# COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

#### AT YALE UNIVERSITY

Box 2125 Yale Station New Haven, Connecticut 06520

COWLES FOUNDATION DISCUSSION PAPER NO. 656R

Note:

Cowles Foundation Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. Requests for single copies of a paper will be filled by the Cowles Foundation within the limits of supply. References in publications to discussion papers (other than mere acknowledgement by a writer that he has access to such unpublished material) should be cleared with the author to protect the tentative character of these papers.

OPTIMAL LABOR CONTRACTS AND THE ROLE OF MONETARY POLICY

IN AN OVERLAPPING GENERATIONS MODEL

Russell Cooper

September 1984

# OPTIMAL LABOR CONTRACTS AND THE ROLE OF MONETARY POLICY IN AN OVERLAPPING GENERATIONS MODEL

Russell Cooper\*

Cowles Foundation Box 2125 Yale Station New Haven, CT 06520

December 1982

Revised September 1984

<sup>\*</sup>I am grateful to Costas Azariadis for helpful discussion on this topic over the last few years. In addition, Oliver Hart provided useful comments and questions at the start of this research, and Bill Lang's careful reading of this manuscript is much appreciated. Finally, I am grateful to seminar participants at Yale, the University of Pennsylvania, the University of Western Ontario, the University of Wisconsin, the University of Toronto, the 1983 Summer Econometric Society Meetings and last but not least, my colleagues at the 1983 NBER Summer Institute for their helpful comments. Financial support from the National Science Foundation (SES 82-08899) is gratefully acknowledged. All remaining errors are my own.

# Introduction

The seminal paper by Lucas [1972] provides an elegant model in which monetary policy has real effects despite the existence of rational expectations and market clearing. In that model, agents observe prices which are noisy signals of current monetary shocks. Due to this imperfect information, agents respond to nominal shocks and a Phillips curve emerges as an equilibrium phenomenon. This paper spawned an extensive literature of both a theoretical (see, for example, Lucas [1973], Sargent-Wallace [1975], Azariadis [1981]) and empirical (see, for example, Barro [1977a], [1978] and Mishkin [1982]) nature. One of the more controversial results from this line of research concerns the inability of the monetary authorities to use feedback rules to systematically influence real economic behavior.

Fischer [1977], Phelps-Taylor [1977] and others have commented on the importance of the market clearing assumption in obtaining these policy implications. Fischer examines an economy in which labor contracts substitute for spot markets as a means of trading labor services. The paper shows that feedback rules can have real affects in the presence of long-term contracts. These results are obtained, however, in a model of labor contracts which is not explicitly derived from optimizing behavior of the agents in question. This is in contrast to the rigorous attention to microeconomic foundations in Lucas [1972], and Azariadis [1981]. Hence, the models of Fischer and others, while certainly useful in a variety of applications, can be criticized (as in Barro [1977b]) for not providing a micro-theoretic basis for contracts. As a consequence, policy results from these models are subject to the well-known "Lucas critique" (see Lucas [1976]).

The purpose of this paper is to investigate the role of monetary policy in a general equilibrium model with labor contracts. In order to do so, we extend the work of Azariadis [1975] and Baily [1974] on optimal labor contracts to an environment with both real and nominal shocks. The insight of these first papers on implicit contracts was that risk neutral firms could operate as insurance companies for risk averse workers in an economy with incomplete markets for the sharing of real risks. In a partial equilibrium model with shocks to the firm's technology and a constant general price level, an optimal contract establishes a rigid wage. If the general price level is random as well, Azariadis' model predicts real wage rigidity. Hence, money would be neutral if we were to imbed these contracts in a general equilibrium setting.

This result of real wage rigidity requires that both the nominal and real shocks be costlessly observable to all parties. In a situation such as that proposed by Lucas [1972], imperfect information can limit the formation of contingent contracts. The resulting indexation to observable variables (such as the price level) may lead to money non-neutralities. If this indexation is costly, then setting a fixed nominal wage will not necessarily be suboptimal.

Section II of this paper describes a simple overlapping generations model composed of risk neutral firms and risk averse workers who live for two periods. As is well known, this model provides for the

holding of money as a store of value. Firm's technology as well as the money supply are random. We first characterize the equilibrium when all trades take place in spot markets. In equilibrium, the consumption of the workers is independent of nominal shocks but dependent on the shocks to technology in both periods of life.

Section III investigates the risk sharing role of fixed nominal wage contracts. Recent papers by Fischer [1977], Gertler [1982] and Phelps-Taylor [1977] explore the consequences of nominal wage rigidity but offer little explanation of their existence. This rigidity can shield workers from the real shock in youth as in the earlier papers by Azariadis and Baily. However, the consumption of workers depends on the monetary shocks of youth; i.e. money is not neutral. Whether labor services are traded with fixed nominal wage contracts or in spot markets will depend on the relative variability of the real and nominal shocks.

Section IV considers the indexation of wages to the price level.

We characterize the optimal degree of indexation and show that indexed contracts will dominate fixed nominal wage contracts despite the imperfections in the information concerning the shocks. These results are contrasted with those of Gray [1976]. In order to explain nominal wage rigidity it is therefore necessary to introduce a cost for contingent contracts. Nonetheless, money will still be non-neutral as long as prices do not fully reveal the nominal shocks.

Section V discusses monetary policy. As suggested by Lucas [1976], the choice of market structure (spot, fixed nominal wage, contingent) is strongly influenced by monetary policy. Also, we show that real allocations are independent of the monetary authorities choice of a multiplicative feedback rule. This extends the results of Sargent-Wallace

[1975] and Azariadis [1981] to the contracting setting. Finally, we show that a non-stochastic monetary policy is optimal. These results are contrasted with those of "equilibrium business-cycle" models.

Section VI summarizes our results and discusses future research. In particular, while this model helps in understanding nominal wage rigidity, it does not produce a Phillips curve as the labor supply decision is inelastic. Generalizations in this area are necessary.

## II. Overview of the Model and Spot Market Equilibrium

This paper follows Azariadis-Cooper [1982] (hereafter A-C) by extending the overlapping generations model (Samuelson [1958], Cass-Yaari [1966]) to include labor contracts. In A-C, we investigated an economy with random shocks to endowments and showed that non-contingent claims could assist in the allocation of real risks. In fact, when no production distortions arose, these claims supported a <u>first-best</u> allocation of risks. This paper extends their earlier model by including nominal shocks and considering a production rather than an exchange economy.

The structure of the model is quite simple. Each generation consists of N workers and F entrepreneurs. Agents live two periods and each period a new generation is born. Time extends from zero to infinity and there is a single consumption good and a sole store of value, money. In any time period there will be 2(N+F) agents alive.

In youth, the N workers supply their unit of leisure time inelastically in the labor market. These agents use the money balances earned in youth to purchase consumption goods in old age. They do not consume in youth. This captures the notion that agents do not generally consume their own output. Workers have identical preferences represented by u(c) which is strictly increasing and strictly concave.

Entrepreneurs (equivalently, firms) also live for two periods. In youth, these agents employ workers to produce output. They possess a stochastic technology  $f(\ell, \tilde{s})$  where  $\ell$  is the level of employment and  $\tilde{s}$  is a random variable. The technology shock takes values in the interval  $[\underline{s}, \overline{s}]$  where  $\underline{s} > 0$ . We assume  $f_{\ell}(\ell, s) > 0$  and  $f_{\ell\ell}(\ell, s) < 0$  for all  $\underline{s}$ . Higher values of  $\underline{s}$  lead to more productivity so that  $\underline{f}_{\underline{s}}(\ell, s) > 0$  and  $\underline{f}_{\ell s}(\ell, s) > 0$  are appropriate. Entrepreneurs take the money balances earned in youth and consume in old age. We assume that all firms are risk neutral.

In addition to  $\hat{s}$  , there is a nominal shock to the economy,  $\hat{x}$  . The aggregate money stock follows

$$\hat{\mathbf{m}}_{t} = \mathbf{m}_{t-1} \hat{\mathbf{x}}_{t}.$$

We assume that  $\hat{x}$  takes values in [x,x] where x>0. These injections and withdrawals of money are in proportion to existing money holdings and occur at the beginning of each period. Both x and s are i.i.d. and are independent of each other.

The informational assumptions of the model are very important. Only firms observe the technology shock  $\hat{x}$  and only old agents observe  $\hat{x}$  (since they are the only ones receiving the transfer). These informational asymmetries serve to limit the formation of contingent markets. Agents born in period t are assumed to know the money supply of the previous period,  $m_{t-1}$ .

Before proceeding, it is important to emphasize and explain some of the assumptions of the model. Since there is only one commodity and no savings or labor supply decisions, the model contains no real dynamics. As stated above, the overlapping generations structure is then simply a

way to introduce money into the model in which it serves as a store of value. Since labor supply is constant, the non-neutrality of money takes the form of a redistribution effect across workers and firms. We have also assumed that there are two classes of agents differing in their preferences over risk and their initial endowments. While it might be preferable to derive these differences endogenously, we have taken a short-cut here by imposing these differences as a means of highlighting the risk sharing characteristics of labor contracts. Finally, the assumption of asymmetric information regarding aggregate shocks may seem a bit severe. The point is, of course, simply to capture (in a tractable manner) the view that agents are uncertain concerning the source and permanency of the price movements they observe.

We begin our analysis by considering an equilibrium with spot markets for both goods and labor services. This equilibrium is characterized by two price functions:  $p(s_t, x_t, m_{t-1}) \equiv p_t(s, x)$  for goods and  $w(s_t, x_t, m_{t-1}) \equiv w_t(s, x)$  for labor. With F firms and  $N = \gamma F$  workers who supply a unit of labor time inelastically, labor market equilibrium requires

$$p_{t}(s,x)f_{\ell}(\gamma,s) = w_{t}(s,x) . \qquad (1)$$

In the goods market, the total demand for goods is simply  $m_t$ . The total supply of goods in money terms is  $p_t(s,x)Ff(\gamma,s_t)$ . Hence, equilibrium requires

$$p_{t}(s,x)f(\gamma,s) = \frac{m_{t-1}^{x}t}{F}.$$
 (2)

These two market clearing equations completely characterize the equilibrium

since the supply and demand decisions in this economy are so trivial. Defining  $\phi(s)=f_{\ell}(\gamma,s)/f(\gamma,s)$  and  $g(s)=f(\gamma,s)$ , the equilibrium conditions are

$$w_t(s,x) = \frac{\phi(s_t)m_{t-1}x_t}{F}$$
 (3)

and

$$p_t(s,x) = \frac{m_{t-1}^x t}{g(s)F}$$
 (4)

From (3) and (4), we can determine the consumption of a generation t worker in a spot market equilibrium as

$$c^{s}(s_{t}, s_{t+1}) = \frac{w_{t}x_{t+1}}{p_{t+1}} = \phi(s_{t})g(s_{t+1})$$
 (5)

In this equilibrium money is neutral and the consumption of workers (as well as firms) is independent of the money transfer. This is, of course, another example of the neutrality of proportional money transfers in an economy with flexible prices. Consumption will generally depend on  $s_t$  from variations in worker's share of money balances  $(\phi(s_t))$  and on  $s_{t+1}$  due to  $g(s_{t+1})$ . While both prices and wages depend on the inherited money supply due to the permanency of the monetary shocks, consumption is a stationary function of  $s_t$  and  $s_{t+1}$ .

It should be noted that if the production function satisfies stochastic constant returns to scale (e.g. the uncertainty is multiplicative) then  $\phi(s) = f_{\ell}(\gamma)/f(\gamma)$  is independent of s. As noted as well by Diamond [1967] and Newbery-Stiglitz [1982], assumptions of multiplicative uncertainty can have strong implications for constrained optimality.

In this model, if  $\phi(s)$  is independent of s, there will be <u>no</u> incentive for workers and firms to sign a labor contract. Hence, we will generally assume that  $\phi$  depends on s and examine how the extent of this dependence affects market structure.

In summary, a spot market equilibrium is characterized by a price function for goods and a wage function for labor. Real allocations are independent of the nominal shock but, in general, will depend on the productivity shocks.

#### III. Fixed Nominal Wage Contracts

As in the optimal labor contracts literature, the real risks in consumption may be shifted from the workers to the risk neutral entrepreneurs. In our earlier paper, A-C, we focused on the shifting of the price risk in old age. In this section, we focus on shifting the wage variability due to the technology shock and discuss the shifting of price risk in Section V.

As long as the productive function does not exhibit multiplicative uncertainty, workers of generation t face randomness in consumption due to  $\mathbf{s}_{t}$ . Suppose that instead of selling labor services in spot markets, the agents establish a contract market for labor which opens  $\underline{\mathbf{ex}}$  ante. In this market, a contract wage,  $\overline{\mathbf{w}}_{t}$ , is set and is independent of realizations of  $\mathbf{s}$  and  $\mathbf{x}$ . These are one-period contracts which yield units of money not promises to delivery consumption next period. Since workers have zero utility if unemployed, the optimal contract would specify a money payment,  $\overline{\mathbf{w}}_{t}$ , and full-employment in all states of nature.

Equilibrium in the  $\underline{ex}$  ante contract market implies that the expected  $\underline{real}$  wage bill under spot labor markets  $(w_{t})$  and the expected  $\underline{real}$  wage

bill under rigid wages  $(\overline{w}_t)$  are equal. 4 That is

$$\overline{\mathbf{w}}_{t} \mathbf{E} \begin{bmatrix} \mathbf{x}_{t+1} \\ \mathbf{p}_{t+1} \end{bmatrix} = \mathbf{E} \begin{bmatrix} \mathbf{w}_{t} \mathbf{x}_{t+1} \\ \mathbf{p}_{t+1} \end{bmatrix} . \tag{6}$$

The expectations in (6) are taken in period t <u>before</u>  $(s_t, x_t)$  are known but with the knowledge of  $m_{t-1}$ . Using (3) and (4) and letting  $k \equiv 1/E(1/x)$  and  $\overline{\phi} \equiv E_g \phi(s)$ ,

$$\overline{w}_{t} = m_{t-1} k \overline{\phi} / F. \tag{7}$$

So the equilibrium contract wage is a constant proportion of the inherited money stock. We can continue to use (4) as the <u>equilibrium price function</u> for our new economy since the demand and supply for goods is independent of the manner in which labor services are traded.

Using (7), the consumption of a worker in generation t in the rigid nominal wage economy,  $\mathbf{c}^{RW}$ , is

$$c^{RW}(x_t, s_{t+1}) = \frac{\overline{w_t} x_{t+1}}{p_{t+1}} = \frac{kg(s_{t+1})\overline{\phi}}{x_t}$$
 (8)

In this expression, we see that consumption is now independent of the technology shock in youth. This is the benefit of the rigid nominal wage contract. However, the cost of this insurance is that consumption depends on the nominal shock: money is not neutral.

Comparing (5) and (8) we see that spot and contract markets offer different types of insurance. By classical neutrality results, spot markets offer insurance against nominal risks but leave agents open to real risks. Contracts which specify fixed nominal wages insure workers against their firm's productivity shocks but leave workers open to nominal

risks. When only real shocks exist (as in the papers by Azariadis and Baily), it is obvious that contracts are desirable. Once we introduce nominal shocks, the welfare ordering of the two setups will depend on the relative importance of the real and nominal shocks.

We can denote the expected utility from spot markets and rigid wage contracts by  $EU(c^S)$  and  $EU(c^{RW})$  respectively where the consumption functions are defined by (5) and (8). The simplest way to compare these expected utilities is to consider mean preserving spreads of the random variable  $z \equiv 1/x$ . That is, define

$$y = \mu_z + \lambda(z - \mu_z)$$

where  $\mu_z = E(z)$  and  $\lambda \ge 0$  is a parameter we will vary. Expected utility in the contract case is then

$$EU(c^{RW}) = EU(ykg(s_{t+1})\overline{\phi}) = EU((kg(s_{t+1})\overline{\phi})(\mu_z + \lambda(z - \mu_z))) . (9)$$

It is easy to see that  $EU(c^{RW})$  is decreasing and concave in  $\lambda$  .

As shown in Figure 1, for  $\lambda$  close to 0 EU(c<sup>RW</sup>) > EU(c<sup>S</sup>) since the insurance against real shocks obtained by the contract dominates the loss of insurance against the nominal shock. As  $\lambda$  increases, the welfare ordering switches as the variability of the nominal shock increases. Likewise, we could also consider mean-preserving spreads of the real shock to produce a comparable argument.

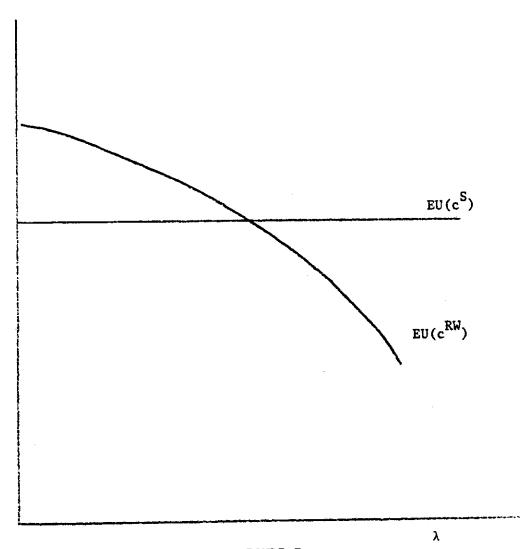


FIGURE I

## IV. Contingent Contracts

While workers do not observe realizations of s or x independently in either the spot market or contract market equilibria, the goods market price revealed information on these shocks. Hence workers may have an incentive to link wages to prices in the form of indexed contracts. This issue has been explored by Gray [1976] in a macro-model with a quadratic loss function over deviations of output from its "desired" level.

Although contracts can be written contingent on the spot price, we will not consider contracts contingent on  $(\hat{s}, \hat{x})$ . From the results of Azariadis [1983], Chari [1983], Cooper [1983], Green-Kahn [1983] and Grossman-Hart [1981], we know that the asymmetry in the information on s is not sufficient to rule out contracts contingent on its realization. That is, the workers and the firm could agree on a contract to induce the firm to reveal  $\hat{s}$ . In our model, if the workers know  $\hat{s}$  they also know  $\hat{x}$  since  $p_t(s,x)$  is publicly observable. In order to induce the firm to announce  $\hat{s}$  truthfully, the contract can no longer specify full employment in all states of nature. Hence, to obtain this information, layoffs must occur. This interesting possibility is currently under investigation.

For now, we concentrate on price-contingent contracts. In a general equilibrium setting, Svensson [1981] discusses the existence of an equilibrium with price-contingent contracts. However, as discussed in Azariadis-Cooper [1981], an equilibrium may not exist. When agents trade price-contingent claims, the equilibrium price function is distorted. In some cases, the trading of these claims destroys all the information conveyed by the price system. 5

In this model with workers and firms, we have seen that the equilibrium price function is <u>independent</u> of the way in which labor services are traded. Hence, the use of price-contingent claims will <u>not</u> distort the information conveyed by the prices.

Denoting realizations of the spot price in period t by  $p_j$ ,  $j=1,2,\ldots,J$ , a price-contingent contract will specify a wage schedule,  $w_t(p_j)$ , and an employment schedule. As before, the optimal contract will be full-employment. Without loss of generality we can write  $w_t(p_j) = \overline{w}_t + q_t(p_j)$ . Here  $\overline{w}_t$  is the equilibrium fixed nominal wage from the previous section and  $q_t(p_j)$  is a price-contingent transfer. As before, competition for workers guarantees that

$$E\left[\frac{q_t(p_t)x_{t+1}}{p_{t+1}}\right] = 0.$$
 (10)

Here the expectation is taken with respect to  $p_t$ ,  $x_{t+1}$  and  $p_{t+1}$ . The optimal contract is determined by solving

$$\max_{\substack{\mathbf{q}_{t}(\mathbf{p}_{i})}} \mathrm{EU} \left( \frac{(\overline{\mathbf{w}}_{t} + \mathbf{q}_{t}(\mathbf{p}_{i})) \mathbf{x}_{t+1}}{\mathbf{p}_{t+1}} \right)$$

subject to (10). Letting  $\,\lambda\,$  be the Lagrange multiplier on (10), the solution satisfies, for all  $\,p_{i}$  ,

$$E\left\{\frac{\mathbf{x}_{t+1}}{\mathbf{p}_{t+1}}\mathbf{U}'\left(\frac{(\overline{\mathbf{w}}_{t}+\mathbf{q}_{t}(\mathbf{p}_{j}))\mathbf{x}_{t+1}}{\mathbf{p}_{t+1}}\right)\bigg|\mathbf{p}_{j}\right\} = \lambda E\left\{\frac{\mathbf{x}_{t+1}}{\mathbf{p}_{t+1}}\bigg|\mathbf{p}_{j}\right\}$$
(11)

In these expressions the expectations are taken over  $x_{t+1}$  and  $p_{t+1}$  given an observation on the current price level. While realizations of  $\hat{x}$  are independent over time, current prices (as in Lucas [1972]) do reveal

information on current realizations of  $\hat{x}$  which affect <u>future</u> prices. Eq. (11) represents efficient risk-sharing given the information solution.

In special circumstances, the solution to (11) reduces to either the spot or contract market solution.

Proposition 1. If  $\hat{x}$  is degenerate, then the solution to (11) is the spot market solution. If  $\hat{x}$  is degenerate (11) yields the contract market solution.

<u>Proof.</u> If  $\hat{s}$  is not random, then observing  $p_j$  is equivalent to observing x. Using (2), (11) reduces to

$$\left\{ \frac{\hat{g}}{m_{t}^{F}} U' \left( \frac{(\bar{w}_{t} + q_{t}(p_{j}))\hat{g}}{m_{t}^{F}} \right) \right\} = \frac{\lambda \hat{g}}{m_{t}^{F}} .$$
(12)

Here  $\hat{g} \equiv g(\hat{s})$  when  $s = \hat{s}$  with probability one. From (12), we see that consumption is completely stabilized (i.e. independent of  $x_t$  or  $x_{t+1}$ ) and

$$w_{t}(p_{j}) = \overline{w}_{t} + q_{t}(p_{j}) = \left[\frac{\overline{m}_{t-1}x_{t}F}{\hat{g}}\right]\overline{c}$$
(13)

where  $\overline{c}$  is the worker's level of consumption. This corresponds to the spot market solution (3) when  $\overline{c} = \hat{g}\phi(\hat{s}) = f_{\hat{k}}(\gamma,\hat{s})$ . So workers, in equilibrium, get paid and consume their marginal product. This is a case of complete indexation of the wage to the nominal shock.

At the other extreme, assume that there is no nominal shock. Hence the current price reveals no information on the future price. From (1) and (11) this implies that  $q(p_j) = 0$  for all  $p_j$  and hence the solution is a constant nominal wage. This corresponds to the case investigated by Azariadis and Baily.

In related research, Gray [1976] showed that if there is no real shock, complete indexation will occur as in this model. However, when there are only real shocks, Gray shows that partial indexation is generally optimal while our results show that no indexation will arise. The difference disappears if we assume a perfectly inelastic labor supply schedule in Gray's model.

When both shocks are present, neither of these two extremes will generally occur. The optimal degree of indexation will reflect, among other things, the information conveyed by spot prices. If  $p_j$  reveals both s and x, then wages will be indexed to x and consumption will be independent of the nominal shocks to the economy. This could occur, for example, if  $\hat{s}$  and  $\hat{x}$  were discrete.

<u>Proposition 2.</u> When spot prices reveal <u>both</u> s and x ,  $w_t(p_j)$  is proportional to  $m_t$  .

<u>Proof.</u> When p reveals s and x, then in equilibrium (11) becomes

$$E_{s_{t+1}} \left\{ \frac{g(s_{t+1})}{m_{t}^{F}} U^{T} \left[ \frac{(\overline{w}_{t} + q_{t}(p_{j}))g(s_{t+1})}{m_{t}^{F}} \right] \right\} = E_{s_{t+1}} \left[ \frac{g(s_{t+1})}{m_{t}^{F}} \right] . (14)$$

Since  $x_t$  is known, (14) implies that consumption is <u>independent</u> of  $m_t$ . Hence  $w_t(p_j)$  must be proportional to  $m_t$ .

In each of the cases examined thus far, money is neutral. Once p(s,x) is not revealing, however, monetary shocks will affect agent's consumption. If  $\hat{s}$  and  $\hat{x}$  are continuous random variables, prices will generally not be revealing. As in Lucas [1972] and the subsequent literature, agents are therefore unable to determine  $\hat{s}$  and  $\hat{x}$  independently.

Proposition 3. If p(s,x) is not revealing, then money is not neutral.

Proof. Using (2), we can rewrite (11) as

$$E_{\mathbf{x_{t},s_{t+1}}} \left\{ \frac{g(s_{t+1})}{m_{t-1}x_{t}^{F}} U' \left( \frac{(\overline{w}_{t} + q_{t}(p_{j}))g(s_{t+1})}{m_{t-1}x_{t}^{F}} \right) \middle| p_{j} \right\} = \lambda E \left[ \frac{g(s_{t+1})}{m_{t-1}x_{t}^{F}} \middle| p_{j} \right]. \quad (15)$$

To see that consumption must depend on  $x_t$ , we simply note that since p(s,x) is not revealing, there must be more than one combination of (s,x) yielding a given  $p_j$ . From (15),  $\overline{w}_t + q_t(p_j)$  is determined by  $p_j$ . In equilibrium, the consumption of a generation t worker with price-contingent contracts,  $c^{pc}$ , is given by

$$c^{pc} = \frac{(\overline{w}_{t} + q_{t}(p_{j}))g(s_{t+1})}{m_{t-1}x_{t}F} .$$
 (16)

From (16), as  $x_t$  varies for a given  $p_t$ ,  $c^{pc}$  must vary as well. Hence money can not be neutral.

Finally, we need to characterize  $w_t(p_j)$ . How does  $w_t$  vary with  $p_j$ ? Is it ever the case that  $w_t$  is a constant as in the contract market equilibrium? It is rather difficult to compute dw(p)/dp from (15) without further simplifications of the model. In particular, we need to specify how the conditional density of x varies with p.

Letting F(x|p) be the conditional distribution of x given p, we follow Lucas [1972] and assume that  $dF(x|p)/dp \le 0$ . That is, increases in p shift the cumulative distribution of x to the right. So observing a high p increases the likelihood of a large nominal shock.

Hence, an increase in p will lead to a reduction in the right-hand side of (15). The effects of an increase of p on the left-hand

side of (15) depends on the sign of V'(c) where  $V(c) \equiv cU'(c)$  and c is consumption. This term, as discussed in Lucas [1972], influences the slope of the Phillips curve in a model with elastic labor supply. One can show that V'(c) = U'(c)(1-R(c)) where R(c) is the Arrow-Pratt measure of relative risk aversion.

Proposition 4. If R(c) > 1 for all c and dF(x|p)/dp < 0, then w(p) increases in p.

<u>Proof.</u> If dF(x|p)/dp < 0, then the right-hand side of (15) falls with p. If R > 1 then the left-hand side of (15) will increase with p. To maintain (15) for all p, w(p) must increase with p.

From this proposition it should be clear that the magnitude of dw/dp will depend on R. The larger is R, the more w will vary with p. When R = 1 for all c, it is easy to see that the left-side of (15) is independent of p so that w(p) is set to keep the expected real wage constant for all p. 7 For R < 1, it is conceivable that as p varies the two sides of (15) may change by the same amounts so that w(p) =  $\overline{w}$  for all p. This will obviously not hold in general. In addition, dw/dp will depend on the information conveyed by p on (s,x).

With respect to optimal market structure, since the equilibrium price function does not depend on the choice of contract, the use of price contingent contracts will always dominate (at least weakly) the use of fixed nominal wage contracts in terms of expected utility. Except for the special case in which  $w(p) = \overline{w}$  satisfies (15), this will be a strict domination. So, in an  $\overline{ex}$  ante contracting equilibrium, firms

will be forced, by competition, to offer workers price contingent agreements rather than rigid wages. Hence, at least in this model, the only means of explaining the use of rigid nominal wages is by assuming a cost of contingent contracting. If these costs exist, then we would observe the use of nominal wage contracts when the loss of the information contained in p is not too costly: i.e. when the nominal shock is not too variable. As the variability of the nominal shock increases, we might see indexation arise as a means of incorporating into a contract the more valuable information on  $\hat{x}$  contained in the spot price.

### V. Monetary Policy

Using the characterizations of equilibrium for the different market structures, we can now examine the role of monetary policy. In particular, we can make explicit comparisons of expected utility under alternative monetary policies to determine the optimal policy.

First we consider the spot market equilibrium. Since money is neutral in this case, the choice of policy is obviously irrelevant. Only when the choice of a monetary rule affects the distribution of the money shock and alters the market structure (see Proposition 1) will monetary policy have real affects.

If, however, labor services are traded through fixed nominal wage contracts, the results are quite different. First, as noted in Section III, real consumption will depend on the nominal shock. Given this non-neutrality, we can address questions of a feedback rule and optimal monetary policy.

One of the primary results of the model of Sargent-Wallace [1975] was the neutrality of a known feedback rule. While that model considered

the effects of money on employment rather than redistribution effects, the difference is not important. Using a macro-model where output depends on the difference between actual and expected prices, Sargent-Wallace found that the distribution of real output was independent of any feedback rule which was public information. Azariadis [1981] used a simplified version of Lucas' [1972] model to show that the results of Sargent-Wallace did not necessarily hold in general. By altering the information conveyed by market prices, non-stochastic monetary policy can affect the information sets of agents and hence have real affects.

In any time period t, we denote by  $I_{t-1}$  the information contained in the history of the economy through period t-1.  $I_{t-1}$  will include past realizations of  $\hat{s}$  and  $\hat{x}$ . A monetary feedback rule determines the current value of x as a known function of past information and a stochastic term; i.e.,

$$x_t = \beta(I_{t-1}, \mu_t)$$

where  $\beta(\cdot)$  is a time invariant function,  $I_{t-1}$  is the information set and  $\mu_t$  is an i.i.d. random variable. We do not allow the monetary authorities to have better information than the agents so that  $s_t \notin I_{t-1}$ .

The Sargent-Wallace result would imply that real allocations are independent of the  $\beta(\cdot)$  which is chosen. Assume for the moment that

$$\beta(I_{t-1}, \mu_t) = \hat{\beta}(I_{t-1})\mu_t . \tag{17}$$

When the monetary policy follows this feedback rule, we find that real allocations are independent of  $\,\hat{\beta}\,$  .

Proposition 5. If  $\beta(I_{t-1}, \mu_t) = \hat{\beta}(I_{t-1})\mu_t$ , then real allocations are independent of  $\hat{\beta}$ .

Proof. Using this feedback rule, (7) becomes

$$\overline{w}_{t} = \frac{m_{t-1} \hat{\beta}(I_{t-1}) E_{\phi}(s) (1+\gamma)}{E(1/\mu)}$$
(18)

Using (8), consumption is

$$c^{RW} = \frac{g(s_{t+1})E[\phi(s)]}{\mu_t E[1/\mu]}.$$

This proposition then lends support to the Sargent-Wallace result. From (18), we see that the deterministic part of  $\beta(\cdot)$  is treated as if it was a part of the inherited money stock,  $m_{t-1}$ . Since allocations are independent of the past money stock, Proposition 5 should not be too surprising. As discussed in Azariadis [1981], a multiplicative feedback rule such as (17) is really equivalent to a change in the unit of accounts. Since their models are in log terms, the results of Sargent-Wallace, Fischer [1977], Gertler [1982] and others are consistent with Proposition 5.

Once the feedback rule loses this multiplicative structure, the neutrality results fail. This is shown in the following proposition.

<u>Proposition 6.</u> If  $\beta(I_{t-1}, \mu_t)$  does not satisfy (17), then real allocations will depend on the choice of feedback rule when labor services are traded in the contract market.

Proof. Using a general feedback rule, consumption of generation t workers is

$$\mathbf{c}^{\mathrm{RW}} = \frac{\overline{\phi}\mathbf{g}(\mathbf{s}_{\mathsf{t}+1})}{\mathbf{E}[1/\mathbf{x}] \cdot \mathbf{x}_{\mathsf{t}}} = \frac{\mathbf{g}(\mathbf{s}_{\mathsf{t}+1}) \mathbf{E}_{\mathsf{s}} \phi(\mathbf{s})}{\mathbf{E}\left[\frac{1}{\beta(\mathsf{I}_{\mathsf{t}-1}, \ \boldsymbol{\mu}_{\mathsf{t}})}\right] \beta(\mathsf{I}_{\mathsf{t}-1}, \ \boldsymbol{\mu}_{\mathsf{t}})} \ .$$

As long as  $\beta(\cdot, \cdot)$  is <u>not</u> multiplicative, consumption will depend on the feedback rule chosen.

As in the multiplicative case,  $\overline{w}_t$  reflects the forecastable portion of  $x_t$ . However, once the feedback rule is <u>not</u> multiplicative, the choice of a rule is not equivalent to a change in the unit of account and the neutrality result will not generally hold. A similar result was obtained in Azariadis [1981].

Before discussing optimal monetary policy, we consider the case where labor contracts are price contingent. If prices are revealing, then policy changes which do not alter this property will be neutral. If policies do alter the information structure (as in the examples of Azariadis [1981]), then these policies will not be neutral. When feedback rules are multiplicative, then from (15) even when prices are not revealing, real allocations are independent of  $\hat{\beta}$ . Once feedback rules lose this multiplicative form, consumption, and the degree of indexation, will depend on the rule chosen.

Finally, we can determine an optimal monetary policy for our economy. Our intent so far has been to compare alternative markets for the trading of labor services. Since only trivial allocative decisions are made in this economy, we have compared the equilibria under these market structures with respect to their ability to share the real and nominal shocks. Our informational assumptions have restricted the set of available contracts and allowed a role for monetary policy. In this incomplete market setting, we can show that the choice of a monetary policy can affect the risk sharing capability of the market structures. Intuitively, the appropriate choice of monetary policy can substitute for missing claims markets.

Proposition 7. A non-stochastic monetary policy can support a constrained optimal allocation.

<u>Proof.</u> If monetary policy is non-stochastic then labor services will be traded via fixed wage contracts (or, equivalently, there is no indexation of the price-contingent contracts). From (8), workers of generation t have consumption of

$$c = E[\phi(s)] \cdot g(s_{t+1}) .$$

Since firms are indifferent with respect to the structure of the market for labor services, and consumers bear no risk in youth, this must be a constrained optimum.

We use the term constrained optimum here since we have not allowed the sharing of consumption risk arising in old age. Following Azariadis-Cooper [1982], we could allow non-contingent claims for the sale of the consumption good. In old age, workers would purchase these claims which would be issued by old entrepreneurs at a price of E(1/p). In this case, old workers would shed all their risk due to  $s_{t+1}$  to the old risk neutral agents. Hence, a non-stochastic monetary rule combined with the non-contingent trade of both goods and labor would yield an optimal allocation of risks.

Proposition 7 lends strong support to the view that non-stochastic monetary policies are desirable. In this model, it allows for a more efficient sharing of real risks. It should be noted that in a version of the Lucas model, Polemarchakis-Weiss [1977] and Azariadis [1981] show that a non-stochastic rule need not be optimal. This was attributed to the lack of insurance markets and Proposition 7 supports this view.

A final point on the role of monetary rule is worth noting. If, for some reason, the monetary authorities choose a stochastic monetary

process, they are constrained by the agent's choice of market structure. For example, when contingent contracts are costly and the monetary shock is not too variable, agents may trade labor services through fixed nominal wages. In such a situation, we know that monetary policy is not neutral. If the monetary authorities try to take advantage of this non-neutrality, a change in policy which increases the variability of the nominal shock may induce agents to start trading in spot labor markets. This would make monetary policy completely neutral. The point, as first suggested by Lucas [1976], is that the structure of the economy depends on monetary policy and this dependence should be incorporated into any policy formulation.

#### VI. Conclusion

The purpose of this paper was to investigate the role of alternative contractual settings in the allocation of real and nominal risks. Using a model with trivial production and consumption decisions we were able to compare the expected utility for agents of spot market trades, fixed nominal wage contracts and indexed contracts. Unlike the earlier results of Azariadis [1975] and Baily [1974], no clear welfare ordering between spot markets and fixed nominal wage contracts arose. Costless indexation to a price function was shown to be the dominant market structure. Once indexation was costly, fixed nominal wage contracts could emerge as an optimal market structure.

Using this model, we also discussed optimal monetary policy. Only multiplicative feedback rules were shown to be neutral. A totally non-stochastic monetary policy can substitute for missing claims markets and support, along with non-contingent claims markets for labor and goods, an optimal allocation of risks.

The major extension of this research will be to introduce a nontrivial labor supply choice. By doing so, we hope to generate a Phillips curve in the market structures for which money is not neutral. The other point of introducing this additional decision concerns the domination of price-contingent wage contracts. Once labor supply is elastic, output and hence the equilibrium price function will not necessarily be invariant to the choice of contract. Hence, when we enlarge the workers' choice set by allowing indexation we may also distort the price function in a way which reduces lifetime expected utility. Discussions of this possibility are motivated by the work of Hart [1975] and Green [1981]. This would provide an alternative to the transactions costs argument for the existence of fixed nominal wage contracts.

24

#### **FOOTNOTES**

- This means of injecting money is quite popular (see, for example, Lucas [1972]). If money transfers are not proportional then monetary policy creates direct wealth effects and will not be neutral. In this paper, we investigate the non-neutrality of proportional transfers as a means of distinguishing these channels of monetary policy. This also facilitates a comparison of the results in this paper with those in Lucas [1972] and the subsequent literature.
- <sup>2</sup>I am grateful to Bennett McCallum for pointing out some confusing elements in the notation used in a previous version of the paper.
- <sup>3</sup>This prohibition against contracts denominated in consumption goods is motivated by the observation that workers and firm shareholders are both spatially and temporally separated so that trades between these groups are undertaken with money.
- To see this, note that firms are indifferent between hiring in the spot market and in a contract market if (6) holds since the employment level is constant across states. Hence if each firm wants  $\gamma$  workers in the spot solution, it will demand the same number of workers when (6) holds. We also need to assume that bankruptcy will not be a problem. That is, in each state of nature the firm must have adequate money balances to pay  $\overline{w}_t$  to each of its workers. We assume that  $N\overline{w}_t \leq \overline{w}_{t-1}\overline{x}$ . Using (6) and (7) this requires  $\gamma k \phi \leq \overline{x}$  so that we can always set  $\overline{x}$ ,  $\gamma$  and the production function to ensure this condition holds. See the recent paper by Farmer [1983] for a discussion of the bankruptcy problem.
- $^{5}$ This possibility of non-existence is similar to that discussed in Kreps [1977].
- <sup>6</sup>I am grateful to Gary Fethke for discussions on this point.
- <sup>7</sup>Bill Lang deserves credit for this observation.
- $^8$ As another special case, quadratic preferences combined with normally distributed random variables will yield an indexation rule which depends on the relative variances of the real and nominal shocks. In general, as observations of p convey more information about x, we would expect the degree of indexation to increase.
- Here we used the maximization of expected lifetime utility for a representative risk averse agent subject to an expected utility constraint on the firm's as the planners problem. Since our interest is in the sharing of risks in youth, this is the appropriate problem. See Peled [1982] for a discussion of the choice of objective problem for a planner.

#### REFERENCES

- Azariadis, C. "Implicit Contracts and Underemployment Equilibria,"
  Journal of Political Economy 83 (1975): 1183-1202.
- Review 71, no. 5 (1981): 946-960.
- "Employment with Asymmetric Information," Quarterly Journal of Economics 98 (1983): 157-173.
- and R. Cooper. "On Predetermined Prices," CARESS Working Paper #81-08 (1981).
- and . "Predetermined Prices and the Allocation of Social Risks," mimeo, 1982 (forthcoming Quarterly Journal of Economics).
- Baily, M. "Wages and Employment under Uncertain Demand," Review of Economic Studies 41 (1974): 37-50.
- Barro, R. "Unanticipated Money Growth and Unemployment in the United States," American Economic Review 67 (1977a): 101-15.
- . "Long-Term Contracting, Sticky Prices and Monetary Policy,"

  Journal of Monetary Economics 3 (1977b): 305-316.
- . "Unanticipated Money, Output and the Price Level in the United States," Journal of Political Economy 18 (1978): 549-80.
- Cass, D. and M. Yaari. "A Re-examination of the Pure Consumption Loans Model," Journal of Political Economy 74 (1966): 353-367
- Chari, V. V. "Involuntary Unemployment and Implicit Contracts," <u>Quarterly</u>
  <u>Journal of Economics</u> 98 (1983): 107-123.
- Cooper, R. "A Note on Overemployment/Underemployment in Labor Contracts under Asymmetric Information," <u>Economics Letters</u> 12 (1983): 81-89.
- Diamond, P. "The Role of a Stock Market in a General Equilibrium Model with Technological Uncertainty," <u>American Economic Review</u> 57 (1967): 759-773.
- Farmer, R. E. A. "Implicit Contracts with Asymmetric Information and Bankruptcy: The Effect of Interest Rates on Layoffs," 1983 (forthcoming Review of Economic Studies).
- Fischer, S. "Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule," <u>Journal of Political Economy</u> 85, no. 1 (1977): 191-205.
- Gertler, M. "Imperfect Information and Wage Inertia in the Business Cycle," Journal of Political Economy 90, no. 5 (1982): 967-987.

- Gray, J. 'Wage Indexation: A Macroeconomic Approach,' Journal of Monetary Economics 2 (1976): 221-235.
- Green, J. "Value of Information with Sequential Futures Markets," Econometrica 49, no. 2 (1981): 335-358.
  - and C. Kahn. "Wage-Employment Contracts," Quarterly Journal of Economics 98 (1983): 173-189.
- Grossman, S. and O. Hart. "Implicit Contracts, Moral Hazard and Unemployment," American Economic Review 71 (1981): 301-308.
- Hart, O. "On the Optimality of Equilibrium When the Market Structure Is Incomplete," Journal of Economic Theory 11 (1975): 418-443.
- Kreps, D. "A Note on 'Fulfilled Expectations' Equilibria," <u>Journal of Economic Theory</u> 14 (1977): 32-43.
- Lucas, R. "Expectations and the Neutrality of Money," <u>Journal of Economic</u> Theory, 4 (1972): 103-124.
- . "Econometric Policy Evaluations: A Critique," in The Phillips

  Curve and Labor Markets (eds. K. Brunner and A. Meltzer), CarnegieRochester Conference Series on Public Policy, Amsterdam: North
  Holland, 1976, pp. 19-46.
- Mishkin, F. "Does Anticipated Monetary Policy Matter? An Econometric Investigation," Journal of Political Economy 90 (1982): 22-51.
- Newberry, D. and J. Stiglitz. "The Choice of Techniques and the Optimality of Market Equilibrium with Rational Expectations," <u>Journal of Political Economy</u> 90, no. 2 (1982): 223-246.
- Peled, D. "Informational Diversity over Time and the Optimality of Monetary Equilibria," Journal of Economic Theory 28 (1982): 255-275.
- Phelps, E. and J. Taylor. "Stabilizing Powers of Monetary Policy under Rational Expectations," <u>Journal of Political Economy</u> 85, no. 1 (1977): 163-190.
- Polemarchakis, H. and L. Weiss. "On the Desirability of a Totally Random Monetary Policy," <u>Journal of Economic Theory</u> 15 (1977): 345-350.
- Samuelson, P. "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money," <u>Journal of Political Economy</u> 66 (1958): 467-482.
- Sargent, T. and N. Wallace. "Rational Expectations, the Optimal Monetary Instruments, and the Optimal Money Supply," <u>Journal of Political Economy</u> 83 (1975): 241-254.
- Svensson, L. "Efficiency and Speculation in a Model with Price-Contingent Contracts," Econometrica 49 (1981): 131-152.