

Monopoly Provision of Product Quality
With Uninformed Buyers*

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ABSTRACT

This essay is concerned with a monopolist's incentives to provide high quality goods when some of its customers cannot observe quality prior to purchase. We show that if all buyers have the same tastes for quality, the monopolist will not try to take advantage of the poorly informed. When tastes differ, however, some quality randomization may become profitable as a means to loosen binding self-selection constraints. The profitability of randomization is shown to depend upon the relative degrees of risk aversion of the buyers and on the convexity of the firm's cost of quality function. We view our results as pointing to some potential benefits from imperfect quality control.

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I. Introduction

In an important recent contribution to the self-selection literature, Mussa and Rosen [1978] investigate the behavior of a monopoly supplying goods of varying quality to heterogeneous agents. In their model, consumers differ in their preferences over price and quality combinations. While the monopolist knows the distribution of these immutable characteristics across agents, it is assumed that an individual's type is not public information. Lacking this information, the monopolist cannot perfectly discriminate between customers; nevertheless, it does prove profitable for the firm to offer a set of price-quality pairs that (at least partially) sort consumers. Mussa-Rosen demonstrate that the self-selection process leads to distortions in the quality sold to the agents relative to what they would receive in a full information (or a first-best) environment. These distortions generally take the form of lower quality levels.

In related work, Chan and Leland [1982], Cooper and Ross [1984] and Farrell [1982] have investigated the behavior of competitive markets in which some (but not all) consumers are uninformed about the quality of goods being offered. The main question addressed in this literature concerns the degree to which prices convey information from informed to uninformed buyers.

In this essay, we bridge these literatures by considering the problem faced by a monopolist producing a product with (potentially) variable quality in a market with uninformed consumers. This is essentially a problem of quality control. The observation that products are generally not of uniform quality is usually attributed to the fact that quality control is costly. Our examples point out that there may be gains from imperfect quality control even when a firm can costlessly reduce all variability in the quality of its products. An imperfect competitor can profitably offer a distribution of qualities at a given price as an efficient means of distinguishing between agents with different preferences over the product. This requires, of course, that at least some of the buyers be imperfectly informed about the product's quality at the time of purchase. As in the studies cited above, prices will still convey some information about the distribution of quality.

The results of this analysis therefore have some bearing on the literature concerned with the role of randomization as a sorting device.¹ When the monopolist offers a distribution of quality at each price, the uninformed consumers see this as a form of randomization. We characterize the conditions under which randomization is profitable as a function of: (i) consumers' risk aversion, (ii) the relative numbers of agents of different tastes, and (iii) the curvature of the firm's cost-of-quality function.

II. Overview of the Model

In this model, we consider a simple monopoly market for a good which can be produced with varying levels of quality. Quality is viewed here as a good, something that everyone would prefer more of. Consumers, each of whom buys either one unit or none of this good, will have differing tastes for quality, in general. These preferences can be represented by:

$$(1) \quad V_i(p, q) = U_i(q) - W_i(p) \quad .$$

Here i is an index of consumer types, q an index of quality and p represents price. We assume that the $U_i(q)$ are increasing and concave and that the $W_i(p)$ are increasing and convex. Through some of what follows it will be convenient to simplify preferences even further, letting $W_i(p) = p$. This allows us to interpret $U_i(q)$ as a willingness-to-pay function or as a reservation price.

In the following section we begin by studying equilibrium in this market when agents are homogeneous in terms of tastes. By assumption, the firm does not know whether particular customers are informed or uninformed about product qualities. We explore its incentive to try to take advantage of its uninformed customers. While the uninformed cannot observe a particular unit's quality before purchase (i.e. the product is an experience good), they do know the conditional distribution of quality given price, $F(q | p)$, of

the units actually sold in equilibrium. They also observe all prices in the market. The informed customers, on the other hand, costlessly observe all prices and qualities.

Section IV considers the monopolist's problem when there is more than one taste-type of agent. We do not allow the firm to observe a buyer's type, so it must let customers self-select from among its price-quality offerings. For most of this discussion we work with a two taste-type example ($i=1,2$) that illustrates most simply the interesting properties of the multi-type equilibria. For this example we order the consumers' tastes in such a way that the second class of buyers will be the relative quality preferers, i.e. $U_2'(q)/W_2'(p) > U_1'(q)/W_1'(p)$ for all p and q .² We denote by N_i the total number of agents of type i , and we let θ_i represent the proportion of those buyers who are informed.

The monopolist is risk neutral over income and obtains profits $\Pi = p - C(q)$ for each unit of quality q sold at price p . $C(q)$ is the assumed constant average (and marginal) cost of producing a unit of quality q . $C(q)$ is further assumed to be (strictly) increasing and convex in q , i. e. $C'(q) > 0$ and $C''(q) \geq 0$. This convexity will prove important since it represents an aversion on the monopolist's part to quality variation, just as the concavity of $U(q)$ yields a similar aversion on the part of consumers.

The monopolist's strategy consists of choosing prices and quality distributions for each of the four (when there are that many) types of consumers. To emphasize an important point, since uninformed consumers cannot observe quality directly, the monopolist can be viewed as offering them a distribution. We denote by \bar{q}_j the random quality offered at price p_j .³ An informed agent purchasing at price p_j will, of course, select the good of highest quality available at that price, we call this quality q_j .⁴ Finally, let \bar{q}_j be the expected quality at price p_j .

One important aspect of this problem concerns the equilibrium concept.⁴ In the analysis of competitive behavior, it was natural to view firms as taking the distributions of quality at each price as given. Hence, a rational expectations equilibrium occurs when consumer conjectures on the conditional distribution of quality are confirmed by firm behavior.

In the monopoly setting, it is not clear that the "simultaneous move" view of equilibrium is appropriate. We view the monopolist as choosing a distribution of quality at each price which consumers take as given when they make decisions. This Stackelberg approach is consistent with the monopolist choosing a technology which, based on the resources devoted to quality control, yields a distribution of qualities. With this interpretation, the existence of uninformed agents may influence the firm's expenditures on quality control.

The Stackelberg approach also helps to avoid one of the problems of using randomization as a sorting mechanism. If the firm could influence the distribution of quality after agents disclosed their taste-types, then in general, the firm would have an incentive to do so. If a firm precommits to a particular distribution of quality, this problem can be circumvented.

III. Provision of Quality With Homogeneous Tastes

As a starting point, we quickly review the solution to the monopolist's problem when all agents are informed (i.e. $\theta=1$). The monopolist chooses a price and a product quality to maximize profits subject to consumers obtaining a minimum utility level, \bar{V} , which is available to them elsewhere. Since agents are all informed, there is obviously no benefit to offering a distribution of quality. Formally, the monopolist solves

$$(2) \quad \begin{array}{ll} \text{maximize} & [p - C(q)] \\ & p, q \\ \text{subject to} & V(p, q) \geq \bar{V}. \end{array}$$

It is easy to see that the solution to this problem, (p^*, q^*) , must involve a binding constraint:

$$(2.a) \quad V(p^*, q^*) = \bar{V}$$

and that the chosen quality, q^* , satisfies

$$(2.b) \quad C'(q^*) = U'(q^*)/W'(p^*) .$$

As expected, the monopolist extracts all of the available consumers' surplus and leaves agents indifferent between staying in the market and obtaining the utility level \bar{v} .

Given this solution, we ask whether the monopolist can increase its profits when some of the consumers are uninformed. The existence of informed and uninformed consumers adds a dimension for the separation of agents. In general, the monopolist could offer goods of varying quality at a number of different prices. Informed agents would choose over prices knowing they would select the maximal quality offered at each price. As before, the uninformed only know the distribution of quality at each price. They will choose to buy at the price offering the greatest expected utility, recognizing that there is some chance that they will get "ripped off", that is, receive a lower quality than the informed buyers. The following proposition indicates, however, that the monopolist cannot improve on the solution to (2) even though there exist some uninformed consumers.⁵ The proof, which is quite straightforward, is available from the authors upon request.

Proposition 1: If some agents are uninformed about product quality but all agents have identical tastes, then the monopolist will not randomize quality and the solution will be identical to that of (2).

This result means that randomization of quality will not provide a means of extracting additional surplus from consumers with identical tastes. Using (2.a) and (2.b), the monopolist already extracts all consumers' surplus. With convex costs and concave preferences, randomization is a cost to both the monopolist and to consumers. However, as discussed in the next section, once consumers differ in tastes as well as information type, the monopolist may find it profitable to offer a lottery over quality in order to separate consumers by taste-type.

IV. Heterogeneous Agents--Randomization as a Sorting Mechanism

The results of the previous section illustrate that, despite informational diversity across agents, the monopolist will choose not to randomize quality. In this section we extend the analysis to agents with differing preferences and discuss the role of quality randomization in this expanded environment.

To begin, suppose that the economy was composed entirely of informed agents of two taste-types with preferences described by (1). The second type (i.e. $i=2$) will be assumed to be the relative quality preferers--the "high" taste-types. Clearly, the monopolist could not profit by randomizing quality since the informed agents would only select the highest quality items at each price. Hence, the firm chooses (p_i, q_i) for $i = 1, 2$ to

$$(3) \quad \text{maximize } \sum N_i [p_i - C(q_i)]$$

subject to

$$(3.a) \quad U_i(q_i) - W_i(p_i) \geq U_i(q_k) - W_i(p_k) \quad \forall i, k$$

and

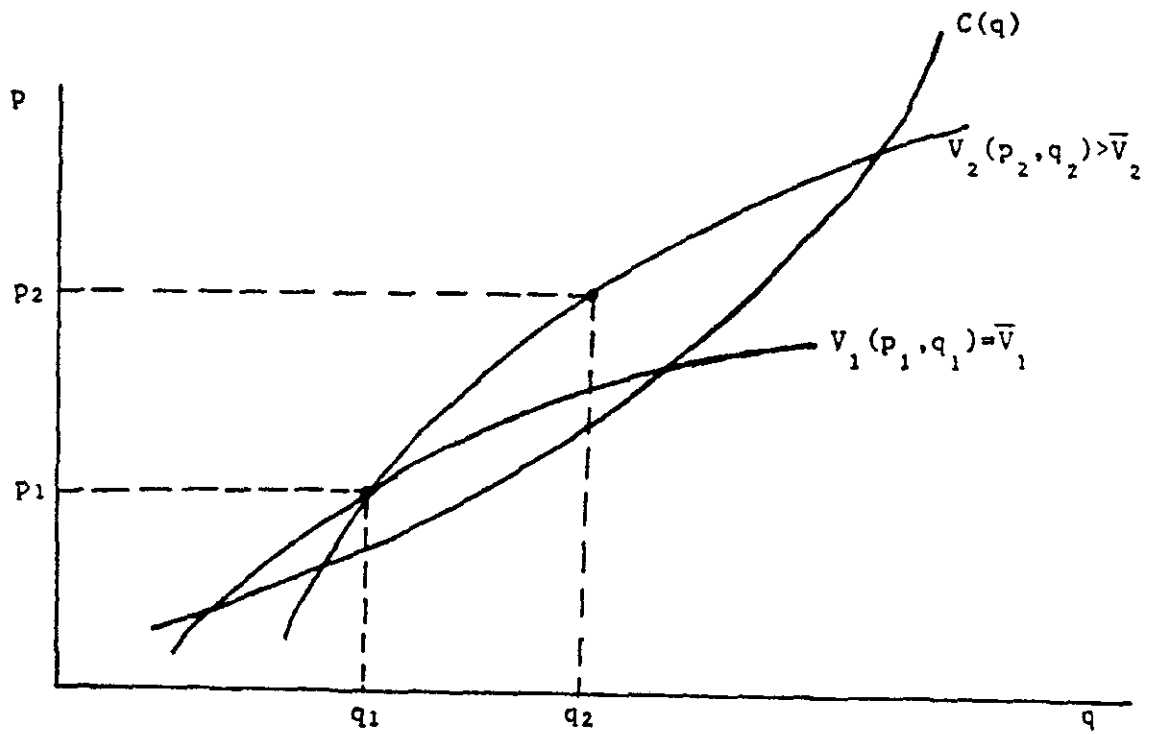
$$(3.b) \quad U_i(q_i) - W_i(p_i) \geq \bar{V}_i \quad \text{for } i = 1, 2.$$

This is a standard self-selection problem of the type originally investigated by Mussa and Rosen. The self-selection constraint appears as (3.a) while the monopolist is also constrained by each agent's option of leaving the market and obtaining \bar{V}_i elsewhere.

In an environment of full information about agents' characteristics and no resale, (3.a) is irrelevant and the solution to (3) is to produce quality efficiently and ensure that (3.b) is binding for all i . When all only imperfect information is available to the monopolist so that agents' types are not public information, the solution to (3) is quite different. Mussa-Rosen show that (3.a) will be binding for $i=2, k=1$ and that (3.b) will be binding only for $i=1$. The monopolist extracts all the surplus from the low taste-types but due to the self-selection constraints, the high taste agents receive positive surplus. Such a solution is depicted in Figure One.

When all agents are informed about product quality, the monopolist will be unable to improve on this solution. However, the existence of uninformed agents will provide a

FIGURE ONE



basis for randomization as a means of extracting further surplus from the high taste-type agents.

In the most general form of randomization, the monopolist could offer lotteries over quality in both the high and low price markets. However, there are only costs and no benefits to randomizing at the high price -- i.e. there are no self-selection constraints to relax. Hence we concentrate on randomization in the low-price market.

To illustrate the gains to randomization, we consider an example in which $C(q)$ is a strictly convex cost schedule and in which all consumers are uninformed. A monopolist again faces agents of two taste-types. There are N_1 "low-taste" agents with preferences $V_1(p,q) = bq - p$ and N_2 "high-taste" agents with preferences $V_2(p,q) = U(q) - p$, where $U'(q) > b$ for all q and $U''(q) < 0$. The monopolist is risk neutral with respect to profits but risk averse with respect to quality since costs are strictly convex.

Following Mussa-Rosen, the non-randomized solution will involve zero consumer's surplus for the low-taste agents and self-selection constraints binding in only one direction: the high-taste agents are indifferent between their own pair, (p_2^*, q_2^*) , and that intended for the low-taste agents, (p_1^*, q_1^*) . The purpose of randomization is to relax this self-selection constraint to extract additional surplus from the high taste-types.

One convenient form of randomization is to allow the monopolist to control a mean-preserving spread over quality in the low price market. We denote by \tilde{q} this random quality where

$$\tilde{q} = q_1 + t\tilde{e} \quad .$$

Here \tilde{e} equals 1 and -1 equiprobably. The monopolist, as before, chooses q_1 and also controls the variability of quality by its choice of t . Formally, the monopolist solves

$$(4) \quad \underset{p_1, q_1, t}{\text{maximize}} \quad N_2[p_2 - C(q_2)] + N_1[p_1 - E_e C(q_1 + te)]$$

$$(4.a) \quad \text{subject to: } bq_1 - p_1 \geq 0$$

$$(4.b) \quad U(q_2) - p_2 \geq E_e U(q_1 + te) - p_1$$

$$(4.c) \quad U(q_2) - p_2 \geq 0$$

$$(4.d) \quad t \geq 0 \quad .$$

In this problem, constraints (4.a) and (4.b) are the individual rationality and self-selection constraints which were binding in the non-randomized solution. With randomization possible, we need to ensure that high-taste agents obtain non-negative surplus as well -- hence we include (4.c). Finally, t must also be non-negative as in (4.d). Using ϕ , δ , μ , γ as multipliers for (4.a), (4.b), (4.c) and (4.d) respectively, the first-order conditions for (4) imply

$$(5.a) \quad N_1 \cdot E_e C'(q_1 + te) = \phi b - N_2 \cdot E_e U'(q_1 + te)$$

and

$$(5.b) \quad N_1 \cdot E_e(eC'(q_1+te)) = -N_2 \cdot E_e(eU'(q_1+te)) + \gamma .$$

These are the derivatives with respect to q_1 (5.a) and t (5.b) under the assumption that (4.c) is not binding.⁶ Our interest is in whether $t > 0$ in the optimal solution to (4) -- i.e. whether randomization is profitable.

Proposition 2: A sufficient condition for randomization in this example is

$$(6) \quad N_1 A^m(q_1^*) \leq N_2 A^c(q_1^*)$$

where $A^m(q_1^*)$ is the monopolist's absolute degree of risk aversion at q_1^* and $A^c(q_1^*)$ is that for the high-taste consumer.

Proof: To show that (6) is a sufficient condition for randomization, we rewrite (5.b) as

$$(7) \quad N_1 [C'(q_1+t) - C'(q_1-t)] = -N_2 [U'(q_1+t) - U'(q_1-t)] + \gamma$$

since \bar{e} takes on the value of -1 and 1 equiprobably. A Taylor series expansion of $C'(q_1+t)$ and $U'(q_1+t)$ around $C'(q_1-t)$ and $U'(q_1-t)$ respectively (dropping terms above the second degree) leaves

$$(7') \quad N_1 \cdot C''(q_1-t) = -N_2 \cdot U''(q_1-t) + \gamma \geq -N_2 \cdot U''(q_1-t)$$

Now, we investigate whether $t = 0$ satisfies (5.a) and (7'). Setting $t = 0$, (5.a), $U'(q) > b$ for all q and $\phi = N_1 + N_2$ together imply

$$C'(q_1^*) < U'(q_1^*) \quad .$$

From this and (7') with $t = 0$, we have the ratio

$$(8) \quad N_1 \cdot C''(q_1^*)/C'(q_1^*) > -N_2 \cdot U''(q_1^*)/U'(q_1^*)$$

Using the Arrow-Pratt measure of risk-aversion,

$$N_1 A^m(q_1^*) > N_2 A^c(q_1^*)$$

is a necessary condition for $t = 0$. If (6) holds, then obviously $t = 0$ will not satisfy (5.a) and (5.b). Q.E.D.

Expression (6) has a very intuitive appeal. $N_1 A^m(q_1^*)$ measures the cost to the monopolist of randomization in the low-price market. Alternatively $N_2 A^c(q_1^*)$ measures the local gains to the monopolist in terms of price increases in the high-taste market that randomization will yield. If the monopolist is locally more risk averse and low taste agents more numerous, randomization will not pay.

While Proposition 2 gives a sufficient condition for randomization, it does not characterize the optimal t^* . Intuitively, if (6) is satisfied at $t = 0$, randomization will continue until either the monopolist becomes more risk-averse than the consumers or until (4.c) becomes binding. That is, as the monopolist increases t , p_2 is increased as well to ensure that (4.b) remains binding. As long as the monopolist remains less risk averse, the only constraint on t will be that (4.c) will eventually limit the

randomization. So, for example, if $C(q)$ is linear, the randomization will continue until the high-taste agents have zero surplus.

This illustrates one of the tradeoffs associated with randomization in this model. Here the convexity of the cost schedule imparts a loss to the monopolist when randomization takes place. Even when the monopolist's costs are linear, there are "costs" to randomization if: (i) there are some informed low-taste agents or (ii) some of the low-taste agents are risk averse. Under these alternative circumstances, one could characterize the necessary and sufficient conditions for randomization as well.

V. Conclusions

The two most important results derived using this simple model can be summarized briefly.

(i) When the monopolist sells to only one taste-type of consumer it has no incentive to try to trick the uninformed into buying low quality some of the time. By forcing them to take this risk, the firm only raises its costs and lowers their willingness-to-pay.

(ii) When the firm sells to more than one taste-type of buyer, randomization of quality may be useful as a tool to relax binding self-selection constraints. This will only be profitable under certain conditions, however. Among the necessary conditions: the uninformed must be relatively

numerous, the (relative) quality preferers must be relatively more risk averse to quality gambles and the firm's unit costs must not be too convex in quality. If these conditions are met, the gains from better sorting can outweigh the costs of randomization.

We close with a remark about whether such randomizing behavior by imperfect competitors trying to sort customers is actually observed in practice. There is certainly a different and universal source of quality variation in the real world that is due to the costly nature of quality control. No firm can afford to completely eliminate variance in the quality of its output. Even in a world with costless quality control, our results imply that a firm may still not find it profitable to eliminate variance in the quality of its output. In general, reducing variance then has two costs, the usual costs of quality control (e.g. more inspections) plus the costs associated with maintaining the desired sorting of customers. While not complicating the story we tell here, we must admit that this will greatly complicate empirical work aimed at testing this theory. The researcher will first need to estimate expenditures on quality control for a fictitious environment in which the self-selection constraints are not binding before comparing them to actual expenditures.

Notes

¹ See, for example, Matthews [1983], Stiglitz [1982] and Prescott and Townsend [1984].

² This assumption ensures that the indifference curves in p - q space will cross once at most.

³ In general, we would not expect this distribution to be a complex one: the firm could simply offer some units for sale that were of a lower quality. This quality will be as low as will fool the uninformed, perhaps so low that such units are properly called "junk". The informed can avoid this quality but the uninformed, selecting randomly from the units offered at a particular price, will get junk with a positive probability. The whole distribution is then described by the two quality levels (high and junk) and the probability that the uninformed get the junk.

⁴ We are grateful to Oliver Hart for emphasizing to us the importance of this difference.

⁵ A similar result, in a model of warranties, was obtained by Grossman [1981].

⁶ $\phi = N_1 + N_2$ and $\delta = N_2$ come from the first-order conditions with respect to p_1 and p_2 .

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