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THE MANY PROPERTIES OF MONEY:

A STRATEGIC MARKET GAME ANALYSIS

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1. INTRODUCTION

Among the major properties of a money are that it can serve as (1) a
numeraire, (2) a means of exchange, (3) a store of value, and (4) a source
of liquidity. Among the lesser properties are that it should be easy to
transport and identify, it should be durable, easily divisible, hard to
counterfeit and easy to store. A possibly desirable property is that it
is an anonymous "bearer instrument," but the price of anonymity is that it
is hard to recover if it is stolen. A personal check which is bank money
can be stopped and is more personal than a $100 bill.

The properties of a money are systemic and strategic and are most
naturally formalized by means of strategic market games. Here we concentrate
on the four major properties noted above and include comments on the cost
of liquidity and the role of various assets as near monies.

2. THE BID OFFER OR COURNOT STRATEGIC MARKET GAME

For clarity and simplicity we confine ourselves to a series of models involving only exchange. Let there be n types of traders and m commodities. A trader of type i has a utility function of the form $\varphi(x_1^i, x_2^i, \ldots, x_m^i)$ which is assumed to be concave and differentiable. Each trader of type i has an endowment of $(a_1^i, a_2^i, \ldots, a_m^i)$.

If only the mth commodity serves as a money then there are $m-1$ markets where j $(j = 1, \ldots, m-1)$ is exchanged for m . A strategy by an individual of type i is described by a vector of dimension $2(m-1)$ consisting of money bids and offers $(b_1^i, b_1^i, \ldots, b_{m-1}^i, q_{m-1}^i)$ where:

$$(1) \quad \sum_{j=1}^{m-1} b_j^i \leq a_m^i , \quad b_j^i \geq 0 \quad \text{and} \quad 0 \leq q_j^i \leq a_j^i , \quad \text{for} \quad j = 1, \ldots, m-1 .$$

The final endowment of a commodity j $(j = 1, \ldots, m-1)$ is given by:

$$(2) \quad x_j^i = a_j^i - q_j^i + b_j^i/p_j \quad \text{where the price} \quad p_j \quad \text{is:}$$

$$(3) \quad p_j = \sum_{i=1}^{n} b_j^i / \sum_{i=1}^{n} q_j^i \quad \text{if} \quad \sum_{i=1}^{n} q_j^i \neq 0 \quad \text{for} \quad j = 1, \ldots, m-1$$

$$= 0 \qquad \qquad \text{if} \quad \sum_{i=1}^{n} q_j^i = 0 .$$

The final endowment of money is given by:

$$(4) \quad x_m^i = a_m^i - \sum_{j=1}^{m-1} b_j^i + \sum_{j=1}^{m-1} p_j q_j^i .$$

The price formation mechanism involves the bid and offer of quantities. Markets always clear. Dubey and Shubik (1978) have studied this model in

detail and established the existence and relationship of the equilibrium points of the strategic market game with a finite number of traders to the model of a competitive exchange economy. An alternative model where individuals name buying and selling prices has also been studied (Dubey and Shubik (1980)). This model is more closely related to the Bertrand-Edgeworth approach to oligopolistic markets.

Let $b^i = (b_1^i, b_2^i, \ldots, b_{m-1}^i)$, $q^i = (q_1^i, q_2^i, \ldots, q_{m-1}^i)$ then each individual of each type will have a payoff function of the form

$$(5) \quad \Pi_i(b^1, q^1, b^2, q^2, \ldots, b^n, q^n) = \varphi_i(x_1^i, x_2^i, \ldots, x_m^i)$$

where equations (2) and (4) express the x_j^i in terms of the strategic variables.

3. MONEY AS A NUMERAIRE

In the model in Section 2 above there are only $m-1$ prices which are determined in the markets. It is quite natural to set $p_m = 1$ using money as a numeraire. However if instead of the strategic market game with $m-1$ markets we had defined a strategic market game with complete markets, i.e. in which all goods are money a new difficulty in the selection of a numeraire appears.

If all goods can be exchanged directly for all other goods all are immediately liquid. Figures 1a and b show the markets in an economy where one or all four goods are money.
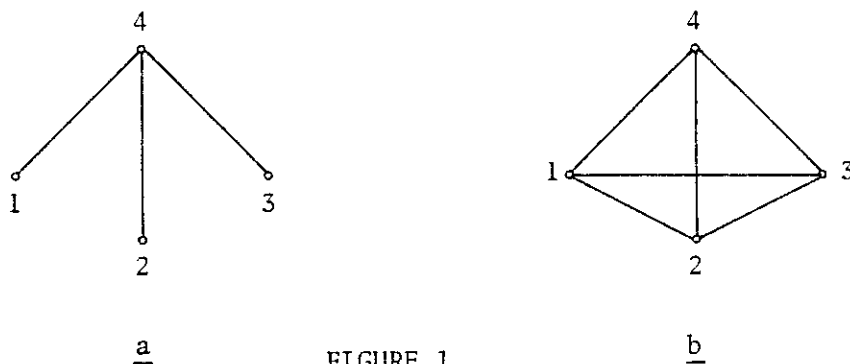
a   FIGURE 1   b

With complete markets a strategy is of dimension $m(m-1)$ where in each of the $m(m-1)/2$ markets an individual of type i can offer b^i_{jk}, a quantity of j for k and b^i_{kj}, a quantity of k for j. There are $m(m-1)/2$ exchange rates determined by the mechanism. In particular out of equilibrium there is no reason why arbitrage opportunities should not exist thus $p_{ij}p_{jk} = p_{ik}$ is not necessarily true out of equilibrium. But selecting say m as the numeraire would require setting $m-1$ prices $p_{m1}, p_{m2}, \ldots, p_{m,m-1} = 1$ not just one price (see Shubik, 1985).

4. MONEY AS A MEANS OF EXCHANGE

The definition of a money as suggested by Figure 1a is that it is a commodity with complete simple markets, i.e. it can be exchanged directly for all other goods. If all markets exist all goods are money. Thus the concept of money as a means of exchange is essentially strategic. If there is only one good which is a money it enters the strategy sets of the players nonsymmetrically as is shown in (1).

The concept of a market as an institution or a process is most radically simplified here by specifying a simple one shot price formation mechanism. The concept of "thickness of a market" is made clear by the volume of trade on both sides of the market (see Dubey and Shubik, 1978).

5. MONEY AS A STORE OF VALUE AND A SOURCE OF LIQUIDITY

A basic difficulty in the interpretation of the general equilibrium model of production and exchange (see Debreu (1959) for instance) is that it is essentially static. All resources and individuals appear at $t = 1$ and disappear at $t = T$. At $t = 1$ there is no prehistory that matters and nothing is owed to the past. At $t = T$ the game is over, the score is calculated and everything disappears.

A fully satisfactory model should connect the time slice being studied both to the past and the future. Here we just concern ourselves with the future to illustrate the store of value of money. The easiest (and most oversimplified) model is that of an exchange economy where the monetary good (which could be fiat) is the only durable.

We have to consider the distinction between stocks and flows. Implicit in the utility function evaluation is the idea that we are evaluating consumption or the worth of flow. If all goods in a one period exchange economy are nondurable then there is nothing left at the end for salvage and no need to distinguish between stocks and flows. If the means of exchange or money is a commodity such as gold we must distinguish three uses and values for it. It has a consumption value per period if used for example as a consumer durable such as jewelry. It has a transactions technology value if used as a means of exchange. As it is a durable it also has a "salvage" value or storage of value worth in further use. All three of these features can be modeled and analyzed as follows.

Let there be $m+1$ commodities during trade and one commodity left over after. The first $m-1$ commodities are not durable. The m and $m+1^{st}$ are durable, say for example gold in jewelry and monetary forms. Although the costs of the manufacture of jewelry from monetary gold and

vice versa can be significant we leave out manufacturing costs. As our

concern is with the concepts of store of value and liquidity we modify the

game in Section 2 to include an extra move where the individuals decide

at the start of the game to divide their gold into two piles, one for

consumption use and the other for monetary use. The gold for consumption

use yields value in the utility function but cannot be utilized for monetary

purposes until a period has elapsed. The gold designated for money is used

in exchange, but yields no consumption value for the period when it is used

in exchange. Thus the cost of liquidity is measured in foregone consump-

tion.

After trade, if we were modeling a multistage process, the monetary

and consumption use gold would be pooled and the process repeated. Here

we pool the two types of gold and assign to them a salvage value which in

essence provides a link to the future and permits us to perform a sensitivity

analysis to see how the solution to the static or one period model is in-

fluenced by changes in the expected valuation of money (and other durable

assets).

Figure 2 shows the two stage game. The first stage is to separate

their gold into two piles, i.e. select their level of liquidity. The

second stage is general exchange. The $P_{1,\ldots,n}$ notation at each vertex
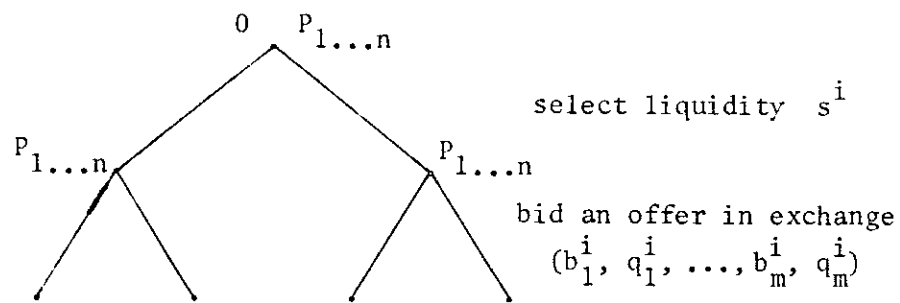
indicates that all move simultaneously.

select liquidity s^i

bid an offer in exchange

$(b_1^i, q_1^i, \ldots, b_m^i, q_m^i)$

FIGURE 2

A strategy by a trader of Type i may at its simplest be regarded as being of the form $(s^i; b_1^i, q_1^i, \ldots, b_m^i, q_m^i)$ where the s^i is a number and the b_j^i, q_j^i are functions of the s^i .

The utility function of an individual of Type i is given by

$$(6) \qquad \phi_i = \varphi_i(x_1^i, \ldots, x_m^i) + V_i(x_{m+2}^i) .$$

In general the argument of V_i should include all durables. Furthermore if credit instruments are considered it is possible that after trade an individual might be unable to pay what he owes. If this is the situation then $V_i(\cdot)$ must be defined for negative as well as positive values.

The function $V_i(\cdot)$ which does not appear in the treatments of general equilibrium provides a critical link between statics and dynamics. Furthermore it provides the means for performing a sensitivity analysis with the general equilibrium model as the special case where $V_i(\cdot) = 0$.

A moment's reflection should be sufficient to have one conclude that money is by no means the only store of value, and by many definitions may not be the best. But for the purposes of the discussion of the properties of money (properties some of which may be shared by other goods or financial instruments) the expression in (6) stresses money as a store of value in the salvage function $V_i(x_{m+2}^i)$.

A reasonable question to ask is what shape or other properties should be ascribed to $V^i(\cdot)$. If we were going to develop the full dynamic programming analogy, for example if we were to consider a T time period model with the payoff to a trader of Type I given by:

$$(7) \qquad \sum_{t=1}^{T} \beta^{T-1} \varphi_i(x_{1,t}^i, \ldots, x_{m,t}^i) + \beta^T V_i(x_{m+2,T}^i)$$

where β is to be interpreted as the "natural discount" rate and all goods except money are nondurable we would expect V_i to be continuous and concave. If however the scenario that we attach to the salvage value includes expectations the concavity of V_i would depend on risk aversion as well as diminishing marginal utility.

6. LIQUIDITY: BRIDGING FINANCE OR SPECULATION

There are two, often interwoven, but essentially different features of economic life which call for money and financial instruments. They are uncertainty and the transactions technology. If there were no exogenous uncertainty whatsoever money and financial instruments would be invented to cut down the costs of a transactions technology. Furthermore endogenous uncertainty would appear as efficiency might call for aggregation which destroys information and hence creates uncertainty.

Here we limit our discussion to the transactions technology, ruling out exogenous uncertainty because it is desirable to try to isolate the many different functions of a money.

The act of exchange consumes time and resources. It also requires mechanisms to guarantee, supplement or substitute for trust. Does A deliver the goods to B before B has paid; or vice versa? Or does A

hold the goods in one hand and B the money in one hand so that they can grab simultaneously? If A delivers the goods to a third party and B picks them up and pays the third party who finances the period between when A relinquishes the goods and when he gets paid.

When an individual deposits money in a bank, the distinctions among the various accounts such as a savings account, a straight deposit account, a NOW account and so forth are in terms of liquidity. How long is the money tied up. Liquidity is a strategic concept as is illustrated by the French financial expression of "masse de manueuvre." Liquidity provides instantaneous fire power. When there is no uncertainty present an individual should be able to preplan how much liquidity he needs when. If liquidity has a cost in terms of foregone value of some sort then in a world without uncertainty each individual will maintain only the liquidity needed for the bridging finance to cover trade.

One view of the role of commercial banking is that its prime function is to provide the financing of riskless production and trade. The time and coordination features of production and exchange involve the float between when A writes a check on his account and B finally gets it credited to his. This may take from a few microseconds to several days. An importer buys shoes from Italy. The time between the paper transaction and the actual delivery may be several weeks. A production process may take several months or even years before the cash flow turns positive. All of these require a bridging of time between receipts and payments.

In contrast with these relatively prosaic considerations, if uncertainty is present an individual may hold cash balances to take advantage of random mispricing, the spotting of bargains, the opportunities created by the insolvency or bankruptcy of others. Furthermore reserves of relatively

high liquidity may be held for precautionary reasons to be able to meet
unforseen payments.

As trading technology, laws and habits change our ideas about the mini-
mum period for which financing is required changes. Thus some years ago
the period of grace between contract and delivery of financial instruments
or money may have been several days. Today money may be borrowed by the
hour.

In the simplest formal model if we do not resort to continuous time
we may consider a single time period of undefined, but finite length Δt.
When we consider direct or eventual trade between two individuals we must
distinguish several cases:

(a) <u>Direct barter</u>: (i) one on one trade where as a good first approxi-
mation there is a simultaneous exchange with a double coincidence
of wants. A and B are immediately able to utilize the items
exchanged. (ii) a variant is that both lose the use of all goods
involved in exchange for Δt. Often but not always a good for
sale is not available for use.

(b) <u>Bilateral trade via an intermediary</u>: A may effect an exchange
with B by employing the services of an intermediary. The picture
or antique must be available for inspection and could even be
transported to a gallery or store. A house may be occupied, but
the owner must make it available to be seen by prospective clients.
In retail or wholesale trade in general the stock yields no con-
sumption value to the seller. It is an intermediate production
good.

In essence trade is a productive process. An attempt to cap-
ture the nuances of timing of trade in a one period model is to

force it onto a bed of Procrustes. However for simplicity it is worth doing so.

(c) "Anonymous" trade matched by an intermediary: The type of trade illustrated by the model in Section 2 is mass anonymous market trade. The mechanism which could be a stock or commodities exchange aggregates bids and offers. Trader A does not find out the identity of B and does not particularly care. In this type of market both sides tend to surrender their goods or instruments before either obtains value for value sent (see Baruch (1972) for an estimate of the amounts in float some years ago).

Table 1 presents a summary of some of the considerations for various goods and instruments as to liquidity, stores of value and related properties. We assume m goods and services plus one or more of the first four items.

	Number of markets	Market efficiency	Anonymous	Liquidity	Cost of liquidity	Sources of store of value
Commodity money	$m+3$	High among all of these financial instruments	yes	complete	foregone utility or interest	commodity use, custom & law as money
Fiat money	$m+3$		yes	complete	foregone interest	custom & law
Bank money	$m+3 \leq$		not quite	almost complete	foregone interest	custom & law
Government notes & bills	$\simeq 3$		not quite	"near money"	foregone interest	custom & law
Commodities	$\simeq 3$	variable from high to low	variable from not quite to individual	near money to highly illiquid	foregone use while being marketed & costs of exchange	commodity use, if durable
Services	$\simeq 3$					---

TABLE 1

Commodity, fiat and bank money are, for many purposes, virtually inter-changeable. Government notes and bills can be changed quickly into some form of money but it does require a two stage process to use them to buy anything else. A money is required. Bank money on occasion is not quite as acceptable in some markets as cash or fiat.

Although there may well exist various barter markets for some goods and services (for example professionals may trade services under the title of professional courtesy) as a good first approximation we may consider only one market between most commodities and money. Three markets are noted to account for the three manifestations of money.

The column entitled market efficiency provides a crude summary of in-formation on thickness, and transactions technology. Money markets may be close to the dream of an efficient competitive market. The markets for paintings or antiques are far from the ideal.

Anonymity is an important property of an efficient market and a use-ful feature in capital flight. Cash, gold bars, dollar bills and bearer bonds are more anonymous than checks or owner registered bonds.

Liquidity is complete when an instrument can be exchanged for all other items in the economy and the transactions costs generated on the payments side of the market are low. Thus money buying art is liquid but not vice versa.

The cost of liquidity is measured in terms of alternative values fore-gone. If an interest rate ρ exists and Δt of time earning interest must be given up then the cost is $\rho \Delta t$. If monetary gold cannot be simul-taneously used for jewelry then the consumer use foregone is a measure.

The store of value aspect of a commodity money is broken down into its commodity worth and an extra value due to its use as a money. Thus

fiat money has no commodity value but nevertheless as long as the rules enforce its use in further exchange it enters into the salvage value.

Life is not pure economics and although some economic purists may want a strictly economic explanation of what keeps the value of a fiat money the answer lies elsewhere. Institutions, law and custom involving taxes, prison, social pressure, torture, death and other unpleasantries provide enough incentives for most of us to refrain from counterfeiting or from refusing to accept legal tender.

7. ON EFFICIENCY IN EXCHANGE

One of the key attractions of the general equilibrium model of exchange is that not only does a market clearing price system exist but it is efficient.

When we model the economy as a game with a salvage value function several problems appear. (1) The mechanism of the game may constrain the feasible set of outcomes. (2) For finite numbers of traders efficiency may be lost due to oligopolistic effects. (3) Even with a continuum of traders efficiency may be lost due to money shortage constraints and (4) there is no a priori guarantee that with a continuum of traders the noncooperative equilibria would lie on the efficient surface of the feasible set of outcomes.

In this section we observe that the model suggested in Section 4 cannot be efficient as it stands. This is immediately obvious if we consider that if there are several individuals with no gold to start with they are unable to trade at all. In order to remedy this situation we must introduce a money market. The alternative designs for such a market have been discussed elsewhere (Shubik (1985)). Figure 3 indicates the three stage process.
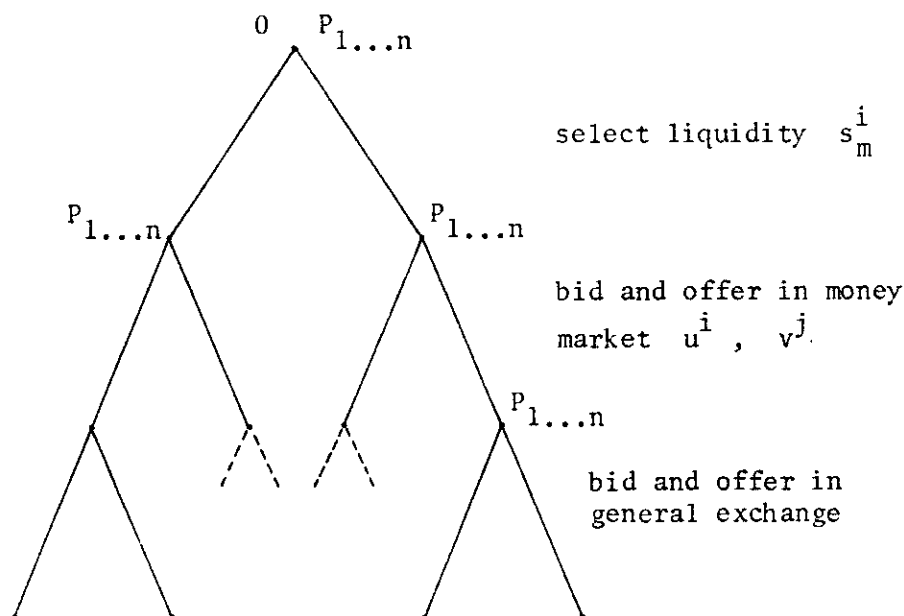
FIGURE 3

An individual of Type i begins with $(a_1^i, \ldots, a_m^i)$ where a_m^i is his

initial supply of gold. He divides his gold into two piles $a_m^i - s_m^i$ and

s_m^i where $s_m^i \leq a_m^i$. In the second stage we must invent a new instrument,

the personal I.O.U. note. Those who wish to borrow bid I.O.U. notes redeem-

able in gold at the end of trade. But out of equilibrium it is possible

that the writer of an I.O.U. note may not be able to satisfy the claims

against him hence we need to specify a default or bankruptcy procedure in-

cluding security conditions if there is more than one creditor. These

nasty but necessary details are discussed elsewhere (Shubik, 1985).

A strategy for an individual of Type i is

$$(s^i, u^i, v^i, b_1^i, q_1^i, \ldots, b_m^i, q_m^i) .$$

As all individuals are insignificant in size we may describe the opti-

mization process of each by an action based on the assumption that expected

price will not be influenced by an individual action. Thus i wishes to

maximize

$$(8) \quad \varphi_i(a_1^i - q_1^i + b_1^i/p_1, \ldots, a_m^i - q_m^i - s^i + b_m^i/p_m) + V_i(x_{m+2}^i)$$

$$\text{where} \quad p_j = \sum_{i=1}^{n} b_j^i / \sum_{i=1}^{n} q_j^i \quad \text{for} \quad j = 1, \ldots, m$$

$$= 0 \quad \text{if} \quad \sum_{i=1}^{n} q_j^i = 0$$

$$(9) \quad p_{m+1} = 1 \quad \text{the price of monetary gold.}$$

The rate of interest is given by

$$(10) \quad 1+\rho = \sum_{i=1}^{n} u^i / \sum_{i=1}^{n} v^i \quad (\text{if} \quad \sum_{i=1}^{n} v^i = 0 \quad \text{then} \quad \rho = 0 \quad \text{and the I.0.U.s are returned}),$$

where the u^i are bids of I.O.U. notes and the v^i are offers of monetary gold

$$(11) \quad x_{m+2}^i = s^i - v^i + u^i/(1+\rho) - \sum_{j=1}^{m} b_j^i + \sum_{j=1}^{m} p_j q_j^i .$$

The expression (11) is only accurate under the circumstances of no default otherwise the term $u^i/(1+\rho)$ will be adjusted to reflect the loss by creditors.

It is straightforward to observe that if $V(\cdot) = 0$, i.e. if no value is attached to left over assets then trade cannot be efficient in the sense that a noncooperative equilibrium (NE) coincides with a competitive equilibrium (CE). In this case there will be no lending as the money for loan would better be used immediately as a utility yielding nonmonetary gold. But if all are to be paid for their goods in monetary gold of no future value no one will offer any goods for sale.

The writings of Aristotle through to Arab and medieval church theology opposed the paying of interest with statements such as "the userer sells nothing which actually belongs to him, but he sells only time, which belongs to God" (Thomas of Chobham quoted by LeGoff, 1979). Yet it is the combination of store of value and consumption foregone that enables a money to be used in exchange through time and for it to command a rate of interest. It must be noted that unlike price which is an exchange ratio between money and a commodity at time t and has dimension of quantity of money over the quantity of good, the rate of interest is a flow or a price of borrowing money per unit of time.

Furthermore as the price of borrowing money is a time price for liquidity, if there is no foregone cost to obtaining liquidity then the price of borrowing could easily be zero. In a stationary state with no uncertainty, no time discount in the utility functions, no transactions costs and a fixed supply of fiat money which must be returned to the agency of issue at the end of trade (a finite horizon) it can be shown (Dubey and Shubik, 1977, 1978) that there will be an equilibrium with zero interest rate. In this situation the church philosophers, Aristotle, the precepts of Islam and economic theory all come together and no userer need go to hell as he lends without charge.

It should be noted that at a zero rate of interest hoarding and saving cannot be distinguished. A negative rate of interest is ruled out by strategic considerations, an individual can always make at least the utility value of consumption gold each period.

Although without a salvage value function there can be no trade in the one period model this is not true for a T time period model, even for T = 2 because in the first period monetary gold will have an ending

value after one period of its consumer worth in the second period, thus as $T \to \infty$ the implicit value of gold is the sum of its discounted value in consumption.

Suppose that we consider trade for T periods where each period individual of Type i receives an endowment of $(a_1^i, \ldots, a_{m-1}^i)$. All of these commodities are nondurable, but the endowments are renewed each period. At period 1 each individual of Type i receives an amount a_m^i of gold which is completely durable. The goal of each individual is to maximize

$$\sum_{t=1}^{T} \beta^t \varphi_i(x_1^i, x_2^i, \ldots, x_m^i) .$$

The steady state CE will utilize all resources including $a_m = \sum_{i=1}^{n} a_m^i$ units of gold in consumption each period. But the steady state NE for $T \to \infty$ will have part of the gold supply always used as money.

The steady state NE is efficient in the context of the feasible set of the trading mechanism. But in comparison with competitive equilibrium it yields the same prices and payoffs as the CE of an economy with the monetary gold thrown away. The specific conditions and proofs will be given in a separate paper.   The efficient NE may involve borrowing and lending.

The strategic market game which might be regarded as better would be one in which fiat money replaces gold, but for this the store of value property of the money must be an artifact of law and society.

Following the basic principle of discussing the simplest of models first, production has been omitted from our discussion. However, trade itself is a form of production. A radio sitting in a department store is as much an intermediate good as is a part for assembly sitting in a factory.

Possibly the important distinction is quantitative more than qualitative. The circulating capital requirements for primary industry and manufacturing are far larger than for retailing. In a modern economy the stock of land, buildings, intermediate goods and other durables with eventual consumption value may easily outweigh in market value the total value of trade for final assumption. Furthermore unlike monetary gold which cannot be simultaneously available for consumption, with many pledged assets it is possible to have one's cake and eat it. Thus one can live in a mortgaged house or drive a car bought on credit.

Given the availability of a large array of stores of value we do not need the extremes of pure fiat money or straight gold money. Paper backed by durable assets provides a basis for a paper money and credit system where the cost of trust is minimized. The efficiency of the fiat money system with paper backed by law, appears better than the strange efficiency of the gold system where for hostage purposes a large stack of gold must be held in limbo. But better still land, buildings, intermediate goods and other durables can simultaneously serve consumption, production and hostage purposes without the paradox of requiring the store of an asset that could be usefully employed otherwise.

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