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PLAUSIBLE OUTCOMES FOR GAMES IN STRATEGIC FORM

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by

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INTRODUCTION

This is the first in a projected series of papers on solutions to games in matrix and extensive form. The predominant solution concept in the literature is that of the noncooperative equilibrium put forward by Nash (1951).

The major virtue of a noncooperative equilibrium is that it satisfies a form of circular stability or self fulfilling prophecy. If i thinks that j will follow his noncooperative equilibrium strategy then i's best response is to select his noncooperative equilibrium strategy and vice versa.

The well known Prisoner's Dilemma game provides both an easy example and considerable experimental evidence that the noncooperative equilibrium strategies are frequently selected.

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a

I \ II	1	2
1	5,5	-1,6
2	6,-1	0,0

b

I \ II	1	2
1	5,5	-63,60
2	50,-45	0,0

c

I \ II	1	2
1	5,5	-5,20
2	20,-5	0,0

c

I \ II	1	2
1	b_1,b_2	d_1,a_2
2	a_1,d_2	c_1,c_2

TABLE 1

Four versions of the Prisoner's Dilemma or "near Prisoner's Dilemma (Table 1c) are shown in Table 1. The games portrayed in 1a and 1b have a unique equilibrium point with payoffs of (0,0) arising from strategies (2,2). Game 1d also has a unique equilibrium point if $a_1 > b_1 > c_1 > d_1$; $a_2 > b_2 > c_2 > d_2$; $a_1 + d_1 < 2b_1$ and $a_2 + d_2 < 2b_2$. In game 1c $a_i + d_i = 20 - 5 > 2b_i = 15$ has one pure strategy equilibrium point like the others plus a correlated mixed strategy equilibrium where the players play (1,2) or (2,1) with equal probabilities if they can precommit.

Even limiting ourselves to the 2×2 matrix game it is easy to construct games with 1, 2, 3 or 4 pure strategy equilibria. Tables 2a-d provide examples.

a

I \ II	1	2
1	5,5	4,3
2	3,4	2,2

b

I \ II	1	2
1	2,1	0,0
2	0,0	1,2

c

I \ II	1	2
1	3,6	4,6
2	3,6	0,0

d

I \ II	1	2
1	6,8	3,8
2	6,4	3,4

TABLE 2

In Game 2a there is a jointly optimal pure strategy equilibrium at (1,1) yielding (5,5). Game 2b has two pure strategy equilibria and a mixed strategy equilibrium where I uses a mixed strategy of (2/3, 1/3) and II uses (1/3, 2/3) and the expected payoff to each is (2/3, 2/3). If they could correlate their strategies so that they could play (1,1) and (2,2) each with 1/2 they could obtain a payoff (3/2, 3/2).

Game 2c has a class of equilibria where I uses his first strategy and II mixes with probabilities $(p, 1-p)$ where $0 \leq p \leq 1$; and similarly II uses his first strategy and I mixes with $(p, 1-p)$ where $0 \leq p \leq 1$.

In game 2d any mix for either player will be an equilibrium strategy. All four pure strategy pairs form noncooperative equilibria.*

In Table 3 games with equilibria which dominate others are shown. In 3a there are three pure strategy noncooperative equilibria at (1,1),

	1	2	3
1	10,10	0,0	0,0
2	0,0	5,5	0,0
3	0,0	0,0	1,1

a

	1	2	3
1	10,10	-6,0	-6,0
2	0,-6	5,5	0,-6
3	0,-6	-6,0	1,1

b

TABLE 3

(2,2) and (3,3) with payoffs (10,10), (5,5) and (1,1). Game 3b has the same equilibria. However in game 3a the safety level associated with any equilibrium is zero but in game 3b the safety level associated with (1,1) is -6 but the safety level with (2,2) is zero.

We may observe from the above examples that the NCE may or may not be unique, symmetric or Pareto optimal.

Table 4 illustrates that the existence of an NCE is not perturbed

*Frequently we shall use the abbreviation NCE for noncooperative equilibrium.

by considerable changes in the structure of the payoff matrix. A matrix of general size $m \times n$ is illustrated. Suppose that a_{ij} is the largest element in the row i and b_{ij} is the largest element in the column j. Then regardless of any changes made to the $mn - m - n + 1$ elements which do not appear in either row i or column j the pair of strategies (i,j) form an equilibrium pair with payoffs (a_{ij}, b_{ij}).

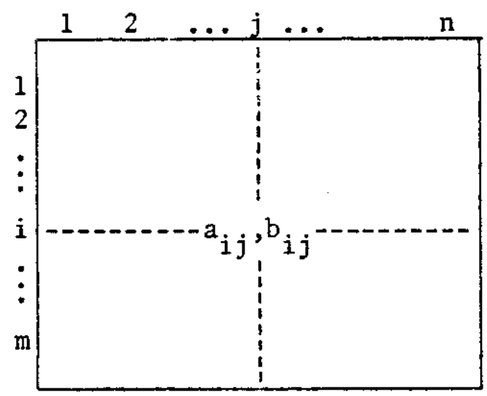

TABLE 4

The changes in the payoffs elsewhere may create new NCEs whose payoffs could dominate the payoffs (a_{ij}, b_{ij}) but even this would not disturb the stability of (i,j) as an NCE unless stability conditions beyond that of self-fulfilling prophecy are specified.

2. WHAT DO WE WANT OF A SOLUTION?

2.1. Normative or Behavioral Solutions

Traditionally game theory solutions have been divided into normative and positive or behavioral solutions. The first set of solutions are prescriptive. Rational people are advised to behave in a particular way, or to accept certain axioms of behavior as a guide. For example the value proposed by Shapley (1953) offers axioms for fair division. Various bargaining procedures have been axiomatized.

The core (see Shubik, 1982, Ch. 6) has been suggested as a solution which satisfies subgroup rationality for all sets of players in a game.

In contrast with the core and value much of the discussion concerning the noncooperative equilibrium solution has stressed best response as being a reasonable way to behave in situations with no direct communication. Furthermore some evidence can be mustered that undergraduates, or engineers or others tend to play one-shot Prisoner's Dilemma games in such a way that the NCE is a reasonably good predictor (see Rapaport and Chammah, 1965). Yet although this is true there is overwhelming evidence that as the entries in even a 2×2 matrix are varied; the briefings manipulated and a host of other factors controlled the NCE as a predictor leaves much to be desired (see Rapaport et al., 1975).

In contrast with a behavioral defense of the NCE Harsanyi and Selten (1982) offer a resolutely normative argument for the "rational selection of a single equilibrium point."

2.2. The Game and Rational Players

Without going into detail, there are four major game representations used in most investigations. The various solutions which have been suggested, in general, are related to one or possibly two of these representations. Underlying each is a large set of implicit and explicit assumptions. In essence the perceptive book of Schelling (1960) where it criticizes game theory is in fact devoted to a critique of the inappropriate use of game models for the study of some strategic problems where certain implicit and explicit assumptions did not apply.

The four major representations of a game of strategy are:

(1) The finite extensive form

(2) The strategic or normal form

(3) The cooperative or coalitional form

and (4) Some variant of an infinite extensive form.

The cooperative form is not a process model. Von Neumann and Morgenstern (1944) explicitly abstracted any considerations of costs or timing of bargaining in their discussion of the characteristic function. Edgeworth (1881) in his discussion of bargaining did the same. In essence the cooperative form is noninstitutional. There is no way one can deduce the specific form of the rules of the game from the cooperative form.

All other three representations noted are process oriented. The two extensive form representations spell out moves and information. The strategic form surpresses a great deal of structure but nevertheless explicitly reflects the rules of the game.

None of the representations are able to treat adequately the role of language. In many aspects of human behavior there is a delicate interplay between words and deeds. Items such as contract, threat and bluff depend upon this interplay and the strategic modeler is faced with the problem that in many of the strategic situations of society the rules are not rigid but depend upon the broader context in which the game is embedded.

The first three representations noted address situations with a well defined beginning and end. Board games or card games fit nicely into this category. But many aspects of politics, economics and life in general do not. There is no definite end, and the beginning may be lost in history. The fourth game representation which allows for the possibility of games of indefinite length opens up the possibility for considering neither normatively cooperative nor noncooperative solutions, but quasi-cooperative solutions whose stability derives from there always being the chance that there will be enough time left to settle accounts.

Associated with the cooperative form are the value, core, stable set, nucleolus, kernel and bargaining set solutions. Associated with the finite extensive form and strategic forms are many variants of noncooperative equilibrium and minimax solutions. The infinite horizon extensive form opens up the possibility for defining and describing many quasi-cooperative and behavioral solutions.

Underlying virtually all of formal mathematical game theoretic analysis is an extremely austere nonsocialized abstract model of the intelligent, calculating rational decisionmaker. Without passion the homo ludens of much of game theory is a colorless, sexless, classless, ageless calculating device who knows what it wants and what constitutes its set of strategies. The assumption of external symmetry made explicitly or implicitly states that any feature distinguishing Player A from Player B must be formally modeled in the game otherwise all features are assumed to be the same. Thus when the game theorist is contrasted with the social psychologist we find that the former tends to be concerned with predicting the outcomes resulting from situations involving identical individuals with different resources and positions while the latter tends to consider outcomes involving different individuals who may start with the same resources.

Much of game theory has been devoted to suggesting what an individually rational, intelligent, nonsocialized calculating consciously goal oriented individual should do when confronted with a well-defined game of strategy.

How successful or useful this approach is cannot be answered without reference to context. Hence we turn to an explicit consideration of both the context and purpose of the models.

3. WHAT ARE THE CONTEXTUAL ASSUMPTIONS?

3.1. Noninstitutional Statics

Much of the success of modern economic theory and political science has been in the investigation of situations involving faceless crowds of individual agents. In particular the attractiveness and apparent power of the modern theory of the price system and mass markets comes from the attenuation of much personal interaction. The essence of decentralization is that individuals need not think about other individuals, but plan their actions against a mechanism called the market. Personality is irrelevant, individual power except to inflict self-harm is nonexistent and special information is of fleeting worth.

Under the appropriate assumptions a large array of different models and solutions all lead to the mass market price system (for a survey see Shubik, 1984). It is possible to construct game models in both strategic and cooperative form and have the NCE, value core and other solutions predict the same set of outcomes in what appears to be a virtually institution free context.

Unfortunately what may hold for a mass market under special circumstances does not hold if there is even one agent of substantial size. The various structures of the mechanism influence outcome and the possibilities for individual signaling and threat may appear.

3.2. Finite Process Models

If one is to understand the structure of strategic interaction even for as few as two individuals the salient features of the game must be spelled out. The extensive form does this in detail and the strategic form does it in a somewhat aggregated manner via the concept of strategy.

In the context of the society, polity or economy the construction of a game in extensive form requires the implicit specification of the institutions and laws of the society. They are described in the rules of the game. Thus when we try to model trade as a game in extensive or strategic form we can start to identify the basic features which distinguish and describe markets, banks, clearing houses and other economic institutions.

When the situation to be modeled is a diplomatic negotiation, a revolution or a mass march we tend to find that our lack of substantive knowledge and the difficulties encountered in sorting out psychological, socio-pyschological, economic, legal, political and other factors make the task of specifying a plausible extensive form difficult and even of dubious worth. The perceptive essay or even a simulation may provide better tools for analysis.

Because of the difficulty in being able to construct extensive form models of many "soft science processes" we run the danger of gross oversimplifications in order to force them upon our methodological bed of Procustes. In particular it is for this reason that we must approach all interpretations of results from simple experimental games such as the Prisoner's Dilemma with great circumspection.

Von Neumann and Morgenstern (1944) in the first chapter of their book warned that the construction of a game theory dynamics might pose considerable difficulties. They stressed that they felt that it was desirable to explore the statics first. They stressed a cooperative theory. But in doing so not only did they surpress the dynamics they even removed the description of the rules of the game by the device of using the characteristic function.

Before one tries to develop a full dynamics, the description of the game in strategic or extensive form provides an understanding of the structural bounds on play. The strategic form which by the device of the strategy collapses the finite extensive form into a matrix or one shot game surpresses much of the structure but not as much as the coalitional form. The noncooperative equilibrium solution applied to a game in strategic form may be regarded as a static solution. All move simultaneously, beyond that time plays no role and the path of play is irrelevant.

3.3. The Infinite Future: Markovian Dynamics

The von Neumann-Morgenstern theory deals with games with a specific beginning and finite end. The analogy with formal games has already been noted. If we wish to construct models which appear to be better approximations of many societal, political and economic processes we need to extend the horizon to an indefinite future. The cost of doing so is to complicate the concept of solution and change the mathematical requirements.

Two natural classes of model which have been considered are repeated games with a stochastic ending or with a discounted payoff. The repeated matrix game offers experimental possibilities in either of these forms. An attractive candidate for a solution to a stochastic game (see Heyman and Sobel, 1984) is an NCE involving strategies which are only dependent on the current state.

When we consider applications of stochastic games however we must ask what phenomena can be best represented. I suggest that in virtually all applications there are several important distinctions which should be made. They are:

Two person: face-to-face

" " : anonymous

Two institutions

One individual and an institution

Few individuals face-to-face

" " anonymous

Many individuals anonymous without group identity or affiliation

" " " with " " " "

The formal models most amenable to analysis are two person games and many person games. It is hard to justify, except on an *ad hoc* basis the assumption that in situations involving two individuals interacting over time anonymity is reasonable and history and personality do not matter.

Fortunately for the applications of duels and antagonistic games in general the assumptions are justified. Furthermore in economic analysis if we believe that the assumption of a mass market with no large agent is justified then the dynamics of such a market may be studied as though it were a collection of individuals each facing his own dynamic program.

In virtually all other instances history, personality and institutions appear to matter. These cannot be ignored even by the experimenter using the simplest of matrix games. The players bring their personalities, mindsets, socialization and training with them and both these and the initial briefing must be taken into account.

Partially in jest, partially seriously John Kennedy of the Department of Psychology at Princeton noted that given control of the briefing an experimenter should be able to get virtually any of the results he wants.

3.4. The Infinite Past: History Matters

In experimenting with as simple a game as that shown in Table 5 some individuals acting as Player 1 select their first strategy and others

		1	2
Player	1	2,1	0,0
	2	0,0	1,2

TABLE 5

select their second. Each easily supplies a rationalization; one of the variety "strategy one is best for me" the other "I think my opponent may be greedy hence I am safer playing strategy two in order to get some payoff."

A briefing telling all players that their competitors are greedy and stubborn appears to influence the outcome.

How are we to control or initialize the initial expectations or subjective probabilities attached by players concerning the nature and behavior of their competitors. One way of doing this is by the initial briefing and this may involve telling a player that he has taken over from a previous player while his competitor is still the same. The new player is then supplied with a history of k periods of play. For example one briefing for the game in Table 5 might be: "During the last 100 periods (2,2) has been played all the time." Another briefing would be "(1,1) and (2,2) have been played alternately for as long as we can remember and (1,1) was played last time."

How far back into history we want to go or need to go appears to be a matter of understanding the problem and its context. It is not merely a problem in methodology or mathematics. It is here that revenge, national pride and other factors regarded as irrelevant, irrational or uninteresting in an economics oriented decision theory appear.

3.5. Does Language Matter?

One school of thought has it that "a barking dog never bites" and "sticks and stones will break my bones but names will never hurt me." Another school takes threat, slander, innuendo and promises as serious. Many of the examples in brinksmanship and bargaining used by Schelling (1960) depend delicately upon words as deeds. Sometimes deeds are best interpreted as part of the conversation. Someone is shot with a foot over the border just to convey the message that we mean that we do not want anyone to cross the border.

Formal game theory does not provide us with a way to encode speech and gesture as moves. In a mass market you cannot argue with the tape, but in a thin market you can argue with the sellers. In a disarmament conference words and gestures are part of the play.

We do not know how to code language into strategies. But at least in experimental games we can introduce a limited set of messages as formal moves. For example consider the game portrayed in Figure 1. We may interpret the first four moves as messages from Player 2 to Player 1 concerning what he intends to do if Player 1 selects 1 or 2. They can be read as:

$$\text{If } P_1 \text{ selects 1 then } \tfrac{1}{2}\text{; if } P_1 \text{ selects 2 then } \tfrac{1}{2}.$$

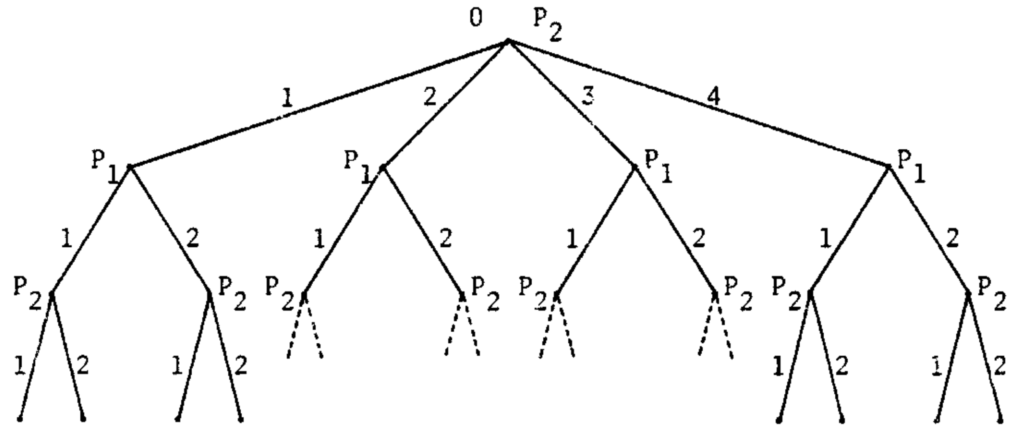

FIGURE 1

We might also include a fifth alternative in which no message is sent. If we include this then the total number of strategies available to P_2 is $5\cdot(2)^{10}$ or 5,120.

Griesmer and Shubik ran a pilot study of a game with this structure in 1962 but I am not aware of the results of any systematic study of games with messages.

It must be noted that in a two person constant sum game, language plays no role except psychological. The only words are deeds. In mass societies individuals can still send simple signals to large groups by wearing badges, campaign buttons, concentration camp numbers or uniforms. Even with large numbers many interactions are binary between two individuals who have to find out if they are friend, foe or neutral.

4. THE SOCIAL SCIENCE SHOPPING LIST

In this section a sketch is given of some of the factors we need to consider when we try to apply strategic analysis to a host of different conflict and cooperation scenarios.

4.1. Historical, Biological, Chronological and Ordinal Time

Cooperative game theory is timeless. This is one of the major reasons why interpreting experiments based on the characteristic function is so difficult. If negotiations take several hours this expenditure of effort may influence perceived payoffs.

The finite game in strategic form is timeless. All players have one (possibly enormously complex) move and all move simultaneously.

The finite game in extensive form has ordinal time. Moves are sequenced but there is no measure of elapsed time. In essence the game tree is event oriented. Actions and the sequencing of actions count, not the time involved.

Yet chess championship games have time limits. Furthermore we frequently wait for decisions to mature or tempers to cool or even for time to heal wounds and to soften or obliterate some memories.

Repeated games or stochastic games tend to be represented with a fixed clock. Each period measures some unspecified Δt and there may be many periods. When a discount factor is introduced as is the case for business games and many economic models the (usually fixed) time period is a quarter or a year.

A key factor distinguishing many problems in the behavioral sciences is the length of time involved in a process. Elapsed time appears to be related to whether decisions or acts are instinctive, consciously thought through, unconscious or habit guided. Qualitatively new problems have been posed by the existence of nuclear missiles where decisions to loose mass destruction must be made by a handful of individuals in less time than most people need to decide to buy a new lawnmower.

The bias of many economists and operations researchers has been towards decision problems over relatively short times, say a few weeks to a few years. It is in this zone where many environmental factors, habits, customs and laws can be regarded as constant. The decisionmaking takes place within the arena of the economy and for the most part concentrates on conscious decisionmaking.

Decisions to marry, have children, commit suicide, kill, declare war, found the National Socialist party, go on a hunger strike, move the tribe westward from the Urals, die at Massada all may have some element of conscious economic decisionmaking to them. But not only is there more, the time scale, scope and context of each is significantly different from the others. The will of a group or a species or a set of genes to survive may be measured on an even longer time scale and may depend far more on instinctive than calculated decisionmaking.

At the level of the more specific, in individual and international relations how long does it take to form trust, respect and consistent beliefs? How long does it take to destroy them and rebuild them? It has been said that "if your friend betrays you once it is his fault, if he betrays you twice it is your fault." Is this merely a matter for ordinal time Bayesian updating or is a more complex process description called for?

4.2. Players and Population

One of the most powerful and useful assumptions in the construction of game theoretic models is the assumption of external symmetry. All personal attributes not specified are assumed to be the same. For many problems the model of the player without personal attributes acting as a principal in an institution free environment may be a reasonable approximation. But for virtually any political or international strategic problem the players are fiduciaries acting through bureaucracies. We use phrases such as "the Russians want" or "the State Department intends." It is easy to deal with such phrases in rhetoric or in essay form but it is extremely difficult to produce useful formal models of the State Department as a player with preferences.

In the literature of operations research we can find titles such as "Solveable Nuclear Wars" (Dalkey, 1965) or "The Statistics of Deadly Quarrels" (Richardson, 1960). The simple model or special statistic serves to call attention to and provide analysis for a special and possibly important point. But nuclear wars are not solveable and deadly exchanges may be grossly misrepresented by body counts. How usefully we can represent whole nations as actors depends heavily upon the question at hand.

The basic distinctions concerning individuals is are they acting as principal agents or as fiduciaries for others. But in the study of strategic behavior what simplification is to be made as to what constitutes a player is critical. Political scientists study "the games nations play." Are institutions actors or should we model them as games within games set in a larger context? At the least we need to distinguish the indivdiual, the informal group, the formal group, various institutions and nations.

One possible modification to the assumption of external symmetry among the players is to consider a population with different arrays of attributes such as hawk or dove, thus as has been considered in biological models we might interpret mixed strategy equilibria as arising from chance encounters with different behavioral types. The recent work of Axelrod (1983) is oriented in this way.

4.3. Preferences

Much has been written about individual preferences. Only three points are stressed here. The first concerns how to describe preferences for organizations or institutions if they are to be treated as players.

The second point appears to be of importance both in the context of political and economic life and in military matters. That is the distinction between personal risk and risk taken on when acting as an agent or fiduciary for other people's money and lives. The economic theory of agency attempts to explain the behavior of agents strictly in terms of economic organizational structure which provides the structure of incentives. Yet the socio-psychological and sociological features of loyalty, honesty, morale, responsibility, pride and other factors appear to play an important role in determining the behavior of generals, civil servants and corporate presidents. The responsibility assumed in sending troops into battle does not appear to be usefully portrayed primarily in terms of economic analysis.

The third point is that in my opinion not enough stress has been laid upon the importance attached to survival in individual preferences.

4.4. Psychological Limits

The survey on decisionmaking and decision theory of Abelson and Levi (1983) provides a relatively comprehensive coverage of some of the problems seen by the psychologist in analyzing decisionmaking. These include limits to memory, limits to calculation, faulty perception and the importance of problem representation in influencing decisions.

Possibly the most important open question at the core of strategic analysis is how individual form subjective probability estimates and how they update them. The experimented evidence that they do not appear to use Bayesian updating does not invalidate the logic of Bayes, but suggests two hypotheses. Individuals may be somewhat less than logical and could benefit from training. The way individuals often use new information is not merely to update and modify odds but to reorganize their perceptions of the causal structure of the system being considered.

4.5. Socio-psychological and Other Criteria

In our search for solutions we need to ask what considerations must be taken into account. How rich must the models be to account for the phenomena we feel to be of critical significance in a process. For example in much of economic theory evaluation and judgment are taken as given, or if there are two individuals with equal resources and risk preferences the one who has less uncertainty concerning evaluation will perform better. Yet the best securities analysts do not appear to be the best investors. Perception and calculation do not appear to be the same as perception, calculation, commitment and decisiveness. Yet even in economics it is precisely where the numbers are few and the stakes are high that factors seek as the courage of one's convictions count.

It is a monument to the success of economic theory that so much can be squeezed from the parsimonious assumptions of given preferences, many rational actors, initial wealth and technology. But it appears that in spite of the economic components to society, politics and war the parsimony of economic theory is not sufficient to provide good explanations elsewhere.

An informal list is presented in Table 6 to indicate some of the factors which are regarded by different social scientists as relevant to decisionmaking. Many of the words such as loyalty, hope, faith are catch-all names for a highly complex set of attributes. Yet when we try to explain strategic behavior there is some context in which each item noted

Individual Psychological Factors	Socio-Psychological Factors	Political Factors	Sociological Factors	Mathematical & Philosophical Solution Properties
preference	specific concern for		social conscience	domain of decision
risk & uncertainty pref.	others	political belief	social position	payoff transformation
tolerance of ambiguity	envy		wealth	local/global solution
self-concern	revenge	nationalism	nationalism	union of games
fear	hate	militarism	militarism	uniqueness
judgmental bias	love			independence of games
self-control	trust	ideology	bureaucracy	symmetry
motivation	greed			safety level
aspiration	honesty	power	religion	external symmetry
instinctive behavior	suspicion			certainty equivalence
survival	sacrifice for young	patriotism	hero/martyr	insufficient reason
age				irrelevant alternatives
sex	signalling			continuity
habit	coordination		societal norms & values	ordinal/cardinal utility
faith	cooperation			limit behavior
sloth			loyalty	
knowledge			custom	value
coding & editing			justice	
memory			tradition	efficiency/optimality
span of attention				
intelligence				stability
rationality		fiduciary norms	fiduciary norms	equilibrium & best response
"cleverness"				
talent				
experience				
commitment				
determination				

TABLE 6

is a factor of consequence. Revenge and envy may not enter into consideration when buying a pound of bacon; but they do when the decision is made to continue a vendetta.

The shopping list is clearly even larger, for example, health and demographic features such as a species innate drive to reproduce have not been included. The overall psychological concerns on perception and cognition are only partially covered.

An important constructive use of theory and gaming experiments is to isolate why and where intuitively important concepts fit into our models and explanation of behavior. Thus we may take a concept such as revenge or envy and ask what is the simplest game in which we would be able to attribute motivation to such factors. It is with this in mind that I suggest that the very success of much of game theoretic thought and experimental gaming may come from their apparent lack of success in being able to answer what is a solution of high predictive value for how individuals will play a one shot or many period two person matrix game.

There is no paradox and no pessimism to this observation. We have a language, a methodology and the possibility to perform some experiments of interest. The noncooperative equilibrium and minimax solutions do not appear to be particularly useful as predictors in general even though they may be quite good in certain contexts. Our problem is to find better solutions and to justify or explain the influence of different contexts.

4.6. A Caveat on Purposeful Modeling

Good modeling calls for (1) clarity of purpose, (2) parsimony, (3) relevance and (4) analytical feasibility. Analogy and example can offer considerable aid in gaining insight. But they can also be devices to mislead by false analogy and special or pathological example. In the context of game theoretic reasoning these dangers are easy to illustrate. It is well known among social psychologists that the running of a simple game with the same mathematical structure in each instance but with different scenarios will lead to different behavior (see the Ph.D thesis of R. Simon, 1967). It is also clear that whole books and hundreds if not thousands of articles have been devoted to the Prisoner's Dilemma game with little argument or discussion devoted to how typical or valuable an experimental game it is and how generalizable are results obtained from experiments using it.

Experimental games may only reflect a few of the factors in command and control systems for nuclear weapons. These systems may manifest a highly different competitive decision structure than political conflict and certainly than competition in mass markets.

Game theory offers abstract models for the study of conflict and cooperation. But the abstraction sufficient to illustrate mass markets may not stretch to mass warfare, murder or even to a Potlach. The offering of solution concepts for context free games played by hypothetically personality free players is a useful exercise in normative game theory but it is not the only approach. Even at the philosophical level individuals are at best idealized as machines with finite capacity hence there are some basic problems to be faced in even defining individual rational behavior.

4.7. Death, Triumph and Disaster

Prior to discussing matrix games one further basic warning is in order. In using matrix games even as analogies in the discussion of topics such as war, diplomacy or any situations involving high or low risk and items of high value such as death the very basis of justification for assigning expected subjective valuation of outcomes is at stake. Kahneman and Tversky (1973) have suggested a π function for subjective probabilities which is not well behaved at the extreme ranges, overestimates low probabilities and underestimates high probabilities.

The act of formulating an abstract matrix game and presenting it to experimental subjects without a detailed discussion of what the abstract von Neumann-Morgenstern expected utilities mean to the players hides many of the key problems in understanding the linkages among psychological socio-psychological and cultural phenomena and the abstractions of game theory.

5. THE SEARCH FOR MEASURES

The remarks here are confined to games in matrix form played by individuals acting on behalf of themselves. Do we have a reasonable theory as to how they will be played if they are played once?

The question being asked here is considerably less ambitious than any of the burning questions concerning military, political or organizational behavior. As a start it is not even at the level of complexity of "do Russian, Chinese, English and American students play matrix games differently?"

5.1. The Probable Outcome

The noncooperative equilibrium as illustrated in the examples in Table 2 is not necessarily unique. When it is unique and involves only pure strategies the prediction of the theory is clear. A specific outcome is predicted with probability of one. When the points satisfying the optimum response conditions are not unique we need extra conditions in order to select among them. We may follow Nash's suggestion and require those conditions to be satisfied:

(1) Best response

(2) Equal value

and (3) Interchangeability of strategies

	1	2	3
1	3,3	0,0	3,3
2	1,2	-1,-1	2,1
3	3,3	0,0	3,3

a

	1	2	3
1	3,3	0,0	0,0
2	0,0	2,2	0,0
3	0,0	0,0	1,1

b

TABLE 7

In Table 7a the four pure strategy equilibrium points all have the same value of (3,3). The strategies for each player are interchangeable being (1,1), (1,3), (3,1) or (3,3). In Table 7b the three equilibrium points have neither the same value nor can strategies be interchanged hence in the sense of satisfying the three criteria the game has no solution.

If we wished to impose the extra condition that:

(4) Choice among NCE compatible strategies is limited to NCEs whose payoffs are not dominated by other NCEs.

then in Table 7b there is one NCE at (1,1) with values (3,3)

For a noncooperative solution neither the equal value nor the interchangeability conditions appear strongly justified as normative or behavioral conditions, whereas condition (4) is justified on both grounds.

Limiting ourselves to the best response assumption alone, what prediction should we make in a one shot game? If the game has a unique pure strategy equilibrium such as in games 8a and 8b we should predict that outcome with certainty. In both cases it is (2,2). The game in Table 8c a

	1	2
1	5,5	-5,10
2	10,-14	0,0

b

	1	2
1	-5,-5	2,2
2	3,2	5,5

c

	1	2
1	-1,1	1,-1
2	1,-1	-1,1

d

	1	2
1	-1,1	0,0
2	0,0	-1,1

TABLE 8

has a unique mixed strategy equilibrium point where each player randomizes with probabilities (1/2, 1/2) on the two pure strategies available to each. Thus the predicted outcome will be .25 on (1,1), .25 on (1,2), .25 on (2,1) and .25 on (2,2) (abbreviated to (.25, .25, .25, .25)) or displayed in matrix form as in Table 9a:

a

	1	2
1	.25	.25
2	.25	.25

b

	1	2
1	2/9	4/9
2	1/9	2/9

TABLE 9

The game in Table 8c is zero sum and the unique equilibrium point is also a saddlepoint. The game in Table 8d is also zero sum and has the same equilibrium point hence the same expected outcomes. But it is not a symmetric game. It is biased against Player 1, he expects losses. If the only criterion is best response this should make no difference.

Without further conditions than best response we have no way of predicting outcomes if there is more than one NCE. Possibly the weakest extra condition we could add is:

(5) The selection by the other player of any one of his NCE strategies is equiprobable.

Consider the game illustrated in Table 10a. It has two pure strategy and one mixed strategy equilibrium points. What should we predict

	1	2
1	2,1	0,0
2	0,0	1,2

a

	1	2
1	2000,1	0,0
2	0,0	1000,2

b

TABLE 10

as probable outcomes? The equilibria are (1,1), (2,2) and mixtures of (2/3, 1/3) and (1/3, 2/3). Suppose that we had a large sample of pairs of players what frequency of outcomes do we expect? If an individual as a game Player 1 really believed in external symmetry and that it* faced an opponent who would randomly select a strategy associated with an NCE it would expect a frequency of:

$$\frac{1}{3}(1) + \frac{1}{3}\left(\frac{1}{3}\right) + \frac{1}{3}(0) = \frac{4}{9} \text{ for strategy 1}$$

and $\frac{1}{3}(0) + \frac{1}{3}\left(\frac{2}{3}\right) + \frac{1}{3}(1) = \frac{5}{9}$ for strategy 2

But if it believes this, his best response is to play 1 and obtain 8/9. Any Player 1 will do so and this can be deduced by any Player 2 who will react against the expectation of Player 1's behavior.

In actual one-shot plays of this game some Player 1's rationalized their choice with "I am greedy and he should be able to figure that out hence I choose 1"; others chose 2 saying "I guess my opponent is greedy so by choosing 2 at least I will get something." This suggests that the

*I adopt the Chinese convention or the convention that the strategic player may be a machine, a human or other organism who can best be referred to generically as "it."

nonsymmetry is being treated as a personality nonsymmetry.

Short of imposing extra socio-psychological conditions we have no way to argue in favor of (1,1) or (2,2). The only equilibrium point which is consistent with the assumption of insufficient reason and the full symmetry, i.e. the assumption that in a game that is intrinsically symmetric (the names of players can be interchanged leaving the game structure the same) is the mixed strategy NCE where we can assume that both populations of Players 1 and 2 are the same and each expects the other to randomize with Players 1 using (2/3, 1/3) and Players 2 using (1/3, 2/3). This gives an expected outcome shown in Table 9b.

Our conclusion is that condition (5) cannot be used in conjunction with (1) to form consistent expectations that are socio psychologically neutral. The principle of insufficient reason leaves only the mixed strategy.

An extra condition which appears to be reasonable is (6) given below:

(6) In a symmetric two person matrix game the only socio-psychologically neutral NCE is a symmetric one.

Applying conditions (1), (4) and (6) to the games in Tables 7a and 10a we would predict (1,1) with certainty for 7a and the distribution shown in 9b for the game in Table 10a.

In Table 10b the payoffs to Player 1 have been multiplied by 1000 in comparison with 10a. The NCEs are still all the same. The game is symmetric if we assume condition (7a) but not so if we assume (7b):

(7a) Cardinal but not comparable utilities are assumed, i.e. games are left unchanged by linear transformations of utility scales,

(7b) Cardinal utility scales and interpersonal comparisons are assumed.

Even with interpersonal comparisons do we expect the same behavior in games 10a and 10b? Condition (1) gives all three NCEs, condition (4) is not relevant and as the game is not symmetric under (7b) we need some other convention for guidance. But the mere fact that individuals choose to compare the size of payoffs is a socio-psychological phenomenon. Thus it would appear to be reasonable to seek an extra criterion from this source.

The mathematical and philosophical shopping list has been sketched in Table 6. Only a few factors have been discussed here and a discussion of the others is deferred for further projected work. A few general observations remain to be made. Harsanyi in his work both on games with incomplete information and his tracing procedure (see Harsanyi, 1975) as well as his work with Selten on the selection of a unique NCE (see Harsanyi, 1982) is resolutely non-psychological and non-socio-psychological. Yet the concept of the playing of a one-shot matrix game *in vitro* by individuals with neither a psychological profile nor any sociological appears to be somewhat pathological. The Harsanyi tracing procedure selects a unique NCE but only as a function of initial subjective expectations and these expectations appear to be formed in part from socio-psychological, cultural and other factors.

The initial formulation of expectations about the nature of one's competitor in any experimental matrix game is based upon the briefing given, the context of the experiment and the various backgrounds and experiences of the players. At best a rationalistic attempt to pick out one equilibrium point does no more than provide a benchmark. Players do not appear to work out sociology-free and psychology-free infinite regressions. The weight of experimental evidence shows a scattering of outcomes selected over even simple 2×2 games. The best response argument for the one-shot game provides some limitations to expected outcomes, but, for example the reasons why I might be willing to predict a high frequency of (1,1) outcomes to players in the game in Table 11a is not merely because it is

	1	2	3
1	10,10	9,9	9,9
2	9,9	0,0	0,0
3	9,9	0,0	0,0

a

	1	2	3
1	10,10	9,9	9,9
2	9,9	$10+\varepsilon,10+\varepsilon$	-10,-10
3	9,9	-10,-10	0,0

b

TABLE 11

best response but because it is (1) Best rèsponse, (2) Pareto optimal, (3) symmetric and (4) high safety level. The choice between (1,1) and (2,2) in the game in Table 11b is less clear. Although $(10+\varepsilon,10+\varepsilon)$ dominates (10,10), the safety level for (10,10) is better. A matrix such as 11a requires little social psychological insight to make a prediction; 11b may require more and 10a still more.

5.2. The Coefficient of Concern

One way to begin to reflect socio-psychological interlinkages is via the utility functions of the players. In particular, as a first approximation we might consider:

$$\Pi_i = P_i + \sum_{j \neq i} \theta_{ij} P_j$$

P_i is the individual's utility function in a one person framework and θ_{ij} is a measure of the coefficient of concern of i for the welfare of j. For many purposes it appears to be reasonable to separate the evaluation of rewards to an individual in isolation and rewards to an individual in the context of others also obtaining rewards. An example is provided in 5.3.

Various settings of the θ_{ij} yield the analoges of the independent utility games of:

$$\text{Joint maximum} \quad \theta_{ij} = 1 \quad \text{for all} \quad i, j$$

$$\text{Noncooperative equilibrium} \quad \theta_{ii} = 1 \quad \theta_{ij} = 0 \quad i \neq j$$

$$\text{Best the average} \quad \theta_{ii} = 1 \quad \theta_{ij} = \frac{-1}{n-1} \quad i \neq j$$

This type of transformation has been reasonably well known among game theorists and provides a way of recasting the extremes of cooperation and competition as noncooperative games. The first formulation of the joint maximum in this form appears to be that of Edgeworth (1881).

The point to be stressed here is that much of the power of economic theory and the usual way of thinking about noncooperative equilibria stresses conscious individualistic behavior with little attention to how concern for others has or has not been tucked into the utility functions (see Dubey and Shubik, 1984). It is resolutely non-sociological. Compassion, greed, spite and revenge have no roles.

5.3. Envy Begins at One-Ply, Revenge at Two-Ply

In Table 6 a host of factors pertaining to different disciplines were suggested. Can they all be illustrated or isolated in the construction and play of one stage or repeated matrix games?

I suggest that it may be fruitful to attempt to construct minimally complex games to illustrate these factors. Tables 12a and 12b provide two examples.

	1	2	3
1	3,10000	0,0	2,2
2	0,0	0,0	2,2
3	2,2	2,2	2,2

a

	1	2	3
1	5,5	-5,10	-20,-1
2	10,-5	0,0	-20,-1
3	-1,-20	-1,-20	-15,-15

b

TABLE 12

In Table 12a the equilibrium point at (1,1) dominates all other NCEs. But if Player 1 envies Player 2 it may be happier to settle for 2 each than to "give" Player 2 an extra 9,998 in return for obtaining 3 instead of 2.

Consider the game in Table 12b. Played once both (2,2) and (3,3) are NCEs and (2,2) dominates (3,3) in payoffs. Suppose this game is played twice by a highly cooperative but revenge oriented player the strategy "play 1 at first round; if "other" plays 1 then play 2 second round otherwise play 3 seems as "rational" if not more rational than play 2 on both occasions.

6. WHERE TO FROM HERE?

The development of economic game theory especially for mass markets has been to some extent an essay in the study of strategic decisions where psychology and social-psychology is minimally important. Much of the search for solution concepts for n-person games has had a normative bent based upon the abstraction of the culture-free, personality-free, society-free rational individual.

A cogent argument for utilizing this model of the decisionmaker is because it is analytically easier and better defined than models with limited capacity and perception. In spite of the rhetorical attractiveness of Simon's "Satisficing man" there is a Will-o-the-Wisp quality to attempts to produce formal models and to define the meaning of rational behavior for the decisionmaker as a finite device interacting with other finite devices. We are forced to raise many of the basic questions posed in artificial intelligence and must confront the possibility that as soon as we postulate individuals who can never know as much as society as a whole,cultural norms and societal conventions become necessary devices to code into manageable size the vast body of data, information and knowledge which the single individual cannot master.

My suggestion is that the time is ripe for the development of context specific theories of decisionmaking with stress upon the distinctions and interlinks among estimates of exogenous and endogenous uncertainty and the actual taking of responsibility for decisions. In particular the employment of context-free game theoretic models in the study of international relations, arms control and other bargaining must be done with great circumspection. The use of simple analogies may obliterate or distort or distract from our understanding of the process at hand.

In parallel with stress upon context, the very pathological simplicity of the matrix game provides an experimental device for the posing of questions and design of experiments not in game theory alone but in the array of other behavioral aspects to strategic decisionmaking.

I suspect that the way to blend strategic behavior with behavioral bias is to consider players as managers running idiosyncratic agents who they do not fully control. The manager is strategic but some of his agents may be behaviorially limited in their choices if they get the move.

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