COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

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COWLES FOUNDATION DISCUSSION PAPER NO. 732

TESTING THE RANDOM WALK HYPOTHESIS:
POWER VERSUS FREQUENCY OF OBSERVATION

by

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December, 1984

November 1984

Testing the Random Walk Hypothesis: Power versus Frequency of Observation

by Robert J. Shiller and Pierre Perron[1]

Power functions of tests of the random walk hypothesis versus stationary first order autoregressive alternatives are tabulated for samples of fixed span but various frequencies of observation. For a t test and normalized β test, power is found to depend, for a substantial range of parameter values, more on the span of the data in time than on the number of observations. For a runs test, power rapidly declines as the number of observations is increased beyond a certain point.

How does power of tests of the random walk hypothesis depend on the frequency of observation? Those who test the random walk model for prices of speculative assets often use very many observations, as data may be readily available on monthly, weekly or even daily basis. It is often casually asserted that, with so many observations, power of tests ought to be very high.

Power functions for an important class of alternatives will

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1. Yale University. This research was supported by the U. S. National Science Foundation (SES-8408565), the Sloan Foundation, and the Universite de Montreal. The authors are indebted to P. C. B. Phillips, Stephen A. Ross and N. E. Savin for helpful discussions.

be tabulated here where the span or range of the data (measured in years, say, whether observations are annual, monthly or daily) is held fixed and the number of observations is varied by changing the frequency of observation. We shall see that over a substantial range of parameter values it is more useful to think of power of a t-test or normalized beta test as depending on the span of the data rather than the number of observations, and that the power of a runs test will be destroyed by too many observations.[2]

Null and Alternative Hypotheses

The null hypothesis and alternative hypothesis to be considered are given by the stochastic differential equations:

(1) H_0: $dp_t = \sigma_0 dw_t \qquad t > 0$

(2) H_1: $dp_t = -\tau(p_t - \mu)dt + \sigma_1 dw_t \qquad -\infty < t < \infty, \quad \tau > 0$

where w_t is a unit Wiener process. The null hypothesis has two nuisance parameters: the standard deviation σ_0 and the starting value p_0. The alternative hypothesis, taking p_0 random, has the continuous time autoregressive parameter τ, and two nuisance parameters: the standard deviation σ_1 and the mean of the

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2. In the finance context, our results augment those of Merton [1980] that frequent sampling enables accurate estimation of the risk of a security but not of its expected return.

stochastic process μ.

The null hypothesis is that p_t is a random walk without a constant term, i. e., a random walk without drift. This null hypothesis has been frequently asserted in the finance literature with regard to prices of speculative assets. The hypothesis means that price p_t can never be described as "too high" (i. e., that it can be expected to fall in the future) or "too low" (i. e., that it can be expected to rise in the future). Many other, more complicated, variations on the null hypothesis are of course, also in the finance literature, and it is a matter for further research to see to what extent the simple results obtained here extend to these other hypotheses.

The alternative hypothesis here makes p_t what is called an "Ohrnstein-Uhlenbeck" process. In the finance context this alternative is the simplest alternative to the random walk hypothesis for securities prices: price p_t has a mean μ and tends to revert back to it in such a way that the expected change is linear in the difference of p_t from its mean. In Shiller [1981] this alternative model was referred to as a "fads" model. In this model, prices move because of repeated investor fads (represented by the innovation in equation 1) but these fads are gradually forgotten. Forgetting takes place with an exponential decay pattern as commonly modelled by mathematical psychologists (See for example Bartholomew [1967]). Thus τ might be thought of as a parameter reflecting human memory retention, as estimated by psychologists and marketing researchers. If the information set consists of current and lagged p_t then the alternative (as well as null) hypothesis shows the "martingale-like behavior" that Sims [1984] argued, from arbitrage considerations, ought to obtain if there is steady information flow.

The alternative hypothesis H_1 is equivalent for $t \geq 0$ (all our observations will have $t \geq 0$) to specifying that equation 2 holds for $t > 0$ and that p_0 is normally distributed with mean μ and variance $\sigma_1^2/(2\tau)$. Thus, the hypotheses in effect assume a fixed startup under the null and a random startup under the alternative. We could not have assumed a random startup under both the null and the alternatve since a random startup for H_0 (for which in effect $\tau = 0$) would imply that the variance of p_0 would be infinite. If we had used a fixed startup for both the null and the alternative then under H_1 it will be seen that the power function would depend on the difference between p_0 and μ (something we would rather not assume knowledge of in the security price application and elsewhere).

If sampled at discrete points of time at intervals of length h the processes are:

(3) H_0: $\quad p_{ht} = p_{h(t-1)} + u^{(h)}_{ht} \qquad t = 1,2, \ldots$

(4) H_1: $\quad p_{ht} = \mu + \beta_h(p_{h(t-1)}-\mu) + v^{(h)}_{ht}$

$\qquad t = \ldots -2, -1, 0, 1, 2, \ldots$

where $u^{(h)}_{ht}$ is normally distributed with mean zero and variance $h\sigma_0^2$ while $v^{(h)}_{ht}$ is normally distributed with mean zero and variance $\sigma_1^2(1-\exp(-2\tau h))/(2\tau)$. The discrete time autoregressive parameter β_h is equal to $\exp(-\tau h)$. Regardless of the sampling interval h, the alternative hypothesis is that the process p_{ht} is first-order autoregressive around the mean μ.

Test Statistics

We are given 1+S/h observations p_0, p_h, p_{2h}, . . . , p_S where h is the sampling inverval and S is the span of the data.

The t test whose power we shall investigate employs the conventional t statistic to test whether the slope coefficient equals one in a regression of p_{ht} on a constant term and $p_{h(t-1)}$ with T = S/h observations:

$$(5) \quad t = \left[\sum_{t=1}^{T} (p_{h(t-1)} - \bar{p}_{-h})^2\right]^{.5} (\hat{\beta} - 1)/s$$

where

$$(6) \quad \hat{\beta} = \sum_{t=1}^{T} (p_{h(t-1)} - \bar{p}_{-h}) p_{ht}$$

$$\left[\sum_{t=1}^{T} (p_{h(t-1)} - \bar{p}_{-h})^2\right]^{-1}$$

(7) $\hat{\mu} = \bar{p}_{0} - \hat{\beta}\,\bar{p}_{-h}$

(8) $\bar{p}_{-j} = T^{-1} \sum_{t=1}^{T} p_{ht-j}, \quad j = 0, h$

(9) $s^2 = (T-2)^{-1} \sum_{t=1}^{T} (p_{ht} - \hat{\mu} - \hat{\beta}\, p_{h(t-1)})^2.$

This t statistic is not distributed as student's t under the null hypothesis, is not asymptotically normal and is indeed very badly approximated by the normal (see Nankervis and Savin [1983]). Thus, the critical value for the t statistic in this test will be based on the empirical distribution of the statistic under the null.

The normalized beta test statistic B as used in Evans and Savin [1981, 1984] is proportional to the $\hat{\beta}$ from (6) above:

(10) $B = (T(T-1)/2)^{.5}(\hat{\beta} - 1)$

The proportionality factor $(T(T-1)/2)^{.5}$ is, under the null hypothesis, the square root of the reciprocal of the diagonal element corresponding to $\beta = e^{-\tau h}$ of the inverse of the information matrix (White [1958]).

The conventional runs test statistic has been used by many researchers to test the random walk hypothesis for speculative asset prices. A run is defined as a sequence of price changes $p_{ht} - p_{h(t-1)}$ all of the same sign. The number of runs r in the sequence of T observed price changes is thus the number of times a price change is followed by a price change of the opposite sign plus one. Under the assumption that the total number n_1 of positive price changes and n_2 of negative price changes ($n_1 + n_2 = T$) are given and that all permutatations of these price changes occur with equal probability then the mean m_r and standard deviation σ_r of the number r of runs are:

$$(11)\quad m_r = 2n_1n_2/T + 1$$

$$(12)\quad \sigma_r = \sqrt{2n_1n_2(2n_1n_2-T)/(T^2(T-1))}$$

The test statistic as used for example by Fama [1965] to test the random walk hypothesis for stock prices is given by: [3]

$$(13)\quad K = (r + .5 - m_r)/\sigma_r$$

It can be shown using a theorem in Mood [1940] that the ratio K is (under H_0) asymptotically (where n_1/T is kept constant as T is increased) normal with zero mean and unit variance.

The effect of changing nuisance parameters, under both H_0 and H_1, is that of adding a constant and/or mutiplying by a constant for all observations. Since all of our tests are independent of both scale and location, all tests are similar tests. A similar test is defined as a test for which the probability of rejecting does not depend on nuisance parameters. Evans and Savin [1984], who nested the hypotheses by assuming a fixed startup under both H_0 and H_1, had the problem that standard

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3. Fama [1965] used a formula for K which allows separately for +, − and 0 price changes and which reduces to this formula if there are no zero price changes. See Wallis and Roberts [1956].

tests are nonsimilar. Dickey and Fuller [1981], however, showed that the t-test for the coefficient of lagged p in a regression of p on a constant, lagged p and t is a similar test . Still, the test is not similar over alternatives of the form $p_t = -\beta(p_t - \mu) + bt + v_t$, in that the power depends on b.

Method

Power functions for a size .05 test were tabulated with 40,000 replications for number of observations T = 8 to 512 and with 10,000 replications for T = 1024.

All simulations were carried out on a CDC Cyber 915 at the Universite de Montreal. The N(0,1) random deviates $u^{(h)}{}_t$ in (3) and (4) were obtained from the subroutine GGNML of the International Mathematical and Statistical Library (IMSL) package, version 9.1.

In the first step the empirical distributions under the null hypothesis of the t-statistic and of the normalized β statistic were computed with 20,000 replications. The series p_{ht} were generated from (3) for t = 1,...,T starting from the initial condition $p_0 = 0$. The significance points were taken from the sorted arrays. Critical values for the t-test and normalized β test were taken as the 2.5 and 97.5 percentage points of the empirical distributions. The numbers were carried to the second step with 5 digit accuracy. Critical values for the runs test were not taken from an empirical distribution but under the assumption that the statistic K was normally distributed.

In the second step the empirical power functions of the tests using the significance points from step one were computed with 20,000 replications (except for T = 1024 which used 10,000). Under the alternative the series p_{ht} were generated from (4) for $t = 1,\ldots,T$ starting from the initial conditon $p_0 = u_0/(1-\exp(-2\tau h))^{.5}$.

This procedure was repeated twice (except for T = 1024 and for the one-tailed tests in the appendix) and the tables show the averages of the two power computations.

Results

In the tables power is shown only for a single value of τ ($\tau = .2$) but powers of tests for $\tau = .2*2^j$ can be found by reading j rows down from the power for $\tau = .2$. In each table, we can read how power depends on the number of observations for a fixed span (reading across rows), how power depends on the span for a fixed number of observations (reading down columns) or how power depends on the number of observations when, as is usually assumed, span is proportional to observations (reading along diagonals from upper left to lower right). Some of the numbers in the tables correspond approximately with power computations in the literature (Dickey [1976], Dickey and Fuller [1979], Evans and Savin [1984], Fuller [1976] and Nankervis and Savin [1984]) and in these cases our results were roughly confirmed.[4]

Table 1 shows a tabulation of the power of a size .05 two-tailed t-test for various data spans and numbers of observations. When the span of the data is very short, (or equivalently, τ is very small) the t test has no power regardless of the number of observations, as indicated by the first three rows of Table 1. In this region the test is in fact slightly biassed (the power is less than the size of the test of .05). Such a bias in the case of a fixed startup at the mean alternative was noted before by Nankervis and Savin [1983].

In the rows corresponding to the spans of 32 or 64 power rises substantially between 8 and 16 observations and slightly between 16 and 32 observations but shows only slight gains when the number of observations is increased further, even to 1024. Thus, for example, if we have 32 years of annual data where τ = .2 the power of the test is .079. If we switched to bi-weekly data, increasing the number of observations by a factor of 26, the power of the test rises only to (interpolating the numbers in the table) about .087.

Table 2. shows a corresponding tabulation for the two-tailed normalized beta test. As with the t-test, increasing the number of observations while holding span fixed may produce

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4. See Appendix. Earlier computations of power functions, however, did not assume a fixed startup under the null and random startup under the alternative and are thus not strictly comparable.

negligible increases in power. That is, for this test there is negligible power for spans of 8 or 16; the power function for S = 32, 64 or 128 is fairly flat for T > 32.

Table 3 shows a corresponding tabulation for the two-tailed runs test, using the test statistic K with critical value at the .05 level equal to ±1.96. This test has negligible power for spans of 8 or 16 regardless of the number of observations. For spans in the range of 8 to 256, the power is greatest at 32 observations, and falls off practically to the size of the test as the number of observations is increased to 1024.

Tables showing power for one-tailed tests are in the appendix.

Conclusion

We have seen that it is wrong to presume that power of tests of the random walk hypothesis must be high just because there are very many observations. If we increase the number of observations by increasing the frequency of observation, that is, if we move to the right along rows in the tables, power may increase only slightly or even decrease. In all the tables shown here, power does not approach one in the limit as we either move to the right along rows or move down along columns, but approaches one as a limit only if we move diagonally down from upper left to lower right, i. e., increasing both span and number of observations.[5] Some of the relevant asymptotic distributions are developed in Perron [1984].

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5. For some values of S or T in the tables the power is given as 1.000. This reflects the fact that the power is very close to one for these values, but not that the power has the limit of one along either rows or columns.

TABLE 1

Power of a Two-tailed t-Test

Number of observations: $T = S/h$

S: span of the data	8	16	32	64	128	256	512	1024
8	.040	.038	.037	.039	.039	.041	.037	.039
16	.042	.042	.043	.044	.045	.045	.044	.043
32	.059	.071	.079	.085	.088	.089	.088	.086
64	.111	.191	.246	.284	.296	.304	.307	.307
128	.175	.470	.701	.824	.872	.892	.901	.907
256	.195	.681	.973	.999	1.000	1.000	1.000	1.000
512	.196	.726	.997	1.00	1.00	1.00	1.00	1.00
1024	.196	.728	.998	1.00	1.00	1.00	1.00	1.00
Lower bound	-3.893	-3.421	-3.271	-3.181	-3.155	-3.137	-3.135	-3.135
Higher bound	0.562	0.379	0.312	0.275	0.240	0.217	0.225	0.246

Note: Table gives power of two-tailed t-test at .05 level against alternative with $\gamma = .2$. Thus, for example, if $\gamma = .2$ where time is measured in years (so that H_1 would predict a rate of decline for p of $.2(p_t - \mu)$ per year) and we have 128 quarterly observations covering a span of 32 years, the probability of rejecting is .088. If $\gamma = .05$ where time is measured in years and we have 128 quarterly observations covering a span on 32 years, then the probability of rejecting is given by .039. The final rows give the critical values of the t-statistic, equation 5.

TABLE 2

Power of a Two-tailed Test
with the Normalized Estimator of Beta

Number of observations: $T = S/h$

S: Span of the data	8	16	32	64	128	256	512	1024
8	.050	.047	.044	.045	.045	.046	.046	.046
16	.058	.059	.057	.058	.060	.060	.061	.058
32	.094	.116	.123	.126	.128	.130	.133	.132
64	.185	.309	.371	.411	.427	.434	.445	.439
128	.283	.644	.843	.925	.953	.964	.969	.971
256	.312	.828	.993	1.00	1.00	1.00	1.00	1.00
512	.313	.860	1.00	1.00	1.00	1.00	1.00	1.00
1024	.313	.862	1.00	1.00	1.00	1.00	1.00	1.00
Lower bound	-6.837	-8.717	-10.174	-10.988	-11.472	-11.693	-11.784	-11.897
Higher bound	0.775	0.486	0.384	0.350	0.297	0.276	0.283	0.315

Note: Table gives power of a two-tailed normalized beta test of the null hypothesis against alternative with $\gamma = .2$. The final rows give the critical values for the statistic B , equation 10.

TABLE 3

Power of a Two-tailed Runs Test

Number of observations: $T = S/h$

S: Span of the data	8	16	32	64	128	256	512	1024
8	.048	.045	.062	.050	.050	.048	.050	.054
16	.056	.044	.065	.050	.049	.049	.050	.055
32	.076	.053	.074	.055	.052	.048	.051	.056
64	.110	.080	.105	.068	.061	.052	.053	.055
128	.141	.139	.189	.123	.094	.072	.063	.059
256	.150	.200	.358	.280	.214	.141	.104	.082
512	.151	.217	.504	.566	.532	.385	.256	.170
1024	.151	.218	.542	.756	.877	.833	.682	.464
Lower bound	-1.96	-1.96	-1.96	-1.96	-1.96	-1.96	-1.96	-1.96
Higher bound	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96

Note: Table gives power of a two-tailed runs test at the .05 level against alternative with $\gamma = .2$. The final rows give the critical values for the runs test statistic K , equation 13.

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Appendix

Not for Publication

I. Power Tables for one-Tailed Tests

Table A-1. Power of a One-Tailed T-Test

Table A-2 Power of a One-Tailed Test with the Normalized Estimator of β

Table A-3 Power of a One-Tailed Runs Test

II. Size of the Tests

Table A-4

III. Comparison with Existing Results in the Literature

Table A-5 Percentage Points of B

Table A-6 Power of Two-Tailed Test Using B

Table A-7 Percentage Points of the T-Statistic (5) under H_0

Table A-8 Power of a Two-Tailed Test Using the T-Statistic (5)

TABLE A-1

Power of a One-tailed t-Test

S: Span of the data	Number of observations: T = s/h 8	16	32	64	128	256	512	1024
8	.057	.059	.059	.062	.063	.061	.064	.058
16	.072	.077	.080	.082	.087	.083	.085	.084
32	.112	.137	.150	.157	.164	.161	.166	.161
64	.201	.327	.398	.446	.468	.464	.482	.468
128	.293	.643	.847	.928	.956	.964	.971	.972
256	.322	.819	.993	1.00	1.00	1.00	1.00	1.00
512	.323	.852	.999	1.00	1.00	1.00	1.00	1.00
1024	.323	.853	1.00	1.00	1.00	1.00	1.00	1.00
Bound	-3.354	-3.050	-2.954	-2.902	-2.876	-2.881	-2.864	-2.879

Note: Table gives power of a one-tailed t-test at .05 level against alternative with $\gamma = .2$. The t-test rejected the null hypothesis if t given by equation 5 was less than the bound shown in the final row.

TABLE A-2

Power of a One-tailed Test
with the Normalized Estimator of Beta

Number of observations: T = s/h

S: Span of the data	8	16	32	64	128	256	512	1024
8	.072	.072	.069	.070	.074	.073	.073	.074
16	.099	.106	.104	.104	.111	.109	.110	.111
32	.173	.208	.216	.221	.232	.229	.233	.233
64	.314	.472	.543	.593	.618	.621	.630	.621
128	.445	.795	.935	.978	.989	.992	.994	.994
256	.479	.920	.999	1.00	1.00	1.00	1.00	1.00
512	.481	.939	1.00	1.00	1.00	1.00	1.00	1.00
1024	.481	.939	1.00	1.00	1.00	1.00	1.00	1.00
Bound	-6.063	-7.543	-8.699	-9.295	-9.558	-9.804	-9.849	-9.924

Note: Table gives power of a one-tailed normalized beta test of the null hypothesis against the alternative with $\gamma = .2$. The test rejected if B given by equation 10 was less than the bound shown in the final row.

TABLE A-3

Power of a One-tailed Runs Test

S: Span of the data	Number of observations: T = s/h: 8	16	32	64	128	256	512	1024
8	.099	.107	.082	.069	.064	.054	.054	.056
16	.105	.120	.089	.075	.065	.056	.057	.060
32	.131	.153	.114	.090	.074	.062	.061	.063
64	.178	.216	.168	.127	.101	.081	.070	.067
128	.223	.326	.283	.223	.164	.123	.094	.086
256	.236	.422	.481	.436	.336	.234	.166	.133
512	.237	.448	.630	.727	.678	.528	.365	.259
1024	.237	.449	.666	.876	.940	.911	.787	.593
Bound	1.645	1.645	1.645	1.645	1.645	1.645	1.645	1.645

Note: Table gives power of a one-tailed runs test at the .05 level against the alternative with $\gamma = .2$. The runs test rejected the null hypothesis if K given by equation 13 was greater than the bound shown in the final row.

TABLE A-4

Size of the Tests

Type of test \ T	8	16	32	64	128	256	512
	Number of observations						
Two-tailed T-test	.051	.048	.048	.051	.051	.052	.050
One-tailed T-test	.048	.048	.050	.049	.051	.048	.048
Two-tailed test with $\hat{\beta}$ normalized	.050	.050	.049	.050	.051	.052	.050
One-tailed test with $\hat{\beta}$ normalized	.048	.049	.048	.048	.052	.049	.048
Two-tailed runs test	.046	.045	.060	.051	.049	.049	.049
One-tailed runs test	.117	.099	.074	.067	.061	.055	.054

Note: Estimates are obtained from 40,000 replications.

III. Comparisons with Existing Results in the Literature

It is difficult to compare our results with those available in the literature for two reasons: 1. our sample size and assumptions about τ do not correspond exactly to those for which others have tabulated power and 2. our hypotheses specify that the starting value of the process is fixed under the null and random under the alternative. We proceed by comparing some of their estimates for similar T and β and by assuming that powers under their fixed startup at μ alternative ought to approximate ours.[6] In all tables, underlined figures are suggested comparisons with ours.

The studies investigating the distribution of B, the normalized estimator of β, include Fuller [1976], Dickey [1976], and Evans and Savin [1984]; the latter by exact numerical computation, the other two by Monte Carlo simulations. Table A-5 compares the results obtained for the bounds used in this paper. By inspection, our results appear consistent with these. Table A-6 contains comparisons of the powers of a test of unit roots against various alternatives obtained by Dickey and Fuller [1979]

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6. Evans and Savin [1984] discuss how the distribution of B is affected by the fixed startup at the mean assumption versus our assumption of a random startup. In the case they present (T = 25, $e^{-T\eta}$ = .9) "the general impression is that the shapes of the two distributions are similar."

and Evans and Savin [1984].

For tests involving the t-statistic previous studies are Fuller [1976] and Nankervis and Savin [1984] for the distibution under the null and Dickey and Fuller [1979] for the two-tailed test constructed by taking equal areas from both tails, again with a fixed startup under the alternative. All power computations for the t-statistic in the literature use Monte-Carlo methods. Tables A-7 and A-8 present the comparisons. Again, the results seem roughly consistent.

TABLE A-5

Percentage Points of B (Equation 10) under H_0

Fuller (1976) (figures adjusted for different normalization factor)

T \ %	2.5	5.0	97.5
24	-10.11	-8.65	0.45
49	-10.99	-9.31	0.37
99	-11.47	-9.64	0.33
249	-11.71	-9.88	0.30
499	-11.87	-9.89	0.30

Evans and Savin (1984

T \ %	2.5	5.0	97.5
25	-9.772	-8.356	0.42
50	-10.781	-9.106	0.357
100	-11.347	-9.521	0.327

Dickey (1976) (as reported by Evans and Savin (1984))

T \ %	2.5	5.0	97.5
24	-9.68	-8.29	0.43
49	-10.81	-9.12	0.36
99	-11.39	-9.54	0.33

Shiller and Perron (1984)

T \ %	2.5	5.0	97.5
16	-8.717	-7.543	0.486
32	-10.174	-8.699	0.384
64	-10.988	-9.295	0.350
128	-11.472	-9.558	0.297
256	-11.693	-9.804	0.276
512	-11.784	-9.849	0.283

TABLE A-6

Power of a Two-tailed Test of H_0 Using Statistic B (Equation 10)

Dickey-Fuller (1979) [Note: fixed start-up at μ under alternative]

alternative β / T	.80	.90	.95	.99
49	.28	.10	.06	.05
99	.86	.30	.10	.05
249	1.00	.96	.43	.06

Evans-Savin (1984) [Note: fixed start-up at μ under alternative]

alternative β / T	.80	.90	.95	.99
25	.066	.033	.025	.024
50	.192	.056	.029	.021
100	.714	.173	.052	.020

Shiller-Perron (1984)

alternative β / T	.819 ($e^{-.2}$)	.905 ($e^{-.1}$)	.951 ($e^{-.05}$)	.988 ($e^{-.0125}$)
16	.059	.047	--	--
32	.123	.057	.044	--
64	.411	.126	.057	--
128	.925	.411	.128	.045
256	1.00	.925	.434	.06

TABLE A-7

Percentage Points of the t-Statistic (5) under H_0

Fuller (1976)

T \ %	2.5	5.0	97.5
24	-3.33	-3.00	0.34
49	-3.22	-2.93	0.29
99	-3.17	-2.89	0.26
249	-3.14	-2.88	0.24
499	-3.13	-2.87	0.24

Nankervis-Savin (1984)

T \ %	2.5	5.0	97.5
25	-3.3	-3.0	0.3
50	-3.2	-2.9	0.3
100	-3.1	-2.9	0.3
250	-3.1	-2.9	0.3

Shiller-Perron (1984)

T \ %	2.5	5.0	97.5
16	-3.42	-3.05	0.38
32	-3.27	-2.95	0.31
64	-3.18	-2.90	0.28
128	-3.16	-2.88	0.24
256	-3.14	-2.89	0.22
512	-3.14	-2.86	0.23

TABLE A-8

Power of a Two-sided Test of H_0 with the t-Statistic (5)

Dickey-Fuller (1979) [Note: used fixed start-up at μ under the alternative]

T \ alternative β	.80	.90	.95	.99
49	.18	.06	.04	.04
99	.73	.18	.06	.04
249	1.00	.89	.28	.04

Shiller-Perron (1984)

T \ alternative β	.819 $(e^{-.2})$	.905 $(e^{-.1})$	.951 $(e^{-.05})$	.988 $(e^{-.0125})$
16	.042	.038	--	--
32	.079	.043	.037	--
64	.284	.085	.044	--
128	.872	.296	.088	.041
256	1.00	.892	.304	.045