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MEASURING MARKET POWER IN U.S. INDUSTRY

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## MEASURING MARKET POWER IN U.S. INDUSTRY

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### ABSTRACT

Non-competitive conduct can be assessed by estimating the size of the markup or Lerner index achieved in a market. The markup implies a price elasticity of demand faced by the representative firm. For a given markup, non-competitive conduct is greater the more elastic is the market elasticity of demand. The ratio of the firm's to the market elasticity is a measure of non-competitive conduct that is insensitive to the value of the monopoly. To implement this measure, both the firm's and the market elasticities of demand must be estimated. Hall shows how to estimate the markup, and hence the elasticity faced by the firm, from the cyclical behavior of productivity. To estimate the market elasticity, an instrumental variables procedure exploiting a covariance restriction between productivity shocks and demand shocks is used. Results for broad sectors of private industry and for non-durable manufacturing industries display a wide range of monopoly power.

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## 1. Introduction

This paper presents some new estimates of the degree of market power in U.S. industry. Non-competitive outcomes can be assessed by comparing price and marginal cost. Using price-cost margins alone to assess market power is problematic because they fail to take into account the value of the monopoly. A monopoly will command a greater margin the more inelastic is the demand for the good. Differences in elasticity of demand should be taken into account in comparing conduct across markets. The measure in this paper does so by normalizing the estimated markup by the elasticity of demand.

In a recent series of papers, Robert Hall<sup>1</sup> suggests that many U.S. industries are not competitive. This result is based on a simple and elegant reinterpretation of the well-known fact that productivity varies pro-cyclically. The reasoning is as follows: As output expands in the trough of a recession, labor increases less than is warranted by a standard production function. Hence, price exceeds marginal cost. Put differently, when output is low, price does not fall enough to allow the sale of goods that can be produced at very low marginal cost. Hall reasons that this finding is only consistent with a high degree of monopoly power. There is little evidence of substantial profits in U.S. industry; he reasons that the monopoly profits are dissipated by chronic excess capacity.

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<sup>1</sup>Hall (1986a, b, c).

In this paper, I extend Hall's framework to yield an independent measure of the degree of monopoly power in an industry. Hall takes perfect competition as his null hypothesis and then measures departures from pure competition. In the formulation in this paper, the polar hypothesis of pure monopoly can be studied. More importantly, if the industry is neither perfectly competitive nor perfectly monopolistic, a measure of the degree of market power can be calculated. The markup implies an elasticity of demand faced by the representative firm in the industry. The ratio of the market elasticity of demand to that implied by the markup is a measure of market power. In the polar case of perfect competition, it equals zero. In the opposite polar case of perfect monopoly, it equals one. In the intermediate range between zero and one, it measures non-competitive conduct.

To implement this measure it is necessary to estimate both the market elasticity of demand and the implied by the markup ratio. I follow Hall in estimating the latter from the cyclicalities of productivity. To estimate the market elasticity of demand, I exploit the covariance restriction between the productivity shock and the disturbance in the demand equation. By definition, an industry productivity shock is uncorrelated with the demand disturbance. The restriction serves to identify the price elasticity of demand. This covariance restriction has an instrumental variables interpretation. The productivity shock is a valid instrument for the price variable in the demand equation. This approach to estimating the elasticity of demand should have wide application beyond its use here in measuring market power.

## 2. Theoretical Framework for Measuring Market Power

In this section of the paper, I review Hall's measure of non-competitive behavior and develop a further measure of the degree of monopoly power.

Under assumptions of constant returns to scale and competition Solow (1957) shows that the total factor productivity residual (the "Solow residual") will measure technological change in a production function. This residual is defined as

$$(1) \quad \Delta \epsilon_t = \Delta y_t - \Delta k_t - \alpha_t (\Delta n_t - \Delta k_t)$$

where  $\Delta y_t$ ,  $\Delta k_t$ , and  $\Delta n_t$  equal the percent change in output, capital, and labor, and  $\alpha_t$  equals the share of labor income in nominal output. The share is measured as  $\alpha_t = W_t N_t / P_t Y_t$  where  $W_t$ ,  $N_t$ ,  $P_t$ , and  $Y_t$  are the levels of the nominal wage, labor, the nominal price level, and real output. Equation (1) applies equally well to the aggregate economy or to a narrowly defined industry.<sup>2</sup> Solow's derivation requires that the technology be constant returns to scale and that technological change be disembodied. Solow's derivation also requires that markets are competitive and labor is variable

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<sup>2</sup>Variables and parameters should all be thought of as subscripted by  $i$  for industry. This subscript is suppressed except where it is needed in a particular context. Productivity has a drift so  $\Delta \epsilon_t$  is not mean zero. Hence the equations in  $\Delta \epsilon_t$  have constants that are suppressed for notational convenience. In the estimates, a constant is included in all the regressions so the estimated  $\Delta \epsilon_t$  are mean zero in the sample.

within the period, so labor is paid its marginal product. Capital need not be paid its marginal product so it can be fixed in advance.

A similar relationship holds under imperfect competition. Define  $X_t$  to be marginal cost. Solow's derivation holds if price does not equal marginal cost, but in equation (1) labor's share in output should be replaced by labor's share in cost. Denote labor's share in cost as  $\alpha_t^c = W_t N_t / X_t Y_t$  and denote the true productivity shock as  $\Delta \varepsilon_t^*$ . Then the true productivity shock is given by

$$(1') \quad \Delta \varepsilon_t^* = \Delta y_t - \Delta k_t - \alpha_t^c (\Delta n_t - \Delta k_t) .$$

Marginal cost is, however, unobservable. Hall suggests assuming that  $\mu = P_t / X_t$  is a constant. Then (1) and (1') can be combined to yield

$$(2) \quad \Delta \varepsilon_t = (\mu - 1) \alpha_t (\Delta n_t - \Delta k_t) + \Delta \varepsilon_t^*$$

where  $\Delta \varepsilon_t$  is the measured Solow residual, where  $\Delta \varepsilon_t^*$  is the true productivity shock, and where  $\mu$  is an unknown parameter to be estimated.

Hall's strategy is then to estimate the markup ratio  $\mu$  and to test the hypothesis that it equals one. For a wide range of industries he is able to reject the null of perfect competition. The markup does not provide a natural metric for the degree of monopoly power in an industry. For example, a monopoly facing relatively inelastic demand will have a larger markup than a monopoly facing more elastic demand. Moreover, it may be desirable to rank industries according to their degree of non-competitive

conduct rather than simply to categorize them as competitive or not competitive. I consider a measure of monopoly power that takes into account the elasticity of demand in the industry. The measure nests the special cases of perfect competition and perfect monopoly.

The markup can be expressed in terms of the elasticity of demand faced by the representative firm in the industry. Hence,

$$(3) \quad \beta^* = \frac{\mu}{1 - \mu}$$

if the representative firm acts as if it faces an elasticity of demand equal to  $\beta^*$ .<sup>3</sup> The value  $-1/\beta^*$  is the Lerner index of market power. Denote the market elasticity of demand as  $\beta$ . If the industry is monopolized, or if its firms collude effectively to duplicate the monopoly outcome, then  $\beta^*$  will equal  $\beta$ . Given estimates of both parameters, it is possible to test the natural null hypotheses of monopoly ( $\beta = \beta^*$ ) as well as that of perfect competition ( $\beta^* = -\infty$  or, equivalently,  $\mu = 1$ ).

Attention need not be confined, however, to these polar cases. The ratio of the market to the firm's elasticities is a natural measure of

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<sup>3</sup>This formula is a simple rearrangement of the first order condition for the one-period profit maximization problem. It will also arise from a discounted present value maximization problem as long as sales today do not affect demand in the future.

monopoly power. Define this ratio as

$$(4) \quad \theta = \beta / \beta^* .$$

The coefficient has a natural interpretation.<sup>4</sup> Under competition  $\theta$  equals zero; under monopoly it equals one. If the industry functions as a Cournot oligopoly, the reciprocal of  $\theta$  measures the number of equally-sized firms. Bresnahan (1987) argues persuasively that the ratio  $\theta$  is an interesting measure of conduct for models where Cournot behavior is not maintained. It normalizes the departure of price from marginal cost by the elasticity of demand. The interpretation as the reciprocal of the number of firms in a Cournot oligopoly provides a useful metric for market outcomes even under different models of firm behavior.

### 3. Identification and Estimation

The strategy is to estimate the markup ratio  $\mu$  and the price elasticity of demand  $\beta$  and then to compute the index of market power  $\theta$ . In this section, I consider instrumental variables procedures for estimating these parameters.

To estimate the markup ratio,  $\mu$ , Hall makes the crucial identifying assumption that the true productivity shock  $\Delta \epsilon_t^*$  has no aggregative

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<sup>4</sup>The distinction between  $\beta$  and  $\beta^*$  has been discussed at least since Chamberlain. It is developed by Bresnahan (1982) and analyzed in detail by Bresnahan (1987).



component. This assumption is specifically at variance with some theories of the business cycle. Specifically, Kydland and Prescott (1982) rely on an aggregate productivity shock as the only driving variable in the business cycle. (It is important to note, however, that some implementations of real business cycle theories, notably that of Long and Plosser (1983), generate cycles with shocks that are independent across sectors.) Hence, Hall's formulation, as he is careful to acknowledge, can shed no light on theories that require an aggregative productivity shock as an essential ingredient in aggregative fluctuations.

Under Hall's assumption, aggregate GNP growth is a valid instrument for  $\alpha_t(\Delta n_t - \Delta k_t)$  in equation (1) on industry data.<sup>5</sup> Some recent work of my own calls into question Hall's assumption in this data. In particular, for many industries it is difficult to reject that marginal cost moves consistently with observed productivity under the hypothesis that observed productivity is true productivity (Shapiro, 1987). In this case, Hall's procedure is invalid. On the other hand, in about half the non-durable manufacturing industries--results for which I will highlight here--my earlier test rejects the null that observed productivity is true productivity. Moreover, it is common practice in industrial organization to assume that year-to-year changes in aggregate output come from demand. Hence, it seems best to remain agnostic; I present results based on Hall's elegant solution of using aggregate GNP growth as an instrument in the markup equation.

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<sup>5</sup>Usually identification is not achieved by disaggregation alone. In this case, at least approximately, disaggregation does lead to identification because of the role of the aggregate demand shock. See the Appendix for details.

This agnosticism need not be nihilism. If there are truly exogenous variables that are correlated with movements in labor input, these can be used as instruments. Valid estimates can be offered without taking a stand on whether or not there is a macroeconomic component to true productivity growth. Oil prices and government purchases (especially military purchases) are candidates for instruments. Estimates that use these will also be presented.<sup>6</sup>

Although one only needs to estimate the elasticity of demand to estimate  $\theta$ , it is helpful to write down the entire demand equation. Consider the following demand equation in percent change form for a given industry:

$$(5) \quad \Delta y_t = \beta \Delta p_t + Z_t \gamma + \Delta \nu_t .$$

Here,  $\Delta y_t$  is the growth rate of output,  $\Delta p_t$  is the growth rate of the relative price, the  $Z_t$  are demand shift variables, and  $\Delta \nu_t$  is a taste shock. To estimate the elasticity of demand, it is necessary to overcome a difficult simultaneity problem. The often intractable identification problem has a straightforward solution in this context. Consider the

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<sup>6</sup>Hall (1987) takes the agnostic position and uses similar instrument lists. But see Hall (1986a, b, c).

following identifying assumptions:

- The productivity shock  $\Delta \varepsilon_t^*$  is uncorrelated with aggregate demand or other aggregate demand shifters such as military expenditure or oil prices.
- The productivity shock  $\Delta \varepsilon_t^*$  is uncorrelated with the taste shock  $\Delta \nu_t$  and with other variables that shift demand.

The first of these assumptions has already been discussed. It is used by Hall to identify  $\mu$  in equation (2). For aggregate demand to be a valid instrument for  $\alpha_t \Delta n_t$  then aggregate demand must be one of the demand shifters  $Z_t$ . For an instrument to be valid it must both be uncorrelated with the disturbance and correlated with the endogenous variable.

The second assumption follows virtually by definition of a productivity shock. Independent movements in technology should be uncorrelated with changes in taste. The identifying assumption is a covariance restriction rather than the more conventional exclusion restriction. Such a covariance restriction has an instrumental variables interpretation (Hausman and Taylor, 1983). The estimated disturbance  $\hat{\Delta \varepsilon}_t^*$  is a valid instrument for price in the demand equation (5). It is certainly correlated with price movements, it is by construction uncorrelated with the aggregate demand component of  $Z_t$ , and it is, by assumption, uncorrelated with the taste shocks  $\Delta \nu_t$ . Implicit is the assumption that the productivity shock is

uncorrelated with the other excluded variables, which are thrown into the error term.<sup>7</sup>

The two-step estimator where the residuals from (2) are used as instruments for (5) is consistent asymptotic normal and is efficient (see Hausman, Newey and Taylor (1987)). Indeed, since the demand equation is just-identified, the instrumental variables estimator is numerically identical to maximum likelihood. Yet, the covariance matrix printed out by standard computer packages is incorrect. The true covariance matrix is a function of nuisance parameters. I compute and report the correct estimates. See the Appendix for details of the computation.

One possible excluded variable, future price changes, could defeat identification. If goods are durable, this variable is relevant for demand and could well be correlated with the productivity shock. The results for the durables are suspect in any case because the flow demand curve is completely inadequate for them. Developing a complete theory of supply and demand for durables under imperfect competition is well beyond the scope of this paper. The results for durables are included only for completeness.

Before presenting the estimates, further consideration of the parameterization of the demand equation is in order. As noted above, only the parameter of the relative price will be estimated. The omitted variables are orthogonal to the instrument, so this procedure yields consistent estimates. Nonetheless, it is worthwhile thinking about the

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<sup>7</sup>Under the identifying assumptions, instrumental variables estimation of  $\beta$  is consistent even if all the  $Z_t$  are omitted. Efficiency could be gained, however, by including some if they are observable. Of course, if they are endogenous, one would need instruments for them.

omitted variables. In particular, aggregate demand is omitted. Note the instrument is orthogonal to this omitted variable not just in theory but by construction as long as aggregate GNP is used as an instrument in the markup equation. Yet, omitting this crucial demand shifter leads to an inefficiency. One way to include it without having to estimate another parameter is to estimate a homogenous demand system. In this case, the left hand side of (5) becomes industry output growth minus aggregate output growth. Hence, the demand equation becomes

$$(5') \quad \Delta y_t - \Delta q_t = \beta \Delta p_t + Z_{1t} \gamma + \Delta v_t$$

where  $\Delta y_t$ ,  $\Delta p_t$ , and  $\Delta v_t$  are, as before, the industry output growth, relative price growth, and demand shock; where  $\Delta q_t$  is aggregate output growth; and where  $Z_{1t}$  are demand shifters excluding aggregate output growth. All the elements of  $Z_{1t}$  except the constant are subsumed into the error term so only  $\beta$  and the intercept are estimated. Note that the orthogonality of aggregate output growth to the instruments implies that the estimate of  $\beta$  will be numerically the same whether (5) or (5') is estimated. The standard errors will, however, be affected. The demand equation actually estimated is (5') where all of the demand shifters (except the constant) are subsumed into the disturbance.

#### Gross output versus value added

The data used to estimate both the markup and the demand equation will be on a value-added rather than a gross price basis. Hence, the estimated

parameters are not comparable to conventionally estimated ones. Hall (1986a) gives the formula to convert the value-added markup to the ratio of gross price to marginal cost ( $\bar{\mu}$ ). If  $\gamma$  is the share of materials in gross output and materials are used in fixed proportions with gross output, then  $\bar{\mu} = \mu/(1+(\mu-1)\gamma)$ . Hubbard (1986) and Domowitz, Hubbard, and Petersen (1987) stress how Hall's empirical results on the value-added markups are substantially larger than the gross-output markups.

The firm's and the market elasticities of demand ( $\beta^*$  and  $\beta$ ) also are estimated on a value-added basis. Again under the assumption that materials are used in fixed proportions with gross output, it is possible to calculate the elasticities on a gross price/gross output basis. Specifically, the gross price/gross output elasticities are  $\bar{\beta}^* = \beta^*/(1-\gamma)$  and  $\bar{\beta} = \beta/(1-\gamma)$ .<sup>8</sup> Note that the correction for both the firm's and the market elasticities are

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<sup>8</sup>Using the value-added deflator rather than the gross output price index in the demand function also adds a multiple of the growth in real materials prices to the disturbance of the demand function. This component of the disturbance is orthogonal to the productivity shock instrument. Changes in materials prices will not bias the measure of total factor productivity as long as materials are used in fixed proportions (the assumption made here) or value-added is measured as a Divisia index (Bruno, 1978, Theorem 2). The GNP data are double-deflated, not Divisia indexes. (The U.S. national income and product accounts are rebased every ten years, not continually.) The bias in using the double-deflated data instead of a Divisia calculation is very small even in the face of materials price changes of the size seen in the 1970s (Bruno and Sachs, 1985, p. 58). Finally, in the estimates where oil price changes are included as instruments for the markup equation, the instrument in the demand equation is orthogonal by construction to this major component of materials price changes.

the same. Hence the measure  $\theta$  need not be corrected for the materials share.

Data are available for materials share in manufacturing. Results on a gross output basis can hence be reported.

#### 4. Data and Results

The markup equation (2) and the demand equation (5') are estimated on a panel of all U.S. private industries. Estimates are presented for both broad sectors and for two digit industries. The two digit industries have the narrowest definitions provided in published National Income and Product Accounts (NIPA) data. The data reflect the comprehensive revisions of the NIPA that were published in December 1985 and are revised as of July 1986. Details are provided in the Appendix. The data are available from 1947 to 1985 except for the labor data, which begin in 1948. The sample period is thus 1949 to 1985 to allow for computing the changes. These data are at a higher level of aggregation than is often used in studies of market power. Consequently, the estimates will be averages of sometimes disparate markets. The advantage of these data over accounting data is that, at least in theory, the NIPA measures the appropriate economic concepts.

The markups equation (2) and the demand equation (5') are estimated by the instrumental variables procedures discussed in the previous section. The elasticity faced by the firm,  $\beta^*$ , and the ratio of the market to firm's elasticity,  $\theta$ , are computed from the estimated  $\mu$  and  $\beta$ . The standard errors of the estimated  $\beta^*$  and  $\theta$  are computed using the first-order Taylor series

approximation, which is valid asymptotically. The covariance between the estimated  $\mu$  and  $\beta^*$  is zero in the sample because of the covariance restriction on the disturbances of the equation.

### Basic Results

Table 1 presents results for broad sectors. The estimates are based on pooled rather than the data for the total sector. Data for the two digit industries are pooled under the corresponding sector. The two digit industries are given in the Appendix. Construction, Wholesale Trade, and Retail Trade have no two-digit sub-industries in these data. The pooling procedure is as follows. The markup equation (2) is estimated by instrumental variables on each sectoral panel of industries. Within the panels, each two digit industry has a different intercept but the same slope coefficient ( $\mu-1$ ). The residuals from these equations are used as instruments for price in the demand equation (5').<sup>9</sup> Again, each demand equation has a different intercept but a common slope coefficient. For both equations, I report the system two stage least squares standard errors.<sup>10</sup> The reported Durbin-Watson statistics are the average for those of the two

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<sup>9</sup>The residuals from the unrestricted estimates of equation (2) where  $\mu$  varies across industries are used as the instruments. This procedure assures that the covariance restriction holds in the sample for each industry.

<sup>10</sup>The standard errors thus are consistently estimated in light of the clear heteroskedasticity across the two digit industries.



digit equations.<sup>11</sup> The pooling has two advantages over estimates carried out on the total data for each sector. First, the estimates are more efficient. Second, it is possible to remove a fixed effect for each industry.

The results of the pooled estimates are presented in Table 1.<sup>12</sup> The first three columns give the estimated markup coefficients, the standard errors of estimates, and the Durbin-Watson statistics for Hall's markup equation (2). The standard errors of the coefficient estimates are given in parentheses. The markups are essentially zero in agriculture, construction, and services. In durable and non-durable manufacturing and in finance, the markups are moderately sized but strongly significant. In mining, transportation, communications and utilities, and trade, the markups are very large. All the markups are estimates with a moderate to very fine degree of precision.

The next three columns give the estimates for the demand equation. The results of the instrumental variables estimation is highly satisfactory. Except for mining, all the estimated elasticities are negative. Except for mining and finance, the elasticities are precisely estimated to be values bounded away from zero.

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<sup>11</sup>The distribution across industries of the Durbin-Watson statistics for non-durables are given in the next table. The distribution of the statistics across the industries is consistent with the hypothesis of no serial correlation. Results for other sectors are similar.

<sup>12</sup>For completeness, Table 1 reports results for durable goods. These should be discounted because, as noted above, the form of the demand function is inappropriate for them.

The next two columns give the values of the implied elasticity  $\beta^* = \mu/(1 - \mu)$  and of the measure of market power  $\theta = \beta/\beta^*$ . There is a one-to-one relationship between  $\mu$  and  $\beta^*$ . Industries with high markups have low implied elasticities. The measure of market power  $\theta$  gives a somewhat different ranking of the competitive than do the markups alone. Mining and finance have estimated market elasticities that are essentially zero. Consequently, they appear to be competitive based on the  $\theta$  measure regardless of the markup. Industries with non-degenerate estimates of the markup also display interesting differences in ranking. For example, transportation and retail trade have similar estimated markups. The industries face different elasticities of demand and hence can be ranked in terms of degree of market power.

The assumption that the slope coefficients in each of the sub-industries are equal underlies the pooling of equations (2) and (5'). For all industries except for durables, one cannot reject these hypotheses using conventionally-sized tests.

Estimates for the individual non-durable manufacturing industries are presented in Table 2. The estimation procedure is the same as for the results in Table 1 except there is no pooling. In several of the industries (food, textiles, apparel, and printing and publishing), the estimated markups are insignificantly different from zero. These industries also have small markups. Other markups are significantly different from zero. Rubber and leather have relatively small markups; paper, chemicals, and petroleum have intermediate markups; tobacco has a very large markup.

The estimated market elasticities all lie between -1.0 and -1.8. Most are estimated very precisely. Using the innovation in industries productivity as an instrumental variable is a promising approach for estimating elasticities of demand.

The last two columns of Table 2 give the implied estimates of  $\beta^*$  and  $\theta$ . The estimates of  $\theta$  display interesting dispersion and yield rankings as to market power that are different from those implied by the markup alone. As in Table 1, all of the point estimates of  $\theta$  fall within the admissible range between zero and one. Tobacco, paper, and chemicals all have values of  $\theta$  insignificantly different from the monopoly outcome and at least three standard deviations different from the competitive outcome. All three industries can be characterized as monopolized despite their very different estimated markups. Apparel and leather have values of  $\theta$  of 0.3 and 0.5 respectively. Moreover, each estimate has standard error such that both the competitive and monopolized outcomes can be rejected. Food and textiles have small estimated  $\theta$ . These industries appear to be competitive, although the point estimates are imprecise. Finally, the standard errors for printing and publishing, petroleum, and rubber are large enough to make it difficult to reject any hypothesis.

#### Alternative Instruments

The previous results are based on the perhaps incorrect assumption that year to year changes in aggregate output are unrelated to productivity shocks. To provide estimates that are robust to failure of this assumption, I present estimates based on alternative instruments. These instruments

should be strictly exogenous. The instruments used are growth in oil prices and military employment.<sup>13</sup> Oil prices have been determined largely by forces independent of the U.S. business cycle. These include the Suez crisis, the OPEC price increases after the Yom Kippur war and after the fall of the Shah of Iran, import quotas, and domestic price regulations. Changes in military employment are largely determined by wars, which again are not caused by U.S. business cycles. On the other hand, oil price changes and wars effect the business cycle. Therefore, they are likely to be adequate instruments.

The results for the pooled manufacturing data with the alternative instrument lists are presented in Table 3. The first line repeats the results from Table 1 for ease of reference. The next three lines show the results for the combinations of the alternative instruments. The final line shows the result with all three instruments. With either oil price or military employment growth alone or jointly as instruments,  $\theta$  is estimated to be 0.1 or 0.2. With any of these combinations of instruments, the estimates of  $\mu$  and hence of  $\theta$  are very imprecise. By comparing the estimates of the last two lines, it is potentially possible to judge whether the use of aggregate GNP growth as an instrument is a valid over-identifying restriction.<sup>14</sup> The point estimates of market power with the alternative instruments are smaller than with aggregate GNP growth as an instrument.

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<sup>13</sup> See the Appendix for sources of these data.

<sup>14</sup> A formal Hausman specification test computed on the last two estimates of the markup and demand equations in Table 3 does not reject the overidentifying restriction.

Yet, those estimates are so imprecise that it is impossible to reject the hypothesis that use of aggregate GNP growth is an instrument is a valid overidentifying restriction.<sup>15</sup>

## 5. Discussion

### Alternative Estimates of the Markup

It is interesting to compare these estimates of market power with those obtained by other techniques. The markup can be estimated from a cost function rather than from the cyclicalities of productivity. Appelbaum (1982), for example, estimates  $\theta$  using the markup from the derivative of an estimated, parametric cost function. He examines three non-durable manufacturing industries: textiles, rubber, and tobacco. His findings for textiles and tobacco agree qualitatively with mine. Textiles are essentially competitive and tobacco has strong market power. He finds rubber is essentially competitive whereas I find that industry to be close to the monopoly outcome, albeit with an imprecise point estimate.<sup>16</sup>

Appelbaum estimates less monopoly power than do I because he finds smaller markups. It is fairly straightforward to explain why Appelbaum's implementation of the cost function yields systematically lower markups than does Hall's non-parametric approach that I follow in this paper. Appelbaum uses a constant returns to cost function where labor, materials, and capital

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<sup>15</sup>The underlying estimates for most of the individual industries using only oil and military employment growth have large standard errors.

<sup>16</sup>Note that rubber is a very concentrated industry (see below).

are all free to vary within the period. Thus, he constrains marginal cost to equal average cost. If an industry shows little profit in excess of the required rate of return on capital, as do most industries, this approach will yield a small markup.<sup>17</sup> Hall's procedure admits the possibility of higher markups in such industries because Solow's formulation allows marginal cost to be below average cost. In the Solow calculation, capital can be quasi-fixed (as it surely is), so the marginal cost curve can have upward slope.

#### Measures of Market Power Compared

It is useful to compare the measures of market power presented in this paper with other measures. Before doing so, it is necessary to put the measures on an equal basis. As discussed above,  $\mu$ ,  $\beta^*$ , and  $\beta$  are on a value-added basis. Table 4 presents the estimates from Table 2 converted to a gross output basis.<sup>18</sup> As Hubbard (1986) and Domowitz, Hubbard, and Petersen (1987) stress, the gross output based markup is substantially lower than the value-added based ones. Moreover, the ranking of the markups changes depending on the materials intensity of the industries.

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<sup>17</sup>U.S. manufacturing industries have low price-cost margins. Domowitz, Hubbard, and Petersen (1986, Table 1) find that profits per output is only about 35 percent in the most concentrated industries and 26 percent overall. Since their calculation of profit includes normal return to capital, their figures imply lower markups than those included in this paper. Any calculation that is based on average cost must similarly yield low markups.

<sup>18</sup>The gross output elasticities should be interpreted with caution because the share of materials may have shift over time.

Table 5 compares a number of measures for the non-durable manufacturing industries. They are the markup coefficient as estimated in Table 2 ( $\mu$ ); the markup adjusted to a gross output basis ( $\tilde{\mu}$ ); the implied elasticity of demand from Table 2 ( $\beta^*$ ); the implied elasticity on a gross output basis ( $\tilde{\beta}^*$ ); the index of market power from Table 2 ( $\theta$ ); and a measure of average concentration ( $CR_4$ ).<sup>19</sup> The cross-sectional standard deviation and simple and rank correlations of these measures are given in Table 5. The simple correlations among all the variables are quite high. Particularly striking is the fairly strong correlation of the concentration ratio with the other measures. Concentration ratios might be very misleading as to market power in cases where the other measures may not. This is especially true if market power arises because of monopolization of local markets. The rank correlations of the measures with concentration are substantially weaker. Much of the leverage for the correlations comes from the extreme observation of tobacco. Excluding tobacco, the correlation of  $\tilde{\mu}$  and the concentration measure falls to 0.40 and that of  $\theta$  and the concentration measure falls to 0.49. Figures 1 and 2 give a scatter of the observations for  $\tilde{\mu}$  and  $\theta$  versus concentration. The important correlation between the measures clearly emerges from the Figures. Aside from petroleum, the scatter diagrams are qualitatively quite similar. The estimates of market power in this paper give a similar picture of the degree of monopolization of U.S. non-durable manufacturing industries than do the concentration ratios.

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<sup>19</sup>The measure is the sales ratio average of four firm concentration ratios based on the four digit data in Rotemberg and Saloner (1986).

### Conclusion

In this paper, I discuss estimation of demand elasticities by exploiting the restriction that demand shocks and technological shocks are independent. Although this technique may have broad implications, it is used here to compare market demand elasticities with those implied by price-marginal cost margins. The price-marginal cost ratio is not, per se, a measure of monopoly power. If an industry has a price-marginal cost ratio that differs significantly from unity, one can reject the hypothesis of pure competition, but one can say nothing else about the degree of monopolization in the industry. A given price-marginal cost ratio will imply more monopoly power the more elastic is demand. Monopoly power must be strong in order to extract high prices in an industry with elastic demand.

The estimated measures of market power correspond to a wide range of conduct. The point estimate for tobacco and chemicals is that they achieve the monopoly outcome. Paper is also close to the monopoly outcome. On the other hand, for apparel and leather, both the monopoly and competitive outcomes can be decisively rejected.

Finally, this paper shows how to estimate the price elasticity of demand by exploiting the covariance restriction between productivity shocks and demand shocks. This covariance restriction is an important step in overcoming the difficulty in identifying the demand curve because of the lack of valid instruments. This technique for estimating the elasticity of demand should have important applications beyond the measurement of market power.



APPENDIX

1. Data

The data used in this paper are described in this Appendix.

Except for the data on the capital stock, they are from the National Income and Product Accounts (NIPA). The data are annual and are available at the two digit level of disaggregation. The NIPA were rebenchmarked in December 1985. At the same time, the base year for the constant dollar calculations was changed to 1982. The data incorporate the July 1986 revisions and were extracted from BEA computer tapes. The industry definitions are based on the 1972 standard industrial classification. In detail, the NIPA data are as follows. Table number refer to those in the NIPA as published in the July Survey of Current Business.

output ( $y_t$ ): Real value-added in 1982 dollars (Table 6.2).

deflator ( $p_t$ ): Value-added deflator (Tables 6.1, 6.2). In the demand equation estimates, the relative price is calculated as the ratio of the industry value-added deflator to the aggregate GNP deflator.

manhours ( $n_t$ ): Hours of full-time and part-time employees (Table 6.11). The industry level manhours data are only available at the sectoral level. To construct hours for the two digit industries, the hours for the sectoral total are divided among the appropriate two digit industries in proportion to the two digit industry's share in full-time equivalent employment (Table 6.7) in the sector.

compensation: Compensation (Table 6.4) is used in the calculation of the labor's share,  $\alpha_t$ . It includes wages and salaries, employers' contributions to social insurance, and other labor income, but not

proprietors' income. The share  $\alpha_t$  is calculated as a Divisia index, that is, it is a moving average of the share at time  $t$  and time  $t-1$ .

The capital stock data are also based on the rebenchmarked NIPA data. They are published in the Survey of Current Business (August 1986, for example) were obtained for this study from computer tape. The capital stock is constant dollar net capital stock of equipment plus structures.

The data for the instruments is as follows: aggregate GNP is from the NIPA Table 6.2 and is measured in constant 1982 dollars. Oil prices are measured as the ratio of the producer price index for crude oil (PPI561) to the GNP deflator. Military employment is full time equivalent Federal military employment (NIPA Table 6.6). All instruments are annual and log differenced.

The two digit industries pooled in the estimates in Table 1 for the respective sectors are as follows. The number of two digit industries is given in parentheses.

Agriculture (2): Farms; Agricultural services, forestry, and fisheries. Mining (4): Metal mining; Coal mining; Oil and gas extraction; Non-metallic minerals, except fuels. Construction (1). Durable manufacturing (11): Lumber and wood products; Furniture and fixtures; Stone, clay, and glass; Primary metal industries; Fabricated metal products; Machinery, except electrical; Electric and electronic equipment; Motor vehicles and equipment; Other transportation equipment; Instruments and related products; Miscellaneous manufacturing industries. Non-durable manufacturing (10): Food and kindred products; Tobacco manufactures; Textile mill products; Apparel

and other textile products; Paper and allied products; Petroleum and coal products; Rubber and miscellaneous plastic products; Leather and leather products. Transportation (7): Railroad transportation; Local and interurban passenger transit; Trucking and warehousing; Water transportation; Transportation by air; Pipelines, except natural gas; Transportation services. Communications and Utilities (3): Telephone and telegraph; Radio and television broadcasting; Electric, gas, and sanitary services. Wholesale trade (1). Retail trade (1). Finance, insurance, and real estate (6): Banking; Credit agencies other than banks; Security and commodity brokers, and services; Insurance carriers; Insurance agents and brokers, and services; Real estate. Services (10): Hotels and other lodging places; Personal services; Business services; Auto repair, services, and garages; Miscellaneous repair services; Motion pictures; Amusement and recreation services; Health services; Legal services; Educational Services

## 2. Econometric Issues

### Disaggregation and Identification

Aggregate output growth will not in general be a valid instrument for estimating the markup equation (2) on disaggregate data. Disaggregation alone typically does not yield identification. If there is an important aggregate demand shock, equation (2) will be approximately identified. Say aggregate GNP growth (denoted  $\Delta q_t$ ) evolves as

$$\Delta q_t = \Delta v_t + \sum_i \omega_i \Delta \varepsilon_{it}^*$$

where  $\Delta v_t$  is an aggregate demand shock and  $\omega_i$  is the share of industry  $i$  in the aggregate. (The shares are constant only for notational convenience.) The inconsistency in estimating  $\mu_i$  that arises because the aggregate is the sum of the industries is

$$(A1) \quad \text{plim } T^{-1}(\hat{\mu}_i - \mu_i) = \omega_i \text{Var}(\Delta \varepsilon_{it}^*) / \text{Cov}(\alpha_{it}(\Delta n_{it} - \Delta k_{it}), \Delta q_t) .$$

The usual problem with identification by disaggregation is that as  $\text{Var}(\Delta v_t)$  vanishes, the inconsistency is of order one. On the other hand, if the variance of the demand shock is not small, the inconsistency is on the order of  $\omega_i$ , which is small.

#### Instrumental Variables Estimator and Covariance Restrictions

Consider estimation of two equations analogous to the markup equation and the demand equation. Data are available on a panel of industries.

Denote them

$$(A2) \quad y_{it}^k = X_{it}^k \delta_i^k + v_{it}^k$$

where  $k = 1, 2$  indexes equations, where  $i = 1, M$  indexes industries, and where  $t = 1, T$  indexes time. In matrix notation we have

$$(A3) \quad y^k = X^k \delta^k + v^k$$

where the  $y^k$  and  $v^k$  are stacked vectors of observations and disturbances and  $X^k = \text{diag}(X_i^k)$ . First consider identification of the parameters. Suppose that the first equation ( $k=1$ ) is at least just-identified as is the case of markup equation. It can be estimated by standard instrumental variables techniques. Suppose that the second equation has two right hand side variables. (In the demand equation they the constant and the price term.) The equation is under-identified by conventional means. That is, no instrument is available. Suppose that the disturbances of the two equations are independent industry-by-industry. That is,  $E(v_i^1 v_i^2) = 0$ . This restriction is sufficient to just-identify the parameters of the second equation. Moreover, it has an instrumental variables interpretation. The residual from the first equation can be used as an instrument for the second equation. This procedure yields consistent, asymptotic normal estimates and, because the second equation is just identified, is equivalent to maximum likelihood (see Hausman, Newey, and Taylor (1987)).

The covariance matrix printed by the computer for the second equation will not be correct. The correct one depends on the estimated parameters and the data from the first equation because the fitted value rather than the true value of  $v^1$  is used as the instrument. Let  $Z_i$  be the instruments for the second equation for the  $i^{\text{th}}$  equation and where  $Z = \text{diag}(Z_i)$ . (In this paper,  $Z_i$  has two columns, the constant and the residuals from the first equation.) The instrumental variables estimator of  $\hat{\delta}_i^2$  is such that

$$(A4) \quad \hat{\delta}_i^2 = \delta_i^2 + (Z_i' X_i^2)^{-1} Z_i' v_i^2 .$$

Note that the second term of the right hand side of (A4) will have an element involving the correlation of  $v^2$  and  $X^1\hat{\delta}^1$  because  $Z$  is dependent on the estimated value of  $\delta^1$ . This term will add to the covariance of the estimate  $\delta^2$ . Define a matrix  $\Omega$  that has elements

$$(A5) \quad \omega_{ij} = v_i^2, X_i^1, \text{cov}(\hat{\delta}_i^1, \hat{\delta}_j^1) X_j^1 v_j^2 .$$

This term is the covariance between the residual from the second equation and  $X^1\hat{\delta}^1$ . Let  $\Sigma$  be the  $M \times M$  covariance matrix of the disturbances of the second equations (industry-by-industry). That is,  $E(v^2 v^2') = \Sigma \otimes I$ . Let

$$S = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

be a matrix that selects elements of  $Z_i$  corresponding to the fitted instrument. The correct variance-covariance matrix of the system instrumental variables estimator of  $\delta^2$  is then

$$(A6) \quad V(\hat{\delta}^2) = (Z'X^2)^{-1} Z' (\Sigma \otimes I) Z (X^2, Z)^{-1} + (Z^2, X)^{-1} Z' (\Omega \otimes S) Z (X^2, Z)^{-1} .$$

The first term is the standard system instrumental variables covariance matrix. The second is the correction term. The resulting standard errors are strictly greater than the ones printed by the standard computer program. In this application, the correction adds ten to twenty percent to the standard errors.

Table 1

Elasticities Implied and Estimated

Pooled, Sectoral Estimates

	Markup Equation			Demand Equation			Implied	
	$\mu-1$	see	dw	$\beta$	see	dw	$\beta^*$	$\theta$
Agriculture	0.0 (0.4)	6.4	1.7	-1.8 (1.0)	14.3	2.3	-96.3 (3294.2)	0.0 (0.6)
Mining	0.9 (0.3)	9.3	1.4	0.1 (0.1)	9.6	2.0	-2.1 (0.4)	--
Construction	0.2 (0.2)	3.9	0.7	-1.0 (0.2)	2.9	1.5	-5.2 (3.2)	0.2 (0.1)
Durable Manufacturing	0.4 (0.0)	5.1	2.2	-1.4 (0.2)	27.5	1.7	-3.5 (0.3)	0.4 (0.1)
Non-durable Manufacturing	0.4 (0.1)	5.0	1.9	-1.3 (0.1)	6.9	1.9	-3.4 (0.6)	0.4 (0.1)
Transportation	1.1 (0.2)	11.4	1.7	-1.0 (0.1)	6.4	1.8	-1.9 (0.1)	0.5 (0.1)
Communications and Utilities	1.3 (0.7)	24.2	1.5	-1.2 (0.4)	178.9	1.8	-1.8 (0.4)	0.7 (0.3)
Wholesale Trade	1.7 (0.6)	3.4	1.6	-1.5 (0.5)	3.0	1.7	-1.6 (0.2)	0.9 (0.3)
Retail Trade	1.2 (0.4)	2.3	2.4	-1.2 (0.3)	2.5	1.8	-1.8 (0.3)	0.7 (0.2)
Finance	0.2 (0.2)	5.5	1.1	-0.1 (0.1)	26.4	1.7	-5.5 (3.1)	0.0 (0.0)
Services	0.0 (0.1)	6.6	1.5	-1.2 (0.3)	22.1	1.8	-26.4 (57.9)	0.0 (0.1)

Table 2

Elasticities Implied and Estimated  
Non-durable Manufacturing Industries

	Markup Equation			Demand Equation			Implied	
	$\mu-1$	see	dw	$\beta$	see	dw	$\beta^*$	$\theta$
Food	0.4 (0.7)	3.3	2.3	-1.0 (0.3)	4.5	2.2	-3.8 (5.2)	0.3 (0.4)
Tobacco	4.0 (1.8)	6.3	1.3	-1.3 (0.3)	8.2	1.8	-1.3 (0.1)	1.0 (0.3)
Textiles	0.3 (0.3)	6.0	1.7	-1.5 (0.5)	8.6	2.0	-4.7 (4.0)	0.3 (0.3)
Apparel	0.3 (0.2)	2.8	1.9	-1.1 (0.3)	3.0	1.4	-4.1 (1.5)	0.3 (0.1)
Paper	1.5 (0.4)	5.5	1.7	-1.5 (0.3)	5.5	2.4	-1.7 (0.2)	0.9 (0.2)
Printing and Publishing	0.4 (0.5)	2.9	1.7	-1.8 (0.6)	3.7	1.6	-3.2 (2.7)	0.5 (0.5)
Chemicals	2.0 (0.7)	6.2	1.8	-1.5 (0.4)	5.3	1.8	-1.5 (0.2)	1.0 (0.3)
Petroleum	1.3 (0.7)	5.9	1.4	-1.5 (1.0)	14.6	2.1	-1.7 (0.4)	0.9 (0.6)
Rubber	0.7 (0.2)	5.2	2.6	-1.8 (1.2)	8.7	1.8	-2.3 (0.3)	0.8 (0.5)
Leather	0.8 (0.4)	5.9	2.4	-1.2 (0.5)	6.7	2.1	-2.3 (0.6)	0.5 (0.2)



Table 3  
 Elasticities Implied and Estimated  
 Pooled Non-durable Manufacturing Industries  
 Alternative Instruments for Markup Equation

Instruments	$\mu-1$	see	dw	$\beta$	see	dw	$\beta^*$	$\theta$
growth in:								
GNP	0.4 (0.1)	5.0	1.9	-1.3 (0.1)	6.9	1.9	-3.4 (0.6)	0.4 (0.1)
oil prices	0.1 (0.3)	14.4	1.8	-1.2 (0.2)	6.7	1.8	-8.6 (17.3)	0.1 (0.3)
military employment	0.2 (0.3)	97.3	1.7	-1.2 (0.2)	22.5	1.9	-5.9 (7.1)	0.2 (0.2)
oil prices and military employment	0.2 (0.3)	6.0	1.9	-1.2 (0.2)	10.6	1.9	-7.6 (11.3)	0.2 (0.2)
GNP, oil prices, and military employment	0.3 (0.1)	4.9	1.9	-1.3 (0.1)	7.4	1.9	-3.9 (0.9)	0.3 (0.1)

Table 4  
Value-Added and Gross Output Parameter Estimates

	value added			gross output			$\gamma$
	$\mu-1$	$\beta^*$	$\beta$	$\bar{\mu}-1$	$\bar{\beta}^*$	$\bar{\beta}$	
Food	0.4	-3.8	-1.0	0.2	-7.1	-2.0	0.466
Tobacco	4.0	-1.3	-1.3	0.5	-2.9	-2.9	0.566
Textiles	0.3	-4.7	-1.5	0.1	-11.2	-3.6	0.582
Apparel	0.3	-4.1	-1.1	0.1	-8.4	-2.3	0.510
Paper	1.5	-1.7	-1.5	0.4	-3.5	-3.1	0.522
Printing	0.4	-3.2	-1.8	0.3	-4.7	-2.6	0.314
Chemicals	2.0	-1.5	-1.5	0.6	-2.8	-2.8	0.455
Petroleum	1.3	-1.7	-1.5	0.2	-5.3	-4.6	0.671
Rubber	0.7	-2.3	-1.8	0.3	-4.4	-3.4	0.462
Leather	0.8	-2.3	-1.2	0.3	-4.8	-2.5	0.519

---

Note: The estimated  $\mu$ ,  $\beta^*$ , and  $\beta$  are from Table 2. The gross output based estimates are derived according to the formulas  $\bar{\mu} = \mu / (1 + (\mu - 1)\gamma)$  (Hall, 1986a),  $\bar{\beta}^* = \beta^* / (1 - \gamma)$ , and  $\bar{\beta} = \beta / (1 - \gamma)$ . The materials shares ( $\gamma$ ) are from Hubbard (1986).

Table 5  
 Alternative Measures of Market Power  
 Non-durable Manufacturing Industries

	$\mu$	$\tilde{\mu}$	$\beta^*$	$\tilde{\beta}^*$	$\theta$	$CR_4$
standard deviation	1.14	0.16	1.21	2.7	0.30	0.19
simple correlation with:						
$\mu$	1.00	0.83	0.76	0.65	0.80	0.68
$\tilde{\mu}$		1.00	0.85	0.87	0.88	0.59
$\beta^*$			1.00	0.92	0.94	0.51
$\tilde{\beta}^*$				1.00	0.84	0.44
$\theta$					1.00	0.62
$CR_4$						1.00
rank correlation with:						
$\mu$	1.00	0.77	1.00	0.77	0.86	0.17
$\tilde{\mu}$		1.00	0.77	1.00	0.77	0.17
$\beta^*$			1.00	0.77	0.86	0.17
$\tilde{\beta}^*$				1.00	0.77	0.86
$\theta$					1.00	0.25
$CR_4$						1.00

Figure 1

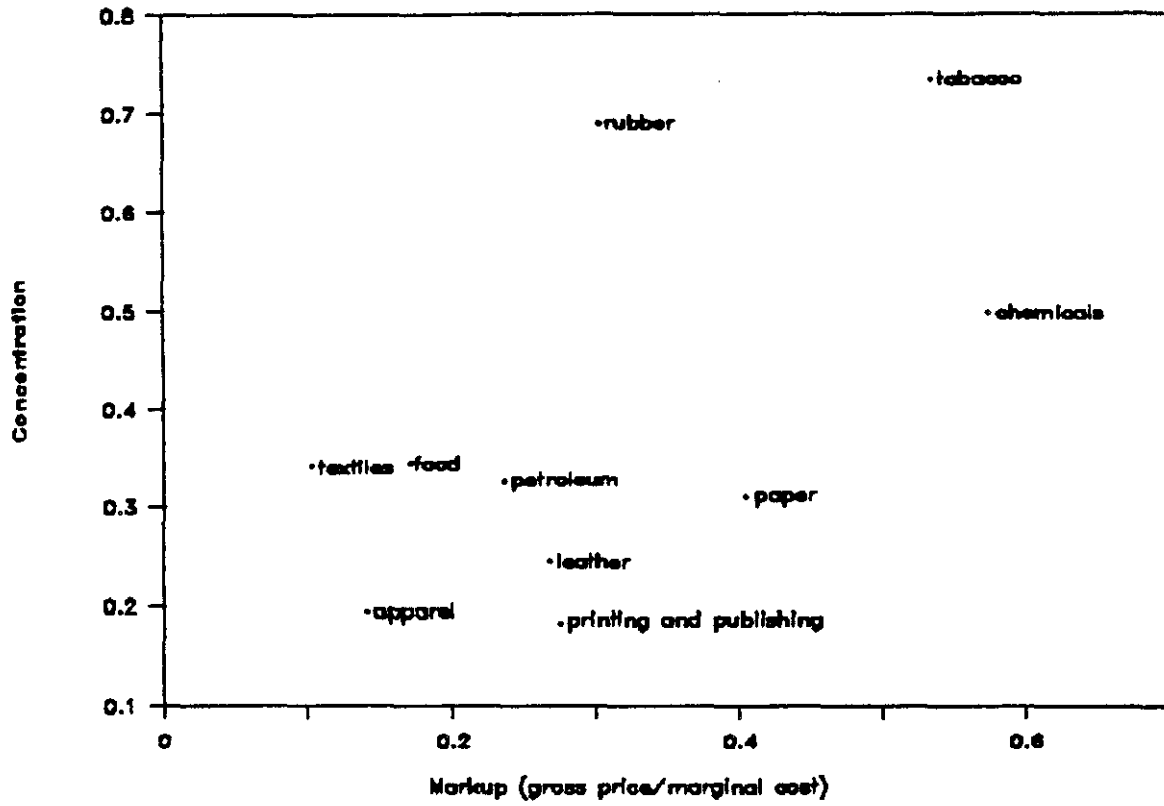
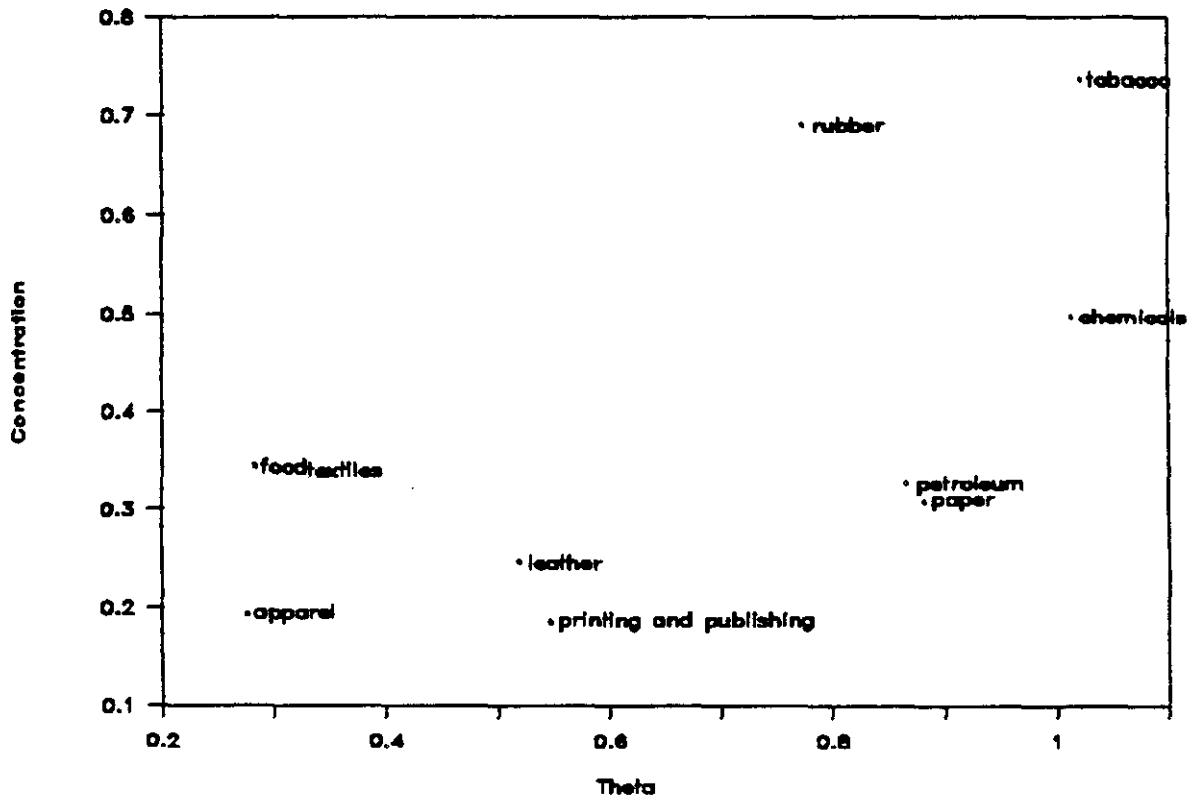


Figure 2



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