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STRATEGIC RESOURCE EXTRACTION: WHEN EASY DOESN'T DO IT

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We have benefited from several lively discussions with Larry Summers.

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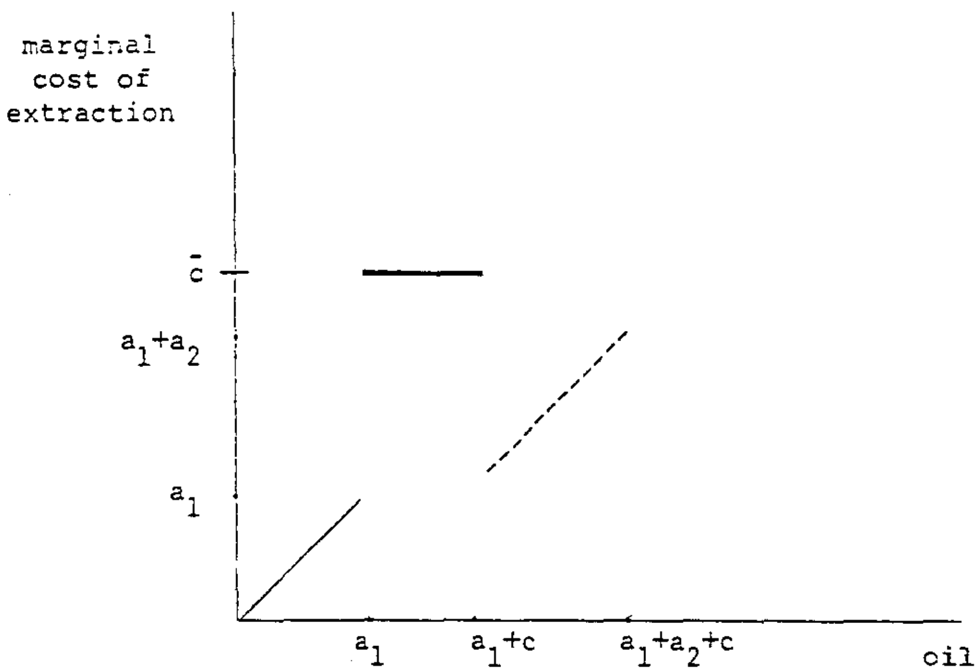
1. Introduction

Suppose that a firm has access to two sources of oil, a deposit with a low but rising marginal cost of extraction and a backstop technology which can produce a limitless supply of a perfect substitute at a high constant cost $\bar{c}$. Can it be rational for the firm to use the expensive mode of production before the marginal cost of extraction of the cheap oil has reached $\bar{c}$? In fact, in many cases expensive substitutes have indeed been produced before the exhaustion of a cheaper resource.

In South Africa the law requires a firm to mine a certain amount of "expensive" gold for every ounce of "cheap" gold it extracts, and in the United States recently, tax subsidies encouraged the extraction of "heavy" oil and the exploration of alternative more costly sources of energy. In both cases, of course, a firm has a direct incentive to use the socially more costly technology, but the question then becomes whether there is any rationale for the governments to make such laws.

Consider a concrete hypothetical example. Let the marginal cost (in labor hours) of extracting the a^{th} level of cheap oil be cumulative and equal to a. Can a rational firm produce $\bar{a}_1 < \bar{c}$ barrels of the cheap oil together with $\bar{x}_1 > 0$ barrels of the expensive oil in the first period? Suppose also that it plans to produce $\bar{a}_2$ barrels of the cheap oil in the second period and nothing else, where second-period marginal cost is $\bar{a}_1 + \bar{a}_2 < \bar{c}$. For instance, can $\bar{c} = 1000$, $\bar{a}_1 = 380$, $\bar{x}_1 = 120$, $\bar{a}_2 = 510$, $\bar{x}_2 = 0$?

Notice that if there is a nonnegative real interest rate at which the firm can lend money, that is, if $R \equiv 1 + r \geq 1$, then the firm can reduce its discounted costs by $\bar{c} - \left(a_1 + \frac{a_2}{R}\right)$ and still produce the same output by replacing one unit of expensive oil with cheap oil. In general, writing total costs in second-period dollars,



heavy line represents oil extracted in period 1

dotted line represents oil extracted in period 2

Diagram 1

$$TC = R[c(a_1) + \bar{c}x_1] + [(c(a_1 + a_2) - c(a_1)) + \bar{c}x_2]$$

$$\frac{dTC}{da_1} - \frac{dTC}{dx_1} = (R-1)(c'(a_1) - \bar{c}) + (c'(a_1 + a_2) - \bar{c}) < 0$$

if $c'(a_1 + a_2) < \bar{c}$ and $c'' \geqq 0$ and $R \geqq 1$ or if $c'(a_1 + a_2) \leqq \bar{c}$ and $c'' > 0$ and $R > 1$.

Variability in future prices further strengthens the argument for extracting cheap reserves first. To see this, consider a firm with two barrels of oil in the ground, with extraction costs of $\bar{c}$ and $a_1 < \bar{c}$. Assume that the interest rate is nonnegative ($R \geqq 1$), and the current price of oil is $p_1 > \bar{c}$. The second-period price is uncertain, $\tilde{p}_2$. To further simplify the problem just have the firm consider strategies under which it extracts one barrel of oil in the first period. It will extract the second barrel in the second period if the realized second-period price, p_2, is greater than the cost of extracting the remaining barrel. If the firm extracts its expensive oil in the first period, its total profits in terms of second-period dollars is $R(p_1 - \bar{c}) + Max(0, p_2 - a_1)$. If the firm extracts its cheap oil in the first period its total profits are $R(p_1 - a_1) + Max(0, p_2 - \bar{c})$. If <u>either</u> $R > 1$ or there is any chance that $p_2 < c$ then there is an advantage in extracting the cheap oil first. How then is it possible to rationalize our example, without taking $R \leqq 1$?[1]

Kemp and Long [1980], addressing exactly the same problem, have in effect made such an assumption, though they showed that it is consistent with a general competitive equilibrium in which every agent has a positive rate of time preference. They point out that if consumption depends solely on the amount of oil produced and if there is no storage, then the only way to save-invest for the future so as to balance consumption across periods is to leave more inexpensive oil in the ground. The idled part of the first-period labor force has nothing better to do than to produce some expensive oil. As we have seen, this can only be brought about in a competitive equilibrium by a nonpositive real rate of interest.

We develop here another possible answer to our questions by showing that even when $R > 1$, a firm in an oligopolistic industry can be rationally maximizing its own profits and yet display the above behavior because it correctly takes into account costs a perfectly competitive firm ignores. These costs, which it is the purpose of this paper to explain, clearly cannot be reduced to the standard argument that a firm's quantity affects price which in turn affects its profits, for that reasoning would not account for why the firm should choose to produce inefficiently.

The analysis which follows is expressed in terms of the perfect[2] Nash equilibria of a Cournot-type game in which each of two identical firms must choose between cheap and expensive production in each of two periods. Our main result is that, if a firm's marginal revenue decreases when its competitor's output increases and some technical requirements are met, then there will be an interval $\bar{I}$ of values of $\bar{c}$ for which there is a perfect Nash equilibrium in which each firm is extracting expensive oil in period one though the marginal cost of cheap oil, even in period two, never reaches $\bar{c}$. Of course if $\bar{c}$ is too high no firm will ever want to produce expensive oil, and if $\bar{c}$ is very low each firm will exhaust the cheap pool and then switch entirely to expensive oil. In the appendix we solve in closed form the model with general linear demand and linear marginal cost (as in our concrete example) and confirm that there is indeed an interval $\bar{I}$ of $\bar{c}$'s for which there is a unique perfect (symmetric) Nash equilibrium, of the desired type.

Suppose that we think of the two firms as domestic and foreign, both selling to consumers in the home country. Let us also suppose that the home country government is interested in its consumer surplus plus the profits of the local firm. We shall show that for the linear model in the above type I equilibrium a government subsidy to the local firm's production of "expensive" oil increases local profits (net of subsidy) and increases total output in both periods, thereby also increasing consumer surplus. On the other hand, a subsidy to the local firm's production of first period cheap oil, meant to improve efficiency through the substitution of cheap for expensive oil, reduces local profits (net of the subsidy) and consumer surplus due to a reduction in second period output.

We hasten to add that there is nothing special about the Cournot game. As we indicate later, our phenomenon could be expected to occur in a variety of other games, perhaps political in nature. Price competitition, in a differentiated products environment, is also consistent with our phenomenon, again assuming the proper connection between increased activity by firm B and the marginal reward to firm A's own activity.

Our results differ from those of Stiglitz and Dasgupta [1980]. In their model individual firms always extract their cheap reserves first. However, they note that small producers face higher marginal revenue than large producers, hence we might find one firm producing a small amount of high marginal cost output while another firm is still extracting a large amount of low marginal cost reserves.

The closest analogue of our phenomenon is to be found in the theory of excess capacity. We also relate our explanation to a rationale for the learning curve.

2. The Strategic Game

Let us define our game $G(\bar{c})$ in both extensive and normal form. In the first period each of two firms A and B must choose quantities of cheap and expensive oil to produce in that period, (a_1, x_1) for A, $(Q_1^A = a_1 + x_1)$ and (b_1, y_1) for B, $(Q_1^B = b_1 + y_1)$, and then the firms must again simultaneously choose in period 2, with full memory of period 1, quantities of cheap and expensive oil (a_2, x_2) and (b_2, y_2). We assume that there is an exogenous demand curve for oil in each period:

$$P_i = D_i(Q_i) \qquad i = 1, 2, \text{ where}$$

$$Q_i \equiv Q_i^A + Q_i^B \equiv (a_i + x_i) + (b_i + y_i) \qquad i = 1, 2$$

that there is no storage,[3] and that A is interested in maximizing $R\pi_1^A + \pi_2^A$ where

$$\pi_1^A = \pi_1^A(a_1, x_1, b_1, y_1) = D_1(Q_1)(a_1 + x_1) - c(a_1) - \bar{c}x_1$$

$$\pi_2^A = \pi_2^A(a_1, a_2, x_2, b_2, y_2) = D_2(Q_2)(a_2 + x_2) - c(a_2; a_1) - \bar{c}x_2$$

while B is interested in maximizing $R\pi_1^B + \pi_2^B$ where

$$\pi_1^B = \pi_1^B(a_1, x_1, b_1, y_1) = D_1(Q_1)(b_1 + y_1) - c(b_1) - \bar{c}y_1$$

$$\pi_2^B = \pi_2^B(b_1, a_2, x_2, b_2, y_2) = D_2(Q_2)(b_2 + y_2) - c(b_2, b_1) - \bar{c}y_2$$

We assume that the costs are cumulative, $c(a_2, a_1) = c(a_1 + a_2) - c(a_1)$, so that (as we have seen) our phenomenon could not occur in competitive equilibrium with $R > 1$. For convenience we have also assumed that the two firms have identical costs.

The strategy space in the game G for firm A is thus

$$S^A = \{(a_1, x_1, \tilde{a}_2, \tilde{x}_2) : a_1 \geqq 0, x_1 \geqq 0, \tilde{a}_2 : \mathbb{R}^4_+ \to \mathbb{R}_+, \tilde{x}_2 : \mathbb{R}^4_+ \to \mathbb{R}_+\}$$

and similarly the strategy space for firm B is

$$S^B = \{(b_1, y_1, \tilde{b}_2, \tilde{y}_2) : b_1 \geqq 0, y_1 \geqq 0, \tilde{b}_2 : \mathbb{R}^4_+ \to \mathbb{R}_+, \tilde{y}_2 : \mathbb{R}^4_+ \to \mathbb{R}_+\}$$

where the functions $\tilde{a}_2$, $\tilde{x}_2$, $\tilde{b}_2$, $\tilde{y}_2$ are defined on the 4-vector (a_1, x_1, b_1, y_1). The payoff to firms A and B is defined on $S^A \times S^B$ by the above formulas.

We shall calculate all the perfect Nash equilibria of the game $G(\bar{c})$, that is, all the Nash equilibria that have the property that the four-vector $(a_2 = \tilde{a}_2(a_1, x_1, b_1, y_1), \tilde{x}_2 = x_2(a_1, x_1, b_1, y_1), \tilde{b}_2 = b_2(a_1, x_1, b_1, y_1)$ $\tilde{y}_2 = y_2(a_1, x_1, b_1, y_1))$ is a Nash equilibrium for the game $G_2(a_1, x_1, b_1, y_1, \bar{c})$ obtained by restricting, in the obvious way, the game G to what happens only in the second period, for all choices (a_1, x_1, b_1, y_1). Notice that the second-period game G_2 depends on the first-period outcome only through the effect of a_1 and b_1 on second-period costs, $c(a_2, a_1)$, $c(b_2, b_1)$. Hence the functions $\tilde{a}_2$, $\tilde{x}_2$, $\tilde{b}_2$, $\tilde{y}_2$, and therefore $\tilde{Q}^A_2 = \tilde{a}_2 + \tilde{x}_2$, $\tilde{Q}^B_2 = \tilde{b}_2 + \tilde{y}_2$ will depend only on a_1 and b_1. Moreover, it is also easy to see that A's profits depend on B's total outputs, Q^B_1, Q^B_2, not on their division into cheap and expensive oil, so that we can write $\pi^A_1 = \pi^A_1(a_1, x_1, Q^B_1)$ and $\pi^A_2 = \pi^A_2(a_1, a_2, x_2, Q^B_2)$.

In section 6 we shall give the closed form symmetric solution to the game arising when demands and marginal costs are linear, that is, $p_i = d - eQ_i$ and $c(a_1) = \frac{1}{2} a_1^2$ and $c(a_2; a_1) = (a_1 + \frac{1}{2} a_2)a_2$. Our phenomenon is more general, however, and so we make only the following assumptions (which are also satisfied, for example, by demand with constant elasticity $P = dQ^{\beta}$ where $-\frac{1}{2} < \beta < 0$, if demand is slightly perturbed at the boundary where $Q = 0$).

Assumptions

(A0) All variables a_i, x_i, b_i, y_i must be chosen so that $(a_i + x_i)$ and $(b_i + y_i)$ lie in the interval $[0, \bar{Q}]$.

(A1) D_i is three times continuously differentiable, $D_i(Q) \geqq 0$, $D_i'(Q) < 0$, and $D_i''(Q)Q^A + 2D_i'(Q) < 0$ for all $(Q^A, Q^B) \in [0, \bar{Q}] \times [0, \bar{Q}]$, $i = 1, 2$, $Q = Q^A + Q^B$.

(A2.1) For all $(Q^A, Q^B) \in [0, \bar{Q}]^2$,
$|D_i''(Q)Q^A + 2D_i'(Q)| > |D_i''(Q)Q^A + D'(Q)|$ and

(A2.2) if $Q^A = \frac{1}{2}Q$, then $D_i''(Q)Q^A + D_i'(Q) < 0$.

(A3) c is three times continuously differentiable,
$c'(a) > 0$, and $c''(a) > 0$ for all $0 \leqq a \leqq 2\bar{Q}$.

(A4) $c(a_2, a_1) = c(a_1 + a_2) - c(a_1)$, so in particular

$$\frac{\partial c(a_2, a_1)}{\partial^2 a_2} = \frac{\partial}{\partial a_1} \frac{\partial}{\partial a_2} c(a_2, a_1) = c''(a_1 + a_2) > 0.$$

Assumption (A1) says that if A produces more his marginal revenue $MR = D'Q^A + D$ decreases, while (A2.2) requires that when $Q^A = Q^B$ an increase in B's output also reduces A's marginal revenue. When $Q^A \neq Q^B$ an increase in B's output can raise or lower A's marginal revenue, provided (in accordance with (A2.1)) that this effect is smaller in magnitude than that caused by the same increase in A's own output. Assumption (A3) says that marginal costs are increasing and (A4) maintains that costs are cumulative (otherwise our problem is uninteresting).

The reader should be aware that there is nothing special about the Cournot game that makes our result possible. Indeed one could replace the entire demand-price apparatus with simple reward functions $W_i^A(Q_i^A, Q_i^B)$, $W_i^B(Q_i^A, Q_i^B)$, letting payoffs be given by

$$\pi^A = R[W_1^A(Q_1^A, Q_1^B) - c(Q_1^A)] + [W_2^A(Q_2^A, Q_2^B) - c(Q_2^A, Q_1^A)]$$

$$\pi^B = R[W_1^B(Q_1^A, Q_1^B) - c(Q_1^B)] + [W_2^B(Q_2^A, Q_2^B) - c(Q_2^B, Q_1^B)]$$

We would need to assume, for C = A or B and i = 1 or 2, that $(\partial W_i^C/\partial Q_i^C) > 0$, $(\partial^2 W_i^C/\partial^2 Q_i^C) < 0$, $|(\partial^2 W_i^C/\partial^2 Q_i^C)| > |(\partial^2 W_i^C/\partial Q_i^A Q_i^B)|$, as in (A1) and (A2.1), and most crucial, $\text{sign}(\partial W_2^A/\partial Q_2^B) = \text{sign}(\partial^2 W_2^B/\partial Q_2^A Q_2^B)$, as in (A2.2). For example, it might be, contrary to the above Cournot hypothesis, that the welfare of A improves as a result of increased production by B, $(\partial W^A/\partial Q^B) > 0$. Then we could still justify inefficient production by A along the lines of this paper if and only if an increase in future production by A increased the marginal reward to B of its own future production $(\partial^2 W_2^B/\partial Q_2^A Q_2^B) > 0$. The most natural setting for $(\partial W^A/\partial Q^B) > 0$ is when A and B produce complements rather than substitutes. Then, as we shall see, an oil cartel A might produce expensive reserves early on if higher future production of oil could be expected to make it easier for the large car industry B to sell more cars later on. The consequence would be higher future produciton of large cars by B and therefore a higher future price of oil for A for any second period quantity chosen.

There is also no reason why quantities need be the strategic variables; we could derive the same qualitative results in a model with price competition and differentiated commodities (see Bulow, Geanakoplos and Klemperer [1983]).

Due to the complicated nature of our model (two techniques of production and two time periods) we must make additional technical assumptions to guarantee the existence of global Nash equilibria. There are several ways of accomplishing this; (A5) and (A6) indicate one way. The reader may find the linear model analyzed in the appendix a more persuasive argument for the naturalness of our phenomenon.

(A5) $D_2(\bar{Q}) > c'(\bar{Q})$ and $D_2'(\bar{Q} + Q^B)\bar{Q} + D_2(\bar{Q}) < 0$ for all $0 \leqq Q^B \leqq \bar{Q}$.

(A6.1) $D_2'(\bar{Q})\,\frac{1}{2}\bar{Q} + D_2(\bar{Q}) > c'(\frac{1}{2}\bar{Q})$.

(A6.2a) $D_1(0)$ is sufficiently high.

(A6.2b) $D_1'(\frac{1}{2}\bar{Q})\,\frac{1}{4}\bar{Q} + D_1(\frac{1}{2}\bar{Q}) < c'(\frac{1}{2}\bar{Q})$.

(A6.3) For $0 \leqq Q \leqq \frac{1}{4}\bar{Q}$, $c''(Q)$ is sufficiently high compared to the derivatives of D_2 and $c''(\hat{Q})$, $c'''(\hat{Q})$, for $\hat{Q} \geqq \frac{1}{2}\bar{Q}$.

(A6.4) $D_1''(Q_1^A + Q_1^B)Q_1^A + 2D'(Q_1^A + Q_1^B)$ is sufficiently negative, for $0 \leqq Q_1^A, Q_1^B \leqq \frac{1}{4}\bar{Q}$.

(A5) sets boundary conditions which allow us to use calculus. (A6) assures the existence of an equilibrium with the right second-order conditions. Assumptions (A1) - (A4) are crucial to the explanation of our phenomenon. (A5) and (A6) can be thought of simply as a blueprint to constructing examples that are not only local equilibria, but global as well.

It is evident that in equilibrium neither player will choose a_2 (or b_2) so great that $\frac{\partial c(a_2, a_1)}{\partial a_2} > \bar{c}$. Hence without loss of generality we can define the second-period costs by the single function

$$\hat{c}(Q_2^A, a_1, \bar{c}) = \min_{a_2} [c(a_2, a_1) + \bar{c}(Q_2^A - a_2)]$$

$$0 \leq a_2 \leq Q_2^A$$

We can thus further simplify our notation for second-period profits to $\pi_2^A = \pi_2^A(a_1, Q_2^A, Q_2^B)$. Notice that $\hat{c}$ is continously differentiable everywhere and three times continuously differentiable except where $\frac{\partial \hat{c}(Q_2^A, a_1, \bar{c})}{\partial Q_2^A} = \bar{c}$. Of course our purpose is to show that the same sort of simplification cannot be made with the period 1 costs. When it is necessary to emphasize that the solutions depend on the exogenous parameter $\bar{c}$, we shall include that as an argument in our functions as well.

3. <u>An Explanation</u>

We shall show in the next section that the solution to the second-period game $G(a_1, b_1, \bar{c})$ is always unique and that the functions $\tilde{a}_2$, $\tilde{x}_2$, $\tilde{b}_2$, $\tilde{y}_2$, and hence $\tilde{Q}_2^A$ and $\tilde{Q}_2^B$, are twice differentiable functions of $(a_1, b_1, \bar{c})$ except possibly when $c'(a_1 + \tilde{a}_2) = \bar{c}$ or $c'(b_1 + \tilde{b}_2) = \bar{c}$. The crucial result is that $(\partial\tilde{Q}_2^B/\partial a_1) > 0$ if $a_1 = b_1$ and $c'(a_1 + \tilde{a}_2) < c$, that is, an increase in output by firm A today will cause B to produce more in the future, for that gives rise to a strategic cost which is absent from competitive models.

Consider the optimization problem faced by firm A, given B's plan $\bar{Q}_1^B = \bar{b}_1 + \bar{y}_1$ and the functions $\tilde{Q}_2^A$ and $\tilde{Q}_2^B$. Suppose for now that $\bar{c}$ is very large (so we do not need to worry about nondifferentiability). A will

$$\max_{a_1, x_1} \; R\pi_1^A(a_1, x_1, \bar{Q}_1^B) + \pi_2^A(a_1, \tilde{Q}_2^A, \tilde{Q}_2^B)$$

by calculating the derivatives with respect to a_1 and x_1.

$$* \qquad R\frac{\partial\pi_1^A}{\partial a_1} + \frac{\partial\pi_2^A}{\partial a_1} + \frac{\partial\pi_2^A}{\partial Q_2^A}\frac{\partial\tilde{Q}_2^A}{\partial a_1} + \frac{\partial\pi_2^A}{\partial Q_2^B}\frac{\partial\tilde{Q}_2^B}{\partial a_1} \qquad \text{and}$$

$$** \qquad R\frac{\partial\pi_1^A}{\partial x_1} = R[MR_1^A - \bar{c}], \qquad \text{where } MR_1^A = D_1^1(Q_1)Q_1^A + D_1(Q_1)$$

Observe first that $\tilde{Q}_2^A$ is defined to be optimal in the second-period game, hence $(\partial\pi_2^A/\partial Q_2^A) = 0$ (the boundary condition assures us of an interior solution $Q_2^A > 0$ in the second period) so we can rewrite *

$$* \quad R \frac{\partial \pi_1^A}{\partial a_1} + \frac{\partial \pi_2^A}{\partial a_1} + \frac{\partial \pi_2^A}{\partial Q_2^B} \frac{\partial \tilde{Q}_2^B}{\partial a_1}$$

$$= R[MR_1^A - c'(a_1)] - \left(\frac{\partial c(\tilde{a}_2, a_1)}{\partial a_2} \right) + \frac{\partial \pi_2}{\partial Q_2^B} \frac{\partial \tilde{Q}_2^B}{\partial a_1}$$

$$= R[MR_1^A - c'(a_1)] - (c'(a_1 + \tilde{a}_2) - c'(a_1)) + \frac{\partial \pi_2^A}{\partial Q_2^B} \frac{\partial \tilde{Q}_2^B}{\partial a_1}$$

$$* \quad = R[MR_1^A] - (R-1)c'(a_1) - c'(a_1 + \tilde{a}_2) + \frac{\partial \pi_2^A}{\partial Q_2^B} \frac{\partial \tilde{Q}_2^B}{\partial a_1}$$

where in all of the above $\tilde{a}_2 = \tilde{a}_2(a_1, b_1, \bar{c} = \infty)$. If we equate ** and * we will get exactly the formula we had in the introduction, plus the term $s(a_1, b_1) = -(\partial \pi_2^A/\partial Q_2^B)(\partial \tilde{Q}_2^B/\partial a_1)$. Note that in both the competitive and oligopolistic models, a correct calculation of marginal cost for a_1 involves not only $Rc'(a_1)$, but also the additional cost in the second period, $c'(a_1 + \tilde{a}_2) - c'(a_1)$.

What distinguishes the oligopolistic model is the additional term we shall refer to as the <u>strategic</u> <u>cost</u> $s(a_1, b_1)$. We know that $(\partial \pi_2^A/\partial Q_2^B) < 0$, since more production by B involves a lower price to A. We shall also show in the next section that at a symmetric equilibrium $(\partial \tilde{Q}_2^B/\partial a_1) > 0$ if $c'(a_1 + \tilde{a}_2(a_1, b_1, \bar{c})) < \bar{c}$. That is, if A weakens his position in the second period, then B will act more aggressively by producing more. Hence the strategic cost is positive. This positive strategic cost of "cheap" oil together with its cost of extraction may in fact make it just as expensive as the "expensive" oil (which plays no strategic role, hence bears no strategic cost). The paradoxical behavior we have spoken of becomes rational when the strategic cost is taken into account.

Let us be a bit more precise. If both * and ** were equal to 0 at $\bar{a}_1 = \bar{b}_1 > 0$ and $\bar{x}_1 = \bar{y}_1 > 0$ then we would by definition be satisfying the conditions for a local symmetric Nash equilibrium, and it would be of the desired type, provided also that $c'(\bar{a}_1 + \tilde{a}_2(\bar{a}_1, \bar{b}_1, \bar{c})) < \bar{c}$. Of course, we began by taking $\bar{c}$ very large, so that ** = 0 could not possibly hold, but we shall show in the next section how to construct an interval of $\bar{c}$'s, and associated $(\bar{a}_1, \bar{b}_1, \bar{x}_1, \bar{y}_1)(\bar{c}) >> 0$ at which * = 0 and ** does hold.

The strategic cost s drives a wedge between the correctly measured social cost of extraction, $Rc'(a_1) + (c'(a_1 + \tilde{a}_2) - c'(a_1))$ and the correctly measured individual cost of extraction, $Rc'(a_1) + (c'(a_1 + \tilde{a}_2) - c'(a_1)) + s(a_1, b_1)$ of cheap oil. This accounts for the socially inefficient but individually rational "overproduction" of expensive oil.

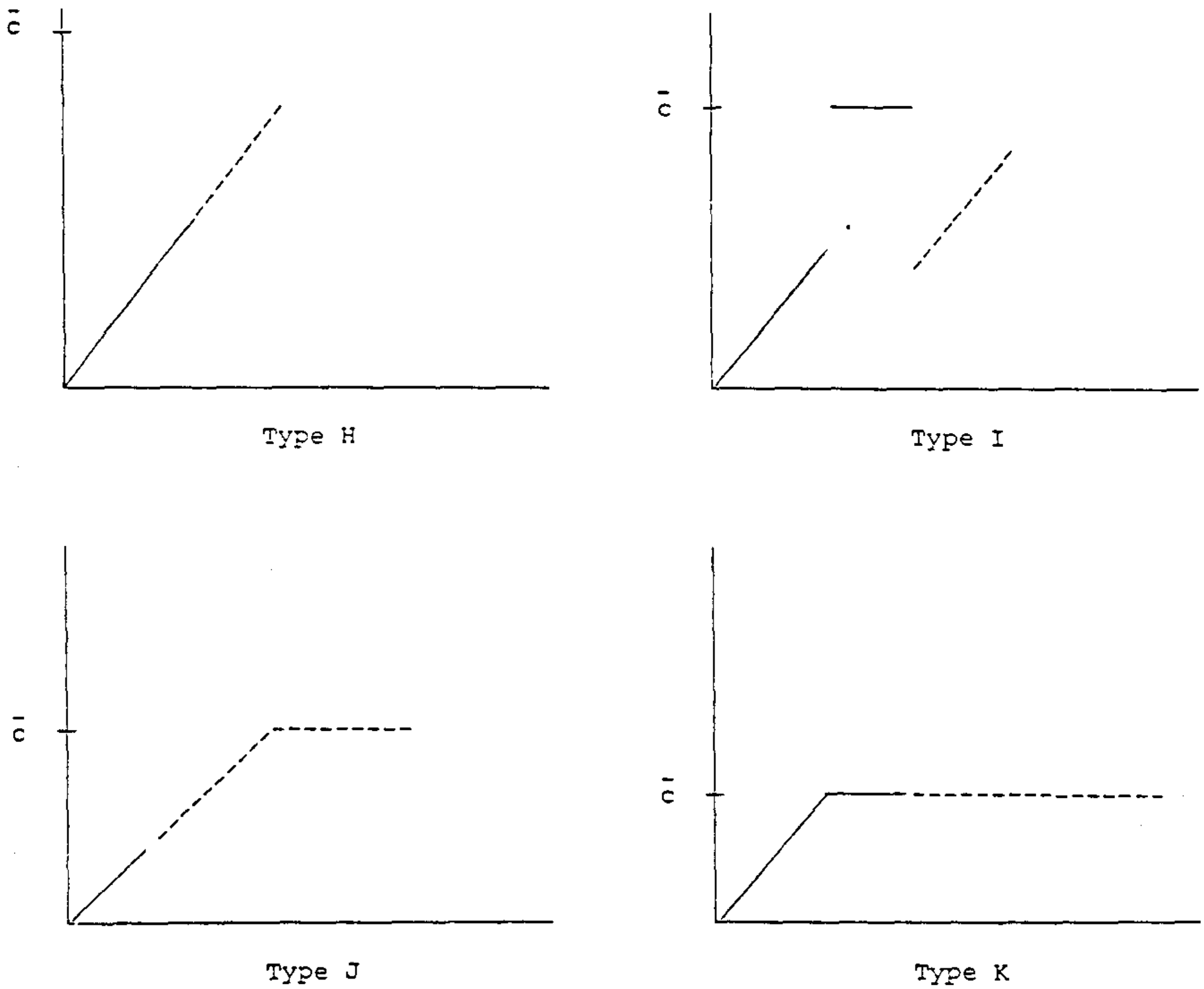
It is possible to draw an analogous rationale for the so-called "learning curve" theory of business strategy which is now fashionable among business strategy consultants. If the marginal cost curve of a firm is cumulative across periods and <u>decreasing</u>, then a given firm A has an extra incentive to produce more in period 1 not only because that lowers its costs in period 2 but also for the strategic reason that it discourages the production of output by competing firms in the second period.

We shall not attempt to classify equilibria in the general case nor do we estimate the amount of expensive oil produced. However, when (inverse) demand $D_i(Q) = d - eQ$ is linear and stable and cost $c(Q) = \frac{1}{2} Q^2$ is quadratic, then we can explicitly calculate all the possible equilibria. It turns out that there are only four possible equilibria, which can be characterized by the following simple diagram. For very high $\bar{c}$, the firms will not use expensive oil. For an interval $\bar{I}$ of values of $\bar{c}$, the firms will overproduce expensive oil due to the strategic cost in the unique symmetric, perfect, Nash equilibrium. For lower values of $\bar{c}$, the firms will exhaust the cheap pool, either in the first period or the second, and then produce expensive oil.

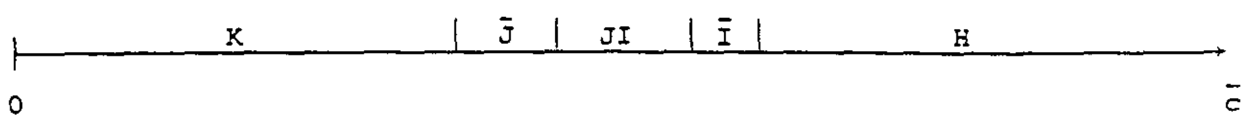
Consider, in a type $\bar{I}$ equilibrium, a small subsidy or tax credit of τ dollars for every unit of the alternative (expensive) energy produced by firm A in period 1. Since a_1 and x_1 were chosen optimally, taking into account the second period effects, the resulting changes $\Delta G_1 < 0$, $\Delta x_1 > 0$, in a_1 and x_1 will have negligible effects on A's profits (net of the subsidy). On the other hand, the reduction in A's marginal costs will, for exactly the reasons explained above, cause $\Delta Q_1^B = \Delta b_1 + \Delta y_1 < 0$, which raises A's first period profits. In the linear case, we must also have $\Delta b_1 > 0$, which raises A's second period profits as well. We know that $\Delta b_1 > 0$, since $\Delta a_1 < 0$ implies that with the same b_1, B would reduce second period production, $\Delta b_2 < 0$. But, that in turn implies that the true marginal cost to B of first period cheap oil would be less than $\bar{c}$ (in the linear case), and hence that B chooses $\Delta b_1 > 0$.

Finally, consider a subsidy τ to firm A's first period production of inexpensive oil. Since this affects intramarginal costs and not marginal costs, it will have no effect on first period total outputs Q_1^A, Q_1^B, but it will result in the substitution $\Delta b_1 = -\Delta y_1 < 0$, $\Delta a_1 = -\Delta x_1 > 0$ which is unfavorable for A's second period profits.

We can draw one more lesson from the linear model. In a type $\bar{I}$ equilibrium (with stable demand across periods and R = 1), the price will be falling across time, even though the marginal costs are rising. This is again because the correctly measured first period marginal cost is equal to the second period marginal cost plus the strategic cost.



Heavy lines denote the marginal cost of each unit extracted in the first period; dashed lines do the same for the second period.



Each equilibrium type occurs only throughout the similarly labaled interval. In JI, both J and I occur. In $\bar{I}$, I is the unique global SPNE.

Diagram 2

4. A Numerical Example

A specific numerical example further clarifies the intuition of our model. Let $D_i(Q) = d - eQ = 8500 - 5Q$ for $i = 1, 2$, let $c(Q) = \frac{1}{2}Q^2$ and let $R = 1.03$ and $\bar{c} = 1000$. Then the unique equilibrium is of type I:

$$a_1 = b_1 = 378.88$$
$$x_1 = y_1 = 121.12$$
$$a_2 = b_2 = 507.57$$
$$x_2 = y_2 = 0.00$$

First, we will confirm this is a local equilibrium by showing that (1) MR = MC in period 1 for expensive reserves; (2) MR = MC in period 2 for cheap reserves; and (3) marginal revenue equals the marginal cost plus the "strategic" cost of extracting an extra unit of cheap reserves in period 1.

(1) First-period marginal revenue for each firm is $d - 3e(a_1 + x_1) = 8500 - 15(378.88 + 121.12) = 1000$, the marginal cost of the expensive output.

(2) Second-period marginal revenue is $d - 3ea_2 = 8500 - 15(507.57) = 886.45$. Second-period marginal cost is $a_1 + a_2 = 378.88 + 507.57 = 886.45$.

(3) The marginal cost of a cheap unit in period one is the change in the present value of total costs due to increasing period one output by one unit and holding second-period output constant. The cash cost in period one is 378.88. Also, because the firm extracts a \$378.88 unit in period one it must take out an extra \$886.45 unit in the second period, to hold that period's output

constant. The extra cost, \$886.45 - \$378.88 = \$507.57, must be multiplied by 1/R to \$507.57 × (1/1.03) = \$492.79. The marginal cost of a cheap unit is therefore \$378.88 + \$492.79 = \$871.67.

The strategic cost of increasing first-period output can be measured by observing how one's competitor responds to your producing an extra first-period unit, and calculating the effect of that response on your profits (e.g., $(\partial\pi_2^A/\partial b_2)(\partial b_2/\partial a_1)$). In this example an increase of 1 unit in a_1 causes an increase of $\frac{e}{(3e+1)(e+1)} = \frac{5}{96}$ units in b_2. The effect of a 5/96 unit increase in b_2 is to lower the price received for the 507.57 units of a_2 by 5(5/96) = 25/96. Discounting, the present value of the strategic cost of increasing a_1 is therefore $\frac{1}{1.03}\left(\frac{25}{96}\right)(507.57) = 128.33$.

Summing the marginal cost of \$871.67 and the strategic cost of \$128.33 makes the firm's private cost of extracting a unit of cheap reserves equal to \$1000, which is marginal revenue. The strategic cost creates a wedge so that units with no strategic cost but a marginal cost of \$1000 (the expensive reserves) are profitable.

Finally, we must ask if this local equilibrium is a global equilibrium. Given b_1 and y_1, can firm a do better by <u>not</u> producing symmetrically? Decreasing a_1 cannot be a profitable strategy. A decrease in a_1 will decrease b_2, causing firm b's second-period marginal cost to remain below c. Firm a's marginal cost will be even lower. Therefore, any reduction in a_1 only leaves the firm in the same local region, and within that region the symmetrical strategy is profit maximizing.

How about increasing a_1? Locally, an increase will reduce profits, but if output is expanded sufficiently there is eventually no strategic cost to increasing a_1. The reason is that second-period marginal costs will have reached their limit of c. If marginal revenue is still above marginal cost at that point some of the losses incurred by locally expanding output could be recouped. (This strategy involves taking out all cheap reserves before any expensive: since second-period marginal costs are c anyway, there is no strategic reason to increase the present value of total costs via early extraction of expensive reserves.)

However, note that in the numerical example if firm a increases a_1 so that second-period marginal costs are c, then second-period output will be (from p. A-3, (i, ii)) 492.88. If marginal costs in period 2 are c = 1000 and second-period output $a_2 + x_2 = 492.88$ then $a_1 \geq 507.12$. However, if $a_1 = 507.12$, $x_1 = 0$, $b_1 = 378.88$, $y_1 = 121.12$ then marginal revenue for firm a is $d - 2e(a_1 + x_1) - e(b_1 + y_1) = 928.81$. But marginal cost is $507.12 + \frac{492.88}{1.03} = 507.12 + 478.52 = 985.64$. The firm is therefore losing money at this point on marginal first-period output, even ignoring strategic costs. Therefore, the firm will choose to reduce a_1 below 507.12 and $a_1 + a_2$ will be less than 1000. However, this reduction puts us back in the original local region, where the firm's profits are only maximized if it reverts to the symmetrical equilibrium.

What happens in our example if firm A is subsidized by a small amount, say, \$15 per unit, for extracting expensive oil, with the costs of the subsidy made up by a lump sum tax? The new equilibrium outputs are:

$a_1 = 361.01$	$b_1 = 376.97$
$x_1 = 140.99$	$y_1 = 122.03$
$a_2 = 509.52$	$b_2 = 506.86$
$x_2 = 0$	$y_2 = 0$

The present value of firm A profits rises by 1797, the added productive inefficiency compensated for by the drop in $b_1 + y_1$ and $b_2 + y_2$. Consumer surplus rises by over 5000 in period one and about 6300 in period two, as total industry output rises in each period. The subsidization of expensive output is quite profitable.

If the government adopts a policy of subsidizing cheap output in the first period by $15 per unit, to encourage the substitution of cheap for expensive first period output, the new equilibrium is:

$$a_1 = 396.43 \qquad x_1 = 103.57 \qquad a_2 = 505.49$$
$$b_1 = 377.59 \qquad y_1 = 122.41 \qquad b_2 = 508.63$$

The result is bad on all counts: the present value of A's profits declines by roughly $800 and the consumer surplus, constant in the first period, drops by over $5000 in the second period.[4]

5. Proof for the General Case

Proposition 1: For every choice of $(a_1, b_1) \in [0, \bar{Q}]^2$ and $\bar{c} \in (0, \infty)$ the game $G_2(a_1, b_1, \bar{c})$ has a unique equilibrium $\left(\tilde{Q}_2^A(a_1, b_1, \bar{c}), \tilde{Q}_2^B(a_1, b_1, \bar{c})\right)$. Moreover, if $a_1 = b_1$, then $\tilde{Q}_2^A = \tilde{Q}_2^B \geqq \frac{1}{2}\bar{Q}$.

Proof: Assumptions (A1), (A3) and (A4) guarantee existence through a routine fixed point argument, and also the existence of a symmetric equilibrium if $a_1 = b_1$. (A6.1) guarantees at least one symmetric equilibrium with $\tilde{Q}_2^A = \tilde{Q}_2^B \geqq \frac{1}{2}\bar{Q}$. To demonstrate uniqueness, suppose that we have two equilibria (Q_2^A, Q_2^B) and $(\mathring{Q}_2^A, \mathring{Q}_2^B)$ and that $Q_2^A > \mathring{Q}_2^A$. From (A3), (A4) we have that

$$\frac{\partial \hat{c}(Q_2^A, a_1, \bar{c})}{\partial Q_2^A} \geqq \frac{\partial \hat{c}(\mathring{Q}_2^A, a_1, \bar{c})}{\partial Q_2^A}$$

hence we must have that $D'(Q_2)Q_2^A + D(Q_2) \geqq D'(\mathring{Q}_2)\mathring{Q}_2^A + D(\mathring{Q}_2)$ in order that marginal revenue equals marginal cost. But from (A1) $Q_2^A > \mathring{Q}_2^A$ tends to reduce marginal revenue, hence from (A2.1) we must have that Q^B changes more than Q^A, that is $|Q_2^B - \mathring{Q}_2^B| > |Q_2^A - \mathring{Q}_2^A|$. But now we have a contradiction, for starting with firm B we can argue that $|Q_2^A - \mathring{Q}_2^A| > |Q_2^B - \mathring{Q}_2^B|$.

Q.E.D.

Proposition 2 shows that the outcome of the second-period game $G_2(a_1, b_1, \bar{c})$ depends differentiably on a_1, b_1, and $\bar{c}$. Moreover, it shows that if for some triple $(a_1, b_1, \bar{c})$ firm A is not exhausting its supply of cheap oil, and if $a_1 = b_1$, then an increase in a_1 will increase B's second-period output,

$$\frac{\partial Q_2^B(\bar{a}_1, \bar{b}_1, \bar{c})}{\partial a_1} > 0.$$

Proposition 2.1: The second-period equilibrium function $(Q_2^A, Q_2^B) = (\tilde{Q}_2^A, \tilde{Q}_2^B)(a_1, b_1, \bar{c})$ is twice continuously differentiable at $(a_1, b_1, \bar{c})$ provided that

$$\frac{\partial c(Q_2^A, a_1)}{\partial a_2} \neq \bar{c}, \text{ and } \frac{\partial c(Q_2^B, b_1)}{\partial b_2} \neq \bar{c}.$$

Hence so is the second-period equilibrium price $\tilde{p}_2(a_1, b_1, \bar{c}) = D(\tilde{Q}_2^A + \tilde{Q}_2^B)$.

Proposition 2.2: Let $(a_1, b_1, \bar{c})$ be a point of differentiability of $(\tilde{Q}_2^A, \tilde{Q}_2^B)$ and let $(Q_2^A, Q_2^B) = (\tilde{Q}_2^A, \tilde{Q}_2^B)(a_1, b_1, \bar{c})$. (1) If $\frac{\partial c(Q_2^A, a_1)}{\partial a_2} < \bar{c}$, then $(\partial\tilde{Q}_2^A/\partial a_1) < 0$, $(\partial\tilde{Q}_2^A/\partial\bar{c}) = (\partial\tilde{Q}_2^B/\partial\bar{c}) = 0$. Moreover, $(\partial\tilde{Q}_2^B/\partial a_1) < |(\partial\tilde{Q}_2^A/\partial a_1)| < 1$. If also $a_1 = b_1$, then $(\partial\tilde{Q}_2^B/\partial a_1) > 0$. (2) If $\frac{\partial c(Q_2^A, a_1)}{\partial a_2} > \bar{c}$, then $(\partial\tilde{Q}_2^A/\partial a_1) = (\partial\tilde{Q}_2^B/\partial a_1) = 0$. If also $a_1 = b_1$, then $(\partial\tilde{Q}_2^A/\partial\bar{c}) = (\partial\tilde{Q}_2^B/\partial\bar{c}) < 0$ and $(\partial\tilde{p}_2/\partial\bar{c}) > 0$.

Proof: The proof of 2.1 and 2.2 is a straightforward application of the implicit function theorem and Cramer's rule for solving linear systems of equalities. Since (A5) eliminates boundary solutions, (A1) and (A4) guarantee that a necessary and sufficient condition for Q_2^A and Q_2^B to be a second-period equilibrium, given $(a_1, b_1, \bar{c})$, is that both

$$\frac{\partial\pi_2^A}{\partial Q_2^A}(Q_2^A, Q_2^B, a_1, \bar{c}) = D'(Q_2)Q_2^A + D(Q_2) - \frac{\partial\hat{c}(Q_2^A, a_1, \bar{c})}{\partial Q_2^A} = 0$$

$$\frac{\partial\pi_2^B}{\partial Q_2^B}(Q_2^A, Q_2^B, b_1, \bar{c}) = D'(Q_2)Q_2^B + D(Q_2) - \frac{\partial\hat{c}(Q_2^B, a_1, \bar{c})}{\partial Q_2^B} = 0$$

If $c(Q_2^A, a_1) \neq \bar{c}$ and $c(Q_2^B, b_1) \neq \bar{c}$, then $\hat{c}$ is twice differentiable and we can write the matrix

$$M = \begin{pmatrix} \dfrac{\partial \partial \pi_2^A}{\partial Q_2^A \, \partial Q_2^A} & \dfrac{\partial \partial \pi_2^A}{\partial Q_2^B \, \partial Q_2^A} \\[2ex] \dfrac{\partial \partial \pi_2^B}{\partial Q_2^A \, \partial Q_2^B} & \dfrac{\partial \partial \pi_2^B}{\partial Q_2^B \, \partial Q_2^B} \end{pmatrix}$$

$$= \begin{pmatrix} D''Q_2^A + 2D' - \dfrac{\partial^2 \hat{c}(Q_2^A)}{\partial^2 Q_2^A} & D''Q_2^A + D' \\[2ex] D''Q_2^B + D' & D''Q_2^B + 2D' - \dfrac{\partial^2 \hat{c}(Q_2^B)}{\partial^2 Q_2^B} \end{pmatrix}$$

From (A1) and (A2.1) we know that in each row the diagonal term is negative and larger than the off-diagonal term. Hence M is invertible and its determinant, $|M|$, is positive. Hence by the implicit function theorem $(\tilde{Q}_2, \tilde{Q}_2^B)$ is differentiable and its derivatives can be calculated by solving:

$$M \begin{bmatrix} \frac{\partial \tilde{Q}_2^A}{\partial a_1} & \frac{\partial \tilde{Q}_2^A}{\partial \bar{c}} \\ \frac{\partial \tilde{Q}_2^B}{\partial a_1} & \frac{\partial \tilde{Q}_2^B}{\partial \bar{c}} \end{bmatrix} = - \begin{bmatrix} \frac{\partial}{\partial a_1} \frac{\partial \pi_2^A}{\partial Q_2^A} & \frac{\partial}{\partial \bar{c}} \frac{\partial \pi_2^A}{\partial Q_2^A} \\ \frac{\partial}{\partial a_1} \frac{\partial \pi_2^B}{\partial Q_2^B} & \frac{\partial}{\partial \bar{c}} \frac{\partial \pi_2^B}{\partial Q_2^B} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial}{\partial a_1} \frac{\partial \hat{c}}{\partial Q_2^A} & \frac{\partial}{\partial \bar{c}} \frac{\partial \hat{c}}{\partial Q_2^A} \\ 0 & \frac{\partial}{\partial \bar{c}} \frac{\partial \hat{c}}{\partial Q_2^B} \end{bmatrix}$$

Let $\frac{\partial c(Q_2^A, a_1)}{\partial a_2} < \bar{c}$. Then $\frac{\partial \partial \hat{c}}{\partial a_1 \partial Q_2^A} = \frac{\partial \partial c}{\partial a_1 \partial a_2} > 0$ and $\frac{\partial \partial \hat{c}}{\partial \bar{c} \partial Q_2^A} = \frac{\partial \partial \hat{c}}{\partial \bar{c} \partial Q_2^B} = 0$.

From Cramer's rule $(\partial \tilde{Q}_2^A / \partial a_1) = \frac{(\partial \partial \hat{c} / \partial a_1 \partial Q_2^A)}{|M|} M_{22} < 0$ since $M_{22} = (\partial \partial \pi_2^B / \partial Q_2^B \partial Q_2^B)$ < 0 from (A1) and (A4). Moreover, from Cramer's rule it follows immediately that $(\partial \tilde{Q}_2^A / \partial \bar{c}) = (\partial \tilde{Q}_2^B / \partial \bar{c}) = 0$. Similarly from Cramer's rule we have that

$$\frac{\partial \tilde{Q}_2^B}{\partial a_1} = \frac{- M_{21} \frac{\partial \partial \hat{c}}{\partial a_1 \partial Q_2^A}}{|M|} > 0$$

if $a_1 = b_1$, for $M_{21} = M_{12} = D''Q_2^A + D'(Q_2) < 0$ if $Q_2^A = Q_2^B$ by assumption (2.2). We saw in proposition 1 that $a_1 = b_1$ implies $Q_2^A = Q_2^B$. Moreover, whether or not $a_1 = b_1$,

$$\frac{\partial \tilde{Q}_2^A}{\partial a_1} + \frac{\partial \tilde{Q}_2^B}{\partial a_1} = \frac{\partial \partial \hat{c}}{\partial a_1 \partial Q_2^A} \frac{1}{|M|} (M_{22} - M_{21}) < 0$$

and therefore total output in the second period declines if a_1 increases. Since $|M_{22}| > |M_{21}|$, $|(\partial\tilde{Q}_2^A/\partial a_1)| > |(\partial\tilde{Q}_2^B/\partial a_1)|$. Finally, a quick calculation reveals that

$$|M| = -\frac{\partial\partial c^A}{\partial^2 a_2}\left(D''Q_2^B + D' - \frac{\partial^2 c^B}{\partial^2 b_2}\right)$$

$$- \frac{\partial^2 c^B}{\partial^2 b_2^2}(D''Q_2^B + 2D') + D'\left(D''(Q_2^A + Q_2^B) + 2D'\right) + D'D'$$

which is bigger, since every term is positive, than its first term, which is $\frac{-\partial\partial c^A}{\partial^2 a_2} M_{22}$. Hence $|(\partial\tilde{Q}_2^A/\partial a_1)| \leq 1$.

Finally, if $\frac{\partial c(Q_2^A, a_1)}{\partial a_2} > \bar{c}$, then $(\partial\partial\hat{c}/\partial a_1\partial a_2) = 0$ and from Cramer's rule we derive at once that $(\partial\tilde{Q}_2^A/\partial a_1) = (\partial\tilde{Q}_2^B/\partial a_1) = 0$ and since now

$$\frac{\partial}{\partial\bar{c}}\frac{\partial\hat{c}}{\partial Q_2^A} = \frac{\partial\partial\hat{c}}{\partial\bar{c}\partial Q_2^B} = 1$$

we have that

$$\frac{\partial\tilde{Q}_2^A}{\partial\bar{c}} = \frac{\begin{vmatrix} 1 & M_{12} \\ 1 & M_{22} \end{vmatrix}}{|M|} = \frac{M_{22} - M_{12}}{|M|}$$

If $a_1 = b_1$, then $Q_2^A = Q_2^B$ and $M_{12} = M_{21}$ and so by (A1) and (A2.1) this expression is negative. Note that we have explicitly calculated the expressions for all the derivatives of $(\tilde{Q}_2^A, \tilde{Q}_2^B)$ and that they are themselves continuously differentiable.

Q.E.D.

At the points of nondifferentiability, the left- and right-hand derivatives exist and are differentiable. Hence in particular, the functions $\frac{\partial s(a_1, b_1)}{\partial a_1}$ and $\frac{\partial s(a_1, b_1)}{\partial b_1}$ are bounded. They depend only on D_2' and higher derivatives and on $c''(q)$ where $q \geqq \frac{1}{2}\bar{Q}$ (by (A6), output is always at least $\bar{Q}$). Therefore it is consistent to assume, for $R > 1$ fixed, that $(R-1)c''(a_1) < -\frac{\partial s(a_1, b_1)}{\partial a_1}$, and $(R-1)c''(a_1) < -\left[\frac{\partial s(a_1, b_1)}{\partial a_1} + \frac{\partial s(a_1, b_1)}{\partial b_1}\right]$ for all $0 \leqq a_1 \leqq \frac{1}{4}\bar{Q}$.

Proposition 3: Suppose (A1) - (A6) hold and $R > 1$ but not too big. Then there is a nontrivial interval of values for $\bar{c}$ such that each game $G(\bar{c})$ has a symmetric, perfect, Nash equilibrium in which both firms produce expensive oil without ever exhausting the supply of cheap oil.

Proof: Recall that we must satisfy the following three conditions, at a point $\bar{a}_1 = \bar{b}_1 > 0$, $\bar{x}_1 = \bar{y}_1 > 0$:

$$* \qquad R[MR_1^A] - (R-1)c'(\bar{a}_1) - c'(\bar{a}_1 + \tilde{a}_2) - s(\bar{a}_1, \bar{b}_1) = 0$$

$$** \qquad R[MR_1^A - \bar{c}] = 0$$

$$*** \qquad c'(a_1 + \tilde{a}_2) < \bar{c} \quad , \quad \tilde{a}_2 = a_2(\bar{a}_1, \bar{b}_1, \bar{c})$$

Conditions * and ** are the first-order conditions for $\bar{a}_1 > 0$ and $\bar{x}_1 > 0$, condition *** assures us that the cheap source of oil is not exhausted.

Let us begin by setting $\overline{\overline{x}}_1 = \overline{\overline{y}}_1 = 0$. Expression * does not depend on $\bar{c}$ so long as $\hat{a}_2(\bar{a}_1, \bar{b}_1, \bar{c}) < \bar{c}$. So let us temporarily fix $\bar{c} = \infty$. Notice that from assumption (A5), if $\overline{\overline{a}}_1 = \overline{\overline{b}}_1 = 0$, * is surely positive (the expression $s(a_1, b_1)$ does not depend on D_1, so (A5) is a consistent hypothesis. From proposition 2 we know that $s(a_1, b_1)$ is bounded). Similarly, if $\bar{a}_1 = \bar{b}_1 = \frac{1}{4}\bar{Q}$, the * is surely negative. Hence there is some $\overline{\overline{a}}_1 = \overline{\overline{b}}_1 < \frac{1}{4}\bar{Q}$ in which it is precisely 0.

Recall that for $a_1 = b_1$, the strategic cost $s(a_1, b_1)$ is positive, so let us set $\overline{\overline{c}} = D_1'(\overline{\overline{a}}_1 + \overline{\overline{b}}_1)\overline{\overline{a}}_1 + D(\overline{\overline{a}}_1 + \overline{\overline{b}})$. Then we have

$$*' \qquad R\overline{\overline{c}} - (R-1)c'(a_1) - c'(\bar{a}_1 + \bar{a}_2) - s(\bar{a}_1, \bar{b}_1) = 0$$

If R is not too large, this implies that $\overline{\overline{c}} > c'(\bar{a}_1 + a_2)$. Hence in fact $\tilde{a}_2 = \hat{a}_2(\overline{\overline{a}}_1, \overline{\overline{b}}_1, \bar{c} = \infty) = \hat{a}_2(\overline{\overline{a}}_1, \overline{\overline{b}}_1, \overline{\overline{c}}) = \tilde{Q}_2^A(\overline{\overline{a}}_1, \overline{\overline{b}}_1, \overline{\overline{c}})$. All that remains is to set $x_1 = y_1 > 0$.

Consider reducing $\bar{a}_1 = \bar{b}_1$ slightly below $\overline{\overline{a}}_1 = \overline{\overline{b}}_1$ in *. If $c''(a_1)$ is sufficiently higher than $\frac{\partial s(\overline{\overline{a}}_1, \overline{\overline{b}}_1)}{\partial a_1} + \frac{\partial s(\overline{\overline{a}}_1, \overline{\overline{b}}_1)}{\partial b_1}$ (which depends on D_2 and $c''(\overline{\overline{a}}_1 + \tilde{a}_2)$, that is, on $c''(q)$ for $q \geqq \frac{1}{2}\bar{Q} + \overline{\overline{a}}_1$ by (A6.2), so again (A6) is consistent), then this makes the second set of terms less negative. Note from proposition 2, that a drop in a_1 reduces $a_1 + a_2$ and a decline in b_1 further reduces a_2, so $c'(a_1 + a_2)$ is also growing smaller. This can be offset in *' by a decrease in $\bar{c}$ below $\overline{\overline{c}}$, which keeps the expression *' equal to 0. Now, the difference between * and *' is that in the former expression the marginal revenue of the first period occurs, whereas in the second it is replaced by $\bar{c}$. Since $\bar{c}$ has dropped and $\bar{a}_1 = \bar{b}_1$ has declined, thereby raising marginal revenue [(A1), (2.2)], it follows that marginal revenue is greater than the new $\bar{c}$. But now we can uniquely choose $\bar{x}_1 = \bar{y}_1 > 0$ in order to bring first-period marginal revenue down to $\bar{c}$. The first-order conditions for a Nash equilibrium with $\bar{a}_1 = \bar{b}_1 > 0$, $\bar{x}_1 = \bar{y}_1 > 0$, $\bar{a}_2 = \bar{b}_2 > 0$, and $c'(\bar{a}_1 + \bar{a}_2) < \bar{c}$ are satisfied. Moreover, the implicit function theorem assures us that the analysis holds true on an interval of values of $\bar{c} < \bar{\bar{c}}$.

To verify that we are at a Nash equilibrium, let us consider changes by firm A away from $(\bar{a}_1 = \bar{b}_1, \bar{x}_1 = \bar{y}_1)(\bar{c})$. Reducing a_1 (holding $b_1 = \bar{b}_1$) doesn't help, so long as $(R-1)c''(a_1) < -\frac{\partial s(a_1, b_1)}{\partial a_1}$, which is our assumption (6.3). Notice that $\frac{d(a_1 + \tilde{a}_2)}{da_1} < 1$, so we maintain $a_1 + \tilde{a}_2 \geq \frac{1}{2}\bar{Q}$. Also, the third term $c'(a_1 + \tilde{a}_2)$ drops as a_1, and therefore (by proposition 2) $a_1 + a_2$ drops. Increasing a_1 loses money for precisely the same reason until the point at which $c'(a_1 + \tilde{a}_2) = \bar{c}$. Here there is a sudden drop in marginal cost, since the strategic cost $s(a_1, \bar{b}_1)$ jumps to 0. But by that point we claim * is less than 0 even without the $s(a_1, \bar{b}_1)$ term. For if $\bar{c}$ is close enough to $\bar{\bar{c}}$, then $\bar{x}_1$ is almost 0. Hence after a_1 rises slightly, with x_1 being lowered at the same rate to keep $MR_1^A = \bar{c}$, x_1 will hit 0. Further increases in a_1 will cause MR_1^A to fall at the rate $D_1''(a_1 + \bar{b}_1)a_1 + 2D_1'(a_1 + \bar{b}_1)$. If this is sufficiently high then in the time it takes $c'(a_1 + \tilde{a}_2)$ to get to $\bar{c}$, $MR_1^A - (R-1)c'(a_1)$ will have fallen by at least $s(\bar{a}_1, \bar{b}_1) - \bar{c} - \left(c'(\bar{a}_1 + \tilde{a}_2(\bar{a}_1, \bar{b}_1))\right)$. Further increases in a_1 can only lower *, since once $c'(a_1 + \tilde{a}_2) = \bar{c}$, only the MR_1^A term and the $-(R-1)c'(a_1)$ need be taken into account, and both of those decline as a_1 increases.

Q.E.D.

6. Conclusion

Given a positive real interest rate, neither a monopolist nor a price-taking perfect competitor would begin the production of expensive oil before the marginal cost of the cheap oil reached c. However, we have shown that the extraction of cheap oil by an oligopolist involves an additional strategic cost which may induce the firm to begin production of expensive oil before depleting its reserves of cheap oil. This strategic cost is due to the increased aggressiveness of the rival firms in the subsequent period once the producer's trump cards, his cheap reserves, have been depleted. This strategic cost is by no means special to the simple Cournot model we have considered here.

We might better understand the oligopolist's decision (in a type I equilibrium) by decomposing a unit output of expensive oil into two joint products: a unit of cheap oil and $c - a_1$ dollars worth of an "investment good" which pays off a_2 dollars in the next period and also reduces output by the other firm. Since $R(c - \bar{a}_1) > \bar{a}_2$ it follows that, from the social point of view, the oligopolist has overinvested in the "investment good": by reducing x_1 and increasing a_1 the firm could have maintained the same production of the cheap good and reduced production of the "investment good," which would have been a social improvement.

Recall the well-known proposition[5] that a monopolist facing a potential entrant will invest in excess capacity. Here we have an example of an analogous principle: in an oligopolistic equilibrium (with no entry) all the firms will invest in excess capacity. We are now faced with an interesting question: if the oligopolistic firms are combined into a single monopolist there will be an efficiency gain through the elimination of the wasteful competition via excess capacity; on the other hand there will be an allocative loss as a result of the ensuing monopolistic reduction in output. Which effect dominates? In our oil extraction example it is easy to see that the social end (consumer surplus _plus_ firm profits) is always better served by the two oligopolists than if they were combined into one monopoly. The production of expensive oil, a joint product one of which is socially beneficial and the other socially inefficient, is moreover on balance socially beneficial--a tax which _discourages_ its production would act in the wrong direction.

Footnotes

1. Having suggested that the answer to our puzzle is not obviously to be found in the introduction of uncertainty, we shall continue to assume known costs.

2. Recall that a perfect Nash equilibrium is one which can be solved backwards from the end; it eliminates the possibility of threat equilibria in which one player threatens to "blow up the world" in the last round if all other players are not cooperative in the earlier periods.

3. The no-storage assumption, which later may seem to take on importance (and surely does not make sense in the case of gold), is meant only to simplify the analysis. Without it we would need marginal cost curves (for example, of the form $MC_1 = a_1 + a_1^2$, $MC_2 = a_1 + a_2 + a_2^2$) to capture the idea that extracting gold today not only depletes the reserves, making the rest harder to get, but also is costlier to get the faster the pace of extraction.

4. Of course, second period subsidies are usually profitable, with the subsidy acting as a way to get the domestic firm to precommit to a larger output. The point here is, given whatever can be done in the second period (e.g., nothing), it may well pay to subsidize the first period output of expensive reserves and not subsidize cheap output.

5. See Stiglitz [19].

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Appendix 1

6. The Linear Model

Let $D_i(p) = d - eQ$, $i = 1, 2$, and let $c(Q) = \frac{1}{2} Q^2$.

6a. Solution of the Second-Period Game $G_2(a_1, x_1, b_1, y_1, \bar{c})$

Let us fix the value of $\bar{c} \leq (e+1)/(2e+1)d$.

Let (a_1, x_1, b_1, y_1) be given, so that the game $G_2(a_1, x_1, b_1, y_1)$ is defined by the strategy spaces $S_2^A = \{(a_2, x_2) : a_2 \geq 0,\ x_2 \geq 0\}$ and $S_2^B = \{(b_2, y_2) : b_2, y_2 \geq 0\}$ and the payoffs π_2^A and π_2^B as above. Notice that for fixed $(a_1, x_1, b_1, y_1; b_2, y_2)$ the function π_2^A is strictly concave in (a_2, x_2) and similarly π_2^B is strictly concave in (b_2, y_2). Hence any pair (a_2, x_2), (b_2, y_2) simultaneously solving the following first-order conditions must be an equilibrium for $G_2(a_1, x_1, b_1, y_1)$, and all equilibria can be so obtained.

$$** \quad \frac{\partial \pi_2^A}{\partial a_2} = d - 2e(a_2 + x_2) - e(b_2 + y_2) - (a_1 + a_2) \leq 0, \quad = \text{ if } a_2 > 0$$

$$\frac{\partial \pi_2^A}{\partial x_2} = d - 2e(a_2 + x_2) - e(b_2 + y_2) - \bar{c} \leq 0, \quad = \text{ if } x_2 > 0$$

$$\frac{\partial \pi_2^B}{\partial b_2} = d - 2e(b_2 + y_2) - e(a_2 + x_2) - (b_1 + b_2) \leq 0 \quad = \text{ if } b_2 > 0$$

$$\frac{\partial \pi_2^B}{\partial y_2} = d - 2e(b_2 + y_2) - e(a_2 + y_2) - \bar{c} \leq 0, \quad = \text{ if } y_2 > 0$$

The above formulas have an easy interpretation in terms of marginal revenue (MR_2^A, MR_2^B) and marginal cost.

Observe that the above system depends on the first-period events only through the pair (a_1, b_1). It can be solved explicitly for the functions $\tilde{a}_2(a_1, b_1)$, $\tilde{x}_2$, $\tilde{b}_2$, $\tilde{y}_2$ in the standard way: Choose k of the four unknowns a_2, x_2, b_2, y_2 to be positive, setting the other 4-k equal to zero, and then solve the corresponding k linear equations in ** for the k unknowns. One then calculates the region in (a_1, b_1) space such that the k solutions are all nonnegative and satisfy the remaining 4-k inequalities. For example, if we choose sign $(a_2, x_2, b_2, y_2) = (+, 0, +, 0)$, then we must solve the first and third equalities for $\tilde{a}_2(a_1, b_1)$ and $\tilde{b}_2(a_1, b_1)$, setting $x_2 = y_2 = 0$, and check that $\tilde{a}_2 \geqq 0$ and $\tilde{b}_2 \geqq 0$ and that $\tilde{a}_2$ and $\tilde{b}_2$ also satisfy the second and fourth inequalities. Proceeding in this way we can calculate the equilibrium functions $\tilde{a}_2$, $\tilde{x}_2$, $\tilde{b}_2$, $\tilde{y}_2$ for all $16 = 2^4$ possible sign patterns of (a_2, x_2, b_2, y_2). In fact it is easy to see that only four of these regions cover the entire (a_1, b_1) space. The other regions are either empty (the assumption $c \leqq \frac{e+1}{2e+1} d$ eliminates the regions where one firm produces nothing whatsoever) or else are contained in the boundaries of the basic four regions. The reader can verify that when two regions overlap the corresponding second-period functions $\tilde{a}_2$, $\tilde{x}_2$, $\tilde{b}_2$, $\tilde{y}_2$ are identically defined. Table 1 describes these equilibrium functions and diagram 2 shows the boundaries of the four regions.

Observe that in the interior of each region the functions $\tilde{a}_2$, $\tilde{x}_2$, $\tilde{b}_2$, $\tilde{y}_2$ are differentiable. In what follows we shall be particularly concerned with the pairs (a_1, b_1) where $a_1 = b_1$, which lie on the long diagonal of the box in Diagram 3. All such points lie either in the interior of region (i, i), where $\tilde{x}_2 = \tilde{y}_2 \equiv 0$, so that $(\partial\tilde{x}_2/\partial a_1) = (\partial\tilde{x}_2/\partial b_1) = (\partial\tilde{y}_2/\partial a_1) = (\partial\tilde{y}_2/\partial b_1) = 0$,

Table 1

Region | **Equilibrium** | **Boundary**

Sign(a_2, x_2, b_2, y_2)

(i,i) $(+,0,+,0)$

$$\bar{a}_2 = \frac{(e+1)d - (2e+1)a_1 + eb_1}{(3e+1)(e+1)} \qquad \bar{x}_2 = 0$$

$$\bar{b}_2 = \frac{(e+1)d - (2e+1)b_1 + ea_1}{(3e+1)(e+1)} \qquad \bar{y}_2 = 0$$

$$\text{if } \begin{cases} \dfrac{(e+1)d + e(3e+2)a_1 + eb_1}{(3e+1)(e+1)} \leq \bar{c} \\ \dfrac{(e+1)d + e(3e+2)b_1 + ea_1}{(3e+1)(e+1)} \leq \bar{c} \\ (2e+1)b_1 - ea_1 \leq (e+1)d \\ (2e+1)a_1 - eb_1 \leq (e+1)d \end{cases}$$

(i,ii) $(+,0,+,+)$

$$\bar{a}_2 = \frac{d + \bar{c} - 2a_1}{3e + 2} \qquad \bar{x}_2 = 0$$

$$\bar{b}_2 = \bar{c} - b_1$$

$$\bar{y}_2 = \frac{(e+1)d - (2e+1)\bar{c} + ea_1}{e(3e+2)} - \bar{c} + b_1$$

$$\text{if } \begin{cases} \dfrac{d + 3ea_1}{3e + 1} \leq \bar{c} \\ \dfrac{(e+1)d + e(3e+2)b_1 + ea_1}{(3e+1)(e+1)} \geq \bar{c} \\ a_1 \leq \dfrac{d + \bar{c}}{2} \end{cases}$$

(ii,i) $(+,+,+,0)$

$$\bar{a}_2 = \bar{c} - a_1 \qquad \bar{b}_2 = \frac{d + \bar{c} - 2b_1}{3e + 2}$$

$$\bar{x}_2 = \frac{(e+1)d - (2e+1)c + eb_1}{e(3e+2)} - \bar{c} + a_1$$

$$\bar{y}_2 = 0$$

$$\text{if } \begin{cases} \dfrac{d + 3ea_1}{3e+1} \leq \bar{c} \\ \dfrac{(e+1)d + e(3e+2)a_1 + eb_1}{(3e+1)(e+1)} \geq \bar{c} \\ b_1 \leq \dfrac{d + \bar{c}}{2} \end{cases}$$

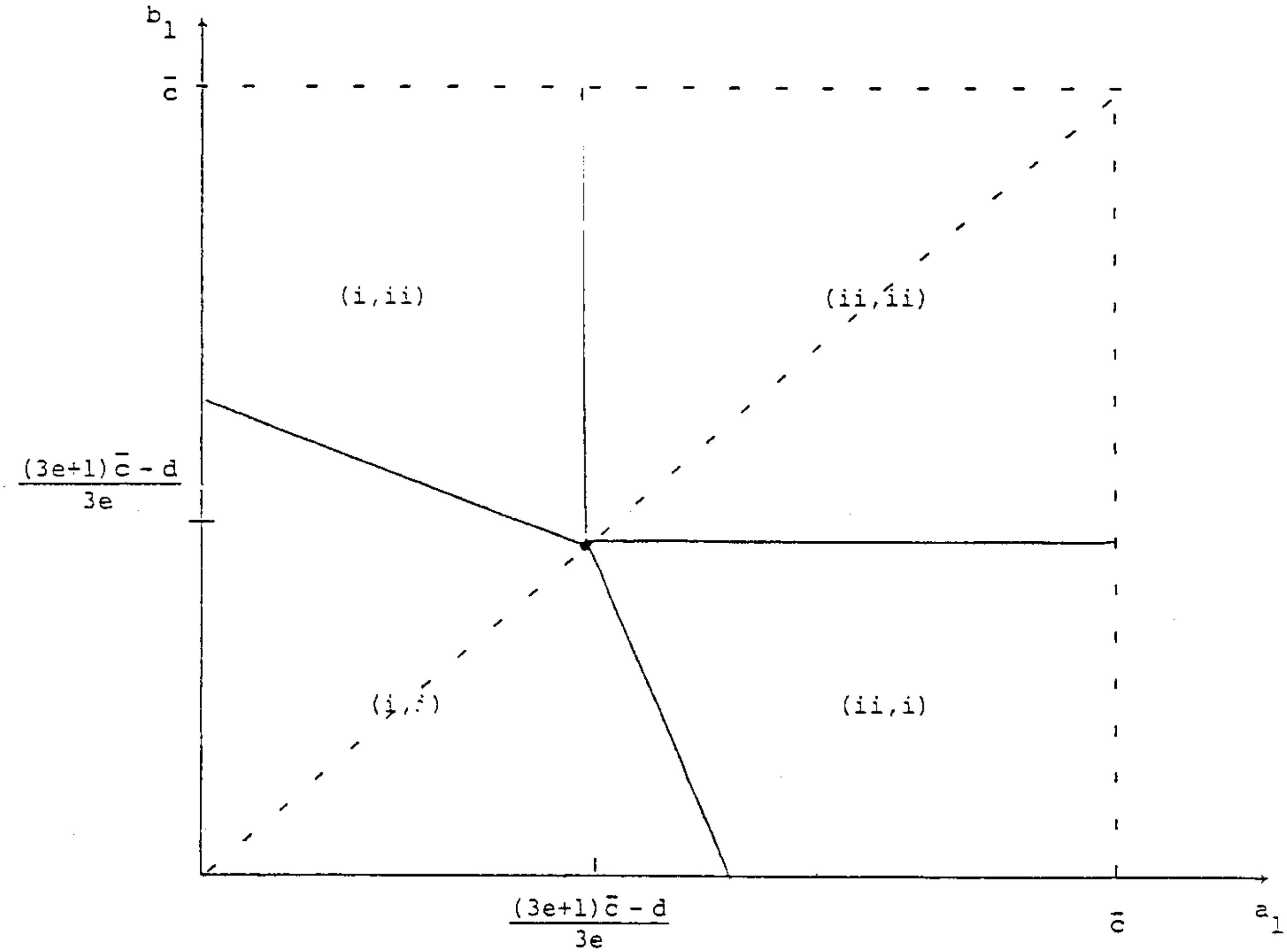
(ii,ii) $(+,+,+,+)$

$$\bar{a}_2 = \bar{c} - a_1 \qquad \bar{b}_2 = \bar{c} - b_1$$

$$\bar{x}_2 = \frac{d - \bar{c}}{3e} - \bar{c} + a_1 \qquad \bar{y}_2 = \frac{d - \bar{c}}{3e} - \bar{c} + b_1$$

$$\text{if } \begin{cases} \bar{c} \leq \dfrac{d + 3ea_1}{3e + 1} \\ \bar{c} \leq \dfrac{d + 3eb_1}{3e + 1} \end{cases}$$

Note: Our formulas apply when a_1 and b_1 are less than or equal to c. If either exceeds c, replace it by $\bar{c}$.



b_1
$\bar{c}$
(i,ii)
(ii,ii)
$\frac{(3e+1)\bar{c} - d}{3e}$
(i,i)
(ii,i)
$\frac{(3e+1)\bar{c} - d}{3e}$
$\bar{c}$
a_1

Diagram 3

or else in the interior of region (ii, ii), where $(\partial\tilde{a}_2/\partial b_1) = (\partial\tilde{x}_2/\partial b_1) = (\partial\tilde{b}_2/\partial a_1) = (\partial\tilde{y}_2/\partial a_1) = 0$, or else (a_1, b_1) is the point $\left(\frac{(3e+1)c - d}{3e}, \frac{(3e+1)c - d}{3e}\right)$ at which the functions $\tilde{a}_2$, $\tilde{x}_2$, $\tilde{b}_2$, $\tilde{y}_2$ are not differentiable. We shall shortly discuss the importance of the foregoing properties of the derivatives along the diagonal $a_1 = b_1$; we shall also show that the particular $a_1 = b_1$ at which the functions $\tilde{a}_2$, $\tilde{x}_2$, $\tilde{b}_2$, $\tilde{y}_2$ are not differentiable cannot be the first-period moves of a Nash equilibrium, and therefore need not concern us. Finally let us recall that the functions $\tilde{a}_2$, $\tilde{x}_2$, $\tilde{b}_2$, $\tilde{y}_2$ are formally defined on all 5-tuples $(a_1, x_1, b_1, y_1, \bar{c})$ even though they do not vary with changes in x_1 or y_1. We shall, in our notation, suppress the dependence on $\bar{c}$.

6b. Necessary Conditions for a Symmetric Equilibrium: Local Equilibria

We are now ready to calculate the perfect Nash equilibria of our game. Since we have uniquely defined $\tilde{a}_2$, $\tilde{x}_2$, $\tilde{b}_2$, $\tilde{y}_2$ on all $(a_1, x_1, b_1, y_1) \in \mathbb{R}_+^4$, we can think of an equilibrium as a four-tuple $(\bar{a}_1, \bar{x}_1, \bar{b}_1, \bar{y}_1)$ such that

$$\underset{a_1, x_1}{\text{Arg Max}} \; [R\pi_1^A(a_1, x_1, \bar{b}_1, \bar{y}_1) + \tilde{\pi}_2^A(a_1, x_1, \bar{b}_1, \bar{y}_1)] = (\bar{a}_1, \bar{x}_1)$$

where

$$\tilde{\pi}_2^A(a_1, x_1, \bar{b}_1, \bar{y}_1) = \pi_2^A(a_1, \tilde{a}_2(a_1, x_1, \bar{b}_1, \bar{y}_1), \tilde{x}_2, \tilde{b}_2, \tilde{y}_2)$$

and

$$\underset{b_1, y_1}{\text{Arg Max}} \; [R\pi_1^B(\bar{a}_1, \bar{x}_1, b_1, y_1) + \tilde{\pi}_2^B(\bar{a}_1, \bar{x}_1, b_1, y_1)] = (\bar{b}_1, \bar{y}_1)$$

where

$$\tilde{\pi}_2^B(\bar{a}_1, \bar{x}_1, b_1, y_1) = \pi_2^B(b_1, \tilde{a}_2(\bar{a}_1, \bar{x}_1, b_1, y_1), \tilde{x}_2, \tilde{b}_2, \tilde{y}_2)$$

Suppose that a point $(\bar{a}_1, \bar{x}_1, \bar{b}_1, \bar{y}_1)$ is an equilibrium of our game, and that the functions $\tilde{a}_2$, $\tilde{x}_2$, $\tilde{b}_2$, $\tilde{y}_2$ are all differentiable there. Since the profit functions π_1^A, π_2^A, π_1^B, π_2^B are always differentiable, it follows that:

$$* \quad R\frac{\partial\pi_1^A}{\partial a_1} + \frac{\partial\pi_2^A}{\partial a_1} + \frac{\partial\pi_2^A}{\partial a_2}\frac{\partial\tilde{a}_2}{\partial a_1} + \frac{\partial\pi_2^A}{\partial x_2}\frac{\partial\tilde{x}_2}{\partial a_1} + \frac{\partial\pi_2^A}{\partial b_2}\frac{\partial\tilde{b}_2}{\partial a_1} + \frac{\partial\pi_2^A}{\partial y_2}\frac{\partial\tilde{y}_2}{\partial a_1} \begin{cases} = 0 & \text{if } \bar{a}_1 > 0 \\ \leq 0 & \text{if } \bar{a}_1 = 0 \end{cases}$$

$$R\frac{\partial\pi_1^A}{\partial x_1} \begin{cases} = 0 & \text{if } \bar{x}_1 > 0 \\ \leq 0 & \text{if } \bar{x}_1 = 0 \end{cases}$$

$$R\frac{\partial\pi_1^B}{\partial b_1} + \frac{\partial\pi_2^B}{\partial b_1} + \frac{\partial\pi_2^B}{\partial a_2}\frac{\partial\tilde{a}_2}{\partial b_1} + \frac{\partial\pi_2^B}{\partial x_2}\frac{\partial\tilde{x}_2}{\partial b_1} + \frac{\partial\pi_2^B}{\partial b_2}\frac{\partial\tilde{b}_2}{\partial b_1} + \frac{\partial\pi_2^B}{\partial y_2}\frac{\partial\tilde{y}_2}{\partial b_1} \begin{cases} = 0 & \text{if } \bar{b}_1 > 0 \\ \leq 0 & \text{if } \bar{b}_1 = 0 \end{cases}$$

$$R\frac{\partial\pi_1^B}{\partial y_1} \begin{cases} = 0 & \text{if } \bar{y}_1 > 0 \\ \leq 0 & \text{if } \bar{y}_1 = 0 \end{cases}$$

We shall call any point $(\bar{a}_1, \bar{x}_1, \bar{b}_1, \bar{y}_1)$ such that $\bar{a}_1 = \bar{b}_1$, $\bar{x}_1 = \bar{y}_1$, and the above relations hold a local equilibrium (a local, symmetric, perfect Nash equilibrium). Putting the first-order conditions in * together with the first-order conditions in ** defines the necessary conditions for a local equilibrium. This system can be solved for the symmetric equilibria using the method applied in the last section. We hold the parameters d, e, and R fixed. For each choice of sign pattern, $\text{sign}(a_1, x_1, a_2, x_2) = \text{sign}(b_1, y_1, b_2, y_2)$ we solve a system of 2k equalities from * and ** in 2k unknowns in terms of $\bar{c}$. We then check for which values of $\bar{c}$ the solutions are nonnegative and satisfy the remaining 2(4-k) inequalities. Once again there are $16 = 2^4$ possible regions in c-space, one for each of the sign patterns $\text{sign}(a_1, x_1, a_2, x_2)$, of which we need concern ourselves with only four. We refer to these regions as H, I, J, and K; the other twelve regions are either empty or consist at most of one point (and are therefore generically unobservable, since $\bar{c}$ is an exogenous parameter). We are most interested in region I.

As compared to section 6a we must deal with two additional difficulties. Since the functions in * are not necessarily concave, we must prove that the conditions necessary for a local equilibrium are also sufficient to guarantee that we have a global Nash equilibrium. Also, the regions I and J overlap in a nontrivial region JI in which the corresponding functions $a_1(\bar{c})$, $x_1(\bar{c})$, etc. from I and J do not agree: in this region JI of c-space there are two local equilibria, one of type I and one of type J.

6c. The Four Types of Local Equilibria

Below in Table 2 we give the results of an easy calculation of the four kinds of local symmetric equilibria obtaining in the regions given by the sign patterns $\text{sign}(a_1, x_1, a_2, x_2) = \text{sign}(b_1, y_1, b_2, y_2) = (+, 0, +, 0)$, $(+, +, +, 0)$, $(+, 0, +, +)$, and $(+, +, 0, +)$, respectively.

Table 2

Type H: $\operatorname{sign}(a_1, x_1, a_2, x_2) = \operatorname{sign}(b_1, y_1, b_2, y_2) = (+, 0, +, 0)$

$$\frac{d[R(3e+1)^2(e+1) + (3e-1)(2e+1)^2]}{R(3e+1)^3(e+1) - (2e+1)^2} \leq c \leq \frac{e+1}{2e+1} d$$

$$a_1 = b_1 = \frac{d[R(3e+1)^2(e+1) - (2e+1)^2]}{R(3e+1)^3(e+1) - (2e+1)^2}$$

$$x_1 = y_1 = 0$$

$$a_2 = b_2 = \frac{d}{3e+1} - \frac{a_1}{3e+1}$$

$$x_2 = y_2 = 0$$

Type I: $\operatorname{sign}(a_1, x_1, a_2, x_2) = \operatorname{sign}(b_1, y_1, b_2, y_2) = (+, +, +, 0)$

$$\frac{d(2e+1)^2}{(3e+1)^2(e+1)R} \leq c \leq \frac{d[R(3e+1)^2(e+1) + (3e-1)(2e+1)^2]}{R(3e+1)^3(e+1) - (2e+1)^2}$$

$$a_1 = b_1 = \frac{Rc(3e+1)^2(e+1) - d(2e+1)^2}{R(3e+1)^2(e+1) - (2e+1)^2}$$

$$x_1 = y_1 = \frac{d-c}{3e} - a_1$$

$$a_2 = b_2 = \frac{d}{3e+1} - \frac{a_1}{3e+1}$$

$$x_2 = y_2 = 0$$

(Cont.)

Table 2 (Cont.)

Type J: $\text{sign}(a_1, x_1, a_2, x_2) = \text{sign}(b_1, y_1, b_2, y_2) = (+, 0, +, +)$

$$\frac{d}{3e+1} \leq c \leq \frac{d[(6e+1)R - 1]}{(3e+1)^2 R - 1}$$

$$a_1 = b_1 = \frac{Rd - c}{(3e+1)R - 1}$$

$$x_1 = y_1 = 0$$

$$a_2 = b_2 = \frac{R((3e+1)c - d)}{(3e+1)R - 1}$$

$$x_2 = y_2 = \frac{d - c}{3e} - a_2$$

Type K: $\text{sign}(a_1, x_1, a_2, y_2) = \text{sign}(b_1, y_1, b_2, y_2) = (+, +, 0, +)$

$$0 \leq c \leq \frac{d}{3e + 1}$$

$$a_1 = b_1 = c$$

$$x_1 = y_1 = \frac{d - (3e+1)c}{3e}$$

$$a_2 = b_2 = 0$$

$$x_2 = y_2 = \frac{d - c}{3e}$$

7. Theorem: Let $d > 0$, $e > 0$, $R > 1$ be given. Then the set $\{c \mid 0 \leq c \leq \frac{e+1}{2e+1} d\}$ can be divided into the four (overlapping) regions H, I, J, K given above, each corresponding to the local equilibria given above. There are no other local equilibria (or global Nash equilibria) for any values of c. If $R > 1$ is sufficiently small then the set $\bar{I} = I \sim J$ is nonempty, and for $c \in \bar{I}$ the unique local equilibrium is a global Nash equilibrium.

Proof: Let us first show that we could not have a Nash equilibrium $(\bar{a}_1 = \bar{b}_1, \bar{x}_1 = \bar{y}_1, \bar{a}_2 = \bar{b}_2, \bar{x}_2 = \bar{y}_2)$ in which $\bar{a}_1 = \bar{b}_1 = \frac{(3e+1)c - d}{3e}$, the only point on the diagonal $a_1 = b_1$ at which one of the functions $\bar{a}_2, \bar{x}_2, \bar{b}_2, \bar{y}_2$ fails to be differentiable. From Table 1 we see that the right derivative $(\partial b_2^+/\partial a_1) = 0$ while the left derivative

$$\frac{\partial \bar{b}_2^-}{\partial a_1} = \frac{e}{(3e+1)(e+1)} > 0$$

because increasing a_1 moves the point (a_1, b_1) into region (iii,ii) while decreasing a_1 moves (a_1, b_1) into region (ii,ii). Consulting Table 1 we also see that $\bar{a}_1 + \bar{a}_2 = c$, hence $(\partial\pi_2^A/\partial a_2) = (\partial\pi_2^A/\partial x_2) = MR_2 - c = 0$ if there is in fact an equilibrium. But there cannot be, for we would then have

$$R \frac{\partial\pi_1^A}{\partial a_1} + \frac{\partial\pi_2^A}{\partial a_1} + \frac{\partial\pi_2^A}{\partial b_2}\frac{\partial\bar{b}_2^+}{\partial a_1} \leq 0$$

and

$$R \frac{\partial\pi_1^A}{\partial a_1} + \frac{\partial\pi_2^A}{\partial a_1} + \frac{\partial\pi_2^A}{\partial b_2}\frac{\partial\bar{b}_2^-}{\partial a_1} \geq 0$$

that is,

$$0 \leqq \frac{\partial \pi_2^A}{\partial b_2} \frac{e}{(3e+1)(e+1)} < 0$$

a contradiction.

The above demonstrates that there can be no symmetric Nash equilibria to our game which are not also local equilibria. Moreover it shows that at a symmetric local equilibrium $(\bar{a}_1, \bar{b}_1)$ must be either in region (i,i) of Diagram 2, in which case $\bar{x}_2 \equiv \bar{y}_2 \equiv 0$ and $(\partial\bar{x}_2/\partial a_1) = (\partial\bar{y}_2/\partial a_1) = 0$, or else in region (ii,ii), in which case $\bar{x}_2 = \bar{y}_2 > 0$ and $(\partial\bar{y}_2/\partial a_1) = (\partial\bar{b}_2/\partial a_1) = 0$. We now proceed to the classification of local equilibria.

Since the first-order conditions * and **, together with the definitions of π_i^A, π_i^B, i = 1, 2 and the behavioral equations derived in Table 2 completely characterize the set of local symmetric equilibria, we shall begin by calculating in all $16 = 2^4$ cases that arise, depending on whether each of the values $a_1 = b_1$, $x_1 = y_1$, $a_2 = b_2$, $x_2 = y_2$ is positive or zero, the set of i's which give rise to such a local equilibrium sign pattern.

We can immediately eliminate seven of these possibilities, for it is impossible that $a_1 = x_1 = 0 = b_1 = y_1$ or that $a_2 = x_2 = 0 = b_2 = y_2$. If there is no output in some period, marginal revenue is d and since the marginal cost of extraction of expensive oil is c < d, either the

$$R \frac{\partial \pi_1^A}{\partial x_1} \leqq 0 \qquad \text{or} \qquad \frac{\partial \pi_a^A}{\partial x_2} \leqq 0$$

condition is violated. We can eliminate three more possibilities by observing that x_1 and x_2 cannot both be positive unless a_1 is positive and a_2 is 0, for if $a_1 = a_2 = 0$, then $(dTC/da_2) - (dTC/dx_2) = -c < 0$ and if $a_1 > 0$, $(dTC/da_1) - (dTC/dx_1) = (a_1/R) + a_2 - \bar{c} = 0$, so $a_1 + a_2 < \bar{c}$, so $a_2 = 0$.

The six remaining possibilities are sign $(a_1, x_1, a_2, x_2) =$ $(+, 0, 0, +)$ or $(0, +, +, 0)$ or type H = $(+, 0, +, 0)$ or type I = $(+, +, +, 0)$ or type J = $(+, 0, +, +)$ or type K = $(+, +, 0, +)$.

Consider first the case where $\bar{a}_1 = \bar{b}_1 = \bar{x}_2 = \bar{y}_2 = 0$ and $\bar{x}_1 = \bar{y}_1 > 0$ and $\bar{a}_2 = \bar{b}_2 > 0$. Such an equilibrium would be an extreme example of the phenomenon we have been trying to illustrate: only expensive oil would be produced in the first period, and only cheap oil in the second period. From Table 2 we know, given $\bar{a}_1 = \bar{b}_1 = 0$, that we would need $\bar{a}_2 = \bar{b}_2 = \frac{d}{3e + 1}$. The first-order conditions would imply that

$$\frac{\partial \pi_1^A}{\partial x_1} = R[MR_1 - c] = 0 \qquad \text{and} \qquad \frac{\partial \pi_2^A}{\partial a_2} = 0$$

From Table 2 we also would have that

$$\frac{\partial \bar{x}_2}{\partial a_1} = \frac{\partial \bar{y}_2}{\partial a_1} = 0$$

Hence we would need

$$\frac{\partial \pi_1^A}{\partial a_1} = R[MR_1 - \bar{a}_1] - \bar{a}_2 + \frac{\partial \pi_2^A}{\partial b_2}\frac{\partial \bar{b}_2}{\partial a_1} \leq 0$$

so again from Table 2,

$$Rc - \bar{a}_2 + (d - e\bar{a}_2)\left(\frac{e}{(3e+1)(e+1)}\right) = Rc - \bar{a}_2 + \frac{(2e+1)de}{(3e+1)(e+1)} \leq 0$$

which is impossible if $R > 1$ and $c \geq \bar{a}_2$.

Second, consider the case where $\bar{a}_1 = \bar{b}_1 > 0$, $\bar{x}_1 = \bar{y}_1 = \bar{a}_2 = \bar{b}_2 = 0$, and $\bar{x}_2 = \bar{y}_2 > 0$. This can indeed occur, but only at a single value of $c = \frac{d}{3e+1}$. For we must have, in such an equilibrium, that $(\partial\pi_2^A/\partial x_2) = MR_2 - c = 0$ and since $\tilde{x}_2$ is never positive unless $a_1 + a_2 = c$, we must have $\bar{a}_1 = c$. For reasons we have given before, if $\bar{x}_2 > 0$, $(\partial\bar{b}_2/\partial a_1) = (\partial\bar{y}_2/\partial a_1) = 0$. With $\bar{a}_1 + \bar{a}_2 = c$, $(\partial\pi_2^A/\partial a_2) = MR_2^A - c = (\partial\pi_2^A/\partial x_2) = 0$. Hence $R[MR_1^A - \bar{a}_1] - \bar{a}_2 = R[MR_1^A - c] = 0$, so $MR_1^A = MR_2^A = c$. This can only happen if $MR_1^A = d - e(2\bar{a}_1 + \bar{b}_1) = c$, i.e., only if $d - e(2c + c) = c$, i.e., only if $c = \frac{d}{3e+1} \in K$. And the equilibrium is precisely that given by type K. The remaining four regions have been analyzed in Table 2.

We defer checking that the boundaries given in Diagram 2 are strictly ordered until Appendix 2. In particular, we show there that for small enough R $\bar{I}$ is nonempty.

Let us now begin the argument that the local equilibria of $\bar{I}$ are Nash equilibria. Suppose we are at a symmetric type I equilibrium ($\bar{a}_1 = \bar{b}_1$, $\bar{x}_1 = \bar{y}_1$) with c in region $\bar{I}$. We must check whether player A, by changing a_1, could increase his profits. As a_1 is increased infinitesimally, there will be no change--that is how we defined a local equilibrium. Small finite changes in a_1 keep the players in region (i,i) in Diagram 1 and, as we shall show, make player A progressively worse off. For a sufficiently large change, when a_1 increases to

$$\alpha = \frac{(3e+1)(e+1)c - (e+1)d - e\bar{b}_1}{e(3e+2)}$$

the point $(a_1, \bar{b}_1)$ moves into region (iii, ii) and here it is possible, since suddenly there is <u>no</u> strategic effect to increasing a_1, that still further increases in a_1 might begin to increase player A's profits again. We shall show, however, that even there

$$\frac{d(R\pi_1^A + \pi_2^A)}{da_1} \leq 0$$

Given $\bar{b}_1$, $\bar{y}_1$, the marginal gain to A from an increase in a_1, keeping $a_1 + x_1 = \frac{d-c}{3e}$ is

$$f(a_1) = R\frac{\partial \pi_1^A}{\partial a_1} - R\frac{\partial \pi_1^A}{\partial x_1} + \frac{\partial \pi_2^A}{\partial a_1} + \frac{\partial \pi_2^A}{\partial b_2}\frac{\partial \bar{b}_2}{\partial a_1}$$

$$= R[MR_1 - a_1] - R[MR_1 - c] - \bar{a}_2(a_1, \bar{b}_1) - s(a_1, \bar{b}_1)$$

Note that at $\bar{a}_1$, $f(\bar{a}_1) = 0$. The best player A can do when increasing a_1 is to offset it by decreasing x_1 so as to hold $(\partial\pi_1^A/\partial x_1) = MR_1 - c = 0$. For $\frac{d-c}{3e} \leq a_1 \leq c$ the marginal gain to A from an increase in a_1 is less than $f(a_1)$:

$$f(a_1) = R(c - a_1) - \bar{a}_2(a_1, \bar{b}_1) - s(a_1, \bar{b}_1)$$

where s is the strategic cost,

$$s(a_1, \bar{b}_1) = \frac{e^2\bar{a}_2(a_1, \bar{b}_1)}{(3e+1)(e+1)}$$

Taking the derivative of $f(a_1)$ yields, for $0 < a_1 < \alpha$,

$$f'(a_1) = -R + \frac{2e+1}{(3e+1)(e+1)}\left(1 + \frac{e^2}{(3e+1)(e+1)}\right)$$

$$= -R + \frac{(2e+1)\left((3e+1)(e+1) + e^2\right)}{(3e+1)^2(e+1)^2} = -R + \frac{(2e+1)^3}{(3e+1)^2(e+1)^2}$$

$$= -R + \frac{8e^3 + 12e^2 + 6e + 1}{9e^4 + 24e^3 + 22e^2 + 8e + 1} < 0 \quad \text{if } R \geq 1$$

Hence $f(a_1) < 0$ for all $\bar{a}_1 < a_1 < \alpha$. Before considering the marginal gain to A from an increase in a_1 when $a_1 > \alpha$, let us first demonstrate that for sufficiently small R, $\alpha > \frac{d-c}{3e}$ for all $c \in \bar{I}$.

$$\alpha = \frac{(3e+1)(e+1)c - (e+1)d - e\bar{b}_1}{e(3e+2)} \geq \frac{d-c}{3e} \qquad \text{iff}$$

$$3(3e+1)(e+1)c - 3(e+1)d - 3e\bar{b}_1 \geq (3e+2) \quad - (3e+2)c \qquad \text{iff}$$

$$(9e^2 + 15e + 5)c \geq (6e+5)d + 3e\bar{b}_1$$

Now, $\bar{b}_1 \leq \bar{b}_1 + \bar{y}_1 = \frac{d-c}{3e}$ where the inequality is strict if $\bar{y}_1 > 0$, i.e., if we are not at the top of region $\bar{I}$. Moreover, $c \in \bar{I}$ implies that $c \geq \frac{d[(6e+1)R - 1)}{(3e+1)^2 R - 1}$. Now, when $R = 1$ we get $c \geq \frac{d[6e]}{9e+6} = \frac{2d}{3e+2}$ where again the inequality is strict if we are not at the bottom of region $\bar{I}$. Hence we have that $\alpha > \frac{d-c}{3e}$ if

$$(9e^2 + 15e + 5)\ \frac{2d}{3e+2} \geqq (6e+5)d + 3e\ \frac{d - c}{3e} \qquad \text{if}$$

$$(9e^2 + 15e + 6)\ \frac{2d}{3e+2} \geqq (6e+6)d \qquad \text{iff}$$

$$3(e+1)(3e+2)\ \frac{2d}{3e+2} \geqq 6(e+1)d$$

$$1 \geqq 1$$

By continuity, for sufficiently small R, $1 < R < \bar{R}$, we must have that $\alpha > \frac{d - c}{3e}$ for all $c \in \bar{I}$.

Consider now that for $a_1 > \alpha$, $(a_1, \bar{b}_1)$ is in region (ii, i) where the marginal benefit to A of an increase in a_1 is

$$f(a_1) = \frac{R\partial\pi_1^A}{\partial a_1} + \frac{\partial\pi_2^A}{\partial a_1}$$

since there is no longer any strategic cost to using a_1, but on the other hand x_1 cannot be further reduced. We need not consider $(\partial\pi_2^A/\partial a_2)(\partial\bar{a}_2/\partial a_1)$, since of course $\bar{a}_2$ is chosen so that $(\partial\pi_2^A/\partial a_2) = 0$. Indeed $\bar{a}_2(a_1, \bar{b}_1) = c - a_1$. Now,

$$R\ \frac{\partial\pi_1^A}{\partial a_1} + \frac{\partial\pi_2^A}{\partial a_1} = R[MR_1^A - a_1] - a_2$$

$$= R[MR_1^A] - (R-1)a_1 - (a_1 + a_2)$$

$$= R[MR_1^A] - (R-1)a_1 - \bar{c}$$

$$< R\bar{c} - (R-1)a_1 - \bar{c}$$

since $MR_1^A < \bar{c}$.

A continuity argument again shows that $f(a_1) < 0$ for all $1 < R < \bar{R}$, $\bar{a}_1 < a_1 \leq c$.

Finally we show that the interval boundaries are ordered as we claimed.

Appendix 2

To show

$$\frac{d[(2e+1)^2]}{(3e+1)^2(e+1)R} < \frac{d[(6e+1)R - 1]}{(3e+1)^2R - 1}$$

or $(3e+1)^2R(2e+1)^2 - (2e+1)^2 < (3e+1)^2(e+1)(6e+1)R^2 - (3e+1)^2(e+1)R$

or $(3e+1)^2[-(2e+1)^2R + (e+1)(6e+1)R^2 - (e+1)R] + (2e+1)^2 > 0$

or $(3e+1)^2[-(2e+1)^2R + (e+1)(6e+1)R^2 - (e+1)R] + (2e+1)^2 > 0$

or $(3e+1)^2[-4Re^2 - 4Re - R + 6R^2e^2 + 7R^2e + R^2 - Re - R] + (2e+1)^2 > 0$

or $(3e+1)^2[(6R^2 - 4R)e^2 + (7R^2 - 5R)e + R^2 - 2R] + (2e+1)^2 > 0$

or $(3e+1)^2[(6R^2 - 4R)e^2 + (7R^2 - 5R)e] + (3e+1)^2(R^2 - 2R) + (2e+1)^2 > 0$

or $(54R^2 - 36R)e^4 + (99R^2 - 69R)e^3 + (57R^2 - 52R + 4)e^2 + (13R^2 - 17R + 4)e$
$+ (R^2 - 2R + 1) > 0$

which is term by term positive for all $R > 1$.

To show

$$\frac{d[(6e+1)R - 1]}{(3e+1)^2R - 1} < \frac{d[R(3e+1)^2(e+1) + (3e-1)(2e+1)^2]}{R(3e+1)^3(e+1) - (2e+1)^2}$$

$$(6e+1)R^2(3e+1)^3(e+1) - (6e+1)R(2e+1)^2 - R(3e+1)^3(e+1) + (2e+1)^2$$
$$\overset{?}{<} (3e+1)^2 R^2 (3e+1)^2 (e+1) + (3e+1)^2 R(3e-1)(2e+1)^2 - R(3e+1)^2(e+1)$$
$$- (3e-1)(2e+1)^2$$

$$R^2(3e+1)^3(e+1)[6e+1 - 3e-1] - R(2e+1)^2[6e+1 + (3e+1)^2(3e-1)]$$
$$- R(3e+1)^2(e+1)[3e+1-1] + (2e+1)^2(3e-1+1) \overset{?}{<} 0$$

$$R^2(3e+1)^3(e+1)3e - R(2e+1)^2(9e^2+3e+1)3e - R(3e+1)^2(e+1)^2(e+1)3e$$
$$+ (2e+1)^2 3e \overset{?}{<} 0$$

$$R^2(3e+1)^3(e+1) - R(2e+1)^2(9e^2+3e+1) - R(3e+1)^2(e+1) + (2e+1)^2 \overset{?}{<} 0$$

When $R = 1$ we have

$$(3e+1-1)(3e+1)^2(e+1) - (2e+1)^2(9e^2+3e+1-1) < 0$$
$$\Leftrightarrow (3e+1)^2(e+1) < (2e+1)^2(3e+1)$$
$$\Leftrightarrow (3e+1)(e+1) < (2e+1)^2$$

By continuity there is some $\bar{R}$ such that for all $1 < R < \bar{R}$, the strict inequality still holds and region I is nontrivial.