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WORKER ASYMMETRIC INFORMATION AND INVOLUNTARY UNEMPLOYMENT

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I. Introduction

Drawing on the well-known capability of asymmetric information to provide insight into apparent *ex post* inefficiencies, recent research has focused on explanations of involuntary unemployment based on imperfect information. In an environment in which a stochastic shock to a firm's technology is unobservable to its workers; Azariadis [1983], Chari [1983], Green-Kahn [1983], and Grossman-Hart [1981, 1983] characterize the impact of this informational asymmetry on employment distortions. Azariadis and Grossman-Hart provide examples of underemployment while Green-Kahn show that overemployment may arise as well. As emphasized in Cooper [1983a] and Hart [1983] the key factors in determining the form of the employment distortion are the worker's preferences over consumption and leisure and the firm's risk aversion. If the firm is risk neutral, then leisure must be an *inferior* good for underemployment to occur. If leisure is a normal good, there appears to be a minimal level of firm risk aversion necessary for underemployment.[1]

This paper considers an alternative environment of worker asymmetric information in which worker's preferences over consumption and leisure are *unobservable* to the firm. There are a number of possible interpretations of this type of model. We might view this uncertainty as arising from the random nature of worker's outside opportunities (see, for example, Hart [1983]). Alternatively, workers may be uncertain, *ex ante*, about the disutility of work as discussed by Carmichael [1983], Hall-Lilien [1979] and Hashimoto and Yu [1980]. More generally, workers can be thought of as facing unknown prices and (other) incomes when negotiating with the firm *ex ante*. In addition, a firm with monopoly power facing a pool of heterogeneous workers will want to sort the workers as in standard problems of "surplus extraction." Though the model presented in Section II relies on uncertainty in a given worker's preferences, the results will hold (with slight changes in the model) under this sorting interpretation as well. Hence we follow the implicit contracts literature by providing an insurance motive for contracting which is driven by worker's aversion to consumption risks. It is this insurance motive, interacting with the incentive problem created by the imperfect information, which produces the employment distortions studied in this paper.

Since the conditions for underemployment in the contracting models with firm asymmetric information outlined by Azariadis [1983], Grossman-Hart [1981, 1983] and Hart [1983] may seem rather restrictive, one of the goals of this paper is to see whether the conditions for underemployment with worker asymmetric information are more acceptable. Section III of the paper shows that when the firm is risk neutral, the normality of leisure is a sufficient condition for underemployment.[2] We also discuss the special case of perfect substitutability of income and leisure (which has been a leading example in the literature) and show that either underemployment or overemployment may occur.

Finally, we consider a situation of bilateral asymmetric information by combining the model of Section III with the models of firm asymmetric information explored elsewhere. Both Hall-Lilien [1979] and Hall-Lazear [1982] discuss the firm-worker problem with bilateral asymmetric information. In these papers, the assumption of risk neutrality on both sides of the contract is made which, following Riordan [1982], will imply that the full-information contract is implementable. In this paper, at least one of the parties is risk averse and some special cases of bilaterial asymmetric information are explored. One interesting example of underemployment occurs when the labor choice is discrete and the optimal dominant strategies contract simplifies to each party having a "veto" over employment. As discussed in this section, a general treatment of this problem is rather difficult and many open questions remain.

II. Worker Asymmetric Information and Implementation of the Full-Information Solution

To begin our analysis, we carefully specify worker preferences and analyze the full-information solution. Risk neutral firms have profits (π) represented by

$$\pi = \ell - \omega \tag{1}$$

where ℓ is the level of employment and ω is total compensation. Since we assume a constant returns to scale technology we restrict our attention to the relationship between this firm and a single worker. We represent a worker's preferences over consumption (equal to ω by assumption of one good) and labor by $U(\omega,\ell,\theta)$ where $\tilde{\theta}$ is a random disturbance to worker's preferences. We assume that, for given θ, preferences are convex and that $U_\omega > 0$, $U_{\omega\omega} < 0$, $U_\ell < 0$, $U_{\ell\ell} < 0$. Consumption is assumed to be a normal good. Define $G(\omega,\ell,\theta)$ as the worker's marginal rate of substitution between compensation and labor, i.e.,

$$G(\omega,\ell,\theta) \equiv \frac{-U_\ell}{U_\omega} .$$

We initially assume that preferences satisfy $U_{\omega\theta} \geq 0$ and $U_{\ell\theta} \leq 0$ with at least one strict inequality implying the single-crossing property of

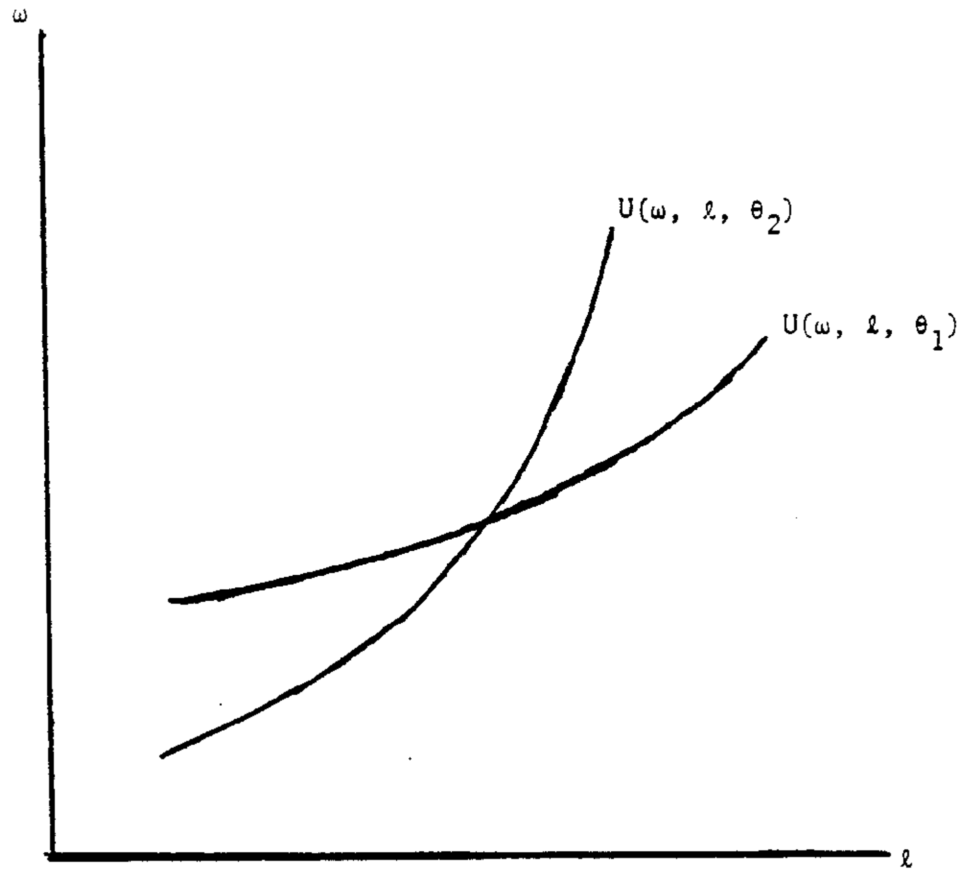


FIGURE 1

Single-Crossing Property of

Worker Preferences: $\theta_2 > \theta_1$

$G_\theta > 0$.[3] That is, for given (ω,ℓ) , the worker's marginal rate of substitution is increasing in θ as shown in Figure 1 so that indifference curves (for alternative θ's) cross only once. We assume that θ takes on values in the interval $[\underline{\theta},\overline{\theta}]$ with $\underline{\theta} > 0$. We denote by $p(\theta_i)$ the probability that θ_i occurs.

Ex ante, the firm and the worker sign a contract $\delta = \{\omega(\theta), \ell(\theta)\}$ specifying compensation and employment for each realization of θ . When realizations of θ are observed by both parties, the optimal full-information contract, $\delta^* = \{\omega^*(\theta), \ell^*(\theta)\}$ solves

$$\max E\{\pi + \lambda U(\omega,\ell,\theta)\} \tag{2}$$

where the expectation is taken over θ and $\lambda > 0$ is an arbitrary weight to parameterize the expected utility possibility frontier. To be feasible, $\ell(\theta) \leq \overline{\ell}$ for all θ where $\overline{\ell}$ is the time endowment of the worker.[4] The interior solution to (2) satisfies

$$U_\omega(\omega,\ell,\theta) = 1/\lambda \quad \text{for all} \quad \theta \tag{3}$$

and
$$G(\omega,\ell,\theta) = 1 \quad \text{for all} \quad \theta\ . \tag{4}$$

Conditions (3) and (4) represent conditions for optimal risk sharing and productive efficiency respectively.

To motivate our results on employment distortions in the next section, we can ask whether or not δ^* is implementable when θ is observed only by the worker. That is, faced with δ^* , would the worker have an incentive to truthfully reveal θ ?[5] The answer can be obtained by using conditions (3) and (4) to implicitly define $\omega^*(\theta)$ as the level of compensation associated with ℓ units of employment time.

So consider an arbitrary θ , $\hat{\theta}$, as shown in Figure 2. The optimal compensation and employment levels associated with $\hat{\theta}$ are determined at the point of tangency between the indifference curves of the two parties as in (3) and (4). We have drawn $\omega^*(\ell)$ flatter than $G(\omega^*(\hat{\theta}), \ell^*(\hat{\theta}), \hat{\theta})$ in the diagram to depict a situation where δ^* is not implementable. It is clear from Figure 2 that the worker will have an incentive to report a θ which will reduce the amount of employment relative to $\ell^*(\hat{\theta})$. It should be equally apparent that θ^* is implementable iff $\omega^*(\ell)$ is tangent to the worker's indifference curve at $(\omega^*(\hat{\theta}), \ell^*(\hat{\theta}))$ for all $\hat{\theta}$ as in Figure 3.[6] Implementation requires

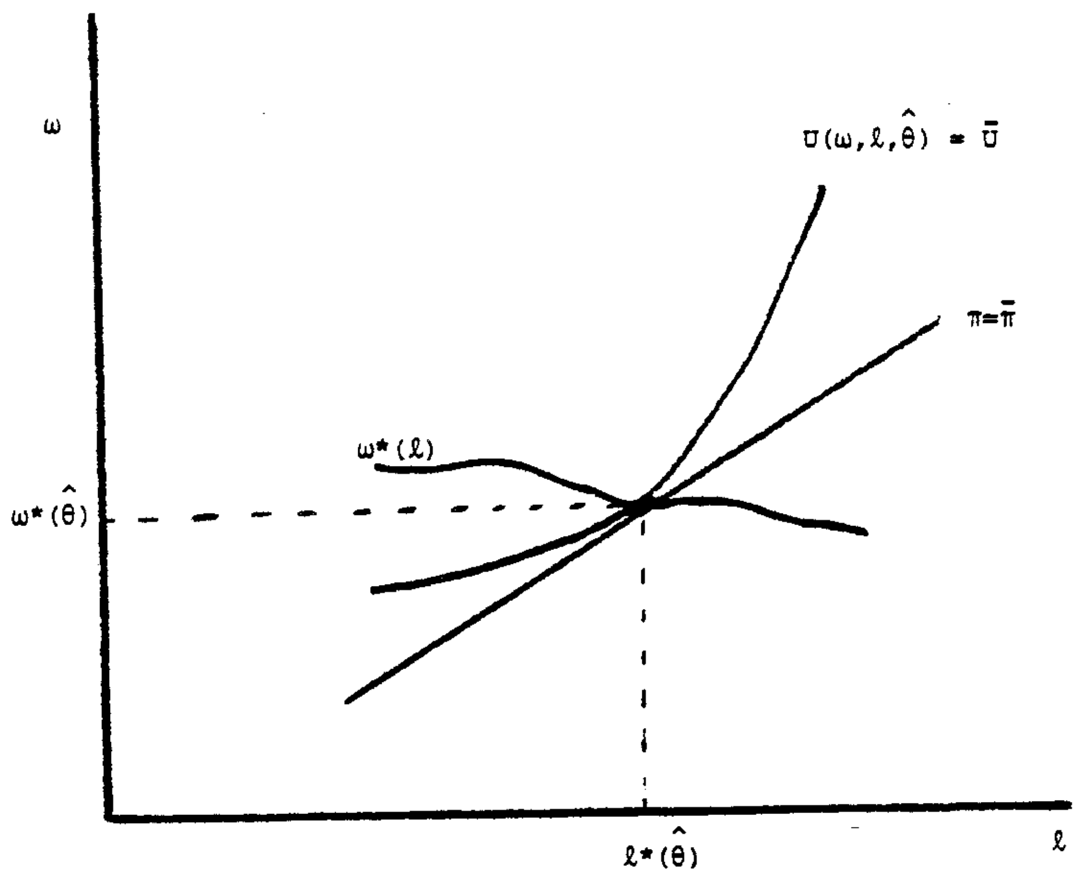
$$U_\omega \frac{d\omega^*(\theta)}{d\theta} = -U_\ell \frac{d\ell^*(\theta)}{d\theta} . \tag{5}$$

As in Cooper [1983a], it will prove useful here to understand the conditions regarding the misrepresentation of θ . In particular, are there interesting conditions under which the worker will lie about the true value of θ to reduce employment as in Figure 2?

Proposition 1: If leisure is a normal good, then a worker, faced with δ^* , will misrepresent θ to reduce employment.

Proof. The proposition clearly holds if the slope of $\omega^*(\ell)$ is less than one for all employment levels. If consumption (as stated above) and leisure are both normal, then differentiation of (3) and (4) yields the result.

Hence we see that the normality of leisure is a sufficient condition for workers to want to reduce their employment under the first-best contract δ^* . Other cases of lying to reduce work will be discussed in the next section for preferences where consumption and leisure are



ω
$U(\omega, \ell, \hat{\theta}) = \bar{U}$
$\pi = \bar{\pi}$
$\omega^*(\ell)$
$\omega^*(\hat{\theta})$
$\ell^*(\hat{\theta})$
ℓ

FIGURE 2

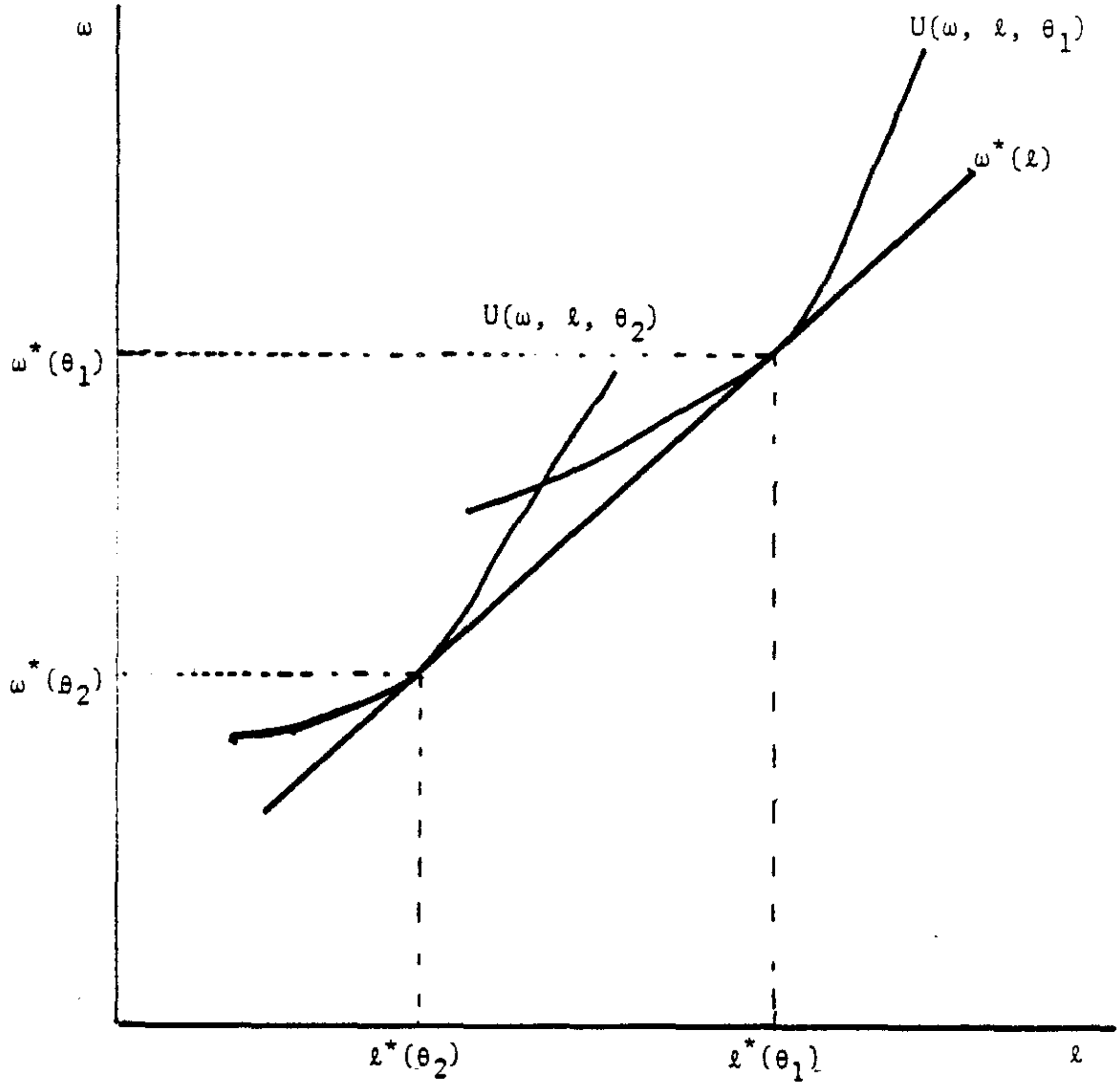


FIGURE 3

perfect substitutes. The implication of Proposition 1 is that the optimal contract under asymmetric information must provide incentives for the worker to be honest about θ and not attempt to reduce employment when leisure is a normal good. We show in the following section that these incentives for truthtelling will create underemployment.

III. Optimal Contract with Worker Asymmetric Information

We now consider the optimal contract when realizations of $\tilde{\theta}$ are observed only by the worker. Following standard practices, contracts under asymmetric information are implemented through a direct revelation game. We define underemployment as occurring when $G < 1$ since both parties could benefit *ex post* from increasing employment and overemployment occurs if $G > 1$ where $G \equiv -U_\ell/U_\omega$ (see Figure 4).

The optimal contract under worker asymmetric information solves

$$\max E\{\pi + \lambda U(\omega,\ell,\theta)\}$$
$$\text{subject to } U(\omega(\theta_i),\ \ell(\theta_i),\ \theta_i) \geq U(\omega(\theta_j),\ \ell(\theta_j),\ \theta_i) \quad \forall \theta_j,\ \theta_i\ . \tag{7}$$

In this problem, we have simply appended incentive compatibility constraints to (1) to ensure truthtelling by the worker.

Proposition 2. If leisure is a normal good, then the solution to (7) yields underemployment for $\theta \in (\underline{\theta},\overline{\theta})$ and efficiency for $\theta = \underline{\theta},\ \overline{\theta}$.

Proof. See Appendix A.

The proof of the proposition analyzes (7) by setting up the associated Hamiltonian problem. The proof uses the condition that $U_{\omega\theta} \geq 0$ as well as the single-crossing property of preferences.

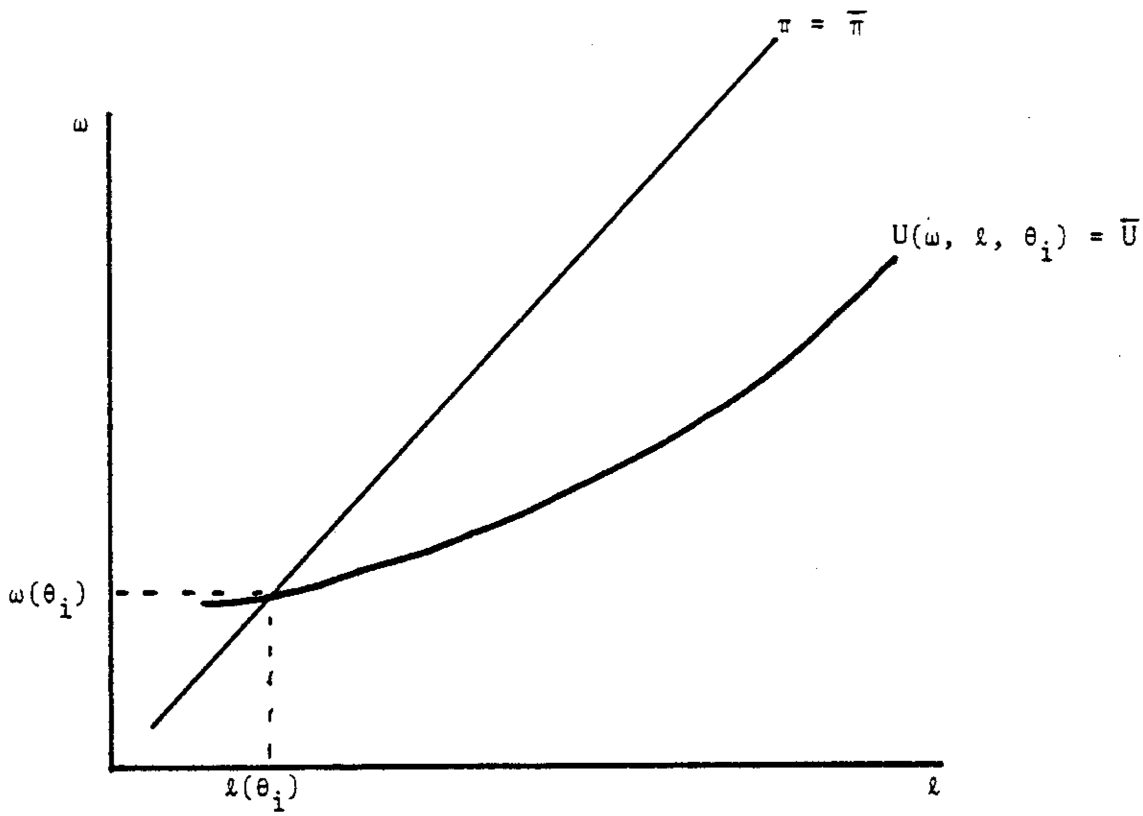


FIGURE 4a. $G < 1$ implies underemployment

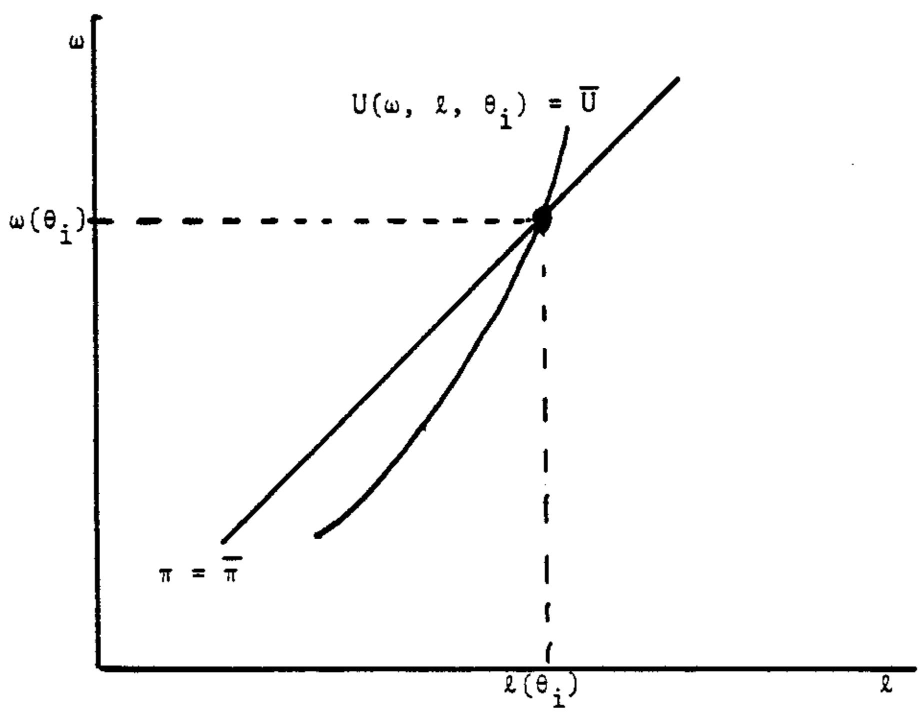


FIGURE 4b. $G > 1$ implies overemployment

There are two additional properties of the solution to (7) worth mentioning. First, as is common in problems with self-selection features, the incentive compatibility conditions imply that $\omega(\theta)$ and $\ell(\theta)$ are monotonically decreasing functions of θ. Second, there is a simple intuitive mechanism for the implementation of the solution to (7). The firm announces a function $\hat{\omega}(\ell)$ and the worker, having observed θ, simply chooses the number of hours he/she wishes to work given $\hat{\omega}(\ell)$. Given this behavior by workers _ex post_, the firm selects $\hat{\omega}(\ell)$ to maximize the objective function in (7).

We have assumed that leisure is normal for all (ω,ℓ,θ) to prove Proposition 2. If leisure is an inferior good, overemployment may arise. We could presumably have regions of overemployment and underemployment if leisure was a normal good for some (ω,ℓ,θ) and an inferior good for other (ω,ℓ,θ).

It may prove instructive at this point to analyze in more detail some special utility functions. These examples will provide some intuition for Proposition 2 and will be useful in our discussion of bilateral asymmetric information as well.

Example 1: Separable Preferences

Here we assume that the worker's preferences are represented by

$$V(\omega) - \theta g(\ell) \tag{8}$$

where $V(\cdot)$ is increasing and strictly concave and $g(\cdot)$ is increasing and strictly convex. This case was initially examined in Cooper [1982]. In the first best contract, $\omega(\theta)$ is constant and $\ell(\theta)$ _decreases_ with θ. The worker would have an obvious incentive to announce that $\bar{\theta}$ occurred regardless of the true value of θ to minimize employment.

In the solution to (7), we find that underemployment occurs since leisure is a normal good if preferences are separable. The reason for this is that the firm must alter $\omega^*(\theta)$ and $\ell^*(\theta)$ to induce the worker to announce the true θ rather than $\overline{\theta}$. Given $\ell^*(\theta)$, there will exist a compensation schedule which will implement this first-best employment rule. However, this will leave the worker facing excessive risks so that adjustments in the employment rule are necessary as well. The solution to (7) balances these adjustments and creates imperfect insurance (i.e., ω is not independent of θ when preferences are separable) and *ex post* underemployment of labor.

Example 2: Perfect Substitutes

Here we represent worker's preferences by $U(\omega - h(\ell,\theta))$. This representation (with θ degenerate) was used by Azariadis [1983] and Grossman-Hart [1981, 1983] to produce examples of underemployment when firms had privately observed shocks to their technology. With these preferences, leisure demand is independent of income. In this specification $h(\ell,\theta)$ represents the monetary value of supplying ℓ units of labor when state θ occurs. Hart [1983] and Moore [1982] analyze the cases of $h(\ell,\theta) = \ell\theta$ and $h(\ell,\theta) = -(\overline{\ell}-\ell)$ where $\overline{\ell}-\ell$ is the leisure consumption. In the first special case, underemployment occurs while overemployment arises for the second example.[7]

With the general $h(\ell,\theta)$ specification, we see that the full-information contract will stabilize $(\omega - h(\ell,\theta))$ as θ varies and that ℓ will decrease with θ. If the firm attempted to implement this contract under worker asymmetric information, the worker will have an incentive to misrepresent his preferences by overstating θ iff $h_\theta > 0$ and by understating θ iff $h_\theta < 0$.

We can analyze this specification of preferences without the assumption maintained in Proposition 2 of $U_{\omega\theta} \geq 0$. The single-crossing property will hold as long as $h_{\ell\theta} > 0$ which we continue to assume. Since the direction of lying under the full-information solution depends on h_θ , it is not too surprising that the direction of the employment distortion will depend on this term as well. When h_θ is positive, then $U_{\omega\theta}$ exceeds zero as well and Proposition 2 holds.

Proposition 3. Suppose that worker's preferences are represented by $U(\omega - h(\ell,\theta))$. Then the optimal contract under asymmetric information has underemployment when $h_\theta > 0$ and overemployment when $h_\theta < 0$.

Proof. See Appendix B.

When leisure demand is independent of income we can have either overemployment or underemployment depending on whether or not "high" θ's are "good" states. If h_θ can not be signed globally, we can have regions of overemployment and underemployment. Hence, we see that unlike the firm asymmetric information problem, it is possible to obtain underemployment with both firm risk neutrality and worker's preferences in which leisure is not inferior.[8]

Having obtained a characterization of the conditions for underemployment in the worker asymmetric information problem we can now address questions concerning bilateral asymmetric information. For readers unacquainted with firm asymmetric information contracting problems, the survey by Hart [1983] and Azariadis-Stiglitz [1983] may be useful references.

IV. Bilateral Asymmetric Information

A natural contracting situation is that of bilateral asymmetric information in which both parties to the contract face future uncertainty. This was the environment initially studied by Hall-Lilien [1979] though their main question concerned the implementation of the full-information employment rule rather than characterizing the optimal incentive compatible contract. In that paper, as well as in Hall-Lazear [1982], both the firm and the worker were assumed to be risk-neutral. As shown in Riordan [1982], and discussed below, the full-information solution can generally be implemented when both parties to the contract are risk neutral. Green-Honkapohja [1981] also consider a model of exchange under bilateral asymmetric information but do not stress the risk-sharing aspects of the problem.

A general characterization of the optimal contract under bilateral asymmetric information is beyond the scope of this paper. That is, I will not present necessary and sufficient conditions for underemployment in a broad class of models. Instead, I will present the general problem, discuss the difficulties in solving it and discuss some (hopefully) illuminating examples.

As in the previous sections of this paper, we represent worker's preferences by $U(\omega,\ell,\tilde{\theta})$. Firm's have profits $\pi = \tilde{s}f(\ell) - \omega$ where $\tilde{s}$ represents a stochastic technology shock, $f(\ell)$ is a concave production function and an increasing, concave function $V(\cdot)$ defines their preferences over income. Under full-information about the realizations of (s,θ) , the optimal contract $\gamma^* = \{\omega^*(s,\theta), \ell^*(s,\theta)\}$ maximizes, for arbitrary $\lambda > 0$,

$$E\{V(\pi) + \lambda U(\omega,\ell,\theta)\} . \tag{9}$$

In (9), the expectation is taken over the joint distribution of (s,θ) .

When realizations of $\tilde{s}$ and $\tilde{\theta}$ are the private information of the firm and worker respectively, a number of important issues arise. First of all, we must choose between alternative equilibrium concepts of dominant strategy, ex post Nash and Bayesian for the revelation game played between the parties. Since preferences of the parties depend only on their own shock, it is well-known that dominant strategy and ex post Nash yield identical solutions.[9] Secondly whether or not $\tilde{s}$ and $\tilde{\theta}$ are independent will be important in the Bayesian equilibrium though irrelevant for dominant strategies.

To make the discussion more precise we define the equilibrium concepts below. Let $\pi(s_k, \theta_j|s_i)$ be the firm's profits under an arbitrary contract when the parties announce (s_k, θ_j) and the firm's true state is s_i . Similarly $W(s_j, \theta_k|\theta_i) = U(\omega(s_j, \theta_k), \ell(s_j, \theta_k), \theta_i)$ and is the worker's utility in state θ_i when the announced states are (s_j, θ_k) .

Definition 1. A contract is dominant strategy incentive compatible if

(i) for each (s_i, θ_j), $\pi(s_i, \theta_j|s_i) \geq \pi(s_k, \theta_j|s_i)$ $\forall s_k$

and (ii) for each (s_j, θ_i), $W(s_j, \theta_i|\theta_i) \geq W(s_j, \theta_k|\theta_i)$ $\forall \theta_k$.

Implementation in dominant strategies thus requires that each party truthfully reveal their true state independent of the announcement of the other party. This is viewed (see, for example, the discussion in d'Aspremont and Gerard-Varet [1979]) as applicable for situations of "complete ignorance" about other parties to the agreement. In dominant strategy incentive compatible contracts, agents need no information at all about the strategies of other agents in the contract. In many circumstances, this simplicity can be quite appealing.

Definition 2. A contract is Bayesian incentive compatible if

(i) for each s_i , $E_{\theta|s_i} V(\pi(s_i, \theta)|s_i)) \geq E_{\theta|s_i} V(\pi(s_k, \theta)|s_i) \quad \forall s_k$

and (ii) for each θ_i , $E_{s|\theta_i} W(s, \theta_i|\theta_i) \geq E_{s|\theta_i} W(s, \theta_k|\theta_i) \quad \forall \theta_k$.

In this situation, each party believes (and these conjectures are correct) that the other party will reveal their true state but neither party knows the other's state. Hence, each party takes conditional expectations over the other's state in deciding what announcements to make. If a contract is Bayesian incentive compatible, then truthful announcements will be made by both parties. For further discussion of this equilibrium concept see Harsanyi [1967], Myerson [1979] and Harris-Townsend [1981].

Determining the optimal contract under bilateral asymmetric information entails solving (7) subject to the incentive compatibility constraints implied by one of the two equilibrium concepts. When θ and s are continuous random variables, the optimization problem is a very difficult double-integral variational problem with partial differential equations as constraints. Even in the case of discrete states, the problem is difficult to solve unless further structure is assumed.

Instead of focusing on these general problems, presented below are two interesting special cases. The key question is whether or not the interaction of asymmetric information on both sides of the contract will strengthen or weaken the case for underemployment.

We first consider a model which extends the analysis of Grossman-Hart [1981] to the bilateral asymmetric information setting.[10] In particular we assume that worker's preferences are represented by

$U(\omega - \tilde{\theta}\ell)$ so that $\tilde{\theta}$ represents a continuous random disutility of work and takes values in $[0,1]$. We also assume that ℓ only takes on the values 0 or 1 to correspond to unemployment and employment states respectively. Finally we maintain the assumption of constant returns to scale so that $\pi = \tilde{s}\ell - \omega$ where $\tilde{s}$ is a continuous random variable in $[0,1]$. We assume that both $\tilde{s}$ and $\tilde{\theta}$ have densities known to both parties.

Grossman-Hart [1981] show that when θ is degenerate, underemployment occurs in the resulting firm asymmetric problem. Incentive compatibility clearly requires that the contract specify only two wages: ω^e if employment occurs and ω^u if employment does not occur. This means that employment states are risky ones for the risk averse firm so that underemployment occurs as a means of avoiding this risk. When $\tilde{s}$ is degenerate, a similar story arises in the worker asymmetric information problem. That is, there will again be two wages and the worker will bear the risk of θ in the employed states. To avoid this risk, underemployment arises.[11] In each of these cases, the party with asymmetric information essentially chooses whether employment will occur or not.

When neither $\tilde{s}$ nor $\tilde{\theta}$ are degenerate, the full-information contract stipulates that employment occurs iff $s \geq \theta$ as depicted in Figure 5. The compensation schedule is set to efficiently share the remaining risks.

The optimal contract under bilateral asymmetric information takes a very intuitive form when dominant strategy implementation is utilized. As in the unilateral asymmetric information case, since $\ell \in \{0,1\}$, compensation can only vary with employment in an incentive compatible contract. To see why, assume that compensation varies while ℓ does not. Then one of the parties would have an incentive to misrepresent

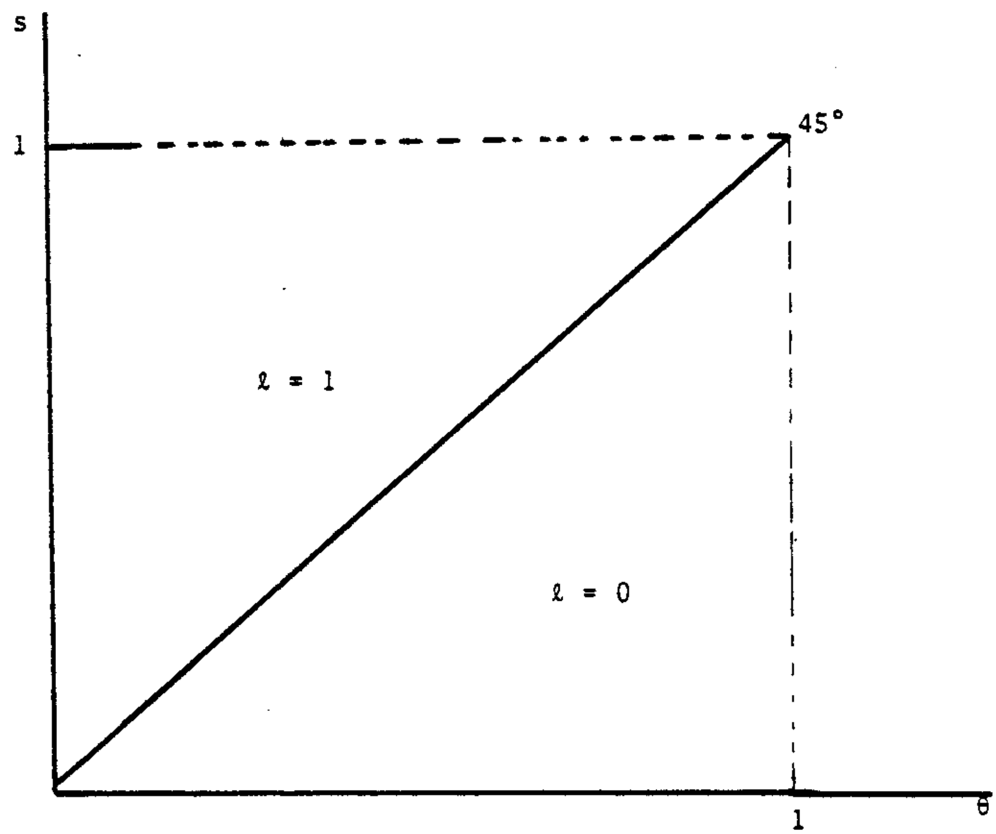


FIGURE 5

Efficient Contract

their true state (by the monotonicity of preferences) for a given announcement of the other party. This violates dominant strategy incentive compatibility. In particular, ω^u is independent of the cause of separation so that severance pay is paid whether the worker quits or is fired. Define d as the difference between the employment and unemployment wages: $d \equiv \omega^e - \omega^u$.

We also impose ex post constraints on the problem so that parties to the agreement are not forced to work against their will. For the worker, this implies that in order for employment to occur $(\ell = 1)$, $d \geq \theta$. Similarly, from the firm's viewpoint, $s \geq d$ is necessary for $\ell = 1$. In the Grossman-Hart analysis and in the worker asymmetric information problem discussed above, these ex post constraints were met.

At this stage, we could formally characterize the optimal contract by choosing over d and one of the wages. However, the underemployment result can be seen directly. In fact the optimal contract in this setting is quite similar to one of the "simple contracts" described by Hall-Lazear. Ex ante the contract sets d . The firm observes $\tilde{s}$ and decides whether it wants to employ workers or not. Similarly, workers observe $\tilde{\theta}$ and decide whether or not they wish to work. From the ex post individual rationality constraints discussed above employment occurs iff $s \geq d \geq \theta$. Comparing Figure 6 with Figure 5, we see that unemployment will definitely occur for $s \geq \theta \geq d$ and $d \geq s \geq \theta$.[12] This is not too surprising given that we had underemployment in both the firm and worker asymmetric information cases discussed above.

However, we do get a different result when worker's preferences are represented by $U(\omega + (1-\ell)\theta)$. In this specification, the value of leisure time is random rather than the random disutility of labor studied

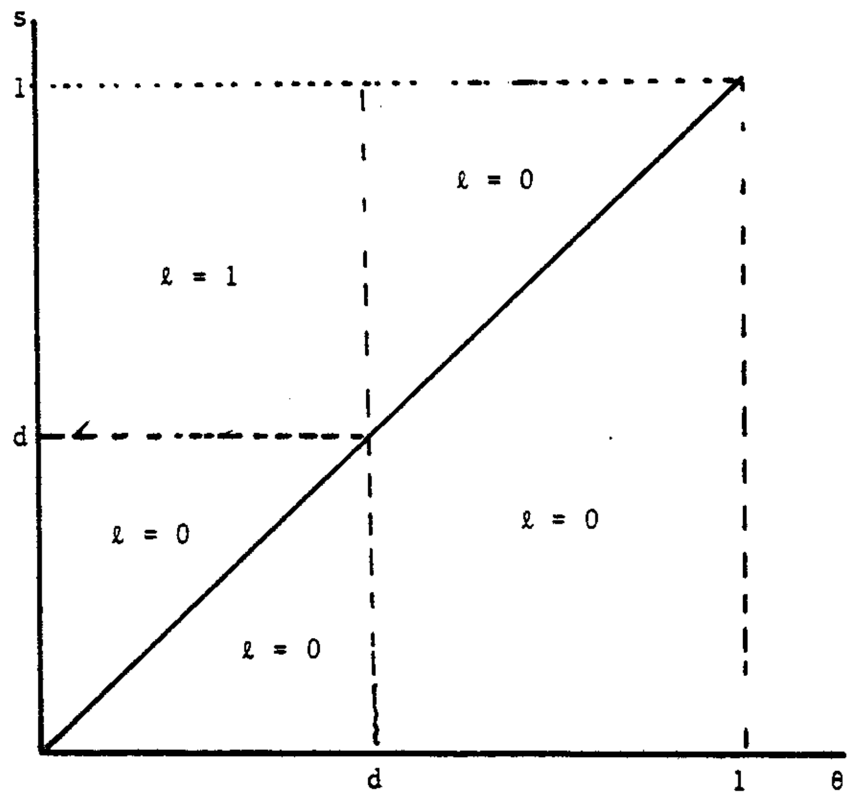


FIGURE 6

Second-Best Contract

above. As discussed by Hart [1983] and Moore [1982] and shown in Proposition 3, the optimal contract with worker asymmetric information yields overemployment since the employment states are less risky than the unemployment states. Again, employment occurs iff $d \geq \theta$.

When we consider the bilateral asymmetric information case, the argument concerning the role of d discussed above holds here as well. Hence, the contract yields underemployment just as in the previous case. Intuitively, with dominant strategy implementation and the *ex post* constraints it only takes the "veto" of one party to set $\ell = 0$. Though workers may wish to create more employment states by increasing d , this only makes it more likely that the firm will not wish to employ any workers.

Hence, these two examples both lead to underemployment. Of course, the difference in preferences will lead to alternative "d's" being chosen.

It should be made clear that this was a very special example in terms of the preferences of the workers, the use of a dominant strategy equilibrium and the existence of *ex post* constraints. While one can argue that the *ex post* constraints are reasonable given the voluntary nature of labor exchanges, they do play a strong role in the analysis.

We now turn to an alternative specification of preferences to look at the difference between dominant strategy and Bayesian implementation. The Bayesian equilibrium concept has been explored most extensively in the public goods literature where separable preferences, linear in the unknown parameter, are often utilized. This linearity is very important in characterizing the solution to the problem and will be used in this analysis as well.[13]

Assume that worker's preferences are represented by $U(\omega) - \theta g(\ell)$ as in Example 1 of the previous section. Furthermore, suppose that $\tilde{\theta}$ takes on the two values θ_1 and θ_2 with $\theta_1 < \theta_2$ and that $\tilde{s}$ also has only two realizations $s_1 < s_2$. Since leisure is a normal good for separable preferences, the firm asymmetric information problem yields overemployment when s_2 occurs and efficiency when s_1 occurs.[14] As noted earlier, the worker asymmetric information problem yields underemployment when θ_2 occurs and efficiency when θ_1 occurs. Hence, in the bilateral asymmetric information problem of maximizing (9) subject to incentive compatibility, we would expect there to be a combination of these results. The following proposition from Cooper [1982] confirms this intuition.

Proposition 4. In the solution to (9), for either dominant strategy or Bayesian implementation,

(i) efficiency occurs in state (s_1, θ_1)

(ii) overemployment occurs in state (s_2, θ_1)

(iii) underemployment occurs in state (s_1, θ_2)

and (iv) the distortion is ambiguous in state (s_2, θ_2).

Proof. See Cooper [1982].

There are two important aspects of this proposition. First, if the states are equiprobable, then the dominant strategy and Bayesian solutions will be identical. When the states are not equiprobable, then the solutions diverge and the Bayesian equilibrium yields higher expected utility for both parties since the set of implementable contracts is larger in the Bayesian case. Secondly, the ambiguity when (s_2, θ_2)

occurs is explained by the fact that the two unilateral asymmetric information cases lead to opposing conclusions on the employment distortion. As discussed in Cooper [1982] whether or not we get underemployment in state (s_2, θ_2) will depend on the variability of s relative to that of θ. If s_2 is close to s_1 then underemployment will occur while if θ_2 is close to θ_1 then overemployment will occur.

Finally, if the firm is risk neutral and worker's preferences satisfy

$$\omega - g(\ell,\theta) \tag{10}$$

where $g_\ell > 0$, $g_\theta > 0$ and $g_{\ell\theta} \geq 0$, then, following Riordan [1982], we can implement the full-information employment rule when s and θ are independent <u>or</u> when these variables are positively correlated.[15] Though Riordan considers a sequential equilibrium, the results hold for the weaker Bayesian equilibrium as well.

Conclusion

The main result of this paper is that the normality of leisure implies underemployment when worker's characteristics are unobservable by the firm. We also demonstrated that when leisure demand was independent of income, either underemployment or overemployment could occur. The type of distortion that occurs when leisure is an inferior good remains an open question.

We also discussed, on a more preliminary basis, the problem of bilateral asymmetric information. The general solution to this problem is quite difficult and will receive more attention in future research. The examples discussed in the paper showed that the case for underemployment may be strengthened if both parties can "veto" an employment state. The example of separable preferences provides a comparison of the two equilibrium concepts.

There are at least two additional problems to consider as extensions of this analysis. First, can we use models of worker and/or bilateral asymmetric information to provide insights into macroeconomic phenomena? This, of course, requires an argument that aggregate shocks are private information to one of the parties to the contract. Secondly, in the bilateral asymmetric information extension of the Grossman-Hart model, we see that severance pay will be independent of the cause of the separation. Given that governments generally pay unemployment insurance to workers who are fired and not to those who quit, it would be interesting to add the government as a third-party to the labor contract and investigate its role in the separation decision.

APPENDIX A: Proof of Proposition 2

To prove Proposition 2, we solve the Hamiltonian problem associated with (7). *Ex post*, the worker announces $m(\theta)$, when θ is the true state of nature, where $m(\theta)$ solves

$$\max_{m(\theta)} U(\omega(m(\theta)), \ell(m(\theta)), \theta) \tag{A.1}$$

Incentive compatibility requires that $m(\theta) = \theta$ for all θ. This implies (see the related discussion in Green-Kahn) that

$$U_\omega \frac{d\omega}{d\theta} + U_\ell \frac{d\ell}{d\theta} = 0 \quad \text{at} \quad m(\theta) = \theta \tag{A.2}$$

and
$$\frac{d\ell}{d\theta} \leq 0 \ . \tag{A.3}$$

(A.3) follows from the second-order conditions to (A.1) due to the single-crossing property of preferences. We will focus on separating solutions so that (A.2) is the incentive compatibility constraint for the problem.[16]

Following standard methodology (see, for example, the presentation in Kamien-Schwartz [1981]), we write the Hamiltonian as

$$H(\omega,\ell,\theta,k) = p(\theta)[(\ell(\theta) - \omega(\theta)) + \lambda U(\omega(\theta), \ell(\theta), \theta)] + \phi(\theta)\left[\frac{-U_\ell}{U_\omega}k\right] + \eta(\theta)k \ . \tag{A.4}$$

Here $k \equiv d\ell/d\theta$ and is the control variable. $\phi(\theta)$ and $\eta(\theta)$ are the costate variables and $p(\theta)$ is the probability that θ occurs. Constraint (A.2) defines $d\omega/d\theta$ as a function of k and the resulting substitution has been made.

The necessary conditions for optimality (where $\dot{x} \equiv dx/d\theta$) are

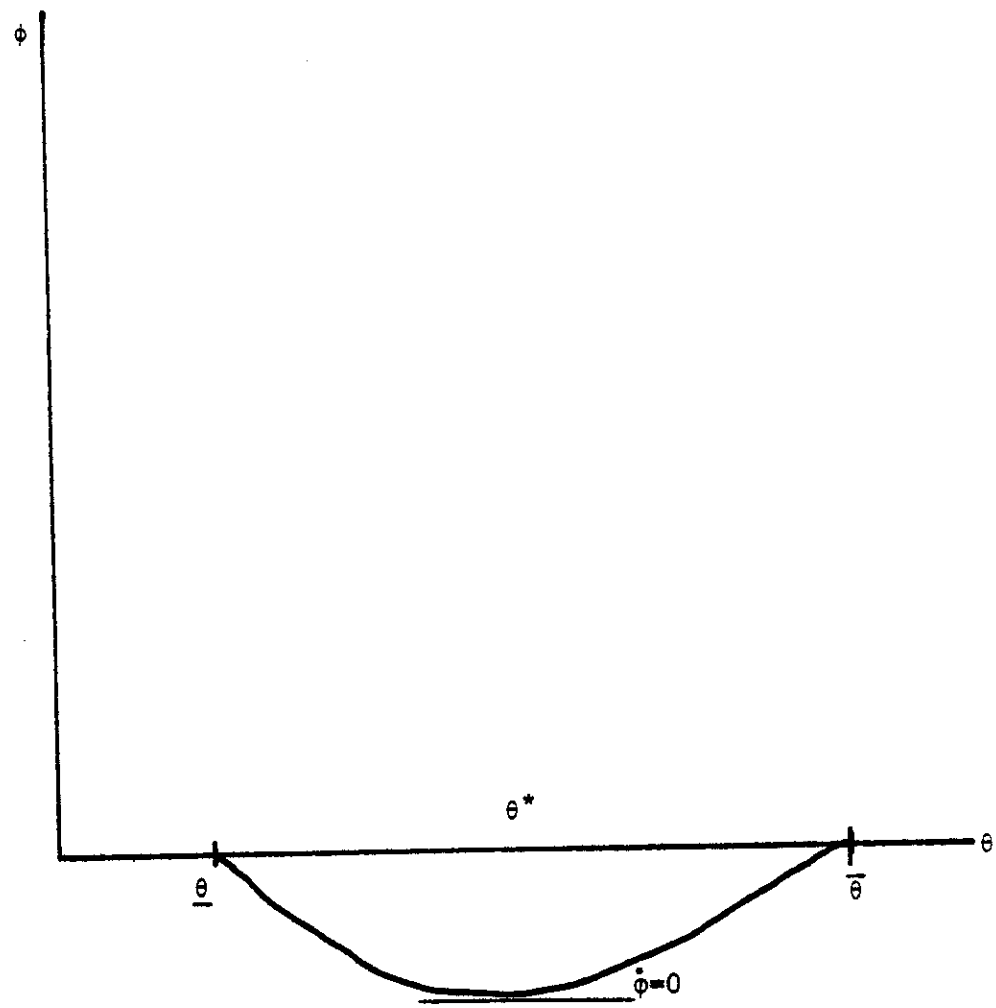


FIGURE 7

$$\phi\frac{U_\ell}{U_\omega} = \eta \tag{A.5}$$

$$\dot{\phi} = p(1 - \lambda U_\omega) - \frac{\phi k}{U_\omega}\left(U_{\omega\omega}\frac{U_\ell}{U_\omega} - U_{\omega\ell}\right) \tag{A.6}$$

$$-\dot{\eta} = p(1 + \lambda U_\ell) + \frac{\phi k}{U_\omega}\left(U_{\omega\ell}\frac{U_\ell}{U_\omega} - U_{\ell\ell}\right) . \tag{A.7}$$

Total differentiation of (A.5) and substitution of (A.6) and (A.7) yields

$$p(1-G) = \phi G_\theta \tag{A.8}$$

where $G \equiv -U_\ell/U_\omega$ and $G_\theta > 0$ by the single crossing property. Hence if $\phi > 0$, $G < 1$ and we have underemployment while the opposite holds for $\phi < 0$. So, to prove the proposition we need to show that when leisure is normal, $\phi > 0$.

Assume that leisure is normal, i.e. $U_{\omega\omega}(U_\ell/U_\omega) - U_{\omega\ell} < 0$, but suppose to the contrary, that $\phi(\theta) \leq 0$ for all θ . Since $\phi(\theta)$ is a continuous function of θ (again see Kamien-Schwartz), there will exist a θ^* such that $\phi(\theta^*) < 0$ <u>and</u> $\dot{\phi}(\theta^*) = 0$ as shown, for example, in Figure 7. From (A.6), when $\theta = \theta^*$, $[1 - \lambda U_\omega(\omega(\theta^*), \ell(\theta^*), \theta^*)] > 0$ as $k < 0$ by incentive compatibility.

The transversality conditions imply that $\phi(\underline{\theta}) = \phi(\overline{\theta}) = 0$. If $\phi(\theta) \leq 0$ for all θ , then $\dot{\phi}(\underline{\theta}) < 0$. From (A.6), this implies that $(1 - \lambda U_\omega(\omega(\underline{\theta}), \ell(\underline{\theta}), \underline{\theta})) < 0$.

Hence,

$$U_\omega(\omega(\underline{\theta}), \ell(\underline{\theta}), \underline{\theta}) > U_\omega(\omega(\theta^*), \ell(\theta^*), \theta^*) . \tag{A.9}$$

The normality of leisure also implies that $U_\omega(\omega(\theta), \ell(\theta), \theta)$

must increase with θ . Total differentiation of U_ω yields

$$\frac{dU_\omega}{d\theta} = U_{\omega\omega}\frac{d\omega}{d\theta} + U_{\omega\ell}\frac{d\ell}{d\theta} + U_{\omega\theta}$$

$$= k\left(U_{\omega\omega}\left(-\frac{U_\ell}{U_\omega}\right) + U_{\omega\ell}\right) + U_{\omega\theta} > 0 \qquad \text{(A.10)}$$

since $d\omega/d\ell = -U_\ell/U_\omega$ from (A.2). With $k < 0$ from incentive compatibility and $U_{\omega\theta} \geq 0$ by assumption, when leisure is normal, $dU_\omega/d\theta > 0$.

Since $\underline{\theta} < \theta^*$, (A.9) and (A.10) are inconsistent so that if leisure is normal, ϕ can not be negative for $\theta \in (\underline{\theta},\overline{\theta})$ as we assumed. This argument can be used to also show that when leisure is normal $\phi(\theta)$ can never be negative. Hence $\phi(\theta) > 0$ for $\theta \in (\underline{\theta},\overline{\theta})$ when leisure is normal so that underemployment occurs. Since $\phi(\underline{\theta}) = \phi(\overline{\theta}) = 0$, we have efficiency in these states.

APPENDIX B: Proof of Proposition 3

To prove Proposition 3, we follow the steps of Appendix A for the preferences of $U(\omega - h(\ell,\theta))$. Using $p(\theta)$ as the density function for θ , the optimal contract

$$\underset{\omega(\theta),\ell(\theta)}{\text{maximizes}} \int_{\underline{\theta}}^{\overline{\theta}} [\pi + \lambda U(\omega - h(\ell,\theta))] \tag{B.1}$$

$$\text{subject to } \frac{d\omega}{d\theta} - h_\ell \frac{d\ell}{d\theta} = 0 \text{ for all } \theta .$$

We can convert (B.1) into a Hamiltonian by letting $\omega(\theta)$ and $\ell(\omega)$ be the state variables and defining $k \equiv d\ell/d\theta$ as the control variable. We define the Hamiltonian as

$$\begin{aligned} H(\omega,\ell,\theta,k) = {} & p(\theta)[(\ell(\theta) - \omega(\theta)) + \lambda U(\omega(\theta) - h(\ell(\theta), \theta))] \\ & + \phi(\theta)[h_\ell k] + \eta(\theta)k \end{aligned} \tag{B.2}$$

where $\phi(\theta)$ and $\eta(\theta)$ are the costate variables.

The necessary conditions for an optimal solution are

$$-\phi h_\ell = \eta \tag{B.3}$$

$$\dot{\phi} = p(1 - \lambda U') \tag{B.4}$$

$$\dot{\eta} = p(\lambda h_\ell U' - 1) - \phi k h_{\ell\ell} . \tag{B.5}$$

Here the arguments have been omitted for convenience and $\dot{x} \equiv dx/d\theta$. Totally differentiating (B.3) yields

$$-\dot{\phi} h_\ell - \phi h_{\ell\ell} k - \phi h_{\ell\theta} = \dot{\eta} . \tag{B.6}$$

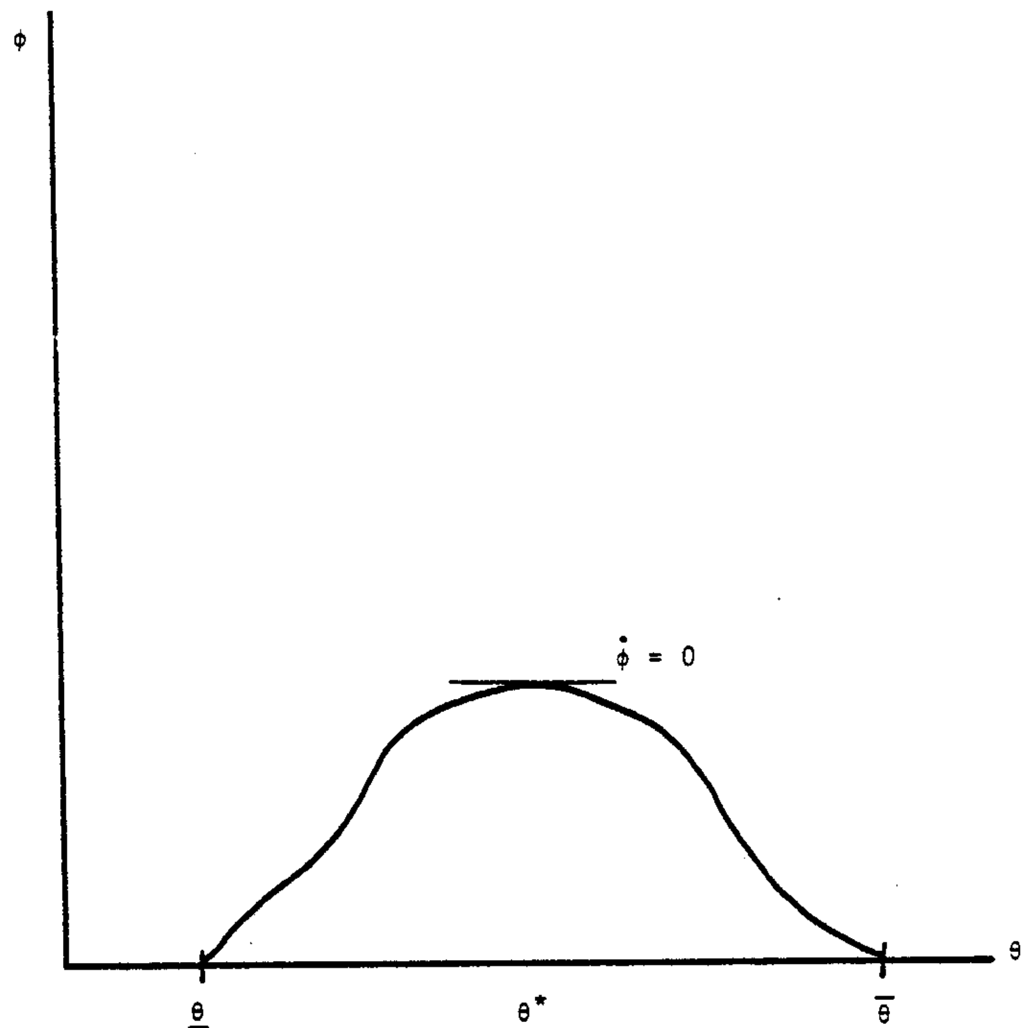


FIGURE 8

Substitution of (B.4) and (B.5) implies that

$$\phi h_{\ell\theta} = p(1 - h_\ell) \; . \tag{B.7}$$

For these preferences $h_{\ell\theta} > 0$ by the single-crossing property. Furthermore $G = h_\ell$ so that $h_\ell < 1$ implies underemployment and $h_\ell > 1$ implies overemployment. From (B.7) we see that the sign of ϕ will determine the direction of this distortion as in Appendix A.

Total differentiation of (B.4) yields

$$\ddot{\phi} = \dot{p}(1 - \lambda U') + p\lambda U'' h_\theta \; . \tag{B.8}$$

We will use (B.8) to construct an argument similar to that used in Appendix A to connect the sign of h_θ to the sign of ϕ as stated in the proposition. To do so we note that at $\dot{\phi} = 0 = p(1 - \lambda U')$, $\ddot{\phi}$ has the opposite sign of h_θ . If $h_\theta > 0$ globally, then $\ddot{\phi} < 0$ when $\dot{\phi} = 0$ as shown in Figure 8. (Here we also use the transversality conditions of $\phi(\underline{\theta}) = \phi(\overline{\theta}) = 0$.) Hence $h_\theta > 0$ implies that $\phi > 0$ so that $h_\ell < 1$ and underemployment occurs. When $h_\theta < 0$, then $\phi < 0$ and overemployment occurs. Of course, if h_θ changes signs then this argument will not work and we shall observe intervals of overemployment and underemployment.

FOOTNOTES

[1]This has not been formalized but is rather a conjecture based on the previous research in this area.

[2]These results require additional assumptions on preferences discussed in the next section.

[3]Here, and in the sequel, G_x is the partial derivative of G with respect to x. See, for example, Riley [1979] and Cooper [1983b] for a discussion of the single-crossing property. Later on we drop the assumption of $U_{\omega\theta} \geq 0$.

[4]We assume that $\bar{\ell}$ is large enough so that an interior solution exists.

[5]For a discussion of implementation under asymmetric information see Harris-Townsend [1981] or Myerson [1979].

[6]This tangency condition is obviously necessary. It is also sufficient given the single-crossing property and monotonicity of worker's preferences. See Appendix A for a complete discussion of implementation.

[7]The intuition behind these results is presented in the following section. Also see the related discussion in Kahn [1982] of severance pay under asymmetric information about employees' outside opportunities.

[8]By continuity, these results will also hold for a slightly risk averse firm.

[9]See the discussion in Cremer-McLean [1982] and Dasgupta, Hammond and Maskin [1979].

[10]I am grateful to Oliver Hart and an anonymous referee for suggesting that I use these special preferences as an example.

[11] This case is not covered by Proposition 2 since we have allowed the firm to be risk averse. However, the proof of this result follows Grossman-Hart [1981] and will not be presented.

FOOTNOTES

(continued)

[12] I am grateful to Lorne Carmichael for referring me to the paper by Hashimoto and Yu [1980] who use a similar diagram in understanding quits and dismissals in a related problem.

[13] See, for example, d'Aspremont and Gérard-Varet [1979] and Laffont-Maskin [1977] which utilize the separability and some linearity.

[14] See Green-Kahn [1983] or Cooper [1982].

[15] As shown by Riordan, $\tilde{s}$ and $\tilde{\theta}$ must satisfy a monotone likelihood ratio condition.

[16] Here we follow Green-Kahn and assume that A.3 is a strict inequality. While some pooling may be optimal, it will not change the distortions arising in the optimal contracts.

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