

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

AT YALE UNIVERSITY

Box 2125, Yale Station  
New Haven, Connecticut 06520

COWLES FOUNDATION DISCUSSION PAPER NO. 691

Note: Cowles Foundation Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. Request for single copies of a Paper will be filled by the Cowles Foundation within the limits of the supply. References in publications to Discussion Papers (other than mere acknowledgement by a writer that he has access to such unpublished material) should be cleared with the author to protect the tentative character of these papers.

INSURANCE, FLEXIBILITY AND NON-CONTINGENT TRADES

Russell Cooper

February 1984

PRELIMINARY DRAFT  
Do Not Quote

INSURANCE, FLEXIBILITY AND NON-CONTINGENT TRADES

by

Russell Cooper\*

Cowles Foundation for Research in Economics,  
Box 2125, Yale Station  
New Haven, Connecticut 06520

October 1982

Revised February 1984

Abstract

This paper considers non-contingent trades through either forward markets or simple contracts. The point of the inquiry is to understand the costs and benefits of trades of this nature. We focus on the tradeoff between insurance (a benefit) and the loss of flexibility in decisions (a cost) as determining properties of trading in forward markets. This tradeoff is also used to explore contract length.

\*Cowles Foundation for Research in Economics, Yale University. Many of the issues raised in this paper resulted from conversations with Costas Azariadis and Michael Riordan. John Bigelow, Philip Dybvig, Jon Eaton, Mark Machina, Richard Nelson and Gary Yohe provided helpful comments for which I am grateful. Financial support from the National Science Foundation (SES 8208899) is gratefully acknowledged.

## I. Introduction

With either a complete set of contingent claims markets (Debreu [1959]) or a full set of securities (Arrow [1964]), the competitive economy is capable of efficiently sharing risks and allocating resources. For purposes of understanding the imperfect nature of insurance arrangements and other allocative inefficiencies, it is often useful to consider economies with incomplete rather than complete markets. This leads naturally to two questions: (1) what are the efficiency properties of incomplete markets models and (2) why are some contingent markets operating while others do not appear to exist?

We know from the results of Diamond [1967], Grossman [1977], Hart [1975] and Stiglitz [1982], for example, that equilibria with incomplete markets will not generally be constrained Pareto optima. That is, a planner facing the same trading restrictions imposed on agents, may be able to improve upon equilibrium allocations arising from incomplete markets.

While the assumption of incomplete markets has provided a rich framework for analysis, the development of models explaining the assumed market structure has lagged far behind. At one extreme, we may consider the non-existence of complicated contingent-trading schemes. This type of incompleteness is generally attributed to informational diversity and the presence of transactions costs. (In light of the work of Harris-Townsend [1981], Myerson [1979] and others one must be a bit careful in specifying the connection between information asymmetries and incomplete markets.)

This paper adopts a somewhat different approach to understanding

market structure. Instead of asking why very elaborate contingent claims markets have not arisen, we focus attention on the non-existence of very simple forward markets.<sup>1</sup> These are markets for non-contingent claims which do not require the revelation of private information for their implementation. Further, the transactions costs associated with these non-contingent claims appear to be minimal. As these markets do assist in the sharing of extrinsic risks (i.e. risks caused by random opportunities as opposed to stochastic endowments or preferences) such as price uncertainty, the non-existence of forward markets in many commodities (labor, currency and agricultural products are exceptions) seems puzzling.

The thesis of this paper is that the key element in understanding whether forward markets will arise is the tradeoff between insurance and flexibility. Risk averse agents facing price variability can obtain insurance through the purchase of non-contingent claims to the consumption good. However, since the terms of these claims are independent of the realization of any relevant random variable, allocative distortions may arise. Hence, non-contingent claims are traded when the insurance gains exceed the loss of flexibility.

The demand for flexibility in uncertain environments has been studied by many authors, including Goldman [1974], Kreps [1979], Marschak-Nelson [1962] and Stigler [1939]. Intuitively, the demand for flexibility is a desire to keep ones "options open" in the face of uncertainty. As discussed below, the trading of non-contingent claims can severely reduce an agent's flexibility, ex post.

Using this tradeoff, we can explore the characteristics of non-contingent claims markets. We do so through a series of examples to illustrate the wide applicability of this framework. Section II of the

paper discusses the existence of these markets by focusing on the consumption decision of a risk averse agent facing price uncertainty. Agents can either purchase the commodity ex post through spot markets or ex ante via non-contingent claims. We show that this choice, and hence the market structure, is determined by the benefits of insurance and the costs of the lost flexibility. Another example of this type was used in Azariadis-Cooper [1984]. If claims markets do operate, we show that the volume of trade is determined by the same factors of risk aversion and the demand for flexibility. This is shown, in Section III, using a model of a firm deciding on the quantities of output to be produced ex ante and ex post. This example also shows that the timing of economic decisions can be related to the flexibility-insurance tradeoff. Finally, in Section IV, we focus on the issue of contract length. A complete set of markets would specify trades of contingent on states and time. In previous sections of the paper, we focused on single-period contracts. Now we consider the optimal length of such an agreement. Non-contingent claims should have maximal length to exploit their insurance characteristics, but by their very nature they ignore realizations of random variables. If these realizations convey information on future distributions, it may be costly to ignore them. Hence, the demand for flexibility and the gains of insurance can determine contract length. Section V includes the conclusions and a discussion of extensions of the research.

In this analysis, no distinction is made between trade in forward markets and the exchange of goods and/or labor services through non-contingent contracts. For our purposes, these transactions are identical. The important aspect of each is that quantity decisions and transfers of money are predetermined.

## II. A Model of Forward Contracts and Consumer Purchases

The prices of many commodities we purchase are random. This implies a demand for forward contracts of these commodities by risk averse agents. Yet, in general, these forward contracts do not exist. For example, if I knew with certainty that in 10 years I would buy a Cadillac and the market price of the car was random, then I would obviously prefer, given my risk aversion, to purchase the car through an actuarially fair forward contract. Yet, if I am uncertain about my future demands, I might be unwilling to commit myself to such a non-contingent purchase.<sup>2</sup> That is, I may want to remain flexible and hence able to respond to changes in relative prices and my own tastes. This is essentially the story put forth by Arrow [1974] in his discussion of incomplete markets. This discussion is also related to the question of whether or not price variability increases expected utility.<sup>3</sup>

Consider a consumer with non-stochastic preferences  $U(x,y)$  where the utility function is increasing and strictly concave in its arguments. Further assume that income,  $I$ , is given and that the price of commodity  $y$  is normalized to one. The price of commodity  $x$ ,  $p$ , is random with a cumulative distribution function  $F(p)$ . This random variable takes on values in the interval  $[\underline{p}, \bar{p}]$  so that  $F(\underline{p}) = 0$  and  $F(\bar{p}) = 1$ .

If our consumer waits until the realizations of  $p$  are known before choosing  $x$ , then this optimal choice  $x^*(p)$  will satisfy

$$pU_1(I - px, x) = U_2(I - px, x) . \quad (1)$$

In this expression  $U_i$  is the derivative of the utility function with respect to its  $i^{\text{th}}$  argument. We denote by  $V^F$ , the expected utility given the flexible choice of  $x^*(p)$ .

$$V^F \equiv E_p U(I - px^*(p), x^*(p)) . \quad (2)$$

Alternatively, our consumer could choose  $x$  prior to the realization of  $p$ . That is the consumer could commit himself to purchase a constant amount,  $\hat{x}$ . This would obviously not be beneficial if  $p$  remained variable, i.e.

$$\max_x U(I - px, x) \geq U(I - p\hat{x}, \hat{x}) \quad \text{for all } p . \quad (3)$$

However, assume that a forward market exists which allows for purchases of commodity  $x$  at a price  $\pi$ . If there are a number of risk neutral agents operating in this market, then, by arbitrage arguments, equilibrium in the forward market implies that  $\pi = E_p$ . Hence, committing oneself to  $\hat{x}$  ex ante will have an insurance gain. Faced with a price  $\pi$ , the consumer selects  $\hat{x}$ , to solve

$$\max_{\hat{x}} U(I - \pi\hat{x}, \hat{x}) .$$

This yields the inflexible utility  $V^I$ .

Our goal is simply to investigate whether or not  $V^I$  exceeds  $V^F$ . The forward market will open iff  $V^I \geq V^F$ . That is, if the risk averse consumers demand claims at  $\pi$ , then trade will occur in the forward market since the risk neutral agents will supply claims at this price. From Jensen's inequality,

$$V^F < U(I - E(px^*), Ex^*) . \quad (4)$$

If  $\frac{dx^*(p)}{dp} \geq 0$ , then (4) becomes

$$V^F < U(I - \pi Ex, Ex) \leq \max_{\hat{x}} U(I - \pi \hat{x}, \hat{x}) = V^I . \quad (5)$$

Hence, when purchases of  $x$  are independent of  $p$  or are positively related (an upward sloping demand schedule)  $V^I$  will always exceed  $V^F$ .

Once  $\frac{dx^*(p)}{dp} < 0$ , as is the usual case, then (4) no longer implies (5) since the  $\text{cov}(x,p) < 0$ . The larger (in absolute value) is this covariance, the more likely is it that  $V^F > V^I$ . In fact, one could use this covariance as a measure of flexibility since it reflects the sensitivity of consumer purchases to relative price changes. It is this covariance which also appears in the literature on welfare aspects of price variability.

To evaluate  $V^F$  and  $V^I$ , we write their difference in a more convenient form.

$$\begin{aligned} V^F - V^I &= \int_p U(I - px^*(p), x^*(p)) dF(p) - U(I - \pi \hat{x}, \hat{x}) \\ &= \left\{ \int_p [U(I - px^*(p), x^*(p)) - U(I - p\hat{x}, \hat{x})] dF(p) \right\} \\ &\quad + \left\{ \int_p U(I - p\hat{x}, \hat{x}) dF(p) - U(I - \pi \hat{x}, \hat{x}) \right\} . \end{aligned} \quad (6)$$

To obtain (6), we simply added and subtracted  $E_p U(I - p\hat{x}, \hat{x})$  from  $V^F - V^I$ . The first term in (6) is the expected gain to flexibility.<sup>4</sup>

Given that we face the random price  $p$ , it tells us the expected value to having  $x$  respond to these realizations. For a given  $p$ , we can expand  $U(I - p\hat{x}, \hat{x})$  around  $U(I - px^*(p), x^*(p))$  to obtain

$$\begin{aligned} U(I - px^*(p), x^*(p)) - U(I - p\hat{x}, \hat{x}) &= (x - \hat{x})(-pU_1 + U_2) \\ &\quad - (x - \hat{x})^2 (p^2 U_{11} - 2pU_{12} + U_{22}) . \end{aligned} \quad (7)$$



Here, all of the derivatives are evaluated at  $x^*(p)$  .

Taking expectations over  $p$  as in (6) and using (1), (7) implies that the first term in (6) is simply

$$-E_p (x^* - \hat{x})^2 (p^2 U_{11} - 2pU_{12} + U_{22}) . \quad (8)$$

By the second order conditions on the choice of  $x^*(p)$  , this term is always positive--i.e. flexibility has its value. Furthermore, from (8) we see that as  $x^*(p)$  is more variable relative to  $\hat{x}$  , the value of flexibility will be greater. Of course, the value of flexibility reflects properties of both preferences and  $F(p)$  .

Looking at the second term in (6), we know from Arrow [1965] and Pratt [1964] that

$$\int_p U(I - px, \hat{x}) dF(p) = U(I - \pi\hat{x} - \phi, \hat{x})$$

where  $\phi$  is a risk premium which is determined by agents aversion to risk with respect to consumption of commodity  $y$  . Hence the second term in (6) is negative and its magnitude determined by  $\phi$  . Whether or not non-contingent trades take place will depend on the demand for flexibility and the risk aversion of agents as in (6). Increases in  $\phi$  and/or reductions in (8) will enhance the likelihood of observing trades of this type.

There are a number of extensions of this simple model. One interesting one allows for random tastes so that preferences are represented by  $U(x,y,\theta)$  where  $\theta$  affects the marginal rate of substitution between  $x$  and  $y$  . An expression similar to (6) can be derived and the same trade-off explored.

In these examples, commodities  $x$  and  $y$  could be interpreted as consumption today and consumption tomorrow with random future prices. Alternatively, we could view the commodities as consumption and leisure with random wages and interpret the forward market as a non-contingent labor contract. Regardless of their interpretation the existence of trades in a forward market will depend on the demand for flexibility and the value of insurance.

### III. Timing of Decisions and Firm Behavior

In the previous model, consumers chose whether or not to make their purchases in spot markets. By assumption, agents were not allowed to first purchase non-contingent claims and then to buy or sell in the spot market. In this section we partly relax this assumption and allow agents to trade in both markets. Our goal is to analyze the volume of trade in the forward market by looking at another example of the flexibility/insurance tradeoff.

Consider the behavior of a firm facing a random output price,  $\tilde{p}$ . As discussed by Applebaum-Lim [1982], models of firm behavior under uncertainty differ in the information available to these agents in making their output choice. Some models, such as Oi [1961], allow the firm to learn the realization of its product price before making a production decision. In other models, Baron [1970] and Sandmo [1971], for example, firms must make output decisions ex ante. This literature was primarily concerned with the effects of price variability on a firm's output decision and welfare.

A firm's choice in the timing of its output decision is obviously closely related to the consumer problem discussed in the previous section. In fact, we could have continued that example here as well and

simply allowed consumers to make ex post purchases in the spot market. To offset the gains to flexibility and hence explain the coexistence of ex post and ex ante firms, Applebaum-Lim introduce cost differences between the types of firms. Following our earlier argument, an alternative way to generate an advantage for ex ante firms is to open a forward market. If the firm is risk averse, then there are insurance gains to producing ex ante and selling output forward.<sup>5</sup> This example differs from the previous case in that we allow firms to sell output in both the spot and forward market. Our interest is in the factors which determine the volume of trade in the forward market.

As before, let  $\tilde{p}$  denote a random output price distributed in the interval  $[\underline{p}, \bar{p}]$  with  $\underline{p} > 0$ . Let  $F(p)$  be the distribution function of  $p$  and  $\pi$  its mean. The risk averse firm has preferences represented by  $U(R)$ , where  $R$  is the firm's profits. We assume that  $U$  is strictly increasing and strictly concave. The firm can sell  $y \geq 0$  units of output, ex post, at the spot price  $\tilde{p}$ . In addition, the firm can commit itself to supply  $z$  units at the arbitrated forward price of  $\pi$ . We continue to assume the existence of risk neutral agents who provide this insurance. The firm's costs  $c(y+z)$  depend on total production and are assumed to satisfy  $c' > 0$ ,  $c'' > 0$ .

The firm's problem is to sequentially choose  $z$  ex ante and then  $y$  ex post. That is, the firm solves

$$\max_{z \geq 0} E_p U[\max_{y \geq 0} (\tilde{p}y + \pi z - c(y+z))] . \quad (9)$$

We can solve this dynamic programming problem by solving, given an arbitrary  $z$ , for the optimal ex post production,  $y^*(p|z)$ . It is straightforward to demonstrate that  $y^*(p|z) = 0$  when  $p \leq c'(z)$  and

otherwise satisfies

$$p = c'(z + y^*(p|z)) . \quad (10)$$

This simply requires that the firm set price equal to marginal cost as long as  $p \geq c'(z)$  . Otherwise, the firm has committed too much output ex ante and produces zero additional output ex post. This is illustrated in Figure 1. For each  $z > 0$  we can find  $\hat{p}(z)$  such that

$$\hat{p}(z) = c'(z) . \quad (11)$$

Here,  $y(p|z) = 0$  for  $p \leq \hat{p}(z)$  . These are the states of overproduction in that the firm wishes to produce less than  $z$  but cannot reverse its ex ante decision. Using (10) and (11), ex ante the firm chooses  $z$  to

$$\text{maximize } U(\pi z - c(z))F(\hat{p}(z)) + \int_{\hat{p}(z)}^{\bar{p}} U(\pi z + py - c(z+y))dF(p) . \quad (12)$$

The first-term represents utility from the overproductive states while the second-term is the expected utility of profits for the states in which production takes place ex post. In (12),  $y = y^*(p|z)$  . Using the envelope theorem,(10) and (11), the necessary condition for (12) is

$$[\pi - \hat{p}(z)]U'(\pi z - c(z))F(\hat{p}(z)) + \int_{\hat{p}(z)}^{\bar{p}} [\pi - p]U'(\pi z + py - c(z+y))dF(p) = 0 . \quad (13)$$

Lemma 1: In the solution to (12)  $\hat{p}(z) < \pi$  .

Proof: Assume, to the contrary, that  $\hat{p}(z) \geq \pi$  . From (10), we see that the second term in (13) is negative. Since  $\hat{p}(z) = c'(z)$  , the first-term in (13) is not positive. Hence (13) cannot hold if  $\hat{p}(z) \geq \pi$  .  $\square$

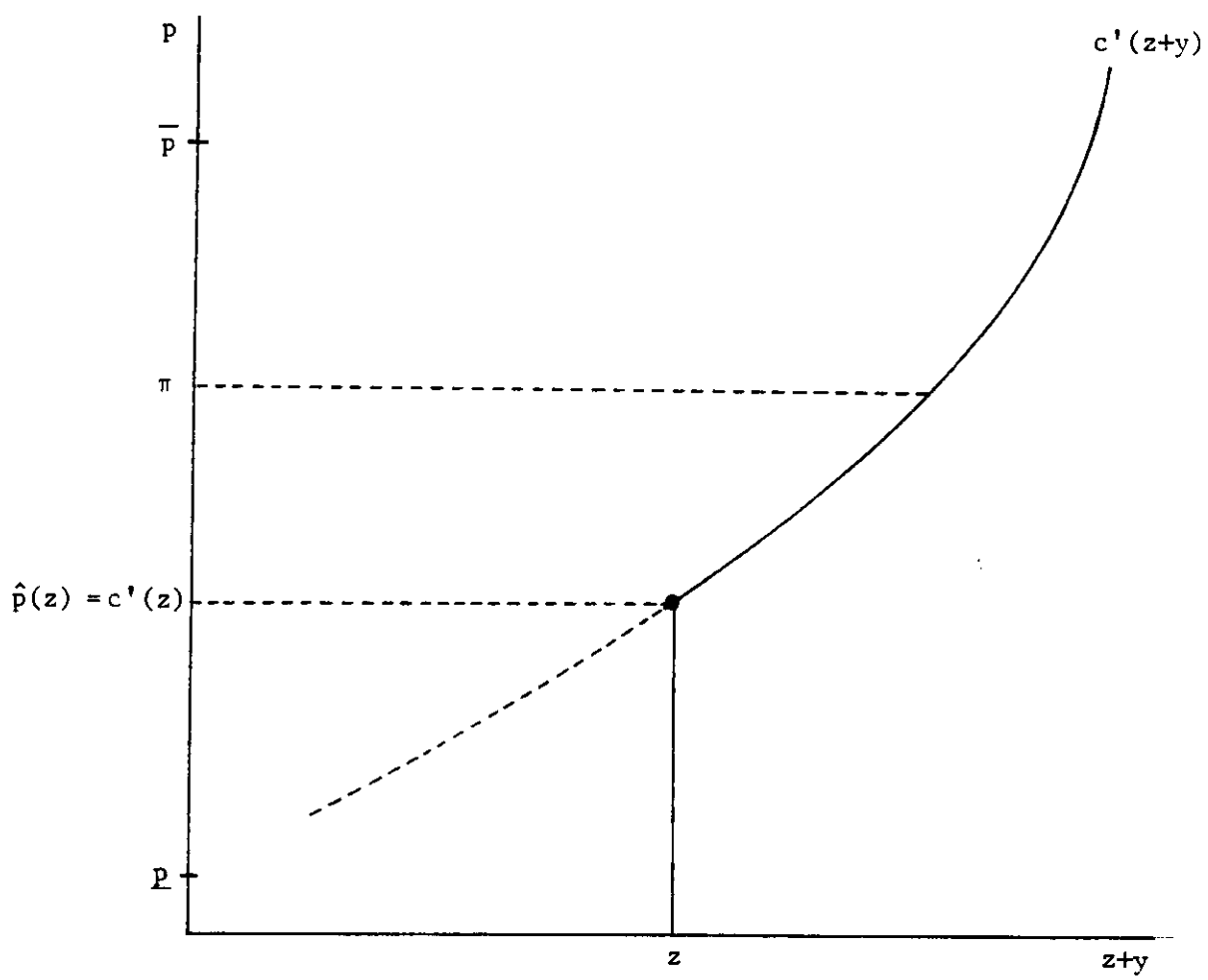


FIGURE 1

Intuitively, the first term in (13) represents the gains to selling commodities forward rather than at relatively low spot prices. The second term represents the losses from selling  $z$  units of output at  $\pi$  rather than at  $p = c'(z + y^*(p|z))$  from (10). If, for example,  $\hat{p}(z) = \pi$  then the firm gains nothing from the marginal unit of  $z$  as  $\pi = \hat{p}(z) = c'(z)$  and loses revenue for  $p \geq \hat{p}(z)$ . Hence the firm sets  $z$  so that  $\hat{p}(z) < \pi$  as in Figure 1. This ensures some gain from the marginal  $z$  when  $p < \hat{p}(z)$  and reduces losses when  $p \geq \hat{p}(z)$ .

Using Lemma 1 and (10), we rewrite (13) as

$$[\pi - c'(z)]U'(R(z))F(\hat{p}(z)) + \int_{\hat{p}(z)}^{\pi} [\pi - p]U'(R(p|z))dF(p) + \int_{\pi}^{\bar{p}} (\pi - p)U'(R(p|z))dF(p) = 0 \quad \text{where } R(z) = \pi z - c(z) \quad (14)$$

$$\text{and } R(p|z) = \pi z + py^*(p|z) - c(y+z).$$

Since  $\pi > \hat{p}(z)$ , we see that the first two terms are positive. These are the gains associated with the sale of output at the forward price  $\pi$  in states when  $\pi > \hat{p}$ . The last term is negative and corresponds to the states in which  $\pi < \hat{p}$ . The choice of  $z$  balances these gains and losses.

The optimal choice of  $z$  will depend, in part, on the risk aversion of the firm. A risk neutral firm would have no incentive to supply commodities forward. The insurance from forward sales is valued by risk averse firms and the trade volume should increase with firm risk aversion.'

Proposition 1: The optimal level of ex ante sales,  $z^*$ , satisfies:

- (i)  $z^* = 0$  if the firm is risk neutral,
- (ii)  $z^* > 0$  for risk averse firms and increases with the firm's risk aversion.

Proof: If the firm is risk neutral,  $c'(0) = 0 < \underline{p}$  and  $F(\underline{p}) = 0$  implies that (14) is met with equality at  $z^* = 0$ . Similarly, it is easy to show that the concavity of  $U(\cdot)$  implies that if the firm is risk averse,  $z^* > 0$ . Setting  $z^* = 0$ , (14) becomes

$$\int_{\underline{p}}^{\bar{p}} U'(py - c(y))(\pi - p)dF(p) . \quad (15)$$

With  $\pi = E(p)$  and profits increasing with  $p$ , (15) must exceed zero and hence  $z^* > 0$ .

Finally, to show that  $z^*$  increases with risk aversion, let  $V(x) = g[U(x)]$  where  $g$  is increasing and strictly concave. From the results of Pratt [1964], the agent with preferences represented by  $V(\cdot)$  is more risk averse than that represented by  $U(\cdot)$ . Let  $z_U^*$  and  $z_V^*$  be the optimal choices of  $z$  for the agent with utility functions  $U(\cdot)$  and  $V(\cdot)$  respectively. We wish to show that  $z_U^* < z_V^*$ .

From  $V(x) = g[U(x)]$ , the optimal choice of  $z_V^*$  satisfies

$$F(\hat{p}(z_V^*))g'[U(R(z_V^*))]U'(R(z_V^*)) + \int_{\hat{p}(z_V^*)}^{\bar{p}} (\pi - p)g'[U(R(p|z_V^*))] \\ \cdot U'(R(p|z_V^*))dF(p) = 0 . \quad (16)$$

To see that  $z_U^* < z_V^*$ , first assume  $z_U^* = z_V^*$ . From the definition of  $y^*(p|z)$ , we know that  $R(p|z) = py^*(p|z) + \pi z - c(y(p|z) + z) > \pi z - c(z) = R(z)$  for all  $p \in (\hat{p}(z), \bar{p}]$ . Since the second term in (16) is negative, and  $g$  is concave, we know that at  $z_U^* = z_V^*$ , (16) exceeds

$$g'[U(\pi z_U^* - c(z_U^*))] \left\{ F(\hat{p}(z_U^*))U'(R(z_U^*)) + \int_{\hat{p}(z_U^*)}^{\bar{p}} (\pi - p)U'(R(p|z_U^*))dF(p) \right\} = 0 . \quad (17)$$

We know that (17) equals zero as (14) implies that the bracketed term is zero at  $z_U^*$ . Since (16) is positive at  $z_V^* = z_U^*$ , the concavity of the programming problem implies that  $z_V^* > z_U^*$ .

□

This illustrates the gains of insurance and the role of risk aversion in determining the volume of forward trades. The costs of these trades is that suppliers cannot profit in high price states. Instead, they are forced to sell their output at  $\pi$  even when  $p > \pi$ . In this sense, forward sales imply a loss of flexibility.

As in the model in the previous section, we might want to identify the factors which influence the magnitude of this flexibility loss. Clearly, the curvature of the firm's cost function will be important. When the marginal cost schedule becomes steeper, the firm's desired quantity response to price changes will fall so that the loss of flexibility will be reduced. More precisely, one can show that convex transformations of the cost function which yield steeper marginal cost schedules, lead the firm to sell more in the forward market.

An alternative means of modeling the demand for flexibility would be to include another source of uncertainty in the firm's profits. For example, if costs are random then a firm will value the opportunity to learn the true output cost before deciding on a production level. Again, selling output ex ante reduces this flexibility. The model in the following section on contract length will formalize this intuition.



#### IV. Optimal Contract Length<sup>6</sup>

In this section of the paper, we consider the determination of contract length. In the models discussed thus far, we have concentrated on single-period forward contracts. Here we show that the same tradeoff between insurance and flexibility will help to determine contract length.

In a series of recent papers, Cooper-Hayes [1983], Dionne [1981], Radner [1981], Rubinstein-Yaari [1980] and Townsend [1982] have shown the gains to long-term relationships. If contracts are of infinite length then problems of moral hazard and adverse selection can be overcome. In addition, long-term contracts provide insurance against price variability.

Despite these incentive effects, contracts are generally of finite duration. In addition, contract length is endogenous as evidenced by the tendency towards shorter contracts in recent years. One explanation of contract length by Gray [1978] assumes the existence of transactions costs associated with contract agreements. Lengthier contracts help to spread these costs over many periods. The cost of long-term contracts in Gray's model is greater uncertainty. Using a quadratic loss function, the optimal contract length is determined.

In this paper, we approach the tradeoff somewhat differently. Here we stress the insurance gains of a long-term arrangement. Risk averse agents facing price uncertainty will demand unconditional claims. Suppliers of the commodity have random costs of production which are correlated over time. With correlated shocks to the cost function, sellers want the agreement to be flexible and to reflect new information. One way to do so is by setting a finite contract length.<sup>7</sup> We show that as the value of flexibility increases, contract length will shorten. In this

way, we again model contract length as the resolution of a tradeoff between insurance and flexibility.

The model we use here is partial equilibrium and is similar to the model in Azariadis-Cooper [1984]. There is a risk averse agent with income  $I$  who spends this income on commodity  $x$ . The price of this commodity,  $p$ , is random so that the consumption of the buyers is random as well. Preferences are represented by an increasing and strictly concave function  $U(I/p)$ . We assume that  $E(1/p) = 1$ .

We do not explicitly model the suppliers in this market though one can imagine a setting where some agents have a random endowment of  $x$ . Assuming this leads to a random supply of  $x$  to the market yields a basis for the price variability. Alternatively, the suppliers of the commodity may have some stochastic elements in their cost functions.

Being risk averse the buyers may prefer to negotiate a contract for the delivery of this commodity at a predetermined price. We assume that there exist risk neutral agents (sellers) who produce and consume  $x$  and are endowed with a unit of leisure time. These agents have preferences over consumption ( $c$ ) and work ( $n$ ) represented by

$$V(c,n) = c - \tilde{\theta}n .$$

Here  $\tilde{\theta}$  is a random disutility term meant to represent a random element in the cost of production. We assume that  $\tilde{\theta}$  is observed only by the seller. These agents produce commodity  $x$  according to the simple technology  $x = n$ . Hence, if these risk neutral agents produce solely for their own consumption (as in a spot market solution) their optimal labor supply,  $n^*(\theta)$ , satisfies

$$n^*(\theta) = \left\{ \begin{array}{ll} 1 & \text{if } \theta \leq 1 \\ 0 & \text{otherwise} \end{array} \right\} .$$

These risk neutral sellers are obviously natural trading partners for the risk averse buyers. In general, their optimal agreement would stipulate money and goods transfers as functions of the random variables  $(p, \theta)$ . Since our goal is to examine the length of simple bilateral contracts, we do not allow any contingencies nor any renegotiation in the contract. One can imagine that the combination of private information about  $\theta$  and other transactions costs limit the contract space. Moreover, we stipulate that contractual commitments can not be met through spot market purchases. This is an extreme assumption meant to highlight the bilateral monopoly created by the contract. Hence, once a seller agrees to a contract, he must produce the promised output in all states. This implies an inefficiency in production since sellers must supply labor when  $\theta > 1$ . We view this inability to set  $n = 0$  for  $\theta > 1$  as a loss of flexibility.

With these restrictions,  $t$ -period contracts simply stipulate a price  $w_t^*$  per unit of the consumption good. The quantity traded per period will therefore be  $q_t = I/w_t^*$  since the buyers only consume commodity  $x$ . Feasibility requires  $q \leq 1$ .

As discussed in Azariadis-Cooper [1984], this inefficiency essentially forces the contract price above the "pure insurance" price of  $E(1/p) = 1$ . That is, actuarially fair insurance will not be available. Depending on the degree of risk aversion of the buyers, the contract may or may not be signed. We use a similar approach here to discuss contract length.

Before characterizing these contracts, we place some additional

structure on the model. We assume that the disutility of labor for the sellers takes on two values  $\{\theta_H, \theta_L\}$  with  $\theta_H > 1 > \theta_L > 0$ . The probability that  $\theta = \theta_H$  in date  $t$  is denoted by  $\pi_t$ . Since our premise is that contract length is determined partly by the value of new information, we assume that  $\pi_t$  follows a simple Markov process. We specify the following stationary transition matrix:

$$\begin{array}{c}
 \theta_t \\
 \swarrow \\
 \theta_{t+1} \left( \begin{array}{cc} \theta_H & \theta_L \\ \theta_H & \pi_{HH} \quad \pi_{LH} \\ \theta_L & \pi_{HL} \quad \pi_{LL} \end{array} \right)
 \end{array} \quad (18)$$

So that if  $\theta_H$  occurs in period  $t$ , then  $\theta_H$  occurs with probability  $\pi_{HH}$  in period  $t+1$  and  $\theta_L$  occurs with probability  $\pi_{HL}$ . For these to be probabilities,  $\pi_{HH} + \pi_{HL} = \pi_{LL} + \pi_{LH} = 1$ . We place no additional structure on the transition matrix at this stage. With  $\pi_1$  as the exogenously given probability of  $\theta_H$  in period 1, we see that

$$\begin{aligned}
 \pi_2 &= \pi_1 \pi_{HH} + (1 - \pi_1) \pi_{LH} \\
 \pi_3 &= \pi_2 \pi_{HH} + (1 - \pi_2) \pi_{LH} \\
 &\vdots \\
 \pi_t &= \pi_{t-1} \pi_{HH} + (1 - \pi_{t-1}) \pi_{LH} .
 \end{aligned} \quad (19)$$

Given this probability structure, we can determine the supply price of a  $t$ -period contract  $w_t^*$ . At this price, sellers will be indifferent with respect to the number of units demanded under the contract. The expected utility to the seller of delivering  $q$  units at a price  $w_t$  in a  $t$ -period contract,  $G_t$ , is

$$G_t = \sum_{\tau=1}^t \pi_{\tau} (w_t - \theta^H)q + \sum_{\tau=1}^t (1 - \pi_{\tau}) [(w_t - \theta^L)q + (1 - \theta^L)(1-q)] . \quad (20)$$

Here we have assumed no discounting. The first-term is the utility when  $\theta^H$  occurs. In this case, the seller produces the required  $q$  units.<sup>8</sup> The sum of the probabilities of state  $\theta^H$  over  $t$  periods is simply  $\sum_{\tau=1}^t \pi_{\tau}$ . The second term in (20) corresponds to realizations of  $\theta^L$ . In these states, the supplier produces  $q$  units as stipulated in the contract and then uses the remainder of leisure time to produce for home consumption. To find the supply price, we solve for  $w_t^*$  such that the choice of  $q$  to maximize (20) is indeterminate. It is straightforward to show that  $w_t^*$  is given by

$$w_t^* = 1 + \frac{\sum_{\tau=1}^t \pi_{\tau} (\theta^H - 1)}{t} . \quad (21)$$

Hence we see that  $w_t^*$  exceeds the "pure insurance" price of  $E(1/p) = 1$  due to the existence of high disutility states. The difference between  $w_t^*$  and 1 depends on the likelihood of  $\theta^H$  occurring and the lost utility from producing,  $\theta^H - 1$ .

Also, from (20), we see the importance of the Markov structure in modeling contract length. Assume there is no correlation in  $\theta_t$ , i.e.  $\pi_{HH} = \pi_{HL}$  and  $\pi_{LH} = \pi_{LL}$ . It is easy to see that  $w_t^*$  will be independent of time since  $\pi_t$  would equal  $t \cdot \pi_1$  so

$$w_t^* = 1 + \pi_1 (\theta^H - 1) \quad \text{for all } t .$$

As discussed in Azariadis-Cooper, a single-period contract will be signed if  $U(I/w_1) \geq EU(I/\hat{p})$ . This depends on the risk aversion of the buyer

and the size of  $\pi_1$  and  $\theta^H$ . If a single-period contract was negotiated, there would be no limit to contract length since  $w_t^* = w_1^*$  for all  $t$ .

Our premise is that the value of integrating new information about  $\theta$  into the contract will shorten its length. This is shown in this model by the property that  $w_t^*$  is an increasing function of time when the  $\theta$ 's are positively correlated over time. Lengthier contracts are more costly when the probability of  $\theta_H$  occurring increases with time. From (21) we see that  $w_t^* > w_{t-1}^*$  iff

$$\frac{\sum_{\tau=1}^t \pi_{\tau}}{t} > \frac{\sum_{\tau=1}^{t-1} \pi_{\tau}}{t-1}$$

or, equivalently,

$$\pi_t > \frac{\sum_{\tau=1}^{t-1} \pi_{\tau}}{t-1}. \quad (22)$$

This has a natural interpretation of the average probability increasing when the marginal probability ( $\pi_t$ ) exceeds the average. Hence, characterizing the evolution of  $\pi_t$  will determine the path of  $w_t^*$ .

From (19), we see that, in general,

$$\pi_t = \pi_{t-1}(\pi_{HH} - \pi_{LH}) + \pi_{LH}. \quad (23)$$

This is a simple difference equation with a positive intercept and a coefficient with absolute value between zero and one. Hence there will exist a stationary probability,  $\hat{\pi}$ , satisfying

$$\hat{\pi} = \hat{\pi}(\pi_{HH} - \pi_{LH}) + \pi_{LH}. \quad (24)$$

At this point, we assume that  $\pi_{HH} > \pi_{LH}$  and, hence, that  $\pi_{LL} > \pi_{HL}$ . Essentially, this assumption means that a given state is more likely next period if it occurred this period. With this correlation, there is a cost in ignoring the information contained in current realizations of  $\vartheta$ . We discuss the implications of relaxing this assumption later.

Lemma 2: If  $\pi_1 < \hat{\pi}$ , then  $\pi_{t+1} \geq \pi_t$ .

Proof: Direct from (23). □

Lemma 3: If  $\pi_{t+1} > \pi_t$  for all  $t > 1$ , then

$$\pi_{t+1} > \frac{\sum_{\tau=1}^t \pi_{\tau}}{t}.$$

Proof: If  $\pi_t$  increases with  $t$ , then  $\pi_t > \frac{\sum_{\tau=1}^t \pi_{\tau}}{t}$ . Hence  $\pi_{t+1} > \pi_t$  is bigger than the average of the other probabilities. □

We now can use Lemmas 2 and 3 to characterize  $w_t^*$ .

Lemma 4: If  $\pi_1 < \hat{\pi}$ ,  $w_t^*$  strictly increases with  $t$  for finite  $t$ .

Proof: From Lemma 3, if  $\pi_{t+1} > \pi_t$  then (22) implies that  $w_{t+1}^* > w_t^*$ . As  $t \rightarrow \infty$ ,  $\pi_t \rightarrow \hat{\pi}$  so that  $w_t^*$  converges to  $\hat{w} \equiv 1 + \hat{\pi}(\theta^H - 1)$ . □

As long as the probability of  $\theta^H$  starts at a low value and  $\pi_{HH} \geq \pi_{LH}$ , the supply price of claims will increase. Intuitively, these conditions guarantee that the probability that  $\theta^H$  occurs increases over time. Hence, to be compensated for their loss of flexibility, sellers must receive a higher contract price.

If there are many identical risk neutral sellers, competition will guarantee that the equilibrium price of a forward contract lasting  $t$  years will equal  $w_t^*$ . Given this price function, buyers will select the length of contract.

We assume that both buyers and sellers will be alive for  $T$  years. A buyer who chooses a contract for  $t$  years will return to the spot market for his purchases thereafter.<sup>9</sup> With this assumption, the expected utility of a buyer of a  $t$ -period contract is

$$V(t) \equiv U\left(\frac{I}{w_t^*}\right) \cdot t + (T-t)E_P U\left(\frac{I}{p}\right) . \quad (25)$$

The costs and benefits of increasing the contract length can be seen from (25). As the length increases, buyers obtain insurance against fluctuations in the spot price,  $p$ , for another period. Yet, with  $w_t^*$  increasing in  $t$ , this additional period of insurance raises the price of insurance over all the periods it was purchased. The contract length is determined by the resolution of this tradeoff. To characterize this choice in our problem, we can choose the time periods small enough so that the  $w_t^*$  prices approximate a continuous function.<sup>10</sup> Taking this as given, the buyers  $\max_t V(t)$ . The equilibrium contract length,  $t^*$ , will then satisfy<sup>11</sup>



$$U\left(\frac{I}{w^*(t^*)}\right) - E_p U\left(\frac{I}{p}\right) = t^* \frac{dw^*}{dt} \frac{I}{(w^*(t^*))^2} U'\left(\frac{I}{w^*(t^*)}\right) \quad (26)$$

The left-hand side of (26) is the gain from an additional period of insurance. The right-hand side is the cost since increasing contract length will increase the price of insurance over all the periods. Using (26), we show that increases in risk aversion will increase the contract length. Intuitively, risk averse agents are willing to pay more for insurance and hence are willing to incur higher costs to increase contract length.

Proposition 2: Contract length increases with the risk aversion of buyers.

Proof: Equation (26) characterizes a tangency between buyer's indifference curves with respect to  $w^*$  and  $t$  and  $dw^*(t)/dt$  as determined from (11) for continuous time. This is shown in Figure 2. At  $E$ , the buyer's marginal rate of substitution equals

$$\frac{U(I/w^*(t)) - E_p U(I/p)}{U'(I/w^*(t))} \cdot t \frac{dw^*}{dt} \frac{I}{w^*(t)^2} \cdot \quad (27)$$

The first term increases with buyer's risk aversion (from a Taylor series expansion) so that the optimal  $t$  will increase as well.

□

Hence we see that the optimal contract length, given the  $w^*(t)$  schedule, is determined by buyer's risk aversion. Contracts are of finite duration (and less than the lives of agents) since  $w^*(t)$  increases with time. We interpret this increase in the forward price as the cost to buyers of the sellers losing their flexibility for an additional period. As the value of this flexibility increases, we would expect contract

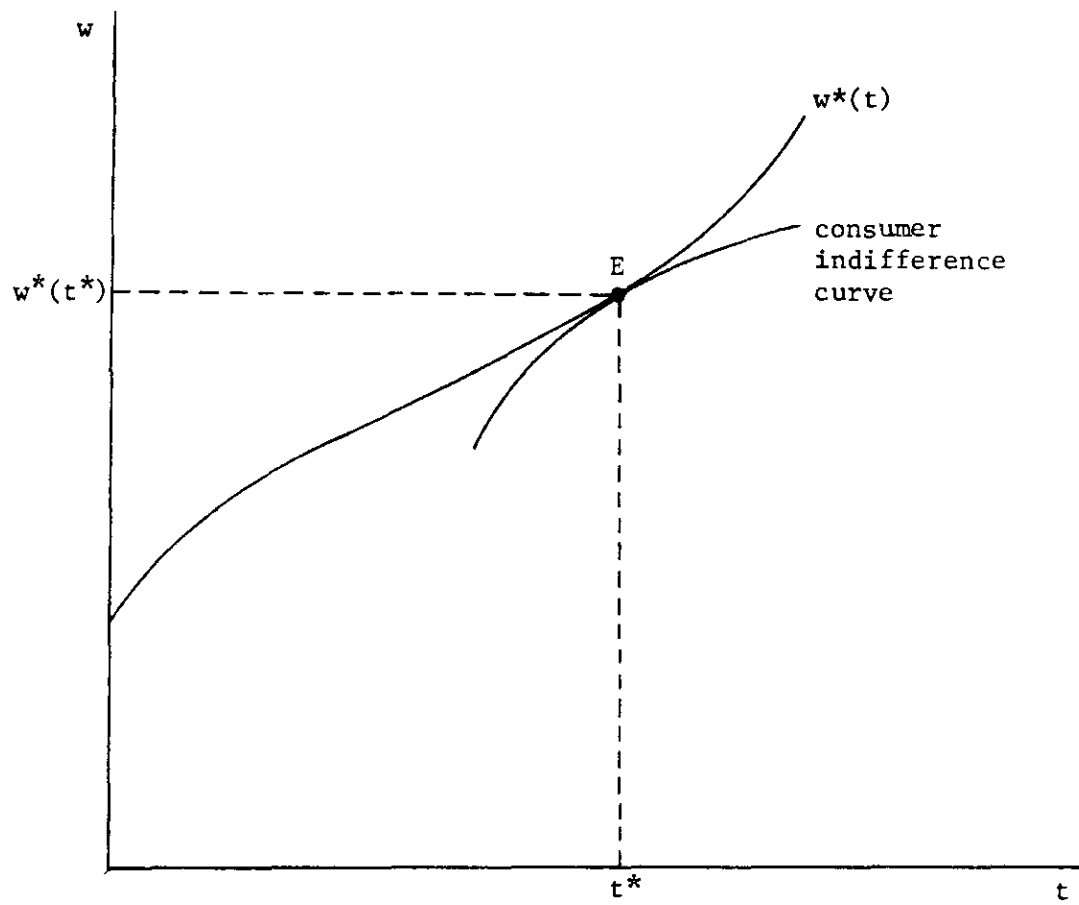


FIGURE 2

length to shorten. This can be modeled by having  $\theta^H$  increase relative to 1.

Proposition 3: As  $\theta^H$  increases, contract length shortens if

$$-\frac{xU''(x)}{U'(x)} \equiv R < 1 .$$

Proof: As  $\theta^H$  increases, we see from (21) that  $w^*(t)$  increases for all  $t$ . Hence the left side of (23) falls due to the monotonicity of  $U(\cdot)$ . If the Arrow-Pratt measure of relative risk aversion is less than one,  $\frac{I}{w^*(t^*)}U'\left(\frac{I}{w^*(t^*)}\right)$  increases as  $w^*(t^*)$  increases. Finally, from (21) we can show that  $\frac{dw^*}{dt} \cdot \frac{1}{w^*(t^*)}$  increases with  $\theta^H$ . Hence the right side of (23) increases with  $\theta^H$ . For (23) to hold  $t^*$  must fall as  $\theta^H$  increases.

□

One important comment about these results is that we have structured the problem so that the shortening of contract length is the only means of incorporating new information. If contracts were contingent on  $\theta$  then a shortened contract length is not necessary. Moreover, if renegotiation is possible ex post, then contract length has an endogenous component as well. This issue warrants further exploration.

Finally, we should note that our assumptions of  $\pi_{HH} > \pi_{LH}$  and  $\pi_1 < \hat{\pi}$  were crucial to our analysis. If  $\pi_{HH} < \pi_{LH}$ , then  $\pi_t$  will follow a cyclical pattern and  $w^*(t)$  will not necessarily be monotone in  $t$ . Similarly, if  $\pi_1 > \hat{\pi}$ , then  $w^*(t)$  will be monotonically decreasing over time.

## V. Conclusion

The purpose of this paper was to explore the existence of trades in simple, non-contingent forward contracts. Using a series of examples, we have shown that the existence of these markets, their volume of trade and the length of the agreements are all determined by the tradeoff between insurance and flexibility. Trades of this type introduce rigidities in prices and quantities. The price rigidities play a positive insurance role while the quantity rigidities are costly distortions. This tradeoff seems rather important in the determination of simple market structures.

The major drawback of this research is its lack of a general structure. As the insurance flexibility tradeoff seems applicable in a variety of circumstances, there ought to be a fairly general representation of this set of problems. The analysis at this stage lacks such generality. Moreover, it might be beneficial to explore these issues in a general equilibrium setting as in Azariadis-Cooper [1984]. Finally, and perhaps most importantly, the models here suffer from a number of restrictions on agent's trades. The point however was to examine the role of non-contingent trades, given the non-existence of contingent trades.

## FOOTNOTES

1. In Azariadis-Cooper [1984], we focus on the risk sharing attributes of forward markets as a prelude to understanding nominal wage rigidity. This research extends many of those ideas to other contexts. Townsend [1978] investigated the optimality of forward markets using a spanning argument. Here we are more concerned with positive aspects of these markets--i.e. when are they active, what factors determine the volume of trade, etc. The fact that forward trades may not arise given the non-existence of other markets is consistent with the results of Hart [1975].
2. This assumes, of course, some imperfections in the resale market.
3. The literature on price variability is lengthy. The original paper by Waugh [1944] was followed by papers by Baron [1970] and Oi [1961] concerning firm behavior, and, more recently, by Eaton [1980]. Some of the same issues are present in the literature on international trade under uncertainty. See, for example, Batra and Russell [1974], Eaton [1979] and Turnovsky [1974]. Our discussion of flexibility directly relates to Eaton's [1980] discussion relating preferences over random prices and substitutability of goods.
4. If  $\hat{x}$  solves  $\max_x E_p U(I - px, x)$ , then this first term is the "expected value of perfect information" as discussed by Dreze-Modigliani [1972]. These authors were concerned with comparing measures of risk aversion between temporal and timeless prospects. As shown in their equation (2.9), the curvature of indifference curves is an important element in this comparison. By the definition

of  $\hat{x}$ , the first term in (6) is not equivalent to that considered by Dreze-Modigliani. A related discussion arises in the paper by Weitzman [1974] concerning price and quantity controls (see Yohe [1977]).

5. Here we assume that the firm is risk averse. This is justified if the firm is owned by risk averse agents in an economy with incomplete capital markets or has a risk averse manager in an imperfect information setting.
6. Conversations with Michael Riordan have been very helpful in developing this section.
7. This is, of course, not the only way to incorporate the new information. Contingent agreements or the renegotiation of an existing contract can substitute for a shorter contract. Here we focus on the choice of contract length and ignore these other possibilities. Future research will focus on a comparison of these alternatives, though a fully contingent contract can clearly nest these other opportunities.
8. The seller takes the earnings from the forward trades and purchases goods in the spot market at  $E[1/p] = 1$ .
9. This admittedly strong assumption guarantees that buyers face risk once their contract expires. This simplification allows us to avoid the dynamic problem of determining the length of future contracts after the current one expires. As in the case of our other assumptions, we are structuring the model to focus on the role of flexibility and insurance in determining contract length.

10. Here we switch to continuous time and the notation  $w^*(t)$  , etc.
11. We assume this is necessary and sufficient.

## REFERENCES

- Applebaum, E. and Lim, C. (1982), "Ex Ante and Ex Post Firms and Stochastic Equilibrium Price Fluctuations," mimeo.
- Arrow, K. J. (1964), "The Role of Securities in the Optimal Allocation of Risk-Bearing," Review of Economic Studies, 31: 91-96.
- Arrow, K. J. (1965), Aspects of the Theory of Risk-Bearing (Helsinki: Yijo Jahanssonin Saatie).
- Arrow, K. J. (1974), "Limited Knowledge and Economic Analysis," American Economic Review, 64: 1-11.
- Azariadis, C. and Cooper, R. (1984), "Predetermined Prices and the Allocation of Social Risks," forthcoming in Quarterly Journal of Economics.
- Baron, D. P. (1970), "Price Uncertainty, Utility and Industry Equilibrium in Pure Competition," International Economic Review, 11: 463-480.
- Batra, R. N. and Russell, W. R. (1974), "Gains from Trade under Uncertainty," American Economic Review, 64: 1040-1048.
- Cooper, R. and Hayes, B. (1984), "Multiperiod Insurance Contracts," Cowles Foundation Discussion Paper No. 673.
- Debreu, G. (1959). Theory of Value (New York: Wiley).
- Diamond, P. (1967), "On the Role of a Stock Market in a General Equilibrium Model with Technological Uncertainty," American Economic Review, 57: 759-773.
- Dionne, G. (1981), "Adverse Selection and Repeated Insurance Contracts," mimeo.
- Dreze, J. and Modigliani, F. (1972), "Consumption Decisions under Uncertainty," Journal of Economic Theory, 5: 308-335.
- Eaton, J. (1979), "The Allocation of Resources in an Open Economy with Uncertain Terms of Trade," International Economic Review, 20: 391-403.
- Eaton, J. (1980), "Price Variability, Utility and Savings," Review of Economic Studies, 47: 513-520.
- Goldman, S. M. (1974), "Flexibility and the Demand for Money," Journal of Economic Theory, 9: 203-222.
- Gray, J. (1978), "On Indexation and Contract Length," Journal of Political Economy, 86: 1-19.
- Grossman, S. J. (1977), "A Characterization of the Optimality of Equilibrium in Incomplete Markets," Journal of Economic Theory, 15: 1-15.



- Harris, M. and Townsend, R. M. (1981), "Resource Allocation under Asymmetric Information," Econometrica, 49: 33-64.
- Harris, M. and Holmstrom, B. (1982), "On the Duration of Agreements," J. K. Kellogg Graduate School of Business, mimeo.
- Hart, O. (1975), "On the Optimality of Equilibrium When the Market Structure Is Incomplete," Journal of Economic Theory, 11: 418-443.
- Kreps, D. M. (1979), "A Representation Theorem for 'Preference of Flexibility'," Econometrica, 47: 565-679.
- Marschak, T. and Nelson, R. (1962), "Flexibility, Uncertainty and Economic Theory," Metroeconomica, 14: 42-58.
- Oi, W. (1961), "The Desirability of Price Instability Under Price Uncertainty," Econometrica, 29: 58-64.
- Pratt, J. W. (1964), "Risk Aversion in the Small and in the Large," Econometrica, 32: 122-136.
- Radner, R. (1981), "Monitoring Cooperative Agreements in a Repeated Principal-Agent Relationship," Econometrica, 49: 1127-1148.
- Ross, S. (1981), "Some Stronger Measures of Risk Aversion in the Small and the Large with Applications," Econometrica, 49: 621-638.
- Rubinstein, A. and Yaari, M. (1980), "Repeated Insurance Contracts and Moral Hazard," mimeo.
- Sandmo, A. (1971), "On the Theory of the Competitive Firm under Price Uncertainty," American Economic Review, 61: 65-73.
- Stigler, G. (1939), "Production and Distribution in the Short Run," Journal of Political Economy, 47: 305-328.
- Stiglitz, J. (1982), "The Inefficiency of the Stock Market Equilibrium," Review of Economic Studies, 49: 241-262.
- Townsend, R. M. (1978), "On the Optimality of Forward Markets," American Economic Review, 68: 54-66.
- Townsend, R. M. (1982), "Multiperiod Contracts and the Gain from Enduring Relationships under Private Information," forthcoming, Journal of Political Economy.
- Turnovsky, S. J. (1974), "Technological and Price Uncertainty in a Ricardian Model of International Trade," Review of Economic Studies, 41: 201-217.
- Waugh, F. V. (1944), "Does the Consumer Benefit from Price Instability?" Quarterly Journal of Economics, 58: 602-614.

Weitzman, M. (1974), "Prices vs. Quantities," Review of Economic Studies,  
41: 50-65.

Yohe, G. (1977), "Comparisons of Price and Quantity Controls: A Survey,"  
Journal of Comparative Economics, 1: 213-233.