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1. THE EXCHANGE ECONOMY

An exchange economy E is characterized by n types of traders and m goods where a trader of type i has a utility¹ function $\varphi_i(x_1^i, \dots, x_m^i)$ and an initial endowment $(a_1^i, a_2^i, \dots, a_m^i)$.

Suppose that this economy has k competitive equilibrium points (henceforth denoted as C.E.s). Possible degeneracies which might give rise to a continuum of C.E.s are set aside. Denote the prices and Lagrangian multipliers associated with the j^{th} C.E. ($j = 1, \dots, k$) by $p^j = (p_1^j, p_2^j, \dots, p_m^j)$ and $\lambda^j = (\lambda_1^j, \lambda_2^j, \dots, \lambda_n^j)$. As prices are homogeneous of order zero we can multiply p^j by a scalar α without

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¹Enough smoothness, for example C_2 , is assumed so that the Lagrangian multipliers will also be a finite set. It is also assumed that the λ_j^i are bounded.

influencing the distribution of goods at the C.E. If p^j is multiplied by α (where $\alpha > 0$) then the associated λ^j must be divided by α . Intuitively all that this says is that if prices rise by a factor of α the marginal utility of income is decreased by $1/\alpha$.

It is well known that there are many different ways one can fix a price or normalize prices in the exchange economy. A particular normalization is chosen here. Furthermore it is suggested that it is a natural choice if one wishes to introduce money explicitly into the model.

We want three properties for a good normalization: (a) It should be intrinsically symmetric, i.e. all goods are treated in the same manner. (b) The normalization equation should be dimensionally sound, i.e. the units on both sides of the equation should match; and (c) There should be a good plausible economic interpretation to the normalization chosen.

Three normalizations are noted. They are:

$$\begin{aligned}
 (1) \quad & \sum_{j=1}^m p_j = 1 && \text{prices} \\
 (2) \quad & \sum_{j=1}^m p_j a_j = 1 && \text{total asset value} \\
 (3) \quad & \sum_{i=1}^n \sum_{j=1}^m \max[(x_j^i - a_j^i), 0] = 1 && \text{total cash flow}
 \end{aligned}$$

The first normalization is one that has been frequently used, it meets the criterion of intrinsic symmetry but fails in dimensional analysis. The dimensions of price p_j are M/Q_j . Thus we have:

$$(4) \quad \frac{M}{Q_1} + \frac{M}{Q_2} + \dots + \frac{M}{Q_m} = ?$$

The second and third normalizations do not face this problem. Instead of (4) we have:

$$(5) \quad \frac{M}{Q_1} Q_1 + \frac{M}{Q_2} Q_2 + \dots + \frac{M}{Q_m} Q_m = M .$$

The normalization is in terms of "money."

As the choice of units for each good is arbitrary we may normalize the total endowment of each good to be one. Thus as $a_j = 1$ for all j equation (2) appears to be the same as (1), but the dimensional analysis shows the difference.

Both (2) and (3) have clear economic interpretations. The first (2) is the total value of all wealth in the economy. The second (3) is the value of all trade or total cash flow. The economic interpretation of the normalization suggests that (2) is a better normalization than (3). All C.E.s are Pareto optimal, but we have no intrinsic way to distinguish among them. This is reflected by using normalization (2) for each. This has the interpretation that total wealth is normalized to be the same at all C.E.s.

2. THE STRATEGIC MARKET GAME WITH FIAT MONEY

The exchange economy E can be reformulated as a strategic market game. The details of the various reformulations are given elsewhere.² In particular the point of concern here involves the introduction of a specified amount of credit or fiat money to monetize exchange. Dubey and Shubik (1979) and Shubik and Wilson (1977) have studied the possibility of

²See Shubik (1973), Shapley and Shubik (1977), Dubey and Shubik (1978, 1980), Dubey and Shapley (1977), Dubey (1982).

introducing a fixed amount M of money to finance trade. When one formulates exchange as a game of strategy using any form of credit or fiat money where there is any possibility whatsoever that an individual will be unable to pay back that which he has borrowed, the rules of the game require that the procedure to be followed in case of default must be specified. This is not a mere institutional detail but a logical necessity. It is however reasonable to expect that one might try to design a default penalty sufficiently harsh to discourage strategic default.

The λ_i^j have an immediate economic interpretation as the marginal utilities to each individual of extra income at C.E. j . In designing a game, if we wanted to discourage strategic bankruptcy we would select a penalty, to be at least as harsh as the largest marginal utility for income

In a game without a credit restriction any penalty $\mu > 0$ will suffice, as the system is self normalizing. Prices can always be inflated to the point that the marginal disutility of any penalty no matter how small is not worth incurring. The dimensions of μ and λ_i^j are utility/money thus inflating prices amounts to increasing the force of the penalty.³ But if the amount of money is fixed in advance an upper bound is placed on the ability to inflate prices⁴ and still obtain a C.E. hence a lower bound is set on the value of the λ_i^j .

By utilizing the normalization (2) at each C.E. j we obtain a set of

³The force of inflation causes immediate problems both in bankruptcy and criminal law. In the latter in particular, the theft of money which in one era is clearly a crime, might be still classified as such even though it should be regarded as a misdemeanor.

⁴Leaving aside variation of velocity. It is suggested elsewhere that velocity is a red herring (Shubik, 1986).

prices (p_1^j, \dots, p_m^j) and a set of Lagrangian multipliers $(\lambda_1^j, \dots, \lambda_n^j)$. The amount of money required to finance trade at C.E. j is given by:

$$(7) \quad \sum_{i=1}^n \sum_{g=1}^m \max[p_g^j(x_g^j - a_g^j), 0] = M_j.$$

If a strategic market game is constructed with a limit on the amount of money of M_j , then in order for the game which we denote by $\Gamma(E, M_j, \mu^j)$, to be able to achieve the C.E. j as an N.E. the bankruptcy penalty μ^j must be selected so that $\mu^j \geq \max(\lambda_1^j, \lambda_2^j, \dots, \lambda_n^j)$. If μ^j is less than the largest λ_i^j then at least one class of individuals⁵ will be motivated to go bankrupt if the prices at C.E. j prevail.

For each C.E. j we can construct a game $\Gamma(E, M_j, \mu^j)$ where the C.E. (and possibly other C.E.s) can be obtained as an N.E.

Two games of the k games $\Gamma(E, M_j, \mu^j)$, $j = 1, \dots, k$ are distinguished; the game with $M_* = \min[M_1, M_2, \dots, M_k]$ and the game with $\mu^* = \min[\mu^1, \dots, \mu^k]$. Both of these games may have many N.E.s but they can each have only one interior N.E. which coincides with a C.E.

The game $\Gamma(E, M_*, \mu^j)$ is the strategic market game associated with the minimal cash flow C.E. of the exchange economy E . All other C.E.s of the exchange economy E require relatively more money (measured as a

⁵It is argued elsewhere (Dubey and Shubik, 1979) that there is no loss of generality in using a function of the form

$$\varphi_i(x_1^i, \dots, x_m^i) + \mu \min \left[\sum_{g=1}^m p_g^i (x_g^i - a_g^i), 0 \right] \text{ instead of } \varphi_i(x_1^i, \dots, x_{m+1}^i)$$

to describe the preference conditions involving bankruptcy conditions punitive enough to prevent strategic bankruptcy. Furthermore although a personal bankruptcy penalty for each trader type can be envisioned calling for (μ_1, \dots, μ_n) rather than a single penalty μ , the single penalty suffices against all types if it is high enough.

percentage of total wealth) than M_x . Hence in the game $\Gamma(E, M_x, \mu^j)$ only one C.E. is feasible.

The game $\Gamma(E, M_j, \mu^*)$ is the strategic market game associated with minimal default penalty C.E. of the exchange economy E . If $\mu^j = \max(\lambda_1^j, \lambda_2^j, \dots, \lambda_n^j)$ then $\mu^* = \min(\mu^1, \mu^2, \dots, \mu^k)$. It is the smallest penalty strong enough for one C.E. to be attainable as an N.E.

An open question is when do these two criteria coincide in their selection of a C.E. Even without an answer to this, minimal cash flow by itself with its implications of minimal liquidity provides an attractive criterion for selecting among C.E.s especially if there were a cost associated with liquidity.

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