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AN APPLICATION TO BELGIUM

by

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ABSTRACT

This paper concerns an application to the Belgian economy of general equilibrium analysis in the presence of downward real wage rigidities. The model aims at explaining the short-run impact of recent income and exchange policies upon employment in Belgium. Mathematical programming techniques are used to compute equilibria.

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1. Introduction

This paper concerns an application to the Belgian economy of general equilibrium analysis with downward real wage rigidities. The paper makes both a methodological and a positive point. The methodological point consists of illustrating how mathematical programming techniques can be used to deal with issues of price rigidity. At the positive level, our model of the Belgian economy aims at describing the short-run impact upon employment of income policies. We show that wage policies are not likely to alleviate unemployment in Belgium unless they occur simultaneously with substantial increases in investment. We also show that attempts at decreasing the trade imbalance have large negative effects on employment.

The traditional approach to computing equilibria is a calculation in the space of commodity prices. See Scarf (1973). The data for this calculation are demand and supply functions expressed in terms of commodity prices. A fixed point algorithm then determines a price vector at which the supply of each commodity exceeds or equals the demand for that commodity.

The computational requirements of fixed point algorithms are quite substantial and severely limit the size of the problems which can be solved numerically. When the number of commodities is large relative to the number of consumers, a traditional fixed point calculation in the space of commodity prices, even if feasible, is cumbersome.

In such cases, the existence of efficient mathematical programming techniques favors a different approach, motivated by a theorem of Negishi (1960), in which the calculation of supply and demand functions is replaced with the solution of a mathematical program. This program optimizes a social welfare function which is a weighted sum of consumer utility functions

subject to material balance constraints. Negishi's theorem states that there exists a choice of welfare weights such that the welfare optimizing solution is an economic equilibrium. A fixed point calculation then searches for these equilibrium welfare weights. This search now occurs in the relatively smaller space of welfare weights. However, each step of the search is computationally more expensive as it requires solving a mathematical program, the number of constraints of which is equal to the number of commodities. This is the approach followed by Dantzig, Eaves and Gale (1979), Dixon (1975), Ginsburgh and Woelbroeck (1981), and Manne, Chao and Wilson (1980), among others.

The methodological contribution of this paper is to illustrate how the Negishi approach can be adapted to a case of price rigidity. See also Keyzer (1981). The difficulty presented by price rigidities in the Negishi approach is that equilibrium prices arise as shadow prices of the material balance constraints in the welfare maximizing program. These shadow prices verify the zero profit conditions for activities in use at the optimum and the no profit conditions for the others. Due to their imputed nature, shadow prices cannot verify any other prior conditions. This raises difficulties for their interpretation as equilibrium prices when these are subject to additional constraints.

The special case of price rigidity considered here is a downward real wage rigidity. Real wages are maintained through supply rationing in the labor market. See Drèze (1975). To compute an economic equilibrium with rationing of the labor supply, the original Negishi calculation needs to be slightly modified. One appends to the Negishi welfare function a term representing the labor rationing valued at the minimum real wage and multiplied by an appropriate weight. A fixed point calculation then

determines both this weight and the other welfare weights. At equilibrium the weight multiplying the labor term is equal to the price index.

We now discuss our application to Belgium. Belgian economic policy has been, during the last years, concerned with three main issues: a growing rate of unemployment, a rapidly deteriorating trade balance, and a lack of investment. Large public deficits, compounded with the increasingly unfashionable flavor of Keynesian economic policies, led public authorities to think of income policies and devaluation as good alternatives. In February 1982, the Belgian government made the decision to devalue and to temporarily put an end to full wage indexation. Wage indexation, which had been effective in Belgium for a very long time, was assumed to be responsible for the persistence of the high unemployment rates observed during the late 1970's and the early 1980's. In early 1984 (two years after the end of full wage indexation), it was claimed that Belgium had recovered its external competitiveness; and there were signs that the trade deficit was slackening. Unemployment had, however, not receded at all; on the contrary, there were some 350,000 unemployed in 1980; there were 525,000 registered in the beginning of 1984 (some 13% of total labor supply). To examine the reasons for this, we have built a small general equilibrium model with downward real wage rigidity for the Belgian economy. The model shows that, in the short run, nothing very different could have happened.

These issues had already been considered by Drèze and Modigliani (1981) under a short-run binding external balance constraint. (At the time of their study (1979), trade imbalance was much less of a problem than later on; Belgium had for a long time run external surpluses.) The conclusion they reached was that, because of capacity constraints, the short-run elasticity of employment with respect to real wages was very low (of the

order -0.2), while it could reach -2 in the long run. Using a completely different and disaggregated approach, the conclusions we draw are surprisingly close to those obtained by Drèze and Modigliani, and stress the importance of investment in reducing Belgian unemployment.

The paper is organized as follows. Section 2 describes our static model of the Belgian economy. Section 3 covers computational and validation issues. Both issues are closely related in our approach. Section 4 concerns our results of wage control policies for the Belgian economy. Appendix 1 presents a dynamic version of our model.

2. The Model

The model describes economic behavior in Belgium as resulting from the transactions among 4 agents (consumers, producers, government, and the rest of the world) of 52 commodities (24 goods and 28 production factors including 24 production capacities, labor and three nonsubstitutable imports).

The institutional framework in which these agents behave is a world of pure competition: every agent makes his decision at given prices and income. With the exception of the wage rate, which is subject to a downward rigidity, all prices are fully flexible and all markets clear.

The model is short-run in nature. It attempts to capture the effect of policy measures within a time span of approximately three years (1977-1980), during which production capacities in the different sectors can be modified through investment. Our static model describes the state of the economy at the end of this three-year time span (1980).

In Appendix 1 we also briefly describe a dynamic version consisting of two time periods. The first time period is the same short-run three-year

span, but is followed by a second ten-year time span, at the end of which all markets, including the labor market, are assumed to be in equilibrium. The dynamic model is conceptually more satisfactory, but is more cumbersome to solve and only confirms the results of our static model. In this first paper, we therefore concentrate on the simpler static model.

The Behavior of Agents

Belgian households are aggregated into a single representative agent with utility function $U(x) + \beta\phi(k)$, x denoting consumption and k terminal capital stocks. The function $U(x)$ is chosen to be $\sum_i \gamma_i \log(x_i - \delta_i)$, leading to the well-known linear expenditure system. See, e.g., Lluch et al. (1977) for the theory and Cherif et al. (1978) for the estimation of the parameters γ_i and δ_i for Belgium. The term $\beta\phi(k)$ represents the discounted sum of utilities of future consumption streams consistent with a short-run capital stock k ; β is the discount factor. For simplicity and as a first approximation, we chose $\phi(k)$ to be linear; as will be discussed later, β is chosen so as to replicate the 1980 investment rate.

The maximization problem describing the agent's choice is

$$\max_{x, \ell, v \geq 0} U(x) + \beta\phi(k)$$

$$\text{subject to } px + pVv \leq w\ell + qk - t - b$$

$$\ell \leq \hat{\ell} - u$$

with $k = \hat{k} + Kv$. The following notation is used:

x = consumption,

ℓ = labor demand,

$\hat{\ell}$ = labor capacity (active population in 1980),

u = unemployment,

\hat{k} = initial (1977) capital stocks,
 k = terminal (1980) capital stocks,
 v = addition to capital stocks,
 p = commodity prices,
 q = capital rents,
 w = wage rate,
 t = government taxes,
 b = short-run current account deficit.

In addition, K is a diagonal matrix with elements κ_j which transforms a 1980 investment flow, v , into an increase of the capital stock between 1977 and 1980, Kv . The assumption behind the construction of the matrix K is that investment changes over this short-run period (1977-80) occur linearly and that terminal capacities are the result of cumulative investments throughout the period. For more details on this treatment of investment in a static model, see, e.g., Ginsburgh and Waelbroeck (1981, pp. 53-63). The matrix V determines the flows required from each vector of origin by the capacity increments, v .

In this agent's problem, the first constraint is the agent's budget constraint; px is the value of consumption, pVv , the value of investment (savings), wl is labor income, qk , capital income, while t and b represent net lump sum transfers to the government and to the rest of the world. Observe that the model does not distinguish between the spending propensities out of capital and labor income. The second inequality, $l \leq \hat{l} - u$, constrains the agent's labor supply and allows unemployment to exist at a positive wage rate; we herewith follow the formulation proposed by Drèze (1975).

Let x^0 , l^0 , v^0 , k^0 be an equilibrium for this agent, given prices p , q , w , lump sum transfers t and b and rationed labor supply $\hat{l}-u$. Furthermore, let $\sigma = pVv^0 - qKv^0$. This equilibrium is also a solution of the following two optimization problems:

$$\max_{x, l \geq 0} U(x)$$

$$\text{subject to } px \leq wl + qk^0 - t - b - \sigma$$

$$l \leq \hat{l} - u$$

and

$$\max_{v > 0} \phi(k)$$

$$\text{subject to } pVv - qKv \leq \sigma$$

with $k = \hat{k} + Kv$. Hence the household's problem decomposes into a "consumer" problem and an "investor" problem. To clarify investment behavior further we examine the Kuhn-Tucker conditions for the latter program. These can be written

$$\sum_i p_i V_{ij} \geq \kappa_j q_j + \frac{\partial \phi}{\partial v_j} / \mu$$

with equality holding if $v_j > 0$ and with μ denoting the multiplier of the investor's budget constraint. The term on the left-hand side of the above inequality represents the cost of a unit of increase of capacity in sector j , while the terms on the right-hand side represent immediate $(\kappa_j q_j)$ and expected $\left(\frac{\partial \phi}{\partial v_j} / \mu\right)$ rents accruing to capital. This shows that the function $\phi(k)$ can also be interpreted as a valuation function for terminal capital stocks.

The agent representing the government chooses a public consumption bundle, g . For convenience this bundle is determined exogenously, $g = \hat{g}$. The government's budget is balanced, $pg = t$, with t denoting taxes.

An activity analysis model describes the productive side of the Belgian economy. The matrix A denotes net outputs of the commodities by the various sectors, the vector λ denotes the labor inputs into the sectors, and A_c is a matrix of inputs of nonsubstitutable or complementary imports into the sectors, i.e., of imports that cannot be substituted for by domestic goods (oil, natural gas, and unclear fuels). Finally, we normalize production capacities to be measured in the same units as production levels, so that capacity constraints can be written $y \leq k$. Producers choose a vector of production levels y that maximizes their profits, $(pA - w\lambda - p_c A_c - q)y$. The coefficients of the matrices A and A_c and of the vector λ are estimated from input-output tables for Belgium; p_c stands for the price vector of nonsubstitutable imports. All the coefficients are borrowed from Cherif et al. (1978).

The last agent is the rest of the world. As is usual in trade theory; his preferences are represented by a trade welfare function; such a function directly relates the utility of a country to its trade (exports and imports), bypassing the detailed representation of its production and consumption activities. For details on this approach, see, e.g., Negishi (1972, p. 32). Let e , m_s and m_c represent, respectively, Belgian exports, imports of substitutable and nonsubstitutable goods. Then, the rest of the world can maximize the (concave) trade welfare function $W(e, m_s, m_c)$ subject to a balance of trade constraint $pe - pm_s - p_c m_c \leq b$, where b is the (possibly zero) Belgian current account deficit.

To specify the trade welfare function for the rest of the world,

one could make the small country assumption and assume that Belgium can export and import any quantities at the world price. Marginal utilities in the rest of the world would then be constant, and the trade welfare function would simply be

$$W(e, m_s, m_c) = p(e - m_s) - p_c m_c$$

with p and p_c denoting world prices. However, such an assumption seems extreme. It is generally believed that export demands from and import supplies to Belgium by the rest of the world are not perfectly elastic, that, for example, Belgium cannot increase its exports unless it also decreases its export prices. The trade welfare function has been chosen as

$$W(e, m_s, m_c) = \sum_i \epsilon_i e_i^{\frac{1}{\xi_i} + 1} - \sum_i \nu_{si} m_{si}^{\frac{1}{\eta_{si}} + 1} - \sum_i \mu_{ci} m_{ci}^{\frac{1}{\eta_{ci}} + 1} .$$

If the rest of the world's marginal welfare with respect to its trade deficit with Belgium, b , is assumed constant, the parameters ξ_i and η_i can be interpreted as price elasticities for exports from and imports to Belgium. What are believed to be reasonable estimates for these elasticities are in Table 1.

TABLE 1
Export and Import Price Elasticities

<u>Exports</u>	<u>Values</u>
Services, Railroad Transportation and Transportation by Roads }	-0.75
Agriculture, Oil Products, Construction, River and Sea Transportation }	-1.00
Processed Food, Textiles, other industry }	-1.50
Steel, Non-ferrous Metals, Non-metallic Products, Chemicals, Paper Products }	-2.00
Metal Working Industries	-2.50
 <u>Imports</u>	
All Industries	-1.00

Market Clearing

On every market so far defined, demand cannot exceed supply. For the goods market the condition reads

$$x + g + Vv + e \leq Ay + m_s .$$

The left-hand side represents total demand (private and public consumption, demand for investment, exports) while the right hand side gives total supply (net production and substitutable imports).

On the market for production capacities, we have

$$y \leq k = \hat{k} + Kv ;$$

again, demand (expressed in units of production) may not exceed supply k , which is equal to initial capacities \hat{k} plus cumulative additions Kv .

Recall that K is a diagonal matrix which transforms investment flows v into terminal capital stocks Kv .

Demand for nonsubstitutable imports, $A_c y$, may not exceed quantities supplied by the rest of the world, m_c :

$$A_c y \leq m_c .$$

Finally, on the labor market, we have

$$\lambda y \leq \ell$$

where λy is labor demanded by producers and ℓ is the labor supply.

Labor Rationing and Downward Wage Rigidity

To allow for unemployment at a positive wage rate, we assume that the real wage rate is downward rigid. With w denoting the nominal wage and $P(p)$ a suitably defined price index, we have that $w/P(p) \geq \underline{w}$, where \underline{w} is a given lower bound on the real wage rate. To ensure market clearing on the labor market, labor supply, ℓ , is rationed. That is, $\ell \leq \hat{\ell} - u$ where $\hat{\ell}$ denotes the full employment labor supply and u is forced unemployment. Consistent with the requirement of an equilibrium with price rigidity, rationing can occur only at the rigid wage, a condition which is expressed mathematically as $(w - \underline{w}P(p))u = 0$.

Economic Equilibrium with Downward Real Wage Rigidity

As a way of summarizing the previous discussion, we now define an economic equilibrium with downward real wage rigidity.

Definition 1: The vectors x (private consumption), g (public consumption), v (capacity increments), e (exports), m_s and m_c (substitutable and nonsubstitutable imports), y (production), together with the scalars t (taxes), b (current account deficit), l (labor supply) and u (unemployment), supported by the price vectors p (goods), p_c (non-substitutable imports), q (capital rents) and w (wage rate) are an equilibrium with a downward rigidity on the real wage rate, if they satisfy the following conditions (for convenience we omit the nonnegativity constraints):

$$(2.1) \quad x, l, v \text{ maximize } U(x) + \beta\phi(k)$$

$$\text{s.t. } px + pVv \leq wl + qk - t - b$$

$$l \leq \hat{l} - u$$

$$\text{with } k = \hat{k} + Kv$$

$$(2.2) \quad g = \hat{g} \text{ with } pg \leq t$$

$$(2.3) \quad y \text{ maximizes } (pA - w\lambda - p_c A_c - qy)y$$

$$(2.4) \quad e, m_s \text{ and } m_c \text{ maximize } W(e, m_s, m_c)$$

$$\text{s.t. } pe - pm_s - p_c m_c \leq b$$

$$(2.5a) \quad x + g + Vv + e - Ay - m_s \leq 0$$

$$(2.5b) \quad p(x + g + Vv + e - Ay - m_s) = 0$$

$$(2.6a) \quad y \leq k = \hat{k} + Kv$$

$$(2.6b) \quad q(y-k) = 0$$

$$(2.7a) \quad A_c y - m_c \leq 0$$

$$(2.7b) \quad p_c (A_c y - m_c) = 0$$

$$(2.8a) \quad \lambda y \leq \ell$$

$$(2.8b) \quad w(\lambda y - \ell) = 0$$

$$(2.9a) \quad w - \underline{w}P(p) \geq 0$$

$$(2.9b) \quad u(w - \underline{w}P(p)) = 0 \quad . \quad \square$$

Some remarks are in order. From the equilibrium conditions we can recover the national accounting identity. Add (2.5b), (2.6b), (2.7b) and (2.8b) and note that, since there are constant returns to scale, profits of firms are zero so that $pAy - w\lambda y - p_c A_c - qy = 0$. Hence, we obtain:

$$(2.10) \quad px + pg + pVv + (pe - pm_s - p_c m_c) = w\ell + qk \quad .$$

This expression reproduces the accounting identity between national expenditure (private and public consumption, investment, and net exports) and national income (labor and capital income). Moreover, if we assume the functions $U(x) + \beta\phi(k)$ and $W(e, m_s, m_c)$ to be strictly increasing, the budget constraints in (2.1) and (2.4) will be satisfied as strict equalities. (All prices will then be positive so that (2.5a), (2.6a), (2.7a), and (2.8a) will hold as equalities.) Adding these equalities also leads to (2.10).

Our second remark concerns the homogeneity of degree zero in prices. The reader will easily verify that this property is indeed verified here

as long as the lump-sum transfers between agents, t and b , are specified in relative terms as well.

This brings us to our final remark, which concerns the issue of income determination among the agents. As discussed above, the allocation of income between consumption and savings is determined endogenously by the utility function of Belgian households. Households are taxed so as to balance the government's budget, government's consumption being exogenous. However, income transfers between domestic and foreign sectors are still undetermined. Long-term equilibrium requires the current account to be balanced. However, this is not necessarily the case in the short run, where imbalances (i.e., $b \neq 0$) may persist over some time (Belgium has been running such deficits since 1975). To account for such situations, we have computed equilibria for various levels of the trade deficit, specified as a fixed percentage of Belgian income ($b = \delta(px + pVv)$).

3. Computation and Calibration

To simplify the presentation, and without loss of generality, we assume that $U(x) + \beta\phi(k)$ and $W(e, m_s, m_c)$ are strictly increasing, so that all inequalities in (2.1) to (2.8) hold as strict equalities. Leaving aside the issue of downward wage rigidity, the simplest way to compute an equilibrium solution to (2.1)-(2.8) is the following: pick prices p , p_c , q and w ; compute solutions to problems (2.1)-(2.4) and check whether they satisfy the market clearing conditions (2.5a), (2.6a), (2.7a) and (2.8a). If so, a solution is obtained. If not, change the prices, and start a new computation. This is the usual search procedure of the Walrasian auctioneer, for equilibrium prices.

Negishi (1960) has shown that the same equilibrium can be obtained

as solution of a mathematical program in which the objective function is a weighted sum of the utility functions of the various agents, while the constraint set consists of the market equilibrium conditions. In Ginsburgh and Van der Heyden (1984), we have extended Negishi's result to deal with downward (or upward) price rigidities and have shown that price rigidities can be handled by appending to the Negishi welfare function a term representing the rationing needed to verify the price constraints. These results show that a solution of the Belgian model can be computed by solving the mathematical programming model (again omitting nonnegativity constraints)

$$(3.1) \quad \max \alpha_1 (U(x) + \beta \phi(k)) + \alpha_2 W(e, m_s, m_c) + \alpha_3 wu$$

$$(3.2) \quad x + Vv + e - Ay + m_s \leq -\hat{g} \quad (p)$$

$$(3.3) \quad y - Kv \leq \hat{k} \quad (q)$$

$$(3.4) \quad A_c y - m_c \leq 0 \quad (p_c)$$

$$(3.5) \quad \lambda y + u \leq \hat{\ell} \quad (w)$$

where $k = \hat{k} + Kv$, and without loss of generality, $\alpha_1 = 1$.

The variables appearing between parentheses at the right of each constraint are the associated shadow prices. The main result contained in Ginsburgh and Van der Heyden (1984) is that a solution of (3.1)-(3.5) is an equilibrium with downward wage rigidity if the shadow prices p verify

$$(3.6) \quad \alpha_3 = P(p) .$$

The reader can check that if this is the case, the solution to (3.1)-(3.5)

ensures $w \geq \underline{wP}(p)$ with $(w - \underline{wP}(p))u = 0$. This leaves us with the determination of the weight α_2 , which is chosen so as to verify a current account deficit

$$(3.7) \quad b = \delta(px + pVv)$$

with δ exogenously given.

We will compute such equilibria for different values of δ as follows: pick weights α_2 and α_3 ; compute a solution to (3.1)-(3.5) and verify whether conditions (3.6) and (3.7) are satisfied. If so, a solution is obtained. If not, change the weights, and start a new computation. The determination of the equilibrium weights is a fixed point problem, which, due to its small dimensional nature, is in our case easily solved by a tâtonnement procedure on the weights.

We now turn to discussing the way a reference solution was generated for 1980. The alternative policy simulations to be considered in Section 4 will be compared to this basic 1980 run and attempt to answer the question "What would have happened if the policy considered had been undertaken approximately three years earlier, i.e., around 1977?"

The major difficulty in computing the base case was the calibration of the parameter β in the utility function $U(x) + \beta\phi(k)$, of the constants ϵ_i , μ_{si} and μ_{ci} of the trade welfare function, and of the lower bound on the real wage rate \underline{w} .

The following procedure was adopted. Exports and imports were set at their 1980 observed values, say \hat{e} , \hat{m}_s and \hat{m}_c , and the following mathematical program was solved

$$\begin{aligned}
& \max U(x) + \beta\phi(k) + \gamma u \\
& \text{s.t. } x + Vv - Ay \leq -\hat{g} - \hat{e} + \hat{m}_s \quad (p) \\
& \quad -Kv + y \leq \hat{k} \quad (q) \\
& \quad A_c y \leq \hat{m}_c \quad (p_c) \\
& \quad \lambda y + u \leq \hat{l} \quad (w) .
\end{aligned}$$

Recall that the vector \hat{k} denotes initial (1977) capital stocks and \hat{l} 1980 full employment. A tâtonnement procedure then solved for weights β and γ verifying both $pVv = \psi(px + pVv)$, with the savings rate ψ set at its 1980 value, and the 1980 unemployment figure of 336,000 workers. Since we know that at equilibrium $\alpha_3 = P(p)$, we then computed \underline{w} in straightforward fashion: $\underline{w} = \gamma/P(p)$. The shadow prices, $p = \hat{p}$, $q = \hat{q}$, $p_c = \hat{p}_c$, and $w = \hat{w}$, of the above program also are the base case (1980) equilibrium prices. If we now make the assumption that at these prices Belgium and the rest of the world were "in equilibrium" (with a given current account deficit), \hat{p} and \hat{p}_c must be proportional to marginal utilities for the rest of the world, i.e., $\hat{p} = \alpha_2 \frac{\partial W(\cdot)}{\partial e} = -\alpha_2 \frac{\partial W(\cdot)}{\partial m_s}$ and $\hat{p}_c = -\alpha_2 \frac{\partial W(\cdot)}{\partial m_c}$. Without loss of generality, we set $\alpha_2 = 1$, and determine the constants in the trade welfare function as follows:

$$\epsilon_i = \hat{e}_i / \hat{p}_i^{\xi_i}, \quad \nu_{si} = \hat{m}_{si} / \hat{p}_i^{\eta_{si}}, \quad \text{and} \quad \nu_{ci} = \hat{m}_{ci} / \hat{p}_{ci}^{\eta_{ci}} .$$

The model reflects the main macro-economic aggregates, as can be seen from Table 2. As an independent check, the reader will observe that the computed income distribution between labor income, wl , and capital income, qk , fits with the observed 1980 distribution, although the 1980

wage rate was calibrated so as only to replicate 1980 unemployment. The computed 1980 current account deficit is overestimated (6.6% versus a 1980 3.7% of Belgian income); notice however that this deficit obtains as the difference between values of the order of 60% of national income, and that the overestimate can thus be considered as small.

TABLE 2

1980 Aggregates: Actual and Simulated (in 10^9 BFr)

	Actual	Simulated
Private Consumption	2220	2359
Public Consumption	647	629
Investment	732	706
Current Account	-128	-223
Total Income	3471	3471
Labor Income	2462	2431
Capital Income	1009	1040
Unemployment (1000 workers)	336	338

4. Wage Control Policies in the Belgian Economy

We examine here the issue of wage controls, a policy issue which has been widely discussed in Belgium during the early 1980's. 100% wage indexation during the 1960's and 1970's made Belgium lose its competitive position on foreign markets. This was considered to particularly hurt the country for it is particularly dependent on trade. Exports represent some

60% of GNP in 1980. In February 1982, the indexation clause was dropped, at the same time as the Belgian currency was devalued. The devaluation became necessary to restore balance in the current account.

The question is to analyze what would have happened by 1980, had such a policy been implemented some three years earlier.

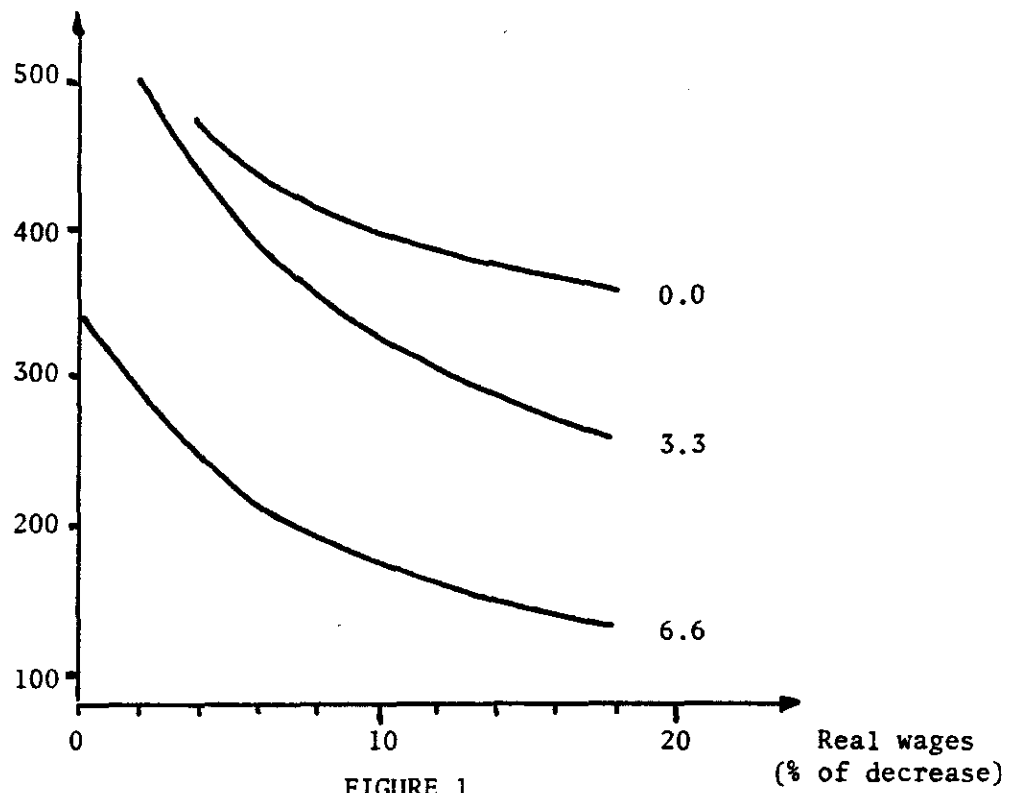
Since we deal with a general equilibrium model, there is no way of simulating a devaluation by manipulating an "exchange rate" between Belgium and the rest of the world. However, our equilibrium model allows us to specify different levels of the current account deficit. We compute equilibria for three such values:

- (a) the pre-devaluation value (a deficit, as calculated by the model, amounting to 6.6% of Belgium's income)
- (b) an equilibrium value (a zero deficit)
- (c) an intermediate value (a deficit representing 3.3% of Belgium's income).

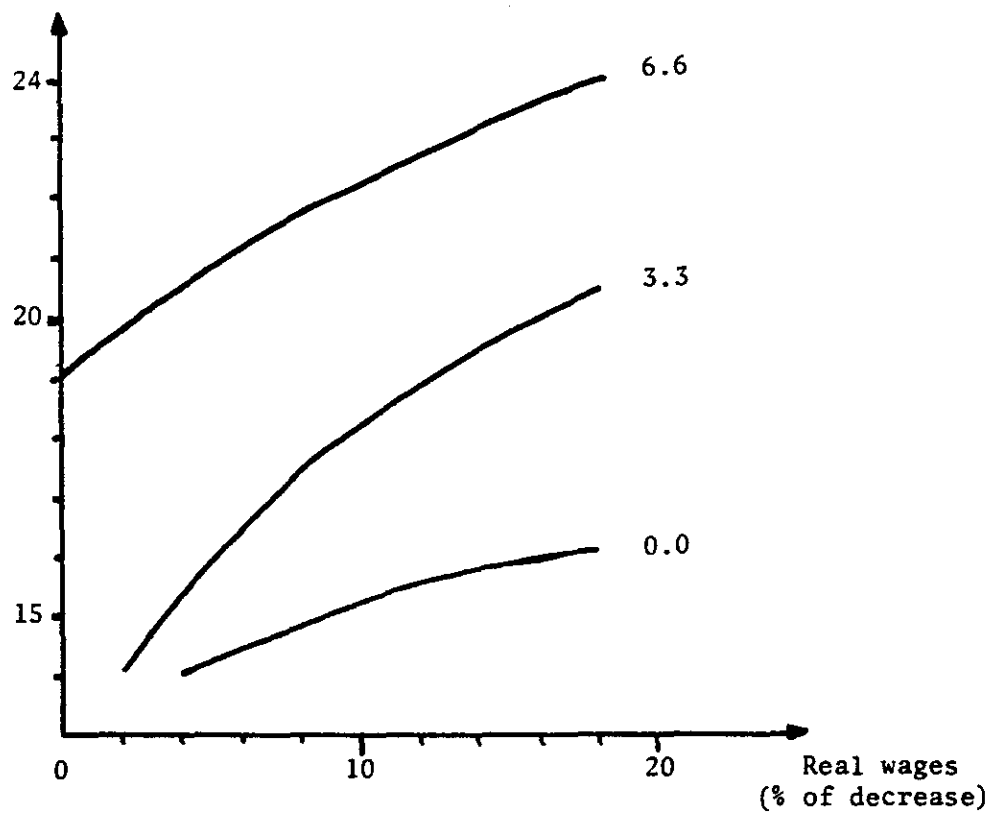
In each of these cases, the real wage was decreased by as much as 18%. The results on some important macroeconomic aggregates are given in Figures 1-3. These show the consequence of a decrease in the real wage (shown on each horizontal axis) on unemployment, the savings-investment rate, and the share of capital income (shown on the successive vertical axis).

We first examine the effects of a decrease in the real wage, at a constant pre-devaluation value of the deficit in the current account, equal to 6.6% of income. Just as suggested by neoclassical economics, there is a decrease in unemployment, but it brings along an increase in the savings rate; this suggests that Belgium may be running short of production capacities and that wage decreases alone, i.e. without increasing capacities, will not alleviate the burden of unemployment. The same conclusion had

Unemployment
(in 1000 workers)



Savings rate
(as a % of income)



Capital income
(as a % of income)

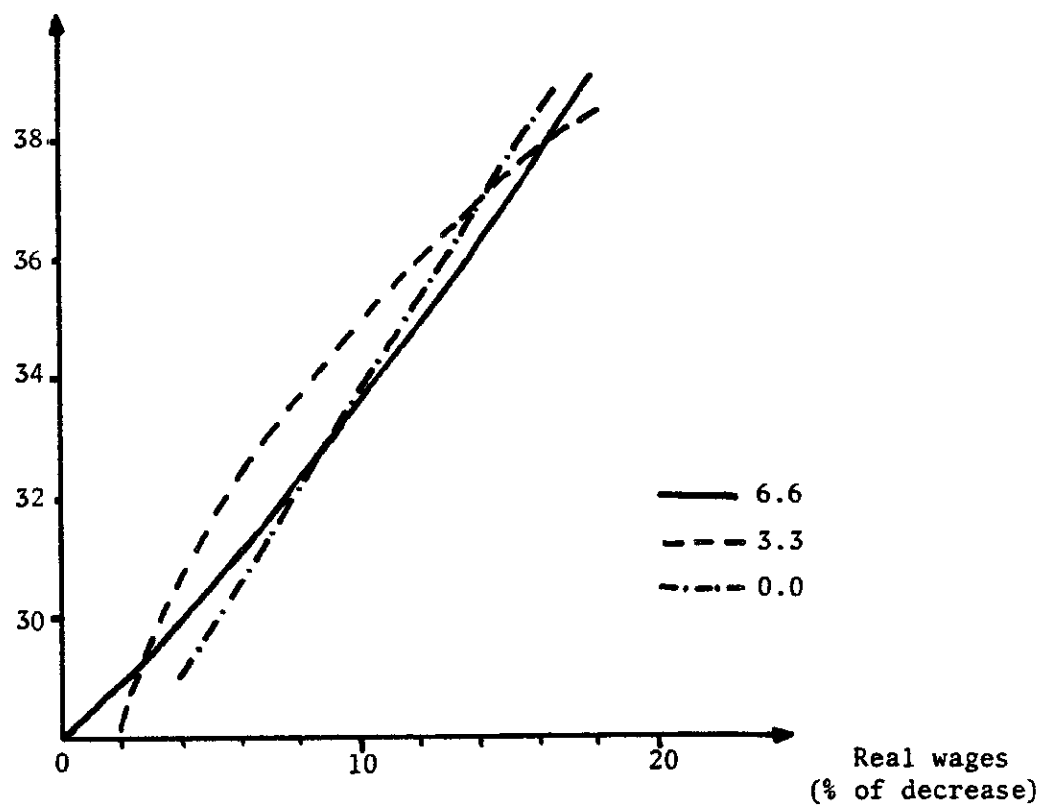


FIGURE 3

been reached by Dreze and Modigliani (1981). Some of the sectoral effects are illustrated in Table 3. It describes the changes between the base case and the extreme case obtained through a wage decrease of 18%. The seven sectors (out of 24) for which details are given account for 75% of the increase in the demand for labor and in the demand for capital. Construction and services account for the largest part. There is no change in the capacity of metal working industries; the large increase in the investment demand for metal products (+51 billion) is met by the rest of the world: exports decrease dramatically (-39 billion), while at the same time imports increase (+16 billion). In the other sectors, the change occurs in the expected direction: production increases, together with exports, while imports fall. But, according to the data fed into the model, entrepreneurs do not find it profitable to invest into the metal working industry, even when the real wage rate falls sharply; or it seems more profitable for them, to invest in other sectors.

The reader will observe that, under the 6.6% constant current account deficit assumption, decreases in the real wage, coupled with increases in the savings rate, would alleviate the unemployment problem, but not eliminate it. For example, a 10% real wage decrease brings unemployment from 8% to 4%. Further reductions in the real wage have decreasing marginal effects on unemployment, as indicated in Figure 1.

It is difficult to assess whether the Belgian economy could have reached such a state since, as mentioned earlier, Belgium devalued its currency in 1982, and almost restored equilibrium in the current account by mid 1984.

This leads us to our other two assumptions on the magnitude of the current account deficit: 3.3% of national income and equilibrium. Figure 2

TABLE 3

Changes between Base Case and 18% Real Wage Decrease; 6.6% Deficit in the Current Account
(Figures in 10^9 BFr and 10^3 workers for labor)

	Production	Final Demand				Intermediate Demand	Sectoral Demand for	
		Consumption	Investment	Exports	Imports		Labor	Capital
1. Agriculture	+22.3	-	-	+3.6	-7.8	+10.9	+2.2	-
2. Metal Working Industries	-	-3.4	+50.6	-39.0	+15.5	-7.3	-	-
3. Non-Ferrous Metals	+26.3	-	-	+6.4	-0.9	+19.0	+7.5	-
4. Processes Food	+14.6	-	-	+2.9	-0.7	+11.0	+9.6	+11.8
5. Textiles	+28.7	-1.0	-	+11.1	-8.9	+9.7	+31.7	-
6. Construction	+34.3	-	+31.4	+0.9	-0.9	+1.1	+42.6	+23.6
7. Services	+33.2	-5.4	+4.7	+9.7	-4.0	+20.2	+49.3	+41.3

shows the dramatic negative effects on the savings rate and, through this, on employment of reductions in the current account deficit (i.e. in the "aid" given to Belgium by the rest of the world). Real wage reductions are still helpful, but the real wage decreases required to attain a given level of unemployment increase substantially as one moves toward equilibrium in the current account. For example, at the current account equilibrium, acceptable real wage reductions (say, less than 10%) still leave 400,000 or more workers unemployed in the short run.

Figure 3 describes the redistribution of income toward capital earners implied by real wage reductions at the constant current account deficit. It is interesting to observe that while national income is redistributed, its real level can be seen to remain approximately constant. Thus, if real income is taken as a proxy for measuring welfare, welfare does not decrease with the real wage. Reductions in the current account deficit, of course, yield different results. For example, there is a 10% loss in real incomes between the 6.6% deficit case and the equilibrium case.

Our simulations are indicative of what actually happened in Belgium during the first half of the 1980 decade. In early 1984, two years after the currency devaluation and the introduction of wage controls, real wages had decreased by 4%. However, unemployment had soared to 500,000 workers, and investment had fallen both in 1982 and 1983, despite the very important income redistribution from labor to capital earners. The results of our model show that, in the short run at least, nothing very different could have happened.

5. Conclusion

We have built a general equilibrium model with real wage rigidity for the Belgian economy. The model allows a study of the effects of real wage reductions and changes in the current account deficit on microeconomic aggregates like sectoral activities, and on macroeconomic aggregates like employment, savings, and national income. We have shown how mathematical programming techniques and a simple tâtonnement procedure can be applied to solve the model. Finally, we have pointed out that results of our model are consistent with the observed short-run impact of recent income and exchange policies in Belgium.

APPENDIX 1

A Dynamic Version of the Model

If the idea of market imperfections (like a downward rigidity of the real wage) and imbalances in certain budget constraints (like in the current account) can be defended in the short run, none of them is an appealing long run concept: clearly the wage rate would have to decrease to clear the labor market and adjustments in the foreign exchange rate would take care of the external deficit.

It thus seems worthwhile to build a two period model: the first is a "short-run" period (say three years) during which imperfections and disequilibria may persist; the second is a "long-run" period (say ten years) during which the invisible hand has the time to play its equilibrating role. In addition, such a two-period model allows a more satisfactory modelling of investment behavior.

We have built such a model using mostly the same "static" parameters for the two periods. The results for the second period are thus not meant to have much predictive power. The purpose of the exercise is to immerse the short run static model into a dynamic setting which has reasonable long term equilibrium properties.

The dynamic model is a straightforward extension of the static model. Its formulation is:

$$\begin{aligned} \max U(x_1) + \alpha_2 W(e_1, m_{s1}, m_{c1}) + \alpha_3 \bar{w}_1 u \\ + \rho [U(x_2) + \beta \phi(k_2) + \alpha_4 W(e_2, m_{s2}, m_{c2})] \end{aligned}$$

$$\begin{aligned}
\text{s.t. } x_t + Vv_t + e_t - Ay_t - m_{st} &\leq -\hat{g}_t & (p_t) \quad t = 1, 2 \\
u_c y_t - m_{ct} &\leq 0 & (p_{ct}) \quad t = 1, 2 \\
y_t - K_1 v_1 &\leq \hat{k} & (q_1) \\
y_2 - K_1 v_1 - K_2 v_2 &\leq \hat{k} & (q_2) \\
\lambda y_1 + u &\leq \hat{\ell} & (w_1) \\
\lambda y_2 &\leq \hat{\ell} & (w_2)
\end{aligned}$$

$$\text{with } k_2 = \hat{k} + K_1 v_1 + K_2 v_2 .$$

Consumers are now described by an agent maximizing an intertemporal utility function, $U(x_1) + \rho(U(x_2) + \beta\phi(k_2))$. ρ denotes the social discount rate, which may be considered as implying a first period savings rate $\psi_1 = p_1 Vv_1 / (p_1 x_1 + p_1 Vv_1)$. The terms $\beta\phi(k_2)$ can be interpreted as the discounted sum of utilities of future consumption after the second period. As in our static model, the weight α_3 is determined so as to meet the short run real wage rigidity, $w_1 = \underline{w}_1 P(p_1)$. The labor market at the end of the second period is assumed in equilibrium.

The remaining weights, α_2 and α_4 , serve the same function as the corresponding weights in the static model. Because of the long run external balance constraint, α_4 is determined by the requirement $p_2(e_2 - m_{s2}) - p_{c2}m_{c2} = 0$. For the first period the situation is similar to that of the static model, in which α_2 is determined to satisfy $p_2(e_2 - m_{s2}) - p_{c1}m_{c1} \leq b$, with $b = \delta(p_1 x_1 + p_1 Vv_1)$, δ being exogenously given.

REFERENCES

- M. Cherif, et al., "Modèle central et modèles de demande finale," Programme National R&D Energie, Services de la Programmation de la Politique Scientifique (Brussels, 1978).
- G. B. Dantzig, B. C. Eaves and D. Gale, "An algorithm for the piecewise linear model of trade and production with negative prices and bankruptcy," Mathematical Programming 16 (1979) 190-209.
- P. B. Dixon, The theory of joint maximization (North Holland, Amsterdam, 1975).
- J. Drèze, "Existence of an exchange equilibrium under price rigidities," International Economic Review 16 (1975) 301-320.
- J. Drèze and F. Modigliani, "The trade-off between real wages and employment in an open economy (Belgium)," European Economic Review 15 (1981) 1-40.
- V. Ginsburgh and L. Van der Heyden, "Equilibrium with government price support policies and a theory of Negishi," unpublished manuscript (1984).
- V. Ginsburgh and J. Waelbroeck, Activity analysis and general equilibrium modelling (North Holland, Amsterdam, 1981).
- Y. Guillaume, Z. Erlich and M. Feron, "Modélisation des interactions économie-énergie," Program National R&D Energie, Services de la Programmation de la Politique Scientifique (Brussels, 1983).
- M. Keyzer, "The international linkage of open exchange economies," Ph.D Thesis, Free University (Amsterdam, 1981).
- C. Lluch, A. Powell and R. Williams, Patterns in household demand and saving (Oxford University Press, London, 1977).
- A. S. Manne, H. P. Chao and R. Wilson, "Computation of competitive equilibria by a sequence of linear programs," Econometrica 48 (1980) 1595-1615.
- T. Negishi, "Welfare economics and existence of an equilibrium for a competitive economy," Metroeconomica 12 (1960) 92-97.
- T. Negishi, General equilibrium and international trade (North Holland, Amsterdam, 1972).