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AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut 06520

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GAMES WITH PERCEPTIVE COMMANDERS BUT LESS PERCEPTIVE SUBORDINATES

Martin Shubik

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by

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1. MODELING LIMITED PERCEPTION

The key assumptions in formal extensive form game theory are that the players know all the rules of the game. These include all details of feasible play and payoffs. Furthermore the assumption of perfect recall is usually made in analysis. Figure 1 shows a simple extensive form for a game with only two strategies for each of two sides.

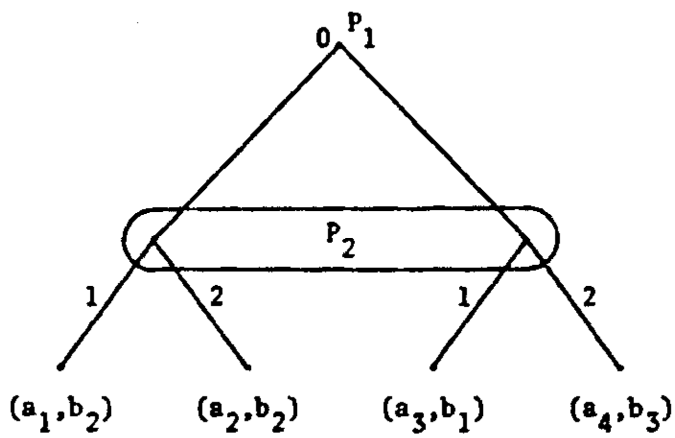

FIGURE 1

*This work relates to Department of the Navy Contract N00014-77-C-0518 issued by the Office of Naval Research under Contract Authority NR 047-006. However, the content does not necessarily reflect the position or the policy of the Department of the Navy or the Government, and no official endorsement should be inferred.

Player 1 moves first from the decision node labeled P_1 and also 0 to indicate that it is the initial node or root of the tree. The moves are branches labeled 1 and 2. We assume that Player 2 moves next. If he is not informed of the move selected by Player 2 he cannot distinguish between the two decision points from which he moves. This lack of knowledge is illustrated by the information (or lack of information) set enclosing the two nodes and labeled with P_2 .

After Player 2 has made his choice both players obtain their payoffs. These are indicated by the pair of numbers at the four terminal nodes of the game tree. The associated strategic form of this game is shown in Table 1.

P_1 \ P_2	1	2
1	a_1,b_2	a_2,b_4
2	a_3,b_1	a_4,b_3

TABLE 1

For simplicity both in exposition and analysis we limit our initial discussion to situations in which the players have preferences described by strict orderings. Thus there are four outcomes to Player 1 which are evaluated as $a_1 > a_2 > a_3 > a_4$. Similarly Player 2 has four outcomes evaluated as $b_1 > b_2 > b_3 > b_4$. Rapaport and Guyer (1966) have calculated that the number of strategically different games of size 2×2 is 78 (accounting for all symmetries).

We now introduce a model of the game played by agents or subordinates of the players. We assume that each agent is not as perceptive as his commander. He can make only two distinctions among the four outcomes whose value can be distinguished by the commanders. We may now describe this model formally. The two players each have 16 moves. A move consists of a vector of the form (x_1, x_2, x_3, x_4) where Player 1 replaces all x_i by either a_1 or a_2 and Player 2 replaces x_i by either b_1 or b_2. Each player has the opportunity to tell his agent how to rank all outcomes using only two values. Thus we can describe the 16 messages of Player 1 as $1 - (a_1, a_1, a_1, a_1)$, $2 - (a_1, a_1, a_1, a_2)$, ..., $16 - (a_2, a_2, a_2, a_2)$ 1 and 16 are the same and involve complete indifference. We may exclude these cases. Thus Figure 2 shows the new game in extensive form

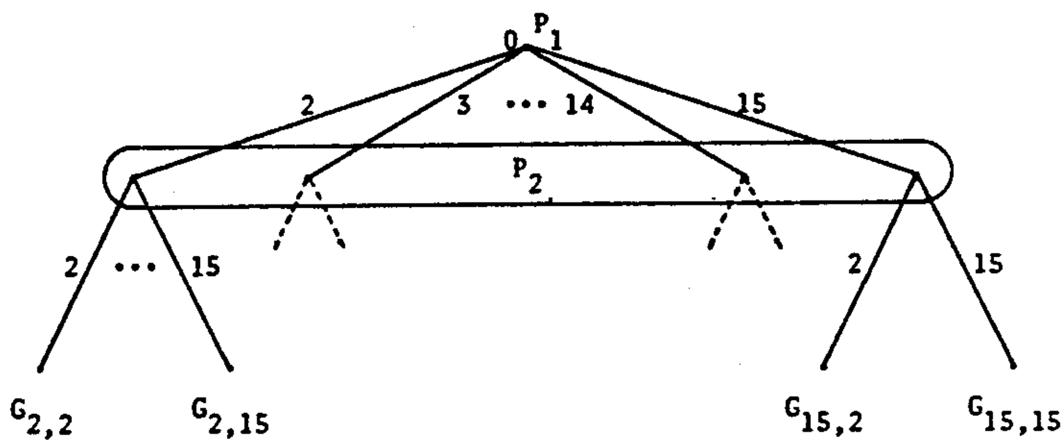

FIGURE 2

The two players simultaneously select one of 14 payoff structures. These lead to 196 subgames which will be played by the two agents. For example the game $G_{2,2}$ is obtained from Table 1 by replacing a_1, a_2, a_3 by a_1 and a_4 by a_2 and similarly b_1, b_2, b_3 by b_1 and b_4 by b_2. This gives the game shown in Table 2.

P_1 \ P_2	1	2
1	a_1,b_1	a_1,b_2
2	a_1,b_1	a_2,b_1

TABLE 2

If each subordinate can only perceive two different payoffs for himself and his competitor then there will be 25 different games in the 196 game structure. Table 3 shows the distinct cases for the payoffs perceived by Agent 1 for Player 1. Similarly there are five cases for Agent 2, giving a total of 25 different 2×2 games with two distinct payoffs each.

a_1	a_2
a_2	a_2

a_1	a_1
a_2	a_2

a_1	a_2
a_1	a_2

a_1	a_2
a_2	a_1

a_1	a_1
a_1	a_2

TABLE 3

Our assumption is that the agents do not have any knowledge or "recall" of the way their commanders see the real problem. They are told what target or outcome values they are to take as given. ("Their's is not to reason why.")

A modeling problem occurs when considering what an agent knows about the values ascribed by the opposing agent. For simplicity in fully defining the first problem we assume that both agents are aware of each other's payoffs but not of their commanders' payoffs. Thus in game $G_{2,2}$ they see the matrix shown in Table 2.

2. SOLUTION THEORY, PERCEPTION AND PRECOMMITMENT

The formal game has been described above, but the solution concept has not been discussed. Central to many discussions of games in extensive and strategic form is the noncooperative equilibrium.

There are many difficulties involving the use of the noncooperative equilibrium as a solution (see Shubik, 1982, Ch. 10, and 1984) there are three sources of difficulty which are noted. They are (1) the indexing problem when equilibria are not unique, (2) mixed strategy equilibria, and (3) the use of perfect equilibria.

As we wish to illustrate some examples where precommitment and acting through unsubtle agents appear to be related, we make the following assumptions. (1) When the last stage subgame does not have a unique equilibrium point there is no natural indexing to carry out the backward induction for a perfect equilibrium without introducing further assumptions based on fact and/or philosophy concerning selection of values. (2) We begin by ruling out mixed strategy equilibria as solutions. (3) We regard the perfect equilibrium solution as reasonable in the sense that the agent's actions are reasonably naturally described as being dependent upon the state in which he finds himself whereas the commander who is setting the environment (the values) for the agent can base his decision on what he expects the agents will do.

3. THE PRICE OF BEING TOO BRIGHT OR SENSITIVE

Suppose that the agent for Player 2 is as perceptive as his commander and shares his assessment of the values of the outcome. All players and agents know this. In terms of the example in Figure 2, Player 2 has lost the strategic option to distort his true payoffs to his agent. This could have an undesirable result. We consider the game shown in Table 4 under the four possibilities (a) the players are both completely perceptive and have cardinal utility measures, (b) the players are completely perceptive but only with ordinal preferences, (c) the players both operate through agents who can only make two distinctions in outcome values to themselves,* (d) Player 2 has a nonperceptive agent, but Player 1's agent is perceptive and all know this.

P_1 \ P_2	1	2
1	4,5	0,2
2	2,0	5,4

a

P_1 \ P_2	1	2
1	b,a	d,c
2	c,d	a,b

b

P_1 \ P_2	1	2
1	a,a	b,a
2	a,b	a,a

$G_{2,2}$

c

P_1 \ P_2	1	2
1	b,a	d,b
2	c,a	a,b

* d

TABLE 4

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*A modeling difficulty appears here which is related to both perception and indoctrination. If the agent shares his commander's perception of value, but has a cruder scale we may restrict the ability of the commander to influence his agent's perceived preference so that the player's strong order is consistent with the agent's weak order thus $a_1 > a_2 > a_3 > a_4$ can be replaced by (a_1, a_1, a_1, a_2) or (a_1, a_2, a_2, a_2) but not (a_2, a_2, a_1, a_1). More important however is, does the commander influence his agent's perception of the opponent's value. How are young Russian or U.S. officers indoctrinated to view how their opposite numbers evaluate outcomes. Do they share the same perceptions as their commander's or are their perceptions obtained from their commanders?

Concerning version (a) there are three equilibrium points. Two pure strategy equilibria at (1,1) and (2,2) and one mixed strategy equilibrium with weightings of (4/7, 3/7) by Player 1 and (5/7, 2/7) by Player 2 yielding 20/7 to each. If we limit the payoffs to ordinal preferences there are two pure strategy equilibria yielding (a,b) or (b,a) .

Table 4c results from a pair of strategies by the players which yield an equilibrium at (2,2) of (a,b) ; but the equilibrium in this game is not unique.

Table 4d yields the best equilibrium for Player 2 (b,a) at (1,1). The lack of perception of 2's agents in contrast with the complete perception of 1's agent is used by 2 to his advantage. An analogy to this situation is noted by Kissinger (1958, p. 97) where he notes "The continued Soviet desire to retain freedom of action by not frightening its own people...." This is tantamount to utilizing the difference between the commander's perceptions and his agent's as a way of enlarging the commander's strategy set.

4. BAYESIAN GAMES, MISPERCEIVED PAYOFFS AND SUBJECTIVE PROBABILITIES

An argument against the analysis above is that the highly intelligent players on each side will not believe that the agents lack perception, are precommited or are anything else but intelligent operators. Any manifestations of indoctrination, limited perception, pathological acceptance of danger are assumed to be possibly false signals sent with intent to deceive. This argument is more empirical than logical. Its validity depends upon our knowledge of the limitations on human perceptions, training, intellectual ability and other decisionmaking constraints. There are many self-interested actions which cannot be taught to a two year old baby, a dog, a cat, an individual with IQ of 80; or a highly intelligent individual with no head for mathematics and a distain for accounts. The opponent looking at the agents of the other side and his own agents faces both an empirical and game theoretical logical problem. He has to have a measure of the uncertainty he attaches to the true nature of the opposing agents and their communication system. If it is true that the opposing command and control system is error prone and inefficient and many of the agents are insensitive with poor perception then the inferences to be drawn will be considerably different from those where the assumption is that what is faced is a group of skilled liars operating in an efficient command and control system.

Harsanyi (1967, 1968a, 1968b) has suggested a formal way of dealing with games with lack of knowledge about the full rules of the game. This involves attaching initial subjective probabilities as to who the real players might be. Formally the game described above can be modified by adding subjective probabilities for each side concerning their assessments as to whether the agents are in fact with limited perception or are hiding their true capabilities (what are the odds that the Doomsday machine really does not work?).

It is suggested here that there are three basic problems with the Harsanyi approach: (1) where do the initial subjective probabilities come from? (2) What is the solution concept to be used and what is its justification? (there are many difficulties with the noncooperative equilibrium in general, the perfect equilibrium in particular--see Shubik (1984)--and the new Harsanyi-Selten (see Harsanyi, 1979) solution concept is open to question); (3) How do we model agents with limited capabilities?

The concept of rational player and agents used in much of game theory is usually defended as a sensible set of assumptions from which to develop a normative theory of behavior. It is sometimes argued that it provides us on occasion with a good enough first order approximation for a behavioral theory.

It is suggested here that once we acknowledge the evidence that the individual does not have perfect perception, that most of us commit elementary errors frequently, such as forgetting; that calculation and planning take time and that in essence every individual has a finite capacity for storing and processing information then the very definition of individual rational behavior is challenged. Leaving aside the model of the individual many of the more interesting and vital problems involving competitive analysis deal with organizations. Even if the agents individually conformed to the model of rational man the command, control and communication system could still generate lack of perception, error and limits to calculation.

5. CONTEXT, CULTURE AND STRATEGIC ANALYSIS

The perceptive work of Kissinger (1958), Wohlsletter (1975), Kahn (1960) and many others was devoted to helping to provide advice and guidance for operational problems set in the context of differing cultures, political systems and technological reality. In all instances the argument was strategic but heavily context constrained. In contrast much of the work in formal game theory has been more or less free from context. In between these two considerably different approaches there has been some work which has posed problems with the difficulties of theory without context and context without sufficient theory. In particular various paradoxes have been raised by Ellsberg (1961) concerning risk and uncertainty; Schelling (1960, 1978) on precommitment, threat communication and mass behavior;

Shubik (1954, 1971) on "truels" or three way fights and escalation and Raiffa (1972) on bargaining.

The work of Schelling (1960) was possibly the earliest to raise serious questions and to point to deep difficulties in applying formal game theory to international relations. But although he created simple paradoxical examples he neither provided a technical critique of the methodology of game theory nor constructed an alternative. Upon first reading the Strategy of Conflict I underestimated the importance of its criticisms because I concentrated upon the methodological weaknesses of the book and its lack of formal apparatus. However upon reconsidering this book the germ of the critical message was there. The formal apparatus of game theory with the implicit assumption of fully rational actors and total knowledge of all of the immediate rules of the game does not adequately model the problems of diplomatic strategy where context, culture and decision pressures play a role.

Although the critique was made well over twenty years ago, little has been done at a formal level to develop a theory of games for fallable organizations.

The importance of context in the analysis of strategic models cannot be sufficiently stressed. Shubik (1954) posed the problem of three players standing on the corners of an equilateral triangle each with a revolver, each with a known rated shooting accuracy of say .9, .8, .7. The goal of each is survival; who has the best chance to survive? This problem can be analyzed as a noncooperative game and paradoxically one can conclude that "the meek shall inherit the earth" (the noncooperative game is sensitive to the parameters). Yet if a different context is provided it is by no means clear that the model remains uninfluenced (read for example for players A, B and C the British Raj, Imperial Russia and Afghanistan).

The psychologists (see Tversky and Kahneman, 1981, Einhorn and Hogarth, 1981, Janis, 1972 and many others) have queried the decision theory and game theory views of individual treatment of probability; the ways forecasts are prepared and decisions are made and the influence of group pressures.

Newer generations of strategic analysts (see for example Bracken, 1982, Blair, 1985, Lewis, 1980, Steinbrunner, 1984) whose formal knowledge of game theory is higher than their predecessors, are writing. But the gap pointed to by Schelling, Ellsberg and others still exists.

In Sections 1-3 a way was suggested for formally modeling limited perception of agents, as a first step towards a fallabalist's theory of games. There are several other limited agents who merit considering. In particular Selten (1975) introduced the concept of the trembling hand. He used it more or less as a mathematical device to consider the stability of different equilibrium points. The idea behind the trembling hand is simple. Suppose that an agent is faced with selecting one out of ten alternatives. Suppose that he selects number two, we assume that there is some finite probability that his hand trembles and he pushes the wrong button. This device appears to me to be an easy formal way to account for error into our calculations.

The immediate tasks of formal game theorists, social psychologists, political and military analysts are to begin to close the gap between the context free theories of games and risk behavior and the context specific problems of strategy, diplomacy, threat and war. This involves the construction of games where cultural bias, limited perception of agents, errors in systems and overloads play an identifiable operational role.

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