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THE STRUCTURE OF SOCIAL RISK

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THE STRUCTURE OF SOCIAL RISK

It is increasingly recognized that the structure of financial risks interacts with economic or fundamental risks in a way that influence real economic outcomes. Recent work documents, on the one hand, the apparent excessive sensitivity of financial markets to economic shocks (see especially Shiller (1979)); and on the other hand the close dependence of investment and economic variables on financial variables.

One of the central developments to analyze such interactions has been portfolio theory, particularly the capital asset pricing model (CAPM). This field has been extremely fertile, and has seen an outpouring of both theoretical and empirical work. Unfortunately, virtually all the work has been directed at a very narrow set of concerns--stock market performance. Thus virtually every major firm has been extensively studied and has its own "beta" estimates from numerous beta vendors.

To our knowledge, however, there has been no attempt to apply these tools to the economy as a whole. The present paper is a preliminary attempt to make such estimates. The first section develops the theory; the second outlines the data; the third presents the estimates; while the last turns to the implications.

I. Deriving the Capital Asset Equation

1. Fundamentals

It is useful to begin with the more complicated but more satisfactory model--the consumption capital asset prices model (CCAPM) developed by Breeden (1979). Assume that there is a representative consumer who receives a stream of labor income with certainty and must allocate it over time to maximize his utility. Thus he maximizes the expected value of

$$(1) \quad U = \sum_{v=0}^T \delta^v u(c_v)$$

subject to

$$(2) \quad W_{t+1} = W_t (1 + \sum_i \gamma_{i,t} r_{i,t}) + L_t - c_t$$

$$(3) \quad \sum_i \gamma_{i,t} = 1$$

where δ is the subjective discount factor, $u(\cdot)$ is the one period utility function, c_v is consumption, w_t is wealth, $\gamma_{i,t}$ is the share of asset i in the portfolio, $r_{i,t}$ is the one-period real rate of return on asset (i.e. the nominal return less the rate of inflation on consumer prices), and L_t is labor income.

An optimal path may be constructed by considering variations in consumption between period 0 and period t , say $t = 1$. In such a case, an optimal plan is one in which an increment to asset i has zero value. Thus, withdrawing a unit of consumption and investing in asset i yields a change in welfare of

$$\Delta U = -u'(c_0) + E\{\delta u'(c_1)(1 + r_{i,1})\}.$$

Thus for an optimal plan

$$(4) \quad u'(c_0) = E\{\delta u'(c_1)(1+r_{i,1})\}$$

or

$$(5) \quad 1 = E\{S_1(1+r_{i,1})\}$$

where $S_t = \delta^t u'(c_t)/u'(c_0)$ is the marginal rate of substitution between present and future consumption.*

Now, equation (5) cannot be directly estimated because it contains an unobserved variable, S_1 . There are two approaches that can be followed, depending on one's tastes: (1) we can use a rate of return proxy for S_1 , which gives us the standard CAPM model derived in Section I.2 below. (2) or we can assume the form of utility function for the consumption CAPM (or Breeden version) of Section I.3 below.

2. The CAPM Variant

The multiperiod CAPM model has been investigated by Merton (1972) and others. He has shown that, under limited circumstances such as quadratic utility, the relation between returns are

$$(6) \quad \hat{r}_i = r_f + b_i(\hat{r}_m - r_f)$$

where the hats represent ex ante (or anticipated) returns, r_i is the return on the i^{th} asset; r_m is the return on the market; and the b_i 's are the ratios of the ex post covariance of r_m and r_i to the variance of r_m .

*Equation (5) was derived by Breeden (1979) as well as Grossman and Shiller (1980).

There are major strengths and weaknesses to this approach. The major strength is that all variables in (6) are observable. The major weaknesses are two: first, as Breeden has shown, the model is incorrect for intertemporal allocation decisions except under limited conditions. Second, it is not at all obvious what the appropriate market rate is.

The CCAPM (Breeden) Variant

To obtain a theoretically more satisfactory result, we can work with the exact equation in (5). Assuming a very short time period, and assuming the present is $t = 0$, we can rewrite (5) as:

$$(7) \quad 1 + E(r_{i,1}) = \frac{1 - \text{cov}(r_{i,1}, S_1)}{E(S_1)}$$

If a riskfree asset exists, with yield r_f , we know that

$$(8) \quad 1 + r_f = \frac{1}{E(S_1)} .$$

(If no riskfree asset exists, r_f would be the "zero-beta portfolio"--the minimum variance portfolio that has zero covariance with S_1 .)

Substituting (8) into (7) yields:

$$(9) \quad 1 + E(r_{i,1}) = (1 + r_f)[1 - \text{cov}(r_{i,1}, S_1)] .$$

To make equation (9) operational requires finding a proxy for S_1 . The most tractable approach is to take the first order approximation to the logarithmic Taylor series (or equivalently, to assume $u(c)$ is a power function implying a constant relative risk aversion). Thus

$$(10) \quad u(c_t) = \frac{1}{1-\alpha} c^{1-\alpha} .$$

From this we obtain

$$(11) \quad S_t = \delta^t c_t^{-\alpha}, \quad \infty \geq \alpha \geq 0.$$

Using (11) we can then estimate (9) directly.

II. Empirical Approach

A. Data

In what follows we will estimate both models outlined in Section I for the aggregate U.S. economy over the 1967-81 period. We assume that labor income (or "human capital") is exogenous, non-tradable, and non-random. All other assets are assumed to be divisible and traded in competitive markets with negligible transactions costs. The following, drawn from the national balance sheet for 1972, gives the totals and shares of the market portfolio:

Asset	Value (billion)	Share (percent)
Corporate fixed capital		
Equipment	510.7	} 41.5
Structures	669.1	
Inventories	543.2	
Housing	1819.2	42.0
Government bonds		
Less than 1 year	87.15	3.0
Greater than 1 year	77.7	3.0
Consumer durables	452.7	10.0
Monetary base*	100.6	---*
	\$4259.2	100.0%

*Ultimately omitted from the portfolio; not included as share.

Throughout this paper, the market portfolio shares will be assumed constant at these levels. The observations are monthly.

The return on the above assets were calculated as follows: the market value of corporate fixed capital is equal to the market value of corporations. Taxes are ignored. It is assumed that housing and consumer durables are bought and sold at replacement cost ($q = 1$) on perfect frictionless used goods markets. Thus the only deviation of market value from replacement cost is for corporate capital.

There are clearly numerous difficulties with these assumptions.

One important problem is the assumption that households can diversify away own-risk in their houses and consumer durables. In addition, the assumption that $q = 1$ for all assets but corporate assets is extreme. In both of these cases, however, the data do not allow another, more realistic approach.

In calculating the results, we have employed the observed market returns. The following identifies the major data sources and techniques for constructing the series.

1. Corporate Fixed Capital: The market returns for fixed capital are calculated as a weighted average of the returns on corporate equities, short corporate bonds and long corporate bonds. The weights were determined by 1975 Flow of Funds corporate balance sheet. The return on equities was determined by the monthly holding returns on the Standard and Poor 500. This is equal to the interpolated quarterly dividend yield plus the monthly capital gains. The returns on short and long bonds were assumed to equal the return on their respective government counterparts discussed below.

2. Housing: Returns are calculated as the sum of the service returns on housing plus the monthly capital gains. Imputed service returns were estimated by Jorgenson and Fraumeni (1980). Capital gains were derived from the home purchase component of the CPI (PZU221).

3. Long Term Government Bonds: Rates are estimated from the yield on 20 year treasury bonds plus the implicit capital gains. Capital gains are calculated using the consol formula.

4. Short Term Government Bonds: Return is the 90 day treasury bill rate.

5. Consumer Durables: Rate is calculated as the sum of the capital gains and service returns. Service returns are again taken from Jorgenson

and Fraumeni. Capital gains are estimated from the consumer price index series on used automobiles (PZU46) (in 1974 approximately 40% of consumer durable expenditures were on automobiles).

6. Monetary base is omitted from the market portfolio due to the lack of reliable data on service returns.

7. In calculating the real interest rates, the monthly CPI excluding home purchase and home interest costs is used (PU22X).

III. Results: CAPM

It will be useful to proceed in a sequence of steps to investigate possible biases or misspecifications. We start with the simplest specification and move to more complete ones.

Model 1. Standard CAPM

The standard model is the following:

$$(12) \quad \hat{r}_{i,t} = r_{f,t} + \beta_i (\hat{r}_{m,t} - r_{f,t})$$

where all variables are as before, but the hats over variables indicate the rates of return are ex ante, or anticipated. To obtain estimates we substitute the ex post returns $(r_{i,t})$, where

$$(13) \quad \hat{r}_{i,t} = r_{i,t} + e_{i,t}$$

which yields

$$(14a) \quad r_{i,t} = r_{f,t} + \hat{\beta}_i (r_{m,t} + e_{m,t} - r_{f,t}) - e_{i,t}$$

$$(14b) \quad r_{m,t} = \sum_i r_{i,t} \gamma_i$$

$$(14c) \quad e_{m,t} = \sum_i \gamma_i e_{i,t}$$

Using OLS estimation on (14) will provide appropriate estimates of the β_i , but there are two potential problems, both arising from inflation: First, the returns are nominal rather than real; second no assets are risk-free, because of the price level uncertainty. Nevertheless, we present the results in line 1 of Table 1 to compare with the corrected version.

Model II. Real returns

The next step allows for the fact that the price level is uncertain. This estimates equation (14), but uses real returns rather than nominal returns. The results are shown in Table 1, line 2.

Model III. No riskless asset

The final step accounts for the fact that there is, in fact, no perfectly riskless asset. The modification of the CAPM procedure for such a case proceeds as follows. It can be shown (again under the restrictions of quadratic utility or continuous time) that the optimal portfolio is a mixture of the market portfolio and a "zero-beta" portfolio. The latter is the minimum variance portfolio that has no correlation with the market. In this case, the excess return on an asset is

$$\hat{r}_i = \hat{r}_z + \beta_i (\hat{r}_m - \hat{r}_z) .$$

The construction of the zero-beta portfolio proceeds as follows. Let $\gamma^z = (\gamma_1^z, \dots, \gamma_n^z)$ be the weights on the zero beta portfolio, while $\gamma^m = (\gamma_1^m, \dots, \gamma_n^m)$ be the weights on the market portfolio, s_{ij}^2 is the covariance between r_i and r_j . Then the zero beta portfolio is found as the solution of the following problem

$$(15) \quad \left\{ \begin{array}{l} \min_{\{\gamma^z\}} \sum_{i,j} \gamma_i^z s_{ij}^2 \gamma_j^z \\ \text{subject to} \\ \sum_{i=1}^n \gamma_i^z = 1 \\ \text{and} \\ \sum_{i,j=1}^n \gamma_i^m s_{ij}^2 \gamma_j^z = 0 \end{array} \right.$$

To find these, we first estimate a stationary variance-covariance matrix \underline{S} for our n risky assets, and $\underline{e} = (1 \dots 1)$. We then have

$$\min_{\{\underline{w}^z\}} \underline{Y}^z \underline{S} \underline{Y}^{z'}$$

subject to

$$\underline{Y}^m \underline{S} \underline{Y}^{z'} = 0$$

$$\underline{Y}^z \cdot \underline{e} = 1$$

Forming a Lagrangean

$$(16) \quad \min_{\{\underline{Y}^z\}} L = \underline{Y}^z \underline{S} \underline{Y}^{z'} + m_1 (\underline{Y}^m \underline{S} \underline{Y}^{z'}) + m_2 (\underline{Y}^z \underline{e}' - 1)$$

from which we obtain the following solution:*

$$(17) \quad \begin{bmatrix} 2\underline{S} & \underline{S} \underline{Y}^{m'} & \underline{e}' \\ \underline{Y}^m \underline{S} & 0 & 0 \\ \underline{e} & 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{Y}^{z'} \\ m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

One of the interesting results of this experiment is to determine how the zero-beta asset differs from the hypothetical riskless asset. Up to now we have taken the riskless asset to be Treasury bills.

The estimation provides for a zero-beta portfolio that has 112% short bonds, -2% capital, -10% housing, and virtually no consumer durables.

*This procedure is suggested in Weston and Copeland (1980), p. 173.

The pattern of betas, shown in line 3 of Table 1, shows very little change from line 2, indicating that accounting for the risky nature of nominal short-term bonds does not change the structure of social risk. It is also comforting to note that the zero-beta portfolio is not ridiculously leveraged.

The results shown in line 3 of Table 1 is our preferred CAPM model, as it includes the most realistic set of assumptions concerning the structure of risks. Comparing this with the other CAPM models (lines 1 and 2) the results are extremely robust. They indicate that the social risk is very high on both capital, equities, and long term bonds; and small but positive on housing and consumer durables.

Subperiod Results

As there have been numerous structural changes in the last 15 years it is useful to estimate the model over subperiods. Table 2 divides the sample into three subperiods:

- I. Pre-OPEC (1967-72)
- II. OPEC to Quantitative Accord of October 1979 (1973-79)
- III. Post Quantitative Accord (1980-1981)

The estimates shown are for the real and nominal returns (corresponding to lines 1 and 2 of Table 1).

The results are quite instructive. The structure of risk premia did not shift greatly after the OPEC oil shock--there is a suggestion of a rise in the risk premium on capital but little else.

The major change occurred after the Quantitative Accord of October 1979. According to these estimates the beta on long term bonds rose from around 1.5 in the 1973-78 period to 2.9 after the Accord. (The reason for this, as well as possible implications, are outlined in W. Nordhaus, "Are Real Interest Rates Really High?" , April 1982, processed.)

TABLE 1

Estimated Betas for the CAPM Model,
Different Specifications, 1967-81

(Figures are estimated betas, with standard errors in parentheses.)

	Capital	Equities	Long Bonds	Housing	Consumer Durables	Short Bonds
1. Nominal	2.0 (.06)	2.8 (.16)	2.1 (.17)	.18 (.05)	.38 (.16)	--
2. Real	2.0 (.06)	2.8 (.16)	2.1 (.16)	.17 (.04)	.37 (.16)	--
3. Zero-Beta	1.9 (.06)	2.7 (.16)	2.0 (.10)	.25 (.04)	.40 (.15)	.05 (.004)

TABLE 2
 Estimates of Betas for Subperiods, Models 1 and 2

<u>Period</u>	<u>Capital</u>	<u>Equities</u>	<u>Government Longs</u>	<u>Housing</u>	<u>Consumer Durables</u>
Model 1. Nominal Returns					
1967-1972	2.1 (.10)	3.0 (.28)	2.1 (.27)	.08 (.07)	.39 (.30)
1973-1979(H1)	2.0 (.10)	3.3 (.23)	1.5 (.21)	.21 (.07)	.45 (.27)
1979(H2)-1981	1.9 (.10)	1.9 (.40)	2.9 (.52)	.27 (.12)	.13 (.21)
Model 2. Real Returns					
1967-1972	2.1 (.10)	3.0 (.28)	2.1 (.26)	.08 (.07)	.39 (.30)
1973-1979(H1)	2.0 (.10)	3.3 (.24)	1.5 (.20)	.21 (.07)	.45 (.26)
1979(H2)-1981	1.9 (.12)	1.9 (.41)	2.9 (.52)	.27 (.12)	.19 (.21)

IV. Results of Consumption CAPM

In estimating the structure of social risk for the CCAPM approach, we are in even more poorly charted waters than for CAPM. Recall that the basic relation is

$$(18) \quad 1 + E(r_{i,1}) = (1 + r_f)[1 - \text{cov}(r_{i,1}, S_1)]$$

where S_1 is the marginal rate of substitution between assumption of period 1 and period 0.

In our empirical estimates, we assume that utility can be described by the power function:

$$(19) \quad u(c_t) = \frac{1}{1-\alpha} c_t^{1-\alpha}, \quad \infty < \alpha \leq 0$$

so using the notation of equations (1) and (19):

$$(20) \quad S_t = \delta u'(c_1)/u'(c_0) = \delta \left(\frac{c_1}{c_0} \right)^{-\alpha}$$

$$(21) \quad 1 + E(r_{i,1}) = (1 + r_f) \left\{ 1 - \text{cov} \left[r_{i,1}, \delta \left(\frac{c_1}{c_0} \right)^{-\alpha} \right] \right\}.$$

To implement (21) requires an estimate of consumption. Since the appropriate concept is consumption services, we use the National Income Accounts real consumption expenditures on services and nondurables. This appropriately includes the services of housing, but excludes the services of other consumer durables.

We can obtain the risk premium from equation (21) by using market weights (γ^m). Averaging over all elements of the portfolio:

$$\begin{aligned}
 (22) \quad 1 + E(r_{m,1}) &= 1 + \sum \gamma_i E r_{i,1} = (1 + r_f) \left\{ 1 - \sum \gamma_i \text{cov} \left[r_{i,1}, \delta \left(\frac{c_1}{c_0} \right)^{-\alpha} \right] \right\} \\
 &= (1 + r_f) \left\{ 1 - \text{cov} \left[r_{m,1}, \delta \left(\frac{c_1}{c_0} \right)^{-\alpha} \right] \right\}
 \end{aligned}$$

Using this approach we can directly estimate the parameters of our assumed utility function. From equations (8) and (20) we have

$$(23) \quad 1 + r_f = \frac{1}{E[\delta (c_1/c_0)^{-\alpha}]} .$$

Taking our alternative values of α , we can calculate, from the ex post value of r_f and c_1/c_0 , an estimate of the rate of discount on utility ($\rho = 1/\delta - 1$), percent per annum.*

<u>Value of α</u>	<u>Value of Discount Rate, ρ (percent per annum)</u>
1/2	-1.1
1	-2.7
2	-5.8
4	-11.7
10	-27.5
200	-99.99

It is clear that only very modest values of α are consistent with plausible rates of discount and observed real returns on safe assets.

From (22) we can each calculate the risk premium on the market portfolio or on individual assets:

* The formula is $\rho = \frac{1}{\delta} - 1$, where $\delta = \frac{E(c_1/c_0)^\alpha}{1 + r_f}$

$$(24) \quad E(r_{m,1}) = r_f - \text{cov} \left[r_{m,1}, \delta \left(\frac{c_1}{c_0} \right)^{-\delta} \right] + o^2$$

where o^2 are second order terms. Using (23) and (24) we show in Table 3 the estimated risk premia.

The striking feature of Table 3 is the very low estimates of the risk premia. Only when the risk aversion reaches absurdly high levels (say $\alpha = 200$) does a risk premium of significant size appear. The reason for the extremely modest risk premia is quite simple: the variation in the rate of growth of real consumption is extremely modest. Over the period 1967 to 1981, the highest quarterly growth rate was 1.6 percent, while the lowest was -0.6 percent. Even if these changes were highly correlated with market returns (which they are not), the resulting risk premia would be miniscule.

One important question is whether the extremely modest risk premia associated with the Breeden approach can be attributed to imprecision associated with the estimated covariances between returns (r_i) and the marginal utility of consumption $[(c_t/c_{t-1})^{-\alpha}]$. This question can be addressed by estimating the variances of the covariance estimates from which the risk premia are calculated.

This question is discussed briefly in the appendix. As is shown there, the covariance estimates for the Breeden model are extremely well determined, and it is extremely unlikely that a set of risk premia anywhere like those in lines 1 or 2 of Table 5 below could be generated by chance generate the observed data.

In addition, it is useful to compare the relative risk premia of different assets. This allows for the fact that there is uncertainty about the ex ante returns or the shape of the utility function. Table 4 compares the relative ranking of social risk for the preferred CAPM (model 3) and the CCAPM model with $\alpha = 2$. (The relative risk premia differ very little

TABLE 3

Risk Premia from CCAPM Model, Equation (29)
(percent per annum)

<u>Value of Risk Aversion (α)</u>	<u>Rate of Time Preference ($1/\delta - 1$)</u>	<u>Risk Premium on: Capital</u>	<u>Market Portfolio</u>
1	-2.7%	.025%	.01%
4	-11.7%	.10%	.056%
200	-99.9%	10.5%	4.2%

TABLE 4

Relative Risks under Two Approaches
(market risk = 100)

	<u>CAPM (Model II)</u>	<u>Consumption CAPM ($\alpha = 2$)</u>
Market	1.00	1.00
Capital	2.0	1.8
Long Bonds	2.1	1.8
Housing	.17	.03
Consumer Durables	.37	.47
Equities	2.8	2.6

as α changes in the CCAPM model.)

The major point is that, while the overall levels of the risk premia may differ, the relative risk under the two approaches are extremely close.

Collecting the results of the two models, we can calculate the structure of returns under our two models. These are shown in Table 5. The ex post returns are simply the averages over the sample period (1967 to 1981) of the real returns on the various assets and on the market portfolio.

The ex ante returns are calculated by a different technique. In the CAPM model, for corporate fixed capital, we calculate ex ante returns under the assumption that future returns be equal to the current post tax rate of profit on corporate capital calculated in the Economic Report of the President, 1982, Table B-88. The return on the zero-beta portfolio is equal to its ex post value. The ex ante returns on all other assets are taken from the estimates shown on line 3 of Table 1. In addition, we show in Table 6 the ratio of the risk premia on the major assets under the two approaches.

For the consumption CAPM model, on the other hand, we calculate the ex ante returns implicit in equation (24). The striking results of these calculations are the incompatibility of the three approaches.

TABLE 5

Ex Ante and Ex Post Risk Premia, 1967-81
(real returns, percent per annum)

	<u>Market</u>	<u>Corporate Capital</u>	<u>Asset Housing</u>	<u>Long Bonds</u>	<u>Consumer Durables</u>	<u>Equities</u>
1. <u>Ex post</u> *	1.7 (.96)	-2.2 (2.1)	5.4 (.46)	-5.5 (2.9)	5.8 (2.0)	-1.1 (3.3)
<u>Ex ante</u>						
2. CAPM	4.3	8.6	.73	8.0	1.59	11.4
3. CCAPM ($\alpha = 4$)	.06	.10	.02	.10	.03	.14

*Figures in parentheses are standard deviations.

TABLE 6

Excess Risk Premium on Different Assets
[ratio of risk premium under CAPM
to them on CCAPM ($\alpha = 4$)]

<u>Assets</u>	<u>Excess Risk</u>
Market	72
Capital	86
Long Bonds	86
Equities	81

V. Conclusion

The results presented here are extremely tentative, for numerous reasons. The data are not completely adequate; misspecifications and biased plague the econometric estimates; the implementation of the CCAPM model relies on a highly simplified parameterization of the utility function. In addition, we assume that the results are from market equilibrium, which is probably tenable for financial market but highly dubious for the consumption-investment decision. Notwithstanding these shortcomings, the following results are suggested.

1. There are two different theoretical approaches to the measurement and testing of the structure of social risk--the empirical CAPM model and the consumption (or Breeden) CAPM model. Each is conditional on slightly different assumptions, and each can be tested empirically. In addition, the results of the two approaches can be compared with the observed returns on different assets.

2. Table 5 above shows the estimated returns from the three different approaches. Even over this very long sample period (180 months), the ex post returns on different assets are implausible as estimates of the underlying ex ante risk premium on risky assets like equities and long term government bonds.

3. The pattern of risk premiums generated by the two theoretical approaches, shown in Table 4, are quite similar. Capital and long term bonds appear to be relatively risky assets; while most other assets are, from a social point of view, relatively safe.

4. A portfolio of short-term bonds (such as 90 day Treasury bills), is a safe investment even taking into account uncertainty about the price level. When more sophisticated "zero-beta" portfolios are constructed,

such as those shown in Table 1, there is virtually no change in the structure of social risk or in the estimated risk premiums.

5. There has been a significant change in the structure of social risk since the Federal Reserve introduced its new operating procedures in October 1979. As is shown in Table 2, the undiversifiable risk of long-term bonds since 1979 has been doubled over the prior 6 years. Moreover, the riskiness of long term bonds since 1979 has actually been larger than the riskiness of equities by a factor of almost two.

6. For the consumption CAPM model, as derived by Breeden, estimates of the level of social risk are extremely small relative to historical risk premia or to estimates of the standard CAPM model. Even when implausibly high values are assumed for the measure of risk aversion (α equal to 2 to 10), the valuation of risk in the market is much higher than should arise from the consumption-oriented approach. For example, when $\alpha = 4$, the estimated ex ante risk premium on the market is seventy times greater than the estimates from the consumption approach. This wide divergence in risk premiums under the consumption CAPM and other approaches arises for all the risky assets, as is shown in Table 6.

7. The very low risk premiums under the consumption approach (along with related results in Shiller (1978) and Grossman and Shiller (1980)) raise profound questions about whether any of the CAPM models are even remotely accurate methods of describing the way markets measure, price, and allocate social risks. The most obvious potential defect in the consumption approach is the assumption that consumers are in intertemporal consumption-savings equilibrium. There is, of course, a massive and persuasive body of evidence that there are significant departures from the permanent income/life cycle models--for good reasons, such as liquidity

constraints, or for less satisfying reasons, such as pervasive uses of rules of thumb. Even accepting this evidence (which is not universally done), it is nonetheless puzzling how the observed patterns of co-movement of consumption and returns could be explained.

References

- Black, Fischer, "Capital Market Equilibrium with Restricted Borrowing," Journal of Business, July 1972.
- Breeden, Douglas, "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment," Journal of Financial Economics 7, September 1979.
- Copeland, T. E. and J. F. Weston, Finance Theory and Corporate Policy, Addison-Wesley, 1980.
- Fama, Eugene F., Foundations of Finance, Basic Books, New York, 1976.
- Grossman, Sanford J. and Robert Shiller, "The Determinents of the Variability of Stock Market Prices," AER, 71, May 1981, pp. 222-227.
- Jorgensen, Dale W. and Barbara M. Fraumeni, "The Role of Capital in the U. S. Economic Growth, 1948-1976," in Capital, Efficiency and Growth, ed. George M. von Furstenberg, Ballinger, 1980
- Kendall, M. G. and A. Stuart, The Advanced Theory of Statistics, Volume 1, Hafner Publishing, 1958, New York.
- Mehra, Rajnish and Edward C. Prescott, "A Test of the Intertemporal Asset Pricing Model," FRB of Minnesota Staff Report 81.
- Merton, Robert C., "An Intertemporal Capital Asset Pricing Model," Econometrica 41, 1973, pp. 807-87.
- _____, "Optimum Consumption and Portfolio Rules in a Continuous Time Model," Journal of Economic Theory, 1971, pp. 373-413.
- Sharpe, William, "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," Journal of Finance, September 1964, pp. 428-442.
- Shiller, Robert, "The Volatility of Long Term Interest Rates and Expectations Models of the Term Structure," Journal of Political Economy, December 1979, 87, pp. 1190-1229.
- Ross, Stephen, "The Current Status of the Capital Asset Pricing Model," Journal of Finance, 33, No. 3, June 1978, pp. 885-901.

Appendix. Precision of Estimated Risk Premia
in Breeden Model

The text contains estimates of the risk premia--where the risk premia are calculated from estimates of the covariances of returns with either returns or with the estimated marginal utility of consumption. It is difficult to estimate the confidence associated with these estimates because the covariances do not follow commonly tabulated distribution.

To obtain an impression of the precision associated with the estimated risk premia, we have calculated and shown in Table A-1 the relevant variances. Column (1) shows the estimated value of the covariances between marginal utility of consumption and returns; these are the basis of the risk premium estimates shown in Table 3 of the text.

Column (2) of Table A-1 shows the sample variance of column (1). These were calculated using the formula:

$$\text{var (cov}(x,y)) = \frac{1}{N} E[(x-\bar{x})^2(y-\bar{y})^2] - [E(x-\bar{x})(y-\bar{y})].$$

(This approach is justified in Kendall and Stuart [1958], pp. 234-235.)

Column (3) shows the required value of the covariance (i.e. the required value of the estimate shown in column (1)) that would be necessary to reconcile the estimated risk premia with the ex ante estimates of the risk premia in line 2 of Table 5 above.

As the table shows, the variances of the covariances [shown in column (2)] are at least two orders of magnitude smaller than the covariances themselves [shown in column (1)]. Using the Tchebychev

Table A-1. Estimated Values and Variances of Estimated Values of
of the Covariance Between Marginal Utility of Consumption $[(c_t/c_{t-1})^{-4}]$
and Returns (r_i) , Basis points per annum.

[All figures are natural numbers times 10^{-8}]

	(1) Estimated Value of Covariance	(2) Sample Variance of Estimated Value	(3) Required Covariance Value to Reconcile with Zero-Beta Estimate
Market	-5,000	3.5	-350,000
Capital	-8,400	12.	-690,000
Equity	-12,000	27.	-900,000
Gov't Longs	-8,300	32.	-840,000
Con. Dur.	-220,000	9.5	-830,000
Housing	-16,000	49.	-60,000

Inequality (which is extremely conservative if the distribution is anywhere near normal), the probability that the true covariances differ from the sample covariances by a large enough magnitude to reconcile the results in column (1) with the CAPM ex ante results shown in column (3) is less than 10^{-4} . While the Tchebychev relation does not strictly hold, since the variance is estimated rather than known a priori, it is nevertheless strongly suggestive of the precision of the result.