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Econometric Modeling as Information Aggregation

by

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ABSTRACT

The information contained in the forecasts from two econometric models can be compared by regressing the actual change in the variable forecasted on the two forecasts of the change. We do such comparisons in this paper, where the forecasts are based only on information through the period prior to the first period of the forecast. If a model's forecast is statistically significant in such a regression, we conclude that the model captures information not in the other model whose forecast is also included in the regression.

The models studied include the Fair model, vector autoregressive (VAR) models estimated by ordinary least squares, vector autoregressive models estimated with Litterman priors, and a new class of models, which we call "autoregressive components" (AC) models. The AC models divide GNP into components and estimate an autoregressive equation for each component.

Our results show that the Fair model's forecasts contain information not in the forecasts of the VAR and AC models. The AC models contain no information not in the Fair model, which indicates that the Fair model uses all the useful information in the components. The VAR models contain information not in the Fair model for the four-quarter-ahead forecasts but not the one-quarter-ahead forecasts. The best AC model contains information not in the best VAR model, which indicates that there is useful information in the components that the VAR models are not using. The best VAR model contains information not in the best AC model for the four-quarter-ahead forecasts but not the one-quarter-ahead forecasts.

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ECONOMETRIC MODELING AS INFORMATION AGGREGATION

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I. Introduction

Structural econometric models often make use of large information sets in forecasting a given variable. The information sets used in large-scale macroeconometric models are typically so large that the number of predetermined variables exceeds the number of observations available for estimating the model. Estimation can proceed effectively only because of the large number of a-priori restrictions imposed on the model, restrictions that do not work out to be simple exclusion restrictions on the reduced form equation for the variable forecasted. The a-priori restrictions make the model an aggregator of information.

Are the a-priori restrictions used in large models useful in producing derived reduced forms that depend on so much information? In other words, do large models use the large amount of information usefully, for purposes of forecasting, or is most of the information extraneous? This paper proposes a procedure for answering this question and applies the procedure to the problem of forecasting U.S. real GNP for the period 1976-III to 1986-II.

One cannot answer the above question by doing conventional tests of the overidentifying restrictions in a model. These restrictions might be wrong in important ways and yet the model may be a useful aggregator of information. Even ignoring this point, however, one cannot perform such tests with most large-scale models because, as noted above, there are not enough observations to estimate unrestricted reduced forms.

We propose to answer this question by comparing a structural model (considered as a structure with an estimation procedure and a forecasting algorithm) with various atheoretical models, representing certain extremes in model structure, using encompassing tests similar to those proposed by Davidson and MacKinnon (1981), Hendry and Richard (1982), Chong and Hendry (1986), and Mizon and Richard (1986).[1] Such tests enable us to discover whether forecasts from a model contain information not included in forecasts from another model. The regressions estimated as part of the testing procedure may also be used to produce a combined forecast from the individual model forecasts, along lines advocated by Granger and Newbold (1986) and others. With such a procedure we can learn whether large-scale econometric model forecasts, in our applications the Fair model (1976) forecasts, carry information not in the broad list of major macroeconomic variables, such as are used in the vector-autoregressive forecasting models. Such a procedure also allows us to verify whether simple information aggregators, the autoregressive components (AC) models to be defined below, carry information relevant to forecasting not included the same broad list of major macroeconometric variables. The AC models, which aggregate a great deal of information when the number of components is large, are extreme versions of the large-scale econometric models in the sense that the reliance of the large-scale models on lagged values of the components of GNP is carried much further and all other variables other than lagged values of GNP are eliminated.

[1]See also Nelson (1972) and Cooper and Nelson (1975) for an early use of encompassing-like tests.

II. The Procedure

Let ${}_{t-s}\hat{Y}_{1t}$ denote a forecast of Y_t (in our application, log real gross national product at time t) made from model 1 using information available at time t-s and using the model's estimation procedure and forecasting method. Let ${}_{t-s}\hat{Y}_{2t}$ denote the same thing for model 2. The parameter s is the length ahead of the forecast, $s > 0$. Note that the estimation procedure used to estimate a model and the model's forecasting method are considered by us as part of the model; we take no account of these procedures here. We consider the following regression equation:

$$(1) \quad Y_t - Y_{t-s} = \alpha + \beta({}_{t-s}\hat{Y}_{1t} - Y_{t-s}) + \gamma({}_{t-s}\hat{Y}_{2t} - Y_{t-s}) + u_t .$$

If neither model 1 nor model 2 contains any information useful for s-period-ahead forecasting of Y_t, then the estimates of β and γ should both be zero. In this case the estimate of the constant term α would be the average s-period-change in Y. If both models contain independent information for s-period-ahead forecasting, then β and γ should both be nonzero. If both models contain information, but the information in, say, model 2 is completely contained in model 1 and model 1 contains further relevant information as well, then β but not γ should be nonzero. (If both models contain the same information, then the forecasts are perfectly correlated, and β and γ are not separately identified.)

The procedure we are proposing in this paper is to estimate equation (1) for different models' forecasts and test the hypothesis H_1 that $\beta = 0$ and the hypothesis H_2 that $\gamma = 0$. H_1 is the hypothesis that model 1's forecasts contain no information relevant to forecasting s periods ahead not in the constant term and in model 2, and H_2 is the hypothesis that model 2's forecasts contain no information not in the constant term and in model 1.

Our testing procedure is similar to the C-test of Davidson and MacKinnon (1981) -- which is a special case of the "Wald encompassing test" of Mizon and Richard (1986) -- but it differs from this procedure in a number of important ways.

First, in our procedure the tests will be done for values of s greater than one. Davidson and MacKinnon, along with many others, have focussed attention exclusively on one-period-ahead forecasts.[2] The information content of models may differ depending on forecast horizon, as we will see below.

Second, the C-test restricts β and γ to sum to one. In our application this restriction does not seem sensible. As noted above, if both models' forecasts are just noise, the estimates of both β and γ should be zero. Also, say that the true process generating Y_t is Y_t equal to X_t + Z_t, where X_t and Z_t are independently distributed. Say that model 1 specifies that Y_t is a function of X_t only and that model 2 specifies that Y_t is a function of Z_t only. Both forecasts should thus have coefficients of one in equation (1), and so in this case β and γ would sum to two.

Third, the C-test restricts the constant term α to be zero.[3] Again, in our application this restriction does not seem sensible. If, for example, both models' forecasts were noise and we estimated equation (1) without a constant term, then the estimates of β and γ would not generally be zero when the mean of the dependent variable is nonzero.

[2]Their doing so was dictated by their setup of the model, wherein multi-period forecasts are not defined.

[3]Chong and Hendry's (1986) formulation of (1) also does not contain a constant term, although they do not constrain β and γ to sum to one.

Fourth, we require that forecasts for period t contain only information through period t-s for a s-period-ahead forecast. Davidson and MacKinnon do not require this. They assume (for s = 1) that the models that gave rise to ${}_{t-1}\hat{Y}_{1t}$ and ${}_{t-1}\hat{Y}_{2t}$ are $Y_t = f(X_{t-1},\delta_1) + \epsilon_{1t}$ and $Y_t = g(Z_{t-1},\delta_2) + \epsilon_{2t}$, respectively, where X_{t-1} and Z_{t-1} are vectors of exogenous variables known at t-1, δ_1 and δ_2 are vectors of parameters, and ϵ_{1t} and ϵ_{2t} are NID error terms. In their paper, the forecasts are ${}_{t-1}\hat{Y}_{1t} = f(X_t,\hat{\delta}_1)$ and ${}_{t-1}\hat{Y}_{2t} = g(Z_{t-1},\hat{\delta}_2)$, where $\hat{\delta}_1$ and $\hat{\delta}_2$ are ML estimates produced over the same sample over which (1) is estimated. If we were dealing with single-equation models, we would replace Davidson and MacKinnon's one-period forecasts with ${}_{t-1}\hat{Y}_{1t} = f(X_t,\hat{\delta}_{1t-1})$ and ${}_{t-1}\hat{Y}_{2t} = g(Z_{t-1},\hat{\delta}_{2t-1})$, where we require that $\hat{\delta}_{1t-1}$ and $\hat{\delta}_{2t-1}$ are based only on observations through period t-1. In practice, of course, we are dealing with multiple-equation models and with s possible greater than one, but we always require that a model's s-period-ahead forecast only be based on coefficient estimates and other information through period t-s. We reestimate the model through period t-1 for each new beginning period t of the forecast.

Using rolling estimation forecasts is important because in so doing we are producing the actual forecasts one would make with the model as time progresses. Such real time forecasts may have different properties from forecasts made with a model estimated with future data. If the model is misspecified (e.g., parameters change through time), then the rolling estimation forecasts (where estimated parameters vary through time) may carry rather different information from forecasts estimated over the entire sample. Also, some models may use up more degrees of freedom in estimation than others, and with varied estimation procedures it is often very difficult to take formal account of the number of degrees of freedom used up. In the extreme case where there were so many parameters in a model that the degrees of freedom were completely used up when it was estimated, it would be the case that $Y_t = {}_{t-s}\hat{Y}_{1t}$ and there would be a spurious perfect correspondence between the variable forecasted and the forecast. This would cause $\beta = 1$ in (1) whether or not model 1 were a good model. One can guard against this degrees of freedom problem by requiring that no forecasts be within-sample forecasts.[4]

Fifth, we do not assume that u_t is identically distributed, as do Davidson and MacKinnon. It seems quite likely that u_t is heteroskedastic. If, for example, $\alpha = 0$, $\beta = 1$, and $\gamma = 0$, u_t is simply the forecast error from model 1, and in general forecast errors are heteroskedastic. Also, we will be considering four-period-ahead forecasts in addition to one-period-ahead forecasts, and this introduces a third order moving average process to the error term in equation (1).[5] We correct for both heteroskedasticity and the moving average process in the estimation of the standard errors of the coefficient estimates. We use the procedure given by Hansen (1982), Cumby Huizinga and Obstfeld (1983), and White and Domowitz (1984) for the estimate of the asymptotic covariance matrix of the estimate $\hat{\theta}$ of the

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[4]Nelson (1972) and Cooper and Nelson (1975) do not require the forecasts to be based only on information through the previous period. Chong and Hendry (1986) do, however, require this. In their procedure the models that give rise to the forecasts are estimated using sample period 1 through T and their regression analogous to equation (1) is run using the sample beginning in period T+1.

[5]The error term in equation (1) could, of course, be serially correlated even for the one-period-ahead forecasts. Such serial correlation does not appear to be a problem with any of the models we study here, however, and we have assumed it to be zero. One should not, of course, uncritically apply procedures such as ours to all models, as Granger and Newbold (1986) have warned.

parameter vector $\theta = (\alpha\ \beta\ \gamma)'$ in (1). Define X as the $T \times 3$ matrix of variables, whose row t is $X_t = [1, ({}_{t-s}\hat{Y}_{1t} - Y_{t-s}), ({}_{t-s}\hat{Y}_{2t} - Y_{t-s})]$, and let $\hat{u} = Y_t - Y_{t-s} - X_t\delta$. We estimate the covariance matrix of $\hat{\theta}$, $V(\hat{\theta})$, as:

$$(2) \quad V(\hat{\theta}) = (X'X)^{-1}S(X'X)^{-1} \quad ,$$

where

$$(3) \qquad S = \Omega_0 + \sum_{j=1}^{s-1}(\Omega_j + \Omega_j') \ ,$$

$$(4) \qquad \Omega_j = \sum_{t=j+1}^{T} (u_t u_{t-j})X_t'X_{t-j},$$

where $\hat{\theta}$ is the ordinary least squares estimate of θ and s is the forecast horizon. When s equals 1 the second term on the right hand side of (3) is zero, and the covariance matrix is simply White's (1980) correction for heteroskedasticity.

III. The Models

As noted above, we want forecasts from models that are based only on information through the period prior to the beginning of the forecast period (through period t-s for a forecast for period t). There are four ways in which future information can creep into a current forecast. The first is if actual values of the exogenous variables for periods after t-s are used in the forecast. The second is if the coefficients of the model have been estimated over a sample period that includes observations beyond t-s. The third is if information beyond t-s has been used in the specification of the model even though for purposes of the tests the model is only estimated through period t-s. The fourth is if information beyond period t-s has been used in the revisions of the data for periods t-s and back, such as revised seasonal factors and revised benchmark figures.

The VAR, AC, and AR models discussed below have no exogenous variables, and so there are no exogenous-variable problems for these models. The way we have handled the problem for the Fair model is to add autoregressive equations for the exogenous variables to the model. For each exogenous variable in the model an eighth-order autoregressive equation (with a constant term and time trend included) has been postulated. When these equations are added to the model, the model effectively has no exogenous variables in it. This method of dealing with exogenous variables in structural models was advocated by Cooper and Nelson (1975) and McNees (1981). McNees, however, noted that the method handicaps the model: "It is easy to think of exogenous variables (policy variables) whose future values can be anticipated or controlled with complete certainty even if the historical values can be represented by covariance stationary processes; to do so introduces superfluous errors into the model solution." (McNees, 1981, p. 404).

For the coefficient-estimate problem, we use rolling estimations for all the models. For the forecast for period t, we estimate the model through period t-s; for the forecast for period t+1, we estimate the model through period t-s+1; and so on. By "model" in this case we mean the model inclusive of any exogenous-variable equations. The beginning observation is held fixed for all the regressions; the sample expands by one observation each time a time period elapses.

The third problem -- the possibility of using information beyond period t-s in the specification of the model -- is more difficult to handle.

Models are typically changed through time, and model builders seldom go back to or are interested in "old" versions. We have, however, attempted to account for this problem in this paper regarding the Fair model. We consider the version of the Fair model that existed as of the second quarter of 1976.

We have done nothing about the data-revision problem in this paper. The data that have been used are the latest revised data. It would be extremely difficult to try to purge these data of the possible use of future information, and we have not tried. Note that it is not enough simply to use data that existed at any point in time (say period t-s) because data on the s-period-ahead value (period t) are needed to estimate equation (1). We would have to try to construct data for period t that are consistent with the old data for period t-s.

We now discuss the various models used for the tests in this paper.

The Fair Model (FAIR)

The first version of the Fair model was presented in Fair (1976) along with the estimation method and method of forecasting with the model. This version was based on data through 1975 I. One important addition that was made to the model from this version was the inclusion of an interest rate reaction function in the model. This work is described in Fair (1978), which is based on data through 1976 II. The version of the model in Fair (1976) consists of 26 structural stochastic equations, and with the addition of the interest rate reaction function, there are 27 stochastic equations. There are 106 exogenous variables, and for each of these variables an eighth order autoregressive equation with a constant and time trend was added to the model. This gave a model of 133 equations, and this is the version that was used.

For the rolling estimations, the first estimation period ended in 1976 II, which is the first quarter in which the model could definitely be said to exist. This allowed the model to be estimated 40 times (through 1986 I).

Because the version of the Fair model used here existed as of 1976 II, because it has in effect no exogenous variables, and because it is estimated via rolling estimation, the forecasts from it can be said to be forecasts that are truly based only on information through the period prior to the first period of the forecast (except for the data revision problem).[6] This may be the first time that a large model this old has been tested.

The VAR Models (VAR4, VAR4P1, VAR4P2, and VAR4P3, VAR1, and VAR2)

We consider six VAR models in this paper. The first, VAR4, estimated by ordinary least squares, is the same as the model used in Sims (1980) except that we have added the three-month Treasury bill rate to the model. There are seven variables in the model: real GNP, the GNP deflator, the unemployment rate, the nominal wage rate, the price of imports, the money supply, and the bill rate. All but the unemployment rate and the bill rate

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[6]This statement needs to be qualified slightly. Although the structural stochastic equations used for the Fair model are exactly as in Fair (1976) and (1978) -- same left hand side and right hand side variables and same functional forms -- the data revisions in the National Income Accounts since 1976 have required slight modifications to some of the identities in the model. Also, the identities in Fair (1976) for the government sector are for the total government sector, whereas in the version used here there are separate identities for the federal government sector and the state and local government sector. This disaggregation of the government sector does not affect anything except that it means that there are more exogenous variables (and thus more exogenous-variable equations) in the version used here than there were in Fair (1976).

are in logs. Each equation consists of each variable lagged one through four times, a constant, and a time trend, for a total of 30 coefficients to estimate.

The next three VAR models -- VAR4P1, VAR4P2, and VAR4P3 --have Bayesian priors imposed on the coefficients of VAR4. We impose the Litterman prior that the variables follow univariate random walks. The standard deviations of the prior take the form

$$S(i,j,k) = \gamma g(k) f(i,j)(s_j/s_i), \tag{5}$$

where i indexes the left-hand-side variable, j indexes the right-hand-side variables, and k indexes the lag. s_i is the standard error of the unrestricted equation for variable i. VAR4P1 imposes parameter values that imply fairly loose priors. They are: 1) $f(i,j) = 1$ for all i and j, 2) $g(k) = 1$ for all k, and 3) $\gamma = .2$. VAR4P2 imposes parameter values that imply much tighter priors: 1) $f(i,j) = 1$ for $i = j$, $f(i,j) = .5$ for $i \neq j$, 2) $g(k) = k^{-1}$, and 3) $\gamma = .1$. VAR4P3 is the same as VAR4P2 except that $f(i,j) = .2$ for $i \neq j$, which implies even tighter priors than for VAR4P2. The parameter values for VAR4P2 are those imposed by Litterman (1979, p. 49).

The fifth VAR model, VAR2, uses only the first two lags of each variable, for a total of 16 coefficients in each equation. The sixth model, VAR1, uses only each variable lagged once, for a total of 9 coefficients. No priors were imposed on VAR2 and VAR1; they were estimated by ordinary least squares.

Each VAR model was estimated 40 times using the same sample periods as were used for the Fair model. Each model was then used to make 40 forecasts of real GNP.

The AC Models (AC-6, AC-13, AC-17, AC-48, AC-E6, AC-E13, AC-E17, and AC-E48)

Time series models like VAR models typically ignore the components of GNP. For example, the VAR models used in this paper contain no components. The current model used by Sims (serial) for forecasting includes only the component nonresidential fixed investment. Including many components in a VAR model rapidly uses up degrees of freedom, and this is undoubtedly one of the main reasons the components are seldom used. A possible alternative to the VAR approach, but one that also does not use much economic theory, is to model each of the components of real GNP by a simple autoregressive equation (but not real GNP itself) and then determine real GNP as the sum of the components (i.e., by the GNP identity).

We have considered eight AC models in this paper, all estimated by ordinary least squares. The models are first distinguished by whether they include 6, 13, 17, or 48 components. Increasing the disaggregation of the components allows one to examine how much additional useful information for forecasting purposes is contained in the more disaggregated components. Each equation for a component contains the first eight lagged values of the component, a constant, and a time trend. None of the AC equations is in log form. For the AC-6, AC-13, AC-17, and AC-48 models, these are all the variables included in the equations. For the AC-E6, AC-E13, AC-E17, and AC-E48 models (E for "extended"), the first four lagged values of real GNP are added to each equation. This allows for an impact of aggregate economic activity on each component and uses up only four degrees of freedom per equation. The components used for each model are listed in the Appendix. The 17 component models (AC-17 and AC-E17) use the same components as does the Fair model. The same sample periods and procedures were used for the

AC models as were used for the Fair and VAR models except the for the 6, 13, and 48 component models the beginning quarter for the estimation periods was 1961 I rather than 1954 I.[7]

The AC models are of interest in two respects. First, if the Fair model turns out to dominate the VAR models (which it does), it is of interest to know if this is due simply to the fact that the Fair model is dealing with the lagged components of GNP. If it is as simple as this, then the AC models, which are caricatures of the large-scale models as information aggregators and which carry the inclusion of lagged components even further, might do even better, and this can be tested. Second, the AC models are to some extent competitors of the VAR models within the class of non theoretical models, at least regarding the predictions of GNP. Both models are based on very little economic theory. It is thus of interest to see if one type of model dominates the other.

The Autoregressive Models (AR4 and AR8)

AR4 and AR8 are simple benchmark models, estimated by ordinary least squares. For AR4 real GNP was regressed on its first four lagged values, a constant, and a time trend. For AR8 real GNP was regressed on its first eight lagged values, a constant, and a time trend. The same sample periods were used here as were used for the Fair and VAR models.

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[7]This choice was dictated by the available data. Fortunately, the results do not appear to be sensitive to the use of the later beginning quarter. For the 17 component model the estimation periods could begin in 1954 I, and so two versions of AC-17 and AC-E17 were estimated, one for the periods beginning in 1954 I and one for the periods beginning in 1961 I. The two versions gave very similar results.

IV. The Results

The results comparing the Fair model to the other models are presented in Table 1. The sample period used for the one-quarter-ahead results is 1976 III - 1986 II, for a total of 40 observations. The sample period for the four-quarter-ahead results is 1972 II - 1986 II, for a total of 37 observations. Remember that each observation for a model's forecast is based on a different set of coefficient estimates of the model -- the rolling estimation. Remember also that for the Fair model all exogenous variable values are generated from the autoregressive equations; no actual values are used. Finally, remember that the estimated standard errors of the coefficient estimates are corrected for heteroskedasticity and (for the four-quarter-ahead results) for the moving average process of the error term.[8]

The results in Table 1 are fairly easy to summarize. The coefficient estimate for the Fair model forecast is always significant at the 5 percent level for both the one-quarter-ahead and four-quarter-ahead results. None of the coefficient estimates for the other models' forecasts is significant at this level for the one-quarter-ahead results. The regression gives a fairly large weight to the AR4 and AC-E17 forecasts, although these are not quite statistically significant. For the four-quarter-ahead results the only significant estimates (aside from those for the Fair model) are for the VAR models. The results across the different VAR models are fairly close,

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[8] In three cases for the four-quarter-ahead results the matrix $V(\hat{\theta})$ was singular or nearly singular. In these three cases we assumed a second order MA process for the error term instead of a third order, which solved the problem. Had this been a more wide spread problem, we would have used one of the estimators in Andrews (1987), but this seemed unnecessary given only three failures. The three failures are FAIR versus VAR4, FAIR versus VAR4P1, and AC-E13 versus VAR1.

TABLE 1

Fair Model Versus the Others: Estimates of Equation (1)

Other Model	One-Quarter-Ahead Forecasts; Dependent Variable is $Y_t - Y_{t-1}$; Sample Period = 1976III-1986II: const	FAIR $_{t-1}\hat{Y}_{1t}-Y_{t-1}$	OTHER $_{t-1}\hat{Y}_{2t}-Y_{t-1}$	SE	DW	Four-Quarters-Ahead Forecasts; Dependent Variable is $Y_t - Y_{t-4}$; Sample Period = 1977II-1986II: const	FAIR $_{t-4}\hat{Y}_{1t}-Y_{t-4}$	OTHER $_{t-4}\hat{Y}_{2t}-Y_{t-4}$	SE	DW
VAR1	-.0034	1.05	-.10	.00915	2.02	-.0135	1.06	.20	.0134	1.66
	(1.20)	(4.51)	(0.38)			(2.22)	(6.54)	(3.20)		
VAR2	-.0029	.89	.14	.00912	2.02	-.0135	1.08	.18	.0135	1.59
	(1.00)	(3.47)	(0.69)			(2.02)	(6.05)	(2.73)		
VAR4	-.0031	.86	.14	.00908	1.93	-.0164	1.07	.20	.0130	1.62
	(1.06)	(3.53)	(0.89)			(2.64)	(6.12)	(3.24)		
VAR4P1	-.0033	.91	.11	.00914	2.02	-.0159	1.08	.19	.0132	1.64
	(1.07)	(3.83)	(0.70)			(2.49)	(6.04)	(2.82)		
VAR4P2	-.0031	.89	.17	.00913	2.06	-.0145	1.04	.29	.0133	1.61
	(1.05)	(3.56)	(0.64)			(2.45)	(6.04)	(2.87)		
VAR4P3	-.0041	.87	.37	.00910	2.08	-.0209	1.07	.49	.0135	1.57
	(1.15)	(3.73)	(0.89)			(3.82)	(6.68)	(2.40)		
AC-6	-.0041	.94	.16	.00913	2.10	-.0201	1.13	.22	.0140	1.50
	(1.11)	(3.98)	(0.67)			(3.08)	(6.85)	(1.58)		
AC-13	-.0034	.99	.02	.00918	2.04	-.0193	1.18	.16	.0144	1.45
	(0.92)	(3.93)	(0.05)			(2.42)	(7.49)	(0.63)		
AC-17	-.0031	1.00	-.04	.00917	2.03	-.0164	1.22	-.00	.0145	1.42
	(0.84)	(4.07)	(0.14)			(1.87)	(8.68)	(0.00)		
AC-48	-.0039	.97	.10	.00915	2.04	-.0191	1.19	.14	.0143	1.48
	(1.15)	(4.00)	(0.50)			(2.95)	(7.58)	(0.95)		
AC-E6	-.0042	.91	.20	.00910	2.14	-.0195	1.15	.17	.0142	1.51
	(1.23)	(3.69)	(0.80)			(2.69)	(8.02)	(1.19)		
AC-E13	-.0043	.81	.40	.00899	2.16	-.0171	1.17	.10	.0145	1.41
	(1.35)	(2.81)	(1.05)			(2.40)	(7.36)	(0.47)		
AC-E17	-.0045	.76	.42	.00894	2.31	-.0168	1.20	.04	.0145	1.42
	(1.41)	(2.79)	(1.50)			(2.50)	(6.62)	(0.17)		
AC-E48	-.0047	.86	.36	.00894	2.21	-.0168	1.21	.03	.0145	1.43
	(1.42)	(3.64)	(1.65)			(2.66)	(7.43)	(0.22)		
AR4	-.0084	.96	.59	.00885	2.47	-.0168	1.22	.01	.0145	1.42
	(2.01)	(4.51)	(1.87)			(1.18)	(9.30)	(0.03)		
AR8	-.0067	.98	.38	.00901	2.32	-.0144	1.22	-.05	.0145	1.41
	(1.72)	(4.48)	(1.33)			(1.78)	(9.78)	(0.27)		

Notes: Y = log of real GNP.
t-statistics in absolute value are in parentheses.
See text for discussion of estimation methods.

with perhaps VAR4 performing the best.

The results in Table 1 are thus rather striking. They provide strong support for the hypothesis that the a-priori restrictions imposed by the Fair model lead the model to be a useful aggregator of information. The significance of the VAR forecasts for the four-quarter-ahead results indicates that some information is in the VAR forecasts that the Fair model is not using for the four-quarter-ahead forecasts, but this is the only significantly negative aspect of the results for the Fair model.

Table 2 compares the VAR models with the AC and AR models. It is hard from Table 1 to pick out which AC and VAR model performs the best because the models are so dominated by the Fair model, but Table 2 provides more ability to discriminate. Regarding the AC models, the best results in terms of significant coefficient estimates are obtained for the 13 and 17 component versions and for the extended (AC-E) versions. In other words, going from 6 to 13 or 17 components does help, but going beyond this does not seem to add further useful information, and adding the lagged values of real GNP to the equations seems to add useful information.

The best performing AC model in Table 2 is probably AC-E17, and so consider the comparisons of AC-E17 with the VAR models. AC-E17 performs better than any VAR model for the one-quarter-ahead results. No VAR model coefficient estimate is significant for these comparisons, although some are close to being significant. For the four-quarter-ahead results the VAR models perform about as well as does AC-E17. The best fit is for AC-E17 versus VAR4P2, where the t-statistic for VAR4P2 is 5.58 and the t-statistic for AC-E17 is 5.59. In other words, both AC-E17 and the VAR models appear to contain independent information useful for forecasting four quarters

TABLE 2

VAR Models Versus AC and AR Models: Estimates of Equation (1)
C = constant, V = VAR Model, A = AC or AR Model
One-Quarter-Ahead Forecasts
Dependent variable is $Y_t - Y_{t-1}$
Sample period = 1976III-1986II

	VAR1			VAR2			VAR4			VAR4P1			VAR4P2			VAR4P3		
	C	V	A	C	V	A	C	V	A	C	V	A	C	V	A	C	V	A
AC-6	.0029	.14	.37	.0034	.39	.17	.0019	.36	.23	.0022	.36	.23	.0025	.56	.16	-.0004	.94	.21
	(0.98)	(0.59)	(1.84)	(1.20)	(2.18)	(0.74)	(0.62)	(2.05)	(0.97)	(0.70)	(2.21)	(0.99)	(0.83)	(2.24)	(0.71)	(0.10)	(2.39)	(0.93)
		[.01057]			[.01014]			[.01003]			[.01020]			[.01011]			[.01008]	
AC-13	.0035	.21	.28	.0039	.43	.10	.0023	.38	.18	.0023	.40	.21	.0028	.60	.11	-.0004	1.01	.19
	(0.95)	(0.94)	(0.74)	(1.12)	(2.39)	(0.25)	(0.66)	(2.13)	(0.44)	(0.64)	(2.37)	(0.51)	(0.78)	(2.38)	(0.29)	(0.10)	(2.52)	(0.49)
		[.01068]			[.01018]			[.01008]			[.01024]			[.01013]			[.01011]	
AC-17	.0026	.25	.32	.0027	.43	.23	.0014	.39	.25	.0011	.42	.31	.0016	.61	.24	-.0018	1.05	.30
	(0.67)	(1.05)	(1.05)	(0.74)	(2.49)	(0.75)	(0.39)	(2.31)	(0.82)	(0.29)	(2.60)	(1.01)	(0.41)	(2.50)	(0.80)	(0.38)	(2.66)	(0.99)
		[.01065]			[.01013]			[.01005]			[.01019]			[.01008]			[.01005]	
AC-48	.0039	.23	.20	.0040	.43	.10	.0026	.39	.13	.0027	.40	.14	.0025	.60	.16	-.0008	1.04	.22
	(1.48)	(1.00)	(1.09)	(1.69)	(2.38)	(0.50)	(1.00)	(2.21)	(0.69)	(1.06)	(2.43)	(0.73)	(0.92)	(2.48)	(0.81)	(0.22)	(2.68)	(1.14)
		[.01067]			[.01017]			[.01007]			[.01024]			[.01009]			[.01004]	
AC-E6	.0019	.14	.48	.0022	.36	.32	.0012	.33	.33	.0014	.32	.34	.0014	.50	.31	-.0011	.87	.33
	(0.57)	(0.55)	(1.89)	(0.69)	(1.91)	(1.31)	(0.39)	(1.81)	(1.27)	(0.45)	(1.81)	(1.29)	(0.44)	(1.96)	(1.27)	(0.27)	(2.19)	(1.39)
		[.01037]			[.01000]			[.00992]			[.01009]			[.00997]			[.00995]	
AC-E13	.0000	.13	.82	.0003	.31	.69	-.0007	.30	.70	-.0005	.28	.71	-.0003	.42	.67	-.0023	.73	.68
	(0.03)	(0.58)	(2.54)	(0.09)	(1.88)	(2.12)	(0.21)	(1.87)	(1.99)	(0.14)	(1.86)	(2.06)	(0.07)	(1.85)	(2.03)	(0.57)	(2.05)	(2.10)
		[.00988]			[.00958]			[.00946]			[.00963]			[.00958]			[.00957]	
AC-E17	-.0008	.09	.83	-.0005	.27	.70	-.0015	.28	.71	-.0011	.24	.72	-.0009	.36	.96	-.0026	.62	.69
	(0.22)	(0.40)	(3.26)	(0.15)	(1.64)	(2.89)	(0.47)	(1.82)	(2.66)	(0.35)	(1.67)	(2.73)	(0.27)	(1.54)	(2.74)	(0.67)	(1.74)	(2.87)
		[.00967]			[.00944]			[.00930]			[.00949]			[.00946]			[.00945]	
AC-E48	.0011	.14	.84	.0013	.32	.51	.0003	.31	.51	.0006	.29	.52	.0004	.47	.51	-.0019	.82	.52
	(0.35)	(0.63)	(3.29)	(0.50)	(1.85)	(2.63)	(0.11)	(1.82)	(2.36)	(0.21)	(1.83)	(2.51)	(0.15)	(2.05)	(2.75)	(0.50)	(2.27)	(2.88)
		[.01009]			[.00978]			[.00967]			[.00985]			[.00971]			[.00969]	
AR4	-.0003	.21	.63	.0004	.39	.48	-.0021	.38	.62	-.0005	.36	.49	.0004	.54	.39	-.0014	.92	.31
	(0.07)	(0.88)	(1.65)	(0.11)	(2.17)	(1.31)	(0.52)	(2.44)	(1.69)	(0.11)	(2.17)	(1.26)	(0.10)	(2.03)	(1.00)	(0.32)	(2.03)	(0.76)
		[.01044]			[.01001]			[.00978]			[.01010]			[.01003]			[.01008]	
AR8	.0021	.23	.36	.0025	.42	.24	.0003	.39	.35	.0017	.40	.23	.0023	.59	.15	.0002	1.05	.06
	(0.50)	(0.94)	(0.96)	(0.63)	(2.31)	(0.67)	(0.08)	(2.36)	(0.97)	(0.43)	(2.28)	(0.60)	(0.53)	(3.10)	(0.55)	(0.06)	(2.29)	(0.15)
		[.01063]			[.01013]			[.00999]			[.01023]			[.01012]			[.01014]	

TABLE 2 (continued)

Four-Quarter-Ahead Forecasts
Dependent variable is $Y_t - Y_{t-4}$
Sample period = 1977II-1986II

	VAR1			VAR2			VAR4		
	C	V	A	C	V	A	C	V	A
AC-6	.0124	.45	.25	.0127	.37	.31	.0073	.36	.32
	(1.55)	(2.03)	(0.62)	(1.69)	(1.82)	(0.81)	(1.06)	(2.13)	(0.87)
		[.0230]			[.0236]			[.0236]	
AC-13	.0142	.51	.19	.0164	.44	.19	.0100	.43	.21
	(1.16)	(2.52)	(0.32)	(1.30)	(2.23)	(0.31)	(0.93)	(2.59)	(0.34)
		[.0232]			[.0240]			[.0240]	
AC-17	.0049	.50	.49	.0079	.44	.47	.0020	.42	.46
	(0.54)	(4.65)	(1.45)	(0.86)	(3.79)	(1.27)	(0.23)	(4.81)	(1.25)
		[.0225]			[.0235]			[.0235]	
AC-48	.0199	.56	-.04	.0234	.52	-.09	.0163	.51	-.10
	(1.98)	(2.59)	(0.09)	(2.20)	(2.32)	(0.19)	(1.85)	(2.73)	(0.20)
		[.0233]			[.0219]			[.0241]	
AC-E6	.0101	.45	.31	.0106	.37	.36	.0054	.36	.36
	(1.00)	(3.26)	(1.22)	(1.01)	(3.00)	(1.57)	(0.52)	(3.40)	(1.50)
		[.0227]			[.0234]			[.0234]	
AC-E13	.0003	.39	.78	.0016	.33	.81	-.0027	.31	.80
	(0.03)	(6.58)	(4.58)	(0.13)	(4.25)	(3.86)	(0.22)	(5.78)	(3.48)
		[.0199]			[.0207]			[.0209]	
AC-E17	-.0015	.42	.74	-.0003	.36	.76	-.0049	.34	.76
	(0.14)	(7.03)	(5.12)	(0.03)	(3.84)	(4.20)	(0.43)	(4.65)	(3.59)
		[.0201]			[.0209]			[.0210]	
AC-E48	.0111	.47	.29	.0131	.40	.30	.0081	.39	.28
	(1.43)	(4.61)	(1.59)	(1.69)	(3.68)	(1.69)	(1.02)	(4.08)	(1.41)
		[.0226]			[.0234]			[.0236]	
AR4	.0075	.54	.29	.0069	.48	.36	-.0017	.46	.41
	(0.41)	(4.74)	(0.67)	(0.40)	(4.15)	(0.79)	(0.09)	(5.35)	(0.80)
		[.0232]			[.0240]			[.0240]	
AR8	.0212	.55	-.06	.0162	.49	.12	.0075	.47	.17
	(1.18)	(5.08)	(0.16)	(0.98)	(4.32)	(0.37)	(0.44)	(5.55)	(0.45)
		[.0233]			[.0241]			[.0241]	

	VAR4P1			VAR4P2			VAR4P3		
	C	V	A	C	V	A	C	V	A
AC-6	.0082	.36	.30	.0095	.69	.19	-.0052	1.03	.34
	(1.17)	(1.93)	(0.79)	(1.24)	(1.92)	(0.44)	(0.53)	(1.63)	(0.89)
		[.0237]			[.0222]			[.0230]	
AC-13	.0107	.43	.21	.0110	.75	.13	-.0059	1.17	.32
	(0.96)	(2.42)	(0.33)	(0.95)	(2.33)	(0.21)	(0.57)	(1.96)	(0.56)
		[.0240]			[.0223]			[.0234]	
AC-17	.0024	.43	.47	.0016	.72	.45	-.0155	1.22	.59
	(0.27)	(4.63)	(1.26)	(0.17)	(3.84)	(1.24)	(1.43)	(3.25)	(1.64)
		[.0235]			[.0218]			[.0226]	
AC-48	.0170	.51	-.09	.0155	.82	-.07	-.0013	1.28	.07
	(1.86)	(2.46)	(0.18)	(1.64)	(2.43)	(0.16)	(0.13)	(2.00)	(0.15)
		[.0241]			[.0224]			[.0237]	
AC-E6	.0062	.36	.36	.0074	.67	.25	-.0070	1.05	.38
	(0.60)	(3.19)	(1.43)	(0.74)	(2.79)	(0.89)	(0.58)	(2.28)	(1.43)
		[.0235]			[.0220]			[.0228]	
AC-E13	-.0025	.32	.81	-.0020	.57	.73	-.0145	.96	.82
	(0.21)	(6.73)	(3.68)	(0.19)	(7.50)	(5.18)	(1.12)	(4.63)	(6.05)
		[.0207]			[.0185]			[.0199]	
AC-E17	-.0044	.34	.76	-.0036	.60	.69	-.0171	1.01	.77
	(0.39)	(5.10)	(3.71)	(0.37)	(5.58)	(5.59)	(1.40)	(4.38)	(6.12)
		[.0210]			[.0197]			[.0202]	
AC-E48	.0083	.39	.29	.0078	.69	.25	-.0064	1.12	.34
	(1.05)	(4.14)	(1.44)	(0.95)	(3.49)	(1.20)	(0.57)	(2.86)	(1.66)
		[.0235]			[.0219]			[.0227]	
AR4	.0018	.47	.34	.0106	.78	.09	-.0044	1.31	.11
	(0.09)	(4.90)	(0.64)	(0.54)	(4.21)	(0.18)	(0.21)	(2.93)	(0.17)
		[.0240]			[.0224]			[.0237]	
AR8	.0124	.47	.07	.0218	.80	-.20	.0086	1.36	-.23
	(0.66)	(5.06)	(0.17)	(1.28)	(4.72)	(0.55)	(0.46)	(3.21)	(0.48)
		[.0241]			[.0223]			[.0236]	

Notes: Y = log of real GNP.
t-statistics in absolute value are in parentheses.
Estimated standard errors of the regressions are in brackets.
See text for discussion of estimation methods.
$V = {}_{t-1}\hat{Y}_{1t} - Y_{t-1}$ for one-quarter-ahead results for VAR models.
$V = {}_{t-4}\hat{Y}_{1t} - Y_{t-4}$ for four-quarter-ahead results for VAR models.
$A = {}_{t-1}\hat{Y}_{2t} - Y_{t-1}$ for one-quarter-ahead results for AC and AR models.
$A = {}_{t-4}\hat{Y}_{2t} - Y_{t-4}$ for four-quarter-ahead results for AC and AR models.

ahead. Comparing across the VAR models, VAR4P2 and VAR4P3 are probably the best, although the results are quite close across the models. The results in Table 2 also show that the VAR models dominate the AR models. Clearly the VAR models contain information not in the AR models, but not vice versa.

The results in Table 2 thus indicate that AC models like AC-E17 contain useful forecasting information not contained in even the best VAR model. In other words, there appears to be useful forecasting information in the components of GNP that is not captured in the VAR models, and so within the class of fairly nontheoretical models, AC models appear to be useful alternatives to the VAR models. It is the case from Table 1, however, that the AC forecasts are not statistically significant at the 5 percent level when compared with the Fair forecasts. Although the AC-E17 forecast gets a weight of .42 for the one-quarter-ahead results in Table 1, it is not statistically significant, and for the four-quarter-ahead results the weight on the AC-E17 forecast is much smaller. These results may be interpreted as indicating that the Fair model captures most of the information in the components of GNP.

IV. Conclusion

The procedure proposed in this paper for examining models as information aggregators appears to be useful in comparing the different models. Using this procedure we have learned that the Fair model does very well relative to the other models. The Fair model cannot be dismissed as being based on the same information used in the other forecasts. The fact that the forecasts from the Fair model are significant shows that they are not collinear with the other forecasts and that the differences between the

Fair model and the other models are meaningful. We have also learned that information about components matters. The information about components of the kind incorporated in the AC models is at best of modest benefit in improving the Fair model forecasts. In this sense the useful information in the Fair model not in the VAR or AR models includes information about components of GNP. We have also learned something about how to combine forecasts. While it appears that the VAR and AC forecasts do not contain a lot of information not in the Fair model forecasts for one-quarter-ahead forecasting, it may be that a combination of the forecasts from the Fair and VAR models is useful for four-quarter-ahead forecasting. In future work we plan to use this procedure to examine the informational content of actual ex ante forecasts.

We should conclude with a warning about the interpretation of our results. The fact that one model does well or poorly for one sample period (in our case 1976 III - 1986 II) does not necessarily mean that it will do well or poorly in future sample periods. The results could change if the structure of the economy is changing, which is, of course, true of any econometric result. In our case, however, the results could also change if the magnitudes of the forecast errors of the different models are changing at different rates through time. The errors could, for example, be changing at different rates because the data are providing different rates of improvement of the models' parameters.

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APPENDIX

Components of AC Models

AC-6:

1. Personal consumption expenditures
2. Gross private fixed investment
3. Change in business inventories
4. Government purchases of goods and services
5. Exports
6. Imports

AC-13:

1. Personal consumption expenditures, durable goods
2. Personal consumption expenditures, nondurable goods
3. Personal consumption expenditures, services
4. Gross private fixed investment, nonresidential
5. Gross private fixed investment, residential
6. Change in business inventories, nonfarm
7. Change in business inventories, farm
8. Government purchases of goods, federal
9. Government purchases of goods, state and local
10. Government purchases of services, federal
11. Government purchases of services, state and local
12. Exports
13. Imports

AC-17:

1. Personal consumption expenditures, durable goods
2. Personal consumption expenditures, nondurable goods
3. Personal consumption expenditures, services
4. Gross private fixed investment, nonresidential, firm sector
5. Gross private fixed investment, nonresidential, financial sector
6. Gross private fixed investment, nonresidential, household sector
7. Gross private fixed investment, residential, firm sector
8. Gross private fixed investment, residential, financial sector
9. Gross private fixed investment, residential, household sector
10. Change in business inventories, firm sector
11. Change in business inventories, household sector
12. Government purchases of goods, federal
13. Government purchases of goods, state and local
14. Government purchases of services, federal
15. Government purchases of services, state and local
16. Exports
17. Imports

Note: See Fair (1984) for the definitions of firm, financial, and household sectors. This breakdown is from the Flow of Funds Accounts.

AC-48:

Personal consumption expenditures, durable goods:
1. Motor vehicles and parts
2. Furniture and household equipment
3. Other

Personal consumption expenditures, nondurable goods:
4. Food
5. Clothing and shoes
6. Gasoline and oil
7. Fuel oil and coal
8. Other

Personal consumption expenditures, services:
9. Housing
10. Household operation, electricity and gas
11. Household operation, other
12. Transportation
13. Medical care
14. Other

Gross private fixed investment:
15. Nonresidential structures
16. Nonresidential producers' durable equipment
17. Residential

Change in business inventories:
18. Farm
19. Nonfarm, manufacturing, durable goods
20. Nonfarm, manufacturing, nondurable goods
21. Nonfarm, merchant wholesalers, durable goods
22. Nonfarm, merchant wholesalers, nondurable goods
23. Nonfarm, nonmerchant wholesalers, durable goods
24. Nonfarm, nonmerchant wholesalers, nondurable goods
25. Nonfarm, retail trade, durable goods
26. Nonfarm, retail trade, nondurable goods
27. Nonfarm, other, durable goods
28. Nonfarm, other, nondurable goods

Government purchases of goods and services, federal:
29. Durable goods
30. Nondurable goods
31. Services, compensation of employees, national defense, military
32. Services, compensation of employees, national defense, civilian
33. Services, compensation of employees, nondefense
34. Services, other services
35. Structures

Government purchases of goods and services, state and local:
36. Durable goods
37. Nondurable goods
38. Services, compensation of employees
39. Services, other services
40. Structures

Exports of goods and services:
41. Merchandise, durable goods
42. Merchandise, nondurable goods
43. Services, factor income
44. Services, other

Imports of goods and services:
45. Merchandise, durable goods
46. Merchandise, nondurable goods
47. Services, factor income
48. Services, other