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FORWARD RATES AND FUTURE POLICY:
INTERPRETING THE TERM STRUCTURE OF INTEREST RATES

by

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I Introduction

We were asked to discuss factors influencing the level of interest rates (why are interest rates so high now?) and secondly to discuss factors influencing the slope of the term structure (why are long rates so high relative to short rates?). Perhaps the implication of the order of these questions is that the theory of the term structure is of secondary importance, a refinement of the theory of the level of interest rates. Many people seem to have the idea that the theory of the term structure is an academic theory about which too much has been written. Perhaps the generally benign slope of the observed term structure contributes to this view.

We have a very different view. The Federal Funds rate is where it is only because the Fed is setting it there.¹ Any questions regarding its level today should be directed to professional Fed watchers who specialize in understanding their current thinking. When other factors, such as projected future Federal deficits or possible future international debt crises are introduced into a discussion of the level of interest rates, then the discussion has inherently turned to the level of longer rates and hence to the shape of the term structure. It is not "academic" to compare the pattern of forward interest rates with the supposed timing of the future events.

¹ The Fed can be ascribed responsibility for the level of the Federal Funds rate regardless of whether it is directly targeting reserves or the Funds rate. While the Fed has not officially abandoned its nonborrowed reserves operating procedure adopted in October 1979, we document below a sharp decline in the volatility of the Funds rate since May 1982. Some observers have even claimed that the Fed pursued interest rate stabilization even during the period of high volatility following October 1979.

It has been noted by many authors (for example Cornell [1983], Engel and Frankel [1982], Roley [1982], and Urich and Wachtel [1982]), as well as in the popular press, that in the period following the October 1979 change in Federal Reserve operating procedures, interest rates of all maturities tended to rise when the Federal Reserve announced a higher than expected money stock. Should we interpret this interest rate response to a higher than expected money stock as implying that the Fed will push the Funds rate up in the future, possibly in response to higher future inflationary expectations? Such a theory implies that new information about the projected path of future short-term interest rates is encoded in the response pattern of the term structure.

Recent events have brought about major changes in projected Federal budget deficits. The deficit for fiscal 1983 is now estimated at about one-half the upper limit of the target range announced by the Fed for the growth in total non-financial debt. Ought the large future deficits to have the ultimate effect of greatly increasing future interest rates when the recession is over and thus of tilting up the term structure today?

Bond market participants and observers have suggested that the world debt crisis may be responsible for high long-term interest rates. Is it plausible to suppose that the slope of the term structure reflects information about potential crises and Fed response to them at future dates?

Interest in such questions has been stimulated by a general perception that long-term rates are "too high". What seems to be meant

by this is that long-term interest rates are unusually high given the recent behavior of short-term interest rates (and other variables, such as inflation, which affect long rates). A standard equation which has been used to forecast long bond rates is estimated in Table 1 for the period 1955:1 to 1979:3. The equation is similar to the one estimated in Table 2 Row 3 of Modigliani and Shiller [1973], which has since been used as a structural equation in the MIT-Penn-Social Science Research Council (MPS) model of the United States economy.² The equation was motivated by the idea that distributed lags on both short-term interest rates and inflation rates might represent expectations of the long run path of future interest rates. The out-of-sample prediction errors of the equation are reported in Table 1, and indeed long-term rates have been very substantially higher than predicted in the last two years.

Of course, the interpretation of the equation as structural is subject to the criticism that coefficients in the equation may change when policy changes. This is particularly relevant if the equation is thought to be summarizing rational expectations of future one-period interest rates. Then whenever there is a change in the optimal forecasting equation for short rates, the long rate equation should also change. As we have already noted, there are several ways in which the process generating short rates may recently have changed.

² The major difference is that the equation reported here does not take any account of risk effects. This has no substantial effect on the coefficients.

Thus even under the simple rational expectations interpretation, the recently high level of long-term interest rates is a phenomenon for which there is an overabundance, rather than a shortage, of possible explanations. In this paper we do not endorse any one candidate. Rather, we assess arguments which have been used to eliminate certain explanations.

The most important of these is the expectations theory of the term structure, which confines attention to the forecasting process for short interest rates. The expectations model has been used as a workhorse for many policy discussions. While practitioners have incorporated risk factors in the form of a constant or even slowly moving "risk premium", they have not focussed upon changes in risk as the primary interpretation of interest rate phenomena. Thus, changes in the shape of the term structure are still understood as reflecting a changed outlook for future interest rates. The model says that even though policy authorities can accurately control short-term rates, long-term rates can not be directly affected by changes in current policy. The setting of policy instruments today will change the yields on longer maturity assets only insofar as they influence a long weighted average of expected future policies.

The simple expectations theory, in combination with the hypothesis of rational expectations, has been rejected many times in careful econometric studies (for example Roll [1970], Sargent [1972], Shiller [1979], and Hansen and Sargent [1981]). But the theory seems perennially to reappear in policy discussions as if nothing had happened to it. It is uncanny how resistant superficially appealing theories in

economics are to contrary evidence. We are reminded of the Tom and Jerry cartoons that precede feature films at movie theatres. The villain Tom the cat may be buried under a ton of boulders, or blasted through a brick wall (leaving a cat-shaped hole), or flattened by a steam roller. Yet seconds later he is up again plotting his evil deeds.

Apparently those who are interested in practical policy discussions feel that there is an element of truth to the theory which survives all of the attacks. One of our objectives here is to help the reader formulate an opinion about the usefulness of the simple expectations model. For this purpose we shall compare the model with an alternative which we call the "tail wags dog" theory. This says that long-term interest rates may overreact to information relevant only to short-term rates.³

In the following section, we develop a linear analytical framework in order to make use of the simple expectations model of the term structure. Our linearization is important because there exist multiple nonlinear representations of the expectations theory.⁴ Rather than choose one version over another, we employ a linearized model which approximates all the nonlinear models. Despite the existence of a

³ According to Arrow [1982], research in both psychology and economics suggests the pervasiveness of such behavior.

⁴ For example, the theory that expected one-period holding period yields are equal on bonds of all maturities is different from the theory that all one-period forward rates equal expected future one-period spot rates. Except under perfect certainty, these and several other variants of the expectations theory contradict each other because mathematical expectations cannot be passed through nonlinear functions. For a detailed analysis of the role of nonlinearities in the expectations theory of the term structure, see Cox, Ingersoll, and Ross [1981].

vast literature on the term structure, the simple linearized expectations model of a term structure of coupon bonds has never been completely described. We take the opportunity here to fill this void and to suggest how well the linearization fares with the recent higher volatility of interest rates.

The linearized expectations model will then be used to interpret the effects of money stock announcements on long bond rates. The linearized model enables us to specify more accurately than heretofore how the effect of money surprises is distributed across forward rates at various horizons and maturities. In addition, the model allows us to ask whether the response of forward rates has been appropriate or whether the market has over- or under- reacted to the announcements. Thus we can compare the simple expectations theory with a model of the "tail wags dog" variety in which long rates overreact to money announcements.

Discussion of other influences on the term structure does not have the benefit of regular announcement data. We shall however present some exploratory work regarding the effects of other factors: credit volume, the increased volatility of interest rates, and the declining duration of long bonds with a higher level of interest rates.

II A Linearized Model of the Term Structure of Interest Rates

In this section we present new expressions for linearized holding period yields over all intervals for bonds of all maturities and linearized forward rates of all maturities applying to all future time periods. Our approach to computing forward rates is different from the existing state of the art approach, proposed by McCulloch [1971], in that our forward rates are yields to maturity on coupon bonds rather than pure discount bonds. Almost all longer term debt instruments pay coupons, and it is important to take this structure of payoffs into account. For example, our forward rates may be compared directly with subsequent spot yields to maturity on coupon bonds as quoted in financial markets. The inability to make such comparisons with conventionally defined forward rates has hampered previous work. Our new formula to compute approximate forward rates is linear in yields to maturity, which links it to a linear expectations model of the term structure. The derivations of the linearizations are straightforward but messy. These and other details are available from the authors in a technical appendix.

The standard linearized expectations model for discount bonds expresses the yield on an i -period discount bond as an unweighted average of expected future yields on i 1-period discount bonds. This model can be described as having a "flat" weighting scheme. The model which we outline below for coupon bonds expresses the yield on an i -period coupon bond as a weighted average of expected future 1-period rates, where rates further in the future are given less weight. The reason for this "declining" weighting scheme is that part of the value

of a coupon bond is derived from coupon payments which will be made in the near future. The coupon bond can be thought of as a package of discount bonds, only one of which has the full maturity of the coupon bond.

The basic model that we shall consider is given in Shiller [1979], [1981]:

$$(1) \quad R_t^{(i)} = \sum_{k=0}^{i-1} W(k) E_t R_{t+k}^{(1)} \quad \text{where } W(k) = g(1-g)^k / (1-g^{i-k}), \quad 0 < g < 1.$$

Here, $R_t^{(i)}$ is the yield to maturity on an i -period bond at time t , E_t denotes mathematical expectation conditional on publicly available information at time t , $W(k)$ ($k=0, \dots, i-1$) are "weights", and g is a constant discount factor.⁵ We will write the discount rate associated with it as \bar{R} ; then $g=1/(1+\bar{R})$. For expositional simplicity, we have left out of (1) any risk premium, although it is customary to modify (1) for empirical work by adding to the right-hand side of the equation a term, $+V_i$, where the risk premium term V_i may depend only on i and is constant through time.⁶ With this simplification, the i -period rate is a weighted average of expected one-period rates. As described intuitively above, the weights decline monotonically in k and sum to 1 [$W(0) + W(1) + \dots + W(i-1) = 1$]. The weighting structure is of the truncated exponential or truncated Koyck variety. For large k

⁵ In the expressions in the text, interest rates are quoted as a rate per period, not percent. Thus, for example, if the time period is monthly and the one-year treasury bill rate is 6% then $R_t^{(12)} = .005$. In the tables, however, rates are quoted in percent per annum. In all expressions superfluous parentheses are used to distinguish superscripts from exponents.

⁶ The constant "risk premium" is due to Hicks [1939].

(very long bonds) the truncation is so far in the future that we can disregard the truncation and for perpetuities ($i=\infty$) there is no truncation.

Equation (1) can most easily be understood by use of the concept of duration (Macaulay [1938]), which is intended as a better measure of how "long" a bond is than its time to maturity. The duration of an i -period bond with yield to maturity \bar{R} is defined by

$$D_i = (gc_i + 2gc_i + \dots + ig_i c_i + ig_i) / (gc_i + g^2 c_i + \dots + g^i c_i + g^i)$$

where $g=1/(1+\bar{R})$, c_i is the coupon rate of the bond (as a fraction of the principal repaid at maturity), and the denominator is the price of the bond as a fraction of par. Thus the duration of a pure discount bond is its time to maturity but the duration of a coupon bond is less than its time to maturity, reflecting the coupon payments which are made earlier. The higher the level of interest rates for a given maturity i the more the future is discounted and thus the shorter the duration. We shall speak here of the duration of an i -period bond as that of a par bond of maturity i , that is a bond whose coupon c_i is \bar{R} . Then, from Macaulay's formula with $c_i = \bar{R}$ we have

$$(2) \quad D_i = (1-g) / (1-g) \quad \text{where } 0 \leq i$$

It then follows that the model (1) makes the i -period yield $R_t^{(i)}$ equal to the present value, discounted by \bar{R} , of future 1-period rates over the maturity of the bond divided by the duration D_i .⁷ Alternatively, (1) may

⁷ The mean of the distribution $W(k)$ defined in (1) is $g/(1-g) - ig^{**i}/(1-g^{**i})$. For large i , this mean is approximately equal to $D_i - 1$. The difference of one arises because the summation in (1) is from 0 to $i-1$ rather than from 1 to i . For small i , the mean is ap-

be described as setting the i -period rate to a duration weighted average expected future interest rates of any maturities which cover the period t to $t+1$ (see Section III below).

The above model is a good approximation to the various expectations models of the term structure if interest rates are not so variable that nonlinearities become important. That is, we suppose $R_t^{(i)}$ lies in the vicinity of \bar{R} for all i and t and that the bonds carry coupons, in periods $t+1, t+2, \dots, t+i$, at a rate (coupon over principal) c_i which is in the vicinity of \bar{R} . We denote a j -period holding yield for an i -period bond $(i>j) H_t^{(i,j)}$. This is the yield (expressed as a rate per period) to buying an i period bond at time t and selling it at time $t+j$ when the bond has become an $i-j$ period bond. The holding-period yield is computed as the yield to maturity of an asset for which one pays the price of an i -period bond at time t , receives the coupons of the i -period bond and is finally paid the "principal" at time $t+j$ which is the price of the bond at time $t+j$. The holding-period yield depends, therefore, on $R_t^{(i)}$, $R_{t+j}^{(i-j)}$, and the coupon on the bond. If, however, the implicit expression for $H_t^{(i,j)}$ is linearized around \bar{R} for all arguments of the expression (i.e. if we take a Taylor expansion of $H_t^{(i,j)}$ in terms of $R_t^{(i)}$, $R_{t+j}^{(i-j)}$, and c_i all around \bar{R} truncated after the linear term) we get a simple approximate holding period yield:

$$(3) \quad h_t^{(i,j)} = \frac{D_t^{(i)} - (D_t - D_{t+j}) R_{t+j}^{(i-j)}}{D_j} \quad 0 < j \leq i$$

proximately $(D_i - 1)/2$.

where D_i and D_j are the i - and j -period durations given by (2). Note that the coupon rate drops out of the expression altogether. In the implicit formula for holding period yields coupon effects are of "second order". Also note that expression (3) is homogeneous of degree 1 in $R_t^{(i)}$ and $R_{t+j}^{(i-j)}$, i.e. the constant term drops out of the expression. Note finally that in the special case $j=1$, i.e. when the holding period is one period long, the expression reduces by substitution of (2) into (3) to that in Shiller [1979].

The m -period forward rate applying to period $t+n$ is computed from the term structure of interest rates at time t . If we can both borrow and lend at the rates given in the term structure then it is possible to arrange a portfolio which guarantees for us a price of a bond at time $t+n$ maturing at time $t+m+n$. The procedure is to buy at time t an $m+n$ period bond and to issue bonds of maturities $1, 2, 3, \dots, n$ so that the total value of the portfolio at time t is zero, and so that the value of the stream of payments on issued bonds exactly equals the coupon received on the $(m+n)$ -period bond over all intervening periods $t+1, t+2, \dots, t+n-1$. The net effect, then, will be to lock in for us a contract to lend at time $t+n$, to receive coupons from $t+n+1$ to $t+m+n$, and be paid back at $t+m+n$.

The yield to maturity on this m -period loan will be called the n -period ahead m -period forward rate $F_t^{(n,m)}$. This forward rate can be computed from $R_t^{(m+n)}$ and $R_t^{(n)}$ as well as all other rates $R_t^{(1)}$, $R_t^{(2)}$, \dots , $R_t^{(n-1)}$ and coupons $c_1, c_2, \dots, c_{n-1}, c_n, c_{n+m}$ of the various bonds. If, however, one linearizes the complicated implicit expression for the forward rate around \bar{R} for all the arguments a simple linearized approximation $f_t^{(n,m)}$ to the forward rate $F_t^{(n,m)}$ results:

$$(4) \quad f_t^{(n,m)} = \frac{D_{m+n} R_t^{(m+n)} - D_n R_t^{(n)}}{D_{m+n} - D_n} \quad 0 < m, 0 \leq n$$

where D_{m+n} and D_n are durations of $m+n$ - and n -period bonds as given by (2). This expression depends only on $R_t^{(m+n)}$ and $R_t^{(n)}$, and not on $R_t^{(1)}$, $R_t^{(2)}$, ..., $R_t^{(n-1)}$, nor on $c_1, c_2, \dots, c_{n-1}, c_n$ or c_{n+m} . The effect of these yields and coupons is again of second order, and drops out of a linearization. Note also that when $m=1$, so that the forward rate is a one-period rate, then using (2) this expression reduces to that in Shiller [1981]. As \bar{R} goes to 0, the bonds become discount bonds, D_m approaches m , D_n approaches n , and we have

$$f_t^{(n,m)} = \frac{(m+n)R_t^{(m+n)} - nR_t^{(n)}}{m} \quad 0 < m, 0 \leq n$$

which for $m=1$ is the conventional linear approximation (e.g. Roll [1970]).

The model (1) then implies:

$$(5) \quad E_t h_{t,t}^{(i,j)} = R_t^{(j)} \quad 0 < j \leq i$$

$$(6) \quad f_t^{(n,m)} = E_t R_{t+n}^{(m)} \quad 0 < m, 0 \leq n$$

Thus, expected linearized holding period yields on all bonds for all holding intervals are equal to the corresponding spot rates and lin-

earized forward rates for all future time periods and all maturities equal the corresponding expected spot rates. Moreover, either (5) or (6) implies (1): that is, subject to the linearizations of forward rates and holding period yields all versions of the expectations theory of the term structure can be reconciled.

Of course, the quality of the linear approximation is an important consideration in judging how well the models have been unified by the linearization. The recent increased variability of interest rates has slightly diminished the accuracy of the linearization. Table 2 shows some data on the quality of approximation over the recent period of volatile interest rates. The point of linearization is generally the mean level of interest rates over the sample period, but in rows 4 and 6 it is varied to the minimum and maximum of the three-month Treasury bill rate over the sample.⁸ This change has little effect on the one-year ahead one-year forward rate or the accuracy of the linearization. It has somewhat greater effect on the linearized twenty-year ahead ten-year yield.⁹

⁸ It might be thought desirable to linearize around the current level of the long-term interest rate at each moment of time. This would produce closer approximations to true holding period yields or forward rates. Unfortunately, such an approach is not compatible with a linear time consistent expectations model of the term structure. Therefore we work with a time-invariant point of linearization throughout this paper, although the point chosen does depend on the particular sample period.

⁹ Varying the point of linearization is equivalent to altering the assumed duration of long bonds. Ando and Kennickell [1983] have recently emphasized that the change in the level of interest rates has altered duration, and thus the behavior of long rates given the path of expected future short rates. We use the linearized model to evaluate this argument below.

One implication of (6) which is important for our purpose is that the sequence of linearized forward rates $[f_{t-n}^{(n,m)}, f_{t-n+1}^{(n-1,m)}, f_{t-n+2}^{(n-2,m)}, \dots, R_t^{(m)}]$ is a martingale, i.e. each element in the sequence equals the expected value conditional on information at that time of the subsequent element in the sequence. However, contrary to a popular misconception, the theory does not imply that long-term interest rates themselves are martingales or random walks. The martingale property of forward rates implies that the change $f_t^{(n,m)} - f_{t+s}^{(n-s,m)}$ over any time interval s of linearized forward rates applying to a particular time $t+n$ should depend only on information coming available between t and $t+s$ and not on any information available before t . If both s and n are very small relative to m then $f_t^{(n,m)} - f_{t+s}^{(n-s,m)}$ is approximately the same as $R_t^{(m)} - R_{t+s}^{(m)}$. Hence a test of the "random walk" character of interest rates by regressing changes of interest rates over very short intervals on lagged information is also, subject to an approximation, a test of this model.

Because of the martingale property, the m -period interest rate can be regarded as the sum of uncorrelated random variables which are changes in forward rates:

$$R_t^{(m)} = (R_t^{(m)} - f_{t-1}^{(1,m)}) + (f_{t-1}^{(1,m)} - f_{t-2}^{(2,m)}) + (f_{t-2}^{(2,m)} - f_{t-3}^{(3,m)}) + \dots$$

According to the expectations theory, if some information variable such as the money supply announcements can explain changes in forward rates well, then it can explain interest rates as well. If a particular money stock announcement has an effect on forward rates on a given day then we do not expect the subsequent changes in forward rates to offset this impact.

Unfortunately, it is easier to test whether the money supply announcements have an effect than that they are not subsequently offset. By concentrating on a very small time interval s (say $s =$ one day over which the money supply announcement is made) it is possible to form a powerful test against the null hypothesis that money supply announcements have no effect. Assessments of such announcement effects, often called "event studies" in finance, are very popular because they often get significant results. They make sense if we really believe that the variable in question is a martingale.

However, lacking information about when the effect might be offset, it does not make sense to single out any short time interval over which subsequent changes in forward rates might be compared with past money stock innovations. A reasonable way of ascertaining whether the effect of money stock movements is only transient would be to regress the sum of all the changes in forward rates between $t-n$ and t (i.e. $R_t^{(m)} - f_{t-n}^{(n,m)}$) on the money stock innovation made known at $t-n$ and we do this below. The problem is that for any but the smallest n the variance of $R_t^{(m)} - f_{t-n}^{(n,m)}$ is much higher than the variance of the overnight change in the forward rate. Thus, such a test will have very little power.

III Effects of Changing Duration on the Behavior of Long Bond Rates

The linearized model presented in the previous section has an immediate application to the term structure equation of Table 1. That equation was motivated by the idea that people's expectations of future short interest rates can be represented by distributed lags on economic variables. The linearized expectations model states that forward rates equal expected future spot rates, and that the long bond rate is a weighted sum of forward rates. This implies that the long bond rate can be written as a linear function of current and lagged variables which are relevant for forecasting short interest rates.

The predictive performance of such an equation could deteriorate for any of several reasons. The forecasting rule for short rates could change, or forward rates could deviate from expected spot rates, or the weighting scheme which translates forward rates into the long bond rate could change. Ando and Kennickell [1983] have recently studied the term structure equation of the MPS model. Since 1977 this equation has displayed a deterioration in fit and in the last two years has underpredicted the long bond rate in the same way as the equation of Table 1. Ando and Kennickell attribute the decline in fit to the last of the three factors mentioned above. They argue that higher interest rates have reduced the duration of long-term bonds, so that a 20 year bond rate today might behave as a 10 year bond rate used to do. Thus they interpret the change as a shift in the relation between long-term bond rates and forward one-period rates, rather than as a shift in risk premia or in the parameters of the forecasting equation for one-period rates. Indeed, the duration of a 25 year bond

as given by (2) above has fallen from 16 years when interest rates were 4% to 9 years when interest rates are 12%.

Ando and Kennickell reestimate the MPS equation over the sample 1955-3 to 1981-4 allowing for a distributed lag on short interest rates whose coefficients depend on short rates in such a way that the distributed lag is shortened when short rates are high. This equation fits recent observations with standard errors of only 10-20 basis points, and has been entered in the MPS model in place of the old equation for the long rate.

While the shorter distributed lag with more recent observations is suggestive that the decline in fit is due to the decline in duration, there is a more direct test of this effect. We constructed linearized forward rates $f_t^{(i_{j-1}, i_j - i_{j-1})}$ ($j = 1, 2, \dots, 6$) from the observed term structure for maturities $i_0, i_1, i_2, \dots, i_6$ of 0, 12, 24, 36, 60, 120, and 240 months. Using the Table 1 sample period 1955:1 to 1979:3, we linearized about the mean of the 20 year bond rate, 5.4% per annum, and then reconstructed a "duration-corrected" 20 year bond rate $R_t^{(240)}$ according to

$$R_t^{(240)} = \frac{1}{D'_{240}} \sum_{j=1}^6 (D'_j - D'_{j-1}) f_t^{(i_{j-1}, i_j - i_{j-1})}$$

where D'_j are computed using the new point of linearization $\bar{R}=12.4\%$ which is the mean 20 year bond rate from 1979:4 to 1982:4.

We reestimated the same term structure equation with $\overset{\circ}{R}_t^{(240)}$ in place of $R_t^{(240)}$ making no other changes in the procedure, and used this to predict $R_t^{(240)}$ out of sample.¹⁰

The results of our procedure are reported in Columns 4 and 6 of Table 1. The duration-corrected equation does perform better in predicting $R_t^{(240)}$ out of sample than the uncorrected equation; its average prediction error in 1981 and 1982 is 125 basis points as compared with 165 basis points for the original equation. In the worst quarter, 1981:4, the duration correction reduces the underprediction by 53 basis points. Nevertheless, it is clear that the change in duration is insufficient to explain the recent altered behavior of long-term interest rates. This follows from the fact that the term structure has not consistently had a steep downward slope in the last three years. Such a slope is necessary if a reduction in duration is to raise the long term rates constructed from a given set of forward rates. Thus the change in the behavior of long bond rates must be due in part to a change in the relation of current and lagged short rates to forward rates. Such a change could occur either because of a shift in the best forecasting rule for short interest rates, or because of risk effects on forward rates given expected future spot rates.

¹⁰ Note that if we had used the same \bar{R} to compute D and D' , then $\overset{\circ}{R}_t$ would equal R_t , and we would have obtained the same result as from the straightforward estimation of the Table 1 equation.

IV Evaluating the Linearized Expectations Model

We are interested here only in describing whether there is an element of truth to the simple expectations model of the term structure which allows for a constant (through time) risk premium for each maturity. As we have noted, such a model is widely used to interpret current events. Since the power of empirical econometrics is limited, the alternative of confirming a structural model explaining time variations in risk premia, however desirable, may be unrealistic.

The model (1) has already been extensively tested, in effect, for short interest rates. With short maturity bonds, the decay of the weighting scheme in (1) is sufficiently small that the difference between $W(k)$ in expression (1) and a flat weighting scheme $W(k)=1/i$, $k=0,1,\dots,i-1$, might be disregarded. Moreover, for obligations issued with maturity of no more than a year there are generally no coupons, so a flat weighting scheme is appropriate anyway. Rigorous empirical work on an expectations model regarding much longer maturities (10, 20, or 30 years), on the other hand, is relatively scarce, and for these maturities a flat weighting scheme would be highly inappropriate. It is important to distinguish tests of the model based on short maturities from tests based on longer maturities, since it is plausible that the expectations theory might work much better for the shorter maturities.

Rather than survey the extensive literature on the term structure, we will instead merely offer a simple display illustrating the predictive content of the simple expectations model. Does a term structure which, after correcting for a constant liquidity premium,

slopes upward for a higher maturity actually portend higher interest rates for the future? Does the result depend on how far in the future we are looking? The answer to this simple question has not been emphasized in the empirical literature on the term structure.¹¹ We will show here that changes in interest rates do not bear a positive relation to the predicted change and that, as Macaulay [1938] first noted, long rates tend to move in the opposite direction from the predicted change.¹²

A few preliminary remarks are in order about the relation of our work to previous research on the term structure. If we have all the available high quality data for the United States, then we do not need to refer to earlier work; we can run our own tests, which will then benefit from a longer sample period. For our displays involving three and six month treasury bill rates, we are using all the first of the month data available in series form from the Salomon Brothers source An Analytical Record of Yields and Yield Spreads, but we did not make use of additional daily data within the month. We have checked our

¹¹ Much of this literature has, in effect, asked whether a secular increase in short rates has been matched by a similar increase in long rates. The most common test of the model is to regress a spot rate on an appropriately lagged forward rate. But this test has low power against the plausible alternative hypothesis that both short and long interest rates have followed comparable trends. The question noted in the text was studied by Shiller [1979], but for long rates only. We have not been able to find any careful evaluation of this question in the literature, except in a few brief responses to the Shiller [1979] paper.

¹² Macaulay did not document this fact or emphasize it. He thought his more important observation was just the low correlation between forward rates and subsequent spot rates. He did note (p. 33) that: "The yields of bonds of the highest grade should fall during a period when short-term rates are higher than the yields of the bonds and rise during a period in which short-term rates are lower. Now experience is more nearly the opposite".

data against analogous data available for a shorter time interval from the Federal Reserve Board. Earlier studies have not generally exploited richer data sets and in many cases used inferior data (e.g. annual average data when point sampled data are appropriate). It should be pointed out that some studies have used individual bond data which are more extensive and some studies have explored the term structure in other countries. It is not clear, however, that all such additional data ought to be used when markets for some of the bonds are "thin." Our sample of U.S. Treasury issue yields is selected to avoid such a weakness. We mention as a final caveat that some studies have identified anomalous features of the data which we ignore. For example, Roll [1970] claimed that forward rate changes have fat-tailed probability distributions, which can lead to spurious t-tests from a simple regression analysis. Such a distribution makes it possible to observe a significant positive coefficient with one sample and a significant negative coefficient with another.

With these warnings we may look at figure 1, which shows a scatter diagram with quarterly changes in three-month bill rates $R_{t+3}^{(3)} - R_t^{(3)}$ on the vertical axis against the predicted change $F_t^{(3,3)} - R_t^{(3)}$ implied by the three and six month bill rates. The data are quarterly 1959-1 to 1982-3. Measurements are for the first trading day of March, June, September and December. Our model implies that the error terms are uncorrelated in a regression of $R_{t+3}^{(3)} - R_t^{(3)}$ on a constant and $F_t^{(3,3)} - R_t^{(3)}$ and the theoretical slope coefficient should be one. The intercept reflects a constant risk premium. The line shown is drawn through the sample means with a slope of one and is not the estimated

regression line. The estimated regression line (Table 3 row 1) has a negative slope.

The negative slope is apparently due primarily to recent observations. We did not attempt to correct for heteroscedasticity over the full sample period, since the recent increase in the volatility of interest rates is extreme and does not represent the continuation of a historical trend.¹³ We did adopt a method suggested by Hansen and Hodrick [1980] which allows us to use the full sample of monthly data by correcting the error term for serial correlation.¹⁴ Our application of this "Corrected OLS" estimation method over the 1959-1 to 1982-10 period is reported in Table 3, Row 2. Use of monthly rather than quarterly data has little effect on our point estimates but does increase their precision. Subsequent results with 3-month bill rates in Table 3 are based on monthly data.

To check the conjecture that the expectations model might perform better in the period before the introduction of new Federal Reserve operating procedures in October 1979, we shortened our sample and repeated our regression tests with and without a heteroscedasticity correction (Table 3, Rows 3 and 4). The slope coefficient is now posi-

¹³ Glejser [1969] suggests a simple method for correcting for error variance which follows a steady time trend. The absolute values of the residuals from a preliminary regression are regressed on a constant and time; the reciprocals of the fitted values are used as weights in a second regression. Mishkin [1978], [1983] has applied this method to an earlier sample period. For the full sample 1959-82, we found that it gave almost all the weight to two or three early observations and therefore produced highly erratic results.

¹⁴ Quarterly changes in interest rates "overlap" when sampled monthly. Therefore the error term follows a third-order moving average process, and the estimated coefficient standard errors must take this into account.

tive; but it is insignificantly different from zero and significantly different from one. This result is not altered by further shortening of the sample period to exclude the unusual period of volatile interest rates in 1974-5 (Table 3, Row 5).¹⁵

Figure 2 shows a scatter diagram with changes over a six-month interval in 30 year (360 month) bond yields $R_{t+6}^{(360)} - R_t^{(360)}$ on the vertical axis against the predicted change $(f_t^{(6,354)} - R_t^{(360)})$ implied by the 360-month yield and the six-month yield at the beginning of the semester. The \bar{R} used to compute the linearized forward rates was 6.65% per annum. Once again, our model implies that the error terms are serially uncorrelated in a regression of $R_{t+6}^{(360)} - R_t^{(360)}$ on a constant and $f_t^{(6,354)} - R_t^{(360)}$, and the theoretical slope coefficient should be one.¹⁶ There is thus a sense in which such a simple regression test is the "right" way to test for market efficiency with long-term bonds, though there is also a sense in which "volatility tests" may have more power (Shiller [1981]). The line shown is drawn through the sample means with a slope of one and is not the regression line, which has a negative slope. Thus as before the results of our test

¹⁵The estimates resemble one in a similar regression by Hamburger and Platt [1975] equation 5, who, however, inexplicably conclude in favor of the expectations model. More favorable results for the simple expectations model were found in Shiller [1981], using data on 6 and 12 month 1.5% Treasury notes (Series EA and E0) and a slightly earlier sample. The more favorable results were not due to the slightly different sample or to the slightly different maturities. Using regression diagnostic procedures described by Belsley, Kuh and Welsch [1980], it was found that the results in that paper were heavily influenced by the 1970-I and 1970-II observations. As confirmed by the Commercial and Financial Chronicle for those dates, the yields on these Treasury notes were behaving erratically at that time. As noted in Shiller [1981], quantities of the Treasury notes outstanding fell below \$100 million after 1969, and the "thin" markets made price data less reliable. When the same regressions in that paper were run truncating the sample at 1969-II, they confirmed the negative results for the expectations theory reported here.

oppose the theoretical forecast: when long rates are predicted to rise, they tend in fact to go down.

The predicted six-month change in the 30-year bond rate is approximately the spread between the 30-year bond yield and the six-month rate, divided by the duration (measured in six-month intervals) of a 30-year bond minus one. The estimated coefficient thus depends on the \bar{R} used to compute duration. For example, if we had chosen $\bar{R}=3\%$ rather than 6.65% the duration given by (2) would rise from 14 years to 20 years and the estimated coefficient would be multiplied by $20/14=1.43$. However, changing the duration used could never change the sign of the coefficient.

In this scatter diagram, the variance of the predicted change in long rates is much smaller than the variance of the actual change in long rates, so the power is going to be small against the alternative hypothesis that the slope term is zero. In drawing figure 2, the horizontal axis had to be put on a different scale than the vertical (as evidenced by the flatter appearance of the slope one line shown). Had they been on the same scale, the scatter of points would be so close together horizontally as to be indistinguishable.

The estimated coefficients for 30-year bond rates in Table 2 are always negative, but reflecting the low power of the tests, have extremely large standard errors. Nevertheless, our use of monthly data gives our estimates some additional precision and we are able to reject at the 4.5% level the hypothesis that the coefficient is one in Row 8. This regression corrects for moving average errors but does

¹⁶ We approximated here by ignoring the difference between $R_t^{(354)}$ and $R_t^{(360)}$.

not correct for heteroscedasticity, but when we perform weighted least squares we still reject at the 5.5% level in Row 9. These results confirm for a shorter but more frequently sampled period the regression tests reported in Shiller [1979].

The standard deviation of the actual change in interest rates is about 2.5 times the standard deviation of the predicted change for the short-term interest rates in figure 1 and about 14 times the standard deviation of the predicted change for the long-term interest rates in figure 2. Thus, under the null hypothesis that the true slope coefficient is one the R^2 of the regressions ought to be small: about $1/[(2.5)^2] = 0.16$ for the short rate regression and about $1/[(14)^2] = 0.005$ for the long rate regression. At least for the short rate regression, the R^2 value is large enough that a test of the null hypothesis against an alternative that R^2 equals zero should have some power.

Under the null hypothesis, the standard deviation of the estimated coefficients in an ordinary least squares regression should be about $2.5/[N^{0.5}]$ for the short rate regressions and $14/[N^{0.5}]$ for the long rate regressions, where N is the number of observations. We have 95 observations in figure 1 and 48 observations in figure 2, implying standard errors of roughly 0.25 and 2.0 respectively. Thus, we easily reject in the short rate regression at conventional levels because there is no apparent relation between actual and predicted changes. We also reject at this level for the long rate regressions by switching to monthly observations to get more data and since the estimated coefficient is decidedly negative.

To summarize these results, we do not find evidence in the sample that is encouraging for the hypothesis that the slope of the term structure correctly forecasts of future changes in interest rates. This behavior of long bond rates has straightforward implications for optimal financing decisions of corporations and individuals who are concerned purely with expected returns. For example Row 8 of Table 3 implies that such an investor would prefer to buy 30-year Treasury bonds rather than 6-month Treasury bills whenever the 30-year yield exceeds the 6-month yield by 75 basis points or more.

Over the period 1959 to 1982, such an investor would have been "long" approximately 45% of the time. The slope of the term structure is a sufficiently good predictor of excess returns to overcome the fact that average returns on long bonds have been lower than on short bonds. The current spread of over 200 basis points makes long investments preferable by a considerable margin. By this reasoning, perhaps companies should delay long financing until long rates fall relative to short rates and householders should not switch from floating to fixed rate mortgages until this occurs.¹⁷ It is perhaps surprising only to students of the expectations theory that this advice corresponds to what a naive person might have done without the guidance of a sophisticated model.¹⁸

¹⁷ This prescription is valid only if our results for government securities carry over to private debt.

¹⁸ Some economists have offered suggestions for corporate financing which are designed to achieve tax savings, but which depend crucially on the expectations theory of the term structure. For example, Roger Gordon [1982] suggests that corporations should issue short debt when short rates are high, in order to accelerate tax deductible interest payments. Perhaps it would not be surprising to see few corporate treasurers heeding such advice.

Of course, the participants in the bond market may well be concerned with risk as well as with expected returns, and our rejection of the simple expectations theory could be attributed to variations in "risk premia" which are so considerable as to destroy any information in the term structure about future interest rates. Such an argument may be more in accordance with current theoretical predispositions than one which says markets are "irrational". Lacking any theoretical restrictions on the variation in risk premia, however, we cannot distinguish between these hypotheses.

Two approaches might preserve the relevance of an expectations theory of sorts. One possibility is to model a time varying risk premium in terms of observed data, a method we employ below. Another possibility is to suppose that the risk premium moves slowly enough that extremely short-term movements in yields (as between trading days before and after a money announcement) can still be understood in terms of a simple expectations model. We have no tangible evidence, of course, that the risk premia do not move from one day to the next.

V Time Varying Risk, Interest Rate Variability and Credit Volume

The poor performance of the simple expectations theory might be improved if the model were modified to allow for a time varying risk premium. Suppose we add a risk premium term V_{it} to the model (1). The one-period excess holding return on an i -period bond, $h_t^{(i,1)} R_t^{(1)}$ then equals $D_i V_{it} - (D_i - 1) E_t V_{i-1,t+1}$ plus noise unforecastable at time t . By regressing the difference between $h_t^{(i,1)}$ and $R_t^{(1)}$ on observable and relevant variables, we can then estimate the risk premium. If $i=2$ then $V_{i-1,t+1}$ is zero by equation (1) and the fitted value of the regression divided by D_2 (or roughly 2) may be regarded as an estimated V_{2t} . If i is very large and if we assume $V_{it} = E_t V_{i-1,t+1}$, then the fitted value of the regression is itself an estimate of V_{it} .

A number of earlier studies of the term structure have incorporated as an ad hoc measure of time-varying risk an eight-quarter moving standard deviation of the three-month Treasury bill yield.¹⁹ Row 1 of Table 4 shows that this variable is indeed significant in predicting excess three-month returns on six-month Treasury bills for the sample period 1959-1979. We found that the moving standard deviation was much less successful in predicting excess returns on long bonds (perhaps because of the much higher variance of these returns and their persistent tendency to be negative over the 1959-79 period). Therefore we do not report regressions with this variable for long bond returns.

¹⁹ This tradition seems to have started with Modigliani and Shiller [1973]; the variable has also been used by Ando and Kennickell [1983], Jones and Roley [1982] and Mishkin [1983] among others.

The moving standard deviation proxy for risk is not derived from any well articulated economic theory. In fact, the basic message of standard finance models is that it is not the volatility of asset returns but their covariance with other asset returns or underlying factors which determines their "riskiness". Moreover, we are disturbed by the fact that the risk variable becomes much less significant if the sample period is extended to include the last three years or if we replace a simple moving standard deviation by a predictor of the variance of the innovation in the three-month bill rate.

The weakness of the standard variable which is thought to measure time variations in risk premia suggests that it may be worth seeking alternatives. It is widely believed that the volume of short and long debt issue, the level of rates and the shape of the yield curve are all jointly determined. Economists have long argued about such issues in the form of 'preferred habitat' theories and policy makers have been motivated to employ such ideas in 'twisting' yield curves via government debt management. Recently, Friedman [1977, 1979 and elsewhere] and Roley [1981, 1983 and elsewhere] have pursued a research program which is based on the premise that supply and demand curves for debt are econometrically identifiable. They have sought to isolate factors which shift one curve without affecting the other.

Our more modest goal here is to see if the volume variable predicts risk premia as we have defined them. In Row 2 of Table 4, we regress excess returns on six-month bills on the previous quarter's ratio of short borrowing to long bond issue by U.S. corporations.²⁰

²⁰ This data is ultimately derived from the U.S. Flow of Funds Accounts, but was aggregated for us by Salomon Brothers. Following

This "volume ratio" might be thought to be high when the market perceives greater risk in returns on longer-term assets. The volume ratio is indeed a significant predictor of excess returns over the period 1959-82, and in fact if we include it in a regression along with the moving standard deviation (Row 3) we find that the volume ratio dominates the conventional measure of risk. However, it is not significant over the shorter sample period 1959-79. The volume ratio is significant only at the 10% level in predicting excess returns on long bonds (Row 4). Our success with this crude measure of relative issue volume by maturity suggests that policymakers would benefit from more frequent and accurate collection of data summarizing credit volume.

The fitted values from these regressions give our best estimates of risk premia. How do these estimates change over time? Consider Row 1 of Table 4. The moving standard deviation of annualized three-month bill rates averaged 70 basis points over the 20 year period from 1959 to 1979. In the period 1979-82, it increased from about 140 basis points in early 1979 to a high of over 250 basis points in late 1981 and 1982. According to the regression of Table 4, Row 1, the expected excess holding return on six-month bills should have been 55 basis points higher than the previous average in 1979, and 140 basis points higher in 1982. This implies that the risk premium in the six-month bill rate should have been about 27.5 basis points higher in 1979, and 70 basis points higher in 1982.

the Flow of Funds convention, "short" borrowing is of maturity less than one year, and "long" financing is all other borrowing.

For long bond rates the moving standard deviation proxy for risk is less successful. But we can treat the spread between the 6-month and 30-year rates as a risk proxy, and ask what risk premium was implied by the maximum 1982 spread of 2.5%. The answer to this question is contained in the regression of Table 4, Row 5. There, excess holding returns on 30-year bonds are regressed on the predicted change in 30-year rates. This predicted change variable is just the spread divided by the duration of a 30-year bond (which in the 1959-1979 sample period averaged 13.5 years or 27 6-month time units) minus one. The coefficient of 71, divided by 26, implies that on average when the spread is 1%, the risk premium in the long rate is 2.7%. Given the 1982 maximum spread of 2.5%, our regression suggests that the risk premium peaked at 6.8%. The very large values of the estimated risk premia seem implausible, but of course they are necessary if we are to interpret the term structure in these terms. The risk premium must move enough to more than offset the perverse behavior of the term structure in predicting future long-term interest rates.

Finally, we can ask whether the risk variables of this section "fix up" the expectations model as we had hoped. Rows 6, 7 and 8 of Table 4 represent Rows 3, 1 and 6 from Table 3 with risk variables added.²¹ We hoped that the addition of these variables would bring the coefficient of the predicted change closer to one. In fact, the coefficient is only slightly closer to one for the short rates and farther from one for the long rate.

²¹ Slight sample changes were necessitated by the quarterly measurement of the volume ratio.

VI Money Announcements and the Term Structure of Interest Rates

The tendency for interest rates to rise suddenly after an announcement of an unexpectedly large money stock has been widely noted. Some have interpreted this effect of a "surprise" money announcement as implying that expansionary monetary policy actually raises interest rates through its anticipated impact on inflation rather than lowering them via a liquidity effect as traditional Keynesian theory would predict. In this section we shall describe the money surprise effect and also refine the interpretation of it. We shall argue, for example, that it is quite wrong to interpret the response to money surprises as disproving the Keynesian liquidity effect on interest rates.

In Figure 3 we see displayed on the vertical axis the change in a short term interest rate, versus, on the horizontal axis, the surprise in the money stock for each week between February 1980 and February 1983. In order to calculate the money surprise variable, one needs a measure of the actual and the expected money stock. The money stock selected corresponds to the unrevised narrow measure of money emphasized by the Fed (M1B, renamed M1 in January, 1982). The money stock announcement occurred in our sample on Fridays at 4:10 p.m. The expected money measure employed is the Tuesday median forecast of the money stock from the weekly market survey of Money Market Services, Inc. In the graph, the money surprise is the difference between the actual and expected money stocks.

Several authors, including Cornell [1983] and Roley [1982], have sought to evaluate the efficiency of the Money Market Services forecast. They have shown that the forecast is efficient with respect to

information sets containing the lagged forecasts and current and lagged interest rates. In addition, it generates a lower root mean squared error in prediction than an ARIMA model based on observed money stocks. We do not reproduce these tests but we can confirm that there is no significant serial correlation in the forecast errors.

The vertical axis displays the change in the three-month-T-bill interest rate in basis points from Friday at 3:30 p.m. to Monday at 3:30 p.m. The dark line is a simple regression through the observations. The regression portrayed in this diagram is found Row 3 of Table 5, along with the results for yields ranging from the Federal Funds rate (denoted FF) to the thirty-year Treasury bond yield. The coefficient of the money surprise is measured in basis points per billion dollar surprise, with the standard deviations in parentheses. It is apparent from the table that the money announcement has a significant, albeit declining, effect throughout the entire term structure.²² This reconfirms the work of other authors for our more recent data sample.

Previous authors have stressed the significance of these results. However the R^2 values in the Table 5 regressions do not exceed 0.286. Furthermore, the sample variance of interest rates from Friday to Monday for our sample from 1980;8 to 1983;7 is only one-third of the

²² An alternative measure of the money stock forecast error suggested by Roley was also employed. This involves regressing the Money Market Services forecast error on the change in the 3-month interest rate from Tuesday to Friday (in order to account for new information available between the forecast and the money stock announcement) and to use the residuals from this regression as the exogenous variable in the Table 5 regression. Since the results closely approximate the regressions displayed in Table 5 we do not report them.

weekly variance in interest rates (measured either Friday to Friday or Monday to Monday). Hence, if the yields approximate a random walk over the week, the regressions explain only three to ten percent of the weekly movement in the rates. It is only by concentrating the period of observation closely around the announcement that results of this magnitude can be obtained. This is an example of the general problem with event studies discussed above. Since the R^2 is so low, we might interpret the regressions as showing only that money surprises are sometimes taken by the market as a reason to expect higher interest rates, at other times are believed to indicate lower interest rates, and in our sample the former circumstance occurred more frequently.²³

Other research on the money announcement phenomenon has examined a number of questions which we have left aside. For example, Roley has considered the impact of the change in Fed policy in October 1979 on the money announcement. We have not investigated whether the effect has disappeared in 1982 due to a Fed policy shift in the direction of stabilizing the Federal Funds rate. Roley has also examined potential nonlinearities such as the effect of money stock innovations which lie outside the Fed's identified target bounds for monetary

²³ The money stock innovation effect may seem less impressive when one considers Grossman's observation [1981] that there has been virtually zero correlation between initially announced weekly money stock changes and the final, revised data on changes in money stock. Maravall and Pearce [1980] found that preliminary data on two-month growth rates of M-1 gave wrong signals as to whether M-1 growth rates were in the FOMC tolerance range 40% of the time. They attributed the revision primarily to changing seasonals. Since the Census X-11 program used to deseasonalize data is publicly known, there is still good reason to believe that the forecast errors represent genuine errors in predicting the money supply before deseasonalization.

growth. Urich and Wachtel [1982] also examined the impact of price index announcements on interest rates, but found relatively weak effects.

Perhaps the most important empirical innovation in this section of our report may be found in Table 6. There we present the effect of money surprises on forward rates. These are calculated according to the theory presented in the previous section. Hardouvelis examined the same effect in a recent unpublished paper [1982], but in the absence of a formula for calculating the forward rate on coupon-carrying bonds, he treated the yields on Treasury issues as pure discount bonds. As can be seen from equation (4), this places excessive weight on the long-term interest rate, and becomes approximately like the regressions involving the change in the interest rates themselves as the dependent variable. It should not be surprising, then, that Hardouvelis re-obtained the previous result that the money announcement effect extends throughout the entire term structure out to thirty years. By contrast, Table 6 shows that the money announcement affects significantly the properly calculated linearized forward rates only as far as the five-year-ahead two year rate.²⁴ Indeed, the explanatory power of the regression, as measured by R^2 , tapers off significantly after the two-year-ahead one-year-rate. These results are displayed

²⁴ Once again, the standard errors appear in parentheses. The notation DF (60,24) refers to the change in the 60-month ahead 24-month rate from Friday at 3:30 p.m. to the following Monday at 3:30 p.m. For maturities up to twelve months, the forward rates are computed without the linearization from T-bill yields. For maturities 12 months or higher, the forward rates are computed from Treasury note and bond yields, using a point of linearization of 12.8%. The results of Table 6 are not materially affected if the forward rates are computed with the current 30 year bond rate at each point in time as the point of linearization.

in Figure 4 where the horizontal axis measures years from the present and the vertical axis represents the forward rate changes in basis points per billion dollar surprise. The horizontal segments in the solid line correspond to the forward rates beginning at the left end of the segment and extending to the right end. The shaded area identifies a two standard deviation bandwidth on either side of the point estimate of the effect. The display begins with the one-month T-bill rate and shows all subsequent forward rates out to the twenty-year-ahead ten-year rate.

What are we to make of this money surprise effect? It is instructive first to consider the impact of money surprises on the shortest interest rate, the Federal Funds rate. This requires an awareness of some basic institutional facts about the current regime of monetary control. Under the system of lagged reserve accounting, banks must hold reserves in a given statement week (Wednesday to Wednesday) to cover their deposits of two weeks earlier.²⁵ This lag is the same as the delay in reporting the level of M1. Furthermore, banks can hold their reserves on any day of the statement week so long as the average over the week is at least at the required level (subject to nonnegativity constraints on their balance at the Fed and the potential effects of random shocks).

On any day the Federal Funds rate may be thought of as the price of bank reserves on that day. If we disregard excess reserves (which are generally small), the aggregate demand schedule for reserves on

²⁵ This system is due to be replaced by contemporaneous reserve accounting in 1984. As we shall see, this change will alter the response of the Federal Funds rate to money stock announcements.

Wednesday, the last day of the statement week, is vertical.²⁶ Demand for reserves is completely inelastic because it is determined by reserve requirements arising from deposits of two weeks earlier and by the fraction of reserve requirements already fulfilled on earlier days of the statement week.

The supply schedule for aggregate reserves is determined by the Fed, since the Fed is the only source of aggregate reserves. The Federal Funds rate may be regarded as a proxy for the total cost of borrowing from the Fed. The Funds rate is then the discount rate plus the nonpecuniary costs ("frowns") which are imposed by the Fed on borrowers at the discount window. The supply schedule is upward sloping since "frowns" increase with the amount borrowed.

The intersection of the vertical demand schedule and the upward sloping supply schedule for reserves determines the Federal Funds rate independently of the current demand for money. Indeed some have argued that under lagged reserve accounting the stock of money this week is determined only by the public's demand for money at the current Funds rate, since banks' supply schedule for money is infinitely elastic at this Funds rate.²⁷

At the beginning of the statement week, individual banks are interested in forecasting the Federal Funds rate at the end of the week. They could profit from being able to predict, say, an increase in the Funds rate, since they average their reserves over the week in ful-

²⁶ This also ignores the possible effects of a provision allowing banks to carry excess reserves forward or to make up a reserve deficiency the following week. Such carry-over is limited to 2% of a bank's required reserves.

²⁷ See Hetzel [1982] for a detailed argument along these lines.

filling their reserve requirements and they could plan to hold required reserves early in the week and lend reserves later. Unfortunately, early in the statement week, banks know neither the position of the demand curve for aggregate reserves nor the position of the supply curve for aggregate reserves at the end of the week. The position of the demand curve is uncertain in part because each bank, while it knows its own deposits, does not know the aggregate deposits from two weeks earlier against which the banking system must hold reserves this week. The position of the supply curve is unknown, even though banks may have some idea as to its slope, because banks do not know this week's aggregate nonborrowed reserves.

The announcement of the money stock from two weeks earlier, which occurs in the middle of the statement week, then carries information about aggregate required reserves and the position of the reserve demand curve at the end of the statement week. If the money stock in any week is demand determined, then the money surprise this week is a surprise only about the demand for money two weeks earlier, and not about the supply of money at that time. But it can still be true that the money surprise carries information about the current supply curve for reserves. Since the Federal Funds rate is observed continually, the height of the intersection of current reserve demand and supply curves is known before the announcement and thus information about one curve is also information about the other. A similar point was made by Nichols and Small [1983] in the context of a rather different model.

It is hard to be more precise about the behavior of the Federal Funds rate over the statement week without becoming involved in more intricacies than are appropriate for this paper. A properly specified rational expectations model of the Federal Funds market (and the market is one for which rational expectations seem most appropriate) would have to detail sources of information unfolding over the statement week: for example, the level of reserves from a week ago, announced on Friday. It would also have to take account of the role of the Federal Funds rate in aggregating the private information of individual banks about their own reserve requirements.

We think though that this discussion lends plausibility to several important points. First, the rise in the Funds rate in response to a positive money surprise provides no evidence that a Fed decision to increase reserves will raise short interest rates. Secondly, the money surprise is information both that the demand for money two weeks ago was higher than expected and that the Fed's policy is now more expansionary than expected. Thirdly, the increase in the Funds rate may reflect nothing more than an expected scramble for reserves at the end of the statement week. Finally, the Funds rate should follow an approximate random walk over the statement week. If the Funds rate deviated systematically from a random walk, for example, if it were persistently higher on any day of the week, then fewer banks would want to hold reserves on that day. Since aggregate reserves cannot be reduced (except to the extent that the banking system borrows less at the discount window), such a situation cannot be an equilibrium.²⁸ If

²⁸ Note that this argument does not depend on lagged reserve accounting.

the Funds rate is indeed a random walk, then information received on Friday that the rate will be higher on Wednesday should cause it to increase immediately.

Unfortunately, examination of the Federal Funds rate on a daily basis over the period over the period January 1977 to February 1983 leads to some doubt about the random walk hypothesis. If we partition the sample into three parts, with the central subsample running from October 1979 to May 1982, we observe changes in Funds market behavior. Over the central subsample, there is a strong tendency for lagged changes in yields to have a significantly positive effect on subsequent changes within the same statement week. This effect is less marked in the end subsamples. Indeed, over an 18-month period from October 1979, the sample means on individual days of the week differ significantly and by as much as 80 basis points. The Funds rate tended to be high on Fridays and low on Wednesdays.²⁹

This evidence against the random walk character of the Funds rate may arise from the fact that the period immediately following October 1979 was a "learning period" in which the behavior of the Funds rate was not well established; or it may be that our analysis of the Federal Funds market is oversimplified.

²⁹ The behavior of the Funds rate also suggests that changes have been made in Federal Reserve policy during the period. For example, the standard deviation of rate changes from Friday to Monday rises from 0.085 in the earliest subsample to 0.695 in the central sample and falls once again to 0.242 in the most recent period. This would seem to suggest that interest rate intervention has been restored after an experimental period of reserve targeting. Our analysis of the money announcement effect on the Funds rate has been based on the assumption of reserve targeting.

The behavior of forward interest rates is far more difficult to model convincingly than is the behavior of the Funds rate. There are too many possible explanations of the response to money announcements for any one to be compelling.³⁰ The temptation, of course, is to interpret the behavior of forward rates by using the expectations theory of the term structure. For example, Nichols and Small [1983], observing that a money stock innovation may carry information both about demand shocks and about supply shocks, claim that the effects on longer term interest rates depend on the relative persistence of these shocks. However, we have seen that the simple expectations theory of the term structure is really of no value.

Many people have suspected that the movement of forward rates is determined in another way: namely, by market overreaction to money stock surprises. There may be no good reason for distant forward rates to respond to money surprises and they may respond just because the market habitually moves forward rates in tandem with the Funds rate. This would be an example of the "tail wags dog" theory discussed in the introduction.

We can of course run tests, as we noted above, to check whether market overreaction has occurred in our sample period. Such tests are reported in Table 7. The market has overreacted if the response in interest rates to a money stock surprise is offset later, that is if interest rates tend to return to their previous level. There are two

³⁰ These explanations include the expected inflation effect (Cornell [1983]), the liquidity effect (Nichols and Small [1983]), an eclectic combination of the two (Hardouvelis [1982]), and changes in risk premia caused by uncertainty about Fed policy after a large money surprise (Cornell [1983]).

main types of regression in Table 7. Rows 2 through 4 project changes in forward or spot rates over a week on the preceding money surprise. The expectations theory implies that changes in forward rates should not be predictable on the basis of lagged information; this should also be approximately true for changes in spot rates measured over short intervals of time. Rows 5 through 7 regress realized forecast errors on the last money surprise known to the market at the time the forecast was made. The expectations theory implies once again that such forecast errors should be unpredictable. The regression of Row 1 of Table 7 tests whether the Federal Funds rate response to a money surprise is reversed by the end of the statement week, that is whether the Funds rate is a random walk with respect to the information in the money stock surprise.³¹

It should not be surprising that, as previously described, these tests offer low power and negligible results. Perhaps most curious is the regression in Row 5, which implies that, if anything, the market underreacts to the information in the money announcement. That is, while the three-month ahead three-month forward rate rises by 8.8 basis points immediately following a one billion dollar money stock surprise, the three-month spot rate prevailing three months from now will rise by another 26.6 basis points according to our results.³²

³¹ In contrast to the other tables, the t-statistics (not the standard errors) appear in parentheses. Standard errors are not reported since the dependent variable follows a 4j'th order moving average process. However, since the regression can be reversed to eliminate serial correlation, the t-statistic correctly indicates the presence or absence of a significant relationship between money surprises and subsequent forecast errors.

³² Conrad [1982] has run a regression similar to Row 1 of Table 7 as well as analogous regressions for shorter maturities and horizons

However, given the low significance and conflicting signs of the coefficients in the other tests of Table 7, we do not wish to exaggerate the importance of this regression.

Although we have not had much success in explaining the response of longer term interest rates to money innovations, we still have priors on the impact effect of a Fed decision to ease monetary policy by increasing nonborrowed reserves, without announcing any policy change. For this purpose we do not need a complete theory of the determination of interest rates. We know enough about the institutions of the Federal Funds market to know that the Funds rate would fall immediately.

One might think that this predictable policy effect on the Federal Funds rate would lead to a predictable effect on the long-term interest rates. It is consistent with both the expectations theory and the "tail wags dog" theory to say that the market predictably communicates a portion of the Funds rate change to the long rate. In fact, a weekly regression (from 1977:1 to 1983:7, using 266 observations) of the Friday to Monday change in the 30-year Treasury bond yield on the Friday to Monday change in the Federal Funds rate had an R^2 of only 0.08. Many other factors besides the Funds rate have an impact on the long rate, so that even overnight there is virtually no relation between the two. Not knowing the overnight impact on the long rate, it is harder yet to specify the longer-run impact.

(down to n and m of one week). He concluded as we did that, while the coefficients are not generally significant, the market appeared to underreact to the announcement.

VII Conclusion.

Economics, the saying goes, consists of theories which are not borne out by the data and of observed empirical regularities for which there is no theory. What must be meant by this is that simple theories do not fare well and complicated theories are too numerous. A pessimist might say that this paper is further confirmation of this epigram. The simple theory that the slope of the term structure can be used to forecast the direction of future interest rate movements seems absolutely worthless. Of course, some version of the expectations theory ought to show up in the data if the Fed were to create a large and predictable pattern of short rates. We merely claim that the theory is useless for interpreting the data provided by recent history, and that forecasting interest rates using the slope of the term structure will only be successful if there is a break in the historical interest rate pattern.³³

We postulated a simple alternative to the expectations theory, a "tail wags dog" theory which asserts that long-term interest rates tend to overreact to current information. While we never carefully defined this theory, we ran some regressions which seemed relevant and found no supportive evidence. This suggests that some aspects of the expectations theory can be useful in modelling the term structure. It

³³ This conclusion stands in contrast to Hansen and Sargent (1981): "... for purposes of unconditional forecasting, it may be wise to use a vector moving average constrained by even a false null hypothesis that economizes on the number of parameters to be estimated. In this sense, the [simple expectations] model-constrained results ... could be useful for forecasting even if one respects the evidence which our procedures have turned up against the term structure restrictions." (pp. 50-51)

may be that the hypothesis that agents maximize expected returns, which underlies the expectations model, can be used to eliminate certain naive psychological theories which imply huge short-run profit opportunities. We still think that a psychological theory which does not have this implication may be superior to the simple expectations theory.

We have also seen that the expectations theory can be partially salvaged if plausible measures of time-varying risk premia are introduced and if one is not uncomfortable with a theory which explains the slope of the term structure and the volatility of long-term interest rates primarily by changes in the risk premium. Our risk measures include a new volume-related variable which potentially brings government financing and the effect of deficits into the analysis. The problem is that such risk proxies are rather arbitrary and achieve only a small improvement in the predictive power of the expectations theory. Alternatively, the expectations theory could be useful in explaining very short-run changes in the term structure if risk premia are slow-moving. However, we have no evidence that this is the case.

The phenomenon we documented here, using new linearized expressions, that forward rates several years out respond to money stock surprises, is an empirical regularity for which theories are potentially too numerous (though we think there is a single plausible explanation for the response of the Federal Funds rate). The traditional Keynesian theory of interest rate determination is one possible framework within which the money announcement effect can be explained, and is not discredited by this effect. In fact, we feel that those

previous authors who have argued that the money announcement effect is a "paradox", for which it is hard to find an explanation, have over-rated the phenomenon. The low R^2 in the regression indicates that the market responds in different ways on different days to the same money surprise, but responds positively to a surprise slightly more often. Anyone with imagination ought to be able to think of ten different stories which might explain this behavior. We can of course test whether the response of forward interest rates was "rational", although as we argued such tests are inherently weak. We found in testing one "tail wags dog" theory with three- and six-month rates that the market, if anything, underreacted to surprises.

Despite our skeptical approach, we are able to apply our analysis to the interpretation of recent interest rate behavior. We find that effects of changed duration help to explain why the long rate moves more closely with the short rate than in earlier years. At times when the short rate has recently risen, observers who are unaware of this change may be misled into thinking that the long rate is "too high". Even correcting for this, long bond rates have been unusually high in the last three years. Increases in risk premia probably account for a substantial part of this increase. If one wished to maximize expected short-term return on investment without regard for risk, one would currently prefer long-term bonds to short-term bonds.

References

Ando, Albert and Arthur Kennickell, "A Reappraisal of the Phillips Curve and the Term Structure of Interest Rates", mimeo, University of Pennsylvania, 1983.

Arrow, Kenneth J., "Risk Perception in Psychology and Economics", Economic Inquiry Vol. 20 No. 1 pp. 1-9, January 1982.

Belsley, David A., E. Kuh and R. Welsch, Regression Diagnostics: Identifying Influential Data and Sources of Collinearity, John Wiley and Sons, New York, 1980.

Board of Governors of the Federal Reserve System, New Monetary Control Procedures, Washington, February 1981.

Conrad, William, "Treasury Bill Market Response to Money Stock Announcements Before and After the Change in Federal Reserve Operating Procedures," mimeographed, Board of Governors of the Federal Reserve System, November 1982.

Cornell, Bradford, "Money Supply Announcements and Interest Rates: Another View", Journal of Business, Vol. 56 No. 1 pp. 1-23, January 1983.

Cox, John, John Ingersoll and Stephen Ross, "A Reexamination of Traditional Hypotheses About the Term Structure of Interest Rates", Journal of Finance Vol. 36 No. 4 pp. 769-99, September 1981.

Engel, Charles and Jeffrey A. Frankel, "Why Money Announcements Move Interest Rates: An Answer from the Foreign Exchange Market", unpublished working paper, University of California at Berkeley, January 1982.

Friedman, Benjamin M., "Financial Flow Variables and the Short-Run Determination of Long-Term Interest Rates", Journal of Political Economy, August 1977, pp. 661-689.

Friedman, Benjamin M., "Substitution and Expectation Effects on Long-Term Borrowing Behavior and Long-Term Interest Rates", Journal of Money, Credit and Banking, May 1979, pp. 131-150.

Glejser, H., "A New Test for Heteroskedasticity", Journal of the American Statistical Association, Vol. 64 pp. 316-323, 1969.

Gordon, Roger H., "Interest Rates, Inflation, and Corporate Financial Policy," Brookings Papers on Economic Activity, 1982:1, pp. 461-488.

Grossman, Jacob, "The Rationality of Money Supply Expectations and the Short-Run Response of Interest Rates to Monetary Surprises", Journal of Money, Credit and Banking, Vol. 13 pp. 409-424, November 1981.

Hamburger, Michael J. and Elliott N. Platt, "The Expectations Hypothesis and the Efficiency of the Treasury Bill Market," Review of Economics and Statistics, Vol. 57 No. 2, pp. 190-9, May 1975.

Hansen, Lars Peter and Richard J. Hodrick, "Forward Exchange Rates as Optimal Predictors of Future Spot Rates", Journal of Political Economy Vol. 88 No. 5 pp. 829-53, October 1980.

Hansen, Lars Peter and Thomas J. Sargent, "Exact Linear Rational Expectations Models: Specification and Estimation", Staff Report No. 71, Federal Reserve Bank of Minneapolis, September 1981.

Hardouvelis, Gikas, "Market Perceptions of Federal Reserve Policy and the Weekly Monetary Announcements", unpublished working paper, University of California at Berkeley, 1982.

Hetzl, Robert L., "The October 1979 Regime of Monetary Control and the Behavior of the Money Supply in 1980", Journal of Money, Credit and Banking, Vol. 14 No. 2, pp. 234-251, May 1982.

Hicks, Sir John, Value and Capital, Oxford, 1939.

Jones, David S., and Roley, V. Vance, "Rational Expectations, the Expectations Hypothesis and Treasury Bill Yields: An Econometric Analysis", National Bureau of Economic Research Working Paper No. 869, June 1982.

Macaulay, Frederick, Some Theoretical Problems Suggested by the Movements of Interest Rates, Bond Yields and Stock Prices in the United States Since 1856, National Bureau of Economic Research, New York, 1938.

Maravall, Agustin and David A. Pearce, "Errors in Preliminary Money Stock Data and Monetary Aggregate Targeting," Special Studies Paper Number 152, Board of Governors of the Federal Reserve System, 1980.

McCulloch, J. Huston, "Measuring the Term Structure of Interest Rates", Journal of Business Vol. 44 No. 1 pp. 19-31, January 1971.

Mishkin, Frederic, "Efficient Markets Theory: Implications for Monetary Policy", Brookings Papers on Economic Activity 3, 1978.

Mishkin, Frederic, A Rational Expectations Approach to Macroeconomics: Testing Policy Ineffectiveness and Efficient-Markets Models, Chicago, 1983

Modigliani, Franco and Robert J. Shiller, "Inflation, Rational Expectations and the Term Structure of Interest Rates", Economica pp. 12-43, February 1973.

Nichols, Donald A. and John H. Small, "Why Interest Rates Rise When An Unexpectedly Large Money Stock Is Announced", forthcoming American Economic Review, 1983.

Roley, V. Vance, "The Determinants of the Treasury Security Yield Curve", Journal of Finance, December 1981, pp. 1103-1126.

Roley, V. Vance, "The Response of Short-Term Interest Rates to Weekly Money Announcements", National Bureau of Economic Research Working Paper No. 1001, October 1982.

Roley, V. Vance, "Asset Substitutability and the Impact of Federal Deficits", National Bureau of Economic Research Working Paper No. 1082, February 1983.

Roll, Richard, The Behavior of Interest Rates, Basic Books, New York, 1970.

Salomon Brothers, An Analytical Record of Yields and Yield Spreads, New York, 1983.

Sargent, Thomas J. "Rational Expectations and the Term Structure of Interest Rates", Journal of Money, Credit and Banking Vol. 4 No. 1 pp. 74-97, February 1972.

Sargent, Thomas J., "A Note on Maximum Likelihood Estimation of the Rational Expectations Model of the Term Structure", Journal of Monetary Economics, Vol. 5 No. 1 pp. 133-43, January 1979.

Shiller, Robert J., "The Volatility of Long-Term Interest Rates and Expectations Models of the Term Structure", Journal of Political Economy 87:1190-1219, December 1979.

Shiller, Robert J., "Alternative Tests of Rational Expectations Models: The Case of the Term Structure", Journal of Econometrics 16:71-87, 1981.

Urich, Thomas, and Paul Wachtel, "The Effects of Inflation and Money Supply Announcements on Interest Rates", mimeographed, New York University Graduate School of Business, August 1982.

FORWARD RATES AND FUTURE POLICY: INTERPRETING
THE TERM STRUCTURE OF INTEREST RATES

Figures and Tables

Figure 1: Actual vs. Predicted Short Rate Change

Quarterly data 1959:1 to 1982:3 (95 observations), from the first day of March, June, September, December. Data are taken from Salomon Brothers' Analytical Record of Yields and Yield Spreads. The short rate is the 3-month Treasury bill rate; the predicted change from the term structure is the 3 month ahead, 3-month forward rate minus the current 3-month rate. The forward rate is computed from the current 3- and 6-month rates.

Figure 2: Actual vs. Predicted Long Rate Change

Six-monthly data 1959:1 to 1982:2 (48 observations), from the first day of January and July. Data are taken from Salomon Brothers' Analytical Record of Yields and Yield Spreads. The long rate is the 30-year Treasury bond rate; the predicted change from the term structure is the 6 month ahead, 30-year linearized forward rate minus the current 30-year rate. The forward rate is computed from the current 6-month and 30-year rates.

Figure 3: The Money Announcement Effect, 1980-83

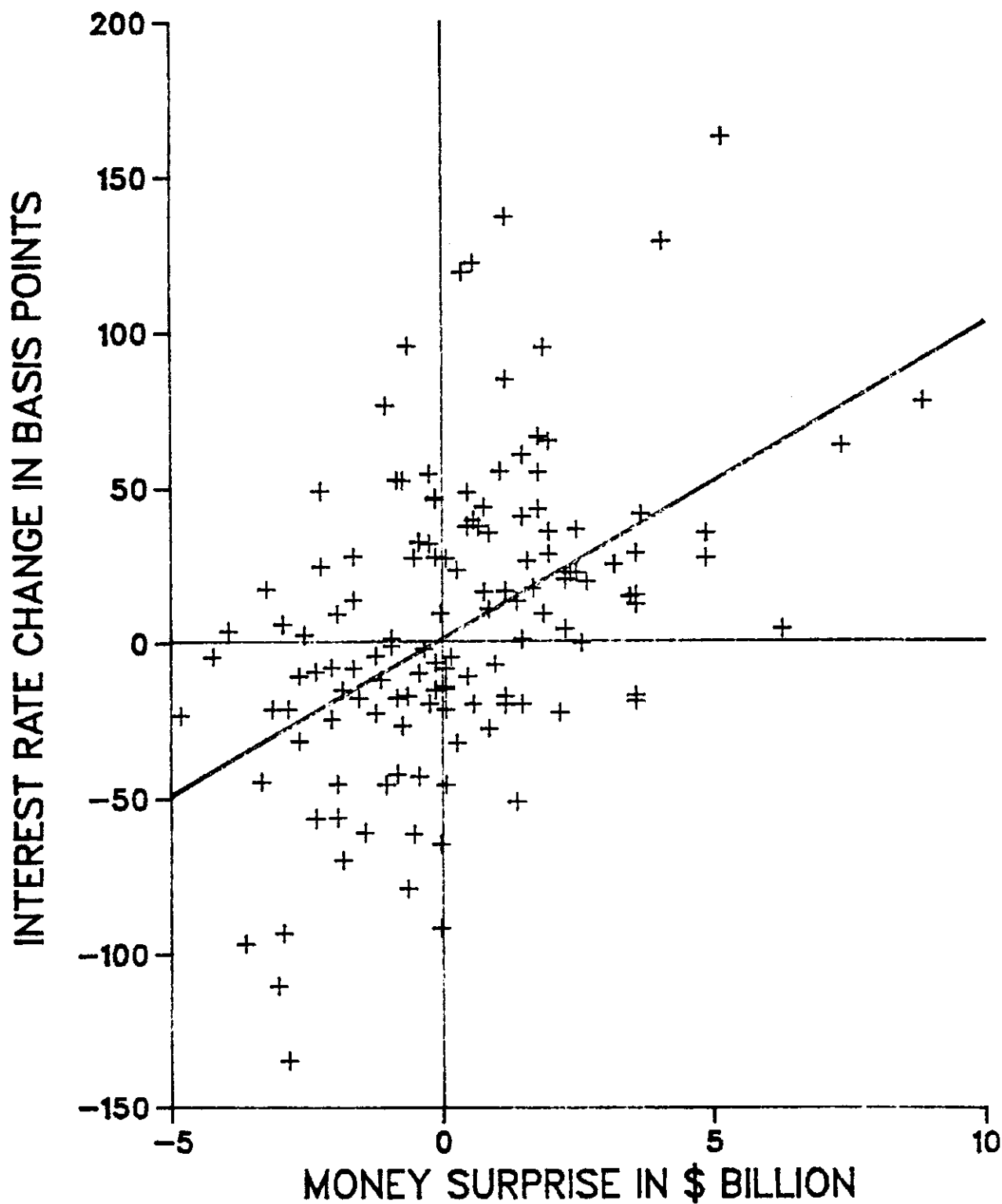
Weekly data 1980:8-1983:4 with some weeks omitted because of holidays (132 observations). Data were provided by the Board of Governors of the Federal Reserve System and Money Market Services, Inc. The interest rate is the 3-month Treasury bill rate. Its change is measured from 3:30 p.m. Friday to 3:30 p.m. Monday. The money surprise variable is the difference between the money stock announced on Friday, in billions of dollars, and the previous Tuesday's median forecast of the money stock from the weekly survey conducted by Money Market Services, Inc. Also shown is the regression line reported in row 3 of Table 5.

Figure 4: Term Structure Response to Money Announcements

Weekly data 1980:8 - 1983:4 (132 observations), as in Tables 5 and 6 and Figure 3. The plot shows the change in the linearized forward rates estimated in Table 6 to result from a one billion dollar money surprise. The change in the 1-year ahead, 1-year forward rate is plotted between years 1 and 2 on the horizontal axis. The change plotted at zero on the horizontal axis is the change in the one-month Treasury bill rate from row 2 of Table 5. The shaded area shows the 95% confidence interval for the estimated response.

FIGURE 3

THE MONEY ANNOUNCEMENT EFFECT, 1980-83



TERM STRUCTURE RESPONSE TO MONEY ANNOUNCEMENTS

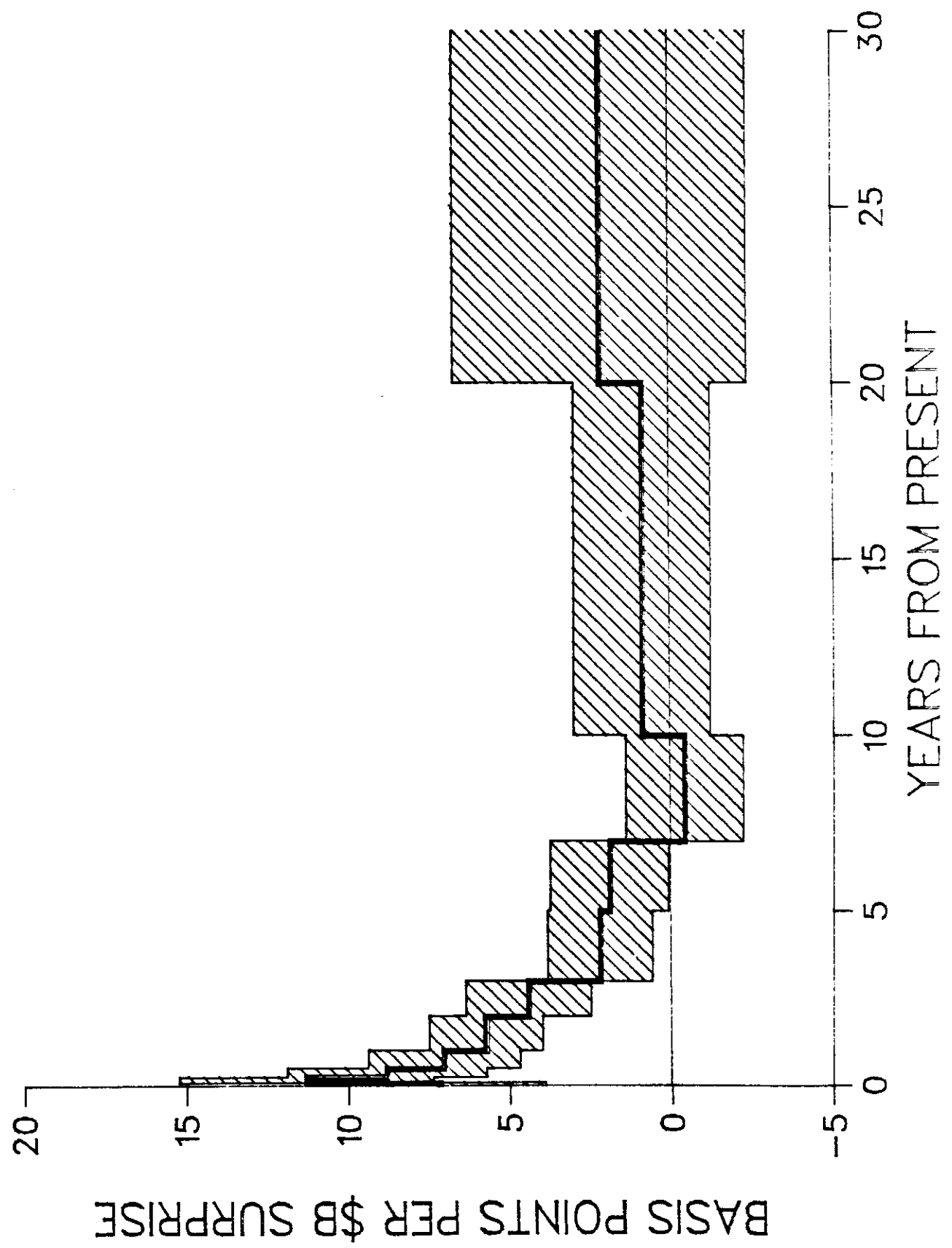


FIGURE 4

TABLE 1

RECENT PERFORMANCE OF STANDARD TERM STRUCTURE EQUATIONS

Dependent Variable	Sample Period	Method	Current		Sum of Lagged		Current Inflation (S.E.)	Sum of Lagged Inflation (S.E.)	R ² (DW)
			Short Rate (S.E.)	Short Rates (S.E.)	Short Rates (S.E.)	Inflation (S.E.)			
20-Year Bond Yield	Quarterly 1955:1-1979:3	19 Lags: 3rd Order Polynomial with end point constraint	1.818 (0.098)	0.261 (0.034)	0.272 (0.053)	0.005 (0.016)	0.379 (0.029)	0.984 (1.192)	
Duration-Corrected 20-Year Bond Yield			1.622 (0.095)	0.336 (0.034)	0.286 (0.052)	0.000 (0.016)	0.319 (0.028)	0.984 (1.322)	
Column:	1	2	3	4	5	6			
Period	Actual Value	Predicted Value: Standard Equation	Predicted Value: Duration-Corrected Equation	Predicted Value: Duration-Corrected Equation	Prediction Error: Standard Equation	Prediction Error: Duration-Corrected Equation			
1979:4	9.29	9.56	9.61	9.61	-0.27	-0.32			
1980:1	10.10	10.34	10.52	10.52	-0.24	-0.42			
1980:2	12.40	11.35	11.68	11.68	1.05	0.72			
1980:3	10.01	10.00	9.84	9.84	0.01	0.17			
1980:4	11.83	10.84	11.04	11.04	0.99	0.79			
1981:1	11.96	11.85	12.30	12.30	0.11	-0.34			
1981:2	12.89	11.65	11.97	11.97	1.24	0.92			
1981:3	13.64	12.30	12.82	12.82	1.34	0.82			
1981:4	15.58	12.54	13.07	13.07	3.04	2.51			
1982:1	14.05	11.73	12.05	12.05	2.32	2.00			
1982:2	13.86	12.14	12.74	12.74	1.72	1.12			
1982:3	14.06	11.92	12.52	12.52	2.14	1.54			
1982:4	11.58	10.39	10.59	10.59	1.19	0.99			

Note: The standard equation is summarized at the top of the table. The correction for duration is described in the text.

TABLE 2

COMPARISON OF LINEARIZED WITH EXACT HOLDING PERIOD YIELDS AND FORWARD RATES

Row	Variable	Sample Period (Observations)	\bar{R}	Mean	Standard Deviation	Correlation of X and x	Minimum	Maximum
1 a	$H_t^{(24,12)}$	Weekly	12.81	9.53	5.11	--	2.65	22.90
b	$h_t^{(24,12)}$	1977-83 (226)	12.81	9.45	5.12	1.000	2.25	22.59
2 a	$H_t^{(360,12)}$	Weekly	12.81	3.43	14.29	--	-18.02	50.29
b	$h_t^{(360,12)}$	1977-83 (226)	12.81	3.50	13.41	0.994	-18.68	45.20
3 a	$H_t^{(240,120)}$	Monthly	6.00	2.74	0.88	--	1.25	6.05
b	$h_t^{(240,120)}$	1953-72 (240)	6.00	2.71	0.87	0.977	0.43	5.89
4 a	$F_t^{(12,12)}$	Weekly	12.81	10.78	2.82	--	6.31	16.72
b	$f_t^{(12,12)}$	1977-83 (278)	12.81	10.78	2.82	1.000	6.32	16.72
c	$f_t^{(12,12)}$		15.65	10.78	2.82	1.000	6.33	16.72
d	$f_t^{(12,12)}$		6.83	10.78	2.82	1.000	6.29	16.71
e	$f_t^{(12,12)}$		0.00	10.78	2.83	1.000	6.26	16.70
5 a	$F_t^{(12,348)}$	Weekly	12.81	10.59	2.16	--	7.69	15.18
b	$f_t^{(12,348)}$	1977-83 (278)	12.81	10.59	2.15	1.000	7.74	15.19
6 a	$F_t^{(240,120)}$	Weekly	12.81	9.41	1.24	--	6.67	14.00
b	$f_t^{(240,120)}$	1977-83 (278)	12.81	9.37	1.11	0.931	7.28	14.34
c	$f_t^{(240,120)}$		15.65	8.67	1.48	0.516	4.26	16.33
d	$f_t^{(240,120)}$		6.83	10.12	1.63	0.817	7.47	13.27
e	$f_t^{(240,120)}$		0.00	10.45	2.01	0.726	7.54	14.45

Notes: Exact holding period returns and forward rates were computed using an iterative procedure, under the assumption that bond coupon rates are R . Linearized holding period returns and forward rates were computed from equations (2) and (3) in the text.

TABLE 3

REGRESSION OF CHANGES IN INTEREST RATES ON CHANGES PREDICTED BY THE TERM STRUCTURE

Row	Dependent Variable	Sample Period (Observations)	Method	Constant (S.E.)	Coefficient of Predicted Change (S.E.)	R ² (DW)
1	$R_{t+3} - R_t^{(3)}$	Quarterly 1959:1 to 1982:3 M, J, S, D (95)	OLS	0.044 (0.049)	-0.212 (0.261)	0.007 (2.205)
2	$R_{t+3} - R_t^{(3)}$	Monthly 1959:1 to 1982:10 (286)	Corrected OLS	0.042 (0.026)	-0.194 (0.194)	0.007 (N/A)
3	$R_{t+3} - R_t^{(3)}$	Monthly 1959:1 to 1979:6 (246)	Corrected OLS	-0.017 (0.017)	0.285 (0.156)	0.030 (N/A)
4	$R_{t+3} - R_t^{(3)}$	Monthly 1959:1 to 1979:6 (246)	Corrected WLS	-0.015 (0.026)	0.265 (0.153)	0.029 (N/A)
5	$R_{t+3} - R_t^{(3)}$	Monthly 1959:1 to 1974:3 (183)	Corrected WLS	-0.012 (0.027)	0.266 (0.179)	0.025 (N/A)
6	$R_{t+6} - R_t^{(360)}$	Semi-Annual 1959:1 to 1982:2 Jan., July (48)	OLS	0.077 (0.052)	-1.205 (2.189)	0.007 (1.516)
7	$R_{t+6} - R_t^{(360)}$	Monthly 1959:1 to 1982:7 (283)	Corrected OLS	0.093 (0.020)	-1.972 (1.678)	0.019 (N/A)
8	$R_{t+6} - R_t^{(360)}$	Monthly 1959:1 to 1979:3 (243)	Corrected OLS	0.081 (0.014)	-1.782 (1.365)	0.032 (N/A)
9	$R_{t+6} - R_t^{(360)}$	Monthly 1959:1 to 1979:3 (243)	Corrected WLS	0.071 (0.030)	-1.697 (1.393)	0.029 (N/A)
10	$R_{t+6} - R_t^{(360)}$	Monthly 1959:1 to 1973:12 (180)	Corrected WLS	0.055 (0.032)	-1.456 (1.785)	0.021 (N/A)

Notes: See text for discussion of Corrected Ordinary Least Squares and Corrected Weighted Least Squares. Standard errors on the constant term are uncorrected in equations estimated by Corrected OLS. The Durbin Watson statistic is not reported for Corrected Least Squares regressions since the error term follows a 3rd or 6th order moving average process and the reported coefficient standard errors take this into account.

TABLE 4

TIME-VARYING RISK PREMIA AND CREDIT VOLUME

Row	Dependent Variable	Sample Period	Method	Constant (S.E.)	Predicted Change (S.E.)	Moving Standard Deviation (S.E.)	Volume Ratio (S.E.)	R ² (DW)
1	$H_t^{(6,3)} - R_t^{(3)}$	Monthly 1959:1-79:6	Corrected OLS	-0.033 (0.023)	--	0.785 (0.185)	--	0.166 (N/A)
2	$H_t^{(6,3)} - R_t^{(3)}$	Quarterly 1959:1-82:4	OLS	0.019 (0.052)	--	--	0.064 (0.019)	0.104 (2.029)
3	$H_t^{(6,3)} - R_t^{(3)}$	Quarterly 1959:1-82:4	OLS	-0.023 (0.069)	--	0.287 (0.302)	0.054 (0.022)	0.113 (2.094)
4	$H_t^{(360,6)} - R_t^{(6)}$	Quarterly 1959:1-82:3	Corrected OLS	-3.661 (1.359)	--	--	0.979 (0.609)	0.039 (N/A)
5	$H_t^{(360,6)} - R_t^{(6)}$	Monthly 1959:1-79:3	Corrected OLS	-2.083 (0.362)	71.803 (35.093)	--	--	0.074 (N/A)
6	$R_{t+3}^{(3)} - R_t^{(3)}$	Monthly 1959:1-79:6	Corrected OLS	0.054 (0.023)	0.510 (0.146)	-0.556 (0.182)	--	0.106 (N/A)
7	$R_{t+3}^{(3)} - R_t^{(3)}$	Quarterly 1959:1-82:4	OLS	0.127 (0.055)	-0.108 (0.223)	--	-0.049 (0.017)	0.086 (2.290)
8	$R_{t+6}^{(360)} - R_t^{(360)}$	Quarterly 1959:1-82:3	Corrected OLS	0.256 (0.062)	-4.759 (2.118)	--	-0.074 (0.025)	0.120 (N/A)

Notes: See text for discussion of Corrected Ordinary Least Squares. Standard errors on the constant terms are uncorrected on equations estimated by Corrected OLS. The Durbin Watson statistic is not reported for these equations since the error term follows a moving average process and the reported coefficient standard errors take this into account.

TABLE 5

Regression of One-Day Change in Interest Rates on Money Stock Surprises.
Weekly Data, 1980-8 to 1983-4, 132 Observations.

<u>Dependent Variable</u>	<u>Constant</u>	<u>Coefficient of Money Surprise</u>	<u>R²</u>
Δ FF	-1.12 (5.21)	7.86 (2.25)	.086
$\Delta R^{(1)}$	7.01 (3.80)	7.13 (1.63)	.128
$\Delta R^{(3)}$	4.25 (3.76)	9.88 (1.62)	.222
$\Delta R^{(6)}$	6.11 (3.49)	9.32 (1.50)	.228
$\Delta R^{(12)}$	1.96 (2.91)	8.16 (1.26)	.245
$\Delta R^{(24)}$.31 (2.20)	6.86 (0.95)	.286
$\Delta R^{(36)}$	1.08 (2.03)	6.13 (0.87)	.275
$\Delta R^{(60)}$	2.68 (1.82)	4.83 (0.78)	.227
$\Delta R^{(84)}$	2.80 (1.67)	4.22 (0.72)	.209
$\Delta R^{(120)}$	2.96 (1.56)	3.36 (0.67)	.161
$\Delta R^{(240)}$	3.12 (1.58)	2.79 (0.68)	.114
$\Delta R^{(360)}$	2.94 (1.53)	2.75 (0.66)	.119

Notes: Δ denotes change between 3:30 p.m. the day of the weekly money stock announcement and 3:30 p.m. on the first trading day after announcement; FF denotes the end of day federal funds rate; $R^{(j)}$ denotes annualized yield to maturity in basis points on bond or bill with j months to maturity. For maturities up to 12 months, the yields are for U. S. Treasury bills. For maturities larger than 12 months, the yields are for U. S. Treasury notes and bonds. The money surprise variable is the difference between the money stock announced on Friday in billions of dollars and the Tuesday median forecast of the money stock from the weekly market survey of Money Market Services, Inc. Standard errors are in parentheses.

TABLE 6

Regression of One-Day Change in Linearized Forward Rates on Money Stock Surprises.

Weekly Data 1980:8 to 1983:4, 132 Observations

<u>Dependent Variable</u>	<u>Constant</u>	<u>Coefficient of Money Surprise</u>	<u>R²</u>
$\Delta F^{(1,2)}$	2.86 (4.54)	11.26 (1.96)	.203
$\Delta F^{(3,3)}$	7.98 (3.57)	8.76 (1.54)	.199
$\Delta F^{(6,6)}$	-2.16 (2.72)	7.01 (1.17)	.216
$\Delta f^{(12,12)}$	-1.29 (2.01)	5.72 (0.87)	.250
$\Delta f^{(24,12)}$	2.94 (2.24)	4.40 (0.96)	.138
$\Delta f^{(36,24)}$	5.91 (1.89)	2.19 (0.81)	.053
$\Delta f^{(60,24)}$	3.27 (2.10)	1.90 (0.91)	.033
$\Delta f^{(84,36)}$	3.69 (2.09)	-0.41 (0.90)	.002
$\Delta f^{(120,120)}$	3.67 (2.43)	0.90 (1.05)	.006
$\Delta f^{(240,120)}$	0.59 (5.25)	2.19 (2.26)	.007

Notes: Δ denotes change between 3:30 p.m. the day of the weekly money stock announcement and 3:30 p.m. on the first trading day after the announcement.

$f^{(n,m)}$ denotes the annualized m-month linearized forward rate in basis points applying to a time n months in the future, which by the expectations theory equals the expected value of R^m n periods hence. The linearization was carried out around an annualized interest rate of 12.8%.

For maturities less than 12 months, the forward rates are computed without linearization from U. S. Treasury bill yields. For maturities 12 months or higher, the forward rates are computed from U. S. Treasury note and bond yields. The money surprise variable is the difference between the money stock announced on Friday in billions of dollars and the Tuesday median forecast of the money stock from the weekly market survey of Money Market Services, Inc. Standard errors are in the parentheses.

TABLE 7

Regression of "Forecast Errors" on Previous Money Stock Surprise .

Longest Weekly Sample in Period 1980:8 to 1983:4

<u>Dependent Variable</u>	<u>Constant</u>	<u>Coefficient of Money Surprise</u>	<u>R²</u>	<u>Number of Observations</u>
ΔFF_t	-20.01 (-1.75)	2.98 (0.61)	0.003	132
$\Delta R_t^{(3)}$	-8.42 (-1.71)	0.21 (0.10)	0.000	124
$\Delta F_t^{(3,3)}$	-11.78 (-2.03)	-3.09 (-1.25)	0.013	124
$\Delta R_t^{(6)}$	-9.04 (-2.11)	-1.16 (-0.64)	0.003	124
$R_{t+3}^{(3)} - F_t^{(3,3)}$	-223.75 (-6.85)	26.63 (1.95)	0.034	110
$R_{t+6}^{(6)} - F_t^{(6,6)}$	-149.35 (-4.15)	14.23 (0.91)	0.008	103
$R_{t+12}^{(12)} - F_t^{(12,12)}$	10.02 (0.23)	-12.21 (-0.71)	0.006	88

Notes: ΔFF_t is the change in the Federal funds rate between the day after the money stock announcement and the last day of the statement week.

$\Delta R_t^{(3)}$, $\Delta F_t^{(3,3)}$ and $\Delta R_t^{(6)}$ are the changes in these variables between the day after the money announcement and the day of the following money announcement.

$R_{t+j}^{(j)} - F_t^{(j,j)}$ is the j-month rate j months after the money announcement minus its forecast in the forward rate on the day after the money announcement.

t statistics are in parentheses.

Standard errors are not reported since the dependent variable follows a 4j'th order moving average process.

FORWARD RATES AND FUTURE POLICIES:
INTERPRETING THE TERM STRUCTURE OF INTEREST RATES

Technical Appendix

by

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John Y. Campbell
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A. LINEARIZATION

I. Linearized Holding Period Yield

Preliminaries. All bonds pay one coupon per period. The price $P_t^{(i)}$ of a bond maturing at time $t+i$ (when it's worth \$1.00) is ex-dividend, i.e., the first coupon received is at $t+1$. Coupons are constant through time, C dollars per period. An i -period bond becomes an $i-k$ period bond at $t+k$.

The yield to maturity $R_t^{(i)}$ is determined from $P_t^{(i)}$, $C_t^{(i)}$ and i implicitly by the requirement that P_t^i is the Pdv of future payments:

$$P_t^{(i)} = \frac{C}{(1+R_t^{(i)})} + \frac{C}{(1+R_t^{(i)})^2} + \dots + \frac{C}{(1+R_t^i)^i} + \frac{1}{(1+R_t^i)^i}$$

so
$$P_t^i = \frac{C}{R_t^{(i)}} + \frac{1}{(1+R_t^{(i)})^i} \left(1 - \frac{C}{R_t^i}\right) \quad (1)$$

k-period holding yield. Define this as the yield to maturity of an asset worth P_t^i at time t , paying coupon C_t^i through time $t+k$ and worth P_{t+k}^{i-k} at time $t+k$. It's defined implicitly by:

$$P_t^{(i)} = \frac{C_t^{(i)}}{(1+H_t^{(i,k)})} + \frac{C_t^{(i)}}{(1+H_t^{(i,k)})^2} + \dots + \frac{C_t^i}{(1+H_t^{(i,k)})^k} + \frac{P_{t+k}^{i-k}}{(1+H_t^{(i,k)})^k} \quad (2)$$

Clearly $H_t^{(i,k)}$ is determined by $C_t^{(i)}$, $P_t^{(i)}$, $P_{t+k}^{(i-k)}$, i and k . Using (1), we can express this in terms of $C_t^{(i)}$, $R_t^{(i)}$, $R_{t+k}^{(i-k)}$, i and k . Expression (2) can be simplified as with (1):

$$P_t^{(i)} = \frac{C_t^{(i)}}{H_t^{(i,k)}} + \frac{1}{(1+H_t^{(i,k)})^k} \left(P_{t+k}^{(i-k)} - \frac{C}{H_t^{(i,k)}} \right) \quad (3)$$

Substituting (1) into (3) and subtracting right hand side from left:

$$0 = \frac{C_t^{(i)}}{R_t^{(i)}} + \frac{1}{(1+R_t^{(i)})^i} \left(1 - \frac{C_t^{(i)}}{R_t^{(i)}} \right) - \frac{C_t^{(i)}}{H_t^{(i,k)}} - \frac{1}{(1+H_t^{(i,k)})^k} \cdot \left(\frac{C_t^{(i)}}{R_{t+k}^{(i-k)}} + \frac{1}{(1+R_{t+k}^{(i-k)})^{i-k}} \left(1 - \frac{C_t^{(i)}}{R_{t+k}^{(i-k)}} \right) - \frac{C_t^{(i)}}{H_t^{(i,k)}} \right) \quad (4)$$

This expression can't be solved for $H_t^{(i,k)}$. This function has a unique solution $H_t^{(i,k)}$ since all terms in present value are positive. We write (4) as

$$f \left(H_t^{(i,k)}, R_t^{(i)}, R_{t+k}^{(i-k)}, C_t^{(i)}, i, k \right) = 0. \quad (5)$$

Linearize (5) around $H_t^{(i,k)} = R_t^{(i)} = R_{t+k}^{(i-k)} = C_t^{(i)} = \bar{R}$ call $\gamma = \frac{1}{1+\bar{R}}$
 shorthand: call these H $R^{(i)}$ $R^{(k)}$ C \bar{R}

$$\frac{\partial f}{\partial R^{(i)}} = - \frac{C}{R^{(i)2}} - i \frac{1}{(1+R^{(i)})^{i+1}} \left(1 - \frac{C}{R^{(i)}} \right) + \frac{1}{(1+R^{(i)})^i} \frac{C}{R^{(i)2}}$$

evaluated at \bar{R} :

$$= \frac{1}{\bar{R}} (\gamma^i - 1), \quad \gamma = \frac{1}{1+\bar{R}}$$

$$\frac{\partial f}{\partial R^k} = - \frac{1}{(1+H)^k} \left[- \frac{C}{R^{(k)2}} - (i-k) \frac{1}{(1+R^{(k)})^{i-k+1}} \left(1 - \frac{C}{R^k} \right) + \frac{1}{(1+R^{(k)})^{i-k}} \frac{C}{R^{(k)2}} \right]$$

at \bar{R} :

$$= - \frac{1}{\bar{R}} (\gamma^i - \gamma^k)$$

$$\frac{\partial f}{\partial H} = \frac{C}{H^2} + k \frac{1}{(1+H)^{k+1}} \left[\frac{C}{R^{(k)}} + \frac{1}{(1+R^k)^{i-k}} \left(1 - \frac{C}{R^{(k)}} \right) - \frac{C}{H} \right] - \frac{1}{(1+H)^k} \frac{C}{H^2}$$

at \bar{R} :

$$= -\frac{1}{R}(\gamma^k - 1)$$

$$\frac{\partial f}{\partial C} = \frac{1}{R^{(i)}} - \frac{1}{(1+R^{(i)})^i R^{(i)}} - \frac{1}{H} - \frac{1}{R^{(k)}(1+H)^k} + \frac{1}{R^{(k)}(1+H)^i} \frac{C}{R^{(k)}} + \frac{1}{(1+H)^k} \frac{1}{H}$$

at \bar{R} :

$$= 0 .$$

Since $f(\bar{R}, \bar{R}, \bar{R}, \bar{R}, i, k) = 0$, the first order Taylor expansion of f is

$$\frac{1}{R}(\gamma^i - 1)(R_t^{(i)} - \bar{R}) - \frac{1}{R}(\gamma^i - \gamma^k)(R_{t+k}^{(i-k)} - \bar{R}) - \frac{1}{R}(\gamma^k - 1)(H_t^{(i,k)} - \bar{R}) = 0$$

sum of coefficients of $\bar{R} = 0$ so \bar{R} drops out

solve for $H_t^{i,k}$:

$$h_t^{(i,k)} = \frac{\gamma^i - 1}{\gamma^k - 1} R_t^i - \frac{\gamma^i - \gamma^k}{\gamma^k - 1} R_{t+k}^{(i-k)}$$

or

$$h_t^{(i,k)} = \frac{(1 - \gamma^i)R_t^i - (\gamma^k - \gamma^i)R_{t+k}^{(i-k)}}{1 - \gamma^k} , \quad k \leq i . \quad (6)$$

Now note that if

$$R_t^{(i)} = \frac{1-\gamma}{1-\gamma^i} \sum_{s=0}^{i-1} E_t \gamma^s R_{t+s}^{(1)}, \quad R_{t+k}^{(i-k)} = \frac{1-\gamma}{1-\gamma^{i-k}} \sum_{s=0}^{i-k-1} E_{t+k} \gamma^s R_{t+k+s}^{(1)} \quad (7)$$

then, substituting (7) into (6)

$$\begin{aligned} E_t h_t^{(i,k)} &= \frac{(1-\gamma)}{1-\gamma^k} E_t (R_t^{(1)} + \gamma R_{t+1}^{(1)} + \gamma^2 R_{t+2}^{(1)} + \dots + \gamma^{i-1} R_{t+i-1}^{(1)}) \\ &\quad - \frac{\gamma^k}{1-\gamma^k} (1-\gamma) E_t (R_{t+k}^{(1)} + \gamma R_{t+k+1}^{(1)} + \dots + \gamma^{i-k-1} R_{t+i-1}^{(1)}) \\ &= \frac{1-\gamma}{1-\gamma^k} E_t (R_t^{(1)} + \gamma R_{t+1}^{(1)} + \dots + \gamma^{k-1} R_{t+k-1}^{(1)}) \\ \therefore E_t h_t^{(i,k)} &= R_t^{(k)} \end{aligned} \quad (8)$$

II. Linearized Forward Rate

The goal is to construct a portfolio, at time t , which has no cost in periods $t, t+1, \dots, t+k-1$; a cost at $t+k$, coupons at $t+k+1, \dots, t+i-1, t+i$; and a final repayment at $t+i$.

We can adopt a simplified notation: P_i refers to $P_t^{(i)}$, etc. This simplification is possible for the linearization of the forward rate because the expression for the forward rate involves only variables at time t .

The payment matrix is:

t+0	P_i	$P_1 X_1$	$P_2 X_2$...	$P_{k-1} X_{k-1}$	$P_k X_k$
1	C_i	$(1+C_1)X_1$	$C_2 X_2$		$C_{k-1} X_{k-1}$	$C_k X_k$
2	C_i	0	$(1+C_2)X_2$		$C_{k-1} X_{k-1}$	$C_k X_k$
3	C_i	0	0		$C_{k-1} X_{k-1}$	$C_k X_k$
⋮	⋮	⋮	⋮		⋮	⋮
k-1	C_i	0	0		$(1+C_{k-1})X_{k-1}$	$C_k X_k$
k	C_i	0	0		0	$(1+C_k)X_k$
k+1	C_i	⋮	⋮		⋮	0
⋮						⋮
i-1	C_i					0
i	$1+C_i$					0

Equations: ① $P_i - \sum_{j=1}^k P_j X_j = 0$

② $C_k X_k + (1+C_{k-1})X_{k-1} = C_i \Rightarrow X_k = \frac{C_i}{C_k} - \frac{(1+C_{k-1})}{C_k} X_{k-1}$

③ $\left\{ \begin{array}{l} C_k X_k + C_{k-1} X_{k-1} + (1+C_{k-2})X_{k-2} = C_i \\ \vdots \\ \text{and so on} \end{array} \right\} \Rightarrow X_j = (1+C_{j-1})X_{j-1}$
 ④ $j = 2, \dots, k-1$

Solving the k equations for x_1, \dots, x_k we find:

$$x_k = \frac{C_i z - P_i}{C_k z - P_k} \text{ where } z \equiv \sum_{j=1}^{k-1} \left(P_j \prod_{n=j}^{k-1} (1+C_n)^{-1} \right)$$

The forward rate $F_t^{(k,i-k)}$ is given implicitly by:

$$L(F, R_1, \dots, R_{k-1}, R_k, R_i, C_1, \dots, C_{k-1}, C_k, C_i, i, k) = 0$$

where

$$L = \left[-(1+C_k)X_k + C_i \sum_{j=0}^{i-k} (1+F)^{-j} + (1+F)^{-(i-k)} \right]$$

where X_k is written in terms of R_1, \dots, R_{k-1}, R_k instead of P_1, \dots, P_{k-1}, P_k using the definition of yield in Part I above.

We wish to linearize L around \bar{R} , i.e. around

$$\bar{R} = F = R_1 = \dots R_{k-1} = R_k = R_i = C_1 = \dots C_{k-1} = C_k = C_i .$$

We will proceed in two steps: in step one we first show that $\partial L / \partial C_n = 0$, $n = 1, 2, \dots, k, i$. Then in step two we will proceed with linearization under the assumption that $C_1 = C_2 = \dots = C_k = C_i = \bar{R}$.

Step 1. For C_n , $n = 1, \dots, k-1$, $\frac{\partial L}{\partial C_n} = -(1+C_k) \frac{\partial X_k}{\partial C_n}$. Clearly,

$\partial X_k / \partial C_n = 0$ if evaluated at \bar{R} , since if $C_i = C_k = \bar{R}$ and $R_i = R_k = \bar{R}$ then $X_k = 1$ regardless of C_1, \dots, C_{k-1} . $\therefore \partial L / \partial C_n = 0$, $n = 1, \dots, k-1$.

To show that $\partial L / \partial C_i = \partial L / \partial C_k = 0$:

First note that at \bar{R} :

$$z = \frac{1}{\bar{R}} \left[1 - \frac{1}{(1+\bar{R})^{k-1}} \right] \quad (A)$$

$$\frac{\partial P_j}{\partial C_j} = \frac{1}{\bar{R}} \left[1 - \frac{1}{(1+\bar{R})^j} \right], \quad \text{all } j \quad (B)$$

Now:

$$\frac{\partial L}{\partial C_k} = -X_k - (1+C_k) \frac{\partial X_k}{\partial C_k} \quad (C)$$

using (A) and (B) we find

$$\left. \frac{\partial X_k}{\partial C_k} \right|_{\bar{R}} = \frac{-1}{1+\bar{R}}$$

substituting in C gives $\partial L / \partial C_k = 0$.

$$\frac{\partial L}{\partial C_i} = -(1 + C_k) \frac{\partial X_k}{\partial C_i} + \sum_{j=0}^{i-k} (1+F)^{-j} \quad (D)$$

using (A) and (B) we find:

$$\frac{\partial X_k}{\partial C_i} \Big|_{\bar{R}} = \frac{1}{R} \left[1 - \frac{1}{(1+\bar{R})^{i-k+1}} \right]$$

substituting in D gives $\partial L / \partial C_i = 0$.

Step 2. We transform X_k so that it is a function of R_1, \dots, R_i rather than P_1, \dots, P_i . To do this, we use

$$P_i = \frac{C}{R_i} + \frac{1}{(1+R_i)^i} \left(1 - \frac{C}{R_i} \right) \quad \text{as in Part I above}$$

$$P_k = \frac{C}{R_k} + \frac{1}{(1+R_k)^k} \left(1 - \frac{C}{R_k} \right)$$

$$Z = C \sum_{j=1}^{k-1} \left[\frac{C}{R_j} + \frac{1}{(1+R_j)^j} \left(1 - \frac{C}{R_j} \right) \right] (1+C)^{j-k}$$

and we have $X_k = \frac{Z - P_i}{Z - P_k}$.

For future reference, we note that when evaluated at the point of linearization (see below),

$$P_i = 1, \quad P_k = 1, \quad Z = \frac{1}{R} \sum_{j=1}^{k-1} (1+\bar{R})^{j-k} = \frac{1}{R} \sum_{j=1}^{k-1} \gamma^{k-j}$$

$$X_k = 1, \quad \text{where } \gamma = \frac{1}{1+\bar{R}} .$$

We can now write our functions as:

$$L(F, R_1, \dots, R_{k-1}, R_k, R_i, C, i, k)$$

$$= \left[-(1+C)X_k + C \sum_{j=0}^{i-k} (1+F)^{-j} + (1+F)^{-(i-k)} \right] = 0 .$$

We linearize this function about the point

$$F = R_1 = \dots = R_{k-1} = R_k = R_i = C = \bar{R} = \frac{1-\gamma}{\gamma} .$$

Now we proceed to take the derivatives of the function w.r.t. each of its arguments.

$$\left. \frac{\partial L}{\partial R_j} \right|_{\bar{R}} = \text{for } j = 1, \dots, k-1 = -(1+C) \frac{\partial X_k}{\partial R_j} = -(1+C) \left[\frac{\partial Z / \partial R_j}{Z-1} - \frac{[Z-1] \partial Z / \partial R_j}{[Z-1]^2} \right] = 0$$

$$\begin{aligned} \left. \frac{\partial L}{\partial R_k} \right|_{\bar{R}} &= -(1+C) \left[\frac{-(Z-1)}{(Z-1)^2} (-\partial P_k / \partial R_k) \right] = -(1+\bar{R}) \left[\frac{1}{(Z-1)} \left[-\frac{1}{\bar{R}} + \frac{1}{\bar{R}} \gamma^k \right] \right] \\ &= -\frac{\gamma^k - 1}{\gamma \bar{R}^{2k-1} \sum_{j=1}^{k-1} \gamma^{k-j} - \gamma \bar{R}} = -\left[\frac{\gamma^k - 1}{\gamma^{k-1} (\gamma - 1)} \right] \end{aligned}$$

$$\left. \frac{\partial L}{\partial R_i} \right|_{\bar{R}} = -(1+C) \frac{1}{Z-1} (-\partial P_i / \partial R_i) = \left[\frac{\gamma^i - 1}{\gamma^{k-1} (\gamma - 1)} \right]$$

$$\begin{aligned} \left. \frac{\partial L}{\partial F} \right|_{\bar{R}} &= -(i-k)(1+F)^{-(i-k)+1} + C \sum_{j=0}^{i-k} (-j)(1+F)^{-j-1} \\ &= -(i-k)\gamma^{i-k+1} + \frac{\gamma-1}{\gamma} \sum_{j=0}^{i-k} j\gamma^{j+1} = \frac{\gamma^{i-k+1} - \gamma}{1-\gamma} \end{aligned}$$

Note that $\frac{\partial L}{\partial F} + \frac{\partial L}{\partial R_i} + \frac{\partial L}{\partial R_k} = 0$ when evaluated at \bar{R} so terms in \bar{R} also drop out. Thus we can set

$$\frac{\partial L}{\partial F}(F - \bar{R}) + \frac{\partial L}{\partial R_i}(R_i - \bar{R}) + \frac{\partial L}{\partial R_k}(R_k - \bar{R}) = \frac{\partial L}{\partial F}F + \frac{\partial L}{\partial R_i}R_i + \frac{\partial L}{\partial R_k}R_k = 0$$

and solve for F .

$$\text{We find } F = \frac{(\gamma^i - 1)R_i - (\gamma^k - 1)R_k}{\gamma^i - \gamma^k}.$$

In the notation of the paper:

$$f_t^{(n,m)} = \frac{(1 - \gamma^{m+n})R_t^{(m+n)} - (1 - \gamma^n)R_t^{(n)}}{\gamma^n - \gamma^{m+n}}, \text{ where } m = i - k, n = k.$$

B. DATA CREATION AND MANIPULATION

We use the following data sets:

- 1) Monthly point-in-time observations of treasury issues.
Most data from 1950, 30-year bonds from 1953;5, 6-month bills from 1959.
Source: Salomon Brothers, Analytical Record of Yields and Yield Spreads
Data file: MONTHLY DATA (monthly base)
- 2) Quarterly point-in-time observations of 3-month and 6-month bills and 20-year bond rates from 1970;1 - 1982;3
Source: Federal Reserve Board
Data file: MONTHLY DATA
- 3) Quarterly observations on volume of short and long gross corporate debt issue, and on the ratio of outstanding short and long debt, 1952-1983.
Source: Salomon Brothers, Inc. (based on Flow of Funds Accounts).
Data file: VOLUME DATA
- 4) Daily interest rate data for all maturities of treasury issues; organized by day of the week 1977-1983.
Source: Federal Reserve Board
Data file: WEEKLY DATA (weekly base)
- 5) Weekly observations on announced M1 and a survey expectation of the announcement, November 1978 - January 1983.
Source: Money Market Services, Inc.
Data file: WEEKLY DATA

Data sets 1 and 2 are used to assess the expectations theory of the term structure (data set 2 provides check on data set 1). Data set 3 is used to study the relation between perceived risk, excess returns and corporate financing methods. Data sets 4 and 5 are used to study the money announcement effect.

Table 1: Standard Term Structure Equations.

Standard equation

The 20 year bond rate was regressed on a constant, the current 3-month Treasury bill rate, the current inflation rate, and polynomial distributed lags of these two variables. A cubic polynomial was estimated over $x(-1)$, $x(-20)$, with the end point constrained to equal zero.

Duration-corrected equation

Equation (1) in the text can be rewritten to express the long rate as a weighted average of forward rates. Suppose we have a sequence of forward rates $f(k)=f(n(k),m(k))$ $k=0\dots K$, which are such that

$$n(k)+m(k)=n(k+1)$$

$$n(0)=0$$

$$n(K)+m(K)=i$$

(that is, which form a non-overlapping exhaustive sequence extending to the horizon i). Then we can write

$$\begin{aligned} R_t^{(i)} &= (1/(1-g)) \sum_{k=0}^K g^{n(k)+m(k)} (1-g)^{n(k)} f(k) \\ &= (1/D) \sum_{k=0}^K [D^{n(k)+m(k)} - D^{n(k)}] f(k). \end{aligned}$$

Note that $g^{n(k)+m(k)} (1-g)^{n(k)} = (1-g)^{\sum_{j=n(k)}^{n(k)+m(k)} j}$, so this expression means

$f(k)$ replaces a sequence of constant one-period rates from $n(k)$ to $n(k)+m(k)$.

We used equation (4) in the text to construct linearized forward rates from the observed term structure, using 5.4% as the point of linearization. Then we recombined these forward rates using the above expression with 12.4% as the point of linearization. This generated an artificial interest rate which can be thought of as the 20-year bond rate which would have been observed had long rates fluctuated about 12.4% over the last 30 years. In the last three years, the artificial interest rate corresponds closely to the actual 20 year bond rate. Finally, we used the artificial interest rate as the dependent variable in the term structure equation.

Table 2 Rows 4, 5, and 6: Calculation of Forward Rates.

The actual forward rates $F(i,j)$ are generated from the implicit definition of forward rates as an i -period ahead yield to maturity on a j -period bond. This definition depends on the coupon values of bonds with all maturities between the present and period i . Since our time unit is six months, in calculating the value of $F(12,j)$ we can represent the intervening asset's coupon with the six month rate. In the case of $F(240,120)$ we need to approximate the coupons on all maturities with six-month increments out to 240 months. The results presented in row 6a select the ten-year bond yield as the approximation. Calculations using the 5-year bond yield as the coupon approximation produced similar results and are not reported.

Since the $F(i,j)$ are only implicitly defined, even with the above approximation, an iterative routine for obtaining yields to maturity as solutions of a polynomial equation is required. Since the coefficients on the polynomial are by definition all positive, the positive root is unique and a simple Newton-Raphson procedure converges rapidly to the solution.

The procedure for generating $F(i,j)$ is as follows:

(1) Obtain the price that must be paid in period i for the return of period $i+j$. This is:

$$X_i = (1+C) \frac{(Z-P_{i+j})}{(Z-P_i)}$$

where
$$P_{i+j} = \frac{C}{R_{i+j}} + (1 - \frac{C}{R_{i+j}}) * (1 + R_{i+j})^{** (-i-j)}$$

$$P_i = \frac{C}{R_i} + (1 - \frac{C}{R_i}) * (1 + R_i)^{** (-i)}$$

$$Z = C \sum_{m=1}^{i-1} \left[\frac{C}{R} + \left(1 - \frac{C}{R}\right) \cdot \left(1 + \frac{R}{m}\right)^{-m} \right] \cdot (1 + C)^{m-i}$$

In all the formulas it is assumed that the coupons remain constant and are approximated by $R_{bar}=12.8\%$ annually (when the linearization takes place around $C=R_{bar}$, the different coupons drop out). In the the case of Z, the yields on the ten year bond are employed as the approximation for the $R(j)$ when the 20-year ahead forecast is made.

(2) Given $X(i)$ one can generate the $F(i,j)$ iteratively as yields to maturity.

This involves repeated estimation of $F(i,j)$ from the following equation (presented without subscripts for simplicity):

$$X - (C + C/(1+F) + C/(1+F)^2 + \dots + C/(1+F)^{j-1} + (1+C)/(1+F)^j) = 0$$

In general, the Newton-Raphson approach for solving a polynomial $G(b)=0$ is

$$b_{i+1} = b_i - \frac{G(b_i)}{G'(b_i)}$$

where b_i is the old guess for the solution. We initiated the search at

$b_i = R_{bar} = 12.8\%$. The procedure converged rapidly (1-4 iterations)

for the tolerance level $-0.00001 < (b_{i+1} - b_i) < 0.00001$.

Below is a sample of the program for calculating forward yields to maturity using PEC (Program for Econometric Computation).

```
PROGRAM YM=MATYIELD COPY;H RBAR J P Y$
GENRV X=RBAR$ @initial guess@
PROC FUNCTION RBAR H J FX FXH X$ @subroutine evaluates function@
```

```

GENRV FX=RBAR*(1+X)*(1-(1-X/RBAR)*(1+X)**(-J))/X$
GENRV FXH=RBAR*(1+X+H)*(1-(1-(X+H)/RBAR)*(1+X+H)**(-J))/(X+H)$
ENDP FUNCTION$
GENRV Y=0$
GENRV I=0$
1 GENRV I=I+1$ @begin iteration@
FUNCTION RBAR H J FX FXH X$ @evaluate function at X and X+H@
GENRV Y=X-H*(P-FX)/(FX-FXH)$ @generate solution@
IF (ABS(Y-X) .LT. (.00001)) $ THEN$ GO TO 2$ @test for convergence@
GENRV X=Y$ @generate new guess@
GO TO 1$
2 PRINTV Y I; @print the result@
FORMAT='(" forward yield =",f8.4, " Iter =",f4.0)'[
END$

```

This program generates a forward yield for a single observation on price P. The user supplies H (equivalent to dX for the numerical differentiation), RBAR, J (equivalent to j in F(i,j)), and P (the price described above). The program returns the forward yield in Y.

Table 2 Rows 1, 2, 3: Calculation of Holding Period Yields.

A similar procedure is employed for calculating the exact holding period yields. Prices are generated using the Rbar in the table. Then a Newton-Raphson iterative procedure is employed. A sample program follows:

```
PROGRAM HR=HOLDINGR; H RBAR K P P2 Y $
GENRV X=RBAR$
PROC FUNCTION RBAR H K FX P2 FXH X$
GENRV FX=RBAR/X+(P2-RBAR/X)/((1+X)**K)[
GENRV FXH=RBAR/(X+H)+(P2-RBAR/(X+H))/((1+X+H)**K)[
ENDP FUNCTION$
GENRV Y=0$
GENRV I=0$
1 GENRV I=I+1$
FUNCTION RBAR H K FX P2 FXH X$
GENRV Y=X-H*(P-FX)/(FX-FXH)$
IF (ABS(Y-X) .LT. (.00001)) $ THEN$ GO TO 2$
GENRV X=Y$
GO TO 1$
2 PRINTV Y I;
FORMAT='(" holding yield =",f8.4, " Iter =",f4.0)'[
END$
```

Here the user supplies H, RBAR (as before), P and P2 (the purchase and sale price), and K (length of the holding period). The program returns the holding yield in Y.

Table 3, Rows 1-5 and Figure 1.

Data set 1. A subsample of the data set is used for the figure: quarterly data 1959:1 to 1982:3 (95 observations), from the first day of March, June, September and December, 3 and 6 month bill rates. In the table this subsample is used for row 1, and the full set of monthly data is used for the other rows.

Data set 1 contains bill rates on a "bond equivalent yield basis". Thus the 3-month yield on a 3-month bill, R33C, can be obtained from the annual bond equivalent yield, TB3, by reverse compounding:

$$R33C=100*((1+TB3/100)**0.25-1).$$

Similarly for the 6-month yield on a 6-month bill, R66C.

Since there are no coupons on Treasury bills, the 3-month forward rate 3 months ahead, F36C (F(3,3)t in the paper) is found as

$$F36C=100*((1+R66C/100)/(1+R33C/100)-1).$$

The term structure prediction of the change in the 3-month rate, PD3C, is

$$PD3C=F36C-R33C.$$

The actual change, D3C, is

$$D3C=R33C(+3)-R33C.$$

These are then annualized by compounding and plotted. The regressions in the table are for non-annualized data (3-month base).

Table 3, Rows 6-10 and Figure 2.

Data set 1. A subsample of this data set is used for the figure: 6-monthly data 1959:1 to 1982:2 (48 observations), from the first day of January and July, 6-month bill and 30-year bond rates. In the table this subsample is used in Row 6 and the full set of monthly data is used for the other rows.

The appropriate time unit for the yield on a bond which pays coupons every 6 months is 6 months. We obtained the 6-month yield on a 30-year bond, R6360C, from the annualized yield, TN30, by reverse compounding:

$$R6360C=100*((1+TN30/100)**0.5-1).$$

Next we applied the linearization formula with 6 months as the time unit:

$$GAMMA=1/(1+R), R=\text{mean of } R6360C$$

$$GAM6360=(GAMMA-GAMMA**60)/(1-GAMMA**60).$$

The term structure prediction of change in the 30-year bond rate is

$$PD6360C=(1-GAM6360)*(R6360C-R66C)/(GAM6360).$$

The actual change is

$$D6360C=R6360C(+6)-R6360C.$$

These are then annualized by compounding and plotted. The regressions in the table are for non-annualized data (6-month base).

Table 4: Risk Variables.

Dependent variables in Table 4 are calculated using the formulas already described. Two new independent variables are introduced from data set 3:

a) The eight-quarter moving standard deviation of the three-month Treasury bill rate. This is calculated, following Ando and Kennickell, as

$$\left[8 \sum_{i=1}^8 (R33C_{-i})^{**2} - \left(\sum_{i=1}^8 R33C_{-i} \right)^{**2} \right]^{**}(0.5)/8$$

This formula does not use the information contained in the current quarter's Treasury bill rate; nevertheless, we used it since it seems to be the conventional measure and, surprisingly, has greater explanatory power for excess holding period returns than the moving standard deviation which includes the current Treasury bill rate. We experimented with the optimal predictor of the variance of the innovation in the three-month bill rate, but found that it performed poorly.

b) The seasonally adjusted ratio of net short borrowing to net long bond issue by U.S. corporations, lagged one quarter. The numerator of this ratio is "short-term business borrowing", a quarterly series provided to us by Salomon Brothers and corresponding to the annual data in the fourth row of Table 1 in their yearly publication "Prospects for Financial Markets". The denominator includes bond issue by foreign as well as U.S. corporations; the equivalent annual series is found in the second row of Table 1 in "Prospects". Net short borrowing has sometimes been negative in this sample period, but bond issue has been consistently positive.

Tables 5, 6 and 7 and Figures 3 and 4.

Data sets 4 and 5. Weekly observations 1980:8 to 1983:4, with weeks omitted containing holiday Mondays or Fridays (132 observations). The money stock was announced on Friday throughout this sample period: we work with Friday and Monday interest rates (prefixed F and M respectively).

Data set 5: Money Market Services data is indexed according to the week in which the money stock was measured. We realigned the series to index them by the week in which the money stock was announced. Money Market Services provided two series, DMR (Friday reported change in M1) and DMF (Tuesday forecast change in M1); our money surprise variable is $SPRISE1=DMR-DMF$. Both changes are measured from an unrevised base.

Data set 4:

a) Bills. The data set records 1,3,6 and 12 month Treasury Bill yields on a discount basis. These are written as MTB1, MTB3 and so forth. We generated the 3-month yield on a 3-month bill, measured on Monday, as

$$MR33C=100*(100/(100-MTB3/4)-1).$$

Similarly for other bill maturities and days of the week. Then we generated the forward rates MF36C and so forth as in the calculations for Figure 1. We annualized yields and forward rates by compounding (annualized rates prefixed by A). Finally we took the changes from Friday to Monday:

$$ADF36C=AMF36C(+1)-AFF36C.$$

These changes are the dependent variables in our regressions.

b) Notes and Bonds. The data set includes 1, 2, 3, 5, 7, 10, 20 and 30 year bond yields. These are written as MTN1, MTN2 and so on. We first reduced all yields to a 6-month time unit, as in the calculations for Figure 2. MTN30 becomes MR160 and so forth, where 60 is the number of 6-month periods to the maturity of the bond and 1 is a superfluous digit.

Next we applied the linearization formula given in the text to calculate forward rates:

$$\begin{aligned} \text{GAMMA} &= 1/(1+R^*), \quad R^* = \text{mean of 6-month yields of all bonds in sample} \\ &= 0.0621 = 12.8\% \text{ when annualized.} \end{aligned}$$

$$\text{MF14060} = ((1-\text{GAMMA}^{**60}) * \text{MR160} - (1-\text{GAMMA}^{**40}) * \text{MR140}) / (\text{GAMMA}^{**40} - \text{GAMMA}^{**60})$$
and so forth, where MF14060 is the 20 year ahead 10-year forward rate prevailing on Mondays. Finally we annualized forward rates by compounding (annualized yields are given in the raw data), and took the change in the annualized rates from Friday to Monday as the dependent variables in our regressions.

C. ECONOMETRIC TECHNIQUES

Corrected Ordinary Least Squares

This procedure is due to Hansen and Hodrick [1980]. These authors demonstrate that an OLS regression with a moving average error generates consistent coefficient estimates. However, the estimated covariance matrix of the coefficient estimates is biased and inconsistent. They propose a procedure which is less efficient than maximum likelihood but far less expensive computationally and probably more robust. This procedure allows the use of the full data set rather than a subsample eliminating overlapping observations. The method involves two stages:

a) An OLS regression using the full data set. Call the vector of coefficient estimates β , and the matrix of regressors X.

b) Correction of the estimated covariance matrix of $\text{SQRT}(T)(\beta-b)$.

This matrix is consistently estimated by

$$T(X'X)^{-1} (X'\Delta X)(X'X)^{-1}$$

where T=number of observations

$\Delta = (T \times T)$ corrected error covariance matrix, as below.

$$\Delta = \begin{bmatrix} M0 & M1 & M2 & \dots & Mk & 0 & 0 & \dots & 0 \\ M1 & M0 & M1 & M2 & \dots & Mk & 0 & 0 & \dots & 0 \\ M2 & M1 & M0 & M1 & M2 & \dots & Mk & 0 & 0 & 0 \\ \dots & M2 & M1 & M0 & M1 & M2 & \dots & Mk & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & Mk & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & Mk \\ Mk & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & Mk & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & Mk & \dots & \dots & M2 & M1 & M0 & M1 & M2 & \dots \\ 0 & 0 & 0 & Mk & \dots & M2 & M1 & M0 & M1 & M2 \\ 0 & \dots & 0 & 0 & Mk & \dots & M2 & M1 & M0 & M1 \\ 0 & \dots & \dots & 0 & 0 & Mk & \dots & M2 & M1 & M0 \end{bmatrix}$$

where M_i , $i=0,1,2,\dots,k$ are estimated as

$$M_i = (1/T) \sum_{t=i+1}^T (\varepsilon_t \cdot \varepsilon_{t-i})$$

where ε_t is the residual from the first stage regression

$k+1$ is the order of the moving average process of the errors.

Corrected Weighted Least Squares

One of the assumptions of the Corrected OLS method is that the dependent and independent variables are jointly stationary and ergodic. For example, the method requires that the equation error variance be constant over time. We found significant time trend in the error variance of our Table 2 regressions, however.

We corrected for this using a method suggested by Glejser [1969]. We did a preliminary OLS regression using all the monthly data. Then we regressed the absolute value of the residuals from this regression on a constant and time. We used the reciprocals of the fitted values from this regression as weights in a second regression: that is, we divided the independent variable, the constant and the dependent variable by the square root of the fitted value. This transformed regression was the starting point for our application of the Hansen-Hodrick method. Note that a weighted least squares (WLS) regression does not have a constant term. This led to some computational complication, but did not otherwise affect our results.