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# INTERTEMPORALLY SEPARABLE OVERLAPPING GENERATIONS ECONOMIES

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## I. Introduction

A complete characterization in standard terms of the equilibria of a "typical" overlapping generations economy is still lacking. In particular we have, so far, no good way of "counting" equilibria or of defining typical economies. Recently Balasko and Shell (1981) showed that when all but a finite number of the generations in an OG economy consist of one log-linear consumer, then generically there is a finite number of equilibria; when every generation takes such a simple form the equilibrium is unique.

In this paper we extend the Balasko-Shell analysis by completely characterizing the set of equilibria of any intertemporally separable overlapping generations (ISOG) economy, that is any OG economy in which some infinite subset $S$ of the generations each consists of a single agent that has an intertemporally separable utility function, $u^t(x_{t1},\ldots,x_{tL},x_{t+1,1},\ldots,x_{t+1,L}) = u^t_t(x_{t1},\ldots,x_{tL}) + u^t_{t+1}(x_{t+1,1},\ldots,x_{t+1,L})$. We use this characterization to define a regular ISOG economy and we show that for regular ISOG economies the number of equilibria can be described as "sequentially finite", that is, for any $T$, the number of sequences $(\bar{p}_1,\ldots,\bar{p}_T)$, $p_t \in \mathbb{R}^L_+$, which can be extended to equilibria $(\bar{p}_1,\ldots,\bar{p}_T,p_{T+1},\ldots)$ is finite. In particular, a small perturbation of any equilibrium price will destroy the equilibrium, no matter how the other prices are changed to compensate. When every agent $ti$ in every generation $t$ in a regular ISOG economy has a utility $u^{ti}$ which gives rise to conventional gross substitutes demands in $2L$ commodities (in particular if there is one Cobb-Douglas consumer per generation) then there is a unique equilibrium. However we also note that for more general regular ISOG economies the number of equilibria, though sequentially finite, may nevertheless be uncountable, even if we restrict ourselves to pareto optimal allocations.

We make precise the notion of typical by arguing that, for fixed utilities, the appropriate topology for the infinite dimensional space $E$ of ISOG economies parameterized by endowments is neither the product topology nor the sup norm topology, but rather what is variously called in global analysis the box or fine or Whitney topology for continuous functions.* The class of regular economies is shown to be typical (or "generic") in the sense that it is an open and dense set in this "fine" topology, and also of probability 1 with respect to the natural infinite product of Lebesque measure. Thus we can simply state our main result: ISOG economies typically have a sequentially finite number of equilibria, though the cardinality of the equilibrium set may, even with probability 1, be uncountable.

Our proposition about the number of equilibria with separability was established already in Geanakoplos-Brown, using nonstandard analysis, where it was shown that the set of equilibria generically formed a 0-dimensional nonstandard manifold, and hence contained a *-finite number of equilibria. The crucial idea needed there is the fact that for $U^t$ separable, the matrix $M = \frac{dx^t_{t1},\ldots,x^t_{tL}}{dp_{t+1,1},\ldots,p_{t+1,L}}$ has rank 1 where $M$ represents the derivative of agent's $t$'s demands at time $t$ with respect to prices at time $t+1$.** Here we obtain the same conclusion by making an elementary observation about the budget constraint in equilibrium and by appealing to conventional generic properties of <u>finite</u> economies.

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* In Balasko-Shell the space of economies could be parameterized by the finite dimensional set of endowments of the non-Cobb-Douglas agents, so no question of the appropriate topology arose.

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** In a paper bearing the same title as this one, Kehoe and Levine, following precisely the idea in Geanakoplos-Brown, showed that the stable manifold of any "regular" steady state is 0-dimensional. Here, as in Geanakoplos-Brown, we deal with economies which differ from generation to generation and with the entire set of equilibria, not just those equilibrium price sequence which converge to a steady state.

We offer our simple result both as an explanation of the Balasko-Shell example and as a guide or limitation to what might be typically true, at least for fixed utilities, as we vary endowments in more general overlapping generations economies.

## II The Model

Each agent $(t,i)$, $t = 1,2,3,\ldots$, $i \varepsilon I_t$, a finite set, is given by a strictly positive vector of endowment, $(e_t^{t,i}, e_{t+1}^{t,i}) \varepsilon \mathbb{R}_+^L \times \mathbb{R}_+^L$, and a twice continuously differentiable, strictly concave, monotonic $(D\,u > 0)$ utility function $u^{t,i}\colon \mathbb{R}_+^L \times \mathbb{R}_+^L \to \mathbb{R}$. Furthermore, each agent $(0,i)$, $i \varepsilon I_0$, a finite set, is given by a strictly positive vector of endowments $e_1^{0,i} \varepsilon \mathbb{R}^L$, and a twice, continuously differentiable, strictly concave, monotonic utility function $u^{0,i}\colon \mathbb{R}_+^L \to \mathbb{R}$. The collection of all the above agents is called $E$. If we take the utilities $u = \{u^{ti}\}$ $t = 0,1,\ldots i\varepsilon I_t$ as fixed, we can parameterize $E$ by the endowments $(E_1, E_2, E_3,\ldots)$ where $E_t \varepsilon \mathcal{E}_t$ is the $L(\#I_{t-1} + \#I_t)$ dimensional vector of time $t$ endowments owned by those born at time $t-1$ and time $t$.

**Definition**: An <u>equilibrium</u> $(p_1,p_2,\ldots), (x_1^{0i},(x_t^{ti}, x_{t+1}^{ti}))$, $t = 1,2\ldots, i\varepsilon I_t$, satisfies $\sum_{i\varepsilon I_{t-1}} x_t^{t-1,i} + \sum_{i\varepsilon I_t} x_t^{ti} = \sum_{i\varepsilon I_{t-1}} e_t^{t-1,i} + \sum_{i\varepsilon I_t} e^{t,i}$, for $t = 1,2,\ldots$ and for each $(t,i)$, $i\varepsilon I_t$, $t = 1,2\ldots$, $(x_t^{ti}, x_{t+1}^{ti})$ maximizes $u^{ti}(x_t, x_{t+1})$ subject to the constraint $p_t\, x_t + p_{t+1}\, x_{t+1} \leqq p_t\, e_t^{ti} + p_{t+1}\, e_{t+1}^{ti}$ and for each $i\varepsilon I_0$, $x_1^{0i}$ maximizes $u^{0i}(x_1)$ subject to the constraint $p_1\, x_1 \leqq p_1\, e_1^{0i}$.

**Definition**: The utility function $u^{ti}$ is called intertemporally separable iff it can be written in the form

$$u^{ti}(x_t, x_{t+1}) = u_t^{ti}(x_t) + u_{t+1}^{ti}(x_{t+1}).$$

Suppose that for some $T$, all the utility functions $u^{Ti}$ $i \in I_T$ are intertemporally separable. It is then possible to imagine splitting each agent $Ti$ into two agents, $(u_T^{Ti}, e_T^{Ti})$ and $(u_{T+1}^{Ti}, e_{T+1}^{Ti})$. This would split the economy $E$ into two disjoint economies, a finite Arrow-Debreu economy $\bar{E}(1,T)$ with $TL$ commodities and an overlapping generations economy $\bar{E}(T+1, \infty)$ beginning at time $T + 1$. Agent $u_T^{Ti}$ in economy $\bar{E}(1, T)$ maximizes $u_T^{Ti}(x_T)$ subject to the constraint $p_T \, x_T \leqq p_T \, e_T^{Ti}$. Similarly agent $u_{T+1}^{Ti}$ in economy $\bar{E}(T + 1, \infty)$ maximizes $u_{T+1}^{Ti}(x_{T+1})$ subject to the constraint $p_{T+1} \, x_{T+1} \leqq p_{T+1} \, e_{T+1}^{Ti}$.*

Finally, observe that if $S=\{T_1 < T_2 < T_3 < \ldots\}$ is an infinite collection of generations such that $u^{ti}$ is separable for all $t \in S$, $i \in I_t$ then we can define a corresponding sequence of finite Arrow-Debreu economies $\bar{E} = \bar{E}(1, T_1), \bar{E}(T_1 + 1, T_2), \ldots$ constructed in the same manner.

<u>Proposition 1</u>: Let $S = \{T_1 < T_2 < T_3 < \ldots\}$ be an infinite set of generations each consisting of <u>one</u> intertemporally separable agent, and no others. Then $(x_1^{0i}, (x_t^{ti}, x_{t+1}^{ti}))$ $t = 1,2,\ldots i \in I_t$ is an equilibrium allocation for $E$ if and only if $(x_1^{0i}, \ldots, (x_t^{ti}, x_{t+1}^{ti}), \ldots x_{T_1}^{T_1 i}),\ (x_{T_1+1}^{T_1 i}, \ldots, x_{T_2}^{T_2 i}),$ $(x_{T_2+1}^{T_2 i}, \ldots, x_{T_3+1}^{T_3 i}), \ldots$ is a sequence of equilibrium allocations for the finite Arrow-Debreu economies $\bar{E} = \bar{E}(1, T_1), \bar{E}(T_1 + 1, T_2), \ldots$ .

<u>**Remark:**</u> In particular if every generation consists of one intertemporally separable agent, then the overlapping generations economy is isomorphic to a sequence of one period L-commodity Arrow-Debreu economies.

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* It would be pedantic to note that agent $ti$ in the economy $\bar{E}(1, T)$, $0 < t < T$ has a utility which is monotonic in only $2L$ of the $TL$ commodities and agents $0i$ and $Ti$ are monotonic in only $L$ of the $TL$ commodities.

**Proof**: Let us begin with an equilibrium $p$, $x$ for the overlapping generation economy. Note that the $0^{th}$ generation is spending all its income on period 1 commodities. By simple accounting in equilibrium it follows that the first generation is spending, in the aggregate, precisely the value (measured by $p_1$) of its first period endowment in period 1, that is since $\left(\sum_{i\in I_0} x_1^{0i} + \sum_{i\in I_1} x_1^{1i}\right) = \left(\sum_{i\in I_0} e_1^{0i} + \sum_{i\in I_1} e_1^{1i}\right)$ and $p_1 \cdot \sum_{i\in I_0} x_1^{0i} = p_1 \cdot \sum_{i\in I_0} e_1^{0i}$, it follows that $p_1 \cdot \sum_{i\in I_1} x_1^{1i} = p_1 \cdot \sum_{i\in I_1} e_1^{1i}$. Arguing inductively we see that for all $t$, $p_t \cdot \sum_{i\in I_t} x_t^{ti} = p_t \cdot \sum_{i\in I_t} e_t^{ti}$. Now for $t\varepsilon S$, there is only one member of the $t^{th}$ generation, and he must satisfy $p_t \cdot x_t^t = p_t e_t^t$ and so, by Walras Law, $p_{t+1}x_{t+1}^t = p_{t+1}e_{t+1}^t$. Thus at the equilibrium prices agent $t\varepsilon S$ would behave in the same way if he were faced with two separate budget constraints, $p_t x_t \leqq p_t e_t^t$ and $p_{t+1}x_{t+1} \leqq p_{t+1}e_{t+1}^t$, rather than the single budget constraint $p_t x_t + p_{t+1}x_{t+1} \leqq p_t e_t^t + p_{t+1}e_{t+1}^t$. Recalling that his utility is intertemporally separable, we can see that if we replaced agent $(u^t,(e_t^t, e_{t+1}^t))$ with two agents $(u_t^t, e_t^t)$, $(u_{t+1}^t, e_{t+1}^t)$ we would preserve maket clearing. The prices $\frac{1}{p_1}(p_1,\dots p_{T_1})$, $\frac{1}{p_{T_1+1}}(p_{T_1+1},\dots,p_{T_2})$, $\frac{1}{p_{T_2+1}}(p_{T_2+1},\dots,p_{T_3})$,... are a sequence of equilibrium prices for the Arrow-Debreu economies $\bar{E}(1, T_1)$, $\bar{E}(T_1+1, T_2)$, supporting the same allocation $x$.

Conversely, suppose that we have a sequence of Arrow-Debreu equilibria for the associated econony $\bar{E}$, given by prices $(\bar{p}_1,\dots,\bar{p}_{T_1})$, $(\bar{p}_{T_1+1},\dots,\bar{p}_{T_2})$,... and an allocation $\bar{x}$. We can consider the price sequence $(\lambda_1\bar{p}_1, \lambda_2\bar{p}_2\ \lambda_3\bar{p}_3,\dots)$ as a possible equilibrium price sequence for $E$, for appropriately defined scalors $\lambda_t > 0$. From the Kuhn-Tucker theorem it follows immediately that if the scaler $\mu_{ti} > 0$ is chosen appropriately for any agent $ti$, he will demand the same bundle when faced by the single constraint $\bar{p}_t x_t + \mu_{ti}\bar{p}_{t+1}x_{t+1} \leq \bar{p}_t e_t^{ti} + \mu_{ti}\bar{p}_{t+1}e_{t+1}^{ti}$, that he demands when faced with two constraints $\bar{p}_t x_t \leq \bar{p}_t e_t^{ti}$ and $\bar{p}_{t+1}x_{t+1} \leq \bar{p}_{t+1}e_{t+1}^{ti}$. For $T \varepsilon S$, there is only one agent, so we can unambiguously write that scalar as $\mu_T$. Let us now recursively define $\lambda_t$ by the rule:

$$\lambda_1 = 1$$

$$\lambda_{t+1} = \begin{cases} \lambda_t & \text{if } t \notin S \\ \mu_t \lambda_t & \text{if } t \varepsilon S. \end{cases}$$

It is evident that $(\lambda_1\bar{p}_1, \lambda_2\bar{p}_2, \ldots)$, $\bar{x}$ is now an equilibrium for E. Q.E.D.

Since the theory of finite economies is well understood, and since we have just shown that a separable overlapping generations economy has precisely the same equilibria as a sequence of finite Arrow-Debreu economies it follows that we can characterize the equilibrium set of the former in terms of the latter. We shall now use the characterization of proposition 1 to define what we mean by a regular and by a generic, separable, overlapping generations economy.

**Definition**: We say that a separable overlapping generations economy E given by utilities $u = \{u^{ti}\}$ and endowments $e = \{e^{ti}\}$ is regular iff it has some decomposition into finite Arrow-Debreu economies $\bar{E}(1, T_1)$, $\bar{E}(T_1+1, T_2), \ldots$ each of which is regular.*

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* Geanakoplos-Brown give a definition of *-regular which applies to any overlapping generations economy, and which therefore does not rely on the decomposition of proposition 1. An economy $E = \{\{u^{ti}\}, \{e^{ti}\}\}$ is called *-regular iff for every T and every price sequence $(\bar{p}_1, \ldots, \bar{p}_T, \bar{p}_{T+1})$ which clears the first T markets the matrix

$$M = \frac{dz_1, \ldots, dz_T}{dp_1, \ldots, dp_T dp_{T+1}}$$

rank TL, where M represents the derivatives of aggregate excess demand in the first TL commodities with respect to prices at times $t = 1, \ldots, T+1$, evaluated at prices $(\bar{p}_1, \ldots \bar{p}_{T+1})$. If the economy is separable, then *-regularity implies regularity in the sense of this paper.

Recall that a finite Arrow-Debreu economy $\bar{E} = (\{u^j\}, \{e^j\})$ is regular iff the matrix $\frac{DZ}{dp}$ of derivatives of excess demands with respect to price has rank $K-1$, one less than the number of commodities, when evaluated at any equilibrium price $\bar{p}$ of $\bar{E}$. Recall the famous theorem of Debreu that any regular Arrow-Debreu economy has a finite number of equilibria and that for fixed $\{u^j\}$ (satisfying our assumptions) the set of endowments $\tilde{E} = \{e_i^j | j = 1,\ldots N,\ i = 1\ldots,K,\ \bar{E} = (\{u^j\}, \{e^j\})$ is regular$\}$ is open, dense, and of probability one in the space of possible endowments $C_K^N(a,b) = \{e \in \mathrm{IR}_+^{NK} | 0 \ll a \leqq e \leqq b\}$ under the usual topology and normalized Lebesgue measure.

Suppose that we have not one but a countable sequence $n = 1,2,\ldots$ of unrelated Arrow-Debreu economies each represented by parameters $E_1, E_2, \ldots,$ $E_n \in \mathcal{E}_n$ for all $n$. What topology should one put on this sequence of parameter spaces $\mathcal{E} = (\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \ldots)$? Given a natural topology $\tau_n$ on each $\mathcal{E}_n$, one obvious candidate is the product topology $\bar{\tau}$, defined to be all sets which are arbitrary unions of sets of the form $\prod_{n=1}^{N} \mathcal{O}_n \times \prod_{n=N+1}^{\infty} \mathcal{E}_n$, where $\mathcal{O}_n$ is open in $\mathcal{E}_n$, $n = 1,\ldots,N$. The difficulty with this topology may be seen at once. Suppose that for each $n$, $A_n$ is open and dense in $\mathcal{E}_n$, with respect to $\tau_n$, but that also for each $n$ there is an $\bar{E}_n \in \mathcal{E}_n \sim A_n$. Then it is easy to see that the set $A = A_1 \times A_2 \times \ldots$ is not only not open in $\mathcal{E}$, with respect to the product topology $\bar{\tau}$, but worse still its complement $\mathcal{E} \sim A$ is dense, since the product topology ignores the tails. If each $\mathcal{E}_n$ were a metric space, with metric $d_n$, we might consider using the sup topology $\tau_\infty$, defined as the arbitray union of open "balls" of the form $B(\hat{E}, r)$, where $E = (E_1, E_2, \ldots) \in B(\hat{E}, r)$ iff $\sup_{n=1,2\ldots} d_n(\hat{E}_n, E_n) < r$. However, even this topology $\tau_\infty$ has too few open sets, for let $\hat{E} = (\hat{E}_1, \hat{E}_2, \ldots) \in A$ satisfy $d_n(\hat{E}_n, \bar{E}_n) \to 0$ in $n$, where $\bar{E}_n$ is an element in $\mathcal{E}_n \sim A_n$. (Such a sequence $\hat{E}_n$ exists by the assumed denseness of $A_n$.) Then there is no ball

$B(\hat{E}, r)$, $r > 0$, which does not contain some element of $E \sim A$. Hence $A$ is not open in $\tau_\infty$.

Thus we are led to consider an even finer topology, often used in global analysis, and variously called the box, or fine, or Whitney (for $c^0$ functions) topology.

**Definiton:** The <u>fine topology</u> $\tau$ for $E = \prod_{n=1}^{\infty} E_n$, is the arbitrary union of sets of the form $O = \prod_{n=1}^{\infty} O_n$, where $O_n$ is open in $(E_n, \tau_n)$ for all $n = 1,2,\ldots$ .*

Let us now also suppose that we have a natural probability measure $\mu_n$ on $E_n$( which may or may not be generated by $\tau_n$). Then we can define the product measure $\mu = \prod_{n=1}^{\infty} \mu_n$ on $E$ in the usual way.

**Definition:** A set $B$ is <u>generic</u> iff it is open and dense with respect to the fine topology $\tau$ and contains a set $B'$ which is $\mu$-measurable and has $\mu$-measure 1.

We are now ready to return to our separable overlapping generations economy with fixed utilities $\{u^{ti}\} = u$. Recall that $S = \{T_1 < T_2 < T_3 < \ldots\}$ is an infinite collection of generations consisting of exactly one intertemporally separable agent. Let $E_i = C_L^m(a,b) = \{e \in \mathbb{R}_+^{mL} | 0 \ll a \leqq e \leqq b\}$ be the set of possible endowments of all the agents in the $i^{th}$ Arrow-Debreu economy $\bar{E}(T_{i-1} + 1,\ldots,T_i)$, where $m = 2 \sum_{t=T_{i-1}+1}^{T_i - 1} \# I_t + \# I_{T_{i-1}} + \# I_{T_i}$ and L is the number of commodities. Let us give $E_i$ the usual normalized

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* It is clear that the fine topology is finer than both the product topology and the sup norm topology.

Lebesgue measure and topology.*

**Proposition 2:** Given a family of separable overlapping generations economies with fixed utilities $u = \{u^{ti}\}$, parameterized by endowments $E = (E_1, E_2, \ldots) \in \mathcal{E} = \mathcal{E}_1 \times \mathcal{E}_2 \times \ldots$, the set $\mathcal{B}$ of regular economies is generic in $\mathcal{E}$ with respect to the product measure $\mu$ and the fine topology $\tau$.

**Proof:** This follows at once form proposition 1, Debreu's theorem for regular finite economies, and the definition of the fine topology.

**Proposition 3:** A regular, separable overlapping generations economy has a sequentially finite number of equilibria, that is for any $T$, the number of vectors $(\bar{p}_1, \bar{p}_2, \ldots, \bar{p}_T)$ which can be extended to equilibria $(\bar{p}_1, \bar{p}_2, \ldots, \bar{p}_T, p_{T+1}, \ldots)$ is finite. Thus any small perturbation of an equilibrium price will destroy the equilibrium, no matter how the other prices are changed to compensate.

**Proof:** Given any $T$ there is a $T_n > T$ with $T_n \in S$. If $(\bar{p}_1, \ldots, \bar{p}_T)$ can be extended to an equilibrium, then in particular it can be extended to $(\bar{p}_1, \ldots, \bar{p}_T, p_{T+1}, \ldots, p_{T_n})$, an equilibrium for $\bar{E}(1, T_1), \ldots, \bar{E}(1, T_n)$. But from Debreu's theorem on regular finite economies, we know that there are only a finite number of these. Q.E.D.

**Proposition 4:** For some separable utilities, $\{u^{ti}\} = u$ and $L \geq 2$, there is an open set $C$ of endowments, with $\mu(C) = 1$, such that for every

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* Our parameterization is canonical if we choose $S$ to be the set of all generations consisting of precisely one intertemporally agent.

economy $(u, E)$, $E \in C$, there are an uncountable number of equilibria, including an uncountable number of pareto optimal equilibria, even if $(u,E)$ is regular for each $E$ in $C$.

**Proof**: Let there be one agent in each generation with utility $u_t^t(x_{t1}^t, x_{t+1}^t) = u(x_t^t) + v(x_{t+1}^t)$. We know that in a finite Arrow-Debreu economy with $L \geqq 2$ goods and 2 agents we can find utilities $u, v$ for the two agents such that for some open set of $A$ of endowments the resulting economy will have 3 equilibria. In fact this can be done even if we require $u = v$. With such a $u$, it follows from proposition 1 that any sequence of one period equilibria will support an overlapping generations equilibrium. But there are $3^{\aleph_0}$, or an uncountable number of such selections. Moreover the reader can easily check that all of these are pareto optimal, since the price sequence will satisfy the Cass-Balasko-Shell condition for optimality. The probability that in a countable number of independent draws an event $A$, of nonzero probability, occurs only finitely often is clearly zero.

Finally it remains to explain why the Cobb-Douglas example computed in Balasko-Shell has a unique equilibrium.

**Definition**: Let $u\colon \mathbb{R}_+^K \to \mathbb{R}$ be a strictly concave, monotonic, $C^2$, utility and $0 \ll e \in \mathbb{R}^K$ be an endowment. We call agent $(u, e)$ a <u>gross substitutes</u> agent iff his derived excess demand $z(p)$ satisfies $\frac{\partial Z_i(p)}{\partial p_j} > 0$ for all $i \neq j$. In case $K = 2L$ and $u(x, y) = u_1(x) + u_2(y)$, we call $(u, e)$ a <u>separable gross substitutes</u> agent iff $(u_1, e_1)$ and $(u_2, e_2)$ are both gross substitutes agents.

**Proposition 5**: Let $E = (\{u^{ti}\}, \{e^{ti}\})$ be a separable overlapping generations economy, that is let there be an infinite set $S$ of generations each consisting of a single agent with an intertemporally separable utility. If every agent $ti$, $t \notin S$, is a gross substitutes agent and if every agent $ti$, $t \in S$ is a separable gross substitutes agent, then $E$ possesses a unique equilibrium.

**Proof:** This follows at once from the decomposition given in proposition 1 and the well-known uniqueness property of finite Arrow-Debreu economies with gross substitute agents.

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