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TWO MISSPECIFICATION TESTS FOR THE SIMPLE SWITCHING  
REGRESSIONS DISEQUILIBRIUM MODEL

by

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for the Simple Switching Regressions Disequilibrium Model

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ABSTRACT

Two specification tests for switching regimes disequilibrium models are developed. The first is an asymptotically locally optimal Lagrange multiplier test of endogeneity of a set of regressors, which takes the convenient form of a LM significance-test of certain regression residuals. The second is a Hausman specification test of the accuracy of regime classification information.

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## Two Misspecification Tests

### for the Simple Switching Regressions Disequilibrium Model

#### Overview

This note presents two tests of misspecification for the simple disequilibrium model (see, for example, Fair & Jaffee (1972) and Goldfeld & Quandt (1975)). Part I presents a Lagrange multiplier test of exogeneity of the price variables in the disequilibrium model. The advantage of this test over the two other members of the "trinity" of classical tests, (Wald and likelihood ratio) is that it only requires estimates of the parameters under the computationally simpler null hypothesis, while it has the same asymptotic distribution as the other two tests both under the null and under a sequence of local alternatives. Moreover it is an asymptotically optimal test, in the sense of being asymptotically locally most powerful invariant.

In Part II we show that a Hausman (1978) test, which compares estimates obtained from two standard disequilibrium estimation methods, can be used to examine the accuracy of information that classifies periods of observation into excess supply and excess demand regimes.

#### Part I: A Lagrange Multiplier Test of Price Exogeneity

A desirable feature of a test of the exogeneity assumption in disequilibrium models is that one need only identify which variables are maintained as exogenous, without having to specify explicitly a price adjustment rule. This is in contrast to the standard technique that tests the significance of the excess demand coefficient in the adjustment equation. The latter procedure is extremely sensitive to correct specification of this additional equation (see Rosen and Quandt (1978)).

For ease of notation we consider the basic disequilibrium model with i.i.d. errors and drop the time subscripts.

$$D = D^* + \varepsilon = X_1 \beta_1 + P \cdot X_1 + \varepsilon \quad (1.1)$$

$$S = S^* + \eta = X_2 \beta_2 + P \cdot X_2 + \eta \quad (1.2)$$

$$Q = \min(D, S) \quad (1.3)$$

D and S represent the "notional" demand and supply functions and Q is transacted quantity.

Let  $Y' = (D, S)$  and  $Y^* = (D^*, S^*)$ . Note that restrictions must be incorporated in  $D^*$  or  $S^*$  for the model to be identified. Our maintained hypothesis will be the exogeneity of the set of ( $k_1$ -row vector of) regressors X ( $E(\varepsilon, \eta | X) = 0$ ) and we want to test for possible endogeneity of the  $k_2$  regressors P. Given that our primary interest is the possible endogeneity of the price variable, we let  $k_2 = 1$ ; this further simplifies notation. Denote by Z the additional set of  $k_3$  exogenous or predetermined variables that do not appear in D and S. The complete set of (maintained) exogenous variables of the model can thus be denoted by  $W = [X, Z]$ . Lagged values of P will in general appear among the predetermined variables in Z. Supposing that P is endogenously determined, we postulate a linear reduced form equation

$$P = W \cdot \delta + \gamma \quad (1.4)$$

We next assume a (trivariate) normal distribution for the vector of disturbances  $(\varepsilon, \eta, \gamma)'$  with mean 0 and covariance matrix

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ & \sigma_\varepsilon^2 \end{bmatrix} = \begin{bmatrix} \Sigma_{11} & \sigma_{\varepsilon\gamma} \\ \text{---} & \sigma_{\eta\gamma} \\ & \sigma_\gamma^2 \end{bmatrix} \quad (1.5)$$

This induces a normal distribution  $f(D, S, P; X, Z)$  for D, S and P.

Let  $d_t$  be defined by  $d_t = 1$  if  $D_t \geq S_t$ , = 0 otherwise. Two ML estimators have been considered in the literature (see Quandt (1982)). The first one,

labeled NOR for "no-observations-on-regimes," does not use the classifying information  $d_t$  but determines implicitly the classification most consistent with the data. We let OR denote the estimation technique that exploits the  $d_t$  information. The model has two endogenous variables,  $Q_t$  and  $P_t$  in the NOR case, and a third one  $d_t$ , the regime classification dummy, in the OR case. Let  $g_1 = \int_0^\infty f(D,Q,P) dD$  and  $g_2 = \int_0^\infty f(Q,S,P) dS$ . The contribution of the  $t^{\text{th}}$  observation to the likelihood is:

$$h(Q,P) = g_1 + g_2 \quad (I.6.NOR)$$

$$h(Q,P,d) = d \cdot g_1 + (1-d) \cdot g_2 \quad (I.6.OR)$$

One can always write  $f(D,S,P) = f(D,S|P)f(P)$ . Hence  $g_1 = f(P) \cdot \int_0^\infty f(D,Q|P) dD$ . Using well known properties of conditional normal distributions (see Johnson & Kotz (1972)), it is straightforward to show that

$$Y|P \sim N(\mu_{Y|P}, \Sigma_{Y|P}), \quad (I.7)$$

where

$$\mu_{Y|P} \equiv Y^* + \frac{1}{\sigma_v^2} \cdot \Sigma_{12} \cdot (P - W\delta)$$

and

$$\Sigma_{Y|P} \equiv \Sigma_{11} - \frac{1}{\sigma_v^2} \cdot \Sigma_{12} \cdot \Sigma_{12}'$$

Thus the hypothesis of exogeneity of price is equivalent to the hypothesis

$$H_0: \Sigma_{12} = 0.$$

The sequence of local alternatives considered is:  $H_1: \Sigma_{12} = K/\sqrt{T}$ , where  $K$  is a  $2 \times k_2$  matrix of constants.

Under  $H_0$

$$Y|P \sim N(Y^*, \Sigma_{11}) \quad (I.7')$$

and the contribution of the  $t^{\text{th}}$  observation to the log-likelihood function

$$\log h = \log( f(P; \delta, \sigma_v^2) ) + \log( \ell(Q; \beta_1, \beta_2, \gamma_1, \gamma_2, \Sigma_{11}) ) \quad (I.8)$$

where  $\ell$  denotes the likelihood contribution for the usual model that conditions

on P as being exogenous. The  $\delta$  and  $\sigma_v^2$  parameters appear only in  $f(P)$ , hence the MLE estimates of  $\beta_1, \beta_2, \gamma_1, \gamma_2$  and  $\Sigma_{11}$  require only the maximization of the usual contribution  $\ell$ .

An LM test of the exogeneity hypothesis can be derived readily by evaluating the gradient of the log-likelihood with respect to  $\Sigma_{12}$  at  $\Sigma_{12}=0$  as follows:

$$g_1 = f(P) \cdot \int_0^\infty \text{const} \cdot |\Sigma_{Y|P}|^{-1/2} \cdot \exp\left(-\frac{1}{2} R_1' \Sigma_{Y|P}^{-1} R_1\right) dD \quad (1.9.1)$$

and

$$g_2 = f(P) \cdot \int_0^\infty \text{const} \cdot |\Sigma_{Y|P}|^{-1/2} \cdot \exp\left(-\frac{1}{2} R_2' \Sigma_{Y|P}^{-1} R_2\right) dS, \quad (1.9.2)$$

where

$$R_1 = \begin{bmatrix} D-D^* \\ Q-S^* \end{bmatrix} - \frac{1}{\sigma_v^2} \Sigma_{12} \cdot (P-W\delta)$$

and

$$R_2 = \begin{bmatrix} Q-D^* \\ S-S^* \end{bmatrix} - \frac{1}{\sigma_v^2} \Sigma_{12} \cdot (P-W\delta).$$

Then

$$\left. \frac{\partial g_1}{\partial \Sigma_{12}} \right|_{\Sigma_{12}=0} = f(P) \cdot \int_0^\infty \text{const} \cdot |\Sigma_{11}|^{-1/2} \exp\left(-\frac{1}{2} \begin{bmatrix} D-D^* \\ Q-S^* \end{bmatrix} \Sigma_{11}^{-1} \begin{bmatrix} D-D^* \\ Q-S^* \end{bmatrix}\right) \cdot \begin{bmatrix} D-D^* \\ Q-S^* \end{bmatrix} \cdot \Sigma_{11}^{-1} dD \cdot (P-W\delta) / \sigma_v^2. \quad (1.10)$$

(For  $\left. \frac{\partial g_2}{\partial \Sigma_{12}} \right|_{\Sigma_{12}=0}$  simply change  $\begin{bmatrix} D-D^* \\ Q-S^* \end{bmatrix}$  to  $\begin{bmatrix} Q-D^* \\ S-S^* \end{bmatrix}$  and  $dD$  to  $dS$ ).

The key result we have used in deriving (1.10) is that

$$\left. \frac{\partial |\Sigma_{Y|P}|^{-1/2}}{\partial \Sigma_{12}} \right|_{\Sigma_{12}=0} = \left. \frac{\partial \Sigma_{Y|P}^{-1}}{\partial \Sigma_{12}} \right|_{\Sigma_{12}=0} = 0 \quad (1.11)$$

because of the presence of the  $\Sigma_{12} \cdot \Sigma_{12}'$  term in  $\Sigma_{Y|P}$ . The required gradient is then obtained from

$$\frac{\partial \log h}{\partial \Sigma_{12}} = \left( \frac{\partial g_1}{\partial \Sigma_{12}} + \frac{\partial g_2}{\partial \Sigma_{12}} \right) / h, \quad (1.12)$$

evaluated at  $\Sigma_{12} = 0$ .

Now consider the artificial model

$$D = D^* + \varepsilon = X_1 \beta_1 + P \cdot \gamma_1 + (P-W\delta) \cdot \zeta_1 + \varepsilon \quad (I.1')$$

$$S = S^* + \eta = X_2 \beta_2 + P \cdot \gamma_2 + (P-W\delta) \cdot \zeta_2 + \eta \quad (I.2')$$

$$Q = \min(D, S) \quad (I.3')$$

that treats the quantity  $(P-W\delta)$  as an extra exogenous variable included on D and S sides, while maintaining the exogeneity of P. The LM test of the hypothesis  $H_0 : \zeta_1 = \zeta_2 = 0$  relies on the gradient  $(\frac{\partial \log \ell}{\partial \zeta_1}, \frac{\partial \log \ell}{\partial \zeta_2})' \Big|_{\zeta_1 = \zeta_2 = 0}$ , where  $\ell$  is the likelihood contribution with all D and S variables assumed exogenous. This gradient is equal (up to scale  $\sigma_y^2$ ) to the gradient (I.12) that defines the LM statistic for the  $\Sigma_{12} = 0$  hypothesis.

We therefore see that the LM test takes the extremely convenient form of a simple LM test of the joint significance of the  $(P-W\delta)$  term on both D and S sides. Since the problem satisfies the MLE regularity conditions, one can test the joint significance of  $\zeta_1$  and  $\zeta_2$  through the asymptotically equivalent (and hence optimal) Wald and LR test-statistics, hence avoiding any need for specialized programming. Operationally, the procedure then amounts to obtaining the vector of residuals from an OLS regression of P on all the exogenous variables of the model. Since the information matrix is block diagonal in  $(\delta, \sigma_y)$  and  $(\beta, \gamma, \Sigma_{11})$  both under  $H_0$  and under a sequence of local alternatives, using the OLS consistent estimates  $\delta$  and  $\sigma_y^2$  does not affect the asymptotic distribution of our test. Note that LM and Instrumented Scores (IS) tests of exogeneity that take the form of significance tests of reduced form residuals included as regressors have been derived also in Newey (1985).

One should note that if more structure is put on equation (I.4), then (locally) more powerful tests can be derived. If, for example, one is willing to assume that the price adjustment rule has a linear excess demand term, as in Rosen & Quandt (1978),

$$P = \alpha(D-S) + Z \cdot \delta' + \gamma' \quad (I.4')$$

then all our discussion remains valid once we incorporate the nonlinear restrictions implied by the reduced form equation for P:

$$P = Z\delta + \gamma \equiv \alpha \cdot \Delta^{-1} \cdot (X_1 \beta_1 - X_2 \beta_2) + \Delta^{-1} \cdot Z\delta + \Delta^{-1} (\gamma' + \alpha(\varepsilon - \eta)) \quad (I.4'')$$

where

$$\Delta \equiv 1 - \alpha(\gamma_1 - \gamma_2).$$

We have explicitly avoided incorporating specific restrictions like the ones implied by (I.4'') in our test because, as evidenced in Rosen & Quandt (1978), estimation that incorporates the restrictions is extremely sensitive to the specification of the price adjustment equation, as the test then becomes a joint test of the exogeneity of P and of the specification of the price adjustment rule.

Finally, we note the following issue regarding Lagrange multiplier tests: the use of the matrix of second derivatives for calculating the quadratic form of the LM test may be preferable to the BHHH (Berndt et al. (1974)) technique of approximating the Hessian by the negative of the outer product of the gradient. This is because Cavanaugh (1985) shows that LM tests using the BHHH matrix are second order inefficient relative to the LM tests that employ the Hessian (and therefore relative to Wald and LR tests also).

## Part II: Tests of the Accuracy of Regime-Classification Information

Consider the canonical two-regime disequilibrium model given by (I.3), with time-subscripts:

$$D_t = X_{1t} \beta_1 + \varepsilon_t \quad (II.1)$$

$$S_t = X_{2t} \beta_2 + \eta_t \quad (II.2)$$

$$Q_t = \min(D_t, S_t) \quad (II.3)$$



Define an indicator variable  $I_t$  as:

$$I_t = 1 \text{ iff } D_t \geq S_t \tag{II.4}$$
$$= 0 \text{ otherwise}$$

Recall that we have denoted by OR the MLE that uses regime classification information in the form of a series  $d_t$ , with the implicit assumption that  $d_t = I_t$ , all  $t$ . The estimator that does not employ such information was denoted by NOR. Subject to standard regularity conditions both the OR and the NOR estimators are consistent (Hartley and Mallela (1977)), while NOR is less efficient than the (fully efficient ML) OR estimator, once  $d_t$  is observed without error. There is very strong evidence that this loss in efficiency by not employing the regime classification can be quite substantial (see Goldfeld and Quandt (1975)).

It is important to note, however, that the consistency of the OR estimator rests crucially on an assumption that the regime classification used is exactly correct. As Lee and Porter (1984) show, the OR estimator is inconsistent if the classification is not exact. Evidently the NOR estimator is consistent irrespective of the validity of the classification, since it does not use it in any way. Let  $H_0$  be the hypothesis, "The dummy variable  $d_t$  describes exactly the true classification of regimes with no error in measurement." The behaviour of the two estimators can then be summarized by

Estimator	$H_0$	$H_1$
OR	Consistent Efficient	Inconsistent
NOR	Consistent Inefficient	Consistent

Hence we have a classic case for a Hausman test of forming the quadratic form of the vector  $\Delta b = (b_{NOR} - b_{OR})$  into the asymptotic covariance matrix of  $\Delta b$ . Given that OR is efficient under  $H_0$ ,  $cov(b_{NOR} - b_{OR}) = 0$  (see Hausman (1978)).

If one is willing to explicitly model the imperfections in the classification information, then (asymptotically locally) most powerful tests can be used in the form of the trinity of LM, Wald and LR tests. For example, Lee and Porter (1984) model  $H_1$  in the following way: Let the true classification be defined by the series  $I_t$  while we possess the series  $d_t$  such that:

	$d_t=1$	$d_t=0$
$I_t=1$	$p_{11}$	$p_{10}$
$I_t=0$	$p_{01}$	$p_{00}$

with  $p_{10}, p_{01}$  not equal to 0.

Assuming these probabilities are fixed, unknown parameters (and are thus independent of the errors  $\varepsilon$  and  $u$ ), the full density  $h(Q_t, d_t)$  can be worked out. An LM test of  $H_0: p_{10} = p_{01} = 0$  can then be constructed easily. This would be asymptotically locally equivalent to Wald and LR tests that employ MLE estimates of  $p_{11}$  and  $p_{00}$  as well, and test  $H_0$  against  $H_1$ . By putting specific structure on  $H_1$ , one is then also able to calculate the (asymptotic local) power of the Hausman test, since one now can obtain an expression for the inconsistency ( $plim \Delta b$ ).

Note, however, that the assumption that  $p_{01}, p_{10}$  do not vary with  $t$ , is very restrictive, and, if invalid, would destroy the optimal properties of the trinity of tests. (For example, this assumption implies that the measurement error cannot depend on the magnitude of excess demand.) The Hausman (H) test,

on the other hand, would remain a valid omnibus test, hopefully powerful along the direction of deviations from  $H_0$  implied by the (unspecified)  $H_1$ . A further advantage of the H test over the asymptotically optimal "trinity" of Wald, LR and LM tests, is that H only requires the usual NDR and OR estimates (of coefficients and covariance matrices), while the members of the "trinity" need specialized software in order to incorporate the  $H_1$  hypothesis. (A counterbalancing consideration is the possible computational difficulty in obtaining the NDR estimator, due to possible unboundedness of and singularities in the likelihood function (Quandt (1982)). If, a priori, the problem of imperfections in regime classification information appears important, it may be worthwhile to try different ways of modelling the errors in the  $d_t$  series (as the independence of the p's from the e's appears inappropriate in most cases). Interesting analogies may also exist between the analysis here and cases of measurement errors on the dependent variable of various limited dependent variables models.

Finally, we ask the question: Are there conditions under which one is better off by using an imperfect classification of regimes than neglecting it altogether? (See analogy with the result that including an imperfect proxy variable creates a smaller inconsistency than dropping it altogether.) In other words, are there conditions on the "severity" of the errors in the regime-classification information, under which the gain in efficiency of using the NDR estimator might outweigh the inconsistency introduced relative to the OR estimator? It is clear that a trade-off would only be possible in finite samples, since as  $T \rightarrow \infty$ , OR is asymptotically biased and inconsistent, while the efficiency gains are only of second order ( $o(1/\sqrt{T})$ ). As a result, an OR that uses inaccurate classification information would asymptotically always have a higher MSE than the NDR estimator.

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