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GAME THEORY

MODELS OF STRATEGIC BEHAVIOR

AND NUCLEAR DETERRENCE

by

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1. ON NUCLEAR DETERRENCE

In everyday usage the verb to deter means "to discourage or restrain from acting or proceeding through fear, doubt, etc....It also means to prevent, check, arrest (Random House Dictionary of the English Language, 1969).¹

Nuclear deterrence broadly refers to deterring a potential aggressor from acting by displaying a resolution to use strategic nuclear weapons against the aggressor. It also implies that the potential aggressor is able to perceive the range of actions that will lead to retaliation and the resolution and the capability to act after the threat has been ignored.

Mutually Assured Destruction is the ultimate in joint threatmanship. It leaves nothing to chance. In its purest form it achieves its apotheosis

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¹I have not checked a modern and a presoviet Russian dictionary, both need to be done. The recent publication by Davis and Stan (1984) indicates that Soviet and U.S. views of deterrence differ.

in Herman Kahn's Domsday Machine (Kahn, 1961). It provides a totally passive threat that occurs with certainty. Most of the world's generals, bishops and political or even academic strategists do not believe in either the Holy Grail or the Unholy Grail of perfect deterrence. Until one is actually dead, even if the odds are over a billion to one there is always a sporting chance.

"Unrealistic hope" may be fantasy. It is also a necessity. Sun Tzu (see Phillips, 1940) said "Now the general who wins a battle makes many calculations in his temple ere the battle is fought." Damon Runyon observed that "The race is not always to the swiftest, nor the battle to the strong, but that is the way to lay your dough." These statements are not inconsistent with hope. Sun Tzu calls for the strategist to do his homework. Runyon advises playing the odds. But while homework can lead to a clearer understanding of the odds, resolve and high morale can change them and a clear comprehension of what is at stake may demonstrate to a protagonist the best choice available is to fight against even apparently overwhelming odds. Runyon was describing the position of a third party betting for money where implicitly the evaluation of winning or losing is straightforward and often a simple linear scale can be assumed for small money bets. Unfortunately nuclear war is not a small money bet. The evaluation of what is at stake in a nuclear war is central and a politically and morally difficult part of the analysis.

1.1. Generals, Politicians, Scientists, Technologists and Scholars

We say therefore War belongs not to the province of
Arts and Sciences, but to the province of social life.

Clausewitz (1832)

Bracken and Shubik (1982) have discussed a broad array of questions and who should be asking and answering them concerning nuclear warfare in order to keep research and analysis in context. Clausewitz understood both the social context and the scientific aspects of the study of war. With the growth of military, bureaucratic and academic establishments, specialists whether they are airforce generals, physicists or professors of political science run the danger of offering dangerously constrained solutions to strategic problems. The sheer volume of potentially relevant factors in the modern world is so overwhelming that for many the biases of highly constrained professional knowledge and biases are preferred to what may be perceived as vacuous generality by those who try to maintain a broad perspective.

This essay offers an exposition of the potential uses of game theoretic reasoning and mathematical models in the study of the prevention of nuclear war. The very essence of any discipline is that it has a body of formal knowledge which requires mastery taking its implicit assumptions of context as given. Thus the study of human anatomy takes individuals as they are, not as they might be on a planet around Alpha Centauri, ten million years from now.

Mathematical disciplines are even more context free. Certain idealizations are taken as defined within the system. Whether the concept of area arose from the practical problem of having to measure fields is a problem for the anthropologist not the mathematician. The symbols are set up and

the rules for manipulating them are specified. Context is irrelevant. But if this system is applied to a specific problem such as national defense, or football or fishing the key preliminary question prior to proceeding with the application is can the context and details of the problem at hand be fitted reasonably into the format of the study being proposed. Otherwise a bed of Procrustes may be being prepared where the distortion and deformation of the reality committed under the name of abstraction kills the value of the study.

It is suggested here that the discipline of game theory is central to further developments in the study of the science of war. But its uses must be set in the context of understanding the art and social processes of war. Formal methodology provides tools for thought not context free operational plans.

Game theoretic analysis provides a formal tool to explore the straightforward extensions of the concepts of individual goal oriented utilitarian behavior in situations involving more than one actor. Without adding realistic considerations such lack of clear goals, poor perceptions, passion and many other human factors many paradoxes are encountered even in the highly constrained attempt to generalized from isolated rational behavior to considering rational behavior among two or more so called rational actors.

Game theory provides a conceptual scheme and a basic accounting language for the analysis of strategy. A great entrepreneur would be a fool to listen only to his accountants; he would probably be a bigger fool not to listen to them at all. In the same way generals and politicians do not need to become expert game theorists, but they would be foolish to ignore an analysis which if it does nothing else points to and isolates many of the

basic problems and paradoxes in strategic behavior.

1.2. The Public Counts in More Ways than One

One of the more disturbing aspects of much literature in Political Science, some military thinking and much popular political discussion is the anthropomorphising of whole countries and the production of stereotypes. Stereotypes provide manageable aggregations. Thus it is often easier to invent a mythical entity called "the Russians" and to discuss policy in terms of "Russia intends" or "America expects" rather than to think in terms of mechanisms and forces within the society as a whole which may or may not lead to the development of a consistent overall behavior.

In the application of formal methods of strategic analysis this error can be manifested in not paying attention to the relevant players. In particular in the context of nuclear war the perceptions of the public as well as the power of weapon systems must be taken into direct consideration when contrasting what the United States, France, the Soviet Union or others may want, fear or intend to do.

Defense policy considerations call for accounting for how at least five groups view the problems. The ability to muster support to a certain extent depends upon how well shared or diverse are the perceptions of (1) the military, political and bureaucratic establishment; (2) the scientific, technical and professional establishment; (3) the nonpolitical leaders of public opinion; (4) the media and (5) the general public.

The questions to be asked of each one are (1) who perceives the dangers, if any. (2) What are the priorities attached to the various problem areas. (3) How strong is the consensus (or lack of consensus) within the

scientific and professional community. (4) Are the professional views and nonprofessional views in concert. (5) What are the estimates (and their reliability) concerning the political feasibility and economic costs of an effective program.

In spite of the natural tendency to construct simple general causal models; or to rationalize the blame for undesirable situations on one or two sources, it is more likely that highly complex phenomena such as society's reaction to the dangers of nuclear war, the realities of nuclear winter or a hothouse effect involve different and complex configurations of many interacting variables which lead to policy formation.

In a democratic society such as that of the United States, governments and bureaus do not change policy direction unless sufficient pressure is built up to make it necessary to do so. Public opinion and pressures count, whether scientifically correct or not. The armchair strategist whether indulging in the writing of essays or the constructing of mathematical models must take into account the distance between the formulation of even the clearest of policies or strategies and their implication in a complex modern society.

1.3. What is Different About Nuclear Strategy?

Tactics is the theory of the use of military forces in combat. Strategy is the theory of the use of combats for the objectives of war....In a word it is easier to make a theory for tactics than for strategy.

Clausewitz (1960, pp. 173 & 190)

Jomini(1947) made three distinctions in his study of the art of war. They were among grand strategy, strategy and tactics. The distinctions are in the context of diplomatic, political and military involvement and scope

in space and time. He isolated strategy from political and social context.²
A category of grand strategy is needed to join the political and military.

When tactics are considered even today, be it in situations involving limited space and time such as in duels or small tank battles there are many reasonably tight and valuable models of combat which can be formulated concentrating on human factors, technology and doctrine. The tactical battle can be studied out of the context of the major war. It appears to be reasonable to ignore political considerations for the purposes of analysis. When we move to theater warfare and to the scope of strategy the separation becomes more dubious. When we go to the overall consideration of the conduct of a war, Grand Strategy clearly has many political, diplomatic and societal components.

With the advent of nuclear warfare and the building of strategic nuclear weapons, technology has created a new and terrifying form of warfare. When viewed from the viewpoint of time constraints nuclear interchange involves "Grand Tactics." The time span for a first and second strike interchange is not far different from a tactical engagement, yet the scope is the world and the payoffs are at the level of national survival if not survival of the species.

A fundamental difference between tactical and grand tactical analysis is that as a first order approximation a tactical engagement can be regarded as an antagonistic game or a game of pure opposition. But even after the first launch has taken place a nuclear war is not a zero sum game. The payoff functions, while not "unthinkable" are not easily susceptible to dispa-

²See Shy (1986, p. 164) for a discussion.

sionate analysis. Dalkey (1965) presented a relatively simple mathematical model of nuclear interchange under the comforting title of "Solvable Nuclear War Models."

The modeling of the command and control system and its interface with political and diplomatic processes from the time of a red alert into the next twenty-four hours hardly exists in the open literature. It is replete with technological, organizational and psychological difficulties. The problem is not so much the black humor of a Dr. Strangelove scenario; but our lack of imagination and ability to generate a menu of other scenarios which are politically, technologically and psychologically acceptable to help isolate and analyze ambiguous command and undefined political factors. The perceptive books of Bracken (1983) and Blair (1985) provide both a technological and organizational insight into the command and control system.

2. GAME THEORY MODELS, GAMES AND SIMULATIONS

The prime purpose at hand is to provide an overview of the worth of game theory modeling, and to a lesser extent the activities of gaming and simulation in the study of war in general and nuclear deterrence in particular. In spite of the caveat that all formal models of war must be placed in context to be of value; before one can place game theory models in context it behooves us to appreciate what the theory is and purports to do without concerns for context.

In this section a brief overview is presented. An attempt has been made to make it reasonably nontechnical and self contained. Those familiar with the basics of game theory may wish to proceed directly to Section 3. Section 3 deals with modeling and raises questions concerning the relevance

and realism of the modeling assumptions made implicitly in the description of players, payoffs and solutions.

2.1. A Brief Tour of Game Theory

The title of game theory is a misnomer which exemplifies the clash between mathematical and non-mathematical cultures. It is basically the mathematical theory of strategy. Von Neuman may be credited with the title derived from the use of chess and Poker as a source for many of the basic concepts. But to the non-mathematician the term game theory has unfortunate connotations such as "it is only a game" implying that it is not serious. By using the word game for the study of military or other strategic exercises the social problem of communication may have been made more difficult. This appears to have been the case in the original introduction of war gaming to the Prussian staff. It was when the chief of staff observed that "It's not a game at all, it's training for war"; that the future of war gaming was assured.

The language of game theory is the language of strategy. It provides a precise way to describe the key elements in situations where there are many actors, with different goals resources and information, each with only partial control over the factors determining the outcome.

2.2. The Structure of the Game

Three different levels of description of a multi-person strategic game were offered by von Neumann and Morgenstern (1944). They are: (1) the extensive form of a game, (2) the normal or strategic form and (3) the coalitional, cooperative or characteristic function form. These three different

representations provide sequentially decreasing amounts of detailed information about the situation being modeled.

The extensive form is the most detailed providing a complete description of process, concentrating on the structure of moves and information. The strategic form suppresses details concerning moves and is designed for the analysis of strategic alternatives and their outcomes. The coalitional form is provided to aid in evaluating the benefits and powers of coalition formation.

In almost all situations of interest there is both a community and divergence of interests. Pure opposition arises for the most part in tactical situations such as duels. Opposition is a binary relationship. Whenever there are three or more parties it is impossible to have opposition among all of them. There is always room for cooperation, even if the only form of the cooperation is for one group to gang up against the rest.

The Extensive Form

Only the simplest of examples are provided here. A detailed, and reasonably precise but not highly mathematical exposition can be found elsewhere (Shubik, 1984). Consider a game where there are two players each of which must choose from a set of two moves. Suppose that Player A makes his selection first, Player B selects his move after A but is not informed of the selection made by A before he has committed himself to his selection. In a second instance we may assume that Player B is informed of A's move prior to selecting his own. The distinction between these two models shows up in the difference in the information sets illustrated in the "game tree" diagrams shown Figures 1a and 1b.

In Figure 1a the game begins at the top node which is labeled with both

a 0 indicating that this is the start and a name P_a indicating that the name of the player called upon to select a move at that choice point is Player A. The two branches leading out of the choice point are indexed so that we have a name for each move. Here they are 1 and 2. They each lead to a new choice point for Player B, each labeled P_b . Player B can choose between two moves labeled 1 and 2 at each choice or decision point. The two choice points (or positions) belonging to B are encircled by a closed curve indicating that they belong to the same information set. This means that as far as B is concerned he cannot distinguish between being at either position as the move selected by Player A is unknown to him at that time. It is of interest to note that time does not enter into this diagram cardinally, but only ordinally. It tells us the order of moves not how long has elapsed between them. In many situations as the length of time increases between moves so does the possibility for information leaks. This requires further modeling.

The only difference between Figure 1a and 1b is that in the first there is one information set drawn around B's two choice points, while in the second there are two separate information sets. These indicate that B is informed of A's move prior to having to move himself.

At the bottom of each terminal branch of each tree there is a pair of numbers. These numbers indicate the payoff to Player A and B respectively. The end of each terminal branch leads to an outcome from a play of the game described (a play can be formally defined as a path from the initial choice point--or root of the game--to the end of a terminal branch).

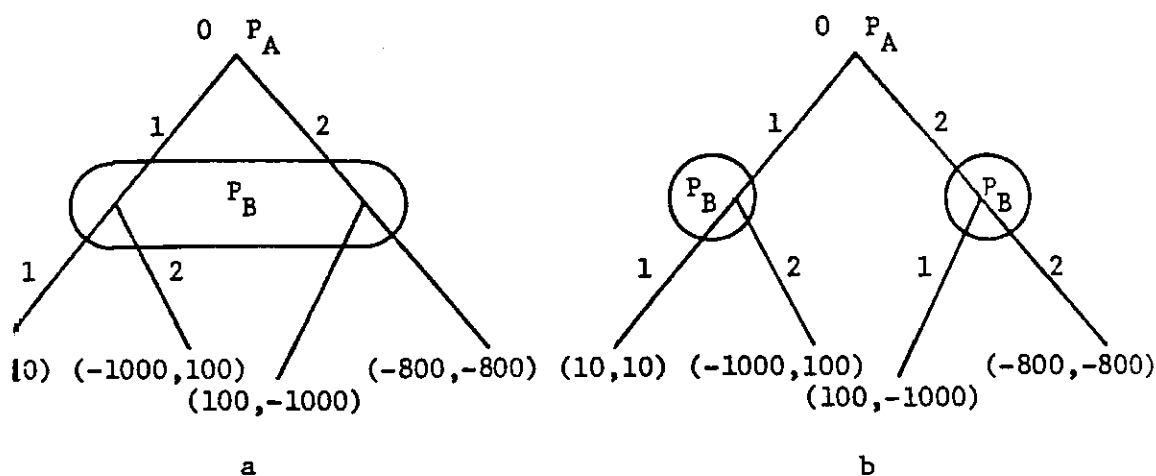


FIGURE 1

It is important to distinguish between an outcome and the value of an outcome. An outcome in a game such as tic-tac-toe or naughts and crosses which has a finite termination is a well defined objective fact. The value of the outcome to each player depends upon the preference system of each player. Thus except for explaining where the preference evaluation comes from the game tree description is an objective or physical description of every possible way a game can be played. Although the logic of the notation enables the mathematician to produce a fully defined description of a game such as chess, the size of the combinatorics involved in general makes the game tree description impractical. Even with the availability of high speed computers the possibilities for an exhaustive survey of alternative strategies are highly constrained. At this point an important lesson can be learned in modeling. Logical considerations permit the development of a notation for the exhaustive description of a game such as chess, technological considerations indicate that the way individuals must play involves some form of aggregation or simplification as the proliferation of alternatives is too great to be considered one by one.

Although the branches which represent moves have been merely numbered with no particular contextual connotations, in an actual game meaning must be attached to the moves. Experimental evidence indicates that a change in context in what appears to be otherwise the same game may influence play (see R. Simon, 1967).

The Strategic Form

The transition from the extensive form, or game tree description of the game to the strategic form is achieved by the reduction of detail so that all that is considered for each player is the set of strategies at his command. In theory this reduction is logically straightforward. In practice it is replete with difficulties. But it has the virtue that the starkness and extreme simplicity of the mathematics helps to illustrate the distinction between a strategy as it is carefully defined in game theory and the meaning of the word strategy as it is used in military and corporate planning. In particular the formal theory makes no distinction between strategy and tactics. Such a distinction appears to require context. One of the difficulties in considering the viewpoint of a Vegetius, Clausewitz, Jomini or other writers on the art and science of war is that as technology and the context of society change so do the definitions of the boundary between strategy and tactics. For many purposes in human affairs it is a good idea to keep definitions relatively vague until there is an operational need for precision. In formal mathematical modeling vagueness is not easy to deal with.

In formal game theory the idea of a strategy is logically simple and straightforward. It is a complete contingent plan which provides an operating rule for all agents under all circumstances that might arise. The two

simple games illustrated in Figures 1a and 1b can be used to make the concept clear. In order to plan for contingencies one needs to be able to recognize that they have occurred. Thus in the game shown in Figure 1a there will be no contingent planning on the behalf of either Player A or B. A is required to move first so his choice is constrained to selecting either move 1 or move 2. B move next, but if he is not informed of A's choice he cannot select an action based upon A's move; thus B's strategy is no more than choosing between his move 1 or move 2. In the game illustrated in Figure 1b matters have become more complicated. The choice problem for A is as it was before, but that for B has changed.

An easy way to understand the nature of the strategic opportunity that faces B is to imagine that Player B is represented by an organization consisting of a commander and two agents. Each agent is located at one of the information sets of B. In order to achieve full flexibility in planning there are four books of instructions that Commander B could issue to his agents. They are as follows;

If A uses Move 1, B uses 1; if A uses Move 2 B uses 1

If A uses Move 1, B uses 1; if A uses Move 2 B uses 2

If A uses Move 1, B uses 2; if A uses Move 2 B uses 1

If A uses Move 1, B uses 2; if A uses Move 2 B uses 2.

The strategic form presents the game as a matrix where the left hand border of the matrix is a list of the strategies available to Player A and the top border of the matrix is a list of strategies available to Player B. Thus in the two examples illustrated, the game shown in Figure 1a gives rise to a 2×2 matrix and the game shown in Figure 1b gives rise to a 2×4 matrix. These are shown in Tables 1a and 1b below. The payoffs illustrated

in the 2×4 matrix are calculated by using the the opposing strategies to determine the outcome.

	1	2		1,1;2,1	1,1;2,2	1,2;2,1	1,2;2,2
1	10,10	-1000,100	1	10,10	10,10	-1000,100	-1000,100
2	100,-1000	-800,-800	2	100,-1000	-800,-800	100,-1000	-800,-800
a			b				

TABLE 1

Although the definition of strategy carries over to games of any size, the number of strategies for a game even as constrained as Tic-Tac-Toe is astronomical.³ A major battle engagement has far more factors and variables than a chess game. Even after brief contemplation it is evident that a general does not work out all contingencies and think in terms of a complete game in strategic form. In general in strategic planning five or six alternatives are already a large number (see Miller, 1956). Thus we suspect that in contrast with the profusion of microscopic details in moves and information which give rise to myriads of formal strategies in a game of any complexity, in military, corporate and other societal planning, strategic planning involves both a high level of aggregation and in most instances, delegation. At some point the plan calls for decentralization of decision-making to the local commander.

At this point a warning must be made. For purposes of exposition the simplest of game trees have been used and these give rise to games in stra-

³See Shubik (1982, p. 38) for an estimate calculated primarily by Shapley.

tegic form which can be represented in one instance by a 2×2 matrix and in the other by a 2×4 matrix. In conversational game theory (see 3.3) below much has been made of the 2×2 matrix. Beyond doubt it is an excellent expository device; but it is highly limited and there is a danger that in some strategic analysis the oversimplification can be misleading.

The Coalitional or Cooperative form

If we were interested only in negotiations and measuring the possibilities for successful accommodation and cooperation between the players we could achieve a simplification beyond that of the strategic form by characterizing for the two player game illustrated above, three numbers. They are the amount that Player A can guarantee for himself without cooperation, the amount that Player B can guarantee without cooperation and the amount that the two of them could achieve jointly. We can read these amounts directly from either of the matrices in Table 1a or 1b. They can be displayed as indicated below:

$$v(A) = -800, \quad v(B) = -800$$

$$v(A,B) = 20.$$

TABLE 2

The function $v(\cdot)$ is known as the characteristic function of a game. The calculation of its values involves both conceptual and computational problems. Confining our remarks to the two person example here, the values attached to what the individuals can achieve alone have been based upon worst case scenarios. What can A absolutely guarantee? This implicitly assumes that it is costless to B to minimize A's payoff. But this is at least

a pessimistic assumption and may be highly counterfactual.

A different but often equally important problem is encountered in defining the value for $V(A,B)$. The way the value of this was obtained was by looking over the matrix in Table 1a (or Table 1b), finding the cell with the joint maximum and adding the two numbers together. But this slips in the implicit assumption that the payoffs are measured in similar units to both sides and that adding payoffs has meaning. This is a highly context specific assumption.

Further comments are made on the cooperative form in Section 2.7 below.

Recapitulating the observations on the different formal representations of a game. The extensive form is the most detailed and process oriented. The strategic form lays stress on the concept of strategy and the direct link of all strategies to payoffs. The coalitional form stresses the combinatorics needed to consider the divergence and community of interests that various parties and groupings of parties have in negotiating outcomes that are mutually satisfactory.

2.3. The Nature of Players

A critically important simplification in formal game theory is the concept of what constitutes a player. Because inasmuch as it is possible the player in a formal game is a bloodless institution-free abstraction it is important to appreciate both the parsimony and the price paid for the parsimony.

The player is an entity or decision making atom endowed with a set of preferences. The great variety that the structure of these preferences may take is indicated in section 2.4. Each player is assumed to be endowed with

perfect comprehension of the structure of the game, no particular psychological attributes, the ability to carry out whatever computations are necessary and no proclivities to making errors.

The key modeling convention is that of external symmetry. Unless it is made specific in the modeling of the game, all properties of all players are assumed to be the same. If one wishes, for example to say that the perceptions of Player A are twice as fine as Player B, or her reaction time is twice as fast as that of Player B then this must be modeled specifically and be reflected in the mathematical formulation. Player A instead of being a human being could be an organization or even a nation state. But if this is so the mathematical formulation which is meant to reflect this feature must be present explicitly in the model.

National defense and war are complicated phenomena and it is easy to leave out critical variables when attempting to present an abstract representation of the phenomena for analysis. Items such as morale, bravery, national will to resist and a host of other factors are difficult to characterize abstractly.

Possibly one of the greatest difficulties in communication between the mathematically inclined modeler, the political analyst and those concerned with operations involves the level of detail selected. There is a constant tug-of-war between adding irrelevant detail and leaving out critical factors.

2.4. Preferences and Utility Theory

More or less in keeping with much of standard economic theory and political theory based on models of the rational utilitarian individual it is frequently assumed that the player in a formal game comes equipped with a well defined set of completely ordered preferences over all outcomes faced in the game. In many instances not only are the preferences for the individual assumed to be completely ordered they may be assumed to be representable by a utility function measured up to a linear transformation. In actuality the strategic problems analyzed by game theory are logically completely separate from questions concerning the representation of individual preferences. One can study the strategic aspects of interlocked decisionmaking making whatever assumptions one chooses about individual preferences. If we wish to deal with aggregates such as a bureaucracy or a nation state as a single player then it may be of considerable importance to deal with a complex representation of the preferences or the choice criterion employed by the aggregate entity regarded as a player. Instead of well ordered preferences obeying conditions such as transitivity other structures may provide for better representations. A survey of some of the alternative assumptions is provided in some detail elsewhere (see Shubik 1982, Chs. 4 and 5). There is a considerable technical literature on utility theory and the representation of preferences and further references are supplied in the Chapters noted above.

The important feature to be stressed here is that in contrast with the prevalence of individualistic economic analysis where the assumptions of reasonably well known and relatively individualistic preferences appear to provide an adequate first order approximation to much of economic reality,

such is not the situation in the analysis of the preference considerations when studying most problems involving defense policy.

2.5. Two Person Constant Sum Games and Solutions

The first formal game model to be analyzed was the two-person zero-sum game.⁴ It is a game with only two players where whatever is won by one side is lost by the other. The sum of net winnings and losses is always zero. There is a slightly broader class of games referred to as games of pure opposition, or in the Soviet literature as "antagonistic games." They play a particularly important role in the concepts of game theory inasmuch as there is a solution concept associated with zero sum games which may be regarded as a sophisticated extension of the concept of individual rationality to a situation involving two players. This is the well known Maximin solution in which because the situation is one of pure opposition it makes sense for each Player to assume that the other is out to do him as much damage as possible. The reason for the validity of this assumption is that the attempt by B to damage A is operationally the same as B trying to maximize his own payoff.

The zero-sum game has direct application in the study of weapons evaluation, force allocation, dueling and other tactical situations where as a good first order approximation the situation can be described as one involving pure opposition. A simple example known as a Colonel Blotto game serves to illustrate the zero sum game. It is named after a mythical Colonel who

⁴There is a minor technical distinction to be made between the class of two-person zero-sum games and two person constant-sum games. However they are strategically equivalent and the distinction is not developed here (see Shubik, 1982, Ch.8).

has two divisions at his disposal and must use them to defend two forts. The enemy has two divisions to use in his attack. If the attacking force is less than or equal to the defending force in size, the defenders win. If the attacking force is larger it wins the fort. Suppose the value of capturing or keeping a fort is 100. We can represent the allocation of the attacking and defending forces to the two forts by a pair of numbers, where the first indicates the divisions assigned to the first fort and the second, to the second.

	(2,0)	(1,1)	(0,2)
(2,0)	200	0	0
(1,1)	0	200	0
(0,2)	0	0	200

TABLE 3

The meaning of the entry in the upper left hand corner of the matrix is as follows. Colonel Blotto defends the first fort with his two divisions and leaves the second fort undefended. The attacker attacks the first fort with both divisions and does not attack the second fort. In this instance the payoff is 200 to Colonel Blotto as he keeps both forts. The payoff to the opponent is -200, but we do not need to note it as it is implicit if we know that the game is zero-sum. The other entries can be interpreted in a like manner.

An emphasis on the zero-sum game comes about naturally for three reasons. It provides a reasonably good approximation to the nature of tactical combat. There is a large body of mathematical methods which enable the analyst to actually calculate what the optimal or Maximin strategy should

be. Furthermore the solution is congenial with the pessimistic viewpoint that calls for the consideration of the enemy's capabilities rather than intentions. The "planning for the worst" or maximin bias may have considerable justification when the situation being studied can be adequately represented by a zero-sum game. However virtually all strategic problems (and for that matter almost every international dealing between nations) require representation by non-constant sum games, i.e. situations which involve a mixture of interests where there is neither pure opposition, nor pure coincidence of interests. The application of a maximin principle to such a game may verge on paranoia as is indicated in Section 2.6.

2.6. Two Person Nonconstant Sum Games

Although a two person game can be used to represent a situation of pure opposition it is easy to illustrate two-person games where there is an intermix of interests. The classical expository example is the so-called Prisoners' Dilemma game. The scenario often told involving two prisoners may or may not be plausible (see Shubik, 1982, Ch. 9). Rather than recount it here the game is given purely abstractly.⁵

There are two players, each has two strategies and the payoffs are indicated in Table 4. If both players use their first strategy each obtains 10. If Player A plays 1 and Player B plays 2 then A obtains -20 and B obtains 30. If A plays 2 and B plays 1 the situation is reversed. A obtains

⁵ An interesting problem in social psychology and game theory is to consider whether there are "natural names" that are suggested by the structure of the 78 ordinally different 2×2 matrices which can be constructed. Are there any universal properties to "the name of the game" or do attempts to name some of these games represent exercises in cultural or occupational or other biases?

30 and B obtains -20. If both play 2 then each obtains 0. It is plain from the structure of the matrix that the employment of strategy 2 dominates the employment of strategy 1 as an optimal reply against whatever the other player chooses. But this exercise of what appears to be clearly individually rational behavior yields both players 0; in contrast if both were willing to play strategy 1 they could obtain 10 each.

	1	2
1	(10, 10)	(-20, 30)
2	(30, -20)	(0, 0)

TABLE 4

The conflict between the temptation to go for 30 and run the risk of ending up with 0 or to follow the normative advice to each accept 10 by both playing strategy 1 is illustrative of a basic debate in the development of the theory of games concerning what constitutes a solution to a nonconstant sum game. A critical division comes in the distinction between normative and behavioristic solutions. Considerations of joint optimality suggest that an advisor might tell the players involved in the game in Table 4 to both select their first strategy because it is jointly optimal and certainly both fair and better to both than obtaining 0 each. Yet there is considerable experimental evidence that this game played once without face to face communication and under a wide variety of briefings tends to be played with both players selecting their second strategy. The pair of strategies (2,2) is referred to as an equilibrium pair of strategies, or to use a geometric analogy they define an equilibrium point. This is referred to as a noncooperative equilibrium in the sense that it exhibits a circular stability based

upon independent reasoning concerning best responses. A pair of strategies constitute an equilibrium pair if were A to be told in advance what B intends to do then A's best response is what he does and vice-versa. The only noncooperative equilibrium point in the game in Table 4 is given by the strategy pair (2,2). This pair would also result from a considerably different interpretation, that is the maximin bias. If A expects B to try to damage him as much as possible he will use his second strategy. Table 5 shows a different game where the joint maximum and a noncooperative equilibrium coincide but the maximin mode of behavior is different.⁶ There is a second noncooperative equilibrium strategy pair using the strategies (2,2) with payoffs of (0,0).

	1	2
1	(10, 10)	(-3, -2)
2	(-2, -3)	(0, 0)

TABLE 5

The joint maximum solution is often too optimistic an assessment. The maximin solution is closely allied to the military bias of pessimism calling for evaluation of the other side's potential laying emphasis on capability rather than cost of and intentions concerning the use of the capability. The noncooperative equilibrium has been used heavily in both conversational and analytical applications of game theory to defense. But the fact that there may be many equilibria or stalemate positions which differ consider-

⁶Technically the maximin calls for the utilization of a mixed strategy (see Shubik, 1982, Ch. 8).

ably in terms of the outcomes and that the same manifested behavior can arise from highly different intentions implies that noncooperative equilibrium analysis must be used with extreme caution.

2.7. The n-Person Problem

In 2.2 the cooperative or coalitional form of the two person game was given. This can be directly generalized to the n-person game for an arbitrary number of individuals. The characteristic function $v(\cdot)$ is a super-additive set function which means that it is a function defined for every set of players that can be formed and that the condition of superadditivity entails the assumption that any two sets of players can jointly obtain at least as much in coalition as they could by independent action. This can be expressed formally as follows: Consider S and T to be two coalitions of players which do not have members in common ($S \cap T = \emptyset$). Then:

$$v(S) + v(T) \leq v(S \cup T) .$$

The number of coalition increases as 2^n for a game involving n players. Thus for a twenty player game there are over a million coalitions to be accounted for. Fortunately when considering models of situations such as alliances or cartels five or six players is often enough. Furthermore special structure often cuts down the number of coalitions which turn out to be critical to any analysis. The calculation of the structure of the characteristic function can be regarded as a presolution in and of itself. These numbers alone provide a guide to the evaluation of the degree of community or opposition of interests among groups.

The apparatus of formal game theory is designed to consider the

implications of the general structure of games of any size including games with masses of small and individually strategically insignificant players (such as a single voter in a presidential election). A class of games called simple games where the value of any coalition is either 0 or 1 appears to be of particular significance in command and control network problems. Surprisingly there turns out to be a close relationship between these games and the work done on the reliability of electrical networks (see Dubey and Shapley, 1979) where the concern is with the conditions under which the network will no longer function.

One of the key difficulties confronted in the application of games in coalitional form comes in deciding how to evaluate threats when trying to evaluate what a coalition can realistically expect to obtain if it acts independently.⁷

2.8. Many Cooperative Solutions

Although three descriptions of an n-person game have been provided, no general proscription has been offered concerning what constitutes a solution to a game. Indeed there is no universally accepted solution concept. For games in strategic form the most utilized solution has been the noncooperative equilibrium. But it is not the only solution, does not give unique predictions and in some instances does not even appear to be particularly

⁷There is even some question as to whether games in coalitional form should be represented by the characteristic function or by a partition function which takes into account the possibility that the players could be split into many subgroups existing at the same time.

reasonable.⁸ These difficulties become even more apparent when one studies games with more than two players.

The coalitional form representation clearly lays emphasis upon the possibilities for bargaining and collaboration. The solutions that have been suggested pick up this feature. The two best known solution concepts for games in coalitional form are known as the core and the value.⁹ The basic idea behind the core is to use the information about the power of all coalitions to locate ways of dividing the joint proceeds in such a manner that no subgroup has the power to overthrow the suggested division by independent action. A simple example illustrate this. Table 6 shows a three person game where all players together can obtain a payoff of 4, but in coalitions of two their payoffs vary as is indicated. An imputation is a set of numbers, one for each player which add up to the amount that all of the players can obtain by cooperation. Thus for example the triad of numbers (.5,1.5,2) is an imputation in this three person game. It adds up to 4. Not only is it an imputation in the game, it is in the core of the game

$$\begin{aligned}v(1) &= 0, v(2) = 0, v(3) = 0 \\v(12) &= 1, v(13) = 2, v(23) = 3 \\v(123) &= 4\end{aligned}$$

TABLE 6

⁸See Shubik, 1982, Chapter 9.

⁹There are many other solutions which have been suggested and analyzed to some degree. These include the stable set, the bargaining set and the kernel (see Shubik, 1982, Chs. 6, 7, and 11).

because there is no coalition that can do better by going out alone. Players 1 and 2 obtain 2 in the suggested imputation. Alone they could obtain 1. Players 1 and 3 together obtain 2.5, alone they could obtain 2. Players 2 and 3 together obtain 3.5, together they could obtain 3. There are many other divisions in the core and there are well developed mathematical techniques for finding them. Unfortunately not all games have nonempty cores. The completely symmetric game shown in Table 7 provides an example:

$$\begin{aligned}v(1) &= 0, v(2) = 0, v(3) = 0 \\v(12) &= 3, v(13) = 3, v(23) = 3 \\v(123) &= 4\end{aligned}$$

TABLE 7

It is easy to check that no matter how they divide the 4 units among themselves it is impossible to divide matters so that each individual obtains at least 1.5, but if they do not obtain this amount there will always be some pair that gets less than 3 and hence would be better off without cooperation. In essence when a core exists all parties are in a position to find a mutual accommodation where they know that going it alone does not pay. When a core does not exist this is an indication that the clash of all parties over the division of joint gain cannot be resolved without modifying claims justified by "we can do better alone." In economic analysis, the great virtue of the competitive price system when it exists is that it lies within the core.

The other widely accepted cooperative solution concept, the value, can be viewed as a combinatoric version of the idea of marginal productivity of the individual. Suppose we had a society in which a coalition structure

leading up to the coalition of the whole were formed randomly and incrementally. For example we might begin with Player 2, then add 1, then add 3. Suppose that we were to consider every way of doing this and credit each player with the increment in value that he adds by joining the coalition at that time. If we multiply all of these increments by the probability that he enters a coalition by a random drawing at any particular instance we obtain an expected value of the worth of a player to the group as a whole. The value is that set of rewards which add to the full product (4 in the example in Table 6) and is divided in proportion to the expected contributions of each player. In this example it happens to be $(5/6, 8/6, 11/6)$. As the game in Table 7 is completely symmetric in the role of all three players the value is $(4/3, 4/3, 4/3)$. But this point is not in the core. There is no core in this game.

Both the core and the value have been utilized and applied at some length to economic analysis. The value has lead to considerable work in voting (see Shubik, 1984, and Shubik, 1982, Chs. 8 and 12). But the static relatively parsimonious models that have proved highly valuable in the development of economic theory do not appear to fit many of the problems in the study of deterrence; although some more or less narrower questions such as network defense are amenable to analysis based on the value solution.

2.9. Dynamics and Incomplete Information

In 2.2 the extensive form of the game was noted. Furthermore comment was made to the effect that the extensive form deals with a full description of process and spells out the nature of moves and information. Thus it would seem that it is the appropriate form to apply to the analysis of bar-

gaining, threats, deterrence and situations involving essentially historical processes. It is here that the formal analogy between parlor games and many human affairs breaks down. The game tree begins at a specific choice point (formally referred to as the root of the tree). But this is a virtually ahistorical representation. Furthermore the tree is finite. The analogy for chess or checkers is clear. But the analogy to Russian-Polish relations is not so clear. In many human affairs there is always a yesterday and a probability that there will be a tomorrow. Not only do history and hope raise fundamental difficulties with the use of the finite game tree. Other problems of considerable concern also exist with the representation. In processes such as bargaining or random search there is no nice clean sequential structure such as that which exists in chess. Moves tend to be, to a great extent in random sequence. A may talk first, but there is a chance that B talks first. In many board and other parlor games the sequence of moves is given more or less rigidly by the rules. In much of bargaining and negotiation not only is the sequence fluid but even the rules are not cut and dried. New ideas and unthought of possibilities may be of importance. The formal language of the game tree can, in theory handle the randomness of moves, but not without complication. Furthermore in fluid situations it is not clear that the strategic analyst is benefitted by trying to catch the type of detail stressed by the extensive form.

There now exists a substantial literature dealing with games which have no finite ending. A certain amount of thought has also been given to trying to portray the presence of history rather than selecting some arbitrary

starting point.¹⁰

Another important limitation in the structure of formal game theory has been that in the original formulation uncertainty enters only in the form of jointly known probability distributions. In other words, all players are meant to share (possibly different refinements of) the same sets of information. There are no subjective differences of opinion concerning the same event. The work of John Harsanyi (1967, 1968a,b) on Games with Bayesian players has offered a way of introducing lack of information. But the cost in mathematical complexity is high. However the literature is growing, both in formal game theory and in application to specific problems such as problems in inspection (see Aumann and Maschler, 1964) and the economics of agency (Arrow, 1985).

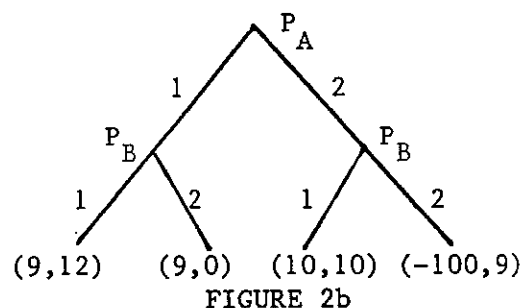
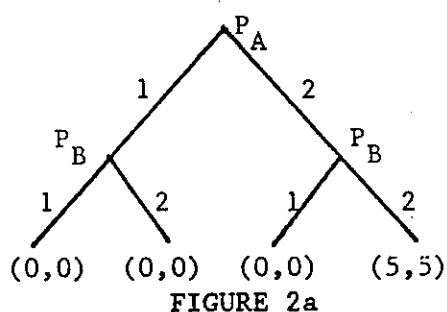
The study of agency theory involves the consideration of relationships between a principal and one or more agents where there is uncertainty and differences in information present. The twin problems of moral hazard and adverse selection appear. An example of moral hazard is where an agent may claim to have performed a service that has not been performed, but the principal has no clear way of knowing. An example of adverse selection is where an offer is made to insure risk to a group. But only those who know that their proclivity to accidents is greater than the norm take the policy. The principal cannot distinguish between the low risk and high risk individuals before issuing the guarantees.

Agency theory is concerned with the design of contracts and incentive systems which will insure best efforts performance from all agents. Unfor-

¹⁰For a discussion of alternative ways for portraying multistage games see Shubik (1982, Ch. 3).

Unfortunately the gap between actual institutional design complete with considerations of morale, sense of responsibility and cultural norms and codes of behavior is hardly reflected in the relatively parsimonious consideration of individualistic economic motivation.

Leaving aside the complexities of analysis with incomplete information, a reasonable question to ask is, is there a consensus among game theorists concerning the appropriate solution concept for a game in extensive form, even when there is perfect information? Figures 2a and 2b show two games. In the first instance by simple backward induction the solution is clear.



Without precommitment they both employ their second strategy. In the game in Figure 2b, if Player A has selected his second move, Player B is confronted with a choice of moves yielding either 10 or 9. As a straightforward utilitarian he should select his first move and obtain 10. This rules out his threat to play his second move thereby obtaining 9 but leaving Player A with -100. The idea behind the concept of a perfect equilibrium (Selten, 1975) is that after the fact it may be "irrational" to carry out a threat hence no rational player will do so. In this game if Player A were to use his first strategy he would end up with 9 regardless of the action of Player B. B on the other hand can select move 1 and obtain 12. If A selects 2 B could have threatened to punish him at little cost by selecting 2. But the strict application of nonhistorical local maximization rules out

B punishing A once deterrence has failed because at that time reprisal is not "plausible."

A basic misunderstanding of many political scientists (for example George and Smoke, 1974, p. 73) is that this type of solution is somehow the game theory solution to such a problem and that the work of Schelling (1960) showed what was wrong with the formal game theorists. It was and still is recognized that there is no universally accepted normative or behavioral theory among game theorists which covers threat, deterrence and precommitment. There are examples, counterexamples and stories. But the pure extension of the model of individual rational man into a context free multistage game does not appear to provide enough structure to suggest a unique satisfactory theory.¹¹

2.10. Experimental, Operational and Educational Games and Simulations

Game theory and gaming are fundamentally different, but highly related topics. A reasonably complete discussion of the different forms of gaming and their uses has been given elsewhere (Brewer and Shubik, 1979). The key element to stress here is that there is a considerable benefit to be gained from attempts to build playable games even if they are not played. Much of game theory is too stylized and complex to provide playable models. Yet command and control systems must be designed so that they can operate even if there is not a PhD in mathematics responsible for every decision point in the system.

¹¹Harsanyi has for many years worked on constructing a theory which selects a unique special equilibrium point for all games, even though they may have a profusion of plain Nash equilibria.

There has been an explosion in the use of simple 2×2 matrix games and other games in experimentation (see for example Rapoport et al., 1976, Teger, 1980, Raiffa, 1982 and the gaming section of Conflict Resolution). At least three lessons can be learned from reviewing this literature even casually. (1) The results obtained appear to depend to a great extent upon the briefing, context and expertise of the players. (2) There is little evidence that a generally accepted normative theory is verified independent of context. (3) There is overwhelming evidence that whatever behavior is postulated, a variety of different behaviors are found distributed statistically across the population of players. Thus although a majority of players may behave "reasonably" in some game there may also be a few who are too timid and a few who are too tough. At the level of application this should warn us against believing that any complex competitive system design which postulates ideal types will function according to the book.

Both the building of and experimentation with games offer an important way to link game theory to competitive phenomena and to provide a device for appreciating the limits imposed by complexity on operating systems.

Operational games, fleet exercises, politico-military exercises all provide useful training and planning devices. They are somewhat removed from formal game theory. Nevertheless can serve to teach, challenge and improve the basic concepts.

Much of the activity in modern gaming, be it for experimentation, teaching or training is computer assisted. The game involving live players virtually always generates a healthy skepticism. No matter how involved the participants become there is frequently a critique involving challenges concerning realism and relevance as to what is going on. This is not the case

with pure simulations.¹²

There is a temptation to take a model such as the purely game theoretic model of Dalkey (1965), already noted and make it realistic and relevant by blowing it up into a fully fledged air battle model on a large computer. One of the dangers in utilizing large scale simulations is that transparency is lost. Unless the modelers are absolutely convinced that their model is a fully adequate representation of the phenomenon to be simulated and that the equations of motion in the simulation are reasonable it is easy to conceal from oneself both structural and behavioral inadequacies in an all computer simulation.

A different use of pure computer models for simulation is to use it for exploration in the development of behavioral theory. Models of bounded rationality are usually extremely hard to analyze purely mathematically. Simulation is required even to explore the implications of the employment of few rules of thumb in a complex environment.

For those who understand the basic conceptual difficulties in selecting what is meant by a solution to a nonzero sum game in extensive form, even leaving aside problems with bounded rationality, simulation is not a substitute for game theoretic investigation, but is clearly complementary. If one is not dedicated to the construction of a purely normative theory, then a broad selection of behavioral descriptions or prescriptions of how to play the game are consistent with a game theoretic solution to a game in

¹²A useful way to obtain a feel for the scope of operational gaming and simulation in the military is to skim the D.O.D. catalogue Catalog of War-gaming and Military Simulation Models, 10th Edition Joint Analysis Directorate of the JCS, JADAM 207-86. The book, The War Game (Brewer and Shubik, 1979) provides a contextual background.

extensive form.

3. MODELING AND THE APPLICATION OF GAME THEORY

3.1. Three Levels of Game Theoretic Application

In discussing the applications of game theory it is desirable to recognize at least three distinct levels of activity. I suggest (a) Conversational; (b) Low church and (c) High church game theory. These somewhat casual terms are not meant to be pejorative but are suggested to provide intuitive distinctions among the highly different uses of a relatively broad body of knowledge.

Conversational game theory could also be described as senior political advisor, essayist and strategic consultant game theory. Its concerns are with models, concepts, ideas, examples, counterexamples, insights, suggestions, new views, paradoxes, highlighting of pitfalls and intuitive appreciation of context. Barry O' Neill, in discussion, has suggested the term proto-game theory which he uses instead of conversational game theory. I am concerned with the prefix "proto" as it implies first or preliminary; an early form of understanding rather than a summary or an encapsulation or simplification for application of a developed body of knowledge.

It is true that once some preliminary insights are grasped, such as the concept of a 2×2 matrix game as the representation of a strategic confrontation, one needs little formal training to generate many insightful analogies and discover many paradoxical situations. But this can be done in two modes. One is where the expositor knows little more formal theory than the elementary examples utilized. The other is where the overall formal theory serves as a basis for editing the advice given.

The term "Low church game theory" is used to cover mathematical modeling applications based on formal game theoretic models where the emphasis is upon the use of the model and its applications to a specific discipline or problem. Thus weapons evaluation models, attrition studies and arms races where a formal mathematical structure may be used to calculate fall into this province that lies between the theorist and the quantitatively inclined individual with operating responsibility.

High church game theory refers to the fundamental mathematical development of the methodology. This covers new formal mathematical structures to represent conflict and bargaining situations. The invention of new solution concepts and the mathematical exploration of their properties. Such an activity may be pursued by individuals with little concern for context or application but it is necessary for the growth of application provided that there is a sufficient flow of ideas and understanding among the practitioners at all three levels. Although such is probably the case in the development of economics, it is not true to as great an extent in the overall study of political science and in the study of defence and deterrence in particular.

3.2. The Strategic Audit

The basis theme of this essay is that game theoretic methods have a considerable contribution to make the study of defense. But that at the highest levels of advice-giving it is modeling and the insights of conversational game theory that must come to the fore. But the value of high level advice needs to be based not only on an understanding of and feeling for context, but also on an in depth appreciation of what calculation and

formal theory have to offer.

In Section 2 a sketch was given of the formal assumptions which go to make up the basic elements of game theoretic analysis. Many of the modeling restrictions assumed in order to produce a well defined mathematical structure do not match our assumptions about the facts of the problems faced in the fog of battle or the miasma of international negotiation. This section provides a check list for the advisor and strategic modeler which connects and contrasts with the check list for the formal mathematical model noted in Section 2¹³ (Shubik, 1968, 1983).

In strategic modeling in general and in the study of deterrence in particular there are seven broad headings for the basic checklist of key features of the model. They are:

1. Scope,
2. Time Frame,
3. Players or strategic actors,
4. Rules of the game and strategy sets,
5. Outcomes, payoffs and goals,
6. Uncertainty and perception,
7. Behavioral assumptions.

A finer breakdown of these elements is suggested.

Rigid rational utilitarian analysis may on the surface appear to fit many models in economic theory and finance. But in the world of strategic planning of any sort the realities are far from this. Table 9 contrasts the difference in stress that the mathematical modeler and game theorist methodologist will have; and the viewpoint of the strategic consultant or

¹³This list has been developed in some detail elsewhere (Shubik, 1983) in the context of corporate planning, but although some important modifications must be made in accounting for differences in long range defense planning and corporate planning, there is a considerable overlap in planning needs. Schelling's (1966) observations on modeling, abstraction and empirical content are closely related to this approach.

1. Scope
 - a. Top strategic: survival at stake,
 - b. Strategic, but below survival,
 - c. Tactical.
2. Time Frame
 - a. Date of analysis,
 - b. Length of period covered,
 - c. Event oriented, fixed clock or an intermix,
 - d. Nature of initial condition assumptions,
 - e. Nature of terminal condition assumptions.
3. Players or Strategic Actors
 - a. Level of aggregation of individuals and institutions,
 - b. The number of strategic players,
 - c. External symmetry assumptions,
 - d. The verification of "strategic dummies."
4. Rules of the Game and Choice Sets
 - a. Relevant political detail: own and other,
 - b. Relevant bureaucratic detail: own and other,
 - c. Relevant technological knowledge: own and other,
 - d. Information and communication conditions,
 - e. Scope and feasible actions.
5. Outcomes, payoffs and Goals
 - a. The clear distinction between physical outcomes and their evaluation,
 - b. Short and long term payoffs,
 - c. Payoffs defined on a finite or indefinite horizon,
 - d. The description and measurement of preference,
 - e. Risk measures,
 - f. Team, group or individual goals,
 - g. Zero sum or nonconstant sum environment.
6. Uncertainty and Perception
 - a. The assessment of the nature of outside uncertainty,
 - b. The belief in and knowability of the payoffs,
 - c. General faith in the model.
7. Behavioral Assumptions
 - a. Rationality or constrained rationality,
 - b. Risk behavior under time pressure and/or stress,
 - c. Instinctive or programmed behavior,
 - d. Problems in perception and comprehension.

TABLE 8

conversational game theorist, whose attitude must be more congenial to those of sociologists, psychologists and historians than mathematicians. Both views are needed, but for different purposes.

MATHEMATICAL GAME THEORY

Rules of the Game;
Impersonal undifferentiated
individuals;
No aspects of socialization;
Fixed well-defined payoffs;
Perfect intelligence;
No learning;
No coding problems.

STRATEGIC ADVICE

Laws and customs of society;
Personal detail may count;
Socialization is implicit;
Payoffs are difficult to define and
may change;
Limited intelligence;
Learning often relevant;
Perception and coding problems important.

TABLE 9

A few comments on the application of this modeling scheme to defense studies are noted.

Scope: The broader the scope, the more difficult it becomes to select relevant variables and specify payoffs. Herman Kahn's catch phrase "thinking the unthinkable" highlights our inability to conceptualize the unexperienced. The suggestion that generals and countries prepare to fight the last war may contain a high element of truth; but it contains no prescription as to how we go about displaying the appropriate imagination to plan for the next one (or its avoidance).

Time Frame: Most of formal game theory analysis is devoted to event rather than clock time. The length of time between moves is hardly studied. Virtually all of the analysis is devoted to the implications of the sequence or ordering of action rather than elapsed time. The game tree lays out sequence not elapsed time. Even in the game tree representation, although it appears to be all-encompassing does not indicate how long the game lasts,

thus it does not reflect an important aspect of the rules of play. The timing of an attack in nuclear warfare involves a matter of minutes (see Bracken 1983, Blair, 1985). Thus the whole concept of studied reply to action has to be reconsidered as the time scale is reduced to decisions which must be made by trained reflex, by instinct or by adherence to preformulated contingency planning. In some sense it would appear that at this point formal game theory might come into its own as an essential part of the subject involves contingent planning. But the proliferation of "what-if" scenarios is exponential and in actuality only a handful of alternative scenarios can be generated in advance, even given high speed digital computers.

In planning and in gaming the description of initial and terminal conditions may have great influence on the analysis. One of the key problems in our historical understanding even to this day is when was the point of no-return crossed in the start of many wars. It may be relatively easy to date the first shot, but that could have even taken place before the point of no return had been reached. Terminal as well as initial conditions are also noted, because (as was perceptively observed by Ikle, 1971) the possibilities of war termination, though frequently covered in the rhetoric of total victory are rarely considered in the more sober analysis of how to enable all parties to extricate themselves from bloodbaths. World War I and the Iran-Iraq war of the 1980s provide examples.

Players or Strategic Actors: As has been noted in Section 2 formal game theoretic analysis can be carried out with virtually any assumptions concerning preferences or goals. The basic strategic viewpoint of game theory is logically independent of the assumptions made about individual preferences. When the players are assumed to be units such as a nation

state or a set of institutions comprising a nation state our ability to even describe the goals of these entities is limited. If complex preference structures are postulated the ability to carry analysis to any depth becomes constrained by the complexity of the model.

Two valuable contributions of methodology to avoiding basic traps in strategic analysis are the explicit consideration of external symmetry and the verification of strategic dummies. It is highly desirable that differences in the assumptions about the players be made explicit. In weapons evaluation for example it is usually assumed that the tank crews on each side are in essence identical. In assessing the war potential in the Middle East one obtains a view that is hardly consistent with the existence of Israel if one weights the Israeli Army, for example, on a one to one basis with the Egyptian Army. What should these weightings be and how do we calculate them?

United States and Soviet military thinking frequently tends to concentrate on bipolar situations where the actors are primarily if not only the Soviet Union and the United States. Strategic analysts frequently make a modeling decision to exclude a nation from consideration as an active player because it does not appear to be sufficiently important. Yet one of the prime lessons to be learned from formal game theory involving three or more players is that it is extremely hard to judge a priori the strategic strength of a player as long as he has some freedom of action. A safer procedure is to include doubtful players into the analysis and then let the analysis establish that they manifest little, if any influence in any of the outcomes.

Rules of the Game and Choice Sets: Institutions and complex operating

systems take on a life of their own. This is well recognized in the political process where there are rarely if ever complete rules written to explain say all the relationships between the House of Lords and the House of Commons. In spite of "catch-22" phrases such as ignorance of the law is no excuse, it is well known that it is impossible for any single individual to know all of the protocol governing even a reasonably well defined complex system. In discussions of command and control systems concern has been voiced over the provincialisation of a system under attack. But this possibility exists in day to day operations where no single individual is in a position to maintain an overview that is much deeper than dangerously superficial.

Outcomes, Payoffs and Goals: The catch phrase "is that a threat or a promise?" serves to illustrate the importance of being able to distinguish an outcome and the value of that outcome to a player. Even in football or horse races, winning the game is not necessarily the same as winning for some players if the game is fixed. A key proposition in effective deterrence is to make sure that each player fully comprehends the value placed by the others on various outcomes.

3.3. Conversational Game Theory

Ellsberg (1961a, 1961b), Rapoport (1960), Schelling (1960, 1966), Shubik (1964, 1968, 1971) and to some extent Raiffa (1982) among others provide examples of conversational game theory. There is an occasional calculation and numerical example, interspersed in a discussion that recognizes the qualitative difficulties in obtaining the appropriate model of human affairs.

Ellsberg (1961a) uses an not fully defined 2×2 payoff matrix to discuss and illustrate problems in sensitivity analysis to consider when changes in weaponry and outcome evaluation are stabilizing or destabilizing. In his paper on risk and ambiguity a key problem in modeling and context is posed strikingly. Suppose that you are told that you must draw a ball from one of two jars. If you select a red ball you win a prize. You are informed that in jar 1 there are precisely 50 red and 50 black balls. In jar 2 you are informed that you are to be told nothing whatsoever about how its contents were selected. Are you indifferent between the two choices?

The work of Schelling (1960) paradoxically simultaneously set forward the uses of conversational game theory, raised fundamental questions which illustrated problems in precommitment, not easily treated in basic game theory, but somewhat misinterpreted the role of the extensive form (Schelling, 1960, Ch. 5). Many of Schelling's perceptive examples were based on 2×2 matrix game illustrations; as has much of the work of Rapoport and Associates. Rapoport's early book (Rapoport, 1960) contains an intermix of simple mathematical models, 2×2 game examples, a comparison between real and theoretical arms races, and many questions and observations on the ethics of debate. For those interested in the problems of contrasting formal game theory with the problems of modeling actual negotiations and bargaining Rapoport's book is a valuable preliminary to reading Raiffa's (1982) work on the art and science of negotiation. This book provides a perceptive blend of case histories, gaming and elementary theory which asks in an operational way, just how useful is professional strategic analysis to those faced with bargains and negotiations.

Two further works are noted as good examples of the mixture of

insight, simple mathematical models and illustrators of the paradoxes and pitfalls in the study of conflict and cooperation. They are the collected essays of Schelling (1984) on choice and consequence and essays in honor of Anatol Rapoport (Diekmann and Mitter, 1986). In both of these the richness of the examples to be gleaned from the 2×2 game are displayed and a mixture of discussion and experimental results concerning paradoxical effects of asocial behavior are noted.

At a more formal level, but important to appreciating the uses and limits of 2×2 games, the study by Rapoport, Guyer and Gordon (1976) provides an exhaustive investigation of the 78 strategically different strongly ordinal 2×2 matrix games which can be constructed. This count is accompanied with a taxonomic discussion of all of the games and experimentation with sensitivity analysis on many specific games.¹⁴ In the taxonomy the Prisoner's Dilemma game is in a class of its own, where an apparently

	1	2
1	(5, 5)	(-12, 10)
2	(10, -12)	(0, 0)

TABLE 10

reasonable definition of individual rational choice conflicts with joint rationality. The argument as can be seen from Table 10 is that for each

¹⁴By strong ordering we mean that ties are ruled out in the description of preferences. If ties are included then the number of different cases, as counted and classified by Powers (1986) turns out to be 726.

player his second strategy is better than his first under every contingency.¹⁵

There are at least several thousand papers which have been written on the Prisoner's Dilemma and on experiments with this game in various forms. Axelrod (1984) has suggested a whole theory of the evolution of cooperation based heavily on this one 2×2 matrix game. In spite of its attraction to experimental social psychologists and to conversational game theorists I find that I am somewhat cautious of the level of generalization that can be fruitfully obtained from results with a single matrix game.

The development of military operations research and the development of formal game theory are each highly different from the needs for strategic analysts to understand the logical complexities of threat and deterrence. Conversational or essay oriented game theoretic thinking which is highly sensitive to context fulfills a vital role. But on the whole the gap between the methodology of game theory and the general understanding in the Political Science profession and at the War Colleges as to what it can contribute has been and is too large.

It would probably be unwise and uneconomic to expect that the Political Science graduate courses and the Staff and War Colleges absorb much in the way of mathematical formalism. Variations on simple games of coordination, the Prisoner's Dilemma, the Dilemma of the Commons, the Battle of the Sexes, the game of Chicken all are richly suggestive and offer valuable analogies and insights, but a deeper understanding of the methodology might increase the value of these examples as both their strengths and limitations are

¹⁵ There is a vector domination of the payoffs of the first strategy by the second.

better understood.

3.4. Low Church Game Theory

The phrase low church game theory is coined to cover the broad area of application where the stress is on obtaining and interpreting the solution to a problem that has been more or less completely formulated mathematically. No attempt is made here to suggest that the classification offered is water tight. For example much of the work of Brams (1984) falls somewhere between what has been termed Conversational and Low Church as does Rapoport's discussion of arms races intermixed with simple mathematical models for which solutions are obtained. There is a large body of heavily mathematical military operations research where the work crosses the boundary in the other direction, that is between Low Church or applied goal oriented analysis and high church or mathematics and methodology oriented game theory where the interest is in theorem building, tool construction and the extension of the methodology itself. Two papers by Shubik and Weber (1981, 1987) provide examples. The first is aimed strictly at application and computation (albeit at a relatively formal level) concerning real problems in network defense and target hardening. The second relates the mathematics of the first to a pure game theoretic problem in understanding the connection between the noncooperative equilibrium solution to one game and the value solution to a related game.

The Mathematics of Conflict (Shubik, 1983) is an edited volume which presents a reasonably exhaustive coverage of many applied tactical game theoretic models encompassing, dueling, Lanchester attrition processes and differential game pursuit and evasion models. Even at the tactical level

modeling problems are considerable and a classical operations research paper on the requirement for the theory of combat by Weiss provides considerable insight into how to proceed. Institutions such as Rand, Institute for Defense Analysis (IDA) and Center for Naval Analysis (CNA) have been the locus of much of the direct military applications of game theory. The overlap of this literature and the literature in political science is limited.

3.5. High Church Game Theory

Game theory as a form of mathematics with its own justification can be studied without concern for context or administrative usefulness. Debate may be held over the relevance of various representations and whether various solutions should be regarded as normative or behaviorally inspired.¹⁶ But these debates can be set aside by the practitioner in his work. However the sponsor of research may wish to direct funding in the direction that is perceived to have the greatest applied payoff (unfortunately it is frequently extremely difficult to make such a judgement with any accuracy).

Solution concepts, such as the stable set pose many mathematical problems. A taste for them is essentially individual and motivated by the aesthetics of doing mathematics rather than solving world problems. Although self rationalizations and rationalizations to funding agencies may be supplied.

It is my belief that purely from an operational viewpoint a consider-

¹⁶ An interesting treatment of this question is provided by L. J. Cohen (1981) who observes that: "The object of this paper is to show why recent research in the psychology of deductive and probabilistic reasoning does not have 'bleak implications for human rationality' as has sometimes been supposed."

able growth in the sponsorship of both pure game theory and of experimental gaming is justified. But the details of an argument as to why the partition function form of a game might be worth considering, or what remains to be learned from further investigations of the core and value solutions would lead us too far afield from the central purpose of this essay which is to consider the possible contributions of game theory in the context of furthering our current understanding of defense and deterrence and considering a more fruitful connection with political science and policy. Suggestions, however are made in Section 5.

4. PROBLEMS IN MODELING DETERRENCE

4.1. The Theory of Threats and Bargaining

The theory of threats and bargaining has been approached from every level of game theory. Perhaps the most valuable for the immediate use of those who consider policy, has been the analogies and paradoxes generated in conversational game theory. These include the Doomsday machine of Herman Kahn as the ultimate example of a precommitted threat and Schelling's (1960, 1976) discussion of many examples. Related to both conversational game theory and the large legal, labor and business literature on bargaining is Raiffa's (1982) experiments and insights on negotiations. Beyond that there is a large formal literature on fair division; and a fast growing literature in game theory applied to economics and to basic game theory on solutions to games in extensive form.

Essentially the work can be broken down into five sections: (1) conversational game theory; (2) formal fair division models; (3) basic solution theory for games in extensive form; (4) models involving automata or con-

strained rationality and (5) specific dynamic models set in context.

The use of game theoretic analogy and examples in essay form is probably here to stay and is part of a process making the consideration of strategic consequences more explicit. Both its production and the validity of its use depends upon the art, care and sophistication of both the producers and consumers.

The adoption of new modes of thought is measured in generations. Basic concepts such as the difference between zero-sum and non-constant sum games, the difficulties in reconciling naively defined individual and joint rationality are gradually making their way into general discourse.

Mainly in the domain of formal game theory there is a large literature dealing with normative models of fair division (see Roth, 1979, for a survey of many of the axiom systems, and Shubik, 1982 for a less technical coverage). The basic models have been those of Nash on the two-person bargaining problem with fixed and variable threats and the extensions of Harsanyi and Shapley to n-person problems with various conditions on both threats and the description of preferences. The details are not of direct concern here, but several aspects of the implicit and explicit assumptions and deductions from this work are of importance to the political scientist.

In order to produce a well defined model it becomes necessary to well-define the concept of threat. This immediately raises basic conceptual problems concerning capabilities and intentions, comparison of preferences and damage exchange rate considerations. In evaluating the plausibility of a threat does one assume a worst case analysis or should the cost of carrying out the threat be considered. If it should be considered how do we measure costs and relative costs? The critical lesson that is learned is

that it is impossible to separate out fair division considerations from measures of power. The fair division axioms invariably involve considerations built up from optimality and symmetry. But before these can be applied, either explicitly or implicitly assumptions are made about initial entitlement, claims, ownership and strategic alternatives available to all individuals and groups. The characteristic function (noted above in 2.8) has within it a host of implicit assumptions concerning the power of groups. Virtually all of this theory is in vitro. It is cut out of time, yet one of the most perplexing problems in the "rightful and just claims" of nations is where does history begin. What are traditional borders and claims, when does the clock start?

Fair division theory is normative and for the most part the individual actors are without personality and are assumed to have well defined preferences.

Both in application to models of economic and other bargaining and in raising basic questions concerning theory development, game theorists have been concerned with the difficulties in defining rational behavior in a world without strictly enforceable precommitment. Bargains and negotiations take place in real time with a haze of uncertainty. The abstract normative cooperative solutions do not fit our needs to understand the processes of negotiation. Other solutions are needed. The noncooperative equilibrium appears to be a candidate.

The noncooperative equilibrium solution seems to have some behavioristic merit in a game in strategic form. It has the property of circular stability, or phrasing it differently if all parties are at an equilibrium point, the system is self-policing. It is in no one's self interest to

deviate. Unfortunately both modeling and solution difficulties appear with attempts to apply noncooperative solutions to bargaining problems. Considerable modeling difficulties are encountered in merely trying to describe bargaining in extensive form. The works of Cross (1969) and Stahl (1972) both provide good coverage and discussion of the problems of modeling.¹⁷ They offer formal bargaining structures and suggest solutions. It is important to stress that it is both difficult and probably undesirable to try to separate completely normative from behavioral considerations. A key problem in virtually all negotiations is to design procedures and institutions which will facilitate the bargaining process.

In dealing with games in extensive form, it is difficult to carry out an analysis that is institution free. The rules of the game provide a description of the carriers of process and are thus in essence elemental institutions. The discussions noted are biased towards economic bargaining and negotiation. In asking how does this literature fit with the problems of international arms negotiations one has to explore both the divergences in essential modeling and the relevance of the solution concept adopted.

A recent thesis by van Damme (1983) provides a handy summary of many of the variations on the theme of noncooperative equilibrium, including perfect, strictly perfect, strong, proper and so forth. The message should be clear. When there is a proliferation of modifications to a solution concept and when the solution is far from unique it usually means that the concepts or the models or both do not provide a satisfactory way to analyze the problem at hand.

¹⁷The more recent work of Rubinstein (1982) is related to the approach of Stahl.

A recent paper by Binmore (1986) entitled "modeling rational players" provides a rogues gallery of many of the paradoxical examples that challenge the formal game theorist attempting to find a satisfactory theory for dynamic games. The splendidly simple paradox posed by Rosenthal is noted as an example to illustrate one among the many difficulties encountered. The game shown in Figure 3 is interpreted as follows Player A moves first

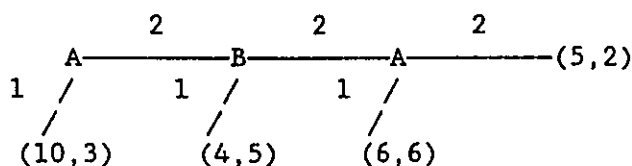


FIGURE 3

If he selects his first move the payoff to him is 10 and to B the payoff is 3. If he selects move 2 B gets the move. If B selects his first move A obtains 4 and B obtains 5. If B selects move 2 the choice goes back to A who can select move 1 with payoffs of (6,6) or select 2 with payoffs of (5,2). Suppose that you are Player B and you are told that A has selected his second move and that you are called on to move, what do you do? If A had been rational you should have never been called upon to move. Thus you must have some form of theory of A's error generation or irrationality if you are required to move.

A different approach to studying sequential play has been suggested by Rubinstein (1986) and applied to the Prisoner's Dilemma. He considers the game as being played by two finite automata where there are maintenance costs which must be paid for every state of the machine that is to be available. Implicit in Rubinstein's model is a "use them or lose them" view of armament. If it costs you to maintain an arsenal that is never actually

physically utilized, but has maintenance costs you will eventually get rid of it.

Smale in earlier work has considered limits on memory and Radner has investigated near (or epsilon) equilibria. But in spite of these promising starts the formulation and analysis of bounded rationality models is only in its early stages.

The message from this survey should be one of cautious hope. The difficulties encountered in constructing good game theory solutions to games in extensive form tell us that to a great extent behavioral approaches, gaming and simulations are complementary with the mathematical approaches. The formal models can help to separate out difficulties and clarify many distinctions. But the relatively simple concept of individual rational man is not able to provide us with a clear theory of behavior in even two person dynamics.

When we turn to the type of problems actually faced in negotiations and threats we see that although all of the concerns dealt with by formal game theory are usually present, there are also many others which are either not touched upon at all or are only present in a formal manner. For example the assumption of symmetry is often used in some form in the mathematical axioms concerning bargaining theories, yet a paper such as that of Foster (1978) points out the considerable technical and conceptual difficulties in even being able to perceive what parity is between two differently structured forces. The message of the value of a three fold approach involving behavioral understanding and gaming; institutional understanding and context relevancy and appreciation of the methodology of game theory is basically present in Raiffa's (1982) work on bargaining.

The clarion call of Simon, Minsky and others over twenty years ago to consider the possibilities and potentials of artificial intelligence turned out to be a call to a battle that was far greater and harder than originally estimated. Considerable progress has been made. But in spite of the popular press glamor of scientific phrases such as "expert systems," the problems in artificial intelligence are Hydra-headed. When one is solved nine or more appear in its place. Critics such as Dreyfus and Dreyfus (1986) may be regarded by some as overly gloomy, but they help to focus our attention on the limits to our understanding of the decision processes of even the single individual.

4.2. Arms Races and Escalation

The study of arms races and escalation provides a reasonably wide and well defined domain for both conversational and low church or plain operations research type of game theory to produce and solve specific models, which even though possibly oversimplified nevertheless illustrate problems of some operational concern. Richardson (1960) was an English meteorologist and physicist dedicated to the promotion of peace studies and the preservation of peace. He engaged in a large statistical study in an attempt to identify the causes of war and in the 1940s and 1950s produced a large data base on the outbreak of war and several formal mathematical models of arms races escalating in full hostilities. His original work was published somewhat after it had been done. Since that time many formal mathematical models of arms races and other resource battles have been built. They also related closely to two large bodies of literature on attrition in fire exchange and to the many models in the study of duopolistic competition in

economics. The early work of Lanchester (1916) provided much of the impetus for the first and Cournot (1838) provided the earliest formal duopoly model.

The models in general are described as simultaneous difference equations with time treated in finite hunks (see for example, Rapoport, 1960); as simultaneous differential equation models with time treated as continuous (Richardson, 1960; Intriligator, 1967; Intriligator and Brito, 1984; Mayer, 1986) or as differential games, stochastic games or parallel dynamic programs.

In spite of the considerable array of mathematical apparatus, the political scientist or defense analyst must be concerned with what is the relevance of these models. Several valuable lessons can be learned.

The mathematics provides useful insights and parables which illustrate that relatively simple processes may contain feedbacks within them which can explode apparently stable systems.

The limitations of our knowledge of mathematics more or less guarantees that the models must be relatively simple when compared with the actual phenomena to be studied. Thus unless the problem involves a reasonably well defined and constrained physical process the results can only be usefully interpreted as holding in an extremely small domain and providing a warning as to the tendency of motion. In many human affairs as the domain changes so does the importance of various driving factors. Thus the elements which launch a depression may be replaced by factors which were not operative at the start.

The mixture of formal analytical models, together with games and simulations have an important conceptual lesson to teach to the social sciences as a whole. The distinction among "game theory" solutions, mechanistic

solutions and behavioral solutions for the study of human behavior in situations involving conflict and cooperation is at best vague. There is no airtight trichotomy. There is no unified party line among high church game theorists as to what constitutes the correct solution to an n-person dynamic game. At best (even with all the assumptions about individual goal oriented conscious rational behavior) there is some agreement concerning solution concepts which seem to fit reasonably well for problems in specific contexts. The context of global nuclear brinkmanship is not one for which there is a unified view.

The formal concept of strategy shows that it is clear that in a process of any complication humans at best employ only a highly limited set of the strategies theoretically available. They cut down by aggregation, delegation, procrastination, lack of perception and many other devices for simplifying action. But unless an individual acts only out of thoughtless habit or instinct, formally "damn the torpedoes, full speed ahead," "Always draw to a Jack," "Do not shoot until you see the whites of their eyes," "shoot first ask questions later" are well defined but relatively simple formal game theoretic strategies.

Simple escalation processes which can be captured by low dimensional fixed differential equations will be called "mechanistic" because the strategies employed exhibit great rigidity, small scope and often give the appearance of wrong headed learning.

Simulations where simple learning conditions have been introduced and where the dynamics of interaction of many rules of behavior can be traced by computer are referred to as "behavioral" because they appear to portray great flexibility (often to the extent that it is extremely difficult to

interpret what can be learned from the output). These two can be interpreted formally as game theoretic strategies. The difference between them and the science fiction view of what is meant by a "game theory strategy" is that the justification given for a behavioral strategy comes from the ad hoc construction of decision rules, defended by observations from social psychology, political science or for other reasons. Whereas the phrase "game theory strategy" is often used to imply optimal strategy. This in turn implies that there exists some well defined superrational way for an individual to play in a dynamic multiperson nonconstant sum game. Although some individuals with a purely philosophical bent (see Harsanyi, 1982) feel that there may be some unifying solution theory which leads to a unique outcome supported by the set of optimal game strategies, this view is not widely shared.

Shubik (1971) offered a simple game known as the dollar auction as an example to illustrate some of the paradoxes of escalation. A dollar is auctioned with the rule that the highest bidder obtains the dollar, but both the highest bidder and the second highest bidder pays. Thus for example if A has bid 95 cents and B has bid a dollar there is an incentive for A to bid over a dollar for a dollar in order to cut losses. Many simple experiments with this game have been conducted (see Teger, 1980) and it is relatively easy to see individuals paying more than a dollar for the dollar. O'Neill (1986a) however offers a solution that is obtained by enlarging the model. He introduces explicitly the size of the resources available. He then goes on to discuss the relevance of the dollar auction to the escalation of international conflict and presents a comparison of the differences between the simple model and the "real thing." This discussion brings to focus the

points made by Beer (1986) on the highly different uses of paradigms, models and metaphors in the production and consumption of the social sciences.

Those developing basic theory and new tools often wish to be more "relevant" than they appear to be in their ability to utilize what they know in examining the pressing real problems of society currently at hand. It is suggested here that it is desirable that paradoxes be examined, examples suggested and metaphors offered. But the caveat should be cum grano salus.

Mathematical economic theory has scarcely recovered from a bout of Catastrophe Theory, which provided liberal metaphors for conversational theorizing over disasters and adequate mathematical technique exercises for high church practitioners.

My prediction is that Chaos Theory will be the next in that fashion show. Both will have had something of value to say and both provide rich sources for metaphor and mathematics. But it is unlikely (though not impossible) that a deep residuum of accumulated understanding of social process will be left by these methods. In a like manner variations on the theme of the Prisoner's Dilemma have generated whole subindustries in game theory applications to the social sciences. It is possible that the Prisoner's Dilemma holds the key to much of our understanding of cooperation and competition. It is also possible that it is a far too simple (or the wrong simple) example to be much more than a source of small experiments, nice mathematical models and suggestive metaphors. Metaphor has much to recommend it (see for example, Brams, 1984) but the interweave among metaphor, model and policy conclusion must be approached with extreme care.

4.3. Crisis Stability

Crisis stability is far too important a problem to be left primarily to mathematical modeling in general and game theory in particular. But there are many special questions and conceptual problems which can be posed where the attempt to specify models of extreme simplicity offer valuable insight. What are some of the key questions? These may well precede the models. For example:

- (1) Could disarmament be as or more destabilizing than increasing armament (in part this is one of the questions investigated by Intriligator and Brito, 1984).
- (2) Does the reliability and accuracy of nuclear weapons increase or decrease potential stability.
- (3) What does Murphy's law as applied to command and control systems do for international security. Is more or less dependence on error safer. (See Bracken, 1983, Blair, 1985.)
- (4) Can we devise structural measures of the degree of potential for cooperation or conflict in any situation.
- (5) Can we develop behavioral measures of stability.
- (6) How good, mutually understood and credible are our first and second strike measures of effectiveness.
- (7) What is the sensitivity of the system to IFF (identification, friend or foe) errors.
- (8) How are nuclear damage payoff functions to be characterized and how do they influence the stability of the game.

All of the questions noted above undoubtedly depend upon the context of the crisis and the systems being considered. But these questions and

several more all can be characterized as sensitivity analysis problems analyzable by means of operations research applied game theory studies. When these questions are asked in context about a specific system, assuming all other factors held constant there is a reasonable chance that qualitative and in some instances quantitative answers rather than allegories can be supplied.

In the debate on appropriations for weapons systems, the menu of what they have versus the menu of what we have is often used. This is the most elementary of military accounting systems. Richelson (1982) discusses the problems with the construction of single number static indicators. Game theoretic analysis at the most stripped down level provides a way to ask better questions concerning the operational meaning of the differences in weapons arrays. Grotte (1980) gives a model for examining stabilizing and destabilizing changes in nuclear force posture based to some extent on the utilization of the work of J. Bracken et al. (1977, see also J. Bracken et al. 1985) on optimal weapons allocations. He encounters the immediate modeling difficulties of how to assess payoffs and how to consider damage exchange rates. Shubik and Weber (1981) in considering network defense against first strike encounter the same difficulties.

O'Neill (1986b) explicitly using a Prisoner's Dilemma matrix has suggested an explicit measure of stability where the numbers in the matrix are based upon the type of detailed computation suggested by Bracken et al. Without even presenting O'Neill's index the factors relevant to the purposes of this discussion can be specified. It is a single number index based upon a single 2×2 matrix game where the numbers are the result of a more or less elaborate more or less well accepted operations research methodology

for providing damage assessment in weapons interchange. Its strength (and the strength of the Grotte, 1980 work as well as the extensions of the earlier paper of Dalkey, 1965 euphemistically entitled "Solvable Nuclear War Models") is that the detailed weapons evaluation and damage exchange problem is acknowledged, separated from the overall assessment, but connected to it. A weakness is that it is doubtful that a special small matrix game reflects enough of the problems of stability and serves as more than a parable for talking about stability.

In summary: (1) the idea of trying to construct stability indices, if viewed only as a methodological discipline is probably worth while; (2) the need to concentrate openly and explicitly on what, if any, overall payoff or utility function is going to be employed is critical. (3) The need to understand that the technical problem of weapons and damage assessment can and should be separated from the much more complex problem of the societal evaluation of the damage is important. (4) The transparency of the models is high and their very unreality helps to warn against misuse.¹⁸

There appears to be a considerable disconnect between the meant to be informed public, including much of the political science community and the military on the role and importance of formal assessment models. There has been for over thirty years a sizeable industry of consultant firm model builders such as Vector, Stanford Research Institute, SAI, Logicon or Mathematica that have been building specific assessment models which are

¹⁸ Even though written over twenty years ago the compendium of misapplications of game theory noted by Wohlstetter(1964) still merits rereading. The article by Intriligator (1982) also provides a useful (but not critical) classification of the role of mathematical models research on conflict theory.

utilized in the evaluation of weapons systems and also appear in shadow form when a major game such as the Global War Game at the Naval War College is played. When these models are used in a major game unless the players are highly experienced professionals in the nature of the warfare being studied, their transparency is lost and the players may be willing to accept dangerously wrong results emerging from the black box as an approximation of reality.

On occasion an article designed for wide dissemination may connect the political science overview with the assessment, thus for example Steinbruner (1984) in a discussion of launch under attack notes a simple command and control assessment model developed by Morawski and Blair at Brookings to perform a sensitivity analysis on retaliation plans.

Two considerably different game theoretic models are noted in concluding this section. Guth (1985) uses an extensive form game with incomplete (and inconsistent) information to model the type of problem that is uppermost in the minds of many western Europeans. The nuclear deterrence situation is modeled with three players, the Soviet Union, the U.S.A. and the Europeans with neither of the first two knowing the true type of the third player at the start.

Shubik (1986) has attempted to formally model the idea of limited perception and to point out the possibility that such a limitation could be used to strategic advantage.

In his book Bueno de Mesquita (1981) and in an article Bueno de Mesquita and Riker (1982) stress the expected utility model for considering war scenarios. But it is at this point that the warnings of formal game theory and of modeling are at their highest. The difficulties with attach-

ing preferences or utility functions or payoffs (as distinct from physical outcome descriptions) to a country as a whole are considerable. One has to justify how and why one can attach a completely ordered preference structure to a nation state or otherwise the payoff structure must be derived from games within the game (i.e. from a subgame representing a single country as several actors) or a much more complex representation of preferences than a complete ordering may be called for.

4.4. Problems with Modeling Deterrence

It has been suggested here that there are many reasonable questions which can be asked for which applied game theory investigations can provide useful answers and insights. Yet when one contemplates scenarios for the start of World War III, it is hard to not envision scenarios involving a small war say in Europe, escalating through blunders, misperception, failures in communication or in the understanding of the context and meaning of the messages being sent. Actions may be required in time spans which permit instinct or rigid programming to dominate.

The model of rational man is a gross low dimensional oversimplification to enable far more complex individuals with limited calculating capability to sweeten their intuitions and improve their insights. Game theoretic analysis has shown clearly that even throwing away all of the complications of personality and society this simple model of man does not provide us with an extension of the concept of rational behavior beyond a situation involving two parties in total opposition.

The coping with the immediate problems of how to think and act concerning the overhanging threat of nuclear destruction and the metaphysics of

deterrence here and now can be helped by game theoretic analysis. However factors such as asymmetries of mindset, individual and organizational risk assessment, coping with system failure, the role of specific leaders and control systems call for the broad array of behavioral science studies in parallel.

Game theory does have a special role to play, but its limitations as well as its strengths must be appreciated. It is about as good a language as we have for discussing and understanding strategy. It provides an underlying methodology, a way to formalize critical questions. In particular it clearly illustrates the current limitations of formal strategic analysis.

This is not a negative comment as it illustrates where and why conceptual links have not yet been forged between the approaches of the other behavioral sciences and the utilitarian economics view of the rational agent.

5. A PROGRAM FOR THE FUTURE

Where do we go from here? The argument above has been made to suggest that game theory in its three forms as well as experimental and operational gaming and mathematical modeling in general all have useful roles to play. This section is devoted to suggesting the type of programs which may have both societal and scientific value on the assumption that national defense and the prevention of nuclear war has an important priority in this society. The raising of the question about the assumption is not merely rhetorical. History teaches that humans adjust to living next to Vesuvius or on top of the Saint Andreas fault, or on an exposed stretch of coast where when the hurricane comes, they will be surprised that it could happen to them even though they were warned. Hope, optimism, foolhardiness and an ignoring of

the suggested odds are all close cousins. A way one stays alive in the trenches is to really believe that no bullet has your name on it. But it is not the only way. Heroism and foolishness, caution and fear appear to be linked in ways we still do not understand.

Life is more than an accounting exercise, but it is often a good idea to understand the accounting even though you defy the odds. Hardware is more visible and easy to justify than research devoted to trying to exorcise the devil you already know and have become used to living with. Nuclear megadeaths, greenhouse effects or nuclear winters, after a few years change from being "unthinkable" to becoming uninteresting everyday household bogeymen. They go from unthinkable to boring and unfundable. There are better and more pressing items to spend money on. The idea that one could even dream of spending the cost of a nuclear carrier task force on basic research instead appears to be foreign to the administrative mind in particular and to the public in general.

Throwing money at a problem is by no means a guaranteed way to solve it (as the Nixon administration's attack on Cancer indicates). Far more important than money alone is the availability of talent and trained personnel. In many endeavors it may be that we are near the bound in our utilization of talent. Thus a few more million in baseball or opera star salaries may do nothing more than bid prices up. There may be no more talent to be had. It is my belief that this is not the case in defense studies.

The Vietnam war destroyed several generations of analysts. They can and must be replaced and reinforced. Institutional support, money and long term educational programs can help. A program for the future is addressed here at three levels. They are: (1) Education and training; (2) Research

and basic theory development and experimentation; (3) Policy, operations and current procedures.

5.1. University and Military Education

The backlash against the war in Vietnam and even earlier the Nixon administration's desire to dispose of the not for profit think tanks changed the attitude and sponsorship of the United States' society as a whole to defense studies.

The intellectual capital devoted to the study of defense in the 1980s appears to be far lower than in the 1950s although in many ways the problems are larger and more pressing.

In particular good interdisciplinary work is difficult to carry out successfully and always has been. Yet the start made at Rand in the 1950s does not appear to have been surpassed by the 1980s. The nature of many defense problems calls for a blend of understanding involving political science, economics, social psychology, engineering, physics, mathematics, operations research, management science and computer science. This breakdown to a certain extent is a close description of the departments in existence at the Rand Corporation in the 1950s and early 1960s. Out of this mixture emerged an impressive list of defense analysts and a body of knowledge. Although technology has moved forward since then, we have primarily been living off intellectual capital rather than building it.

There is a danger that one can fall into the trap of romanticizing the past. The call is to recreate Camelot, ignoring the change in time and circumstances. This discussion is based on the premise that not only is it not possible to recreate the past (except in near parody such as towns like

Williamsburg). It is usually not desirable to do so. The call is not for a Rand of the 1980s or 1990s, but for a program that will be as successful or more successful than the research establishment was in the 1950s.

The choice in education is often presented as though it were a choice between being literate or numerate but not both. If you know the classics you are not a customer for calculus. Yet the very essence of good conversational game theory and good model building in general is an appreciation of both. The numbers are recognized and used. But it is not what the numbers are that counts it is what they mean. In order to understand what they mean it is necessary to be able to do a certain amount of calculation, but it is also critical to be able to set them in context.

Conversational game theory backed up with a decent knowledge and mastery of the basic concepts of formal game theory could improve the intermix of historical literacy with formal concepts of strategic analysis. Such a course (possibly combined with a liberal number of small operational and experimental games) could and should be given at every Academy, War College and major university. The languages of game theory open up a level of strategic literacy and discourse that serves as a basis to test both essays on strategy and formal models. The level envisioned calls for an intermix of the writings of Kahn (1960), Schelling (1960, 1984), Ellsberg (1961a, 1961b) Rapoport (1960), Shubik (1975) and others together with a rigorous, but basically elementary text at a level around that of the work of Shapley and Shubik presented in Shubik (1982).

The understanding of the rigor of formal model building combined with a skepticism of models produces individuals who are able to avoid the demands for spurious realism and detail that often is the earmark of the operator

who lacks the formal training to seek fruitful abstraction.

The type of intermixed operational, experimental and theoretical course given by Raiffa (1982) on bargaining can easily be modified to concentrate more on diplomatic and military contexts.

5.2. Basic Applied Mathematics and Operations Research

The need for more straightforward courses on low and high church game theory, depends upon the defense needs for trained operations research analysts. In my estimation, although desirable, it is not as pressing as the general raising of formal modeling understanding among the military as a whole, the social sciences in general and political science and economics in particular.¹⁹

If the basic problem being addressed is how to upgrade the level of thinking and research on nuclear war prevention and deterrence then there are undoubtedly many valuable damage exchange calculations to make and a large list of reasonably well defined problems in dueling, force structure, search, weapons evaluation, attrition and others which can provide useful inputs. But this type of work succeeds when the concepts and ideas of basic theory are understood qualitatively by those who know how to ask the right operational questions.

The National Academy of Science, the universities and the war colleges will both produce and call forth the support needed for the development of an applied mathematics if they have provided the leadership in developing

¹⁹It is of interest to note that in the Soviet Union, the book by Captain Suzdal (1977) entitled Game Theory for the Naval Officer was produced in an edition of 20,000. Whether it was read or used as a decoration is difficult to assess.

the conceptual tools for analysis together with the ability to relate them to the societal problems at hand.

Even with the general comments noted above several comments on specific problems merit thought. The computer and simulation has not been an unmixed blessing. It is easy to let a simulation be a substitute for thought and analysis rather than a complement to analysis. Equally dangerous is the possibility that the simplifications needed to permit the use of analytical methods result in the application of a dangerously oversimplified analysis to an important applied problem. The paper by Weiss (1983) on tactical combat indicates the difficulties in adequate modeling.

Introducing into the mathematical analysis of dueling or pursuit and search models, the possibilities of time lags, occasional blackouts of information and set up problems increases the mathematical difficulties considerably. Yet if the techniques such as dueling or search and evasion games are to be of direct application, the gap between the analyzed simplicity of the mathematical models and the hardly comprehended complexity of simulations must be closed.

5.3. The Development and Interpretation of Basic Theory

There are many problems in pure game theory that merit support if only for the reason that we need to expand our intellectual inventory to support both low church and conversational applications. Rather than make general observations, several specific problems are noted together with suggestions as to why knowing the answers could be of value to applied work as well as pure theory.

The $2 \times 2 \times 2$ and The 3×3 Games

More or less exhaustive work has been done on the 2×2 game. Every one of the 78 different ordinal games with strong preferences have been considered. Much of the imaginative applications of conversational game theory to defense have come through analogy and example. As noted above Schelling (1976) in a single article suggested 16 2×2 games as relevant to strategic analysis. The listing and investigation of the 726 2×2 games with ties by Powers (1986) required a fair amount of drudgery. But it had some rewarding insights. Introducing zones of indifference helps to model in a relatively simple manner situations in which lack of knowledge or perception may obscure strength of perception. But it does far more, it calls attention to the presence of implicit collusion or antagonism that is obscured in games without zones of indifference. If Player A is indifferent between his pay-offs if he employs strategy 1 or 2, he may then be able to use a secondary criterion Does he at zero cost wish to hurt or help Player B? The game in Table 12a illustrates this.

The game in Table 11b shows a slight variation which raises questions about self interest and safety. In Table 12a no matter what A does his payoff is the same.²⁰ If he plays 1, B obtains a or b. If he plays 2, B obtains c or d. By playing 1 he helps B, by playing 2 he hurts him, at no cost.

²⁰ It can be argued by those schooled in preference theory that the numbers in the matrix should already take into account the interpersonal evaluation of outcomes not only to oneself but to others. Thus the desire to help or hinder is already in the utility representation. Another view is that the numbers represent only the individualistic worth of the outcome ignoring the payoff to the other.

	1	2		1	2
1	(a,a)	(a,b)	1	(a,a)	(b,b)
2	(a,c)	(a,d)	2	(a,c)	(a,d)

TABLE 11a

TABLE 11b

The situation portrayed in Table 11b is somewhat different. If B is trying to maximize his own payoffs clearly his first strategy dominates his second strategy under all circumstances. Given this situation a generous A could play his first strategy and both could obtain the payoffs (a,a). But if A is supercautious, his second strategy dominates the first thus the solution to this game will be strategies (2,1) and payoffs of (a,c).

The $2 \times 2 \times 2$ game is the simplest three person game in strategic form that can be constructed without one of the players being a strategic dummy. Even having everyone fully appreciate or remember all of the payoffs requires that each keep $8 \times 3 = 24$ entries in perception or memory. This is already a fairly large number. A crude large upper bound on the number of strategically different games is $(4!)^3/8 = 1,728$. Are there conceptually new phenomena in terms of competition and collusion which appear? Can we obtain some form of categorization of the strategic differences among broad sets of these games. Our investigation of the 2×2 shows that a large number are intrinsically cooperative and only one has the structure of the pure Prisoner's Dilemma.

There have been several attempts to provide taxonomies over all 2×2 matrix games or to produce an index of the intrinsic degree of community or opposition of interests in the structure of the game. This is a difficult problem even to formalize in a fully satisfactory manner. But it is still an important source of problems which are both of interest in abstract game

theory and of importance in defense analysis.

The 3×3 game is in some senses easier than the $2 \times 2 \times 2$ game, and in other senses worse. There are only 18 payoff values to keep track of as compared with 24. There is only one competitor to track as compared with two. Yet the combinatorics of the case distinctions is somewhat surprising. The crude upper bound on the cases is given by $(9!)^2/(3!)^2$. But this relatively innocent looking number amounts to $(60,480)^2 = 3,657,830,400$. This number is large, but not unmanageable as a starting point for the construction of a way to characterize the important competitive and other structural differences in this class of games. For example, a couple of questions which can be answered generally about all $N \times N$ matrix games with strong ordinal preferences is what is the most cooperative structure possible and what is the most intrinsically competitive structure? Hardin's (1968) parable of the dilemma of the commons is an example of a conversational game theory approach to a question which requires a formal game theoretic investigation, to wit what are the appropriate generalizations of the prisoner's dilemma game to situations with more than two strategies and more than two individuals.

Matrix Games with Weak Orderings

Much of theory development in economics has been devoted to equilibrium and statics. This mode of thought has made considerable inroads into political science and other social sciences. The abstractions considered tend to be either static nonprocess models of static process models. An example from economics illustrates the difference. The central theorem in Debreu's (1959) work on the price system concentrates on existence without concern for specific process; no mechanism of price adjustment is studied or even

needs to be postulated.

A study of voting can be approached independent of process. The Shapley-Shubik power index (Shapley and Shubik, 1954) illustrates this. It is possible however to build process models of both economic exchange and voting. The work of Farquharson (1969) and Shubik (1973) and others on strategic voting and strategic market games provide examples.

When static models of human affairs are constructed, at least two implicit modeling assumptions are often made when process description is avoided. The first assumption is that (at least at a high level of abstraction) details of process should not matter. The second is that individuals have well defined preferences.

The fight between abstraction and institutional detail is an old one and cannot be judged a priori. A good theorist should have a feeling for the level of relevant detail.

The assumption about preferences calls to attention a different type of implicit dynamics. Any consideration of bargaining processes or the social-psychology of choice over time indicates that a great amount of effort is expended searching for information, modifying one's preferences and attempting to clarify differences in evaluation where at one instance in time the differences are not apparent. If one tries to represent this picture of human behavior in a simplified static form one needs to lay stress on indifference in preference between outcomes or inability to compare between some outcomes. At the very least this suggests that the study of strategic situations with a high probability for the presence of ties or weakly ordered preferences is important. Along with this goes the need to consider tie breaking rules and the resolution of indifference as not merely minor insti-

tutional problems which require a slight modification of analysis based on the assumption of clear strongly ordered preferences, but as a key factor in using static models to represent essentially dynamic phenomena.

Information and Equilibria

When a process is reasonably well understood it may be possible to represent it as a game in extensive form. Associated with any game tree is a class of games all represented by the same tree, but with different information conditions. This set of games can be arranged in a partial ordering where for any two games on the same branch of the partial order it is meaningful to say that one game has more information than the other in the sense that the information sets of one are a strict refinement of the sets of the other. Any game in this set of games can be investigated using some solution concept. Hence, for example we could solve each game for its set of pure and mixed strategies. As information is increased it appears that the number of pure strategy equilibria increases and the number of mixed strategy equilibria falls. Can we characterize the changes in the number of equilibrium points as information conditions change.

Solutions and Theory

A major problem of concern to those involved in both applied and formal game theory is the definition and modification of the concept of solution to a game in strategic or extensive form. The leading contender in both conversational and low church game theory has been the noncooperative equilibrium. It is well known that if the Nash (1951) definition is taken in an unadorned manner then many games will have a profusion of equilibrium points. But for purposes of prediction and strategic analysis we would like

to have a way to cut down on this multiplicity of suggested outcomes.

Can we invent reasonable extra criteria which limit our choice? This requires a mixture of modeling and mathematics. The work of Selten (1975) on (the unfortunately named) perfect equilibria has offered a way of selecting among equilibria in sequential games, but a profusion of perfect equilibria may still remain. Sequential equilibria have been suggested by Kreps and Wilson (1982). A coverage of some of the variations on noncooperative equilibria has been given by Shubik (1982, Chapters 9 and 10). The purpose here is not to launch into a discourse on these differences but to pose both the pure and applied game theory problems. Except in games with a great amount of special structure the noncooperative equilibrium solution has little predictive power.²¹ A reasonable question to ask is are there any more basic assumptions or axioms which can be added to cut down the set of noncooperative equilibria. But this question must be posed both at a mathematical and modeling level. The extra conditions may not appear to be reasonable when interpreted either as norms for human behavior or as positive statements about what people do.

It may be that abstract game theory applied to a general set of games must content itself with a patchwork of many competing mathematical structures and solution concepts each of which, in application can only be justified in some ad hoc manner such as "it seems to provide a reasonably good fit to the problem at hand."

The search for a unique equilibrium point for all games, or the right solution concept for everything may best be left to the philosophers and

²¹ And little if any normative properties to recommend it.

theologians. At this stage in the development of both theory and applications it may be that handtailoring might be the most productive. This means that game theory applications, model structures and solution concepts might be specialized in application to the study of nuclear deterrence; the political problems of voting; the study of economic markets; experimental social-psychology, weapons evaluation or military tactics. Such a growth in specialized applications does not displace the need for the basic development of abstract game theory. On the contrary, in the same way as the development of large scale manufacturing creates a larger and more vital market for the machine tool industry, the specialized applications may require a larger back up of formal mathematics.

Error and Solutions

For interesting technical reasons Selten introduced the concept of trembling hand equilibrium into his study of equilibrium points. The basic idea is that any player when called upon to move is subject to a small error, thus when he reaches to press button A, by accident with some small probability he may press button B or C. The game can be solved given this possibility, then we can study the behavior of a limiting set of games where the size of the error is reduced. Although the elegance of the mathematical sensitivity analysis is illustrated by the process of smoothing away the tremble; in actuality in many command and control systems and in network communication error generation is the rule not the exception. Thus a fruitful set of questions might be asked about solution properties applied to games where it is taken as an axiom or at least as part of the modeling that there is some finite threshold of unintentional error to be expected from all decision makers.

Many more specific problems could be listed together with an argument as to why they are of importance both from the viewpoint of theory and application. But probably the major stumbling block to increasing the volume of work is less the presence of problems which are relevant, can be well formulated and might be reasonably tractable, than it is the sociology of scientific research.

The nature of the theory of games is such that the health of its development depends much more than many other branches of mathematics upon the interplay between formal mathematics and at least two levels of application in many different disciplines. This calls for a level of interdisciplinary coordination which at the best of times is difficult to come by. Funding sources tend to like to be able to separate applied mathematics, from social-psychology, political science or economics. Euphemisms such as Decision Sciences or Management Science have been invented in part to give some sense of identity or unity to a mixed bag of pure and applied mathematics, operations research, behavioral sciences and ad hoc model building. The development of Game theory requires cooperation and coordination among highly different practitioners as well as interdisciplinary funding. Neither of these are impossible but neither is easy to nurture. Conversational and formal game theory are far more important to each other for their eventual development than may be recognized by those who specialize in either, but not both.

5.4. The Need for Experimental, Educational and Operational Gaming

The study of strategy calls for the use of several different techniques. Of varying importance to the problems at hand are: (1) the essay (including conversational game theory); (2) free form operational gaming; (3) semi-rigid operational gaming; (4) simulation; (5) experimental gaming and (6) rigid rule gaming and analysis (including formal game theory).

A reasonably detailed discussion of the history and purposes of gaming and simulation has been given elsewhere (Shubik, 1975a, b; and Brewer and Shubik, 1979) and is not repeated here. The uses of operational and educational gaming have been recognized by every major military establishment in the world. An example of the value attached to the importance of gaming is the size of the War Gaming Department at the Center for Naval Warfare Studies at the United States Naval War College.²² But the gap between basic game theory research, the results of social psychology and experimental gaming and military operational and teaching gaming is probably larger now than it was in the 1950s and 1960s. At that time the Systems Development Corporation was spun off from Rand to perform much of the activities in gaming and systems design while Rand still maintained a high level of activity in both experimental gaming and game theory.

In spite of the presence of a war gaming department with between 60 to 80 members, even the Naval War College does not have an experimental gaming and game theory component which at a bare minimum would supply the connections for the flow of insights and understanding between operational gaming and theory and experimentation.

²²See: Annual Report 1985-1986, Naval War College, Center for Naval Warfare Studies.

A group of between 7 to 10 professionals at the PhD. level could be of considerable worth. It would consist roughly of 2 game theorists, 2 experimental social psychologists, 1 or 2 statisticians; 1 or 2 quantitative political scientists and 1 or 2 engineer-physicists with broad weapons systems understanding. Bureaucratic and administrative problems in adequately funding such a group to attract first class professionals may present problems. But it is a group of this variety that is needed to provide the interlink between the universities and defense education and the mathematics and hard and soft sciences approach to defense problems.

Fashion changes. After the seminal work of Goldhamer and Speir (1961) there was an interlude when the Political Military Exercise (PME) flourished as an art form. In the past 10 or 15 years the sweep of fashion carried in with it the all computer simulation, expert systems and the dream of artificial intelligence. It is possible that the current trend is somewhat back in the direction of the PME and free form game. But regardless of sponsorship or lack of sponsorship, the hard questions have hardly been asked, let alone answered. Is the experience more than a mere game. What are the purposes served? Was anything actually learned from the work of Guetzkow et al. (1963) on International Simulation and its relatives? Did the PMEs of Bloomfield (1960) leave lasting results. Were the ingenious small and more mechanized games of Paxson (1972) of lasting value.

An even more important set of questions is did any of this work interface with and influence the development of game theory and experimental gaming and vice-versa. We do not have to be in the loop where not knowing history dooms us to living it again. It can be established that at least the work on formal game theory has been cumulative. We know considerably

more than we did twenty or thirty years ago. It is not that clear that the work on conversational game theory, applied strategic analysis, operational gaming or even experimental gaming has been consistently cumulative. Cute portrayals of paradoxes, clever examples and even profound insights are certainly better than nothing, but they do not constitute a unified and growing body of knowledge.

At this time we have enough trained personnel, but we are missing the focus and the institutional will and means to direct sufficient organizational and other assets to fundamental applied research in bargaining, negotiation, deterrence, experimental psychology, gaming and game theory. Yet the size of funding required is a fraction of the amount we are willing to expend on a broad array of hardware improvements for communications and weapons systems which at best offer only marginal tactical improvements.

A few specific items concerning experimental gaming are noted.

- (1) When does a war game go nuclear? (2) What do we know about gaming and the relationship between real time operations and game time operations?
- (3) What do we know about gaming, briefing and start point and end point effects in trying to model ongoing phenomena?

It is more or less accepted in the war gaming community that war games do not go nuclear with ease. An in depth study of the evidence of war game plays by different national groups investigating what percentage went nuclear, and investigating why would merit support.

An all encompassing problem for gaming, game theory and their applications to war is do we need to make allowances for the interplay between attitude to risk and culture? Would a Japan of today still find the Kamikaze volunteers that it found 45 years ago. Are risk perceptions and

attitudes towards risktaking, to a good first approximation culture free, or is this an important point at which we should abandon the assumption of external symmetry.

5.5. The Psychological Component and Normative Theory

A central problem in the application of game theory to the social sciences in general and to the study of a topic such as deterrence in particular is are our applications meant to be viewed as normative suggestions or behavioral observations. Are we trying to teach ourselves and the Soviets how to behave; or do we take their preferences, perceptions and intentions as given and constrain our role to advising how to behave given these assumptions as facts of life. The "is and ought" aspects of what constitutes a solution to a bargaining problem have already been noted above (Section 4.1). In the last few years there has been an explosion in the growth of models of behavior under asymmetric information and a growth of concern for risk assessment, subjective probability. And there has also been a growth of literature on human behavior concerning events with extremely low probabilities but disastrous consequences attached to them. This includes items such as nuclear reactor failure, atomic war or plague. The literature in psychology, heavily influenced by the contributions of Kahneman, Slovik and Tversky (1982) (see also Tversky and Kahneman, 1981) is replete with examples that individuals do not behave as expected utility maximizers when faced with small probability events with great consequence. A tentative "Prospect Theory" has been suggested which indicates that the human reaction to risk is highly dependent on the framing of the perceptions of the risk. When the same objective material is presented in terms of the

risk of being killed or in terms of the chances for survival different reactions are encountered. That context counts comes as no surprise to the historian, scholar of strategy or operational or experimental gamer. The briefing can easily bias the results of the game.

Leaving aside, for the moment the paradoxical messages from the psychology of individual risk; there is still an open debate in the literature on social psychology concerning risky shift and group behavior. Most of the major decisions in our society are fiduciary decisions. Someone is making decisions involving the money and the lives of others. How does risk behavior of the fiduciary match with risk behavior of the individual acting to his own account? Do fiduciaries acting as a group behave more or less conservatively than they would on the average as individuals? (see Janis, 1982).²³

The third set of open questions comes from the thousands of experiments on matrix games associated with experimental social psychology and game theory. The pioneer work of Rapoport, Guyer and Gordon (1976) in spite of its scope and intensity in its concentration on the 2×2 game in every variant only scratched the surface.

The fact that both game theorists and social psychologists have been involved in the experimentation has highlighted the extremely different but essentially complementary approaches. The game theorists for the most part are not particularly interested in personality differences. Much of the most fruitful mathematical development of the subject has been devoted to the study of how equally intelligent and capable individuals (the assumption

²³ This is not precisely defined.

of external symmetry) will behave when faced with a nonsymmetric environment. In contrast the social-psychologist is frequently most interested in the explanation of differences in performance of players in a completely symmetric game. Their explanation is via nonsymmetries in the nature of the players.

Possibly one of the few clear lessons that emerges from the multitude of experimental matrix games is that briefing and context matter considerably and that in spite of the elegance of the backward induction argument in the finite horizon Prisoner's Dilemma game people do not select the noncooperative equilibrium point over the whole period of play.

Attitudes towards low probability, high consequences risk; the understanding of fiduciary risk and the interpretation of what we have learned from matrix games are three basic problem areas directly relevant to not merely the study of deterrence but also to the collateral problems of command and control of the system which must back up deterrence.

One social scientist's black box is another's livelihood. Thus it is easy for a scholar to be narrow in attempting to explain some aspects of a clearly multivariate, complex, variously coupled man-machine system with political, economic and technological factors varying in importance. It is extremely difficult to be general without risking the dangers of being vacuous or at best superficial.

The Name of the Game

In conversational game theory several 2×2 matrix games have been given names. The three most popular in the rogue's gallery are the Prisoner's Dilemma, the Battle of the Sexes and Chicken. The first has already been illustrated. The other two are shown in Tables 12a and 12b respec-

tively.

	1	2		1	2
1	(0,0)	(2,1)	1	(-5,-5)	(10,-2)
2	(1,2)	(0,0)	2	(-2,10)	(-1,-1)
Battle of Sexes			Chicken		

TABLE 12a

TABLE 12b

The conversational game theory story told for the first involves a husband and wife who want to go to the movies together but each prefers a different movie. The story of Chicken involves two California hotrod drivers coming at each other each with a wheel of his car on the center line of the road. The first one to swerve is chicken. Do the numbers suggest the scenarios? Giving the Chicken, Battle of the Sexes and Prisoner's Dilemma matrices to students with no previous game theory and giving them the three names, asking them to attach the names to the matrices only produced weakly significant results for the Battle of the Sexes. I have repeated this exercise for several years with students in game theory. Do these matrices conjure up the same visions to French, Japanese and Russians? Are there "natural names" and stories to be hung on the other 75 games, or at least some of them and some of the 726 games including ties? For some of these games Kelley has suggested the concept of fate control.

Can we teach the Russians the same code book for how we talk about 2×2 matrix games, or can we teach ourselves how not to indulge in dubious or at least unsubstantiated generalizations about what they mean.

Doublecross is Two-ply and Revenge is Three-ply

One of the great strengths of formal game theory is that it has enabled us to clarify problems which were thought to be purely psychological or sociological. Thus for example bluffing in Poker turns out to be related to the employment of optimal mixed strategies. The various cooperative solutions indicate the limitations of pure economic reasoning in the division of joint product. An avenue of research that has scarcely begun concerns building games in extensive form to find out how complicated matters must become before it is even possible to identify phenomena that we talk about in every day social existence. An example illustrated this point. Each set of branches of a game tree can be regarded as a "ply" or as a search of depth of one. How many moves or ply must a game tree reach before certain phenomena appear. You cannot double cross someone unless there is an implicit or explicit norm already in existence. Thus a game to illustrate an opportunity to doublecross must be at least two-ply. You cannot take revenge on someone unless you have been doublecrossed in some form. Thus revenge requires a game of length at least three. What is the minimum scenario and amount of complexity needed for fear, hope, greed, humiliation, generosity to appear? A research game theory and gaming program in the operational linguistics of threats appears to be both feasible and worthwhile.

It is my belief that revenge represents a far higher and more sophisticated human process than does the playing of a perfect equilibrium point--even though we know that at that point in the game tree where Grey's cities are in ruins it would be irrational to wipe out Green's cities as that would be mere pique after deterrence had failed.

Status and Survival

Ethredge (1986) has discussed the power motive of individuals as a key element in deterrence. There is a natural opportunity for the development of both formal game theory and gaming along two critical dimensions which have been primarily the domain of the social-psychologist and sociologist. They are survival and status. Shubik and Thompson (1959) have suggested a formal game structure which stress the trade off between probability for survival and gain. Shubik (1971) proposed a class of games known as games of status where the payoffs are the ranking of the players in the society. The first class of games draws attention to the important nonsymmetry between gaining and losing when one is near the brink. The second class of games highlights the considerable increase in the competitive structure that appears when the innocent passtime of wealth accumulation is substituted for by status goals.

5.6. Defense Studies, Technology and Game Theory

The phrase rational behavior needs at the very least to be replaced with the phrase "context rational behavior." The suspicion of the businessman and the soldier for the academic, and vice-versa is well earned. The clever and the sophisticated strategy had better fit with the facts relevant to the problem at hand rather than an institution free, process free general theory of rational choice. Cleverness in the design of ploys and counter-ploys will add extremely little to arms control verification and to the design of stable enforcement mechanisms if they are not designed to be in tune with the physical facts. The practical men tend to be overwhelmed with detail, the theorists unconcerned. If we had a good enough theory the luxury

of not being concerned with the details of process might be sustained. But we do not have a good enough theory.

In economics the failure of sterile noninstitutional models of oligopolistic threat and counterthreat, highlighted by better game theoretic techniques to make these exercises easier, faster and more complex has revived a new interest in industrial organization where facts and theory might be considered together.

The parallel between work in oligopoly theory and in the use of competitive models in defense is close. The defense analyst and political scientist should learn from both the successes and failures in the developments in economics. Threats to enter into competition appear to depend upon reaction times, secrecy in information, the possibility of leaks, the verification of information and other time and process factors. The game theorist as advisor to a specific industry beyond supplying a method of thought and observation has little to say unless some ad hoc facts have been absorbed. The 2×2 matrix game, even repeated over time may have little to contribute to the understanding of, say arms control, beyond parable unless it can be shown that it faithfully reflects the lags and uncertainties of actual arms control verification procedures (see Tsipis, Hafenmeister and Janeway, 1986, for example).

5.7. What Does Game Theory Have to Contribute That is Operationally Useful Concerning Deterrence

There is not one, but many problems posed by trying to understand deterrence. There are many disciplines required to investigate the highly different but related parts. Yet whether looking at the political process, the command and control system, arms control bargaining, the hardening of silos or first and second strike scenarios a component of individualistic conscious interlinked strategic decisionmaking is present.

Game theory provides an underlying set of languages and methodology for the study of conflict and cooperation. It is an organizing device and not a substitute for substantive knowledge and understanding of context. Its strengths highlight its limitations. By forcing us to understand the logical implications of trying to push the model of the individual goal oriented rational intelligent actor in multiperson interaction it highlights the need to consider models with limited rationality, and constraints on data processing. The behavioral models of political science are in general not necessarily incompatible with game theoretic thought on dynamics.

"Has game theory lived up to its promise?" This particular rhetorical sentence can be used equally well for a session in self-congratulation or criticism. The more proper qualification is "promise to whom?" With this in mind I close on a moderately optimistic note. The concepts of game theoretic analysis were one of the major new conceptual insights of the twentieth century. The acceptance of new insights is a social process. Around half of a century is not inordinately long for basically new ideas to make their way into commonly accepted knowledge. When newspapers and popular writers start to talk of the "Zero sum problem" or the Prisoner's Dilemma

and when conversational game theory examples proliferate, these serve as an indication that the basic concepts have made their way into the thinking of society as a whole.

The early proponents of "a revolution in thought" often misestimate the straight technical difficulties in modeling and mathematics in providing specific answers to burning (and often ill-defined complex) questions. Game theory per se will not solve many of the problems in deterrence. It has not even solved many of the problems in the economics of oligopoly theory (a subject at which it was first aimed). But it has provided considerable insights and a clarification of the problems. The mere fact that work is proceeding on all three levels (conversational, low and high church) is a sign of progress.

The gap between the acceptance of conversational theorizing in the social sciences and the appreciation of the formal concepts and difficulties encountered in game theory, models of limited rationality and artificial intelligence is important. It needs to be narrowed if the flow of valuable ideas between those with substantive concerns and methodologists is to be increased. Although this is by no means the best of all possible worlds, there are indications that this gap is closing. More stress on the appropriate educational programs could speed up this process.

There are enough low church problems that are sufficiently well defined and are directly relevant to defense problems that support of them is merited (to some extent on an ad hoc basis).

Basic research and new ideas require the growth and backing of a cadre of researchers. But although increased funding is a necessary, it is not always a sufficient condition; the talent has to be available. It is possible

that some of the talent that is currently going into law and finance (in accord with Willy Sutton's Law)²⁴ could be redirected. However extra resources spent on basic research, even if they help to bring forth excellent work may only contribute tangentially to the specific problems of policy advice on deterrence now.

A good overall strategy requires the simultaneous study of the operational problems at hand with the means at hand; the willingness to sponsor and pay for the educational program required and the foresight to risk sufficient expenditure on basic research without unrealistic stress on magic solutions which rarely if ever come directly from basic research, but which in the long run feed off it much as a healthy tree feeds off the soil in which it is planted.

²⁴Q. Mr. Sutton why do you rob banks? A. Because that's where the money is.

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