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MULTIMARKET OLIGOPOLY

BY

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Abstract

Actions a firm takes in one market may affect its profitability in other markets, beyond any joint economies or diseconomies in production. The reason is that an action in one market, by changing marginal costs in a second market, may change competitors' strategies in that second market. We show how to calculate the strategic consequences in market 2, of a change in conditions in market 1 or of a firm's action in market 1. Qualitatively, the same results hold for both simultaneous markets and sequential markets: whether a more aggressive (i.e., lower price or higher quantity) strategy in the first market provides strategic costs or benefits depends on (a) whether the strategy increases or decreases marginal costs in the second market and (b) whether competitors' products are strategic substitutes or strategic complements. The latter distinction is determined by whether more aggressive play by one firm in a market raises or lowers competing firms' marginal profitabilities in that market. We discuss applications to how firms select "portfolios" of businesses in which to compete, to rational retaliation as a barrier to entry, to international trade, and to the learning curve.

Both strategic substitutes competition and strategic complements competition are compatible with either quantity competition or price competition. For example, strategic complements competition arises from price competition with linear demand and from quantity competition with constant elasticity demand.

The distinction between strategic substitutes and strategic complements is also important in other areas of industrial organization. For example, we show that with strategic complements competition firms will strategically underinvest in fixed costs. This contrasts with earlier studies which, focusing on the total profits of potential entrants rather than the marginal profits of established competitors, invariably emphasized the use of excess capacity.

1. Introduction

A firm's opportunities in one market can affect its opportunities in another market--even if its customers and competitors are completely different in the two markets. One implication of this interaction between the markets is that a firm may decline the chance to enter a market in which it can earn revenues in excess of incremental costs, and choose instead to enter markets in which its revenues are less than its incremental costs.

The basic principle is an old one and can be illustrated by numerous examples from military history. In World War II there was much discussion of the so-called two-front war. The question was whether Germany, already committed to fighting the Allies in the West, should attack Russia in the East. It would not do for the Germans to weigh the costs and benefits of a war against Russia alone; correctly calculated costs are those that take into account the fact that an army must nevertheless be maintained on the Western front. What we emphasize here is that even measuring correctly calculated costs against the gains anticipated in the East is not sufficient: what must also be accounted for is the possible changes in the Western Allies' strategy that might be brought about by the news that Germany is simultaneously fighting two wars.[1]

In the industrial organization context, we show that whether taking a new opportunity in a second market increases or decreases the firm's profits in the first market depends on two factors: (1) whether increasing output in the second market increases or decreases marginal costs in the first market (we will refer to these cases, somewhat imprecisely, as increasing and decreasing costs), and (2) whether the firm's product and its competitors' products in the first market are first-order or strategic substitutes or complements.

This latter distinction, which is made precise in Section 3, relates to whether expectations of more aggressive play by one firm in a market lead to less aggressive (strategic substitutes) or more aggressive (strategic complements) play by competing firms in the same market.

We show that one cannot determine whether products are strategic substitutes or complements without empirically analyzing a market. For example, quantit competition and constant elasticity demand may yield strategic complements, but a linear demand curve with the same elasticity around equilibrium will always yield strategic substitutes. With price competition linear demand yields strategic complements with constant costs, but strategic substitutes with sufficiently rapidly decreasing costs.

Our analysis of these two determinants of "strategic cost" can be extended in a straightforward way when the markets are sequential rather than simultaneous. The decision what to do at time 1 must take into account not only market 1 revenue and correctly measured cost (which anticipates market 2 output), but also the effect of market 1 sales on the actions of market 2 competitors. Thus with decreasing marginal costs, for example, firms will set period 1 marginal revenue below marginal cost with strategic substitutes in period 2, but above marginal cost with strategic complements in period 2. In later sections of this paper we apply these results to dumping in international trade and to models of the learning curve.

We also apply these same results to the issue of strategic investment in cap Selling units at marginal revenue below marginal cost in period 1 in order to decrease one's costs of production in period 2, is formally equivalent to investing in capital that will directly reduce production costs in period 2. Thus with strategic substitutes in period 2 we will observe overinvestment in cap

in period 1, whereas with strategic complements in period 2 we will observe underinvestment in capital in period 1. Given the same constant-returns-to-scale production functions, and for industries protected from new entry, industries engaging in strategic substitutes competition will be more capital intensive than monopolistic or competitive industries, which in turn will be more capital intensive than industries engaging in strategic complements competition. This contrasts with the qualitative implications of papers that focus exclusively on the use of "excess capacity" to deter entry.

If a firm faces potential entrants in a market (see, e.g., Spence, 1977, and Dixit, 1979, 1980), as well as established rivals, then it must consider the effects of its actions on the entry decisions of its potential competitors as well as the strategic consequences for the play of its established rivals. We show that the former effect depends on only one of our two factors--cost. Entering a new market hurts competitors in other markets if costs are decreasing, no matter what the strategic relationship between the goods. A potential competitor's decision whether to enter a market depends on his total profit there; how aggressively he plays once he is in depends on his marginal profit. If a firm has decreasing costs, then expanding into more markets will discourage entry but, if products are strategic complements, it will also make established rivals play more aggressively.

The possibly adverse consequences a firm might suffer in one market as a result of entering another market can be thought of as a barrier to entry. A particularly striking example is one where A and B are each monopolists in different markets even though A's marginal costs are less than the marginal revenue it would earn if it began to sell in B's market. A might rightly figure, however, that in the status quo B has no incentive to enter A's

market, yet if A entered B's market and if costs were increasing then B would rationally (not punitively) react by invading A's market.

This paper is organized as follows. In Section 2 we analyze a simple numerical example of Cournot competition between producers of undifferentiated products in which a firm stays out of one market because of the effect on the actions of its competitors in another market. In Section 3 we define and discuss the concept of first-order or strategic substitutes and complements. Section 4 provides a graphical explanation of our basic propositions. They apply to both price and quantity competition with differentiated products. Section 5, which is not essential for an understanding of the remainder of the paper, contains formal proofs of these results in a more general framework, and Section 6 describes a range of applications of our work.

2. Numerical Example

Consider Cournot duopolists A and B choosing quantities q_a and q_b for sale in a single market with inverse demand function $f(q_a, q_b) = 200 - q_a - q_b$. Assume total costs are $c_a(q_a) = 1/2q_a^2$, $c_b(q_b) = 1/2q_b^2$ so that both firms have increasing marginal costs. Then in Cournot-Nash equilibrium $q_a = q_b = 50$, the price is 100, both firms have marginal costs and marginal revenues of 50, and each firm earns $100 \times 50 - 1/2x(50)^2 = 3750$.

Now give firm A access to a new market where it can sell x_a units at a price $g(x_a) = 55$. Assume $c_a(q_a, x_a) = \frac{1}{2}(q_a + x_a)^2$. In the multimarket Cournot-Nash game firm A chooses q_a and x_a and firm B chooses q_b, all simultaneously. On the surface, the opportunity to sell in the new market appears to be profitable for A. Correctly calculated marginal costs for the first unit sold in the new market are 50, and marginal revenue is 55. However, this gain somehow disappears. Multimarket equilibrium requires:

(MR = MC in the shared market for A) $200 - 2q_a - q_b = q_a + x_a$

(MR = MC in the shared market for B) $200 - q_a - 2q_b = q_b$

(MR = MC in A's other market) $55 = q_a + x_a$

The equations solve to $q_a = 47$, $x_a = 8$, $q_b = 51$. The price in the shared market is 102. Firm B's profits have increased to $3901\frac{1}{2}$, but firm A's profits have fallen to $3721\frac{1}{2}$. Selling in the new market hurts firm A.

At first the result that a company can be made worse off by being given access to a new market may seem bizarre. The following intuition may help. Giving A access to the new market increases his effective marginal cost of selling units in the shared market because of the opportunity cost of not selling those units in the new market. A's higher effective marginal cost

means that he will sell fewer units in the shared market for any given quantity that B sells. B therefore increases his output, reducing the profitability of A's sales in the shared market. A thus ends up worse off overall even though the opportunity to reallocate between the two markets can only be helpful for any given output level that B chooses. If B had kept his output unchanged at 50, the extra units A sold in the new market and the rebalancing of his sales between the two markets would have earned A an extra 18½ in profits. However his competitor responded by increasing output from 50 to 51, thereby lowering the price in the shared market by 1 and reducing A's profits by 47. On net A loses 28½.

The example also shows that subsidizing a firm may hurt it. If both A and B were allowed access to both markets but the price in the smaller market was 50, both would sell 0 in this market and earn 3750 in total. Giving A (but not B) a subsidy of 5 per unit sold in the smaller market would then hurt A by leading back to the equilibrium above in which A earns only 3721½. Equivalently, since achieving a cost saving is formally identical to receiving a subsidy, a firm may also hurt its profits by reducing its costs of doing business in one of its markets.

These results remain essentially unchanged if instead of having the markets clear simultaneously they cleared sequentially. Having the monopoly market clear second makes the formal problem identical to what it is in the simultaneous case. If A is allowed to choose its monopoly quantity before the duopoly game is played, he will choose zero so long as the price in the monopoly market, $g(x_a)$, is at or below 56.25. If the monopoly market price is a bit higher, A would find a positive quantity profitable for him in the sequential game--though not as profitable as for B.[2]

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Note that if both firms were able to compete in both markets then with the price in the new market of 55 we get $q_a = q_b = 48\frac{1}{3}$, $x_a = x_b = 6\frac{2}{3}$, and each firm earns profits of $3848\frac{11}{18}$.

Table 1 summarizes the profits made by each firm in the different cases.

Each firm is always better off if it is prevented from competing, or is able to (credibly) precommit to not competing, in the new market.[3]

Finally, note that A entering the new market makes the entry of new competitors into the larger market more likely. Imagine that a third firm C could enter the main market with the same technology as A and B after payment of a fixed charge F, and assume that C anticipates a three-firm Nash equilibrium after entry. It is easy to check that if A and B are competing only in the larger market then C will enter only if F < 2400, but that if A has access to the new market then C will enter for values of F up to $2709\frac{3}{8}$.[4]

A natural question to ask is how dependent is our example on the form of competition? We will show that the Cournot assumption is unnecessary. While the sign of the strategic effect in the main market varies with the nature of competition and other variables, we are able to identify those variables and thus define when an apparently profitable opportunity is really unprofitable, and when an apparently unprofitable opportunity is really profitable. Showing how to calculate the sign of the strategic effect is an important part of our paper, and enables us to show not only the sensitivity of our results to the type of competition, but also the sensitivity of other important results in industrial organization.

Table 1

PROFIT MATRIX FOR NUMERICAL EXAMPLE

		Firm B	
		competes only in larger market	competes in both markets
Firm A	competes only in larger market	3750, 3750	3902, 3722
	competes in both markets	3722, 3902	3849, 3849

(In each pair the first number is A's
profit, the second number is B's.)

3. Strategic Substitutes and Complements

Consider a market in which firm A chooses a strategic variable q^A, and firm B simultaneously chooses q^B. Assume that a higher level chosen for this variable indicates more aggressive play. For example, if firms choose quantities then q^A and q^B can be thought of as the quantities that A and B choose. If, however, firms choose prices then because <u>low</u> prices are a sign of aggressive play then q^A and q^B can be thought of as the <u>inverses</u> of the prices charged by A and B.

We define q^A to be a substitute for q^B if the partial derivative of B's profits with respect to q^A is less than zero, $\partial \pi^B / \partial q^A < 0$, and a complement if the partial is greater than zero. In making the decision of whether to invest a given amount of capital in a market, competitors become less willing to enter if they expect more of a substitute to be produced. We will always assume that q^A and q^B are substitutes.

However, given its capital stock (that is, its marginal cost curve) a competitor must still decide how aggressively it should compete in a market: how much it should produce or how little it should charge. Whether B will compete more or less aggressively when A becomes more aggressive depends not on the effect of A's actions on B's overall profitability, but on the effect of A's actions on B's marginal profitability.

Assuming that the profit function $\pi^B(q^A, q^B)$ is smooth, at a Nash equilibrium $(\bar{q}^A, \bar{q}^B) \gg 0$ we must have that $\partial \pi^B / \partial q^B = 0$ and, if the second-order conditions are strictly satisfied, that $\partial^2 \pi^B / \partial^2 q^B < 0$. It follows that if a small increase in aggressiveness by A reduces B's marginal profit, $\partial^2 \pi^B / \partial q^A \partial q^B < 0$, then B will respond by reducing q^B; the result is a substitution of q^A for q^B. Accordingly, and analogous to our previous definition of substitutes and complements, we give the following.

<u>Definition</u>. We call q^A a first-order or <u>strategic substitute</u> for q^B if $\partial^2\pi^B/\partial q^A\partial q^B < 0$ and a first-order or <u>strategic complement</u> if $\partial^2\pi^B/\partial q^A\partial q^B > 0$.

In the numerical example above, B could predict (as we will see) that q^A (A's quantity) would be reduced as a result of A's marginal costs rising from selling in a new market. Since q^A was a strategic substitute for q^B (B's quantity), B's response was to increase q^B, thus hurting A's profits.

It can easily be shown that in a Cournot (quantity setting) model with linear demand the respective outputs $q^A = Q^A$, $q^B = Q^B$, are strategic substitutes. Indeed, it seems most natural that if an increase in q^A lowers B's revenue (and profit) then it should lower B's marginal revenue (and marginal profit) as well; accordingly, we will think of strategic substitutability as the "normal" Cournot relationship. It is important to realize, however, that such a relationship is not necessary in a Cournot model. Consider, for example, constant elasticity inverse demand $f^B = (q^A + q^B)^{-\alpha}$, $0 < \alpha < 1$, at an equilibrium $\bar{q}^A$, $\bar{q}^B$ in which $\bar{q}^A/\bar{q}^B < \alpha$. The reader can calculate that now firm B regards q^A and q^B as strategic complements $(\partial^2\pi^B/\partial q^A\partial q^B > 0)$: an increase in q^A would, <u>ceteris paribus</u>, also increase q^B. Notice that if A faced the same demand curve $f^A = (q^A + q^B)^{-\alpha}$ then at the same equilibrium, with $\bar{q}^A/\bar{q}^B < \alpha$, A would regard the two goods as strategic substitutes. In what follows we shall always look at strategic substitutability vs. complementarity from the point of view of firm B.[5]

We show in Section 5 that with price competition and linear demand q^A and q^B are strategic complements, provided marginal costs are not declining too rapidly. As a consequence, firm A might sell in a new market that on the surface appears unprofitable, because A's higher marginal

costs encourage B to price higher which gives A an important side benefit in the shared market.

With increasing costs, then, A gets an extra benefit from entering the new market if products are strategic complements, and is hurt if products are strategic substitutes. With decreasing costs the exact opposite results hold: strategic complements may make an otherwise profitable opportunity unprofitable, and strategic substitutes can make an otherwise unprofitable opportunity profitable.

4. Graphical Explanation of Results

Figure 1 shows A's reaction curve XX' and B's reaction curve SS' in the space of q^A and q^B. As before, q^A and q^B can be thought of either as quantities in a quantity competition game or as the inverses of prices in a price competition game. Thus, for example, if B chooses $q^B = 0$, A optimally reacts with $q^A = X_{A'}$. If B chooses $q^B = X_B$, A's best reaction is $q^A = 0$. If q^B is a strategic substitute for q^A everywhere then an increase in q^B always elicits a reduction in q^A, i.e., XX' is downward sloping as we have drawn it. Similarly, assuming strategic substitutability of q^A for q^B is equivalent to assuming that SS' is downward sloping. In Nash equilibrium q^A is a best response to q^B and vice versa, i.e., $N = (N_A, N_B)$ is the unique Nash equilibrium in the system shown.

Consider now a per unit subsidy which decreases A's marginal costs everywhere. Previously, A's marginal revenue equalled his marginal cost along the line XX'. Decreasing A's costs means that marginal revenue now exceeds marginal cost along XX', so that for any given q^B, A must raise q^A to bring marginal revenue and marginal cost back into balance (because A's second-order conditions tell us that his marginal revenue is decreasing faster than his marginal cost at any point on XX'). So A's new reaction curve DD', shown in Figure 2, is everywhere higher than XX', and leads to a new Nash equilibrium at DS = (DS_A, DS_B). Similarly, if A's marginal costs are raised by a per unit tax his new reaction curve II' will be everywhere lower than XX' and the new equilibrium is IS = (IS_A, IS_B).

We want to determine the additional effect that the tax or subsidy has on A's profitability, beyond the direct effect. Recall that in equilibrium A's marginal revenue equals his marginal cost, i.e., at equilibrium, a small

If the demand in the new market is sufficiently weak, then with increasing marginal costs and strategic substitutes (or decreasing marginal costs and strategic complements) A will be worse off overall. If the price in the new market is ε above the marginal revenue A would get by confining himself to one market, then competing in the new market (and selling $O(\varepsilon)$ units there) raises his profits by $O(\varepsilon^2)$. The fact that A will reduce his output in the shared market also has only an $O(\varepsilon^2)$ effect on profitability. However B will increase q^B by $O(\varepsilon)$ reducing A's profits in the shared market by $O(\varepsilon)$ and leaving A worse off in total.

Finally, consider the case in which the markets are played sequentially. If A precommits to a positive sales volume in the new market before playing in the shared market then he is in the situation of Figure 2 or 3. With increasing marginal costs and strategic substitutes, for example, his reaction curve II' is below XX' everywhere, and there is a strategic cost to him in the second (shared) market of producing in the first (new) market. He should take account of this cost by setting marginal revenue in the first market equal to his long-run marginal cost plus the strategic cost.

We now summarize our results so far.

Loose statement of Proposition 1 and Corollary

Suppose A and B are competing in a market with strategic substitutes. If A enters a new market (or is subsidized in another market in which he is already competing), and if A has increasing marginal costs, B will increase his activity in the shared market, hurting A in that market. If demand in the new market is sufficiently weak, A will make lower total profits than previously.

If _either_ the strategic variables are strategic complements _or_ A's marginal costs are decreasing, the effects are reversed. (If we have _both_ strategic complements _and_ decreasing marginal costs, the statement of the proposition is unchanged.)

(We give a more general version of this proposition, which allows for both A and B competing in both markets, in Section 5.)

Loose statement of Proposition 2

Suppose A and B are competing in a single market with strategic substitutes. Giving A a per unit subsidy in that market will help A beyond the direct effect.

If _either_ the strategic variables are strategic complements _or_ A is taxed (rather than subsidized), the effect is reversed.

Loose statement of Proposition 3

Suppose A and B are competing in a market in two different time periods with strategic substitututes. With increasing marginal costs, period 1 marginal revenue will exceed long-run marginal costs.

If _either_ the strategic variables are strategic complements _or_ the marginal costs are decreasing, the effect is reversed.

We can also use Figures 2 and 3 to analyze the effect of a firm's actions in other markets on potential competition in this market. Imagine that B is not yet a competitor in either market but that he can enter the main market for a fixed charge F. If A did not enter the new market, B would anticipate profits in the main (shared) market corresponding to the equilibrium N. If however A entered the new market, B would anticipate profits corresponding to

the equilibrium IS, IC, DS, or DC, depending on whether the new market increased or decreased A's marginal costs in the main market and on whether we have strategic substitutes or strategic complements.

As previously argued, a small change in q^B will have only a second-order effect on B's profits but a small change in the value of q^A has a first-order effect on B's profits. Thus since $IS_A < N_A$, $IC_A < N_A$, $DS_A > N_A$, $DC_A > N_A$, B will anticipate greater profits from entry if A enters another market which increases A's marginal costs, but will anticipate lower profits from entry if A enters another market that decreases A's marginal costs. (Note that this result does not depend on strategic substitutes or complements.) Whether B is made sufficiently much better off or worse off that he changes his decision whether to enter the market depends in part, of course, on the cost of overcoming the barriers to entry (i.e., the size of F). However we can say that if A is uncertain about B's costs of entry, then entering new markets which increase A's marginal costs increases the probability of B's entry.

Loose statement of Proposition 1'

If A is competing in a market, entering a new market which increases A's marginal costs will make the original market more attractive to both current and potential competitors.

If entering the new market decreases A's marginal costs, the effect is reversed.

Finally, we must note a caveat to all the results above. In all the figures we drew A's reaction curve so that it cut B's reaction curve from above. This is not essential for Nash equilibrium, but it is easy to see

that it is essential for a stable Nash equilibrium.[7] By considering how our figures would be changed if A's reaction curve cut from below, the reader can verify that:

Rider to Propositions 1, 2, 3, and 1'

Propositions 1, 2, 3, and 1' apply if the initial Nash equilibrium is stable. If the initial Nash equilibrium is unstable, all the results are reversed.

The next section provides a rigorous mathematical treatment of all these results in a more general framework, but is inessential to an understanding of the remaining sections.

5. A General Model of Price or Quantity Competition

We may, very easily, and quite generally, capture the idea that the only connection between markets $i = 1$ and $i = 2$ is the interrelated cost of production faced by each firm $F = A$ and $F = B$, by writing profits:[8]

$$\pi^F = R_1^F(q_1^A, q_1^B) + R_2^F(q_2^A, q_2^B) - c^F(q_1^A, q_2^A, q_1^B, q_2^B) \ .$$

R_i^F is the revenue of firm F in market i and c^F its cost of production. Since we shall have occasion later to consider per unit output subsidies s (or taxes $-s$) to firm A in market 1, we shall include another term in firm A's profits:

$$\pi^A = R_1^A(q_1^A, q_1^B) + R_2^A(q_2^A, q_2^B) - c^A(q_1^A, q_2^A, q_1^B, q_2^B) + sh(q_1^A, q_1^B) \ .$$

The functions R_i^F, c^F, and h, $F = A, B$, $i = 1, 2$, determine a game with strategic variables (q_1^A, q_2^A), (q_1^B, q_2^B) which can be solved for its Nash equilibria. We will always assume that the variables q_i^F must be chosen in a closed, bounded interval $0 \le q_i^F \le T$, $F = A, B$, $i = 1, 2$. In many of our applications we will assume that B's costs are separable, i.e., its production in market 1 does not affect its costs in market 2, so that

$$\pi^B = R_1^B(q_1^A, q_1^B) + R_2^B(q_2^A, q_2^B) - [c_1^B(q_1^A, q_1^B) + c_2^B(q_2^A, q_2^B)]$$

$$= [R_1^B(q_1^A, q_1^B) - c_1^B(q_1^A, q_1^B)] + [R_2^B(q_2^A, q_2^B) - c_2^B(q_2^A, q_2^B)] \ .$$

In this case B acts as if it were two separate firms B_1 and B_2 and we can say that A faces different consumers and different competitors in the two markets.

We will make the following assumptions:

Assumptions

0)   The functions R_i^F, c^F, and h are twice continuously differentiable and

a Nash equilibrium $\bar{q} = (\bar{q}_1^A, \bar{q}_2^A, \bar{q}_1^B, \bar{q}_2^B) \gg 0$ exists when s = 0. Assume

that evaluated at any q, and with s = 0, the second-order conditions for

optimality hold: $\dfrac{\partial^2 \pi^F}{\partial^2 q_i^F} < 0$, $\begin{vmatrix} \dfrac{\partial^2 \pi^F}{\partial^2 q_1^F} & \dfrac{\partial^2 \pi^F}{\partial^2 q_2^F} \end{vmatrix} > \begin{vmatrix} \dfrac{\partial^2 \pi^F}{\partial q_1^F \partial q_2^F} \end{vmatrix}^2$, F = A, B , i = 1, 2 .

Evaluated at Nash equilibrium:

1)   a)   <u>Substitute Goods</u>: $\dfrac{\partial \pi^F}{\partial q_i^G} < 0$, F $\neq$ G, i = 1, 2 .

b) $\dfrac{\partial}{\partial s} \dfrac{\partial \pi^A}{\partial q_1^A} > 0$.

2)   <u>Weak Stability</u>: Let M be the matrix $\dfrac{\partial^2 \pi^F}{\partial q_i^F \partial q_j^G}$, i = 1, 2 , F = A, B , j = 1, 2 , G = A, B .

Then the determinant $|M|$ is positive.

3)   <u>Increasing Costs</u>: $\dfrac{\partial^2 c^A}{\partial q_1^A \partial q_2^A} > 0$ and $\dfrac{\partial^2 c^B}{\partial q_1^B \partial q_2^B} \geqq 0$.

or

3')   <u>Decreasing Costs</u>: $\dfrac{\partial^2 c^A}{\partial q_1^A \partial q_2^B} < 0$ and $\dfrac{\partial^2 c^B}{\partial q_1^B \partial q_2^B} \leqq 0$.

4)   <u>Strategic Substitutability</u>: $\dfrac{\partial^2 \pi^B}{\partial q_2^A \partial q_2^B} < 0$ and $\dfrac{\partial^2 \pi^B}{\partial q_1^A \partial q_1^B} < 0$.

or

4')   <u>Strategic Complementarity</u>: $\dfrac{\partial^2 \pi^B}{\partial q_2^A \partial q_2^B} > 0$ and $\dfrac{\partial^2 \pi^B}{\partial q_1^A \partial q_1^B} > 0$.

5)   <u>Own Effect Dominance</u>: $\begin{vmatrix} \dfrac{\partial^2 \pi^A}{\partial q_1^A \partial q_2^A} \end{vmatrix} \begin{vmatrix} \dfrac{\partial^2 \pi^B}{\partial q_1^B \partial q_1^B} \end{vmatrix} > \begin{vmatrix} \dfrac{\partial^2 \pi^A}{\partial q_1^B \partial q_2^A} \end{vmatrix} \begin{vmatrix} \dfrac{\partial^2 \pi^B}{\partial q_1^A \partial q_1^B} \end{vmatrix}$.

Quantity Competition

In a Cournot game we can think of the q_i^F as quantities Q_i^F set by the

firms, where revenue is determined by the inverse demand functions f_1^A, f_2^A,

f_1^B, f_2^B: $R_i^F(q_i^A, q_i^B) = f_i^F(q_i^A, q_i^B)q_i^F$, $i = 1, 2$, $F = A, B$ and cost is techno-

logically determined by the functions $\bar{c}^F(Q_1^F, Q_2^F)$, $F = A, B$:

$c^F(q_1^A, q_2^A, q_1^B, q_2^B) = \bar{c}^F(q_1^F, q_2^F)$.[9] For linear demand for differentiated

products, for example, $f_i^F = a_i^F - b_i^F q_i^A - d_i^F q_i^B$, $i = 1, 2$, $F = A, B$, and with

linear marginal costs:

$$c^F(q_1^A, q_2^A, q_1^B, q_2^B) = \bar{c}(q_1^F, q_2^F)$$

$$= e_1^F q_1^F + e_2^F q_2^F \pm r_1^F(q_1^F)^2 \pm r_2^F(q_2^F)^2 \pm r_3^F q_1^F q_2^F$$

where all the constants a_i^F, b_i^F, d_i^F, e_i^F, and r_i^F are positive. It is easy to

verify that our assumptions hold for the linear model, as well as for many

other quantity competition games, once the coefficients are restricted in

accordance with the weak stability hypothesis (Assumption 2).

Assumption 1 says that an increase in B's activity (producing more output)

reduces A's profits and also that a subsidy to A in market 1 increases the

profitability of a marginal increase in A's activity there. For the quantity

game $h(q_1^A, q_1^B) = q_1^A$ and $\partial \pi^A / \partial s \partial q_1^A = 1$.

Assumption 2 is always true if the equilibrium is locally stable. We

will discuss this assumption in more detail after introducing price competi-

tion.

Assumption 3 says that an increase in output by firm A in market 1 either

increases the marginal cost of A's production in market 2 or decreases it.

One or the other must be true if there is to be market interdependence. When

we consider price competition we will always assume that firm B's costs are separable, so that in effect A faces different competitors in the two markets. With quantity competition the analysis is simpler (since $\partial c^F/\partial q_i^A \partial q_j^B = 0$) and so we allow B's costs to be interdependent as well, provided, as is empirically reasonable, that they are increasing or decreasing according to whether A's are increasing or decreasing.

Assumption 4 was discussed at length in Section 3. We call the reader's attention to the analogy between a substitute $\partial \pi^B/\partial q_2^A < 0$ (complement $\partial \pi^B/\partial q_2^A > 0$) and a strategic substitute $\partial^2 \pi^B/\partial q_2^A \partial q_2^B < 0$ (strategic complement $\partial^2 \pi^B/\partial q_2^A \partial q_2^B > 0$) and we also note that quantity competition gives rise to strategic substitutes in the linear model:

$$\frac{\partial^2 \pi^B}{\partial q_2^A \partial q_2^B} = \frac{\partial}{\partial q_2^A} \left[f_2^B + \frac{\partial f_2^B}{\partial q_2^B} q_2^B - \frac{\partial c}{\partial q_2^B} \right] = \frac{\partial f_2^B}{\partial q_2^A} < 0 .$$

Assumption 5 is trivially satisfied for the Cournot case since $\partial^2 \pi^A/\partial q_1^B \partial q_2^A = \partial^2 c^A/\partial q_1^B \partial q_2^A = 0$.

Price Competition

All of our five abstract assumptions apply also to price compeitition with the difference that the "normal" (i.e., linear) case involves strategic complementarity instead of strategic substitutability. As noted above we assume that B's costs are separable for price competition.

We begin by letting the strategic variables q_i^F be the prices P_i^F. (Later we will see that it is easier to think of the q_i as the inverses of prices.) Consider demand functions g_1^A, g_2^A, g_1^B, g_2^B depending on prices P_1^A, P_2^A, P_1^B, P_2^B, so that revenue $R_i^F = g_i^F(P_i^A, P_i^B)P_i^F$ and costs

$$c^F(P_1^A, P_2^A, P_1^B, P_2^B) = \bar{c}^F(g_1^F(P_1^A, P_1^B), g_2^F(P_2^A, P_2^B))$$

where $\bar{c}$ is a standard cost function of quantities (Q_1^F, Q_2^F). We assume that $\partial g_i^F/\partial P_i^F < 0$ and $\partial g_i^F/\partial P_i^G > 0$, $F \neq G$, $i = 1, 2$. For example, the linear case with differentiated products has $g_i^F = \alpha_i^F - \beta_i^F P_i^F + \delta_i^F P_i^G$, $F \neq G$, $i = 1, 2$, $F = A, B$, $G = A, B$.

Observe now that

$$\frac{\partial^2 c^F}{\partial P_1^F \partial P_2^F} = \frac{\partial}{\partial P_2^F}\left[\frac{\partial \bar{c}}{\partial Q_1^F} \frac{\partial g_1^F}{\partial P_1^F}\right] = \frac{\partial^2 \bar{c}}{\partial Q_2^F \partial Q_1^F} \frac{\partial g_2^F}{\partial P_2^F} \frac{\partial g_1^F}{\partial P_1^F},$$

hence $\partial^2 c^F/\partial P_2^F \partial P_1^F$ has the same sign as $\partial^2 \bar{c}^F/\partial Q_2^F \partial Q_1^F$. Thus letting the strategic variables q_i^F be prices P_i^F gives Assumption 3 the same meaning as before.

Note also that in general we could have strategic substitutability or strategic complementarity, i.e., either Assumption 4 or Assumption 4', but in the linear case we have strategic complementarity:

$$\frac{\partial^2 \pi_2^B}{\partial P_2^A \partial P_2^B} = \frac{\partial}{\partial P_2^A} \left[g_2^B + \frac{\partial g_2^B}{\partial P_2^B} P_2^B - \frac{\partial \bar{c}}{\partial Q_2^B} \frac{\partial g_2^B}{\partial P_2^B} \right]$$

$$= \frac{\partial g_2^B}{\partial P_2^A} - \frac{\partial^2 \bar{c}}{\partial^2 Q_2^B} \frac{\partial g_2^B}{\partial P_2^B} \frac{\partial g_2^B}{\partial P_2^A}$$

which is certainly positive if $\partial^2 \bar{c}/\partial^2 Q_2^B > 0$. Even if $\partial^2 \bar{c}/\partial^2 Q_2^B$ is negative, $\partial^2 \pi_2^B/\partial P_2^A \partial P_2^B > 0$ so long as the demand curve $(g_2^B)^{-1}$ has a steeper downward slope than the (declining) marginal cost curve $\partial \bar{c}/\partial Q_2^B$.

We now consider the role of Assumptions 2 and 5, in either the price or quantity competition model. We could have written down Assumptions

2') $|M| < 0$ and

5') $$\left| \frac{\partial^2 \pi^A}{\partial q_1^A \partial q_2^A} \right| \left| \frac{\partial^2 \pi^B}{\partial q_1^B \partial q_1^B} \right| < \left| \frac{\partial^2 \pi^A}{\partial q_2^B \partial q_2^A} \right| \left| \frac{\partial^2 \pi^B}{\partial q_1^A \partial q_1^B} \right|$$

as we did Assumptions 3' and 4'. It is easy to show that generically each assumption or its alternate must hold--that is, for example, only by accident will $|M| = 0$ in equilibrium. In the proof of Proposition 1 given below we show that the signs of these four expressions 2-5 alone are enough to determine the qualitative effect of a subsidy to A in market 1 on the profits of A in market 2. Here we try to justify our emphasis on Assumptions 3 and 4 (increasing vs. decreasing costs and strategic substitutability vs. complementarity) by arguing that 2 and 5 are more natural than their counterparts 2' and 5'.

Assumption 5 is a consequence of the natural hypothesis that own effects are larger than cross effects. Suppose in the price competition model that

$|\partial g_i^A/\partial P_i^A||\partial g_i^B/\partial P_i^B| > |\partial g_i^A/\partial P_i^B||\partial g_i^B/\partial P_i^A|$, $i = 1, 2$ and that

$|(\partial^2 \bar{c}^F/\partial^2 Q_1^F)(\partial^2 \bar{c}^F/\partial^2 Q_2^F)| \geq |\partial^2 \bar{c}^F/\partial Q_1^F \partial Q_2^F|^2$, $F = A, B$.[10] Then Assumption 5

is equivalent for __linear__ demand to

$$\left|\frac{\partial^2 \bar{c}^A}{\partial Q_1^A \partial Q_2^A}\frac{\partial g_1^A}{\partial P_1^A}\frac{\partial g_2^A}{\partial P_2^A}\right|\left|\left(2 - \frac{\partial^2 \bar{c}^B}{\partial^2 Q_1^B}\frac{\partial g_1^B}{\partial P_1^B}\right)\frac{\partial g_1^B}{\partial P_1^B}\right| > \left|\frac{\partial^2 \bar{c}^A}{\partial Q_1^A \partial Q_2^A}\frac{\partial g_1^A}{\partial P_1^B}\frac{\partial g_2^A}{\partial P_2^A}\right|\left|\left(1 - \frac{\partial^2 \bar{c}^B}{\partial^2 Q_1^B}\frac{\partial g_1^B}{\partial P_1^B}\right)\frac{\partial g_1^B}{\partial P_1^A}\right|$$

or

$$\left|\frac{\partial g_1^A}{\partial P_1^A}\frac{\partial g_1^B}{\partial P_1^B}\right|\left|2 - \frac{\partial^2 \bar{c}^B}{\partial^2 Q_1^B}\frac{\partial g_1^B}{\partial P_1^B}\right| > \left|\frac{\partial g_1^A}{\partial P_1^B}\frac{\partial g_1^B}{\partial P_1^A}\right|\left|1 - \frac{\partial^2 \bar{c}^B}{\partial^2 Q_1^B}\frac{\partial g_1^B}{\partial P_1^B}\right|$$

which is always true (given the own effect hypothesis) if the marginal cost

curve $\partial \bar{c}^B/\partial Q_1^B$ is increasing (in Q_1^B) or, if it is decreasing, so long as it is

less steep than the demand curve $(g_1^B)^{-1}$. We have previously mentioned that

with quantity competition, (5) always holds.

Suppose also that the equilibrium $\bar{q} = (\bar{q}_1^A, \bar{q}_2^A, \bar{q}_1^B, \bar{q}_2^B)$ is locally stable,

i.e., if we set $\dot{q}_i^F = +(\partial R/\partial q_i^F - \partial c/\partial q_i^F)$ if the q_i are quantities, or $\dot{q}_i^F =$

$-(\partial R/\partial q_i^F - \partial c/\partial q_i^F)$ if the q_i are prices, then no matter where we start, if it

is sufficiently close to $\bar{q}$, $\frac{d}{dt}\|q(t)\| < 0$. In other words, suppose that each

firm F adjusts when out of equilibrium by moving q_1^F and q_2^F in the directions

that will bring marginal revenue toward marginal cost and that this process

tends towards the equilibrium $\bar{q}$. Then it can easily be shown that Assumption

2 holds, i.e., $|M| > 0$.[11]

Recall the well-known property (called diagonal dominance) that a matrix

M whose diagonal elements are so big that $|M_{ii}| > \sum_{j \neq i} |M_{ij}|$, for all i has

sign $|M| = \text{sign}\prod_i M_{ii}$ (in our case $|M| > 0$, since M is 4×4 and $M_{ii} =$

$\partial^2 \pi^F/\partial^2 q_i^F < 0$ for all i). We leave to the reader the elementary calculation

that with quantity competition, increasing marginal cost curves, and linear demand the diagonal dominance property holds, given the own effect hypothesis: $|(\partial f_i^A/\partial q_i^A)(\partial f_i^B/\partial q_i^B)| > |(\partial f_i^A/\partial q_i^B)(\partial f_i^B/\partial q_i^A)|$, $i = 1$, 2. Even if marginal cost is decreasing, $\partial^2 c^F/\partial^2 q_i^F < 0$, diagonal dominance still holds if the demand curve is at least twice as steep as the marginal cost curve, $|\partial f_i^F/\partial q_i^F| > 2|\partial^2 c^F/\partial^2 q_i^F|$, $F = A$, B, $i = 1$, 2, at $\bar{q}$. Similarly, the reader can verify that diagonal dominance holds for price competition as well, no matter whether marginal costs are increasing or decreasing, provided that the demand curve is at least 4 times as steep as the cost curve, $|(\partial g_i^F/\partial P_i^F)(\partial^2 c^{-F}/\partial^2 Q_i^F)| < 1/4$, and the above own effect hypothesis holds.

Finally, we must check Assumption 1 for the price competition model. Note that for $F \neq G$,

$$\frac{\partial \pi^F}{\partial P_i^G} = P_i^F \frac{\partial g_i^F}{\partial P_i^G} - \frac{\partial c^{-F}}{\partial Q_i^F} \frac{\partial g_i^F}{\partial P_i^G} = \frac{\partial g_i^F}{\partial P_i^G} \left[P_i^F - \frac{\partial c^{-F}}{\partial Q_i^F} \right] > 0$$

in equilibrium. (In equilibrium we deduce $P_i^F > \partial c^{-F}/\partial Q_i^F$ from

$$0 = \frac{\partial \pi^F}{\partial P_i^F} = \frac{\partial g_i^F}{\partial P_i^F} \left[P_i^F - \frac{\partial c^{-F}}{\partial Q_i^F} \right] + g_i^F \text{).}$$

For price competition we let $h(P_1^A, P_1^B) = g_1^A(P_1^A, P_1^B)$ and thus $\partial^2 \pi^A/\partial s \partial P_1^A = \partial g_1^A/\partial P_1^A < 0$. Both of the conditions of Assumption 1 are reversed. Observe, however, that if we make a change of variable transformation $q_i^F = 1/P_i^F$ then none of the Assumptions 2-5 are affected, since they are independent of the units in which goods are measured and they depend on second derivatives. On the other hand, the transformation reverses the signs of the expressions in 1

(since they depend on differentiating with respect to a single strategic variable) thus bringing the price competition model into agreement with the Cournot competition model.

Now that we have established that the same analytic framework can serve for both price and quantity competition, and with q serving henceforth _either_ for quantities _or_ for the inverses of prices, as appropriate, we can state the following general propositions (recalling that with price competition we will take B's costs to be separable):

Proposition 1. Let a multimarket oligopoly satisfy Assumptions 0, 1, 2, 5. Suppose that the commodities are strategic substitutes (4) and that costs are increasing (3). Then a subsidy to firm A in market 1 will increase B's activity in market 2, hurting A there. If either the goods are strategic complements (4') or costs are decreasing (3'), the effect of the subsidy is reversed.

Proof. Note that the proposition applies equally to price or quantity competition. For small changes of s and q around $\bar{s} = 0$ and $\bar{q}$, the second-order conditions will still hold. Hence it suffices to show that we can find $q(s)$, $q(0) = \bar{q}$, such that all the first-order conditions hold. Let us consider the matrix $M = \partial^2 \pi^F / \partial q_j^G \partial q_i^F$:

$$
\begin{array}{cccccc}
 & q_1^A & q_2^A & q_1^B & q_2^B & s_1^A \\
\partial\pi^A/\partial q_1^A & \begin{bmatrix} M_{11} \end{bmatrix} & \overset{\text{cost}}{M_{12}} & \overset{\text{strat}}{M_{13}} & M_{14} & 1 \\
\partial\pi^A/\partial q_2^A & \overset{\text{cost}}{M_{21}} & M_{22} & M_{23} & \overset{\text{strat}}{M_{24}} & 0 \\
\partial\pi^B/\partial q_1^B & \overset{\text{strat}}{M_{31}} & 0 & M_{33} & \overset{\text{cost}}{M_{34}} & 0 \\
\partial\pi^B/\partial q_2^B & 0 & \overset{\text{strat}}{M_{42}} & \overset{\text{cost}}{M_{43}} & M_{44} & 0
\end{array}
$$

In the Cournot case, $M_{23} = M_{14} = 0$. Assumption 5 implies that $|M_{21}M_{33}| > |M_{23}M_{31}|$. When B's costs are separable, $M_{34} = M_{43} = 0 = M_{41} = M_{32}$. Note that M_{21}, M_{12}, M_{34}, M_{43} are negative (positive) if costs are increasing (decreasing) and M_{31} and M_{42} are negative (positive) for strategic substitutes (complements).

It follows at once from the implicit function theorem and $|M| > 0$ that the equilibrium (q_1^A, q_2^A, q_1^B, q_2^B)(s) moves differentiably away from ($\bar{q}_1^A$, $\bar{q}_2^A$, $\bar{q}_1^B$, $\bar{q}_2^B$) for small changes in s around $s = \bar{s}$. By Cramer's rule (and the implicit function theorem) we can calculate any of the four derivatives, such as dq_2^B/ds, by replacing the appropriate column of M, in this case the fourth, with the vector $(-1,0,0,0)'$, taking the determinant of the resulting matrix, and dividing it by $|M|$.

The determinant in question has value

$$
\begin{vmatrix} M_{21} & M_{22} & M_{23} \\ M_{31} & 0 & M_{33} \\ 0 & M_{42} & M_{43} \end{vmatrix} = \left[M_{21}(-M_{42}M_{33}) - M_{31}[(M_{22}M_{43}) - (M_{23}M_{42})] \right]
$$

$$
= \underset{\text{(strat)(cost)}}{(-M_{22})(M_{31})(M_{43})} + \underset{\text{(cost)(strat)}}{(-M_{33})(M_{21})(M_{42})} + M_{31}M_{23}M_{42} \; .
$$

Recall that in Nash equilibrium M_{22} and M_{33} are necessarily negative. Each of the first two expressions has a sign given by the product of a strategic effect term (M_{31} or M_{42}) and a cost term (M_{43} or M_{21}). Assumption 5 guarantees that the third expression is dominated by the second, hence with increasing cost firms and strategic substitutes   a subsidy to firm A in market 1 increases the activity of firm B in market 2, thus hurting firm A there. If the firms were decreasing cost firms, then the subsidy in market 1 would help firm A in market 2 as well.

Evidently if the two pairs of commodities were each strategic complements, then a subsidy to firm A in market 1 would also help its market 2 profits, if costs were increasing. $\square$

Proposition 1'. Let a multimarket oligopoly satisfy Assumptions Q, 1, 2, 5. Suppose that costs are increasing. Then no matter whether commodities are <u>all</u> strategic substitutes (Assumption 4 from A's and B's point of view), or <u>all</u> strategic complements (Assumption 4' from A's and B's point of view), a subsidy to firm A in market 1 will reduce A's own activity in market 2, helping B there. If costs are decreasing, this effect of the subsidy is reversed.

Proof. We can proceed exactly as in the proof of Proposition 1, obtaining

$$
\frac{dq_2^A}{ds} = 1 \begin{vmatrix} M_{21} & M_{23} & M_{24} \\ M_{31} & M_{33} & M_{34} \\ 0 & M_{43} & M_{44} \end{vmatrix}
$$

$$
= M_{21}(M_{33}M_{44} - M_{43}M_{34}) - M_{31}(M_{23}M_{44} - M_{43}M_{24}) \ .
$$

The first big term is negative if costs are increasing, since $M_{21} < 0$ and

from Assumption 0, $M_{33}M_{44} > M_{43}M_{34} \geq 0$. For quantity competition $M_{23} = 0$

$\underset{\text{(strat)}}{\text{and }(\ M_{31}}\ )\underset{\text{(strat)}}{(\ M_{24}}\ )\underset{\text{(cost)}}{(\ M_{43}}\ ) \leq 0$ since the strategic terms have the same sign

and the cost term is nonpositive.

With price competition (and therefore by assumption separable produc-

tion costs for B) $M_{34} = M_{43} = 0$, so $dq_2^A/ds = (M_{21}M_{33})M_{44} - (M_{31}M_{23})M_{44} < 0$

by Assumption 5, if A's costs are increasing, so that $M_{21} < 0$. $\square$

Remark 1. If B's costs are separable, then this proposition depends only on

A's costs, and not at all on the form of competition, unlike Proposition 1.

Remark 2. If either Assumption 2 is replaced by its opposite 2', $|M| < 0$,

or Assumption 5 is replaced by its opposite 5' (which could happen only with

price competition, for which we have also assumed B's costs separable so that

$M_{43} = M_{24} = 0$) then the effects of the subsidy are all reversed.

Remark 3. Propositions 1 and 1' also hold if firm A is the lone producer in

market 1 and a duopolist in market 2, provided we replace Assumption 2 with

the corresponding assumption that the 3×3 matrix M satisfy $\text{sign}|M| = (-1)^3 < 0$.

Corollary. Let firm A, a duopolist in market 2, where goods are strategic

substitutes, have increasing costs. If firm A is given monopoly selling

rights to a new market, 1, with sufficiently small demand, then in the new

equilibrium firm A will make smaller profits than it did in the equilibrium

in which it had no access to the new market. If we have either strategic

complements or decreasing costs the effect is reversed. (If we have both

strategic complements and decreasing costs, A again makes smaller profits

than before.)

Proof. We can apply the method of proof of Proposition 1. Assume that demand in market 1 is just sufficiently high that firm A's marginal cost of producing one more unit, beyond what it is producing in the original one-market equilibrium, is equal to the maximum price it could get by selling that unit in market 1. Consider now the effect of a subsidy to firm A in market 1 (or equivalently the effect of a small increase in demand in market 1) on A's total profits:

$$\frac{d\pi^A}{ds}\bigg|_{s=0} = \frac{\partial\pi_1^A}{\partial q_1^A}\frac{dq_1^A}{ds} + \frac{\partial\pi_2^A}{\partial q_2^B}\frac{dq_2^B}{ds} + \frac{\partial\pi_2^A}{\partial q_2^A}\frac{dq_2^A}{ds} + \frac{\partial\pi_1^A}{\partial s}$$

$$= \frac{\partial\pi_2^A}{\partial q_2^B}\frac{dq_1^B}{ds} < 0 .$$

The above argument follows from the original equilibrium condition that $\partial\pi_1^A/\partial q_1^A = 0 = \partial\pi_2^A/\partial q_2^A$ and since $\bar{q}_1^A = 0$, $\partial\pi_1^A/\partial s = 0$. Finally, $\partial\pi_2^A/\partial q_2^B < 0$ and from Proposition 1, $dq_2^B/ds > 0$. □

We see from the above that the effect on A's profits of a small subsidy may be expressed as the sum of two terms: a direct effect, $\partial\pi_1^A/\partial s = \bar{Q}_1^A$, which would accrue to a perfectly competitive firm or to a monopolist (facing a differentiable demand), and a "strategic" effect, $(\partial\pi_1^A/\partial q_1^B)(d\bar{q}_1^B/ds) + (\partial\pi_2^A/\partial q_2^B)(dq_2^B/ds)$ resulting from the reaction in both markets 1 and 2 of the other firm to A's changed opportunities in market 1. Quite similarly, if the markets were sequential then a marginal increase in q_1^A would have two effects: a direct effect $\partial\pi_1^A/\partial q_1^A = \partial R_1^A/\partial q_1^A - \partial c^A/\partial q_1^A$ and a strategic effect $(\partial\pi_2^A/\partial q_2^B)dq_2^B/dq_1^A$. We now show that the sign of the strategic effect $(\partial\pi_2^A/\partial q_2^B)(dq_2^B/dq_1^A)$ in the sequential game is determined by exactly the same factors--increasing vs.

decreasing costs and strategic substitutability vs. complementarity--that determine the sign of the strategic effect in the simultaneous market game.

Let us assume temporarily that the firms A and B are competing only in market 2, and:

$\hat{0}$) There is a Nash equilibrium $(\bar{q}_2^A, \bar{q}_2^B) \gg 0$ and for $s = 0$ and for all q,

$\partial^2 \pi_2^F / \partial^2 q_2^F < 0$, $F = A, B$.

$\hat{1}$) $\partial \pi_2^F / \partial q_2^G < 0$, $F \neq G$.

$\hat{2}$) $|(\partial^2 \pi_2^A / \partial^2 q_2^A)(\partial^2 \pi_2^B / \partial^2 q_2^B)| > |(\partial^2 \pi_2^A / \partial q_2^B \partial q_2^A)(\partial^2 \pi_2^B / \partial q_2^A \partial q_2^B)|$ holds at all $q_2^A, q_2^B \geq 0$, $s = 0$.

Once again we have a proposition which is true for either price or quantity competition.

Proposition 2. Let firms A and B compete in a single market (call it market 2) under conditions satisfying $\hat{0}$, $\hat{1}$, $\hat{2}$. Then for any sufficiently small subsidy s to the output of firm A there is a unique Nash equilibrium $(\tilde{q}_2^A(s), \tilde{q}_2^B(s)) \gg 0$, and the functions $\tilde{q}_2^A, \tilde{q}_2^B$ of s are differentiable around $s = 0$. Furthermore, if at $(\bar{q}_2^A, \bar{q}_2^B)$ q_2^A is a strategic substitute for q_2^B, then the effect of a small subsidy to A is a decrease in q_2^B and therefore a strategic benefit to A beyond the direct subsidy effect. If q_2^A is a strategic complement for q_2^B, then the strategic effects are reversed. No matter what the strategic relation between the commodities, a subsidy to A will increase q_2^A, thus hurting B.

Proof. We apply precisely the method of the proof in Proposition 1. By hypothesis the second-order conditions are satisfied for all q, if s is small enough, hence the first-order conditions are sufficient to determine equilibrium. Let

$$
\hat{M} = \begin{bmatrix} \dfrac{\partial^2 \pi^A_2}{\partial^2 q^A_2} & \dfrac{\partial^2 \pi^A_2}{\partial q^B_2 \partial q^A_2} \\[20pt] \dfrac{\partial^2 \pi^B_2}{\partial q^A_2 \partial q^B_2} & \dfrac{\partial^2 \pi^B_2}{\partial^2 q^B_2} \end{bmatrix} \, , \quad \text{so} \quad |\hat{M}| > 0 \, .
$$

Once again we have from the implicit function theorem that there is a locally unique solution $(q^A_2(s), q^B_2(s))$ around $s = \bar{s} = 0$ and that

$$
\frac{dq^B_2}{ds} = \begin{vmatrix} \dfrac{\partial^2 \pi^A_2}{\partial^2 q^A_2} & -1 \\[20pt] \dfrac{\partial^2 \pi^B_2}{\partial q^A_2 \partial q^B_2} & 0 \end{vmatrix} \Bigg/ |\hat{M}| = \dfrac{\partial^2 \pi^B_2}{\partial q^A_2 \partial q^B_2} \Bigg/ |\hat{M}| \begin{cases} < 0 \text{ for strategic} \\ \quad \text{substitutes} \\[8pt] > 0 \text{ for strategic} \\ \quad \text{complements} \end{cases}
$$

Note that

$$
\frac{dq^A_2}{ds} = \begin{vmatrix} -1 & \dfrac{\partial^2 \pi^A_2}{\partial q^A_2 \partial q^B_2} \\[20pt] 0 & \dfrac{\partial^2 \pi^B_2}{\partial^2 q^B_2} \end{vmatrix} \Bigg/ |\hat{M}| > 0 \text{ always.}
$$

Global uniqueness follows immediately from the well-known global uni-valence properties of mappings whose 2×2 Jacobians $\hat{M}$ satisfy $\hat{M}_{ii} < 0$ and $|\hat{M}| > 0$. $\square$

We are now in a position to extend our results to sequential markets, 1 and 2. Revenue $R_i^F(q_i^A, q_i^B)$ and costs $c^F(q_1^A, q_1^B, q_2^A, q_2^B)$ are exactly as before. To capture the idea of a sequential game a perfect Nash equilibrium is defined as a 4-vector $((\bar{q}_1^A, \tilde{q}_2^A), (\bar{q}_1^B, \tilde{q}_2^B))$, where $\tilde{q}_2^F$ is a function from $R_+^2 \to R_+$ specifying for every first-period "move" (q_1^A, q_1^B) what firm F will do at time 2, that satisfies two properties:

α) $(q_2^A = \tilde{q}_2^A(q_1^A, q_1^B), q_2^B = \tilde{q}_2^B(q_1^A, q_1^B))$ is a Nash equilibrium for the game restricted only to market 2 in the obvious way, given the choices (q_1^A, q_2^A) from period 1.

β)   Given the functions $\tilde{q}_2^A$ and $\tilde{q}_2^B$, $\bar{q}_1^A$ and $\bar{q}_1^B$ are the optimal choices in period 1.

We make the following three assumptions:

$\hat{0}$)   A perfect Nash equilibrium exists with strictly positive outcomes, $(\bar{q}_1^A, \bar{q}_2^A, \bar{q}_1^B, \bar{q}_2^B) \gg 0$ where $\bar{q}_2^F = \tilde{q}_2^F(\bar{q}_1^A, \bar{q}_2^A)$, F = A, B.

$\hat{1}$)   The same as $\hat{1}$ above.

$\hat{2}$)   The same as $\hat{2}$ above.

Let us now introduce one more term:

<u>Definition</u>. Let $\partial c(\bar{q}_1^A, \bar{q}_2^A, \bar{q}_1^B, \bar{q}_2^B)/\partial q_1^F$ be called the economic or social or correctly calculated economic cost to firm F of increasing q_1^F by one unit in period 1, in equilibrium.

Notice that in period 1 there has not yet been any production in period 2, hence we can distinguish also the "apparent" cost to firm F of increasing q_1^F by one unit in period 1, $\partial c^F(q_1^A, 0, q_1^B, 0)/\partial q_1^F$. In the quantity game where $q_i^F = Q_i^F$ the difference is between $\partial \bar{c}^F(\bar{q}_1^F, \bar{q}_2^F)/\partial Q_1^F$ and $\partial \bar{c}^F(\bar{q}_1^F, 0)/\partial Q_1^F$ and in the price game, assuming that demand is 0 when $q_i^F = 1/P_i^F = 0$, the difference is between $[\partial \bar{c}^F(g_1^F(\bar{q}_1^A, \bar{q}_1^B), g_2^F(\bar{q}_2^A, \bar{q}_2^B))/\partial Q_1^F][\partial g_1^F(\bar{q}_1^A, \bar{q}_1^B)/\partial q_1^F]$ and the same expression with $\bar{q}_2^G$ replaced by 0 for G = A, B. In our next proposition we show that the rational oligopolistic firm does not use either the social or the apparent cost in making its calculations.

<u>Proposition 3</u>. Let $(\bar{q}_1^A, \bar{q}_2^A, \bar{q}_1^B, \bar{q}_2^B)$ be a perfect Nash equilibrium outcome of a two-period game satisfying Assumptions $\hat{0}$-$\hat{2}$. If firm A's costs are increasing and q_2^A is a strategic substitute for q_2^B, then period 1 marginal revenue exceeds correctly calculated economic costs, $\partial R_1^A(q_1^A, q_1^B)/\partial q_1^A >$ $\partial c^A(q_1^A, q_1^B, q_2^A, q_2^B)/\partial q_1^A$. If A's costs are decreasing or if q_2^A is a strategic complement for q_2^B, then the inequality is reversed: $\partial R_1^A/\partial q_1^A < \partial c^A/\partial q_1^A$.

<u>Proof</u>. From Proposition 2 we know that $\bar{q}_2^F(q_1^A, q_1^B)$ is differentiable, since a change in either q_1^A or q_1^B simply acts as a subsidy (or tax) on one or both of the firms in period 2. In equilibrium firm A chooses $q_1^A = \bar{q}_1^A$ by setting

$$\frac{\partial R_1^A(\bar{q}_1^A, \bar{q}_1^B)}{\partial q_1^A} - \frac{\partial c^A(q_1^A, q_2^A, q_1^B, q_2^B)}{\partial q_1^A} + \frac{\partial \pi^A}{\partial q_2^B}\frac{d\bar{q}_2^B}{dq_1^A} = 0 .$$

Proposition 3 now follows from the sign of $d\tilde{q}_2^B/dq_1^A$, which was calculated in Proposition 2. Note that from increasing costs, an increase in q_1^A acts exactly as a negative subsidy to firm A in period 2.

Remark 4. Proposition 3 is almost an exact analogue of Proposition 1.

If costs $\bar{c}^F(Q_1^F, Q_2^F)$ are given by $\bar{\bar{c}}^F(Q_1^F + Q_2^F)$ where $\bar{\bar{c}}^{F'} > 0$ and $\bar{\bar{c}}^{F''} < 0$, $F = A, B$, we call the cost structure a learning curve, after Spence (1981): the more that is produced, the less the marginal cost. Proposition 3 implies that for quantity competition with strategic substitutes, first-period marginal revenue will be less than correctly calculated first-period marginal cost in learning curve situations, whereas with strategic complements the reverse will be true. If costs are given by the function $\bar{\bar{c}}^F(Q_1^F + Q_2^F)$ with $\bar{\bar{c}}^{F'} > 0$ and $\bar{\bar{c}}^{F''} > 0$ then we say that we have a natural resource cost structure: the more that is produced, the harder it becomes to produce more, no matter when the production occurs. The relationship of equilibrium marginal revenue and correctly calculated marginal cost is exactly the reverse of the learning curve situation.

As usual, with price competition our results depend on the separability of B's costs. Relaxing this assumption, the strategic effect of A's first period actions dp_2^B/dp_1^A will have two terms: the familiar term caused by A's changed costs in the second period, and a new term showing how A's changed period 1 price affected B's quantity, and thus B's costs and price in the second period. From proposition 2 we can calculate

$$\frac{dp_2^B}{dp_1^A} = \underbrace{\left(\frac{-\dfrac{\partial^2 \pi_2^B}{\partial p_2^A \partial p_2^B} \cdot \dfrac{\partial \bar{c}^A}{\partial q_1^A} \cdot \dfrac{\partial g_1^A}{\partial p_1^A}}{|\hat{M}|}\right)}_{\text{(old term)}} + \underbrace{\left(\frac{\dfrac{\partial^2 \pi_2^A}{\partial^2 p_2^A} \cdot \dfrac{\partial \bar{c}^B}{\partial q_1^B} \dfrac{\partial g_1^B}{\partial p_1^A}}{|\hat{M}|}\right)}_{\text{(new term)}}$$

In the case of strategic complements for price competition these two
terms have opposite signs (with strategic substitutes the signs are the same).
In the "learning curve" context above, a decrease in A's period 1 price has
the strategic cost of lowering A's second period costs and price, and thus B's
second period price, but also the strategic benefit of lowering B's first period
output, thereby raising his second period costs and price. In general it is
impossible to say which term is larger, but with linear demand and symmetrical
firms the second term dominates. (This result has also been noted by Fudenberg
and Tirole (1983).)

A special case of propositon 3 of particular interest is where the output
of firm F is produced with a constant-returns-to-scale, neoclassical production
technology $Q^F = H^F(K, L)$, $\partial H^F/\partial K > 0$, $\partial H^F/\partial L > 0$, $\partial^2 H^F/\partial K \partial L > 0$, $\partial^2 H^F/\partial^2 K < 0$,
$\partial^2 H^F/\partial^2 L < 0$, $F = A, B$. Suppose that in period 1 each firm can buy capital at
a known cost r. Production and sales take place instantaneously in period 2,
at which time labor is available at a known cost w. Since there are constant
returns to scale there is an efficient $(K/L)^F_{eff}$ ratio determined by r and w,
independent of output, which would be chosen by both a perfectly competitive
firm and a monopolist. Here the situation is different: investing in capital
in period 1 has both a direct effect (the saving in costs) and a strategic effect
(the effect on competitors' output). Since capital lowers marginal costs, the
strategic effect is an additional benefit with strategic substitutes, but a
cost with strategic complements. Thus we have the following corollary to
Proposition 3.

Corollary. Let firms $F = A, B$ produce output from neoclassical production functions $H^F(K^F, L^F)$. Suppose that they install capital in period 1 at price r, and suppose that in period 2 they hire labor at price w and produce output, announcing prices or quantities. If q_2^A and q_2^B are strategic substitutes, then the equilibrium $(\bar{K}/\bar{L})^F > (K/L)_{eff}^F$, that is, there is excess capacity. If q_2^A and q_2^B are strategic complements, then the equilibrium $(\bar{K}/\bar{L})^F < (K/L)_{eff}^F$, that is, there is insufficient capacity.

In particular, the corollary shows that in the linear case capital installation followed by price competition yields an underinvestment equilibrium! In Section 6 we contrast the usual entry deterrence-excess capacity story with our phenomenon of strategic investment against an established competitor. Notice that our result here depends on having a differentiable production function. With a fixed coefficients production function, production could be efficient in equilibrium.

6.   Applications

A.   Product Portfolio Selection

The most obvious application of our results is to the theory of how a firm should select a "portfolio" of businesses in which to compete, in particular a firm producing joint products with strongly interrelated costs.[12]

Proposition 1 tells us in which cases entering a new market helps a firm against its current competitors in other markets, and in which cases the firm is hurt.

Proposition 1' tells us that a firm may enter businesses which are on the face of them unprofitable, in order to deter potential entrants into related businesses in which the firm's costs are now decreased. A firm doing this is engaging in a kind of limit pricing--it is selling goods at what seems to be less than the maximal profit or even at a loss (if the markets are sequential)--but the entry that is being deterred is not entry into the market with the "limit pricing," which might itself be quite secure, but rather entry into other markets in which the firm competes.

Conversely, entering new businesses which increase a company's marginal costs of operating elsewhere (e.g., by increasing the marginal opportunity cost of constrained capital or management skills) makes a company's other markets more attractive to potential entrants. A new competitor may be less likely to enter a company's only market where it can expect an aggressive response, than to enter one of the markets of a diversified conglomerate which may find it cheaper to "move over" for the new entrant by diverting more of its resources elsewhere.

As a possible example of a firm that ignored this cost of diversification, consider the case of Frontier Airlines. In the early 1980s the firm expanded beyond its original Denver hub to capitalize on some apparently profitable opportunities. Many feel that the airline made a tactical error. After Frontier spread itself over several new markets, other airlines began to compete much more aggressively for shares of the Denver market. Some of this new competition may have been inevitable in a changing, deregulated environment, but some of it was probably due to a perceived weakness on the part of Frontier.

B.   Rational Retaliation as a Barrier to Entry

A special case of entry deterrence of particular interest is that in which one firm, firm A, is considering entering the market of another firm, firm B, which is itself a potential entrant into firm A's markets. If at least one of the firms has increasing marginal costs, it may be rational for firm B to retaliate by entering A's markets if A enters his market, but not to enter A's markets otherwise. There are two reasons. First, by Proposition 1', A entering B's market makes A's home markets more attractive candidates for entry, as in our Frontier Airlines example (if A has increasing marginal costs). Second, by Proposition 1, A entering B's market makes B's home market less attractive thus reducing B's opportunity cost of using his resources elsewhere (if B has increasing marginal costs). The story that one firm might do best to avoid another's "territory" for fear of retaliation is not unfamiliar. The point we are making here is that neither does the equilibrium in which each company avoids the other's territory depend on tacit collusion nor is the threat of retaliation one that would be costly for either firm to

carry out. Initially, it would be costly for B to enter A's markets even if

A did not retaliate (so no tacit collusion is necessary to restrain B from

entering A's markets). However, after A has entered B's market it may be

profitable for B to retaliate (so the threat to do so is completely credible

without recourse to additional stories about reputations).[13]

For example, assume that Polaroid had a monopoly in the instant photog-

raphy market and that Kodak had a dominant position in other market segments.

It could be that if Polaroid moved into Kodak's markets and could count on no

changes in competition in the instant market, Polaroid would make more money.

But Polaroid's entering a new market would divert resources from the instant

market, weakening Polaroid there and making entry in the instant market prof-

itable for firms that previously thought they would lose money in instant

photography. Also Polaroid's entry might reduce the attractiveness of some

of Kodak's market segments, encouraging Kodak to look for other opportunities.

Thus a particularly likely firm to suddenly find the instant market profitable

would be Kodak. Polaroid might know that if it entered Kodak's market, Kodak

would respond by entering Polaroid's market, not as an imperfect punishment

for Polaroid but simply because the instant market would be more profitable

to Kodak now. We give a numerical example as a footnote.[14]

C. Strategic Underinvestment in Fixed Costs

Selling units in one market in order to reduce marginal costs in a

second market is formally equivalent to investing in capital that will

directly lower the marginal production costs in the second market. Proposi-

tion 3 tells us that with decreasing marginal costs a firm may sell units at

a loss in the first market in order to prevent entry in the second, if the

markets are sequential. This is equivalent to the familiar result that a firm may overinvest in capital in the first period of a two-period game--i.e., invest beyond the point where an extra dollar's investment in period 1 saves a dollar's expenses in period 2--in order to reduce its marginal cost and hence deter entry in the second period.[15]

If however the firm cannot prevent entry in the second period, then we know that if we have decreasing costs and strategic complements the firm will avoid apparently profitable opportunities in the first period (sell at marginal revenue > marginal cost) in order to make its competitors play less aggressively in the second period; i.e., a firm may underinvest in period 1--stop investing at a point where an extra dollar's investment would save more than a dollar's expenses in period 2--because investing more would make his opponent compete more aggressively in period 2.[16]

Thus ordinarily with price competition and linear demand, the more a firm invests, the lower an entrant's price will be, because greater investment lowers the incumbent's expected price. (If the entrant's marginal cost curve is steeper than his demand, linear demand price competition yields strategic complements and these results are reversed.) A firm may have an "entry deterrence" incentive to overinvest, but if he cannot deter entry then he has a "price war avoidance" incentive to hold back and underinvest. This contrasts with the work of Spence (1977), Dixit (1979, 1980), and others, who focus exclusively on the use of excess capacity to deter entry.[17,18]

As an example of a firm underinvesting, consider the following differentiated products price competition game:

$$q_a = g - ep_a + fp_b , \quad q_b = g - ep_b + fp_a ,$$
$$tc_a = I_a + q_a c_a(I_a) , \quad tc_b = q_b c_b$$

where q is quantity, p is price, g, e, and f are positive parameters with e greater than f, I_a is the investment level chosen by firm A in the first period, c_a is the convex function giving the constant marginal cost yielded by this investment, c_b is B's marginal cost, and tc_a and tc_b are total costs. First A chooses I_a and then the firms compete on price. It is straightforward to show that given c_a, the second-period equilibrium is at

$$P_a = \frac{(2e+f)g + 2e^2 c_a + efc_b}{4e^2 - f^2}$$

and

$$P_b = \frac{(2e+f)g + 2e^2 c_b + efc_a}{4e^2 - f^2}$$

which yield equilibrium profits $\bar{\pi}_i(c_a, c_b)$ of

$$\bar{\pi}_a = \frac{e}{(4e^2 - f^2)^2} \left[(2e+f)g - (2e^2 - f^2)c_a + c_b ef \right]^2 - I_a \qquad (1)$$

$$\bar{\pi}_b = \frac{e}{(4e^2 - f^2)^2} \left[(2e+f)g - (2e^2 - f^2)c_b + c_a ef \right]^2 . \qquad (2)$$

A will choose I_a so as to maximize $\bar{\pi}_a(c_a(I_a), c_b) - I_a$ by setting $(\partial\bar{\pi}_a/\partial c_a)(dc_a/dI_a) = 1$. For concreteness, let us take $q_a = 500 - 5p_a + 4p_b$, $q_b = 500 - 5p_b + 4p_a$, $c_a(I_a) = \max(100, 335 - I_a^{\frac{1}{2}})$, and $c_b = 242$. Regardless of how much A invests, he cannot reduce c_a below 100. Therefore $\bar{\pi}_b$ will always be positive so B's entry cannot be deterred. The reader can convince himself that A will choose $I_a = 60^2$, so $c_a(I_a) = 275$, $\bar{\pi}_a (275,242) - 3600 = 800$, and equilibrium output $q_a = 148$. Notice that A is actually <u>underinvesting</u>.

If he were to maintain output at q_a but invest one more unit, his costs would change by $q_a(\partial c_a/\partial I_a) + 1 \approx -\frac{1}{2}(148)\frac{1}{60} + 1 \approx -.23 < 0$. He doesn't invest more, as we have seen, because that would tend to reduce equilibrium $\bar{p}_b$, which is bad.

Suppose however that A can reduce his marginal costs to a low enough level to deter entry: simply let $c_a = 335 - I_a^{\frac{1}{2}}$. Solving, from (2), $\bar{\pi}_b(c_a, c_b) = 0$ we get, in our numerical example, that $c_a = 61.4$ and $I_a = (273.6)^2$. In fact, A is better off investing $(273.6)^2$ and being a monopolist (his profits are 11,709) than he is investing $I_a = (60)^2$ and allowing B to play. But given that A has invested $I_a = (273.6)^2$ to keep B out, he will as a monopolist produce $q_a = 394.74$. The same amount could have been produced more cheaply with less investment, since the savings in marginal costs from the final dollar of investment $q_a(\partial c_a/\partial I_a) = \frac{1}{2}q_a I_a^{-\frac{1}{2}} \approx (197.37/273.6) \approx .72 < 1$. (We leave these calculations to the reader except to note that to figure the monopoly demand for A's product we always chose p_b (as a function of p_a) at exactly the level that would make $q_b = 0$. In our example that means $p_a = 500 - \frac{5}{9}q_a$.)

Thus in our example, A overinvests if he can deter entry but underinvests if he cannot.[19]

D. International Trade

Dumping in international trade. From Proposition 2 we know that if a firm is selling only in a foreign market, and if the strategic variables are strategic substitutes, then a subsidy to the firm will increase its profits by more than the subsidy. Thus a government concerned only with total national income may subsidize a firm to "dump" its products at low prices in a foreign market.

Proposition 3 showed that firms may sell at marginal revenue below true long-run marginal cost, and this gives us another possible explanation of dumping. The result can easily be extended to show that a firm may price below marginal cost in one period in order to make competitors less aggressive in future periods. For example, if a Japanese firm has decreasing costs over time (as in an industry with a "learning curve") and his American competitor's product is a strategic substitute, the firm may "dump" output in the early stages of a market's development to encourage competitors to either contract operations or withdraw from the market.

Tariffs and quotas that hurt the "protected" firm. Our numerical example in Section 2 not only illustrates Proposition 1 and its corollary that a subsidy in, or entry into, one market may hurt the firm in a second. It also shows (as Proposition 1' suggests) that companies may actually be hurt by measures to protect them from foreign competition in their domestic market if they are also competing in large foreign markets (even without any foreign retaliation). To see this, interpret the two markets in our numerical example as two different countries with the smaller market being firm B's home market and with A as the domestic producer in the larger market. If initially both companies are competing in both markets, then we can see from Table 1 that a trade barrier blocking the import of firm A's products into firm B's home market will actually hurt firm B--its profits fall from 3750 to 3722.[20] An import tax on firm A's products has exactly the same effect of hurting firm B. (A tax of 5 per unit on A's products in B's home market reduces the effective price received by A to 50,and leads to the same equilibrium that is obtained by preventing A from competing in B's home market.)

We doubt that any firm has really been hurt by a tariff on imports into
its home market. However, it may be that the increase in profits from such
tariffs is sometimes small. Who then would actively lobby for a tariff or
subsidy? In most cases such policies will increase the protected firm's
sales volume. If the firm's employees benefit from increased sales, they
may actively support a protectionist policy that has little effect on firm
profitability.

E. The Learning Curve

The learning curve is a multimarket oligopoly in which firms compete in
sequential markets. By discussing the interrelationship of markets through
strategic substitutes and complements we can clarify some issues in the
learning curve literature.

In his seminal paper, Spence (1981) argued that in the simple learning
curve problem with no diffusion of knowledge between firms there is little
difference between the "open loop" Nash equilibrium in which firms choose all
periods' strategic variables simultaneously and the "closed loop" or perfect
Nash equilibrium. Fudenberg and Tirole (1983) emphasized that there is a
significant qualitative difference between the closed and open loop solutions
and that with linear demand quantity competition would lead to greater production
in the first period than the second (in a two period model with a low interest
rate).

Spence examined the two period problem in which industry demand had a
constant elasticity of -1.25, and in which there was no spillover or diffusion
of knowledge between firms. A constant elasticity near -1 meant there was
automatically little strategic substitutability or complementarity: the reader

can confirm that with constant elasticity of -1 (only necessary for the second period) there is no strategic effect in an industry with symmetrical firms and there will be no difference between the open loop and closed loop solutions. However, with a linear demand curve, even if equilibrium second period industry elasticity were -1, products would be strategic substitutes and the Fudenberg-Tirole claim that with quantity competition first period output would exceed second only in the closed loop solution would be borne out. However, even given quantity competition, there is no reason to assume that a real market would exhibit strategic substitutability. To continue with constant elasticity, any inelastic industry demand will yield a symmetrical equilibrium with strategic complements. Strategic complementarity, of course, gives all decreasing cost firms an incentive to produce less in the first period, and thus reverses the Fudenberg-Tirole claim.

With price competition, the situation is more complicated. If both firms have a learning curve, A's price in the first period affects B's marginal cost in the second period. If A charges a lower price in the first period he reduces B's quantity sold there, thereby altering both firms' period one outputs and period two cost curves. In general, with price competition and strategic complements firm's prices may be higher or lower in the first period than in the second period. Given linear demand and symmetrical firms, however, the cross-effect (of A's price on B's current sales and therefore future costs) is dominant: Strategic complementarity means that A's lower first period price (implying lower second period costs and therefore prices) causes B to reduce his second period price, but this effect is overwhelmed by B's lower first period sales raising his second period costs and (therefore) price. In this important special case therefore, prices are lower, and so output higher, in the first period than in the second. (With strategic substitutes prices are always lower in the first period than in the second.)[21]

With quantity competition A cannot influence B's first market quantity.
However, his first period quantity can influence B's second period costs if
the learning curve has "spillovers" (i.e., diffusion of knowledge allows all
firms to learn from any one firm's production). The introduction of the
cross-effect (of A's output on B's subsequent costs) has the dramatic implication
that learning can <u>hurt</u> a firm. Consider a two period learning game where each
firm has constant first period costs of c_1 and constant second period costs of
$c_2(Q_1)$ where Q_1 is first period industry output and $(dc_2/dQ_1) < 0$.[22] That is,
there is a 100 percent spillover so if one firm produces an extra unit in the
first period all firms find their constant second period marginal cost c_2
equally reduced.

Now assume a constant elasticity demand curve of $Q_t = kP_t^{-a}$, where t is a
time subscript. In the open loop equilibrium, each firm produces up to the
point where in the first period marginal revenue equals marginal cost of c_1 +
$(q_2/1+r)(dc_2/dQ_1)$ (where q_2 is the firm's output in period 2 and r is the
interest rate), taking second period output of all competitors is given. Thus
firms always produce more than if the second period did not exist. But in the
closed loop equilibrium, where firms recognize that their first period output
will affect their opponents' second period choice, firms will have much less
incentive to "invest" in reducing second period costs. For example, if a = 1
second period profits will be independent of firms' (equal) costs and hence of
first period output, so firms will set marginal revenue equal to c_1 in period 1.
If industry demand is inelastic (a < 1), then firms will stop producing when
marginal revenue <u>exceeds</u> c_1; i.e., <u>less</u> is produced than if the second period
did not exist. The cost advantage of producing one's own output more cheaply
is swamped by the strategic consequences of the equal cost reductions of
competitors.

Footnotes

1. Recall also that both William of Normandy when he landed in England in 1066, and Cortes when he arrived in Mexico in 1519, destroyed their own ships in order to stiffen the resolve of their troops. Of particular interest to us is the effect that cutting off their escape routes to other countries might have had on their enemies who, knowing that it would be more difficult to defeat these armies than to defeat armies with easy escape routes, would have had less incentive to try.

2. For example, if $g(x_a) = 59.95$, then A will choose $x_a = 6.4$, raising his profits by 11.84 and raising B's profits by 120.92. A sets marginal revenue in the market in which he can precommit above his properly calculated marginal cost, because of the effect on the duopoly market. Similarly in oligopolistic natural resource extraction, if firms' costs rise in cumulative output (i.e., a "reverse learning curve"), firms contract output in early periods so that their marginal revenue exceeds long-run marginal cost.

3. If the two firms could collusively agree that both would produce in both markets, the overall market expansion would increase joint profits. However, if one firm pulled out of the market the other would follow. There have been cases when the firms in a multimarket industry all withdrew from one market within a short period of time, e.g., American makers of convertible automobiles in the mid-seventies. (In the convertibles case we suspect government regulation may have contributed to the mass withdrawal from the market.)

4.   With A in the main market only, the three-firm equilibrium is: $q_A = q_B = q_C = 40$, price = 80, and each firm earns 2400. With A in both markets, $x_A = 25$, $q_A = 30$, $q_B = q_C = 42\frac{1}{2}$, price = 85, and B and C each earn $2709\frac{3}{8}$.

5.   Generalizing our propositions to many firms requires agreement by all firms $F \neq A$ on whether q^A is a strategic substitute or complement.

6.   In our numerical example q^A and q^B were strategic substitutes. The figure below shows A's reaction curve in the shared market before (XX') and after (II') being given access to the new market.

7.   Stability means that if we start close to the equilibrium and the firms adjust q^A and q^B in the directions that bring marginal revenue towards marginal cost, then they will move closer to the equilibrium. In contrast, if q^A and q^B are slightly displaced from an unstable equilibrium and the firms adjust q^A and q^B in the natural way, they will continue to move away from equilibrium.

8.   The analysis can easily be generalized to many firms and many markets.

9.   We always assume that $(\partial f_i^F / \partial q_i^G) < 0$, if $F = G$ or if $F \neq G$.

10.   Equivalently, we could assume that there are units in which to measure goods such that $|\partial g_i^F / \partial P_i^F| > |\partial g_i^F / \partial P_i^G|$ for $F \neq G$, i = 1, 2 and $|\partial^2 c^F / \partial Q_1^A \partial Q_2^A| \leq |\partial^2 c^F / \partial^2 Q_i^F|$, F = A, B, i = 1, 2. If the former were false, then we could raise prices and yet demand for both commodities would rise.
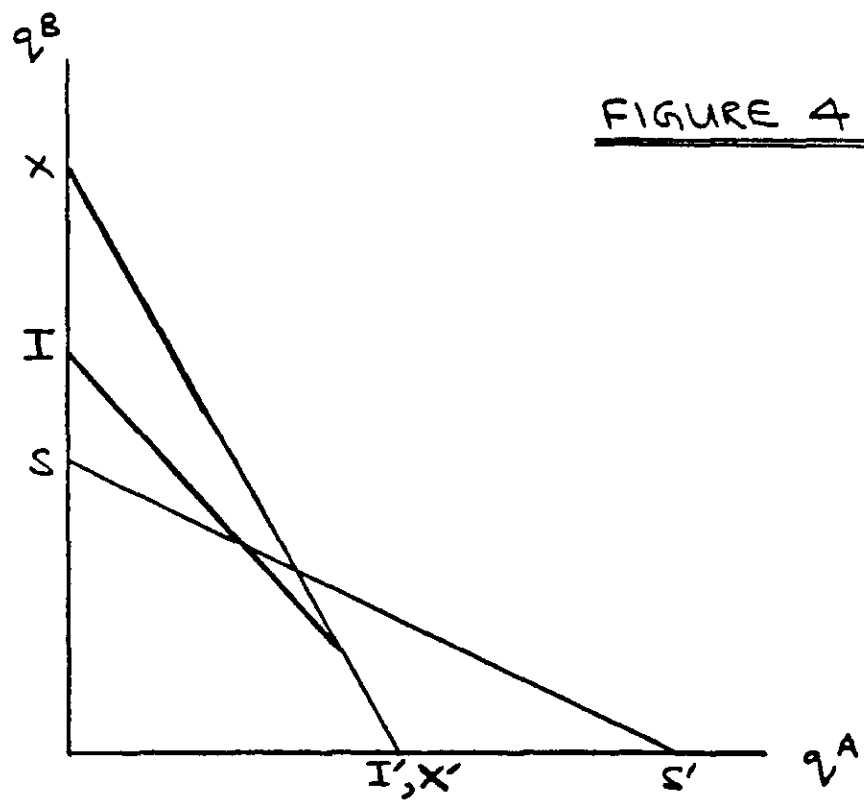
FIGURE 4 (for Footnote 6)

11. Needless to say, there is a long tradition in economics which justi-
fies comparative statics assumptions on the basis of stability.

12. That joint products may give rapidly decreasing marginal costs is
obvious. Constrained capacity may lead to rapidly increasing costs across
substitutes in production in the short run. Rapidly increasing marginal costs
are also possible in the long run if two products, say X1 and X2, can both be
produced as alternative joint products to product Y. If the company is only
able to sell a limited amount of Y, then the total production of X1 + X2 is
effectively constrained.

13. The precise game we are describing has three stages: First A
announces whether he will enter B's market, then B announces whether he will
enter A's market, then the simultaneous market game is played. We consider
only perfect equilibria.

14. That Polaroid may attract new competition into its own markets by
entering other markets has been made plain by our numerical example in Section
2 and by our discussion of Frontier Airlines in the previous subsection. This
depends on Polaroid having increasing marginal costs and thus weakening itself
in its home market. To see that Kodak might find Polaroid's market relatively
more attractive than its own, even if Polaroid has become no weaker there
(e.g., if Polaroid has flat marginal costs), consider the following:

A is initially a monopolist in market 1; B is the incumbent monopolist
in market 2. Each firm faces a fixed cost of 750 of competing in each market.
The inverse demand in each market is $p_i = 100 - q_{ai} - q_{bi}$. Firm A can supply
all its markets as much as it wants at a constant marginal cost of zero. Firm

B has the capacity to produce 60 units at a constant marginal cost of zero, but has a very high marginal cost (say 100) of supplying additional units beyond 60.

If A and B stay in their respective markets each will produce 50 units and sell at a price of 50, earning net profits of $50 \times 50 - 750 = 1750$ each.

B initially has no incentive to invade A's market. If he did, A would produce 40 in the contested market. B would produce 20 in the contested market and 40 in the uncontested market, earning 1700.

If however A enters B's market, B has the choice of remaining in his home market only and selling $33\frac{1}{3}$ for a net profit of $361\frac{1}{9}$, or retaliating by entering A's market. If both A and B are in both markets the equilibrium is that A produces 35 in each market (at a price of 35) and B produces 30 in each. A's profits are 950 and B's profits are 600. Thus B would react to A's entry by entering A's market, but would not have entered otherwise. A, who apparently had an incentive to invade B's market (his profits would have risen from 1750 to $2111\frac{1}{9}$ if B had not retaliated), would have done better to confine himself to one market.

15. Note that it may be possible for the firm to increase investment and lower total variable cost but still raise marginal cost in the relevant range. This case corresponds to the case of increasing marginal costs in Proposition 3, and reverses all the results in this section; e.g., such investment would make potential entrants assume the firm would compete less aggressively and would encourage potential entry, so that the firm would have an incentive to underinvest to deter entry.

16. With strategic substitutes we would have overinvestment, as above. We stated this result formally as a corollary to Proposition 3.

17. Recall also that Dixit showed in his 1980 _Economic Journal_ article that a firm would never hold idle capacity to deter entry, and Kreps and Spence (1983) imply that this result depends on Dixit's assumption of quantity, rather than price, competition. Actually, Dixit's result is independent of the type of competition but it relies on the assumption of strategic substitutes. With strategic complements and Dixit's cost function, a monopolist might hold more capacity to deter entry than he would actually use when he produced.

18. This basic point that with strategic complements a firm's competitors play less aggressively if the firm's costs are higher, has applications in other areas of industrial organization. For example, the limit pricing literature implies that a monopolist may price lower than the pre-entry profit-maximizing price in order to signal low costs and deter entry. However, an important assumption in this literature is that any entrant learns the incumbent's costs immediately after entry. If this assumption is relaxed, then if entry occurs, and with strategic complements, the incumbent would like the entrant to believe that his costs are higher than they really are. Thus the incumbent might, in principle, charge a higher first period price than the monopoly price. Although he has an "entry deterrence" incentive to price low and signal low costs, he has a "price war avoidance" incentive to price high and signal high costs which helps him if entry does occur.

19. Things would be still more complicated in a three-period game in which A invested, then B invested, and then the price (or quantity) competition game was played--this game represents the more realistic assumption that potential entrants may enter a market on a variety of levels. Now whether A over- or underinvests affects B's choice of investment as well as B's choice of price in the final period. While greater investment by A may reduce the profitability of all B's potential investments, there is no particular reason why it should reduce the optimal level of B's investment. For example, if airline A invests in a new generation of jets, airline B may find that its best response is to enter the market with new jets itself.

20. In the numerical example given, the protected market ends up with a lower price than the open market, whereas we normally think of protected markets having higher prices. Slightly different numerical examples give higher prices in the protected market without affecting our other results.

21. As an example of why prices being different in different periods may be important, consider cities that are thinking of auctioning off the rights to their cable TV franchises. For simplicity assume the interest rate is zero. Assume that as more franchises are built, firms learn more and reduce their costs of building new operations. Should customers have their franchises auctioned off first or second? With strategic complements and a strong cross-effect, or with strategic substitutes, customers do better by having their franchise auctioned off first and so may scramble to sell off their rights early.

22. We will assume d^2c_2/dQ_1^2 is sufficiently positive to assure the stability of a symmetric n-firm equilibrium.

References

Brander, J. and P. Krugman, "A 'Reciprocal Dumping' Model of International Trade," Massachusetts Institute of Technology, June 1981, mimeo.

Bulow, J. and J. Geanakoplos, "Strategic Resource Extraction: When Easy Doesn't Do It," Stanford University, May 1983, mimeo.

Dixit, A., "A Model of Duopoly Suggesting a Theory of Entry Barriers," Bell Journal of Economics, Vol. 10, No. 1 (Spring 1979), 20-32.

_____, "The Role of Investment in Entry Deterrence," Economic Journal, (March 1980), 95-106.

Frank, C. R., "Entry in a Cournot Market," Review of Economic Studies, Vol. 32 (1954), 245-250.

Fudenberg, D. and J. Tirole, "Learning by Doing and Market Performance," January 1983, mimeo.

Gale, D. and H. Nikaido, "The Jacobian Matrix and the Global Univalence of Mappings," Mathematische Annalen, Vol. 159 (1965), 81-93. Reprinted in Readings in Mathematical Economics, P. Newman, ed., 1968, pp. 68-81.

Kreps, D. M. and A. M. Spence, "Modelling the Role of History in Industrial Organization and Competition," 1983, mimeo.

Lieberman, M., Unpublished doctoral thesis, Harvard University, 1983.

Milgrom, P. and J. Roberts, "Limit Pricing and Entry under Incomplete Information," Econometrica, Vol. 50 (1982), pp. 443-60.

Panzar, J. C. and R. D. Willig, "Free Entry and the Sustainability of Natural Monopoly," Bell Journal of Economics, Vol. 8, No. 1 (Spring 1977).

_____, "Economies of Scale in Multioutput Production," Quarterly Journal of Economics (August 1977).

Ruffin, R., "Cournot Oligopoly and Competitive Behavior," Review of Economic Studies, Vol. 38 (1971), 493-502.

Seade, J., "On the Effects of Entry," Econometrica, Vol. 48, No. 2 (March 1980), 479-489.

Spence, A. M., "Entry, Capacity, Investment, and Oligopolistic Pricing,"
Bell Journal of Economics, Vol. 8, No. 2 (Autumn 1977).

_____, "Investment, Strategy, and Growth in a New Market," Bell
Journal of Economics, Vol. 10, No. 1 (Spring 1979), 1-19.

_____, "The Learning Curve and Competition," Bell Journal of Economics,
Vol. 12, No. 2 (Autumn 1981), 49-70.

"Where Frontier Lost Its Way," Business Week (February 7, 1983).