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THE USE OF SIMPLE GAMES TO ILLUSTRATE CONCEPTS AND TO PROVIDE EXPERIMENTAL EVIDENCE

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by

M. Shubik*

1. INTRODUCTION

This paper is devoted to a discussion of several simple experimental games used in a series of lectures on game theory. The prime purpose of these games was to raise questions and illustrate problems in the construction of game theoretical models in the social sciences. The students were asked to make choices or to give opinions as to what imputation should be recommended as a solution in a cooperative game.

The games presented were in extensive, strategic and cooperative form. The remarks made here are confined to those in strategic and extensive form. The discussion of the games in cooperative or coalitional form has been given in part elsewhere (Shubik, 1975) and the further results will be presented in a subsequent note.

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2. THE GAMES AND THEIR DIDACTIC PURPOSE

Nine sets of games have been utilized, of which two (Sets 1 and 6) were in coalition form. The remaining seven sets of games together with their purpose are now noted.

The first games in strategic form were Set 2. The games and instructions are shown below:

GAME 2

You are Player 1, i.e., "Row Player." You are playing an unknown, unseen competitor in three two by two matrix games with payoffs as follows:

A	1	2
1	5,5	-9,10
2	10,-9	0,0

B	1	2
1	-10,-10	5,-5
2	-5,5	0,0

C	1	2
1	2,1	0,0
2	0,0	1,2

You are going to play only once. Suppose that the units were x$10, i.e., -9 = -$90. You have to play. How do you play and what are your reasons?

Three informal names have been given to these games, they are:

"Chicken"
"The battle of the sexes"
"The prisoner's dilemma"

Match the names with games A, B, C and give your reasons if you can.

Comment

The students were asked to select strategies for the three 2×2 matrix games prior to any discussion of utility theory, games in strategic form or solution concepts.

The three games are well known illustrative games in both game theory teaching and experimentation. The first is the prisoner's dilemma with a unique NE (noncooperative equilibrium) at (2,2). The second is the game of chicken with two pure strategy equilibria at (1,2) and (2,1) and a mixed strategy equilibrium. The third is the battle of the sexes with pure strategy equilibria at (1,1) and (2,2) and a mixed strategy equilibrium.

The games have been utilized on six occasions on around 160 individuals. The full details of the results are given in the appendix. The percentages selecting Strategy 1 in the three games were 8.5, 44 and 80.6 respectively.

Little attention has been paid to the linkage between verbal and mathematical description. The names attached to the matrices have been constructs of game theorists. Are the names suggestive to those who are not aware of the structures associated with them? The evidence points to a high percentage with the correct identification of the battle of the sexes, but not the others (see appendix).

Game set 3 was addressed to problems involving interpersonal comparisons of welfare. Four games were given to the students. The payoffs in the first three are linear transformations of each other. The fourth games were utilized essentially as a rationality check in 1984 and 1985. It is a zero sum game with a saddlepoint solution.

<u>GAME 3</u>

You are going to play each of the following four games once, each against a different unknown competitor. You are Player 1 selecting a row. Circle your choice.

	A	1	2	B	1	2	C	1	2	D	1	2
Your choice	1	1,2	0,0	1	100,2	0,0	1	1,200	0,0	1	5,-5	-2,2
	2	0,0	2,1	2	0,0	200,1	2	0,0	2,100		10,-10	0,0

(a) What is your choice in A
B
C
D

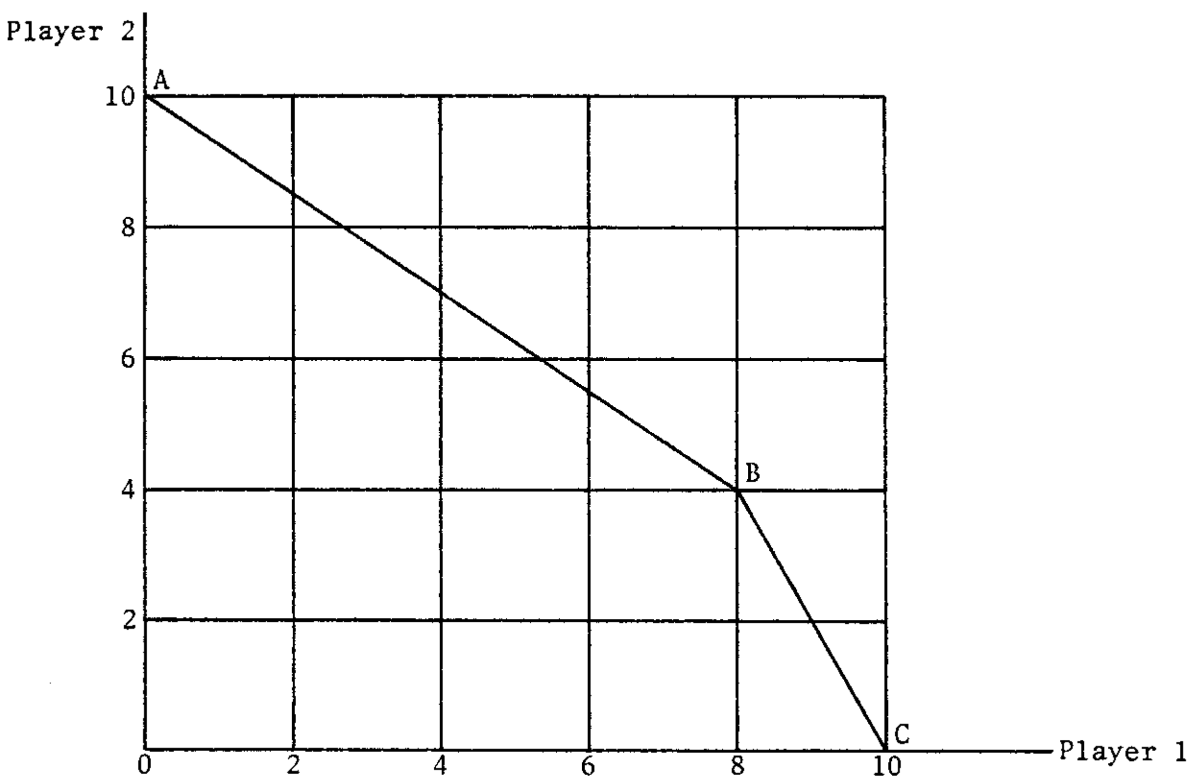
Comment why.

If all players were only concerned with ordinal payoffs all three games A, B, C should be played the same. Over the five years 29.6% of the respondents played all three games the same way. In the two years with the zero sum game added 100% selected the saddlepoint strategy.

These games were used as an introduction to discussion on ordinal or cardinal payoffs and to the possibility of interpersonal comparisons.

Game 4 is a repetition of an experiment originally run by Stone (1958) which calls attention to the importance of the presentation of the game. Schelling (1960) laid stress upon "prominence" or natural cues in some situations.

GAME 4



Players 1 and 2 move simultaneously without knowledge of the other's move. You are Player 2, your strategy is to draw a horizontal line. Player 1 will draw a vertical line. If the two lines intersect within (or on the boundary), that is your payoff. If it is outside, you both obtain zero. Draw in the line you select.

The point B on the efficient surface ABC is prominent. Some of the theory is given in the appendix. This game was used in the class to raise questions concerning the interplay between considerations of equity or fair division and the concept of a prominent point. The results in every one of the five runs were clearly bimodal. 36.2% of the respondents selected the prominent point and 33.3% chose the even split.

In order to open up questions concerning threat and the possibility of precommitment the students were asked in their weekly exercises how they would play if they had the opportunity to move first. Although it was relatively easy to understand the enormous advantage this conferred upon the player nevertheless there was a discussion on leaving a little for the other player.

It appeared that even though the games were presented as one shot anonymous games many students were not willing to regard them as being completely _in vitro_ or out of context. A concern for the payoff to other side, even if it was over the barrel was evinced.

Game 5 which was actually played for money with the students using their own money, was used prior to discussing preference for items involving gambles, the Bernoulli paradox and then the application of game theory models to bidding and auctions.

GAME 5

In class using three special dice a number between 1 and 1,000 will be generated. In this game we will all use actual money. Those who do not wish to bid may submit a message of "no bid." Otherwise, you bid in units of one cent and the highest bidder will receive x cents where $1 \leq x \leq 1{,}000$ in return for paying his bid to the auctioneer.

Do not forget to sign your name. Payment will be made at the end of class.

Explain your bid.

The results were used to raise several basic problems. Do individuals with the same information base assume that they have the same utility functions or if they do not, then how do they think about the utility functions of others prior to bidding? The assumption that in deciding to act under risk that an individual has a clear perception of his own utility function is not clear.

The results over five years show some crude consistency in the shape of the distribution of bids, but the interpretation of this information is not clear.

Game 7 was presented in three versions 7A, 7B and 7C. Version 7A is given below.

GAME 7A

Suppose you were given \$3 on one condition, that you use some of this money to participate in an experiment on competitive bidding. What follows is description of that experiment. After reading the description, you will be asked to state your bid. If you wish, you may explain your decision on the back of this page.

You are bidding for \$9. There are two bidders: you and an expert on bidding strategies (called "Other"), who was given \$9 on the same terms as you were. The bids are sealed, therefore you will not be told what Other's bid was. Likewise, Other has made a bid without knowing yours.

If you bid \$x ($0 < x \leq 3$) and Other bids \$y ($0 < y \leq 9$), you will receive \$9[x/(x+y)] and Other will receive \$9[y/(x+y)]. In other words, you will divide the \$9 between you and Other in proportion to your bids. Thus if both of you bid the same amount, you will each receive \$4.50, regardless of whether you bid 1¢ or \$1.

Please write your bid here (in \$ and/or ¢): ____________________
If you wish to explain your decision, use the back of this page.

Games 7A and 7C differed from 7A in the constraints on funds available for bidding. The first line of the third paragraph of the description of game 7A has the limits on the bids for the player and Other.

For game 7B the limits were ($0 < x \leq 3$) and ($0 < y \leq 3$), and for game 7C $(0 \leq x \leq 9)$ and $(0 \leq y \leq 3)$.

Four years of data for games 7A and 7B are available and three years of data for game 7C (see appendix for data).

The games were selected so that the noncooperative equilibrium solution under all three treatments give the same solution $x = y = 2.25$. The maximin solution for case 7A is $x = 0$ and 7B and 7C is $x = 2.196$. The results indicate that choice was apparently influenced by the levels of resources. In particular the individuals tended to bid more if they had more resources. The table below shows the distribution of bids at or above and below the noncooperative equilibrium.

		7A 3/9	7B 3/3	7C 9/3
	more	12	13	16
bids	2.25	2	3	1
	less	18	19	16

Game 8 was a somewhat more sophisticated version of game 7 where uncertainty concerning the value of the prize is introduced nonsymmetrically. The student is required to make a bid for an object which may either be worthless or is worth $18. Her opponent or competitor knows, but she does not. She knows that the probabilities for the two values are 50:50.

GAME 8

Suppose you were given $9 on one condition, that you use some of this money to participate in an experiment on competitive bidding. What follows is description of that experiment. After reading the description, you will be asked to state your bid. If you wish, you may explain your decision on the back of this page.

You are bidding for a share in the contents of a "black box" which with

probability 1/2 has $18 in it, and with

probability 1/2 has 0 in it.

There are two bidders: you and an expert on bidding strategies (called "other"), who was given $9 on the same terms as you were. The bids are sealed, therefore you will not be told what Other's bid was before you have made your bid. Likewise, Other had made a bid without knowing yours. The minimum unit in bidding is 1 cent. No fractions smaller are recognized.

Although you do not know the contents of the black box, the other bidder does. He has an information advantage over you. In particular, this means that his strategy can be more flexible than your strategy. You must bid $\$x$ $(0 < x \leq \$9)$ regardless of what is in the box. He can bid y_1 $(0 < y_1 \leq \$9)$ if the box is empty and y_2 $(0 < y_2 \leq \$9)$ if it contains a prize of $9.

Your expected payoff is:

$$\Pi_1 = \frac{1}{2}\, 0 \left(\frac{x}{x + y_1}\right) + \frac{1}{2}\, 18 \left(\frac{x}{x + y_2}\right) + 9 - x \,.$$

Please write your bid here (in $ and/or ¢): ______________________________
If you wish to explain your decision, use the back of this page.

The game was used to illustrate how to calculate the value of information comparing the symmetric information with the nonsymmetric information game. The solution and results are presented in the appendix. The average bids in the two years this game has been run were 2.5 and 2.42. The NE is at $x = 2$. Bidding appeared to be higher than that projected

	Above	21
Bid	x = 2	2
	Below	14

by the NE.

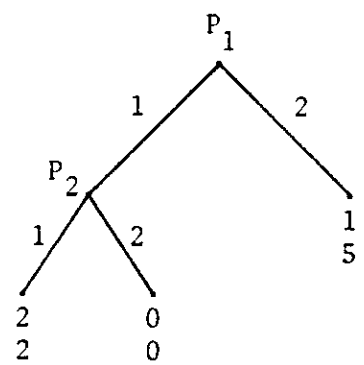
Game set 9 presented three games in extensive form. They were used for two purposes. The first two were to consider perfect equilibrium points and the last game was played in order to pose some questions concerning how to cope with irrational behavior. The three games are illustrated below.

GAME 9

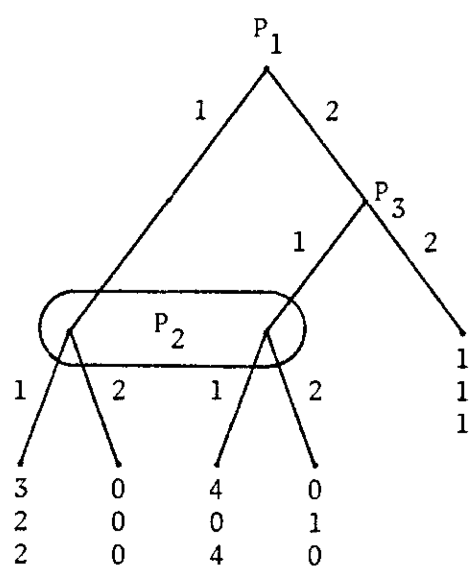
Three games in extensive form are shown below. In each instance you are Player 2 and you are trying to maximize your expected payoff.

In each game you can select move 1 or 2. State which you select and give a reason why.

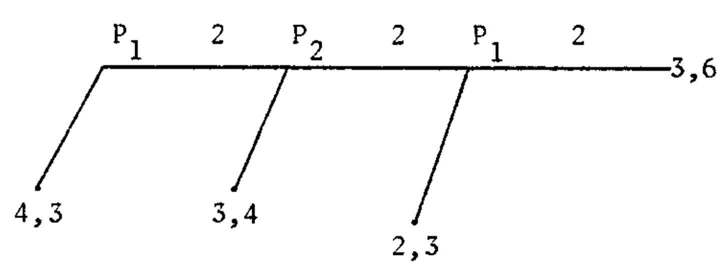
Game 9a



Game 9b



Game 9c



The set of games has been played only once. In the first game 20 out of 21 students playing as Player 2 selected their first move, which is consistent with the perfect equilibrium at strategy pair (1,1).

Game 9b has a somewhat complicated information condition. There are two pure strategy equilibria (1,1,2) and (2,2,2) with payoffs (3,2,2) and (1,1,1). The equilibrium point (2,2,2) is trembling hand perfect. Out of 21 responses for Player 2, 15 selected 1 and 6 selected 2.

The last game 9c was posed in order to raise some questions concerning context. The students were asked to act as Player 2 and to decide what to do if Player 2 is called upon to move. If one examines the game it is clear that Player 1 should opt for selecting move 1 and thus Player 2 should never obtain the move. If he does then he must conjecture that Player 1 has made an error, is otherwise irrational or that he has false information about Player 1. 17 out of 21 students chose move 2 while 4 chose move 1 on the basis that more irrationality might follow. In the appendix the reasons given for their selections are given.

3. CONCLUDING REMARKS

The use of a variety of simple games serves to provide a cost effective device for teaching. The playing of the game is quick and serves to illustrate a host of modeling problems and conceptual points. The running of a game in the classroom clearly raises problems with control. It tends to be sloppy; but over the years it is possible to build up some relatively large sample sizes and obtain enough regularity that consistent patterns begin to emerge. In particular there is no decent normative theory for nonefficient noncooperative equilibrium points. There may be some tendencies towards equilibrium behavior in some contexts, but even at this level of poorly controlled experimentation the effects of interpersonal comparisons, prominence, level of resources and equity considerations appear to be present not as more minor corrections but as competing forces whose influence may vary as a function of context.

The simple games used in a didactic mode appear to not merely aid students in understanding concepts but provide pilot experimental data and challenge the concepts and predictions of the theorists.

APPENDIX

ANALYSIS OF ALL GAMES

ANALYSIS OF GAME 2

You are Player 1, i.e., "Row Player." You are playing an unknown, unseen competitor in three two by two matrix games with payoffs as follows:

A	1	2
1	5,5	-9,10
2	10,-9	(0,0)

B	1	2
1	-10,-10	(5,-5)
	(-5,5)	0,0

C	1	2
1	(2,1)	0,0
	0,0	(1,2)

	Strategy 1	Strategy 2	
RDS*	3	16	19
80	1	15	18
81	4	37	41
83	4	27	31
84	2	13	15
85	0	42	42
Σ	14	150	164

	Strategy 1	Strategy 2	
RDS	6	13	19
80	5	9	14
81	23	17	40
83	16	15	31
84	5	10	15
85	16	26	42
Σ	71	90	161

	Strategy 1	Strategy 2	
RDS	18	1	19
80	11	2	13
81	27	13	40
83	23	8	31
84	10	5	15
85	40	2	42
Σ	129	31	160

You are going to play only once. Suppose that the units were x$10, i.e., -9 = -$90. You have to play. How do you play and what are your reasons?

The circle indicates a pure strategy noncooperative equilibrium.

*Name of experiment

Response

			Chicken	B of S	P.D.	
	A = chicken	83	8 22 19 20 5	3 6 3 8 3	7 6 18 5 9	85
Actual	B = B of S		4 6 7 4 2	12 19 28 20 12	1 9 3 8 3	
	C = P.D.		6 6 13 8 10	3 9 7 4 2	8 19 18 19 5	

1980 upper left
1981 upper right
1983 lower left
1984 lower right
1985 center

Percentage Correct*

	Chicken	B of S	P.D.
1980	38	70.6	47
1981	64.7	55.9	55.9
1983	60.6	62.5	61.3
1984	29.4	70.6	29.4
1985	47.5	73.7	46.2

*All populations undergraduate, MBA, law and graduate students with roughly the same profile except for the last class who were slightly more biased to mathematics and a small exposure to game theory.

	1980	1981	1983	1984	1985	%	Chance
3 right	6	16	16	5	16*	44.4	16.6
1 right	9	12	9	8	15	39.8	33.3
0 right	2	6	0	4	7	15.8	50.0
no reply	1	8	-	4	3	-	-
	18	42	32	21	41		

*Some respondents with previous experience.

Words and numbers: The matching question was to see if "numbers tell the story." There appears to be some weak support for "battle of the sexes" and the percentages with 3 right is considerably better than chance.

ANALYSIS OF GAME 3

Do Interpersonal Comparisons Matter?
Three Simple Games

1. The Games

Three games were considered for 1980, 1981, 1983, a fourth was added for 1984. They are illustrated in Figure 1.

A	1	2
1	1,2	0,0
2	0,0	2,1

B	1	2
1	100,2	0.0
2	0,0	200,1

C	1	2
1	1,200	0,0
2	0,0	2,100

Figure 1

A quick examination of the three games shows that B is constructed by multiplying the payoffs to Player 1 by 100. C is constructed by multiplying the payoffs to Player 2 in game A by 100.

2. The Theory

Do the size of numbers and interpersonal comparisons between the payoffs to Player 1 and Player 2 matter? All of the three matrices in Figure 1 are of the form shown in Figure 2.

	1	2
1	b_1, a_2	c_1, c_2
2	c_1, c_2	a_1, b_2

Figure 2

There are three different fairly straightforward assumptions that can be made concerning preferences and interpersonal comparison of preferences.

(1) We may assume that the only feature that counts is individual ordering of preference. If this were the case, then we should expect that all games are played the same.

(2) We may assume that each player has a measurable scale on the worth of outcomes to him, but is not concerned with the magnitude of the outcomes to the other. If that were the case games A and C would always be played the same way.

(3) We may assume that each player not only takes into account the worth of outcomes to himself or herself, but compares the relative worth of outcomes between the two players. In this instance it is possible that all games are played differently.

Much of economic theory of individual consumer behavior makes the first of the assumptions above. But it is scarcely applied to two person strategic situations.

Analysis of the Games

	1980	1981	1983	1984	1985
Consistent with the first assumption	7/38 or 18.4%	9/32 or 28.1%	9/28 = 32.1%	7/20 = 35%	11/27 or 40.7%
All 2's	3 or 7.9%	5 or 15.6%	8/28 = 28.6%	7/20 = 35%	8/27 or 29.6%
All 1's	4 or 10.5%	4 or 12.5%	1/28 = 3.6%	0	3/27 or 7.4%
Consistent with the second assumption	12/38 or 31.6%	17/32 or 53.1%	18/28 = 64.3%	15/20 = 75%	16/27 or 59.3%
Quid pro+ quo for A and C	14/38 or 36.8%	17/32 or 53.1%	8/28* = 28.6%	0	3/27 or 7.4%
Cautious+ quid pro quo between B and C	7/38 or 18.4%	3/32 or 4.4%	0	0	0

*From 1983 onward instructions stressed a different player for each game. Hence no directly signalled quid pro quo is possible.

*See data following

Game 4 (1984/1985 only)

	1	2
1	5,-5	-2,2
2	10,-10	0,0

(1984) Chose 22 20/20 "saddlepoint rationality test"

(1985) Chose 22 26/26 "saddlepoint rationality test"

3. The Results

Thirty-eight respondents responded as follows:

1980

Respondent	Game A	Game B	Game C	Comment
1	1	2	2	
2	2	1	2	
3	(½,½)	2	1	on BC quid pro quo
4	1	2	1	
5	1	2	1	on B, C quid pro quo
6	2	1	1	A small, B most-likely C follows B
7	2	2	1	on BC quid pro quo
8	1	2	1	on BC quid pro quo
9	2	1	1	what do numbers represent?
10	1	1	1	
11	2	1	2	on BC cautious quid pro quo
12	2	1	1	C "altruism"
13	2	2	1	on BC quid pro quo
14	2	1	2	
15	1	1	1	
16	2	1	2	on BC cautious quid pro quo
17	1	1	1	
18	2	2	2	quid pro quo but no idea of marginal value
19	2	2	1	
20	2	2	1	quid pro quo
21	1	1	2	cautious quid pro quo
22	(½,½)	complex strategy		take turns quid pro quo
23	2	1	1	he's greedy, I am not
24	2	2	2	
25	1	2	2	
26	1	2	2	
27	(½,½)	(½,½)	(½,½)	
28	2	2	2	
29	2	2	1	BC quid pro quo
30	2	2	1	BC quid pro quo
31	2	2	1	BC quid pro quo
32	2	2	1	BC quid pro quo
33	(½,½)	2	1	BC quid pro quo
34	(½,½)	2	1	BC quid pro quo
35	2	1	2	
36	1	1	1	assume other is greedy
37	1	1	1	prefer altruism to greed
38	2	1	2	cautious quid pro quo

1981

Respondent	Game A	Game C	Game B	Comment
1	1	1	2	cautious quid pro quo
2	2	2	2	quid pro quo; no idea of marginal value
3	2	2	1	BC quid pro quo
4	2	2	2	
5	2	2	1	
6	1	1	1	assumes opponent is greedy
7	1	1	1	
8	2	2	1	
9	2	2	2	
10	2	2	1	
11	1	2	1	risk it when it really counts
12	1	2	1	
13	1	1	1	
14	2	2	1	
15	1	2	1	
16	2	2	1	
17	1	1	1	
18	1	2	1	
19	2	1	2	don's risk it when it really counts
20	2	2	1	
21	2	2	2	
22	1	2	1	
23	1	2	1	
24	2	1	1	my opponent is greedy, I am not
25	1	2	1	
26	1	2	2	no consistent reasoning
27	2	2	1	
28	2	2	2	
29	2	2	1	
30	2	2	1	
31	1	1	2	
32	(½,½)	(½,½)	(½,½)	flip a coin

1983

Respondent	A	B	C	Comment
1	2	2	2	I'm greedy, opponent not
2	2	1	1	opponent greedy
3	1	1	2	
4	1	1	2	
5	2	2	1	opponent altruistic
6	2	2	2	
7	1	1	1	opponent greedy
8	2	1	1	I'm greedy A
9	1	1	2	other greedy AB
10	2	1	2	
11	2	2	2	I'm greedy
12	1	2	1	I'm greedy B
13	2	2	2	greedy solution
14	2	2	1	opponent altruistic B
15	2	2	1	quid pro quo BC
16	2	2	2	
17	2	2	2	I'm greedy
18	2	1	1	opponent malicious
19	2	1	2	
20	1	2	1	opponent greedy C
21	1	2	1	quid pro quo BC
22	2	2	1	
23	2	1	2	quid pro quo BC
24	2	1	2	opponent greedy B
25	2	1	2	opponent greedy BC
26	2	2	2	
27	1	2	1	quid pro quo BC
28	2	2	2	

1984

		Game		
Respondent	A	B	C	Comment
1	1	1	2	
2	2	2	2	I'm greedy
3	2	2	2	I'm greedy
4	1	1	1	other greedy
5	2	1	2	other greedy
6	2	1	2	individual with smaller gain is greedy
7	2	2	2	
8	1	1	1	I'm greedy, opponent greedy
9	2	2	1	
10	1	2	1	individual with larger gain is greedy
11	2	2	2	
12	1	1	2	individual with smaller gain is greedy
13	2	1	2	
14	2	2	2	I'm greedy
15	1	2	1	individual with smaller gain is greedy
16	2	1	2	
17	2	2	1	individual with smaller gain is greedy
18	2	2	2	I'm greedy
19	1	2	1	everyone altruistic
20	2	2	2	I'm greedy

1985

Respondent	A	B	C	Comment
1	(½,½)	1	2	don't know/go for 200/
2	2	2	1	other thinks I maximize/same/he will max
3	1	2	2	?
4	2	2	2	maximize own without knowledge
5	2	2	2	maximize own without knowledge
6	1	1	1	assume other maximize
7	2	2	2	maximize own, he must change
8	1	1	1	something better than nothing
9	2	1	2	rather 0 than less/big winner any way/
10	2	2	1	gives me highest/same/gives other highest
11	1	1	1	influenced by information on battle of sexes
12	-	2	1	gain great for me/ do unto others
13	1	2	2	accept 50%/won't accept 2%/he'll compromise
14	2	1	2	rational opponent goes for 1/
15	2	1	2	/2 not worth risk/same reasons
16	2	1	2	difference between 0, 100, 200 important not 0, 1, 2
17	2	2	1	banking on opponent wants something/should let me get 200/vice versa
18	2	2	2	hope he believes I'm greedy
19	2	2	2	I go for max
20	2	1	2	/settle for 100/vice versa
21	2	2	2	no clear choice for any
22	2	2	2	maximize my potential
23	2	2	1	greed first order/same/his greed
24	1	1	2	know 4:1 in favor of 1/max for disadvantaged player
25	2	2	2	maximize self interest
26	1	1	2	assume opponent greedy

Strategies	1980	1981	1983	1984	1985	Σ
111	5	4	1	2	3	15
112	1	2	3	2	3	11
121	3	7	4	3	0	17
122	3	1	-	-	2	6
211	4	1	3	-	0	8
212	6	1	5	4	5	21
221	8	10	4	2	3	27
222	3	5	8	7	8	31
Other	5	1	-	-	2	8
Σ	38	32	28	20	26	144

ANALYSIS OF GAME 4

Prominence or Equity
Or Something Else?

1. The Problem

There are certain games which may be regarded as games involving coordination. The battle of the sexes is one of them. Both strategy pairs (1,1) and (2,2) are better than the other alternatives. But if one cannot

	1	2
1	1,2	0,1
2	0,0	2,1

talk or communicate otherwise how can one coordinate behavior? In society this coordination between anonymous strangers may be brought about by laws, common rules, customs or conventions. A simple example is provided by the "rules of the road." It does not much matter if the convention is to drive on the left or the right as long as all know the convention. The driver on the other side of the road may be a perfect stranger, but you expect him or her to have enough sense to stick to the appropriate side of the road.

The game examined here was one suggested and examined by Stone in 1958. The problem faced in selecting a horizontal (Player 2) or vertical (Player 1) line is that cooperation is called for without communication or joint conventions. There are at least three suggestions as to how this game might be played.

2. Some Theory

1. The Prominent Point or special artifact used to resolve uncertainty. This is point B , (8,4) .

2. Absolute Equity regardless of the shape of the optimal set of outcomes. This is obtained by drawing the 45° line from 0 to the surface. It yields (5.5, 5.5).

3. The Nash-Shapley-Harsanyi cooperative solutions which is based upon sets of axioms which are not presented at this time.

3. The Results

As can be seen from the table below, the point of prominence ("B") was chosen by 16/38 (42%) in 1980 and by 11/32 (34%) in 1981. The even split was chosen by 9/38 (24%) in 1980 and 10/32 (31%) in 1981. Thus, the evidence lends weight to different approaches by the population. The views of all players are not the same, the assumption of symmetry does not hold. There are sizeable numbers supporting the prominent point solution and the equal split.

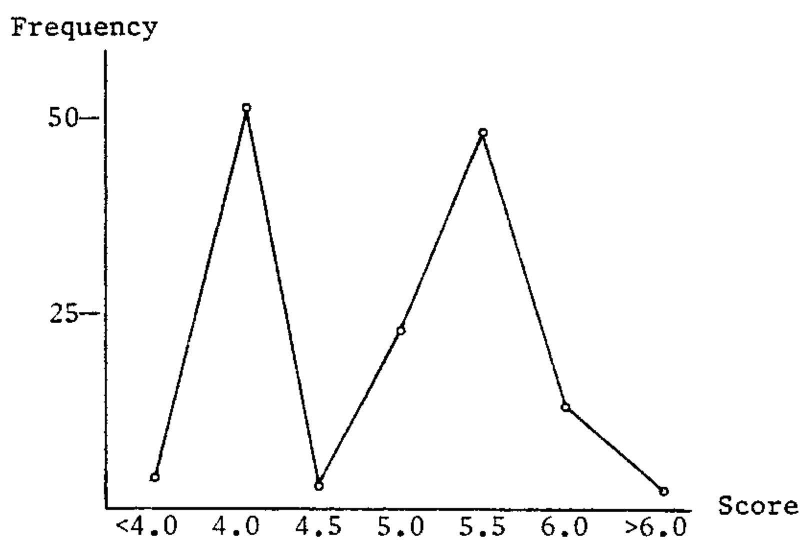
Both in experimental psychology and in military tactics the importance of artifacts or prominences is recognized. Thus, the only tree on a plain or the one bend in a road may serve as focal points. Part of the writings of Schelling (1960) on strategic analyses has been devoted to considering coordination without direct communication, and the role of prominent points or other means to coorelate behavior.

Point	1980 #	1981 #	1983 #	1984 #	1985 #	Σ
< 4.0	0	1	2	0	1	4
4.0	16*	11*	8*	6*	10*	51*
4.5	1	1		1	0	3
5.0	5	9		3	5	22
5.5	9+	10+	12+	8+	8+	47+
> 6.0	5	1	2	1	1	13
6.0	1	0	0	0	0	1
Total	38	32	24	19	25	141

*line through B.

+line through intersection of AB and the bisector of AOC.

GAME 4



ANALYSIS OF GAME 5

Sealed Bidding an Uncertain Prize

1. The Game

Students submitted sealed bids for a prize of anywhere from 1 cent to 1000 cents with equal probabilities for any outcome. Ties were to be resolved by randomization. Bids were to the nearest cent. The expected return in money is 500.5 cents. An unsuccessful bid is assumed to be costless, and the highest bidder pays his bid for the prize.

2. Some Theory

Viewing this game as a one-shot game of strategy let individual i's payoff be P_i his bid p_i and utility for money be $_i(x)$ for x . Let W_i be initial wealth of i .

Thus:

$$P_i = \varphi_i(W_i) \quad \text{if} \quad p_i \neq \max[p_i, \ldots, p_n]$$

$$= \frac{1}{1000}\int_{x=1}^{1000} \varphi_i(x + W_i - p_i)\,dx \quad \text{if} \quad p_i = \max_{p_j}[p_1, \ldots, p_n] \;.$$

We may guess that for a linear utility function for all, there exists a simple symmetric solution where each selects some number $p_i = v$. But if this were so, we would need a tie-breaking rule such as each person in a tie randomizes with probability $1/s$ where s is the number in the tie. Thus, here if all name a price p and all have the same expected value for the prize, then each i selects p_i to maximize:

$$P_i = \frac{1}{n}(500.5 - p) \quad \text{if} \quad p_i = p$$

$$= 0 \quad \text{if} \quad p_i < p$$

$$= (500.5 - p_i) \quad \text{if} \quad p_i > p \;.$$

If all pick $p_i = 500.5$, it is easy to check that this is in equilibrium in pure strategies with a payoff of zero.

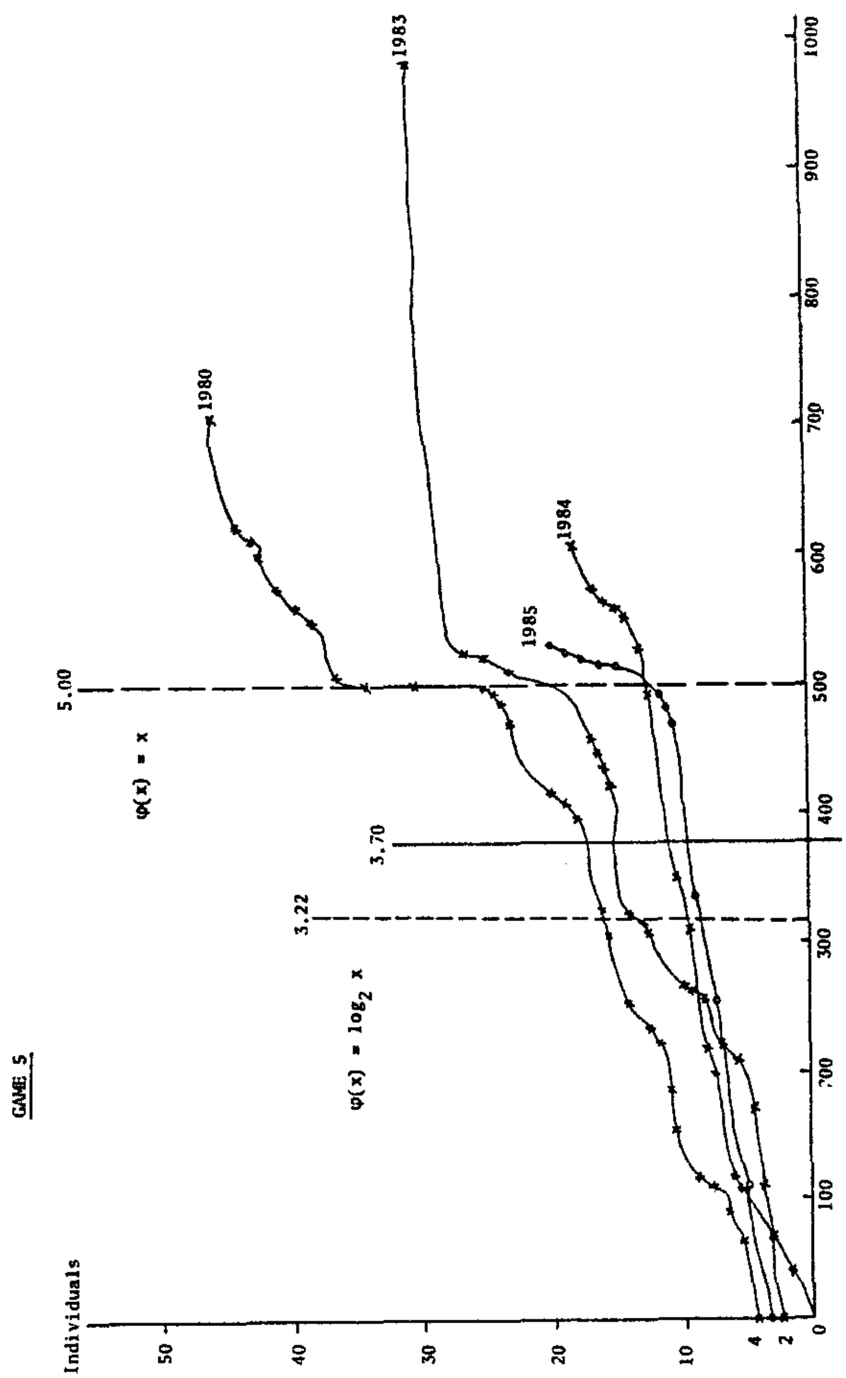

GAME 5
Individuals
50
40
30
20
10
4
2
0
100
200
300
400
500
600
700
800
900
1000
φ(x) = log₂ x
φ(x) = x
3.22
3.70
5.00
1980
1983
1984
1985

FIGURE 1

1980	
Mean Bid	3.73
Median Bid	4.00
# Above 5.00	31.8%
3.22 - 5.00	31.8%
Below 3.22	36.4%

1983	
Mean Bid	3.58
Median Bid	3.34
# Above 5.00	4/23 = 17%
3.22 - 5.00	8/23 = 35%
Below 3.22	11/23 = 48%

1984	
Mean Bid	3.29
Median Bid	3.00
# Above 5.00	6/16 = 37.5%
3.22 - 5.00	2/16 ≐ 12.5%
Below 3.22	8/16 = 50%

1985	
Mean Bid	3.87
Median Bid	4.00
# Above 5.00	27.8%
3.22 - 5.00	27.8%
Below 3.22	44.4%

EXP (linear)	5.00
EXP ($\log_2$)	3.22

ANALYSIS OF GAME 7

A Simple Bidding Game

1. The Game

You have recently played in a game with the following simple form. You have an amount of money A_1 and your competitor has A_2. You bid an amount x where $0 \leq x \leq A_1$ and your competitor bids y where $0 \leq y \leq A_2$ for shares of a prize of a prize worth K. You share the prize in proportion to your bid. Thus, your overall payoff may be described as follows:

$$\Pi_1 = K \frac{x}{x+y} + (A_1 - x) \ .$$

In the three conditions under which you actually played the game the parameter values were as follows:

	K	A_1	A_2
Game C	9	9	3
Game A	9	3	9
Game B	9	3	3

2. Some Theory

You were asked to play once. All played the role of Player 1 against an unknown competitor.

The noncooperative equilibrium can be solved for immediately from

$$\Pi_1 = K \frac{x}{x+y} + A_1 - x$$

$$\Pi_1 = K \frac{y}{x+y} + A_2 - y \ .$$

Taking first derivatives (and checking second order conditions) we find

$$K = \frac{y}{(x+y)^2} = 1$$

$$K = \frac{x}{(x+y)^2} = 1$$

giving $x = y = K/4$ if $K/4 \leq A_1$ or A_2 . For all of the conditions tested here this holds. Thus, for all the treatments with $K = 9$, $x = y = 2\frac{1}{4}$.

The maximin solution offers another explanation of individual behavior. Suppose the Player 1 assumes Player 2 tries to do his worst. This involves spending as much as possible. Thus, we may assume in cases b and c that $y = 3$ and in case a that $y = 9$. We may solve for x by maximizing over x .

$$K \frac{x}{x+y} + A_1 - x \quad \text{where } y \text{ is fixed at } y \ ,$$

or $$x = \sqrt{Ky} - y$$

For cases b and c $x = \sqrt{(9)(3)} - 3 = 3(\sqrt{3} - 1) = 2.196$

For case a $x = \sqrt{81} - 9 = 0$.

GAME 7A (3/9)

Bid	1980	81	83	85
3.00	3	6	1	2
2.25	1	0	0	1
2.00	0	2	0	1
1.75	1	0	0	0
1.50	0	0	2	0
1.00	1	1	1	1
0.75	0	0	0	1
0.50	1	0	1	0
0.01	1	1	3	0
# of Responses	8	10	8	6
AVERAGE	1.81	2.30	0.94	2.00

GAME 7B (3/3)

Bid	1980	81	83	85
3.00	4	4	2	3
2.25	1	0	1	1
2.20	0	0	0	1
2.10	0	0	0	1
2.00	3	1	1	1
1.50	4	0	0	0
1.25	0	1	0	0
1.00	0	0	1	0
0.10	0	0	1	0
0.02	1	0	0	0
0.01	0	2	1	0
	13	8	7	7
AVERAGE	2.02	1.91	1.62	2.51

GAME 7C (9/3)

Bid	1980*	81	83	85
7.00	0	-	1	0
6.00	1	-	1	0
5.00	0	-	1	0
4.50	1	-	0	1
3.01	3	-	1	3
3.00	3	-	1	3
2.25	1	-	0	0
2.20	0	-	0	1
2.10	1	-	0	0
2.00	4	-	0	1
1.50	0	-	0	1
1.00	2	-	0	0
0.99	0	-	1	0
0.10	0	-	0	1
0.01	1	-	2	0
0.00	1	-	0	0
	17	-	8	8
AVERAGE	2.35	-	3.13	2.41

*One student bid $9 but reported an error after the game.

ANALYSIS OF GAME 8

Reaction to Nonsymmetric Information Conditions

You played in a game with extensive form as illustrated in Figure 1.

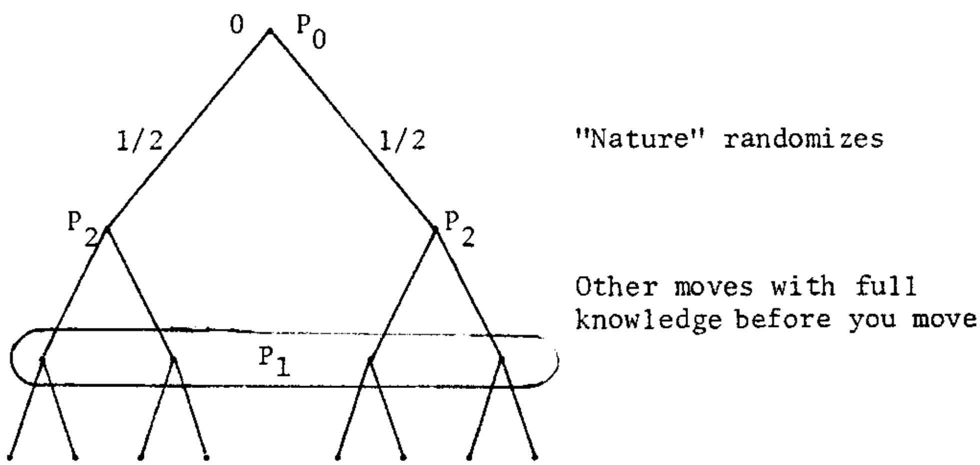


FIGURE 1

Payoffs

Let x = your move (equivalent to your strategy as it involves no contingent planning);

y_1 = other's move, if state 1;

y_2 = other's move, if state 2.

Other's strategy is: (y_1, y_2)

$$\Pi_1 = (1/2)(0)\frac{x}{x+y_1} + (1/2)\frac{18x}{x+y_2} + 9 - x$$

$$\Pi_2 = \begin{cases} (0)\left(\dfrac{y_1}{x+y_1}\right) + 9 - y_1 & \text{if state 1} \\[2ex] \dfrac{18y_2}{x+y_2} + 9 - y_2 & \text{if state 2} \end{cases}$$

By inspection, $\boxed{y_1 = .01}$ as all were required to bid at least one cent.

First order optimization conditions give

$$(1) \qquad \frac{\partial \Pi_1}{\partial x} = \frac{9y_2}{(x+y_2)} - 1 = 0$$

$$(2) \qquad \frac{\partial \Pi}{\partial y_2} = \frac{18x}{(x+y_2)} - 1 = 0 \; .$$

From (1) and (2) $9y_2/18x = 1$ or $y_2 = 2x$ hence $\boxed{x = 2 \; , \quad y_2 = 4}$

$$\Pi_1 = \frac{1}{2}\,\frac{(18)(2)}{6} + 9 - 2 = 10 \quad \text{for a gain of 1}$$

$$\Pi_2 = 8.99 \quad \text{if } 1$$

$$= 17 \quad \text{if } 2$$

or $$\Pi_2 = \tfrac{1}{2}(8.99) + \tfrac{1}{2}(17) = 12.995 \; .$$

<u>What you did</u>:

1983 Average bid = 2.5
(5.04 without .01)

1985 Average bid = 2.42
(3.17 without .01)

Bid ($)	1983 #	1985 #
9	2	1
8.50	1	0
4.75	0	1
4.50	1	1
4.00	0	2
3.75	0	1
3.00	2	1
2.25	0	7
2.00	1	1
1.30	1	0
.01	8	5
	16	21

ANALYSIS OF GAME 9

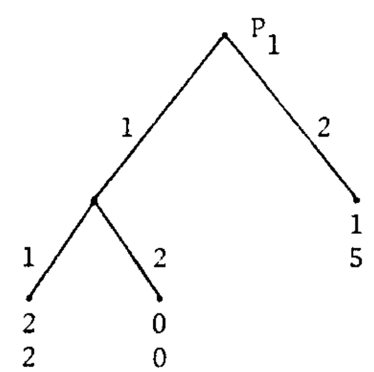
"Rational" Behavior in Games
in Extensive Form

The following three games were played in class after a set of lectures on extensive form. The games and instructions are shown below.

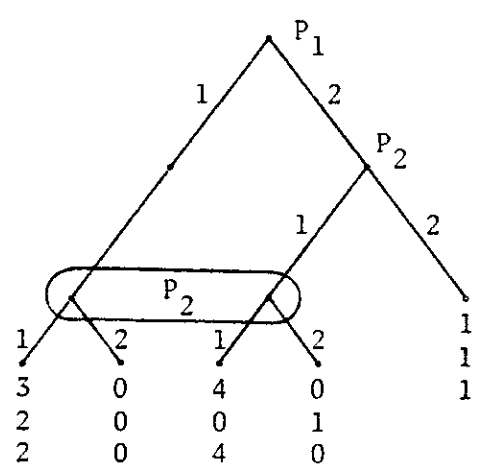
Three games in extensive form are shown below. In each instance you are Player 2 and you are trying to maximize your expected payoff.

In each game you can select move 1 or 2. State which you select and give a reason why.

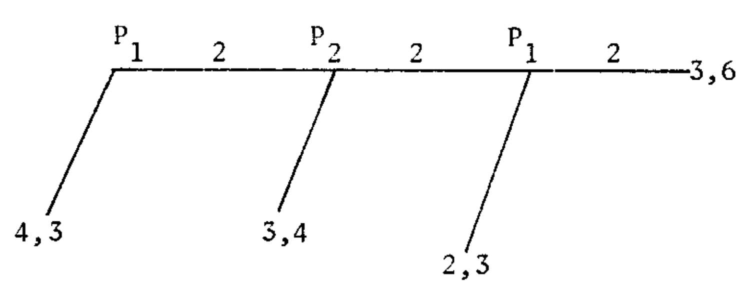
Game 9a



Game 9b



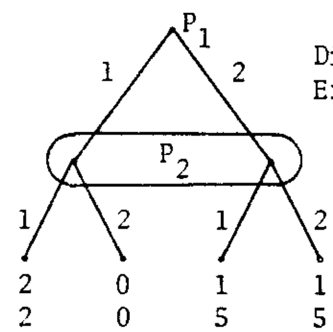
Game 9c



Game 9a

The strategic form of this game is:

		Player 2	
		1	2
Player 1	1	2,2	0,0
	2	1,5	1,5



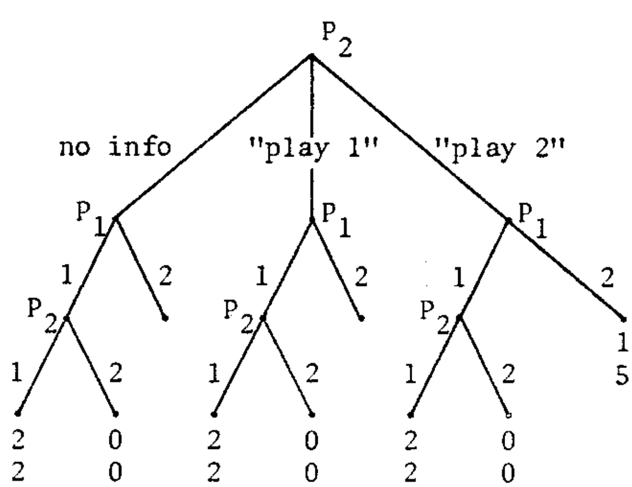
Different Extensive Form

The strategy pairs (1,1) and (2,2) are both (NE) noncooperative equilibrium points. The payoffs are (2,2) and (1,5) respectively. Both NE are perfect (i.e., they are equilibrium in the subgames) in the extensive form shown immediately above, but not in the one you played in.

Out of 21 responses from individuals acting as Player 2, 20 selected 1 and 1 selected 2. The only perfect equilibrium in Game 9a is (1,1) with payoff (2.2).

The one selecting 2 and others observed the problem with having strategy 2 by Player 2 regarded as a plausible threat. Furthermore, how is it to be communicated?

We could model communication formally by considering a game in which Player 2 has 3 choices to start with. He can send no message to Player 1; a message "I am going to play 1" or a message "I am going to play 2." This is shown below.



Game 9b

There are two classes of NEs (with mixed strategies but at an extreme, with no pure strategies).

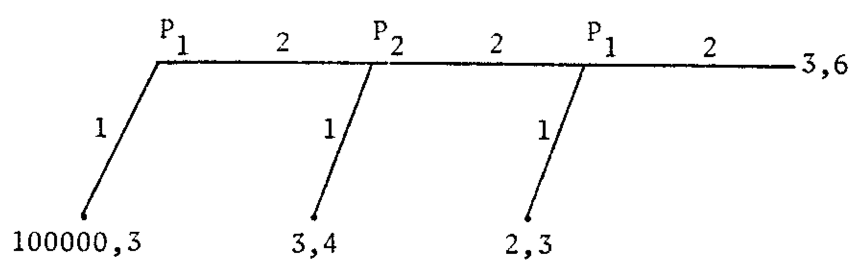
The two pure strategy equilibria are (1,1,2) with payoff of (3,2,2) and (2,2,2) with payoff of (1,1,1). In the second equilibrium the "threat" of Player 2 to play 2 stops him from ever getting the move.

The only "trembling hand perfect equilibrium" is (2,2,2) (see GTSS, Chapter 9, p. 266 for discussion). But we note that its payoffs dominated by the payoff at (1,1,1) thus one cannot make a convincing normative argument for it. (See Selten, 1975.)

Out of 21 responses, 15 individuals acting as Player 2 selected 1 and 6 selected 2.

Game 9c

This game poses a problem in the conceptualization of the assumption of rational behavior. If Player 1 is "rational," he should select 1 and obtain a payoff of 4. If Player 2 is given the move, in this game, he must assume that Player 1 has made an error or is otherwise irrational. Thus, we cannot avoid attaching some form of subjective evaluation of the rationality of one's competitors. In order to emphasize this point, we can compare the game shown below with Game 9c.



Here, if Player 2 is called upon to move, his cognative map of what Player 1 is trying to do must face a severe shock.

Out of 21 individuals selecting as Player 2, 17 chose 2 and 4 chose 1.

Actual Data

Player 2 chooses:	Games 9a	9b	9c
	2	2	2
	1	2	2
	1	1	2
	1	2	2
	1	1	2
	1	1	2
	1	2	2
	1	1	1
	1	2	2
	1	1	2
	1	1	1
	1	1	2
	1	1	1
	1	1	2
	1	1	2
	1	1	2
	1	1	2
	1	1	1
	1	1	2
	1	1	2
	1	1	2
Sample size	21	21	21

Alternative	9a: 1	9a: 2	9b: 1	9b: 2	9c: 1	9c: 2
frequency	20	1	15	6	4	17

Comments

Game 9a

In this game, you don't know whether or not P_1 knows your bid; well, I would say you have to choose 1 in any case to insure reward. No communication--separate sealed bids.

You want P_1 to choose move (2), therefore you should use your intimidation power by choosing (2). It would be nice if you could let P_1 know your intrusion; but if you end up at (0,0) at least you've "educated" P_1 at no price.

I choose option 1, if P_1 gives me the chance, since it is a clear choice between 2 and 0.

I take move 1, because player 1 always has the option to take move 2. I might as well choose what gives me the highest payoff should he choose move 1 (since this does not influence my payoff if he chooses move 2).

P_1 might choose 2 so he'd get 1 rather than 0. But he might want to act or let me get a huge return. I would have every intention of playing 1, so that from that point, we would both gain.

Move 1 maximizes minimum payoff.

Move 1 maximizes payoff.

I select move 1, because I maximize my winnings that way.

P_2 chooses "1" guaranteeing $2 or $5 depending on P_1's play.

Since these games are non-repeating, I just go for maximum return and assume other players will too.

Move 1. It doesn't look like much of a choice.

Move 1. Because if my decision has any effect, i.e., if P_1 chooses #1 then this would give the better payoff.

As P_2 I'd make move 1, because although I could have had $5 had P_2 terminated the game, since he didn't, I'm not going to choose move 2 to spite him 0, because that would be like cutting off my nose to spite my face. I'd want the $2.

Choose 1 because if the game continues until I get to choose, I will be faced with payoffs of 2 for 1 and 0 for 2.

If it were repeated I might pick 2 to force P_2 to pick 2, but with one shot I do better than 1.

Select move 1--I prefer 2 to 0.

I pick move 1 since it gives me best payoff (it's also the best for Player 1).

P_1 will choose 1 also regardless of what P_1 and P_2 is better off if he chooses 1.

I only get control of my own destiny if P_1 chooses move 1. Thus, I need only choose between payoffs of 2 or 0. I choose the higher payoff and, therefore, choose move #1.

P_1 will assume (I hope) that I will maximize to his/her advantage--and I will!

Move 1 to maximize choice between [0,2].

Game 9b

In this case choose option 1 and hope player 1 chooses 1 and does not somehow collude with P_3.

Select move 2. In this way you guarantee the other two players get nothing; and also, assuming greedy opponents (i.e., going for 4) you end a winner.

I believe that P_1 has chosen option 2, so I choose option 2.

I take move 1. Even though I don't know where I am in 2 instances, 0 is the minimum I can get in both 1 and 2, but I open myself to the highest payoff with move 1. In the third instance, I know where I am but have no control over the outcome.

I would suspect that P_3 is more likely to have passed to me than P_1 because his expected payoffs are higher (4,0) rather than (3,0) than P_1's. I would therefore choose 1 in the hope that PP_3 sees that as a maximizing behavior.

Move 1. Maximum potential payoff.

Move 1. This maximizes the payoff for Player 2.

I select move 2 because I believe both 1 and 3 will go for the (4,0,4) payoff. Both 1 and 3 think that if I get a chance to move I will choose 1 to get my best payoff, and thus they will get (4,0,4). I'm not hurt that much if I'm at the other side since its only a difference of 1.

P_2 is only concerned with his payoffs. Assuming P_1 chooses "1" with probability 0.5 and P_3 chooses "1" with probability 0.5, P_2 figures 67% chance that play comes from P_1, 33% that it came from P_3. Expected payoff, choosing strategy 1.

Move 2.

Move 1. I could try to second guess $P_1 + P_3$ but I've found that, that never works.

Move 1. Because it gives me a shot at the bigger payoffs.

I take move 1, because I am assuming that P_1 will try to get as much as possible (either 3 or 4) and will not want to risk passing the ball (so to speak) to P_3 and risk possibly getting 0 or 1.

Choose 1 because I will get either 2 or 0, whereas if 1 chooses 2, I'll get 1 or 0.

Move 1. The choice is not very clear.

Select move 1. I prefer the prospect of 2, 0, to 0, 1. I assume that it is equally likely that P_1 will select 1 as it is that he will select 2.

Move 1. Player 1 will not want to confuse me into picking 2 since we'll get nothing. However, this is not a Styblo equilibrium. If player 1 tells me he picked 2, I'd pick 2. If I told him I'd pick 2, he'd pick 1. My solution isn't very certain. No collusion is possible.

P_1 will choose 1, P_2 will choose 1, P_2 will choose 1, P_1 will choose 1, P_3 gets no choice.

Here I choose move #1. It has a higher expected value. (This may backfire if Players 1 and 3 assume I'm trying to maximize.)

Choose 1. Since the final move is mine, the other players and I must double guess each other. I'll choose the higher payoff, since there is no reason why P_1 should believe I would tend toward one strategy or the other. If it's 50-50 chance, which it seems to be, I'll go for the dough.

Note: Player 1 can do best by selecting 2. Player 2 can do best by selecting 1, but risks getting nothing at that point. Assuming greed on P_1 and P_3's part--choose 2.

<u>Game 9c</u>

In this game, I would choose two as player two; but it's irrelevant because Player 1 will choose 1 right away unless I offer him some of my potential 6 reward.

It would be highly irregular for P_1 not to choose move (1), as my move seems rather trivial. However, if P_1 is stupid enough to choose (2) initially, I'll take move (2) hoping he'll be stupid again.

I choose option 2, since that gives P_1 a choice between receiving 2 or 3 in which case he would choose 3, giving me 6.

It seems highly likely that P_1 will choose move 1 since it insures him the highest payoff. But since he moves first I can't control what move he makes, so this does not influence my choice of moves. If I take move 1, I definitely get 4. Yet if I take move 2, it seems that P_1 would choose the move giving him the highest payoff, which gives me 6. So I'll take move 2 hoping that P_2 won't take the highest payoffs at his first opportunity but will at his second.

P_1 is irrational if he plays 2 unless he expects P_2 to compensate him when P_2 gets 6. Because this must be the case (because otherwise P_1 would not have played 2, 4 is his highest possible payoff). I would play 2 to lead to 3, 6.

It doesn't really matter, but move 2. Pass it back to P_1, and assume that he will maximize. A tad risky, but it is very unlikely that the game will go this far.

If Player 1 chooses option 1, the game is over. But if he chooses option 2, then Player 2, me, would choose option 2 expecting Player 1 to choose option 2 on his next move so as to maximize his remaining payoff chances for being stupid enough to choose move 2 in the first place.

I would choose 2 as Player 2. If I chose 1, I would get a payoff of 4. But, it seems very likely that Player 1 will choose route 2 with payoff (3,6) since this maximizes his payoff given he's at that rate. So most likely with a choice of 2 I will end up with payoff of 6. (Even if I'm not willing to take the risk for a loss of 1.)

P_1 will choose strategy 1 if he's smart. Barring that, P_2 chooses strategy 1 conditional on the fact that P_1 is too dumb to maximize. (If he sent play back to P_1, he'd likely not end up with the "6" payoff because of P_1's stupidity.)

2--but if P_1 has a brain in his head, he would make move 1 and P_2 would never get a chance to play.

Move 1--because if P_1 chose strategy 2 he's doing strange stuff.

Move 2--because P_1 would be most likely to choose #2 following my move thus giving me 6. If I choose #1 I only get 4.

I would assume that P_1 is smart and would terminate the game immediately to get his maximum payoff of 4. If P_1 did not do this, I would make move 1, and terminate the game getting 4 for myself because I would assume that P_1 given a chance to move again would be too stupid to try and get a max of 3 over 2 (and thus giving me 6 over 3), and I'd end up with 3 by his choice rather than 4 by my own.

I choose 2 as Player 2 because if I, my payoff is 4, were as if I choose 2, P_1 will then be faced with a payoff of either 2 or 1 or 3 for 2 so I will choose 2 and I will get 6.

Move 2-- P_1 then has incentive to go for 3, 6 and I get more (unless P_1 is envious). Note: It is not clear why P_1 wouldn't always choose his first move.

Choose move 2--it's in P_1's interest to make a choice which will return 6 (6 > 4) to P_2 .

I chose 2 since I will get at least 3. If Player 1 picks his best, I'll get 6. Unless he's out to get me.

P_2 will choose 1 but it doesn't matter because P_1 will choose 1. If P_1 chooses 2 it means that P_1 "hates" money, thus given another choice P_1 will choose (2,3) rather than (3,6) thus since P_2 can get 4 in (3,4) by choosing 1 if P_1 chooses 2 on his first move, P_2 will choose 1.

I can't see why Player 1 would choose anything other than move 1. However, if given the choice, I'll take move 2.

If P_1 chooses other than move 1, I'll assume this irrational. So if my move is relevant (P_1 chooses move 2), I'll go for the maximum payoff hoping that P_1 is <u>consistently</u> irrational.

Selecting strategy 2 results in an equilibrium at payoff (3.6). Therefore, if you (P_2) get to play (which is unlikely) select strategy 2.

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