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ENOUGH COMMODITY MONEY AND  
THE SELECTION OF A UNIQUE COMPETITIVE EQUILIBRIUM

by

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1. THE EXCHANGE ECONOMY

We consider an economy with  $n$  types of trader and  $m+1$  commodities. Let a representative trader of type  $i$  have endowment  $(a_1^i, a_2^i, \dots, a_{m+1}^i)$  and have his preferences represented by a (twice differentiable) utility function  $\varphi_i(x_1^i, \dots, x_{m+1}^i)$ . Leaving aside degeneracies which might give rise to a continuum of competitive equilibria (henceforth noted as C.E.s) the exchange economy will have some finite number  $k$  of C.E.s where  $k$  can be larger or smaller than either  $n$  or  $m+1$ .

Associated with equilibrium  $j$  are a set of prices and Lagrangian multipliers  $p_1^j, p_2^j, \dots, p_{m+1}^j$  and  $\lambda_1^j, \lambda_2^j, \dots, \lambda_n^j$ .

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## 2. THE STRATEGIC MARKET GAME AND ENOUGH MONEY

Suppose that we reformulate the exchange economy as a strategic market game (see Shubik, 1973; Shapley and Shubik, 1977; Dubey and Shubik, 1978) where the  $m+1^{\text{st}}$  commodity is employed as a means of payment. A strategy by a trader of type  $i$  is of the form  $(b_1^i, q_1^i, b_2^i, q_2^i, \dots, b_m^i, q_m^i)$  where

$$0 \leq q_j^i \leq a_j^i \text{ for } j = 1, \dots, m \text{ and } \sum_{j=1}^m b_j^i \leq a_{m+1}^i, \quad b_j^i \geq 0.$$

It is natural in this strategic market game to set the price of the money at  $p_{m+1} = 1$ . All other prices are determined as exchange rates between each of the other  $m$  goods and the  $m+1^{\text{st}}$  good.

As all purchases are paid for in cash it is possible to attach precise meaning to what is meant by enough money. As the game is a single simultaneous move game where (as is shown in Figure 1) money and goods are

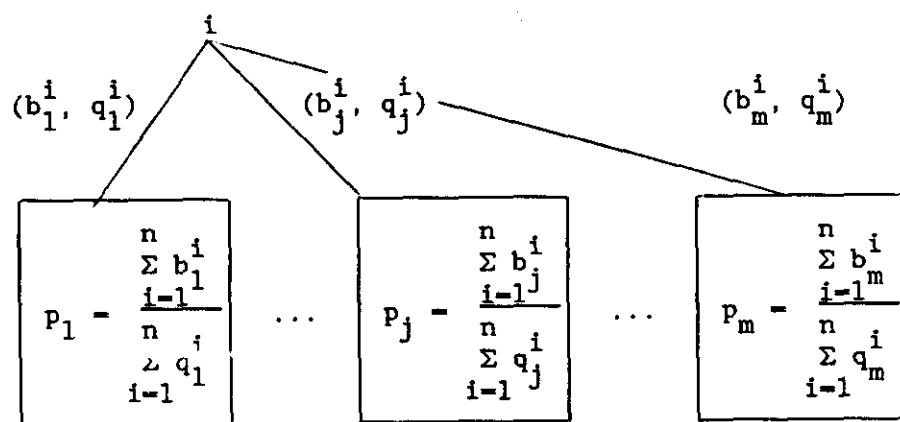


FIGURE 1

simultaneously bid and offered at  $m$  trading posts and  $m$  prices are all simultaneously determined, in essence the trading technology is completely specified. During the game the velocity of money is at most one. Money is

bid in each market.<sup>1</sup> Each individual  $i$  has as his final endowment of good  $j$  ( $j = 1, \dots, m$ )

$$(1) \quad x_j^i = a_j^i - q_j^i + b_j^i/p_j$$

and as he is paid for the goods he has sold

$$(2) \quad x_{m+1}^i = a_{m+1}^i - \sum_{j=1}^m b_j^i + \sum_{j=1}^m p_j q_j^i .$$

We could have considered sequential, or many other forms of market clearance and payment which could permit a different velocity for money feasible because of the different trading technology. For our analysis it does not matter which technology we consider as long as it is well defined so that the volume of payments can be estimated. Enough money to finance trade must be defined with respect to the technology of trade. In the strategic market game described above we may now state the following:

There is enough money in the strategic market game  $\Gamma$  to achieve C.E.  $j$  of the associated exchange economy as a noncooperative equilibrium (henceforth N.E.) of  $\Gamma$ . This requires that:

$$(3) \quad \sum_{k=1}^m p_k^j \max[(x_k^i - a_k^i), 0] \leq a_{m+1}^i \quad \text{for } i = 1, \dots, n .$$

This states that the amount of money on hand is sufficient to finance all

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<sup>1</sup>Division by zero is regarded as yielding zero.

net purchases<sup>2</sup> of  $i$  at the price levels of the  $j^{\text{th}}$  C.E.

There are three possibilities that may occur in a game  $\Gamma$  concerning the presence of enough money to achieve C.E.  $j$  of an exchange economy as an N.E. of  $\Gamma$ .

- (a) There is enough money and it is well distributed.
- (b) There is enough money but it is poorly distributed.
- (c) There is not enough money.

The inequalities in (3) show the conditions for (a). For (b) it is possible that:

$$(4) \quad \sum_{i=1}^n \sum_{k=1}^m p_k^i \max[(x_k^i - a_k^i), 0] \leq \sum_{i=1}^n a_{m+1}^i \quad \text{but for some } i$$

$$\sum_{k=1}^m p_k^j \max[(x_k^i - a_k^i), 0] > a_{m+1}^i .$$

The condition for (c) states that

$$(5) \quad \sum_{i=1}^n \sum_{k=1}^m p_k^i \max[(x_k^i - a_k^i), 0] > \sum_{i=1}^n a_{m+1}^i .$$

If condition (c) prevails the trading technology must be changed, a credit system must be introduced or more money must be added to overcome the shortage if the efficient trade desired is to be attained. This raises further "enough money" conditions which are discussed in Section 3.

If condition (b) is encountered it may be possible to achieve the C.E.

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<sup>2</sup>Implicitly here we are ruling out wash sales where an individual both sells and buys in the same market. But see Dubey and Shubik (1978) and Shubik (1984).

$j$  as an N.E. by modifying the game  $\Gamma$  through the introduction of a money market where the commodity money or "gold" can be lent and borrowed. In this paper this possibility is not discussed further.

If condition (a) is encountered we know that there is enough money in  $\Gamma$  to finance C.E.  $j$  as an N.E. of  $\Gamma$  but this does not tell us anything about whether the other C.E.s can be attained as N.E.s. This point is discussed further in Section 3.

### 3. A CLASS OF GAMES WITH ADDITIONAL MONEY

Suppose that the game  $\Gamma$  does not have enough money in the sense that there does not exist any C.E.,  $j = 1, \dots, k$  for which conditions (a) or (b) are satisfied. We can construct a set of games which we denote by  $\Gamma(s)$  where  $s \geq 0$ . The  $s$  is an additional amount of good  $m+1$  added to everyone's resources. Thus the game  $\Gamma(0)$  denotes the original game (and with it we may associate the original exchange economy  $E(0)$ ). We now wish to consider two features of this set of games and the related set of exchange economies. As  $s$  increases what happens to the set of C.E.s associated with  $E(s)$  and what happens to the enough money conditions on  $\Gamma(s)$ .

We have assumed that for  $\Gamma(0)$  inequality (5) holds. As money is added do we expect that finally enough money will be attained for some  $s^*$ . As  $s$  is increased beyond  $s^*$  will there still be enough money. These questions have answers which clearly depend on the assumptions we make about the properties of the commodity money. Without making specific assumptions about the nature of the intrinsic worth of money as its quantity is increased there is no guarantee that the condition of enough money will ever be achieved.

Intuitively we may feel that no matter how much of a commodity money there is around its marginal utility to the individual relative to the marginal utility of any other commodity should remain above some small finite value. Dubey and Shapley (1977) have suggested that for any resource distribution

$$(6) \quad \frac{\partial \varphi_i / \partial x_{m+1}^i}{\partial \varphi_i / \partial x_j^i} \geq \Delta \quad \text{for all } j = 1, \dots, m; \quad i = 1, \dots, n \quad \text{and all } s$$

This certainly provides a sufficient condition for there eventually to be enough money. If we also believe that there is also a diminishing marginal utility for money as its quantity increases then eventually the utility for money will become approximately linear.

### 3.1. Equilibria Distinguished by Cash Flow

A special structure for the utility function is suggested and analyzed. As a first approximation as an individual becomes richer we explore the implications of the possibility that the income effect attenuates. Specifically we assume that after some level an individual  $i$  has preferences which can be represented by a utility function of the form

$$(7) \quad V_i(\varphi_i(x_1^i, \dots, x_m^i) + x_{m+1}^i) \quad \text{for } i = 1, \dots, n.$$

For the class of games  $\Gamma(s)$  with utility functions of the structure in (7) the associated class of exchange economies has the special property that for all  $s$  all  $E(s)$  will have the same number of C.E.s thus if there were not enough money in the original game  $\Gamma(0)$  as  $s$  is increased eventually the point will be reached where at  $s_1$  one C.E. of the exchange

economy  $E(s_1)$  can be attained as an N.E. of  $\Gamma(s_1)$ . As  $s$  is increased we obtain a sequence  $s_1, s_2, \dots, s_k$  where the game  $\Gamma(s_j)$  is able to attain  $j$  of the  $k$  C.E.s of the exchange economy  $E(s_j)$ . Thus for this class of games there is a level of money  $s_1$  enough to achieve a unique C.E. which can be regarded as the minimal cash flow C.E. There is also a level of money  $s_k$  enough so that all C.E.s can be financed.

### 3.2. Fiat Money and Sensitivity Analysis

In a separate note it has been shown that the introduction of a fiat money and bankruptcy penalty serves to select any number of C.E.s of any exchange economy as N.E.s of the related strategic market game (Shubik, 1986). This is possible a better model of a modern economy than one with commodity money.

Both models, unfortunately leave unanswered the problem of sensitivity analysis if the supply of any nonmonetary commodity is varied. New C.E.s could appear.

## 4. ON ENOUGH MONEY AND MINIMAL CASH FLOW

With a commodity money the minimization of cash flow needs actually releases extra real resources from the needs of trade. The gold can be worn as jewelry rather than being kept liquid for payments. The situation is more complex with fiat money. But in general in most multiperiod economies a positive rate of interest will be present and there may be a loss of earning associated with liquidity. Thus cash flow requirements appear to provide an extra condition to distinguish among equilibria.



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