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A NOTE ON ENOUGH MONEY IN A STRATEGIC MARKET GAME WITH COMPLETE OR FEWER MARKETS

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ENOUGH MONEY IN A STRATEGIC MARKET GAME
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by

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1. MONEY, MARKETS AND TRADE

In several previous papers (Shubik, 1973, Shapley and Shubik, 1977, Dubey and Shubik, 1978a, b, Dubey, Mascolell and Shubik, 1980) the concept of a strategic market game has been developed.

A strategic market game portraying an exchange economy is a game of strategy where market prices are formed by the strategic behavior of the players. Given n players trading in m goods there are many different markets and mechanisms which can be constructed to facilitate trade. We must specify how many markets there are and how each market functions.

In an economy with complete markets, with m goods there will be $m(m-1)/2$ markets. Every commodity is directly exchangeable for every other commodity. There is complete liquidity. One of the several properties of a money is complete liquidity. Concentrating on this property we may define a money as a commodity which can be exchanged directly for all others.

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Thus, as is illustrated in Figure 1a for an economy with $m = 4$ goods, all four may be regarded as monies if there are complete markets and there are m different market structures with $m-1$ markets where only one commodity is a money. These are shown in Figures 1b-1e.

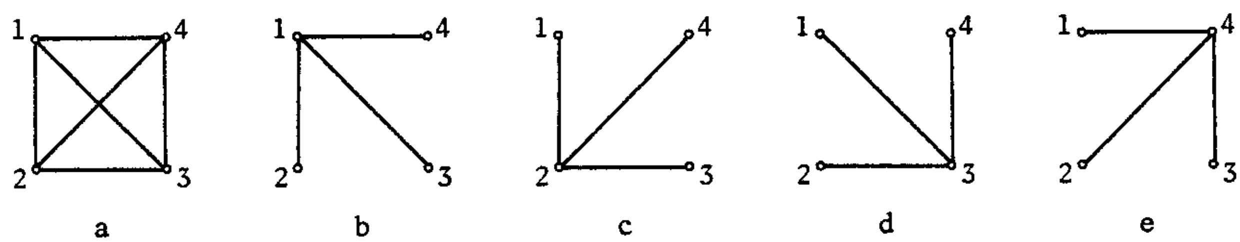

FIGURE 1

In a simple exchange economy with complete markets credit is not needed as all items can be exchanged directly. If only one commodity is a money and there are only $m-1$ markets then if all individuals are required to trade using the money it is possible that efficient trade would be hampered by lack of liquidity. For example, unless there is credit an individual with a plentiful supply of all commodities except that he has no money, will be unable to trade.

In an economy with one money, $m-1$ markets and no credit in order to be able to achieve efficient trade there must be "enough money" held by all traders. The specific meaning of "enough money" depends upon the actual method of trade, the distribution of the money among the traders and the marginal value of the monetary good as a good. In essence "enough money" means that the noncooperative equilibrium solutions to a strategic market game is <u>interior</u>, in other words it is not constrained by lack of liquidity.

For simplicity two specific market mechanisms are described to illustrade the relationship between market structure and liquidity.

The Sell-All Model

We begin with the simplest model of exchange utilizing a single money. The model is also suitable for a worst case analysis inasmuch as it requires the largest amount of money to facilitate trade.

Let commodity m be the money, hence the market structure is as shown in Figure 2. There are n types of trader. A trader of type i is characterized by an endowment density $(a_1^i, a_2^i, \ldots, a_m^i)$ and a utility function $\varphi_i(x_1^i, \ldots, x_m^i)$.

In this simple strategic game trade takes place as follows. All individuals are required by the rules to put up everything for sale except their money. They then all use their money to bid simultaneously in the $m-1$ markets for the $m-1$ goods offered for sale.

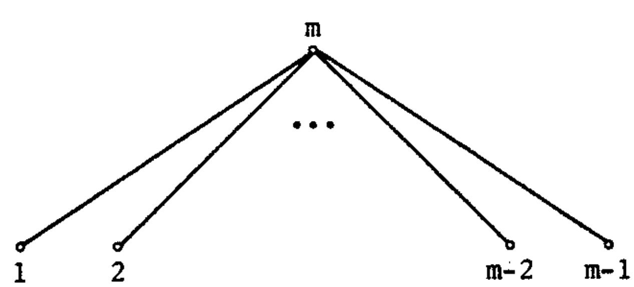

FIGURE 2

We consider a strategy by an individual of type i to be a vector of dimension $m-1$ of the form $(b_1^i, b_2^i, \ldots, b_{m-1}^i)$ where $0 \leq b_j^i$ for $j = 1, \ldots, m-1$ and $\sum_{j=1}^{m-1} b_j^i \leq a_m^i$. A strategy is an allocation of the amount of money to be spent on each of the $m-1$ goods. The constraint imposed on the sum of all bids is that you cannot buy for more money than you have on hand. This is in essence a cash flow constraint. Although the individual will be paid back after trade for resources sold he does not have the use of income from goods not yet sold.

Figure 3 shows the $m-1$ markets and indicates the simple market clearing price formation mechanism. In essence price is determined by the ratio of the amount of money closing the amount of goods offered for sale. The total amount of good j available for sale by all traders of type i is denoted by $\int a_j^i$. We assume that there is a continuum of traders of each type where each type has the same measure and all individuals are of insignificant size compared to the markets as a whole. Let $a_j = \sum_{i=1}^{m} \int a_j^i$ be the total supply of good j ; and similarly let b_j be the total amount of money bid for j . Then price is defined as:

$$p_j = b_j/a_j \ . \tag{1}$$

Thus an individual bidding b_j^i for j obtains

$$x_j^i = b_j^i/p_j \quad \text{for} \quad j = 1, \ldots, m-1 \ . \tag{2}$$

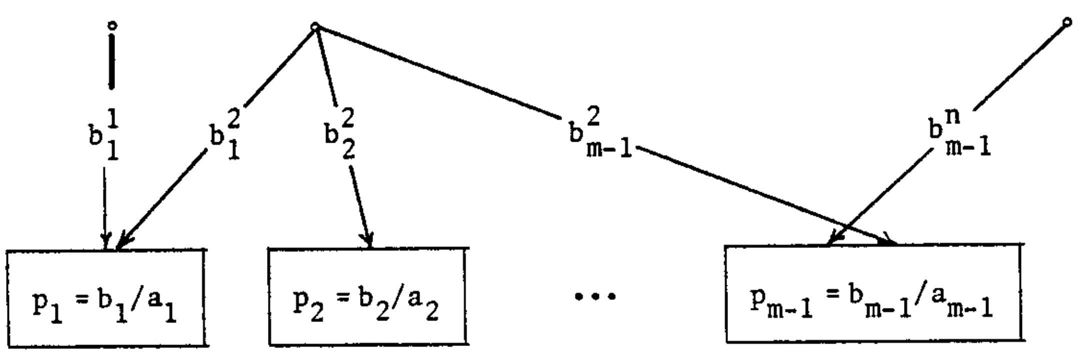

FIGURE 3

After trade has taken place each individual is paid for what he has sold. Thus

$$x_m^i = a_m^i - \sum_{j=1}^{m-1} b_j^i + \sum_{j=1}^{m-1} p_j a_j^i \ . \tag{3}$$

We may attach a price of $p_m = 1$ to the money.

If we have a continuum of traders and any trader is insignificant in size then it has been shown elsewhere (Dubey and Shapley, 1977, Dubey and Shubik, 1978b) that in solving for the noncooperative equilibrium a single individual will not influence price, hence the individual optimization problem appears to be the same as that found in the study of general equilibrium with the additional cash flow constraint. An individual of type i attempts to maximize

$$\varphi_i(x_1^i, \ldots, x_m^i)$$

$$\text{subject to} \quad \sum_{j=1}^{m} p_j(a_j^i - x_j^i) = 0 \tag{4}$$

and

$$\text{subject to} \quad \sum_{j=1}^{m-1} p_j x_j^i \leq a_m^i \,. \tag{5}$$

Even though condition (4) might be satisfied, condition (5) could easily constrain the optimization.

In this simple model we forced all individuals to sell all of their goods thus in the trade that takes place all goods in the economy pass through the market once. This calls for at least an amount of money of:

$$\sum_{j=1}^{m-1} p_j a_j \tag{6}$$

and if a shortage of money is not to influence the optimization, the distribution must also be such that (5) is satisfied for all individuals.

The amount noted in (6) is the largest amount of money needed to finance trade under all one shot trading arrangements. It is an extreme upper bound in the sense that if we allowed sequential trade with payments made before the next round of trade, the money would be able to be used more than once in a period. Here velocity would be a substitute for quantity. Furthermore, although the tax collector might like the idea of forcing all goods through the market, neither individuals nor firms sell all of their nonmonetary assets each year. In general only a fraction of all goods in the economy are sold each year. We can model the possibility that an individual need not sell all by a slight complication of the first model.

The Bid-Offer Model

Instead of bidding only to buy, the individual now is assumed to make offers of how much he or she intends to sell. A strategy which was formerly of dimension $(m-1)$ is now of dimension $2(m-1)$ and is of form $(b_1^i, \ldots, b_{m-1}^i; q_1^i, \ldots, q_{m-1}^i)$ where

$$b_j^i \geq 0 , \quad \sum_{j=1}^{m-1} b_j^i \leq a_m^i , \quad 0 \leq q_j^i \leq a_j^i \quad \text{for} \quad j = 1, \ldots, m-1 . \tag{7}$$

The condition on the q_j^i stops an individual from selling more than he owns. Here, in contrast with the previous model it becomes possible for an individual to consume his own goods without having to buy them back from the market hence the volume of trade may become less. The individual cash flow constraint instead of being as shown in (5) now becomes[1]

$$\sum_{j=1}^{m-1} p_j \max[(x_j^i - a_j^i), 0] \leq a_m^i . \tag{8}$$

[1]Equation (8) ignores "wash sales" which could be important when there are only a finite number of players. See Dubey and Shubik (1978a).

2. THE CHOICE OF MARKETS AND ENOUGH MONEY

If there are only $m-1$ markets we have m candidates which could serve as a money. The inequality displayed in (5) for the sell all model or the inequality in (8) for the bid-offer model will be different for each choice of a money. A natural question to ask is, is it possible for the distribution of resources to be such that any commodity could serve as a money in sufficient supply. The answer is that in the sell-all model it is not possible for all commodities to serve as a sufficient money, but for the bid-offer model it is. The proof of the first is fairly obvious, but is given below, the second can be illustrated by example.

Assertion 1. In the sell-all model it is not possible to find an exchange economy such that any commodity could serve as a sufficient money.

Proof. The proof is by a simple contradiction. Suppose that it were true then for a trader of type i and a particular equilibrium set of prices $p_1, p_2, \ldots, p_{m-1}, 1$, as the equilibrium would not be influenced by which commodity serves as money the following inequalities must hold

$$\begin{aligned}
p_1x_1^i + p_2x_2^i + \cdots + p_{m-1}x_1^i &\leq a_m^i \\
p_1x_1^i + p_2x_2^i + \cdots \qquad\qquad + x_m^i &\leq p_{m-1}a_{m-1}^i \\
&\vdots \\
p_2x_2^i + \cdots + p_{m-1}x_m^i + x_m &\leq p_1a_1^i
\end{aligned}$$

summing we obtain

$$(m-1)\{p_1x_1^i + p_2x_2^i + \cdots + p_mx_{m-1}^i + x_m^i\} \leq \{p_1a_1^i + p_2a_2^i + \cdots + p_{m-1}a_{m-1}^i + a_m^i\} .$$

Now we sum over all types of traders, then as we know that all trade is monetized the sum on types yields

$$(m-1)\{p_1a_1 + p_2a_2 + \dots + a_m\} \leq \{p_1a_1 + p_2a_2 + \dots + a_m\}$$

but for $m > 1$ this is impossible.

Assertion 2. In the bid-offer model it is possible to find an exchange economy such that any commodity could serve as a sufficient money.

This assertion is shown by a simple robust example, where a finite perturbation of the initial holdings still leaves all of the commodities as sufficient monies.

Let there be m types of trader distinguished only by their initial endowments. All traders have a utility function of the form

$$\varphi_i = \prod_{j=1}^{m} x_j^{1/m} .$$

A trader of type j has an endowment of the form

$$(A, A, \dots, (1+\Delta)A, \dots, A)$$

where the $(1+\Delta)$ factor occurs for the j^{th} component.

The unique competitive equilibrium for this market has prices of $(1, 1, 1, \dots, 1)$ and a symmetric final allocation of $\left(\left(1+\frac{\Delta}{m}\right)A, \left(1+\frac{\Delta}{m}\right)A, \dots, \left(1+\frac{\Delta}{m}\right)A\right)$ for all traders. But the total amount of purchases for any type involved in any of the m market structures (each using a different money) is $\frac{(m-1)}{m}\Delta A$, hence if

$$\frac{(m-1)}{m}\Delta A < A$$

there will always be enough money, no matter which is used.

It is easy to see that the general class of such economies does not depend upon the strict symmetry of the utility functions or the strict symmetry of the set of slightly nonsymmetric endowments in the special example.

3. CONCLUDING REMARKS

The use of a commodity money or a fiat money or credit in trade depends upon a vague intermix of trust, law, custom and technology. In the statics of general equilibrium these institutional details only get in the way of clear analysis. In the dynamics of trade (as was well known to Jevons, Fisher, Keynes and many others) these institutional features play a role. Thus the technological limits to the velocity of payments, the desirability, portability, standardization and identifiability of the means of payment all play a role in dynamics. In any attempt to reconcile microeconomic statics with macroeconomic dynamics it is therefore important to help to clarify the abstract meaning of enough money in trade and how it relates the social custom and technology of trade with the noninstitutional static principles of the efficient price system.

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