feedback among theory, observation and experimentation. Experimental game theory is only at its beginning but already some of the messages are clear. Context matters. The game theoretic model of rational man is not an idealization but an approximation of a far more complex creature which performs under severe constraints which appear in the concerns of the psychologists and social psychologists more than in the game theoretic literature.

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